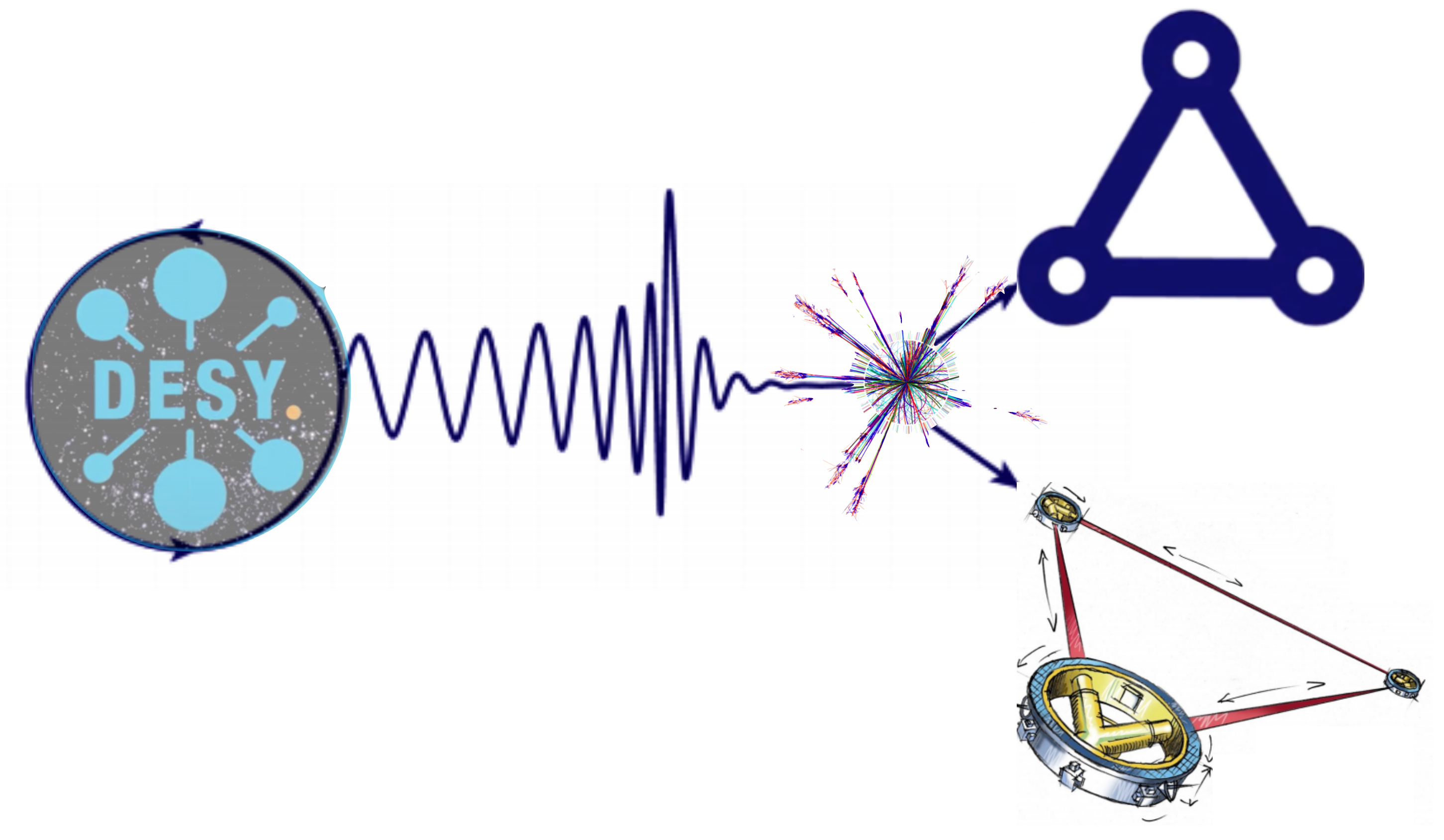
WEFT @ fifth Post N/M order

Status report



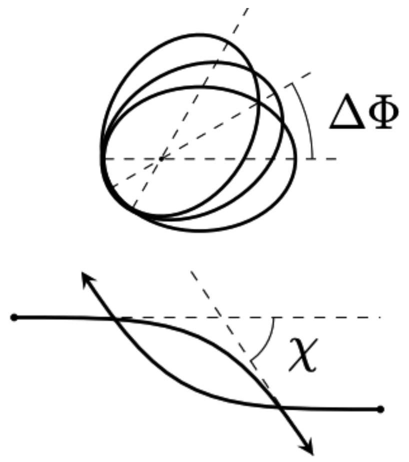
Rafael A. Porto

WEFT approach to GW physics PN PM

Goldberger Rothstein
(NRGR 2006)
RAP (2006)
Goldberger Ross (2009)

Kalin **RAP** **(2020)**

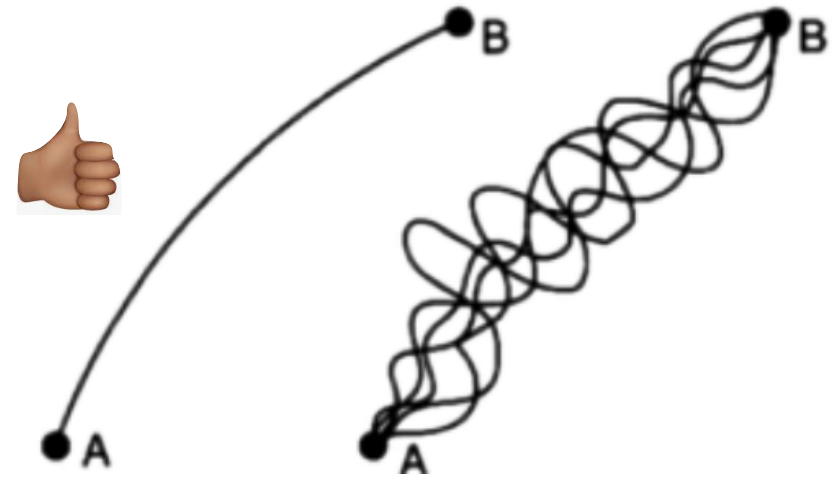
Galley **RAP** et al.
Foffa Sturani et al.
Blumlein et al.

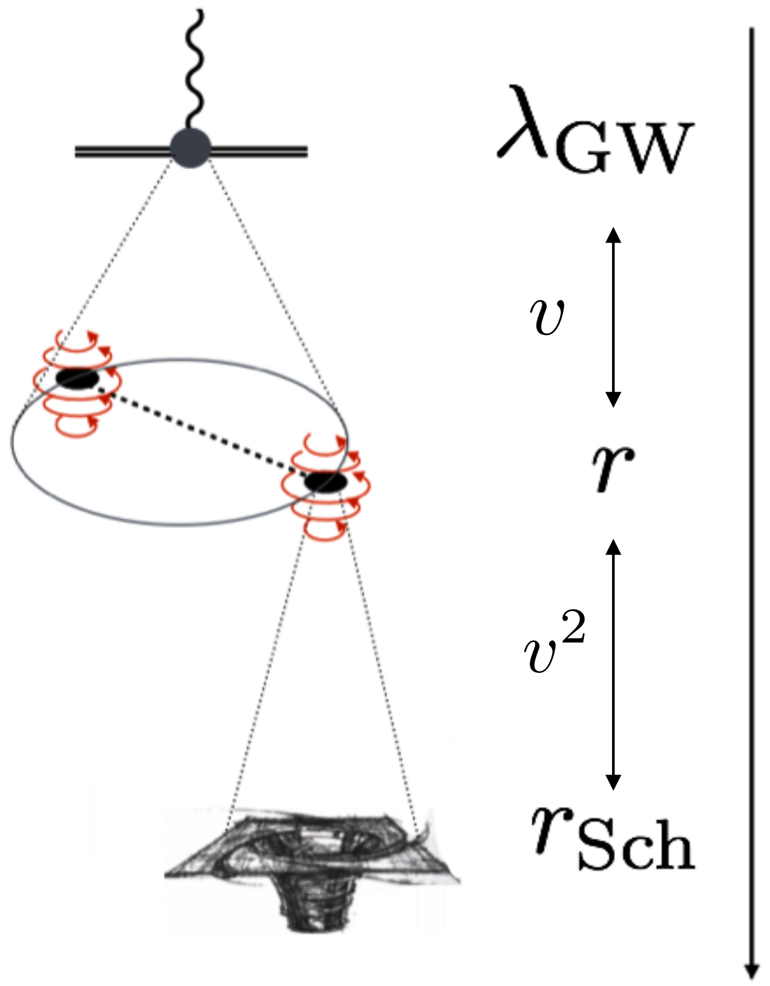


- **Separation of Scales:**

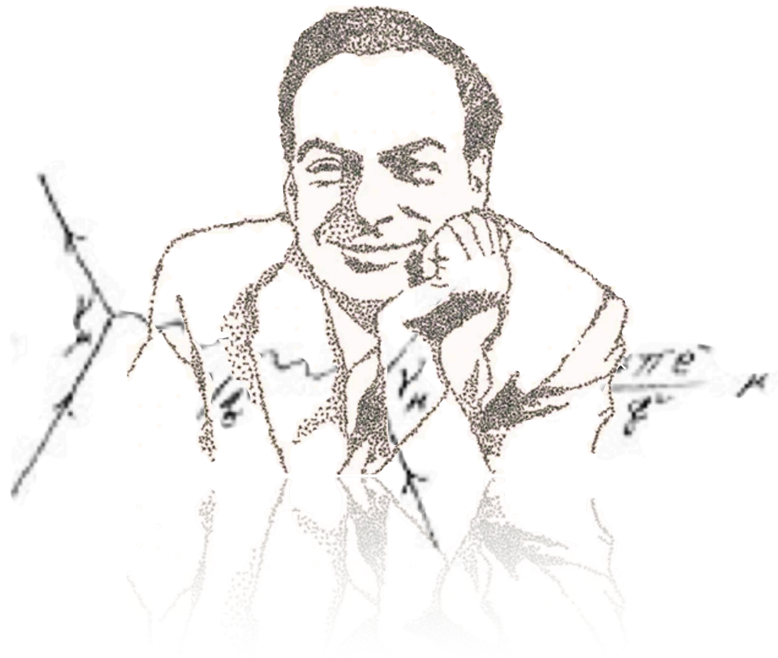
$$r_{\rm Sch} \ll r \ll \lambda_{\rm GW}$$

Classical

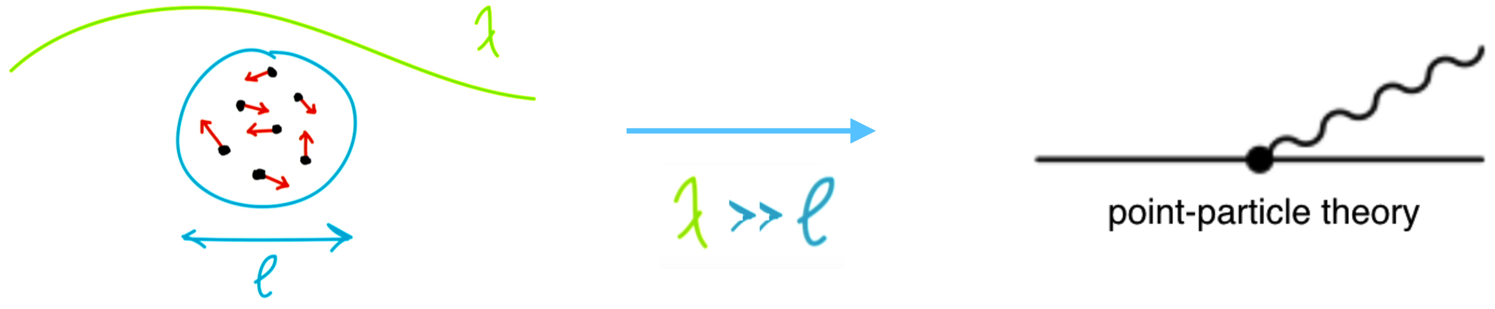




- **Effective Field Theory:**
  Classical effective action (saddle point) *one scale at a time* (method of regions)



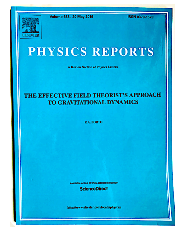
$$e^{iW} = \int D[\lambda_{\rm rad}^{-1}]\, D[r^{-1}]\, D[r_s^{-1}]\, e^{iS_{\rm full}}$$



$$\ldots = \frac{1}{2} Q_{ij} E^{ij} + \ldots$$

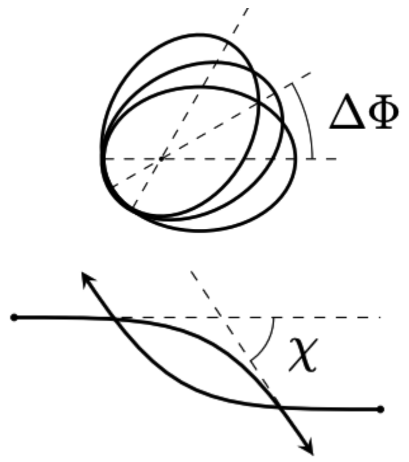
$$S_{\rm pp} = -\sum \frac{m_a}{2} \int d\tau_a\, g_{\mu\nu}(x_a(\tau_a)) v_a^\mu(\tau_a) v_a^\nu(\tau_a) + \ldots \qquad \frac{1}{2}\omega_\mu^{ab} S_{ab} u^\mu$$

The effective field theorist's approach to gravitational dynamics
Physics Reports
Rafael A. Porto
Volume 633, 20 May 2016, Pages 1-104



WEFT approach to GW physics

PN PM



Goldberger Rothstein (NRGR 2006)
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Kalin **RAP** **(2020)**

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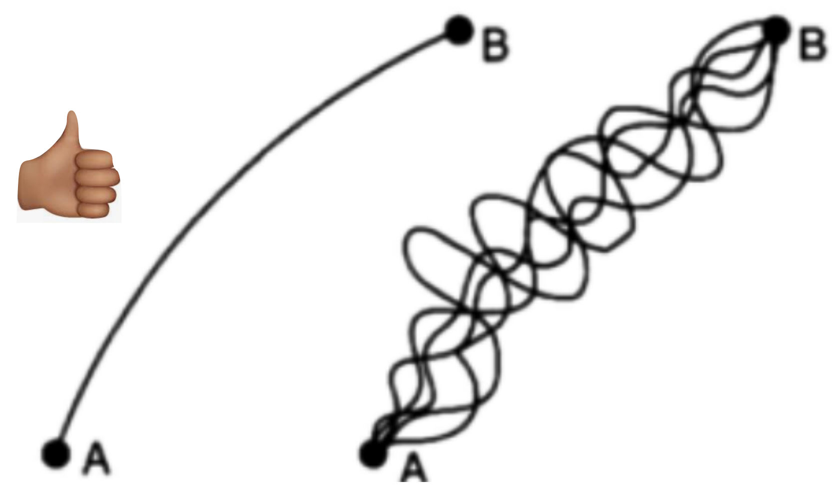
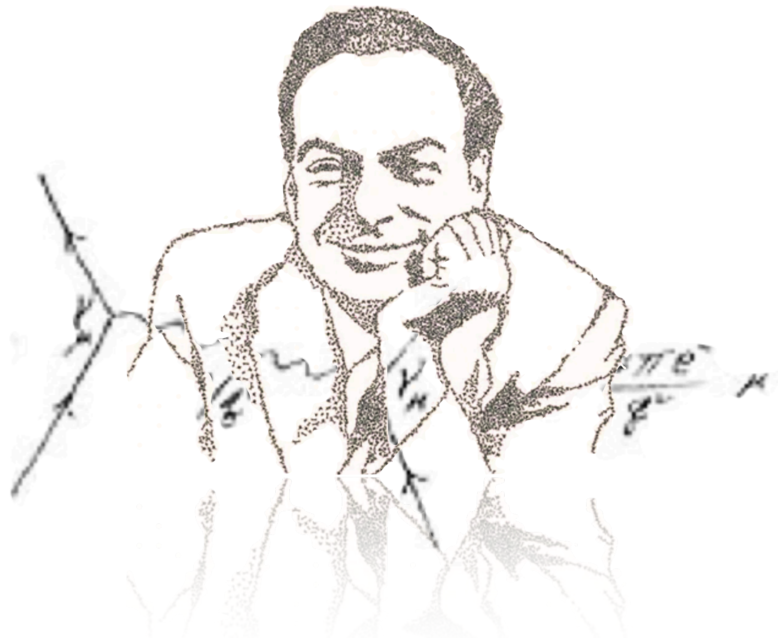
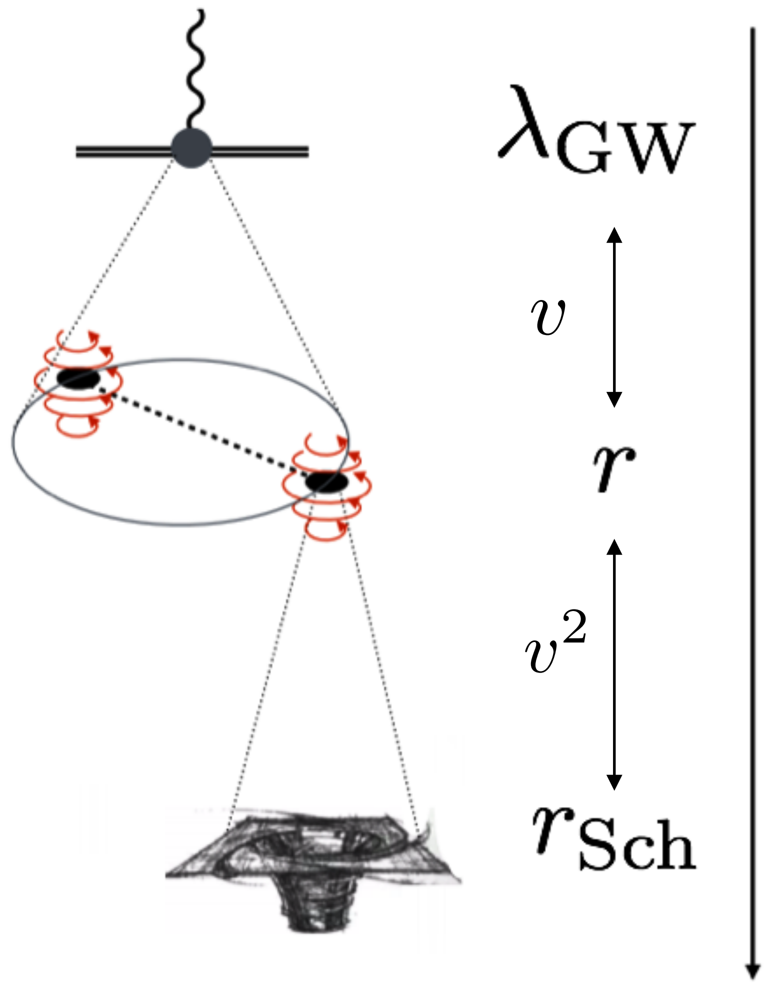




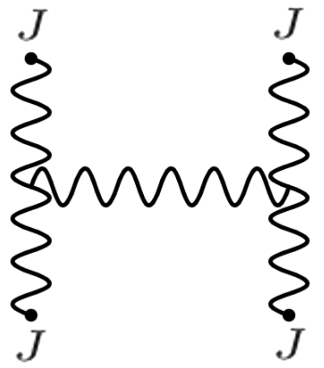


$$e^{iW} = \int D[\lambda_{\rm rad}^{-1}]\, D[r^{-1}]\, D[r_s^{-1}]\, e^{iS_{\rm full}}$$

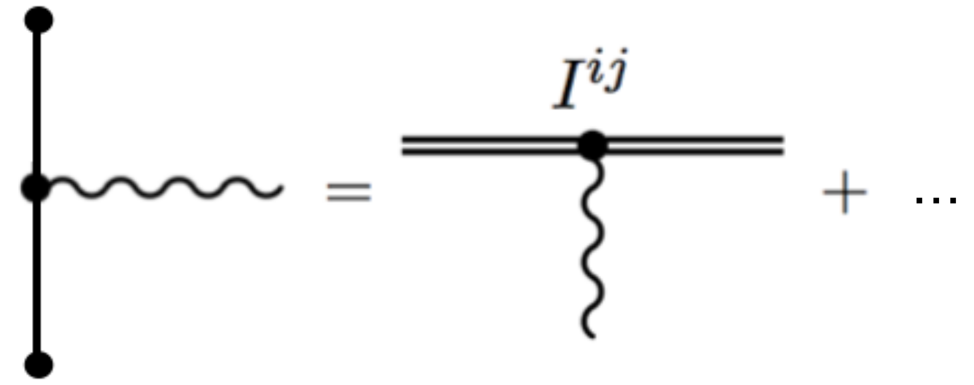
Radiation modes

Potential modes

$$\log \langle 0|0\rangle^J =$$



UV Divergences: localized sources



$$\frac{1}{2} I_{ij} E^{ij}$$

PHYSICAL REVIEW D **110**, 044046 (2024)

Gravitational radiation from inspiralling compact binaries to N³LO in the effective field theory approach

Loris Amalberti[1,2,*] Zixin Yang[1,†] and Rafael A. Porto[1,‡]

Integrate-out potentials and match into binary multipoles

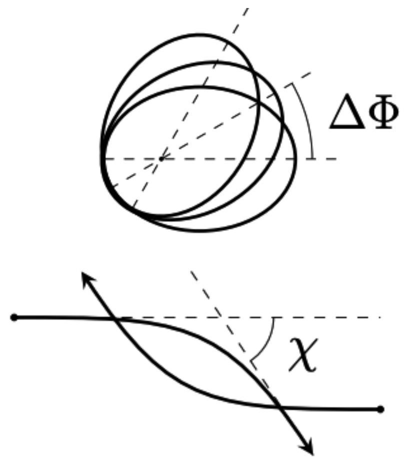
Goldberger Rothstein
(NRGR 2006)
RAP (2006)
Goldberger Ross (2009)

Kalin **RAP (2020)**

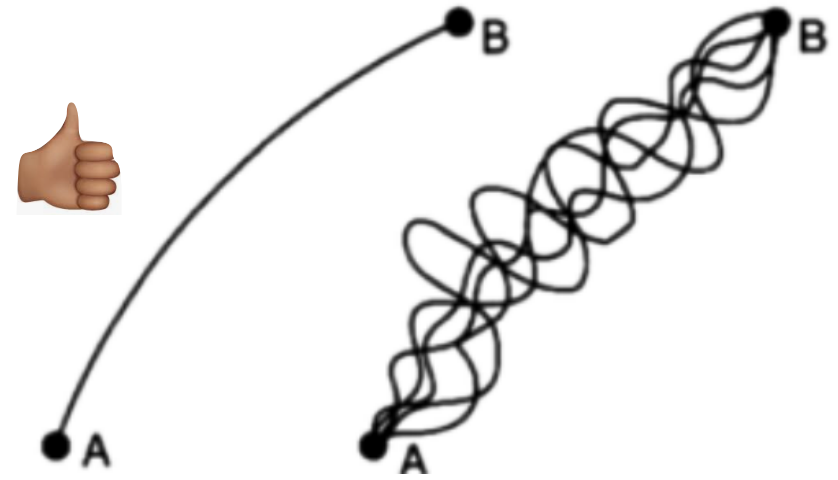
Galley **RAP** et al.
Foffa Sturani et al.
Blumlein et al.

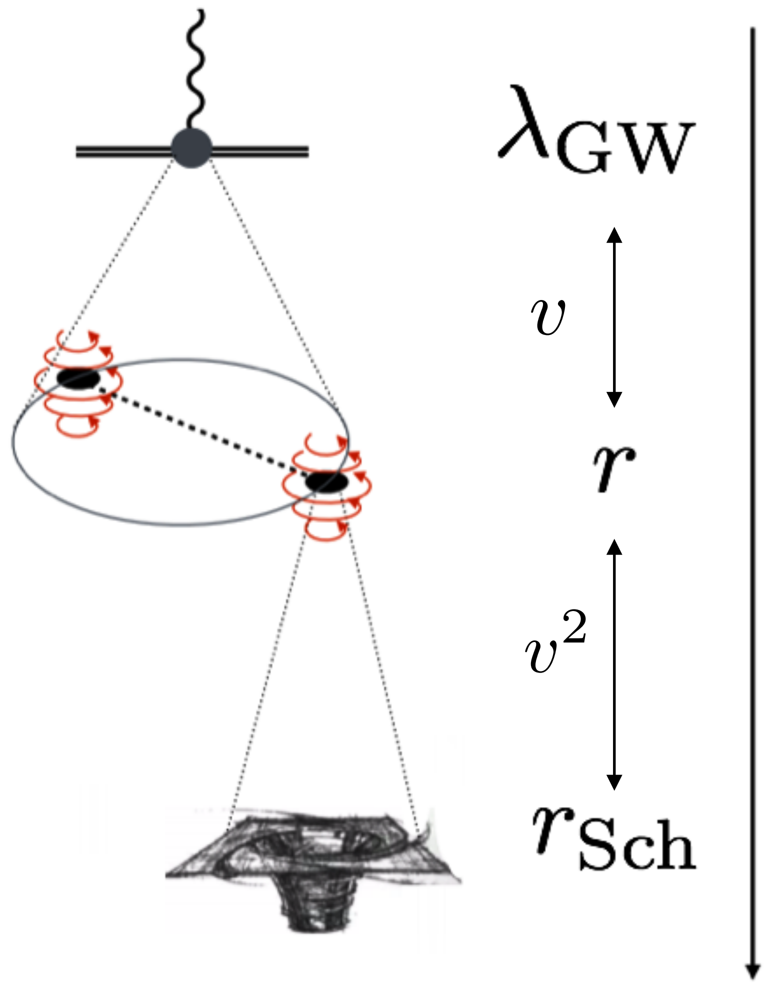
WEFT approach to GW physics

PN
PM



Classical





- **Separation of Scales:**

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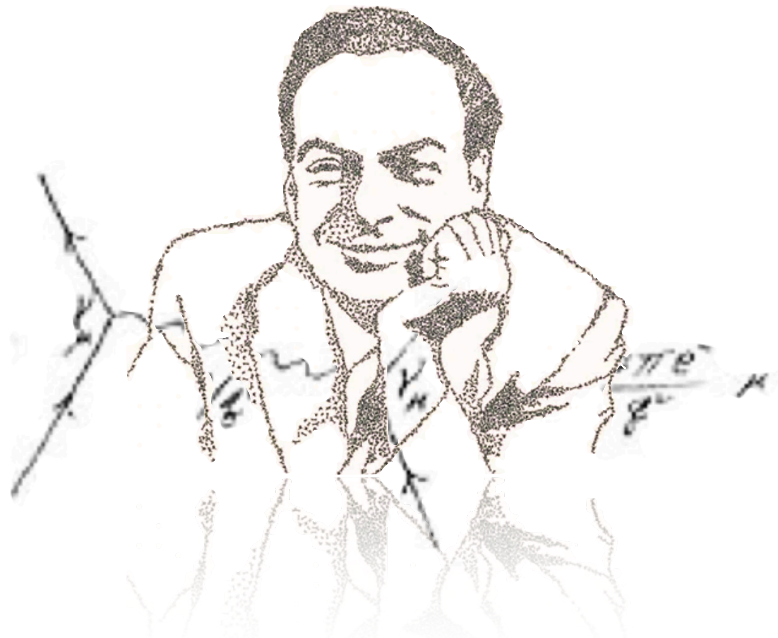
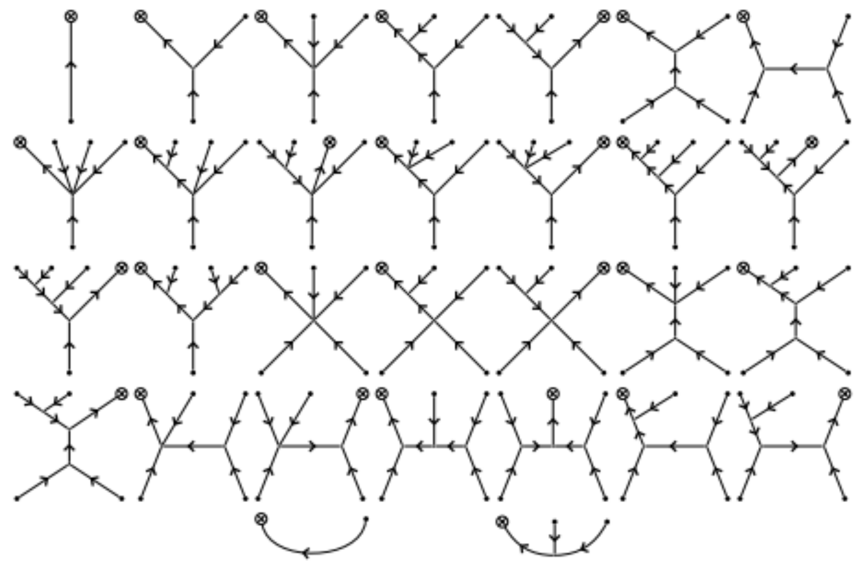
$$e^{iW^{(+,-)}} = \int D[\lambda_{\rm rad}^{-1}]\, D[r^{-1}]\, D[r_s^{-1}]\, e^{iS^{C}_{\rm full}}$$





$$\begin{pmatrix} 0 & -\Delta_{\rm adv}(x-y) \\ -\Delta_{\rm ret}(x-y) & \frac{1}{2}\Delta_H(x-y) \end{pmatrix}$$

$$\Delta p^\mu_{\rm tot} = \Delta p^\mu_{\rm cons} + \Delta p^\mu_{\rm diss}\,,$$

$$\Delta p^\mu_{\rm cons} \equiv \mathbb{R}\Delta p^\mu_{\rm F}$$

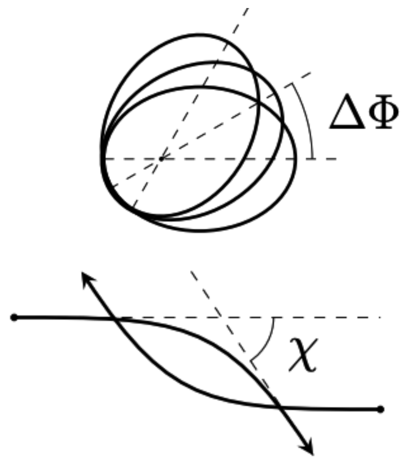
Radiation-Reaction forces

PHYSICAL REVIEW LETTERS **130,** 101401 (2023)

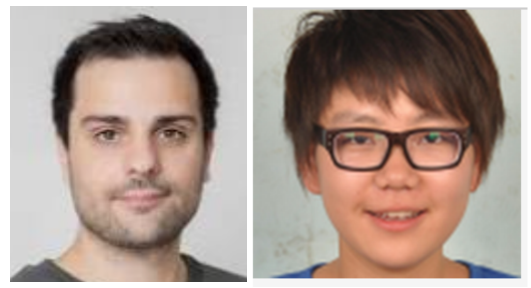
Radiation Reaction and Gravitational Waves at Fourth Post-Minkowskian Order

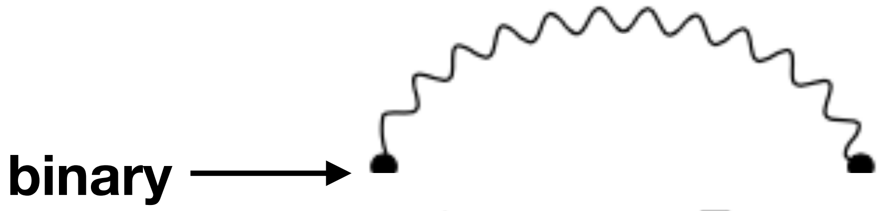
Christoph Dlapa,[1] Gregor Kälin,[1] Zhengwen Liu,[2,1] Jakob Neef,[3,4] and Rafael A. Porto[1]

WEFT approach to GW physics *5PN*

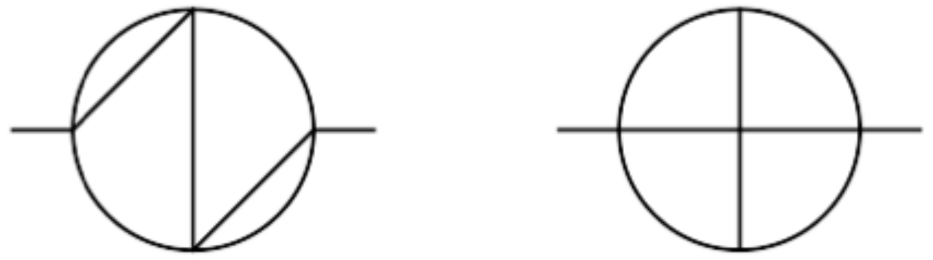


RAP Riva Yang (2024)



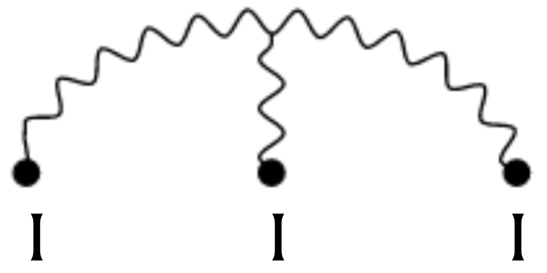


$$-\frac{1}{2}I^{ab}(\lambda)\frac{E_{ab}}{\sqrt{g_{\mu\nu}V^\mu V^\nu}}$$

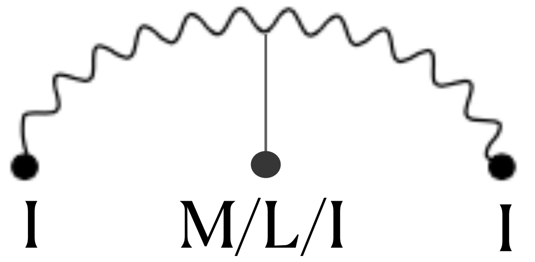


potential region to 5PN

"Memory"



"Tails"



$$\sqrt{g_{\mu\nu}V^\mu(\lambda)V^\nu(\lambda)}M(\lambda)+\frac{1}{2}\omega_\mu^{ab}L_{ab}(\lambda)V^\mu(\lambda)$$

Relative (no-recoil)

$$V^\mu=(1,0,0,0).$$

Failed-Tail and Memories

$$S_{5\rm PN}=S_{(\rm P)}+S_{(\rm RR)}+S_{(\rm T)}-\frac{G^2}{15}\int{\rm d}t\,L_+^{kl}I^{(4)}_{-,kj}I^{(3)}_{+,jl}+\frac{G^2}{30}\int{\rm d}t\,L_-^{kl}I^{(4)}_{+,kj}I^{(3)}_{+,jl}$$
$$+\frac{G^2}{5}\int{\rm d}t\left(\frac{1}{2}I_{-,ij}I^{(4)}_{+,jk}I^{(4)}_{+,ki}-I^{(4)}_{-,ij}I^{(4)}_{+,jk}I_{+,ki}+\frac{1}{7}I^{(2)}_{-,ij}I^{(3)}_{+,jk}I^{(3)}_{+,ki}+\frac{2}{7}I^{(3)}_{-,ij}I^{(3)}_{+,jk}I^{(2)}_{+,ki}\right)$$

Linear Radiation-Reaction

$$S_{(\rm RR)}=-\frac{G}{5}\int{\rm d}t\,I_-^{ij}(t)I_+^{ij(5)}(t)\,,$$

Mass/Energy Tails

$$S_{I_2(\rm T)}=\frac{2M_+G^2}{5}\int\frac{{\rm d}\omega}{2\pi}\omega^6I_-^{ij}(-\omega)I_+^{ij}(\omega)\left[\frac{1}{2\epsilon}+\frac{41}{30}+i\pi\,{\rm sign}(\omega)-\log\left(\frac{\omega^2e^{\gamma_{\rm E}}}{\pi\mu^2}\right)\right]$$
$$-\frac{G^2M_-}{5}\int\frac{{\rm d}\omega}{2\pi}\omega^6I_+^{ij}(-\omega)I_+^{ij}(\omega)\left[\frac{1}{2\epsilon}+\frac{41}{30}-\log\left(\frac{\omega^2e^{\gamma_{\rm E}}}{\pi\mu^2}\right)\right],$$
$$S_{I_3(\rm T)}=\frac{2G^2M_+}{189}\int\frac{{\rm d}\omega}{2\pi}\omega^8I_-^{ijk}(-\omega)I_+^{ijk}(\omega)\left[\frac{1}{2\epsilon}+\frac{82}{35}+i\pi\,{\rm sign}(\omega)-\log\left(\frac{\omega^2e^{\gamma_{\rm E}}}{\pi\mu^2}\right)\right],$$
$$S_{J_2(\rm T)}=\frac{32G^2M_+}{90}\int\frac{{\rm d}\omega}{2\pi}\omega^6J_-^{a|ij}(-\omega)J_+^{a|ij}(\omega)\left[\frac{1}{2\epsilon}+\frac{49}{20}+i\pi\,{\rm sign}(\omega)-\log\left(\frac{\omega^2e^{\gamma_{\rm E}}}{\pi\mu^2}\right)\right]$$

Galley **RAP** et al.
Foffa Sturani et al.
Blumlein et al.

$$M_+=\sum_{a=1,2}m_a\left[1+\frac{v_{a,+}^2}{2}-\frac{1}{2}\sum_{b\neq a}\frac{Gm_b}{r_+}\right],$$
$$M_-=\sum_{a=1,2}m_a\left[1+\boldsymbol{v}_{a,+}\cdot\boldsymbol{v}_{a,-}+\frac{1}{2}\sum_{b\neq a}Gm_b\frac{\boldsymbol{r}_+\cdot\boldsymbol{r}_-}{r_+^3}\right],$$
$$L_-^{ij}=2\sum_{a=1,2}m_a\left(\boldsymbol{x}^{[i}_{a,-}\boldsymbol{v}^{j]}_{a,+}+\boldsymbol{x}^{[i}_{a,+}\boldsymbol{v}^{j]}_{a,-}\right),$$
$$L_+^{ij}=2\sum_{a=1,2}m_a\boldsymbol{x}^{[i}_{a,+}\boldsymbol{v}^{j]}_{a,+}\,,$$
$$I_-^{ij}=\sum_{a=1,2}m_a\left(2\boldsymbol{x}^{(i}_{a,-}\boldsymbol{x}^{j)}_{a,+}-\frac{2}{3}\delta^{ij}\boldsymbol{x}_{a,-}\cdot\boldsymbol{x}_{a,+}\right),$$
$$I_+^{ij}=\sum_{a=1,2}m_a\left(\boldsymbol{x}^{i}_{a,+}\boldsymbol{x}^{j}_{a,+}-\frac{1}{3}\delta^{ij}\boldsymbol{x}_{a,+}\cdot\boldsymbol{x}_{a,+}\right),$$

$$Q_{ij}\equiv\textstyle\sum_a m_a\boldsymbol{x}_a^i\boldsymbol{x}_a^j$$

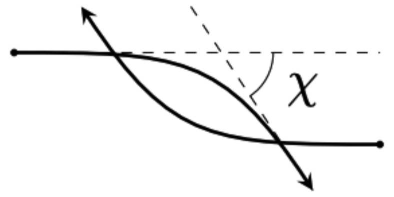
$$\left(\frac{\delta}{\delta\boldsymbol{x}^i_{a,-}}\int S[\boldsymbol{x}_\pm]dt\right)\Big|_{\rm PL}=0$$

$$\frac{\rm d}{{\rm d}t}\frac{\partial L_{(\rm P)}[\boldsymbol{x}_a]}{\partial\boldsymbol{v}^i_a}-\frac{\partial L_{(\rm P)}[\boldsymbol{x}_a]}{\partial\boldsymbol{x}^i_a}=\left[\frac{\partial R[\boldsymbol{x}_\pm]}{\partial\boldsymbol{x}^i_{a,-}}-\frac{\rm d}{{\rm d}t}\frac{\partial R[\boldsymbol{x}_\pm]}{\partial\boldsymbol{v}^i_{a,-}}\right]\Big|_{\rm PL}$$

in the "Physical Limit" (PL), i.e., $\boldsymbol{x}_{a,+}\to\boldsymbol{x}_a$, $\boldsymbol{x}_{a,-}\to 0$,

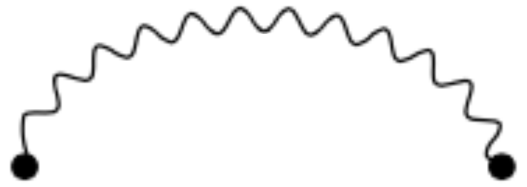
RAP Riva (2026)

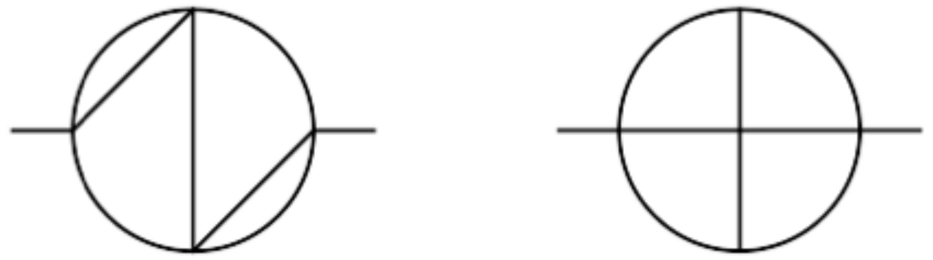
WEFT approach to GW physics *5PN*

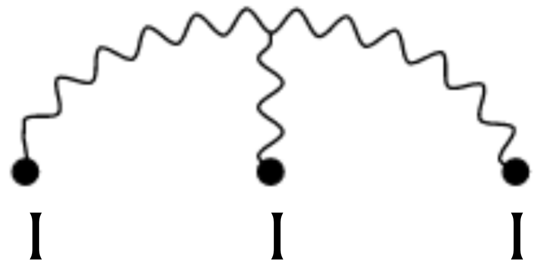


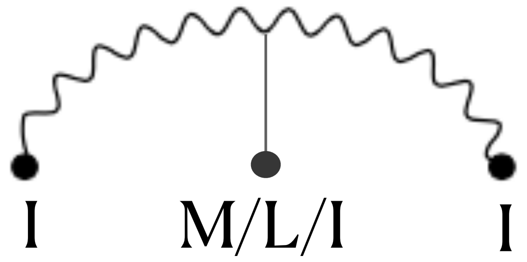
$$\frac{\chi_{\rm rel}}{2} = \sum_n \left(\frac{GM}{b}\right)^n \chi^{(n)}_{b,\,\rm rel} = \sum_n \frac{\tilde{\chi}^{(n)}_{j,\,\rm rel}}{j^n \Gamma^{n-1}}\,,$$

Total even-in-velocity scattering angle







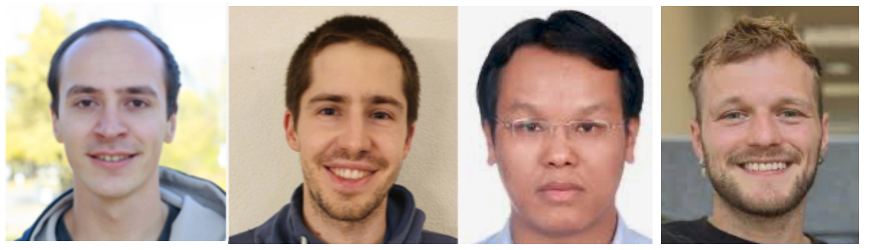


$$\tilde{\chi}^{(4,\nu^2)\rm tot}_{j(\rm even)} = \frac{1491}{400}\pi v_\infty^6\,, \quad ✓$$

$$\tilde{\chi}^{(5,\nu^2)\rm tot}_{j(\rm even)} = \left(\frac{35548627}{21600} - \frac{224057}{1440}\pi^2 + \frac{1408}{45}\log(2v_\infty)\right)v_\infty^5\,,$$

$$\tilde{\chi}^{(6,\nu^2)\rm tot}_{j(\rm even)} = \left(\frac{15036845}{4032} - \frac{401004899}{1146880}\pi^2 + \frac{201}{2}\log\left(\frac{v_\infty}{2}\right) + \frac{2817}{16}\zeta(3)\right)\pi v_\infty^4\,.$$

New benchmark at 5/6PM

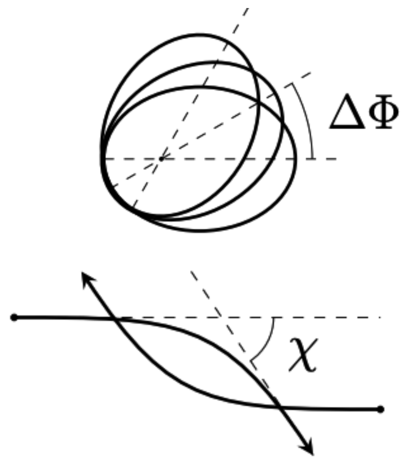


Perfect agreement 4PM

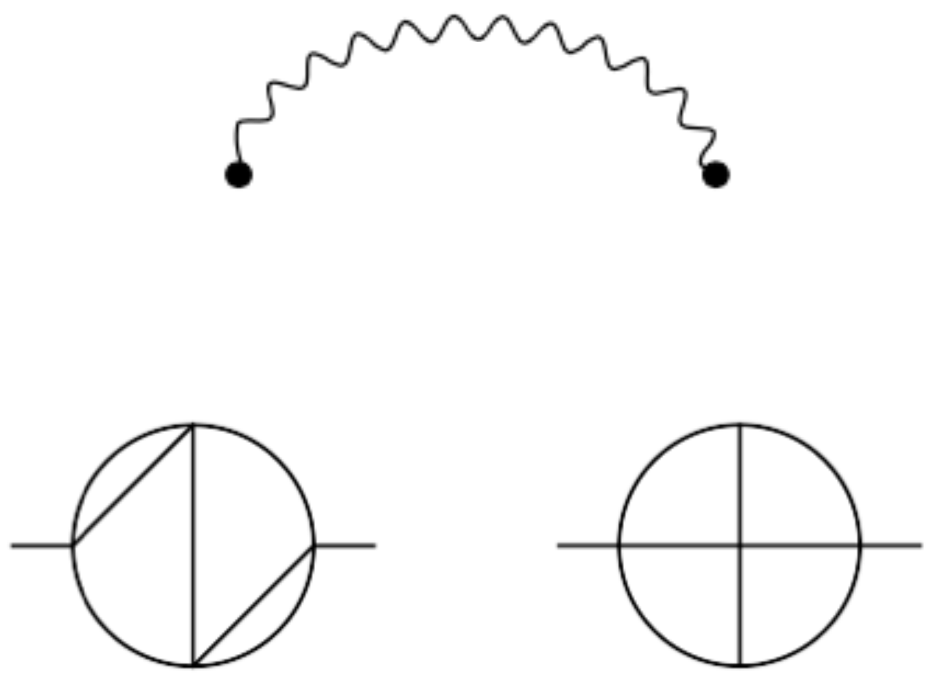
Dlapa Kalin Liu Neef **RAP** **(2022)**

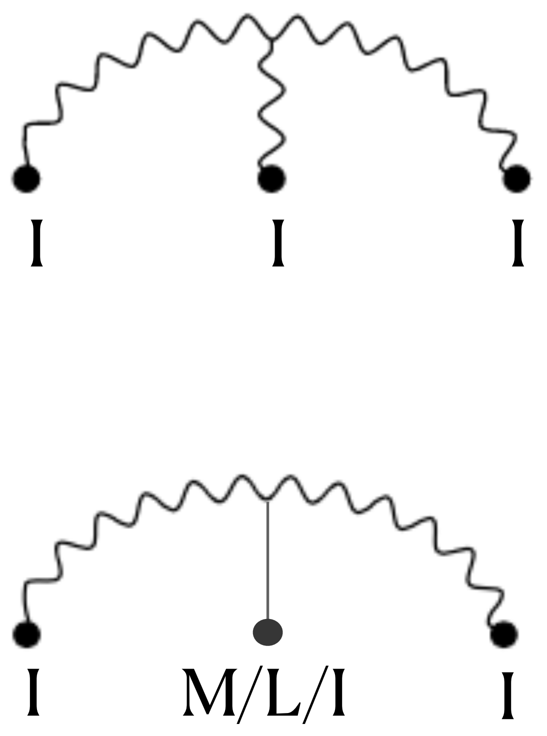
RAP Riva (2026)

WEFT approach to GW physics 5PN



Total even-in-velocity scattering angle





$$\tilde{\chi}^{(4,\nu^2)\mathrm{tot}}_{j(\mathrm{even})} = \frac{1491}{400}\pi v_\infty^6 \,,$$
$$\tilde{\chi}^{(5,\nu^2)\mathrm{tot}}_{j(\mathrm{even})} = \left(\frac{35548627}{21600} - \frac{224057}{1440}\pi^2 + \frac{1408}{45}\log(2v_\infty)\right) v_\infty^5 \,,$$
$$\tilde{\chi}^{(6,\nu^2)\mathrm{tot}}_{j(\mathrm{even})} = \left(\frac{15036845}{4032} - \frac{401004899}{1146880}\pi^2 + \frac{201}{2}\log\left(\frac{v_\infty}{2}\right) + \frac{2817}{16}\zeta(3)\right)\pi v_\infty^4 \,.$$

Can we reconstruct an isotropic-like acceleration?

Not derived from a Hamiltonian!

$$\boldsymbol{a}_{(\mathrm{iso})} = -\frac{GM}{r^2}\boldsymbol{n} + \sum_n \frac{G^n}{r^{n+1}}\Big[\left(\alpha_n(\boldsymbol{v}^2) + \delta_n(\boldsymbol{v}^2)(\boldsymbol{v}\cdot\boldsymbol{n})\right)\boldsymbol{n} + \left(\beta_n(\boldsymbol{v}^2)(\boldsymbol{v}\cdot\boldsymbol{n}) + \gamma_n(\boldsymbol{v}^2)\right)\boldsymbol{v}\Big]$$

$$M\nu\left(\boldsymbol{a}_{(\mathrm{iso})}\cdot\boldsymbol{v}\right)_{\{\alpha_n,\beta_n\}} = 0 - \frac{\mathrm{d}E_{(\mathrm{iso})}}{\mathrm{d}t}$$
$$2M\nu\left(\boldsymbol{r}^{[i}\boldsymbol{a}^{j]}_{(\mathrm{iso})}\right)_{\{\alpha_n,\beta_n\}} = 0 - \frac{\mathrm{d}L^{ij}_{(\mathrm{iso})}}{\mathrm{d}t}$$

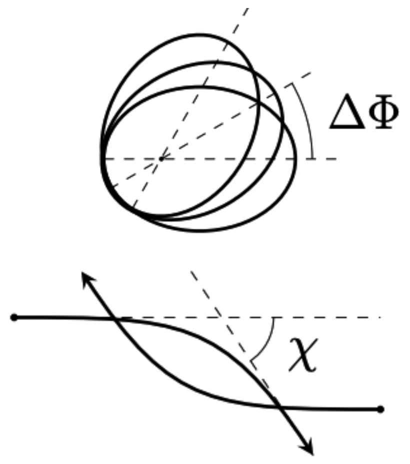
$$\Delta^{(3)}E_{(\mathrm{RR,iso})} = \left(\frac{\bar{\delta}_3 + 4\bar{\gamma}_3 + 6\bar{\gamma}_2}{8}\right)\frac{G^3M^4\nu^2}{b^3} \,,$$
$$\Delta^{(4)}E_{(\mathrm{RR,iso})} = \left(\frac{\bar{\delta}_3 + 5\bar{\gamma}_3 + 7\bar{\gamma}_2}{3}\right)\frac{G^4M^5\nu^2}{b^4} \,,$$
$$\Delta^{(2)}\boldsymbol{L}^i_{(\mathrm{RR,iso})} = (2\bar{\gamma}_2)\frac{G^2M^3\nu^2}{b^2}\varepsilon^{ijk}b^j v_\infty^k \,.$$

$$E^{5\mathrm{PN}}_{(\mathrm{iso})} = +\left(\frac{\bar{\beta}_3}{3}\right)\frac{G^3M^4\nu^3}{r^3}\boldsymbol{v}^6 + \left(\frac{\bar{\alpha}_4 + \bar{\beta}_4 - 2\bar{\beta}_3}{4}\right)\frac{G^4M^5\nu^3}{r^4}\boldsymbol{v}^4$$
$$+\left(\frac{\bar{\alpha}_5 + \bar{\beta}_5 - \bar{\alpha}_4 - \bar{\beta}_4 + 2\bar{\beta}_3}{5}\right)\frac{G^5M^6\nu^3}{r^5}\boldsymbol{v}^2$$
$$+\left(\frac{5\bar{\alpha}_6 - 2\bar{\alpha}_5 - 2\bar{\beta}_5 + 2\bar{\alpha}_4 + 2\bar{\beta}_4 - 4\bar{\beta}_3}{30}\right)\frac{G^6M^7\nu^3}{r^6} \,.$$

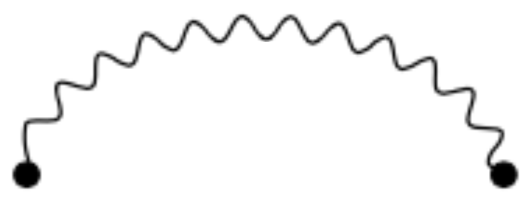
Manohar et al. (2022)

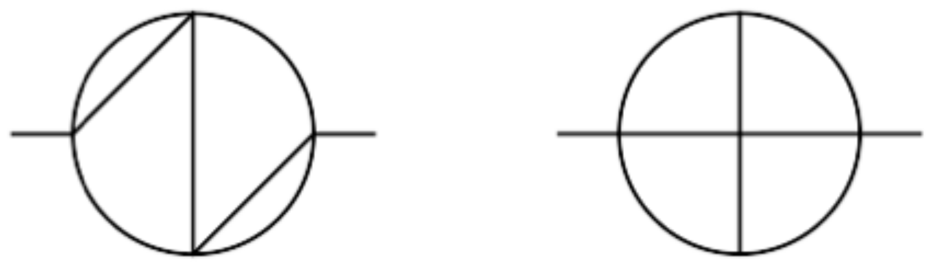
RAP Riva (2026)

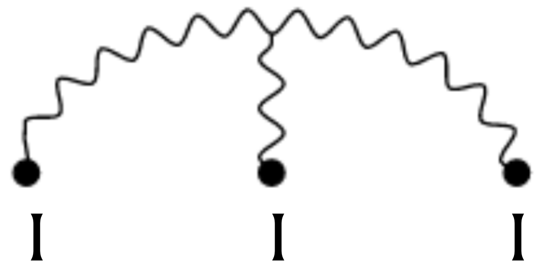
WEFT approach to GW physics 5PN

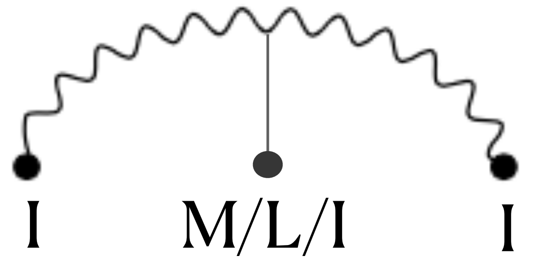


Total even-in-velocity scattering DATA! $\{\chi, \tau, \Delta E, \Delta L_z\}$









$$\Delta^{(4)}\boldsymbol{p}_{(\mathrm{iso})} = \frac{G^4M^5\nu^3}{b^4}\left[\left(\frac{\pi(3\bar{\alpha}_4 - 2\bar{\beta}_3)}{8}\right)\frac{\boldsymbol{b}}{b}\right]v_\infty^3\,,$$

$$\Delta^{(5)}\boldsymbol{p}_{(\mathrm{iso})} = \frac{G^5M^6\nu^3}{b^5}\left[\left(\frac{4(31\bar{\alpha}_4 + 4\bar{\alpha}_5 - 2\bar{\beta}_4 - 18\bar{\beta}_3)}{15}\right)\frac{\boldsymbol{b}}{b} + \left(\frac{\pi(3\bar{\alpha}_4 - 2\bar{\beta}_3)}{4}\right)\frac{\boldsymbol{v}_\infty}{v_\infty}\right]v_\infty$$

$$\Delta^{(6)}\boldsymbol{p}_{(\mathrm{iso})} = \frac{G^6M^7\nu^3}{b^6}\left[\left(\frac{\pi(134\bar{\alpha}_4 + 34\bar{\alpha}_5 + 5\bar{\alpha}_6 - 2\bar{\beta}_5 - 16\bar{\beta}_4 - 68\bar{\beta}_3)}{16}\right)\right.$$
$$\left. + \left(\frac{8(31\bar{\alpha}_4 + 4\bar{\alpha}_5 - 2\bar{\beta}_4 - 18\bar{\beta}_3)}{15}\right)\frac{\boldsymbol{v}_\infty}{v_\infty}\right]\frac{1}{v_\infty}\,.$$

(Notice b and v connection for conservative-like)

$$\tau_{\mathrm{int}} = -\frac{\boldsymbol{v}_+\cdot\boldsymbol{b}_+}{v_+^2} = -\frac{\boldsymbol{b}\cdot\Delta\boldsymbol{v}}{v_+^2} + \int \mathrm{d}t\,\frac{t(\boldsymbol{v}_+\cdot\boldsymbol{a}(t))}{v_+^2}\,.$$

$$\tau^{(4)}_{(\mathrm{iso})} = \left(\frac{\bar{\alpha}_4 + 6\bar{\beta}_3 + \bar{\beta}_4}{8}\pi\right)\frac{G^4M^4\nu^2}{b^3}v_\infty\,,$$

$$\tau^{(5)}_{(\mathrm{iso})} = \left(\frac{61\bar{\alpha}_4 + 4\bar{\alpha}_5 + 28\bar{\beta}_3 + 21\bar{\beta}_4 + 4\bar{\beta}_5}{15}\right)\frac{G^5M^5\nu^2}{b^4v_\infty}\,,$$

$$\tau^{(6)}_{(\mathrm{iso})} = \left(\frac{738\bar{\alpha}_4 + 82\bar{\alpha}_5 + 5\bar{\alpha}_6 - 356\bar{\beta}_3 + 18\bar{\beta}_4 + 22\bar{\beta}_5}{80}\pi\right)\frac{G^6M^6\nu^2}{b^5v_\infty^3}$$

Can we reconstruct an isotropic-like acceleration?

$$\boldsymbol{a}_{(\mathrm{iso})} = -\frac{GM}{r^2}\boldsymbol{n} + \sum_n \frac{G^n}{r^{n+1}}\left[\left(\alpha_n(\boldsymbol{v}^2) + \delta_n(\boldsymbol{v}^2)(\boldsymbol{v}\cdot\boldsymbol{n})\right)\boldsymbol{n} + \left(\beta_n(\boldsymbol{v}^2)(\boldsymbol{v}\cdot\boldsymbol{n}) + \gamma_n(\boldsymbol{v}^2)\right)\boldsymbol{v}\right]$$

Not derived from a Hamiltonian!

$$M\nu\left(\boldsymbol{a}_{(\mathrm{iso})}\cdot\boldsymbol{v}\right)_{\{\alpha_n,\beta_n\}} = 0 - \frac{\mathrm{d}E_{(\mathrm{iso})}}{\mathrm{d}t}$$

$$2M\nu\left(\boldsymbol{r}^{[i}\boldsymbol{a}^{j]}_{(\mathrm{iso})}\right)_{\{\alpha_n,\beta_n\}} = 0 - \frac{\mathrm{d}L^{ij}_{(\mathrm{iso})}}{\mathrm{d}t}$$

$$\Delta^{(3)}E_{(\mathrm{RR,iso})} = \left(\frac{\bar{\delta}_3 + 4\bar{\gamma}_3 + 6\bar{\gamma}_2}{8}\right)\frac{G^3M^4\nu^2}{b^3}\,,$$

$$\Delta^{(4)}E_{(\mathrm{RR,iso})} = \left(\frac{\bar{\delta}_3 + 5\bar{\gamma}_3 + 7\bar{\gamma}_2}{3}\right)\frac{G^4M^5\nu^2}{b^4}\,,$$

$$\Delta^{(2)}\boldsymbol{L}^i_{(\mathrm{RR,iso})} = (2\bar{\gamma}_2)\frac{G^2M^3\nu^2}{b^2}\varepsilon^{ijk}b^jv_\infty^k\,.$$

$$E^{5\mathrm{PN}}_{(\mathrm{iso})} = +\left(\frac{\bar{\beta}_3}{3}\right)\frac{G^3M^4\nu^3}{r^3}\boldsymbol{v}^6 + \left(\frac{\bar{\alpha}_4 + \bar{\beta}_4 - 2\bar{\beta}_3}{4}\right)\frac{G^4M^5\nu^3}{r^4}\boldsymbol{v}^4$$
$$+\left(\frac{\bar{\alpha}_5 + \bar{\beta}_5 - \bar{\alpha}_4 - \bar{\beta}_4 + 2\bar{\beta}_3}{5}\right)\frac{G^5M^6\nu^3}{r^5}\boldsymbol{v}^2$$
$$+\left(\frac{5\bar{\alpha}_6 - 2\bar{\alpha}_5 - 2\bar{\beta}_5 + 2\bar{\alpha}_4 + 2\bar{\beta}_4 - 4\bar{\beta}_3}{30}\right)\frac{G^6M^7\nu^3}{r^6}\,.$$

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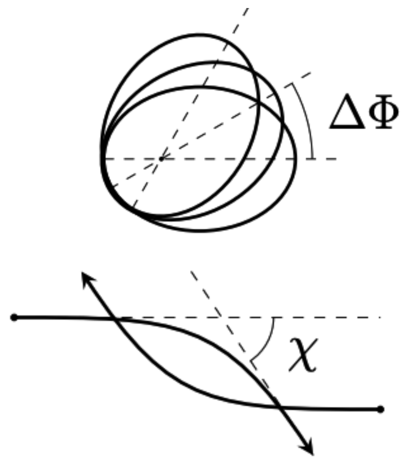
WEFT approach to GW physics 5PN



Total even-in-velocity scattering DATA! $\{\chi, \tau, \Delta E, \Delta L_z\}$

Gauge equivalent to original harmonic EFT calculation

$$\mathbf{a}_{(\mathrm{BT})} = \frac{8G^2M^2\nu}{5r^3}\left[\left(18v^2 - 25(\boldsymbol{v}\cdot\boldsymbol{n})^2\right)(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{n} - \left(6v^2 - 15(\boldsymbol{v}\cdot\boldsymbol{n})^2\right)\boldsymbol{v}\right] + \frac{16G^3M^3\nu}{5r^4}\left[\boldsymbol{v} + \frac{\boldsymbol{v}\cdot\boldsymbol{n}}{3}\boldsymbol{n}\right],$$

$$\delta\boldsymbol{r}_{\mathrm{BT}\to\mathrm{iso}} = -\frac{8}{3}\frac{G^2M^2\nu}{r}(\boldsymbol{v} - (\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{n})$$

$$\Delta^{(4)}\boldsymbol{p}_{(\mathrm{iso})} = \frac{G^4M^5\nu^3}{b^4}\left[\left(\frac{\pi(3\bar{\alpha}_4 - 2\bar{\beta}_3)}{8}\right)\frac{\boldsymbol{b}}{b}\right]v_\infty^3,$$

$$\Delta^{(5)}\boldsymbol{p}_{(\mathrm{iso})} = \frac{G^5M^6\nu^3}{b^5}\left[\left(\frac{4(31\bar{\alpha}_4 + 4\bar{\alpha}_5 - 2\bar{\beta}_4 - 18\bar{\beta}_3)}{15}\right)\frac{\boldsymbol{b}}{b} + \left(\frac{\pi(3\bar{\alpha}_4 - 2\bar{\beta}_3)}{4}\right)\frac{\boldsymbol{v}_\infty}{v_\infty}\right]v_\infty$$

$$\Delta^{(6)}\boldsymbol{p}_{(\mathrm{iso})} = \frac{G^6M^7\nu^3}{b^6}\left[\left(\frac{\pi(134\bar{\alpha}_4 + 34\bar{\alpha}_5 + 5\bar{\alpha}_6 - 2\bar{\beta}_5 - 16\bar{\beta}_4 - 68\bar{\beta}_3)}{16}\right) + \left(\frac{8(31\bar{\alpha}_4 + 4\bar{\alpha}_5 - 2\bar{\beta}_4 - 18\bar{\beta}_3)}{15}\right)\frac{\boldsymbol{v}_\infty}{v_\infty}\right]\frac{1}{v_\infty}.$$

$$\tau_{\mathrm{int}} = -\frac{\boldsymbol{v}_+\cdot\boldsymbol{b}_+}{v_+^2} = -\frac{\boldsymbol{b}\cdot\Delta\boldsymbol{v}}{v_+^2} + \int \mathrm{d}t\,\frac{t(\boldsymbol{v}_+\cdot\boldsymbol{a}(t))}{v_+^2}.$$

$$\tau^{(4)}_{(\mathrm{iso})} = \left(\frac{\bar{\alpha}_4 + 6\bar{\beta}_3 + \bar{\beta}_4}{8}\pi\right)\frac{G^4M^4\nu^2}{b^3}v_\infty,$$

$$\tau^{(5)}_{(\mathrm{iso})} = \left(\frac{61\bar{\alpha}_4 + 4\bar{\alpha}_5 + 28\bar{\beta}_3 + 21\bar{\beta}_4 + 4\bar{\beta}_5}{15}\right)\frac{G^5M^5\nu^2}{b^4v_\infty},$$

$$\tau^{(6)}_{(\mathrm{iso})} = \left(\frac{738\bar{\alpha}_4 + 82\bar{\alpha}_5 + 5\bar{\alpha}_6 - 356\bar{\beta}_3 + 18\bar{\beta}_4 + 22\bar{\beta}_5}{80}\pi\right)\frac{G^6M^6\nu^2}{b^5v_\infty^3}$$

Can we reconstruct an isotropic-like acceleration?

$$\delta_{(\mathrm{FT})}\boldsymbol{r} = -\frac{1}{30}\frac{G^5M^5\nu^2}{r^4}\boldsymbol{n} + \frac{G^4M^4\nu^2}{r^3}\left[\left(\frac{7}{12}\boldsymbol{v}^2 + \frac{53}{15}(\boldsymbol{v}\cdot\boldsymbol{n})^2\right)\boldsymbol{n} - \frac{9}{2}(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v}\right] + \frac{G^3M^3\nu^2}{r^2}\left[\left(-\frac{76}{15}\boldsymbol{v}^4 + \frac{52}{5}\boldsymbol{v}^2(\boldsymbol{v}\cdot\boldsymbol{n})^2 + 4(\boldsymbol{v}\cdot\boldsymbol{n})^4\right)\boldsymbol{n} + \left(\frac{20}{30}\boldsymbol{v}^2 - 16(\boldsymbol{v}\cdot\boldsymbol{n})^2\right)(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v}\right],$$

$$\delta_{(\mathrm{M})}\boldsymbol{r} = -\frac{71}{105}\frac{G^5M^5\nu^2}{r^4}\boldsymbol{n} + \frac{G^4M^4\nu^2}{r^3}\left[\left(-\frac{1679}{630}\boldsymbol{v}^2 - \frac{136}{35}(\boldsymbol{v}\cdot\boldsymbol{n})^2\right)\boldsymbol{n} + \frac{128}{105}(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v}\right] + \frac{G^3M^3\nu^2}{r^2}\left[\left(\frac{44}{5}\boldsymbol{v}^4 - \frac{2022}{35}\boldsymbol{v}^2(\boldsymbol{v}\cdot\boldsymbol{n})^2 + \frac{358}{7}(\boldsymbol{v}\cdot\boldsymbol{n})^4\right)\boldsymbol{n} + \left(\frac{566}{35}\boldsymbol{v}^2 - \frac{706}{35}(\boldsymbol{v}\cdot\boldsymbol{n})^2\right)(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v}\right],$$

$$\delta_{(\mathrm{2RR})}\boldsymbol{r} = \left(\frac{502}{225} - \frac{412}{525}\right)\frac{G^5M^5\nu^2}{r^4}\boldsymbol{n} + \frac{G^4M^4\nu^2}{r^3}\left[\left(\left(\frac{67}{9} - \frac{334}{45}\right)\boldsymbol{v}^2 + \left(\frac{1084}{225} + \frac{712}{45}\right)(\boldsymbol{v}\cdot\boldsymbol{n})^2\right)\boldsymbol{n} + \left(-\frac{314}{45} - \frac{124}{15}\right)(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v}\right] + \frac{G^3M^3\nu^2}{r^2}\left[\left(-\frac{2816}{75}\boldsymbol{v}^4 + \frac{5312}{25}\boldsymbol{v}^2(\boldsymbol{v}\cdot\boldsymbol{n})^2 - 176(\boldsymbol{v}\cdot\boldsymbol{n})^4\right)\boldsymbol{n} + \left(-\frac{256}{3}\boldsymbol{v}^2 + \frac{432}{5}(\boldsymbol{v}\cdot\boldsymbol{n})^2\right)(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v}\right],$$

No Schott-term ambiguity!

$$\boldsymbol{a}^{2.5\mathrm{PN}}_{(\mathrm{RR,iso})} = -\frac{8}{5}\frac{G^2M^2\nu}{r^3}\left[\boldsymbol{v}^2\,\boldsymbol{v} - 3\,\boldsymbol{v}^2(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{n}\right] - \frac{8G^3M^3\nu}{5r^4}\left[3\boldsymbol{v} - \frac{17}{3}(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{n}\right]$$

← Dissipation comes from iterated 'linear' force only!

$$\boldsymbol{a}_{(\mathrm{FT,iso})} = \frac{5}{6}\frac{G^6M^6\nu^2\boldsymbol{n}}{r^7} + \frac{G^5M^5\nu^2\boldsymbol{n}}{r^6}\left[-\frac{49}{20}\boldsymbol{v}^2\boldsymbol{n} + \frac{289}{30}(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v}\right] + \frac{G^4M^4\nu^2\boldsymbol{n}}{r^5}\left[-\frac{23}{3}\boldsymbol{v}^4\boldsymbol{n} + \frac{184}{15}\boldsymbol{v}^2(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v}\right],$$

$$\boldsymbol{a}_{(\mathrm{M,iso})} = -\frac{67}{63}\frac{G^6M^6\nu^2\boldsymbol{n}}{r^7} + \frac{G^5M^5\nu^2\boldsymbol{n}}{r^6}\left[\frac{1951}{630}\boldsymbol{v}^2\boldsymbol{n} - \frac{5476}{315}(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v}\right] + \frac{G^4M^4\nu^2\boldsymbol{n}}{r^5}\left[\frac{1058}{105}\boldsymbol{v}^4\boldsymbol{n} - \frac{5612}{315}\boldsymbol{v}^2(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v}\right],$$

← "Energy-conserving"

$$\boldsymbol{a}_{(\mathrm{2RR,iso})} = \left(\frac{1994}{225} + \frac{228}{175}\right)\frac{G^6M^6\nu^2\boldsymbol{n}}{r^7} + \frac{G^5M^5\nu^2\boldsymbol{n}}{r^6}\left[\left(\frac{1861}{225} - \frac{6}{7}\right)\boldsymbol{v}^2\boldsymbol{n} + \left(\frac{3382}{45} + \frac{100}{7}\right)(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v}\right] + \frac{G^4M^4\nu^2\boldsymbol{n}}{r^5}\left[\left(-\frac{388}{75} - \frac{728}{15}\right)\boldsymbol{v}^4\boldsymbol{n} + \left(\frac{3344}{45} + \frac{704}{15}\right)\boldsymbol{v}^2(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v}\right]$$

$$E^{5\mathrm{PN}}_{(\mathrm{FT,iso})} = -\frac{1}{30}\frac{G^6M^7\nu^3}{r^6} + \frac{31}{60}\frac{G^5M^6\nu^3}{r^5}\boldsymbol{v}^2 + \frac{23}{20}\frac{G^4M^5\nu^3}{r^4}\boldsymbol{v}^4,$$

$$E^{5\mathrm{PN}}_{(\mathrm{M,iso})} = +\frac{7}{27}\frac{G^6M^7\nu^3}{r^6} - \frac{55}{42}\frac{G^5M^6\nu^3}{r^5}\boldsymbol{v}^2 - \frac{1219}{630}\frac{G^4M^5\nu^3}{r^4}\boldsymbol{v}^4,$$

$$E^{5\mathrm{PN}}_{(\mathrm{2RR,iso})} = +\left(\frac{118}{225} - \frac{412}{525}\right)\frac{G^6M^7\nu^3}{r^6} + \left(\frac{643}{225} + \frac{526}{175}\right)\frac{G^5M^6\nu^3}{r^5}\boldsymbol{v}^2 + \left(\frac{3889}{225} - \frac{2}{5}\right)\frac{G^4M^5\nu^3}{r^4}\boldsymbol{v}^4.$$

[including a term from RR-RR!]

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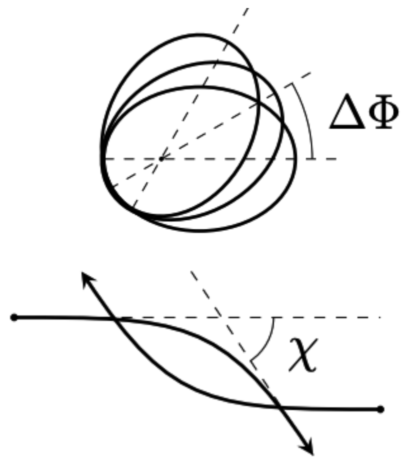
WEFT approach to GW physics 5PN

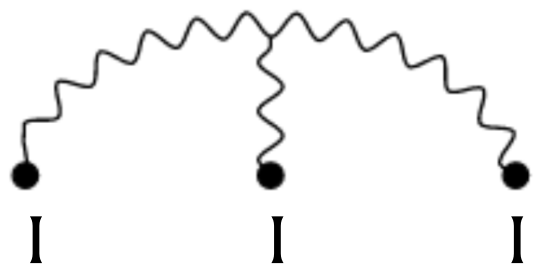


(ON-SHELL) Conservative sector

$$\Delta_{\rm ret/adv}(x) = \Delta_F(x) + \Delta_{\mp}(x)\,, \quad \Delta_{\rm ret/adv}(x) = \int_k \frac{e^{ik\cdot x}}{(k^0 \pm i0^+)^2 - \boldsymbol{k}^2}\,,$$

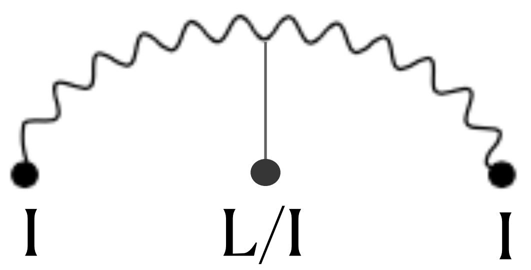
$$\Delta_F(x) = \int_k \frac{e^{ik\cdot x}}{k^2 + i0^+}\,, \quad \Delta_{\pm}(x) = (2\pi i)\int_k e^{ik\cdot x}\delta(k^2)\theta(\pm k^0)\,.$$

$$\Delta_{\rm ret/adv}(\omega) = \mp\frac{i}{4\pi}\omega\,, \quad \Delta_F(\omega) = -\frac{i}{4\pi}{\rm Sign}(\omega)\omega\,, \quad \Delta_{\pm}(\omega) = \pm\frac{i}{2\pi}\theta(\pm\omega)\omega$$



$$S^{\rm cons}_{\rm (FT)} = -\frac{G^2}{30}\int\frac{{\rm d}\omega_1{\rm d}\omega_2}{(2\pi)^2}(i\omega_1^4\omega_2^3)L_{ki}I_{kj}(\omega_1)I_{ij}(\omega_2)\hat{\delta}(\omega_1+\omega_2)$$

$$= \frac{G^2}{30}L_{kl}\int\frac{{\rm d}\omega}{2\pi}(-i\omega^7)I_{ik}(-\omega)I_{il}(\omega)\,,$$



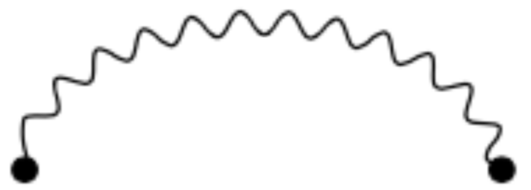
$$S^{\rm cons}_{\rm (M)} = \frac{G^2}{70}\int\frac{{\rm d}\omega_1{\rm d}\omega_2{\rm d}\omega_3}{(2\pi)^3}I_{ij}(\omega_1)I_{jk}(\omega_3)I_{ki}(\omega_2){\rm Sign}(\omega_3){\rm Sign}(\omega_1)\hat{\delta}(\omega_1+\omega_2+\omega_3)$$

$$\times\left(7\omega_1^4\omega_3^4 - 2\omega_1^3\omega_3^3\omega_2^2 + 2\omega_1^2\omega_3^2\omega_2^4\right)\,,$$

$$= -\frac{G^2}{70}\int\frac{{\rm d}\omega{\rm d}\omega_1}{(2\pi)^2}I_{ij}(-\omega)I_{jk}(\omega_1)I_{ki}(\omega-\omega_1){\rm Sign}(\omega){\rm Sign}(\omega_1)$$

$$\times\,\omega^2\omega_1^2\left(2\omega^4 - 6\omega^3\omega_1 + 15\omega^2\omega_1^2 - 6\omega\omega_1^3 + 2\omega_1^4\right)\,,$$

← Double PV integral!



$$S^{\rm cons}_{\rm (RR)} = -\frac{2\pi G}{5}\int\frac{{\rm d}\omega}{2\pi}I_{ij}(-\omega)I_{ij}(\omega)\Delta_F(\omega)\omega^4\,,$$

← 2RR even-in-velocity cons.!

$$\Delta p^\mu_{\rm tot} = \Delta p^\mu_{\rm cons} + \Delta p^\mu_{\rm diss}\,,$$

$$\Delta p^\mu_{\rm cons} \equiv \mathbb{R}\Delta p^\mu_{\rm F}$$

$$S^{\rm cons}_{({\rm RR}^2)} - \frac{2G^2}{25}\int\frac{{\rm d}\omega_1{\rm d}\omega_2{\rm d}\omega_3}{(2\pi)^3}I_{ij}(\omega_1)I_{jk}(\omega_3)Q_{ki}(\omega_2){\rm Sign}(\omega_3){\rm Sign}(\omega_1)\hat{\delta}(\omega_1+\omega_2+\omega_3)$$

$$\times\left(\omega_1^4\omega_3^4 + \omega_1^3\omega_3^3\omega_2^2 - \frac{1}{2}\omega_1^2\omega_3^2\omega_2^4\right)$$

MUST INCLUDE RR-RR!

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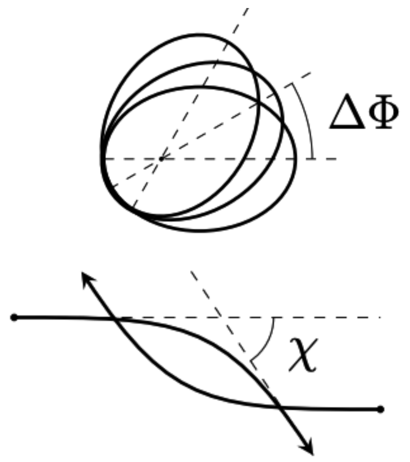
WEFT approach to GW physics 5PN

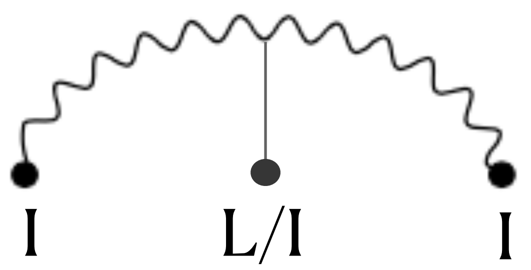


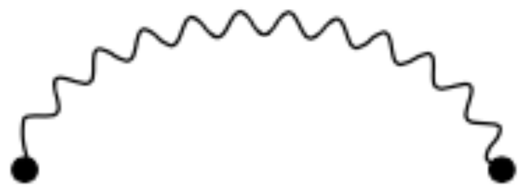
(ON-SHELL) Conservative sector

Tail-like region

$$I^{ij}(\omega) = I^{ij}_{(0)}(\omega) + I^{ij}_{(\mathrm{N})}(\omega) + \cdots ,$$

$$I^{ij}_{(0)}(\omega) = b^{\langle i}b^{j\rangle}\delta(\omega) - i2b^{\langle i}v^{j\rangle}_{\infty}\delta'(\omega) - v^{\langle i}_{\infty}v^{j\rangle}_{\infty}\delta''(\omega)$$





$$S^{\mathrm{cons}}_{(\text{T-like})} = +\frac{17G^2}{150}\int \frac{\mathrm{d}\omega}{2\pi}(-i\omega^7)L^{kl}I^{ki}(-\omega)I^{il}(\omega)$$
$$+\frac{G^2}{5}\int \frac{\mathrm{d}\omega_1\mathrm{d}\omega_2\mathrm{d}\omega_3}{(2\pi)^3}\delta(\omega_1+\omega_2+\omega_3)\Big\{\Big(\frac{\omega_1^3\omega_3^3\omega_2^2}{7} - \frac{\omega_1^4\omega_3^4}{2}\Big)I^{ij}_{(\mathrm{N})}(\omega_1)I^{jk}_{(\mathrm{N})}(\omega_3)I^{ki}_{(0)}(\omega_2)$$
$$+\Big(\frac{2\omega_1^4\omega_3^4}{5} + \frac{2\omega_1^3\omega_3^3\omega_2^2}{5}\Big)I^{ij}_{(\mathrm{N})}(\omega_1)I^{jk}_{(\mathrm{N})}(\omega_3)Q^{ki}_{(0)}(\omega_2)\Big\} ,$$

$$\int \frac{\mathrm{d}\omega}{2\pi}\mathrm{Sign}(\omega)\delta(\omega)f(\omega) = 0$$

Feynman symmetry

RAP Riva Yang (2024)

$$S^{\mathrm{cons}}_{5\mathrm{PN}/4\mathrm{PM}} = S_{(\mathrm{pot})} + S^{\mathrm{cons}}_{(\mathrm{T})} + \frac{G^2}{5}\int \mathrm{d}t\Big\{-\frac{1}{6}L^{kl}I^{(4)ki}I^{(3)li} + \frac{1}{7}I^{(2)ij}I^{(3)jk}I^{(3)ik} - \frac{1}{2}I^{ij}I^{(4)jk}I^{(4)ik} + \frac{2}{5}\Big(-L^{ki}I^{(4)kj}I^{(3)ij} + Q^{ki}I^{kj(4)}I^{(4)ij} + Q^{(2)ki}I^{(3)kj}I^{(3)ij}\Big)\Big\}, \quad (7.2)$$

porated into a local-in-time Hamiltonian (see also [94]). Likewise, at higher orders in G, the expression in (7.2) will incorporate all of the "tail-like" 5PN contributions to the conservative sector, provided the multipole moments with $n \leq 2$ derivatives are kept unperturbed (setting the acceleration to zero). However, starting at 5PM, other conservative (Feynman) memory

$$S^{\mathrm{cons}}_{(\text{T-like})} = \frac{G^2}{5}\int \mathrm{d}t\Big\{-\frac{17}{30}L_{kl}I^{(4)}_{ki}I^{(3)}_{il} + \frac{1}{7}I^{(2)}_{(0),ij}I^{(3)}_{jk}I^{(3)}_{ki} - \frac{1}{2}I_{(0),ij}I^{(4)}_{jk}I^{(4)}_{ki} + \frac{2}{5}\Big(Q^{(2)}_{(0),ij}I^{(3)}_{jk}I^{(3)}_{ki} + Q_{(0),ij}I^{(4)}_{jk}I^{(4)}_{ki}\Big)\Big\}$$

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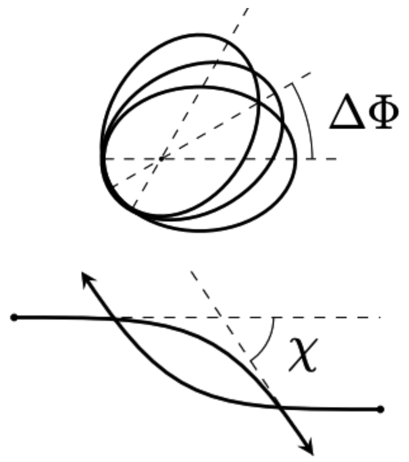
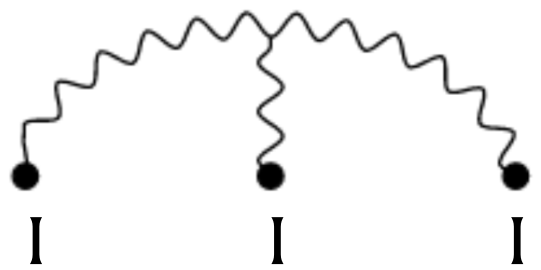
WEFT approach to GW physics 5PN



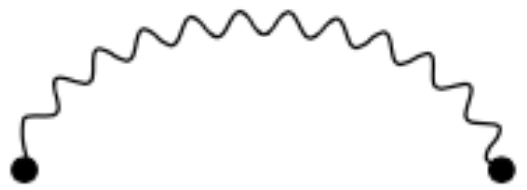
(ON-SHELL) Conservative sector

Memory-like region



$$I^{ij}(\omega) = I^{ij}_{(0)}(\omega) + I^{ij}_{(\mathrm{N})}(\omega) + \cdots ,$$

$$S^{\mathrm{cons}}_{(\mathrm{M\text{-}like})} \supset -\frac{G^2}{70}\int \frac{\mathrm{d}\omega \mathrm{d}\omega_1}{(2\pi)^2} I^{ij}_{(\mathrm{N})}(-\omega) I^{jk}_{(\mathrm{N})}(\omega_1) I^{ki}_{(\mathrm{N})}(\omega-\omega_1)\mathrm{Sign}(\omega)\mathrm{Sign}(\omega_1)$$
$$\times\, \omega^2\omega_1^2 \left(2\omega^4 - 6\omega^3\omega_1 + 15\omega^2\omega_1^2 - 6\omega\omega_1^3 + 2\omega_1^4\right)$$



$$S^{\mathrm{cons}}_{(\mathrm{M\text{-}like})} = \int \frac{\mathrm{d}\omega_1 \mathrm{d}\omega_2}{(2\pi)^2}\mathrm{Sign}(\omega_1)\mathrm{Sign}(\omega_2) A(-\omega_1) B(\omega_1-\omega_2) C(\omega_2)$$
$$= \int \mathrm{d}t_1 \mathrm{d}t_2 \mathrm{d}t_3 \left(-\frac{i}{\pi}\frac{\mathscr{P}}{t_1-t_2}\right)\left(-\frac{i}{\pi}\frac{\mathscr{P}}{t_2-t_3}\right) A(t_1) B(t_2) C(t_3)\,,$$

⟵ Double PV integral!

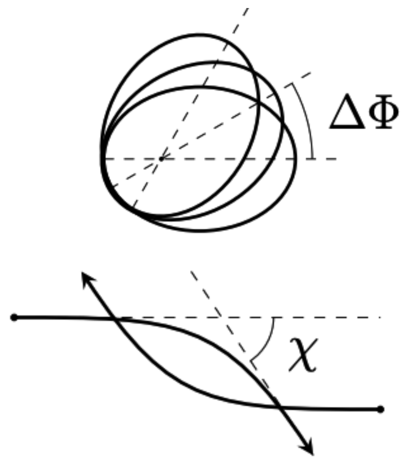
Poincare Bertrand ⟷ $\mathrm{Sign}(\omega)\mathrm{Sign}(\omega_1) = 1 - 2\vartheta(\omega)\vartheta(-\omega_1) - 2\vartheta(-\omega)\vartheta(\omega_1)\,.$

$$S^{\mathrm{cons(PBloc)}}_{(\mathrm{M\text{-}like})} = \int \mathrm{d}t_1 \mathrm{d}t_2 \mathrm{d}t_3 \delta(t_1-t_2)\delta(t_2-t_3) A(t_1) B(t_2) C(t_3)\,.$$

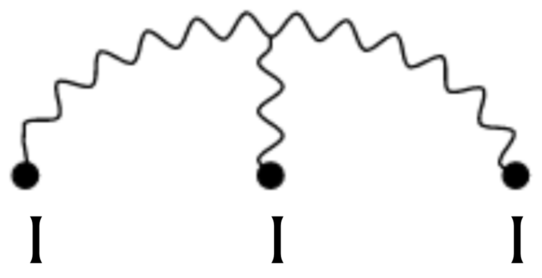
↑ sign is + on this routing!
(after solving frequencies)
Consistent with tail region

RAP Riva (2026)

WEFT approach to GW physics 5PN



(ON-SHELL) Conservative sector

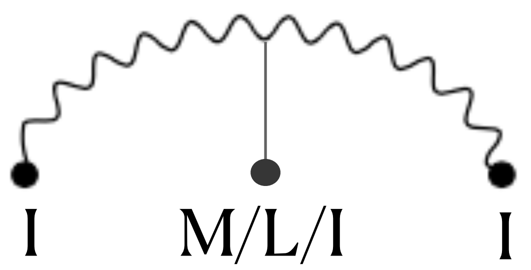


$$S^{\text{cons(PB)}}_{\text{(tot)}} = \frac{G^2}{5}\int \mathrm{d}t\Big\{-\frac{17}{30}L_{kl}I^{(4)}_{ki}I^{(3)}_{il} + \frac{1}{7}I^{(2)}_{(0)ij}I^{(3)}_{jk}I^{(3)}_{ki} - \frac{1}{2}I_{(0)ij}I^{(4)}_{jk}I^{(4)}_{ki} + \frac{2}{5}\Big(Q^{(2)}_{(0)ij}I^{(3)}_{jk}I^{(3)}_{ki} + Q_{(0)ij}I^{(4)}_{jk}I^{(4)}_{ki}\Big)$$
$$+\frac{1}{7}\hat{I}^{(2)}_{ij}I^{(3)}_{jk}I^{(3)}_{ki} - \frac{1}{2}\hat{I}_{ij}I^{(4)}_{jk}I^{(4)}_{ki} + \frac{2}{5}\Big(\hat{Q}^{(2)}_{ij}I^{(3)}_{jk}I^{(3)}_{ki} + \hat{Q}_{ij}I^{(4)}_{jk}I^{(4)}_{ki}\Big) - \frac{1}{7}\hat{I}^{(2)}_{ij}\hat{I}^{(2)}_{jk}I^{(4)}_{ki} + S^{\text{cons(PB)}}_{\text{RR-RR}}\Big\},$$

T-like [M+RR^2]

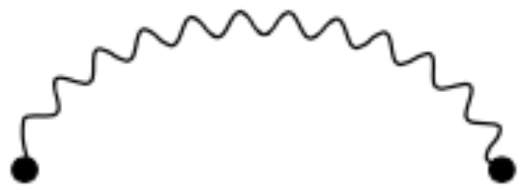
T-like completion (M+RR^2 loc)

nonT-like (M+RR^2 loc)



$$S^{\text{cons(PB)}}_{\text{RR-RR}} = +\frac{2M\nu G^2}{25}\int \mathrm{d}t\Big\{-3\boldsymbol{a}^i_{(\mathrm{N})}\boldsymbol{r}^j_{(0)}I^{(3),jk}_{(\mathrm{N})}I^{(3),ki}_{(\mathrm{N})} + 4\boldsymbol{a}^i_{(\mathrm{N})}\boldsymbol{v}^j_{(0)}I^{(2),jk}_{(\mathrm{N})}I^{(3),ki}_{(\mathrm{N})}$$
$$-\boldsymbol{r}^k_{(\mathrm{N})}\boldsymbol{a}^j_{(\mathrm{N})}I^{(3),ki}_{(\mathrm{N})}I^{(3),ij}_{(\mathrm{N})} - \Big(\boldsymbol{r}^i_{(0)}I^{(3),ij}_{(\mathrm{N})} - 2\boldsymbol{v}^i_{(0)}I^{(2),ij}_{(\mathrm{N})}\Big)\mathcal{E}^{jk}\Big(\boldsymbol{r}^\ell_{(0)}I^{(3),\ell k}_{(\mathrm{N})} - 2\boldsymbol{v}^\ell_{(0)}I^{(2),\ell k}_{(\mathrm{N})}\Big)\Big\}$$
$$+\frac{GM\nu}{5}\int \mathrm{d}t\Big\{\boldsymbol{a}^k_{(\mathrm{N})}I^{(3),ki}_{(\mathrm{N})}\delta_{(\mathrm{RR})}\boldsymbol{r}^i - \Big(\boldsymbol{r}^k_{(\mathrm{N})}I^{(4),ki}_{(\mathrm{N})} - \boldsymbol{v}^k_{(\mathrm{N})}I^{(3),ki}_{(\mathrm{N})}\Big)\delta_{(\mathrm{RR})}\boldsymbol{v}^i$$
$$+\boldsymbol{r}^k_{(\mathrm{N})}I^{(5),k\ell}_{(\mathrm{N})}\Delta_{\mathcal{E}}\boldsymbol{r}^\ell + \Big(\boldsymbol{r}^k_{(0)}I^{(3),k\ell}_{(\mathrm{N})}(t) - 2\boldsymbol{v}^k_{(0)}I^{(2),k\ell}_{(\mathrm{N})}\Big)\mathcal{E}^{\ell i}\Big(\delta_{(\mathrm{RR})}\boldsymbol{r}^i + \Delta_{\mathcal{E}}\boldsymbol{r}^i\Big)\Big\}$$

M-like cons RR-RR!!
(ask me later)



$$\tilde{\chi}^{(4,\nu^2)\text{cons}}_{j\text{(tot)}} = \pi\Big(-\frac{8919}{1400} + \frac{8919}{1400}\Big)v^6_\infty = 0\,,$$

$$\tilde{\chi}^{(5,\nu^2)\text{cons}}_{j\text{(tot)PB}} = \Big(\frac{217866437}{151200} - \frac{224057}{1440}\pi^2 + \frac{1408}{45}\log(2v_\infty) + \frac{38144}{225} - \frac{3584}{1125} + \frac{1024}{675} + \frac{127232}{3375}\Big)v^5_\infty$$
$$= \Big(\frac{48699841}{30240} - \frac{224057}{1440}\pi^2 + \frac{1408}{45}\log(2v_\infty) + \frac{9728}{270}\Big)v^5_\infty\,,$$

$$\tilde{\chi}^{(6,\nu^2)\text{cons}}_{j\text{(tot)PB}} = \pi\Big(\frac{7572253}{2240} - \frac{10812865}{32768}\pi^2 + \frac{201}{2}\log\Big(\frac{v_\infty}{2}\Big) + \frac{2817}{16}\zeta(3) + \frac{5864113}{26880} + \pi^2\frac{2721}{5120}$$
$$-\frac{63005}{5376} - \pi^2\frac{2721}{5120} + \frac{149}{42} + \frac{9427}{120}\Big)v^4_\infty$$
$$= \pi\Big(\frac{96731149}{26880} - \frac{10812865}{32768}\pi^2 + \frac{201}{2}\log\Big(\frac{v_\infty}{2}\Big) + \frac{2817}{16}\zeta(3) + \frac{630661}{8960}\Big)v^4_\infty$$

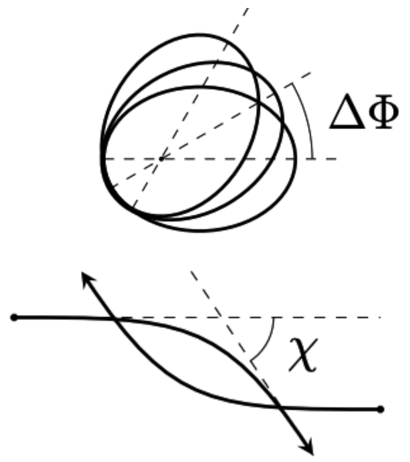
Perfect agreement at 4PM
(consistent with mass scaling)

Dlapa Kalin Liu **RAP** (2021)

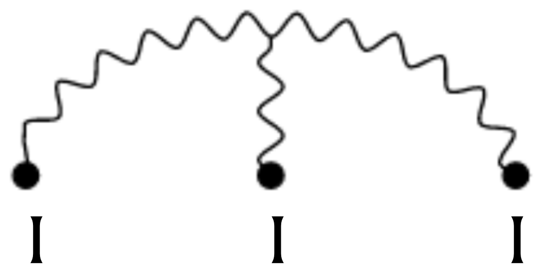
Perfect agreement in
pot + T-like [v vs v_inf]
&
sign mismatch wrt cM=1
in WQFT for M-like!

RAP Riva (2026)

WEFT approach to GW physics 5PN



(ON-SHELL) Conservative sector T & M-like



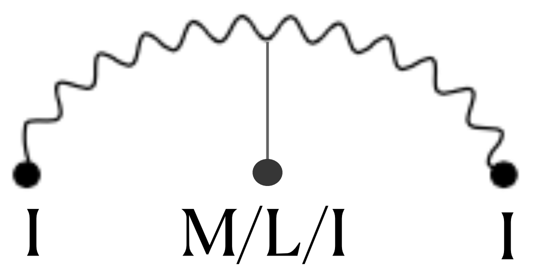
$$\frac{\delta \hat{H}_{(\mathrm{iso})}}{\nu^2} = -\frac{2973}{350}\hat{\boldsymbol{p}}^4\left(\frac{GM}{r}\right)^4 - \left(\frac{617}{150}+\frac{608}{45}\right)\hat{\boldsymbol{p}}^2\left(\frac{GM}{r}\right)^5 - \left(\frac{104833}{25200}-\frac{50299}{8400}\right)\left(\frac{GM}{r}\right)^6 .$$

B2B from scattering angle

Dissipative=Tot - Cons

(isotropic representation)

EOB coefficients (consistent with Tutti-frutti)



$$\tilde{\chi}^{(5,\nu^2)\mathrm{cons,TF}}_{j(\mathrm{tot})} = \left(\frac{1408}{45}\log(2v_\infty) - \frac{365555}{6048} - \frac{4}{15}d^{\mathrm{rat}\,\nu^2}_{5\mathrm{loc}} - \frac{224057}{1440}\pi^2\right)v_\infty^5$$

$$\tilde{\chi}^{(6,\nu^2)\mathrm{cons,TF}}_{j(\mathrm{tot})} = \pi\left(\frac{2817}{16}\zeta(3) + \frac{30161}{192} + \frac{201}{2}\log\left(\frac{v_\infty}{2}\right) - \frac{10812865}{32768}\pi^2 - \frac{15}{32}\left(d^{\mathrm{rat}\,\nu^2}_{5\mathrm{loc}} + a^{\mathrm{rat}\,\nu^2}_{6\mathrm{loc}}\right)\right)v_\infty^4 ,$$

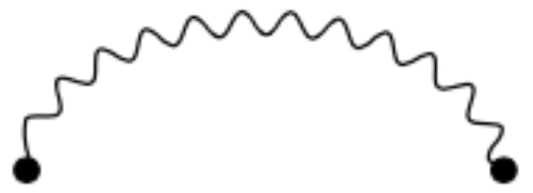
$$d^{\mathrm{rat}\,\nu^2}_{5\mathrm{loc}} = -\frac{394747}{63} - \frac{1216}{9} = -\frac{403259}{63} ,$$

$$a^{\mathrm{rat}\,\nu^2}_{6\mathrm{loc}} = -\frac{13559209}{12600} - \frac{189583}{12600} = -\frac{1718599}{1575} .$$

UNIVERSAL

AND LOCAL IN TIME

DECOMPOSITION!



$$\tilde{\chi}^{(4,\nu^2)\mathrm{cons}}_{j(\mathrm{tot})} = \pi\left(-\frac{8919}{1400} + \frac{8919}{1400}\right)v_\infty^6 = 0 ,$$

$$\tilde{\chi}^{(5,\nu^2)\mathrm{cons}}_{j(\mathrm{tot})\mathrm{PB}} = \left(\frac{217866437}{151200} - \frac{224057}{1440}\pi^2 + \frac{1408}{45}\log(2v_\infty) + \frac{38144}{225} - \frac{3584}{1125} + \frac{1024}{675} + \frac{127232}{3375}\right)v_\infty^5$$

$$= \left(\frac{48699841}{30240} - \frac{224057}{1440}\pi^2 + \frac{1408}{45}\log(2v_\infty) + \frac{9728}{270}\right)v_\infty^5 ,$$

$$\tilde{\chi}^{(6,\nu^2)\mathrm{cons}}_{j(\mathrm{tot})\mathrm{PB}} = \pi\left(\frac{7572253}{2240} - \frac{10812865}{32768}\pi^2 + \frac{201}{2}\log\left(\frac{v_\infty}{2}\right) + \frac{2817}{16}\zeta(3) + \frac{5864113}{26880} + \pi^2\frac{2721}{5120} - \frac{63005}{5376} - \pi^2\frac{2721}{5120} + \frac{149}{42} + \frac{9427}{120}\right)v_\infty^4$$

$$= \pi\left(\frac{96731149}{26880} - \frac{10812865}{32768}\pi^2 + \frac{201}{2}\log\left(\frac{v_\infty}{2}\right) + \frac{2817}{16}\zeta(3) + \frac{630661}{8960}\right)v_\infty^4$$

pi^2 cancel between tail and memory regions!

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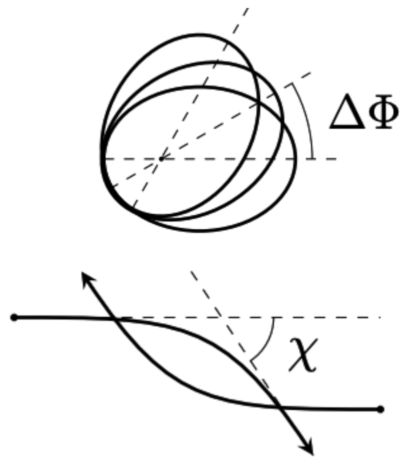
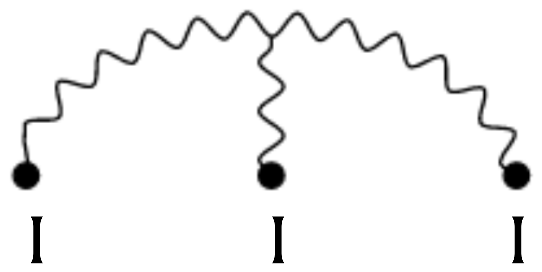


Alternative Memories



$$F^{ij}_{(\mathrm{M})}(\omega) = \int \frac{\mathrm{d}\omega_2 \mathrm{d}\omega_3}{(2\pi)^2} \mathcal{F}(\omega,\omega_2,\omega_3) I^{jk}(\omega_2) I^{ki}(\omega_3) \Delta_{\mathrm{adv}}(\omega)\Delta_{\mathrm{ret}}(\omega_3) + \int \frac{\mathrm{d}\omega_1 \mathrm{d}\omega_2}{(2\pi)^2} \mathcal{F}(\omega_1,\omega_2,\omega) I^{jk}(\omega_1) I^{ki}(\omega_2) \Delta_{\mathrm{adv}}(\omega)\Delta_{\mathrm{ret}}(\omega_1) + \int \frac{\mathrm{d}\omega_1 \mathrm{d}\omega_3}{(2\pi)^2} \mathcal{F}(\omega_1,\omega,\omega_3) I^{jk}(\omega_3) I^{ki}(\omega_1) \Delta_{\mathrm{ret}}(\omega_1)\Delta_{\mathrm{ret}}(\omega_3)\,.$$

Ret/Adv to F

$$F^{ij\,\mathrm{cons}}_{(\mathrm{M})}(\omega) = \int \frac{\mathrm{d}\omega_2 \mathrm{d}\omega_3}{(2\pi)^2} \mathcal{F}(\omega,\omega_2,\omega_3) I^{jk}(\omega_2) I^{ki}(\omega_3) \Delta_F(\omega)\Delta_F(\omega_3) + \int \frac{\mathrm{d}\omega_1 \mathrm{d}\omega_2}{(2\pi)^2} \mathcal{F}(\omega_1,\omega_2,\omega) I^{jk}(\omega_1) I^{ki}(\omega_2) \Delta_F(\omega)\Delta_F(\omega_1) + \int \frac{\mathrm{d}\omega_1 \mathrm{d}\omega_3}{(2\pi)^2} \mathcal{F}(\omega_1,\omega,\omega_3) I^{jk}(\omega_3) I^{ki}(\omega_1) \Delta_F(\omega_1)\Delta_F(\omega_3)\,.$$

γ-3 prescription

$$\Delta_F(\omega_1)\Delta_F(\omega_3)\delta(\omega_1+\omega_2+\omega_3) \to \Delta_{\mathrm{ret}}(\omega_1)\Delta_{\mathrm{ret}}(\omega_3)\delta(\omega_1+\omega_2+\omega_3)\,.$$

Driesse et al. (2026)

Rerouting consistent with PB — Symmetric Adv/Ret combination independent of region!

$$\Delta_{\mathrm{ret}}(\omega_1)\Delta_{\mathrm{ret}}(\omega_3) \to \Delta_{\mathrm{ret}}(\omega_1)\Delta_{\mathrm{ret}}(-\omega_3) = \Delta_{\mathrm{ret}}(\omega_1)\Delta_{\mathrm{adv}}(\omega_3)\,,$$

PB prescription flips the ret*ret term whereas g-3 flips the other two ret*adv pieces **(different overall sign!)**

Key point: Full force **[on the left]** has a definite routing independent of cons. prescription! [**signs must be +!**]

If g=3 singularities cancel out in full answer: Divergence can only be in terms **untouched** by g-3 prescription!

RAP Riva (2026)

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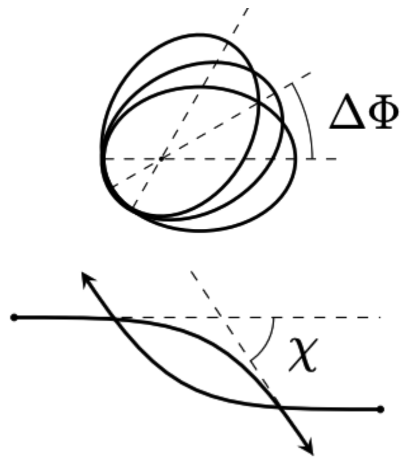
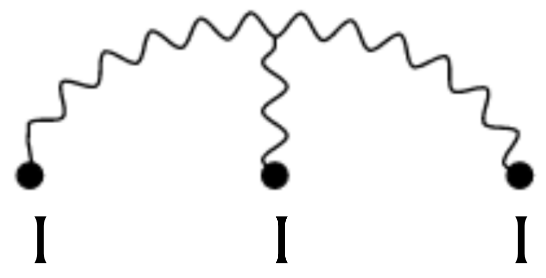


Alternative Memories

$$S^{\rm cons} = -\frac{16\pi^2 G^2}{70}\int \frac{d\omega_1 d\omega_2 d\omega_3}{(2\pi)^3}\,\delta\!\!\!{}^-(\omega_1+\omega_2+\omega_3){\rm Sign}(\omega_1){\rm Sign}(\omega_3)$$
$$\times\, \Delta_{\rm ret}(\omega_1)\Delta_{\rm ret}(\omega_3) I_{ij}(\omega_1) I_{jk}(\omega_3) I_{ki}(\omega_2)\mathcal{F}(\omega_1,\omega_2,\omega_3)\,.$$



$$I^{\rm M}_{1({\rm PB})} = -\frac{1}{15(8\pi)^4\epsilon} + \mathcal{O}(\epsilon^0)\,,$$

$$I^{\rm M}_{2({\rm PB})} = +\frac{5}{6(8\pi)^8\epsilon^2} + \mathcal{O}(\epsilon^{-1})\,,$$

dim. reg. poles cancel!

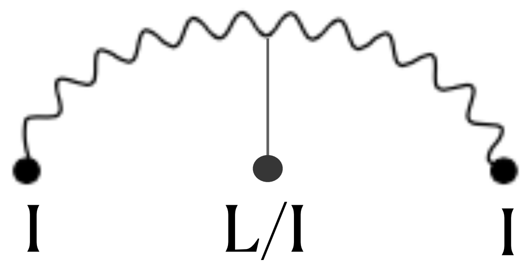
$$S^{\rm cons}_{(\gamma\text{-}3)} = S^{{\rm cons}(\gamma\text{-}3)}_{(\text{T-like})} + S^{{\rm cons}(\gamma\text{-}3)}_{(\text{M-like})} =$$
$$\frac{16\pi^2 G^2}{70}\int \frac{d\omega_1 d\omega_2 d\omega_3}{(2\pi)^3}\,\delta\!\!\!{}^-(\omega_1+\omega_2+\omega_3)\Delta_{\rm ret}(\omega_1)\Delta_{\rm ret}(\omega_3)$$
$$\times\, I^{ij}_{({\rm N})}(\omega_1) I^{jk}_{({\rm N})}(\omega_3)\left[I^{ki}_{(0)}(\omega_2) - I^{ki}_{({\rm N})}(\omega_2)\right]\mathcal{F}(\omega_1,\omega_2,\omega_3)\,.$$

sign flip between tail and memory!

γ-3 prescription

In principle unrelated (?) to singular behavior of conservative sector!

Change the sign of the memory force (does not complete a well defined action!)



$$S^{\rm cons}_{\rm PBloc} = S^{\rm cons}_{(\text{T-like})} + S^{\rm cons}_{(\text{M-like}){\rm loc}} =$$
$$\frac{16\pi^2 G^2}{70}\int \frac{d\omega_1 d\omega_2 d\omega_3}{(2\pi)^3}\,\delta\!\!\!{}^-(\omega_1+\omega_2+\omega_3)\Delta_{\rm ret}(\omega_1)\Delta_{\rm ret}(\omega_3)$$
$$\times\, I^{ij}_{({\rm N})}(\omega_1) I^{jk}_{({\rm N})}(\omega_3)\left[I^{ki}_{(0)}(\omega_2) + I^{ki}_{({\rm N})}(\omega_2)\right]\mathcal{F}(\omega_1,\omega_2,\omega_3)\,,$$

same-sign prescription for tail and memories

(Real) Feynman/PB

Consistent conservative sector across regions

(Equivalent to Ret/Adv symmetric routing)

Most likely singular at g=3 SO WHAT! (v~0.94!)

$$\tilde{\chi}^{(5,\nu^2){\rm cons}}_{j({\rm tot}){\rm PB}} = \left(\frac{48699841}{30240} - \frac{224057}{1440}\pi^2 + \frac{1408}{45}\log(2v_\infty) + \frac{9728}{270}\right)v_\infty^5$$

$$\tilde{\chi}^{(6,\nu^2){\rm cons}}_{j({\rm tot}){\rm PB}} = \pi\left(\frac{96731149}{26880} - \frac{10812865}{32768}\pi^2 + \frac{201}{2}\log\left(\frac{v_\infty}{2}\right) + \frac{2817}{16}\zeta(3) + \frac{630661}{8960}\right)v_\infty^4$$

VS.

$$\tilde{\chi}^{(5,\nu^2){\rm cons}}_{j({\rm tot})(\gamma\text{-}3)} = \left(\frac{9522061}{6048} - \frac{224057}{1440}\pi^2 + \frac{1408}{45}\log(2v_\infty)\right)v_\infty^5\,,$$

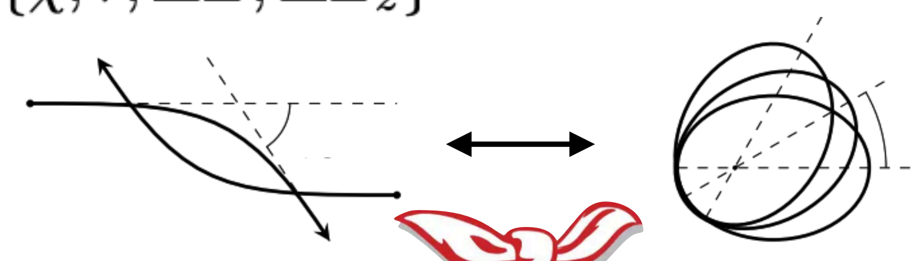
$$\tilde{\chi}^{(6,\nu^2){\rm cons}}_{j({\rm tot})(\gamma\text{-}3)} = \pi\left(\frac{47419583}{13440} - \frac{10812865}{32768}\pi^2 + \frac{2721}{2560}\pi^2 + \frac{201}{2}\log\left(\frac{v_\infty}{2}\right) + \frac{2817}{16}\zeta(3)\right)v_\infty^4$$

extra pi^2

WEFT approach to GW physics 5PN

Alternative (non-H) B2B approach to 2-body problem!

$\{\chi, \tau, \Delta E, \Delta \boldsymbol{L}_z\}$



FT, M and 2RR effects encapsulated in the (isotropic) structure:

$$\boldsymbol{a}^{2.5\mathrm{PN}}_{(\mathrm{RR,iso})} = -\frac{8}{5}\frac{G^2M^2\nu}{r^3}\left[\boldsymbol{v}^2\boldsymbol{v} - 3\,\boldsymbol{v}^2(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{n}\right] - \frac{8G^3M^3\nu}{5r^4}\left[3\boldsymbol{v} - \frac{17}{3}(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{n}\right]$$

Dissipation comes from iterated 'linear' force only!

$$\delta\boldsymbol{a}_{(\mathrm{iso})} = \frac{6259}{630}\frac{G^6M^6\nu^2}{r^7}\boldsymbol{n} + \frac{G^5M^5\nu^2}{r^6}\left[\frac{50783}{6300}\boldsymbol{v}^2\boldsymbol{n} + \frac{3431}{42}(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v}\right] + \frac{G^4M^4\nu^2}{r^5}\left[-\frac{8977}{175}\boldsymbol{v}^4\boldsymbol{n} + \frac{12148}{105}\boldsymbol{v}^2(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v}\right],$$

'Energy' conservation! $-\frac{d}{dt}\Delta\hat{E} = \boldsymbol{v}\cdot\Delta\boldsymbol{a}$

$$\delta\hat{E}_{(\mathrm{iso})} = -\frac{13\nu^2}{378}\left(\frac{GM}{r}\right)^6 + \frac{6389\nu^2}{1260}\left(\frac{GM}{r}\right)^5\boldsymbol{v}^2 + \frac{33809\nu^2}{2100}\left(\frac{GM}{r}\right)^4\boldsymbol{v}^4,$$

$$\delta\hat{\boldsymbol{L}}^i_{(\mathrm{iso})} = \varepsilon^{ijk}\boldsymbol{r}^j\boldsymbol{v}^k\left[\frac{1669}{350}\frac{G^5M^5\nu^2}{r^5} + \frac{3037}{105}\frac{G^4M^4\nu^2}{r^4}\boldsymbol{v}^2\right],$$

Symplectic (local PB) conservative — FT+M+2RR — sector:

Local cons. EOM (`Dissipative'= Tot - Cons)

$$\delta\hat{H}_{(\mathrm{iso})} = -\frac{\nu^2}{50}\left(\frac{2973}{7}\hat{\boldsymbol{p}}^4\left(\frac{GM}{r}\right)^4 + \frac{7931}{9}\hat{\boldsymbol{p}}^2\left(\frac{GM}{r}\right)^5 - \frac{5758}{63}\left(\frac{GM}{r}\right)^6\right)$$

$$\tilde{\chi}^{(5,\nu^2)\mathrm{cons}}_{j(\mathrm{tot})\mathrm{PB}} = \left(\frac{48699841}{30240} - \frac{224057}{1440}\pi^2 + \frac{1408}{45}\log(2v_\infty) + \frac{9728}{270}\right)v_\infty^5$$

$$\tilde{\chi}^{(6,\nu^2)\mathrm{cons}}_{j(\mathrm{tot})\mathrm{PB}} = \pi\left(\frac{96731149}{26880} - \frac{10812865}{32768}\pi^2 + \frac{201}{2}\log\left(\frac{v_\infty}{2}\right) + \frac{2817}{16}\zeta(3) + \frac{630661}{8960}\right)v_\infty^4$$

To g-3 or not to g-3?

$$\bar{d}_5^{\mathrm{loc}} = \left(\frac{331054}{175} - \frac{63707}{512}\pi^2\right)\nu + \left(-\frac{403259}{63} + \frac{306545}{512}\pi^2\right)\nu^2 + \left(\frac{1069}{3} - \frac{205}{16}\pi^2\right)\nu^3,$$

$$a_6^{\mathrm{loc}} = \left(-\frac{1026301}{1575} + \frac{246367}{3072}\pi^2\right)\nu + \left(-\frac{1718599}{1575} + \frac{25911}{256}\pi^2\right)\nu^2 + 4\nu^3.$$

To pi^2 or not to pi^2?

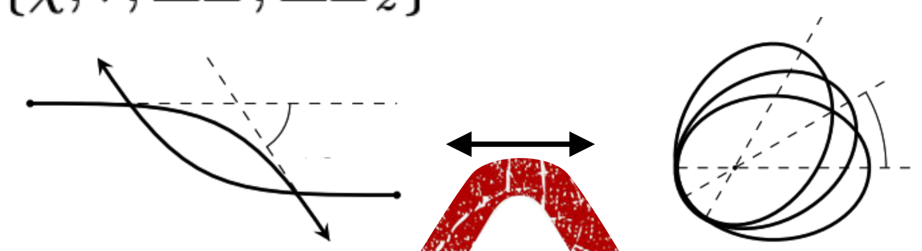
$$d_{5\mathrm{loc}}^{\nu^2(\gamma\text{-}3)} = -\frac{42915}{7} + \frac{306545}{512}\pi^2,$$

$$a_{6\mathrm{loc}}^{\nu^2(\gamma\text{-}3)} = -\frac{742757}{700} + \frac{25911}{256}\pi^2 - \frac{907}{400}\pi^2$$

WEFT approach to GW physics 5PN

Alternative (non-H) B2B approach to 2-body problem!

$\{\chi, \tau, \Delta E, \Delta \boldsymbol{L}_z\}$



Hyperbolic does not fully describe elliptic!

$$S_r^{(\rm nloc)} = -\frac{GE}{2\pi}\int_\omega \frac{dE}{d\omega}\log\left(\frac{4\omega^2}{\mu^2}e^{2\gamma_E}\right)$$

FT, M and 2RR effects encapsulated in the (isotropic) structure:

$$\boldsymbol{a}^{2.5\rm PN}_{(\rm RR,iso)} = -\frac{8}{5}\frac{G^2M^2\nu}{r^3}\left[\boldsymbol{v}^2\,\boldsymbol{v} - 3\,\boldsymbol{v}^2(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{n}\right] - \frac{8G^3M^3\nu}{5r^4}\left[3\boldsymbol{v} - \frac{17}{3}(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{n}\right]$$

Dissipation comes from iterated 'linear' force only!

$$\delta\boldsymbol{a}_{(\rm iso)} = \frac{6259}{630}\frac{G^6M^6\nu^2}{r^7}\boldsymbol{n} + \frac{G^5M^5\nu^2}{r^6}\left[\frac{50783}{6300}\boldsymbol{v}^2\boldsymbol{n} + \frac{3431}{42}(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v}\right] + \frac{G^4M^4\nu^2}{r^5}\left[-\frac{8977}{175}\boldsymbol{v}^4\boldsymbol{n} + \frac{12148}{105}\boldsymbol{v}^2(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v}\right],$$

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$$\delta\hat{\boldsymbol{L}}^i_{(\rm iso)} = \varepsilon^{ijk}\boldsymbol{r}^j\boldsymbol{v}^k\left[\frac{1669}{350}\frac{G^5M^5\nu^2}{r^5} + \frac{3037}{105}\frac{G^4M^4\nu^2}{r^4}\boldsymbol{v}^2\right],$$

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Local cons. EOM (`Dissipative'= Tot - Cons)

$$\delta\hat{H}_{(\rm iso)} = -\frac{\nu^2}{50}\left(\frac{2973}{7}\hat{\boldsymbol{p}}^4\left(\frac{GM}{r}\right)^4 + \frac{7931}{9}\hat{\boldsymbol{p}}^2\left(\frac{GM}{r}\right)^5 - \frac{5758}{63}\left(\frac{GM}{r}\right)^6\right)$$

$$\tilde{\chi}^{(5,\nu^2)\rm cons}_{j(\rm tot)PB} = \left(\frac{48699841}{30240} - \frac{224057}{1440}\pi^2 + \frac{1408}{45}\log(2v_\infty) + \frac{9728}{270}\right)v_\infty^5$$

$$\tilde{\chi}^{(6,\nu^2)\rm cons}_{j(\rm tot)PB} = \pi\left(\frac{96731149}{26880} - \frac{10812865}{32768}\pi^2 + \frac{201}{2}\log\left(\frac{v_\infty}{2}\right) + \frac{2817}{16}\zeta(3) + \frac{630661}{8960}\right)v_\infty^4$$

To g-3 or not to g-3?

$$\bar{d}_5^{\rm loc} = \left(\frac{331054}{175} - \frac{63707}{512}\pi^2\right)\nu + \left(-\frac{403259}{63} + \frac{306545}{512}\pi^2\right)\nu^2 + \left(\frac{1069}{3} - \frac{205}{16}\pi^2\right)\nu^3,$$

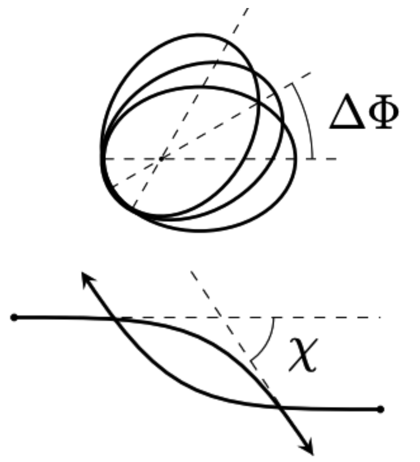
$$a_6^{\rm loc} = \left(-\frac{1026301}{1575} + \frac{246367}{3072}\pi^2\right)\nu + \left(-\frac{1718599}{1575} + \frac{25911}{256}\pi^2\right)\nu^2 + 4\nu^3.$$

To pi^2 or not to pi^2?

$$d^{\nu^2(\gamma\text{-}3)}_{5\rm loc} = -\frac{42915}{7} + \frac{306545}{512}\pi^2,$$

$$a^{\nu^2(\gamma\text{-}3)}_{6\rm loc} = -\frac{742757}{700} + \frac{25911}{256}\pi^2 - \frac{907}{400}\pi^2$$

WEFT approach to GW physics 5PM



Damour et al.,
Blanchet et al.,
Foffa RAP et al.
(2015-19')

0PN	1PN	2PN	3PN	4PN	5PN	6PN

$$G\left(1+v^2+v^4+v^6+v^8+v^{10}+\cdots\right)$$

$$G^2\left(1+v^2+v^4+v^6+v^8+\cdots\right)$$

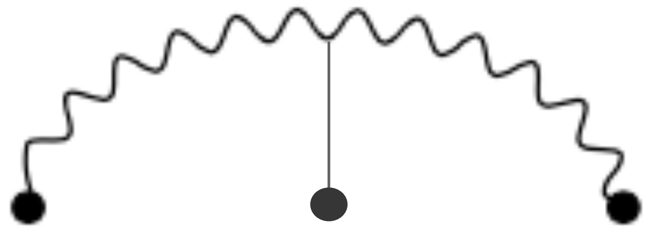
$$G^3\left(1+v^2+v^4+v^6+\cdots\right)$$

$$G^4\left(1+v^2+v^4+\cdots\right) \leftarrow \log v$$

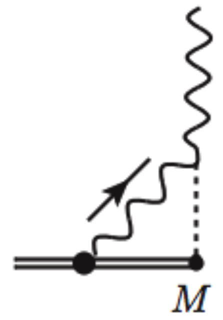
$$G^5\left(1\,\text{-}\right.$$

UNIVERSAL FORM
(IR/UV MIXING + OPTICAL THM)

Standard M-tail



$iA^{(1)}_{\text{tail}}(\boldsymbol{k}) =$



Hyperbolic does not fully describe elliptic!

$$\mathcal{S}_r^{(\text{nloc})} = -\frac{GE}{2\pi}\int_\omega \frac{dE}{d\omega}\log\left(\frac{4\omega^2}{\mu^2}e^{2\gamma_E}\right)$$

$$\bar{E}^{\text{iso,hyp}}_{\text{4PN(4PM)}} - \bar{E}^{\text{iso,ell}}_{\text{4PN(4PM)}} = \nu x^5\left[\frac{37933}{45}+\frac{1036\gamma_E}{45}-\frac{113847608\ln 2}{45} - \frac{1472499\ln 3}{20}+\frac{13671875\ln 5}{12}\right] \simeq 14.94\nu x^5\,,$$

Energetics and scattering of gravitational two-body systems at fourth post-Minkowskian order

Mohammed Khalil,[1,2,*] Alessandra Buonanno,[1,2,†] Jan Steinhoff,[1,‡] and Justin Vines[1,§]

WEFT approach to GW physics 5PM

PHYSICAL REVIEW LETTERS 132, 221401 (2024)

Local in Time Conservative Binary Dynamics at Fourth Post-Minkowskian Order

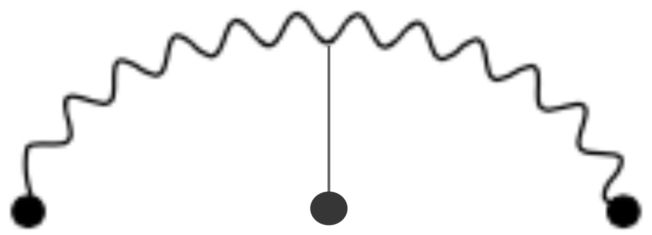
Christoph Dlapa,[1,*] Gregor Kälin,[1,†] Zhengwen Liu,[2,3,‡] and Rafael A. Porto[1,§]

$$\frac{1}{\pi\Gamma}\chi^{(4)(n\mathrm{SF})}_{b(\mathrm{nloc})} = \frac{\nu}{(\gamma^2-1)^2}\Bigg\{h_1 + \frac{\pi^2 h_2}{\sqrt{\gamma^2-1}} + h_3\log\left(\frac{\gamma+1}{2}\right) + \frac{h_4\,\mathrm{arccosh}(\gamma)}{\sqrt{\gamma^2-1}} + h_5\log\left(\frac{\gamma-1}{8}\right) + h_6\log^2\left(\frac{\gamma+1}{2}\right)$$
$$+ h_7\,\mathrm{arccosh}(\gamma)^2 + \frac{h_8\log(2)\,\mathrm{arccosh}(\gamma)}{\sqrt{\gamma^2-1}} + h_9\log\left(\frac{\gamma-1}{8}\right)\log\left(\frac{\gamma+1}{2}\right)$$
$$+ \frac{h_{10}\log\left(\frac{\gamma^2-1}{16}\right)\mathrm{arccosh}(\gamma)}{\sqrt{\gamma^2-1}} + h_{11}\mathrm{Li}_2\left(\frac{\gamma-1}{\gamma+1}\right) + \frac{h_{12}\left[\mathrm{arccosh}(\gamma)^2 + 4\mathrm{Li}_2\left(\sqrt{\gamma^2-1}-\gamma\right)\right]}{\sqrt{\gamma^2-1}}\Bigg\}.$$

nSF order (expanded in Mass ratio)

$$\frac{1}{\pi\Gamma}\chi^{(4)\log}_{b(\mathrm{nloc})} = -2\nu\chi_{2\epsilon}(\gamma) = \frac{-2\nu}{(\gamma^2-1)^2}\left(h_5 + h_9\log\left(\frac{\gamma+1}{2}\right) + \frac{h_{10}\,\mathrm{arccosh}(\gamma)}{\sqrt{\gamma^2-1}}\right),$$

Integration problem depends on two scales (velocity and mass ratio)! EXACT SOLUTION ITERATED ELLPTICS



Apply B2B!

$$\chi^{(4)}_{b(\mathrm{loc})} = \chi^{(4)\mathrm{cons}}_{b(\mathrm{tot})} - \chi^{(4)}_{b(\mathrm{nloc})}$$
$$\chi^{(4)\log}_{b(\mathrm{loc})} = -\chi^{(4)\log}_{b(\mathrm{nloc})}$$

$$i^{4\mathrm{PM}}_{r(\mathrm{loc})} = \frac{2v_\infty^4}{3(\Gamma j)^3}\left(\frac{\chi^{(4)}_{b(\mathrm{loc})}}{\pi\Gamma} + \frac{\chi^{(4)\log}_{b(\mathrm{loc})}}{2\pi\Gamma}\log\frac{j^2}{v_\infty^2}\right)$$

LOGS ARE UNIVERSAL!

$$i^{4\mathrm{PM}}_{r(\log)} = -\frac{E}{(2\pi)M^2\nu}\Delta E_{\mathrm{ell}}(j)\log(-\mathcal{E}) = \frac{2\nu}{3}\frac{(1-\gamma^2)^2}{(\Gamma j)^3}\chi_{2\epsilon}(\gamma)\log(-\mathcal{E}) + \cdots,$$

WEFT approach to GW physics *5PM*

PHYSICAL REVIEW LETTERS **135**, 251401 (2025)

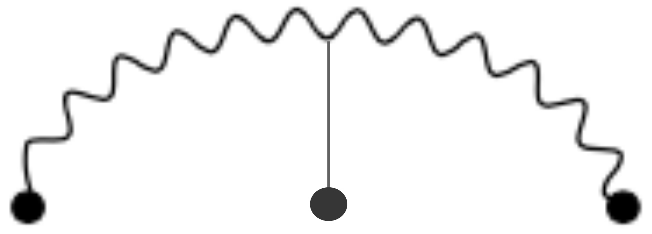
Local-in-Time Conservative Binary Dynamics at Fifth Post-Minkowskian and First Self-Force Orders

Christoph Dlapa[1,*] Gregor Kälin[1,†] Zhengwen Liu[2,3,‡] and Rafael A. Porto[1,§]

$$
\begin{aligned}
\frac{\chi^{(5)(1\mathrm{SF})}_{(\mathrm{nloc})}}{\Gamma\nu} &= \frac{\mathcal{G}_1(x)h_{13}(x)}{25200(x-1)^6x^3(x+1)^6\,(x^2+1)^7} + \frac{\mathcal{G}_2(x)h_{14}(x)}{90(x-1)^6x^3(x+1)^6\,(x^2+1)^7} + \frac{\mathcal{G}_3(x)h_{15}(x)}{12600(x-1)^7x^3(x+1)^7\,(x^2+1)^8} \\
&+ \frac{\mathcal{G}_4(x)h_{16}(x)}{1680(x-1)^7x^5(x+1)^7} + \frac{\mathcal{G}_5(x)h_{17}(x)}{30(x-1)^7x^3(x+1)^7\,(x^2+1)^8} + \frac{\mathcal{G}_6(x)h_{18}(x)}{420(x-1)^8x^3(x+1)^8} \\
&+ \frac{\mathcal{G}_7(x)h_{19}(x)}{336(x-1)^5x^5(x+1)^5\,(x^2+1)^8} + \frac{\mathcal{G}_8(x)h_{20}(x)}{1680(x-1)^7x^5(x+1)^7} + \frac{\mathcal{G}_{10}(x)h_{21}(x)}{4(x-1)^5x^2(x+1)^5} + \frac{\mathcal{G}_{11}(x)h_{22}(x)}{2(x-1)^5x(x+1)^5} \\
&+ \frac{\mathcal{G}_{12}(x)h_{23}(x)}{4(x-1)^5x^2(x+1)^5} + \frac{\mathcal{G}_{13}(x)h_{24}(x)}{4(x-1)^8x^2(x+1)^8} + \frac{\mathcal{G}_{14}(x)h_{25}(x)}{(x-1)^8x(x+1)^8} + \frac{\mathcal{G}_{15}(x)h_{26}(x)}{(x-1)^8x(x+1)^8} \\
&+ \frac{\mathcal{G}_{16}(x)h_{27}(x)}{(x-1)^5x(x+1)^5} + \frac{\mathcal{G}_{17}(x)h_{28}(x)}{4(x-1)^8x^2(x+1)^8} + \frac{\mathcal{G}_{19}(x)h_{29}(x)}{2(x-1)^5x(x+1)^5} + \frac{\mathcal{G}_{20}(x)h_5(x)}{2(x-1)^5x(x+1)^5} \\
&+ \frac{\mathcal{G}_{22}(x)h_{30}(x)}{4(x-1)^5x^2(x+1)^5} + \frac{\mathcal{G}_{23}(x)h_{31}(x)}{4(x-1)^5x^2(x+1)^5} + \frac{\mathcal{G}_{24}(x)h_{22}(x)}{2(x-1)^5x(x+1)^5}\,.
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{\Gamma}\chi^{(5)\,\log}_{b(\mathrm{nloc})} = \nu\Bigg(&\frac{\mathcal{G}_1(x)h_1(x)}{90(x-1)^6x^3(x+1)^6\,(x^2+1)^7} + \frac{\mathcal{G}_3(x)h_2(x)}{30(x-1)^7x^3(x+1)^7\,(x^2+1)^8} + \frac{\mathcal{G}_4(x)h_3(x)}{4(x-1)^5x^2(x+1)^5} \\
&+ \frac{\mathcal{G}_6(x)h_4(x)}{(x-1)^8x(x+1)^8} + \frac{\mathcal{G}_7(x)h_5(x)}{2(x-1)^5x(x+1)^5} + \frac{\mathcal{G}_8(x)h_6(x)}{4(x-1)^5x^2(x+1)^5}\Bigg) \\
+\nu^2\Bigg(&\frac{\mathcal{G}_1(x)h_7(x)}{44100(x-1)^6x^3(x+1)^6\,(x^2+1)^8} + \frac{\mathcal{G}_3(x)h_8(x)}{210(x-1)^7x^3(x+1)^7\,(x^2+1)^9} + \frac{\mathcal{G}_4(x)h_9(x)}{4(x-1)^3x^2(x+1)^5} \\
&+ \frac{\mathcal{G}_6(x)h_{10}(x)}{(x-1)^8x^2(x+1)^8} + \frac{\mathcal{G}_7(x)h_{11}(x)}{4(x-1)^5x^2(x+1)^5} + \frac{\mathcal{G}_8(x)h_{12}(x)}{4(x-1)^5x^2(x+1)^5}\Bigg)
\end{aligned}
$$

Logs are SF-exact!



Answer depends on products of **MPLs** up to **weight 3** with y=1-x and **5 letters** (0,1,2,1+i,1-i)

$$
\begin{aligned}
\mathcal{G}_{20}(x) &= \frac{1}{2}G(2;1-x)G(1;1-x)^2 - G(1,2;1-x)G(1;1-x) + G(0,1,1;1-x) + G(1,1,2;1-x) \\
&+ G(1-i,1-i,1;1-x) + G(1-i,1+i,1;1-x) + G(1+i,1-i,1;1-x) + G(1+i,1+i,1;1-x)\,, \\
\mathcal{G}_{21}(x) &= \frac{3}{2}G(2;1-x)G(1;1-x)^2 - 3G(1,2;1-x)G(1;1-x) + 3G(0,1,1;1-x) + 3G(1,1,2;1-x) \\
&+ G(1-i,0,1;1-x) + G(1-i,2,1;1-x) + G(1+i,0,1;1-x) + G(1+i,2,1;1-x)\,, \\
\mathcal{G}_{22}(x) &= G(2;1-x)G(0,1;1-x) + G(0,1,1;1-x) - G(0,1,2;1-x) - G(0,2,1;1-x) \\
&+ \frac{1}{4}\left[-G(2;1-x)G(1;1-x)^2 + 2G(1,2;1-x)G(1;1-x) - 2G(1,1,2;1-x)\right]\,, \\
\mathcal{G}_{23}(x) &= -G(2;1-x)G(1;1-x)^2 + 2G(1,2;1-x)G(1;1-x) - G(2;1-x)G(0,1;1-x) \\
&+ G(0;1-x)(G(1;1-x)G(2;1-x) - G(1,2;1-x)) + G(2;1-x)G(1,2;1-x) - G(0,1,1;1-x) \\
&+ G(0,1,2;1-x) - 2G(1,1,2;1-x) - 2G(1,2,2;1-x)\,, \\
\mathcal{G}_{24}(x) &= -\frac{1}{2}G(2;1-x)G(1;1-x)^2 + G(1,2;1-x)G(1;1-x) \\
&+ G(2;1-x)(G(1-i,1;1-x) + G(1+i,1;1-x)) - G(1,1,2;1-x) - G(1-i,1,2;1-x) \\
&- G(1-i,2,1;1-x) - G(1+i,1,2;1-x) - G(1+i,2,1;1-x)\,, \\
\mathcal{G}_{25}(x) &= \frac{1}{4}\left[G(2;1-x)G(1;1-x)^2 + 2\left(G(2;1-x)^2 - G(1,2;1-x)\right)G(1;1-x)\right. \\
&\quad \left. -4G(2;1-x)G(1,2;1-x) + 2G(1,1,2;1-x) + 4G(1,2,2;1-x)\right]
\end{aligned}
$$

$$
G(a_1,\ldots,a_n;z) = \int_0^z \frac{dt}{t-a_1}G(a_2,\ldots,a_n;t)\,,
$$

$$
G(\cdot;z) = 1\,,\quad G(\underbrace{0,\ldots,0}_{n};z) = \tfrac{1}{n!}\log^n z\,,
$$

Derive local-in-time counterpart

$$
\chi^{(5)(1\mathrm{SF})}_{b(\mathrm{loc})} = \chi^{(5)(1\mathrm{SF})}_{b(\mathrm{even})} - \chi^{(5)}_{b(\mathrm{nloc})}\,,\qquad \chi^{(5)\,\log}_{b(\mathrm{loc})} = -\chi^{(5)\,\log}_{b(\mathrm{nloc})}\,,
$$

WEFT approach to GW physics *5PM*

PHYSICAL REVIEW LETTERS 132, 221401 (2024)

Local in Time Conservative Binary Dynamics at Fourth Post-Minkowskian Order

Christoph Dlapa,[1,*] Gregor Kälin,[1,†] Zhengwen Liu,[2,3,‡] and Rafael A. Porto[1,§]

All orders in velocity!

Known to 6PN order

$$\hat{H}^{\rm ell}_{\rm 4PM} = \sum_{i=1}^{i=4} \frac{\hat{c}_{i(\rm loc)}}{\hat{r}^i} + \sum_{i=1}^{i=4} \frac{\hat{c}_{i(\rm nloc)}}{\hat{r}^i} + \frac{4\nu^2}{3\hat{r}^4}\frac{(\gamma^2-1)}{\Gamma^2\xi}\chi_{2\epsilon}\log\left(\frac{\hat{r}}{e^{2\gamma_E}}\right)$$

Logs are universal (PN-exact) fixed by radiated flux

Perfect agreement with state of the art in PN!

Energetics and scattering of gravitational two-body systems at fourth post-Minkowskian order

Mohammed Khalil,[1,2,*] Alessandra Buonanno,[1,2,†] Jan Steinhoff,[1,‡] and Justin Vines[1,§]

$$\begin{aligned}
\hat{H}^{\rm ell,iso}_{\rm 6PN(4PM)} &= \hat{H}^{\rm ell,iso}_{\rm 5PN(4PM)} + \left[\frac{33}{2048} + \frac{429\nu^6}{2048} - \frac{3003\nu^5}{2048} + \frac{3003\nu^4}{1024} - \frac{1287\nu^3}{512} + \frac{2145\nu^2}{2048} - \frac{429\nu}{2048}\right]p^{14} \\
&+ \frac{Gp^{12}}{r}\left[\frac{273}{1024} - \nu^6 - \frac{3\nu^5}{2} + \frac{75\nu^4}{4} + \nu^3\left(-\frac{218307}{140} + \frac{10834496\ln 2}{3} + \frac{19775583\ln 3}{140} - \frac{138671875\ln 5}{84}\right)\right. \\
&\quad + \nu^2\left(\frac{10614711}{22400} - \frac{5417248}{5}\ln 2 + \frac{27734375\ln 5}{56} - \frac{59326749\ln 3}{1400}\right) \\
&\quad \left. + \nu\left(-\frac{1860381}{44800} + \frac{1354312\ln 2}{15} + \frac{19775583\ln 3}{5600} - \frac{27734375\ln 5}{672}\right)\right] \\
&+ \frac{G^2p^{10}}{r^2}\left[\frac{441}{256} + \frac{693\nu^6}{512} + \frac{17175\nu^5}{512} - \frac{2505\nu^4}{256}\right. \\
&\quad + \nu^3\left(\frac{1752882443}{134400} - \frac{50772177511\ln 2}{3780} + \frac{15140243287719\ln 3}{2867200} + \frac{15746212109375\ln 5}{3096576} - \frac{1065779114477\ln 7}{442368}\right) \\
&\quad + \nu^2\left(\frac{930216823}{107520} - \frac{188966394467\ln 2}{7560} + \frac{147239183828125\ln 5}{12386304} + \frac{484445052035\ln 7}{589824} - \frac{7125985899279\ln 3}{2293760}\right) \\
&\quad \left. + \nu\left(-\frac{29016839}{35840} + \frac{154094423\ln 2}{72} + \frac{527065116993\ln 3}{2293760} - \frac{96889010407\ln 7}{1769472} - \frac{12533579921875\ln 5}{12386304}\right)\right] \\
&+ \frac{G^3p^8}{r^3}\left[\frac{2805}{512} - \frac{19425\nu^5}{256} - \frac{168131\nu^4}{512} + \nu^3\left(-\frac{5539742599}{120960} + \frac{38790406370519\ln 2}{1786050} + \frac{1009279694921875\ln 5}{877879296}\right.\right. \\
&\quad \left. + \frac{453841966033589\ln 7}{89579520} - \frac{244047465883413\ln 3}{10035200}\right) + \nu^2\left(-\frac{180308862367}{2822400} + \frac{116606471572979\ln 2}{1071630}\right. \\
&\quad \left. + \frac{3680972377512689\ln 7}{358318080} - \frac{448065058976289\ln 3}{40140800} - \frac{1812791824897656625\ln 5}{3511517184}\right) + \nu\left(-\frac{3456473588783}{304819200}\right. \\
&\quad \left.\left. + \frac{369057536315537\ln 2}{9185400} + \frac{607401830370627\ln 3}{80281600} - \frac{1267373911442149\ln 7}{429981696} - \frac{44240036362654375\ln 5}{2341011456}\right)\right] \\
&+ \frac{G^4p^6}{r^4}\left[\frac{2275}{256} - \frac{105\nu^6}{128} + \frac{1855\nu^5}{32} + \left(\frac{146987}{192} - \frac{41\pi^2}{64}\right)\nu^4 + \nu^3\left(-\frac{74\ln r}{5} - \frac{25729\pi^2}{4096} + \frac{5104603957}{60480} + \frac{148\gamma_E}{5}\right.\right. \\
&\quad \left. - \frac{2348423027149\ln 2}{51030} + \frac{8674336284777\ln 3}{286720} + \frac{250707235071713\ln 7}{17915904} - \frac{2232609748046875\ln 5}{125411328}\right) \\
&\quad + \nu^2\left(\frac{197\ln r}{140} - \frac{197\gamma_E}{70} - \frac{104939\pi^2}{16384} + \frac{2714234991803}{16934400} - \frac{126132398166437\ln 2}{1071630} + \frac{763693932388383\ln 3}{8028160}\right. \\
&\quad \left. + \frac{2046237450111171875\ln 5}{3511517184} - \frac{4304025048065071\ln 7}{71663616}\right) + \nu\left(-\frac{5827\ln r}{1008} - \frac{2337139\pi^2}{25165824} + \frac{3571766093993}{76204800}\right. \\
&\quad + \frac{5827\gamma_E}{504} - \frac{616925145960877\ln 2}{3214890} + \frac{525414163807156625\ln 5}{585252864} + \frac{1554400159532395\ln 7}{107495424} \\
&\quad \left.\left. - \frac{144912376553769\ln 3}{4014080}\right)\right]. \qquad \text{(A6)}
\end{aligned}$$

Implemented in the "EOB gauge"

Fourth post-Minkowskian local-in-time conservative dynamics of binary systems

Donato Bini and Thibault Damour

Phys. Rev. D **110**, 064005 – Published 3 September 2024

effective one-body Hamiltonian (in energy gauge). Our computation capitalizes on the tutti frutti approach [D. Bini *et al.*, Novel approach to binary dynamics: Application to the fifth post-Newtonian level, Phys. Rev. Lett. **123**, 231104 (2019)] and on recent post-Minkowskian advances [Z. Bern *et al.*, Scattering amplitudes, the tail effect, and conservative binary dynamics at O(G4), Phys. Rev. Lett. **128**, 161103 (2022); C. Dlapa *et al.*, Conservative dynamics of binary systems at fourth post-Minkowskian order in the large-eccentricity expansion, Phys. Rev. Lett. **128**, 161104 (2022); C. Dlapa *et al.*, Local in time conservative binary dynamics at fourth post-Minkowskian order, **132**, 221401 (2024)].

PHYSICAL REVIEW LETTERS 128, 161104 (2022)

Conservative Dynamics of Binary Systems at Fourth Post-Minkowskian Order in the Large-Eccentricity Expansion

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PHYSICAL REVIEW LETTERS 128, 161103 (2022)

Scattering Amplitudes, the Tail Effect, and Conservative Binary Dynamics at $\mathcal{O}(G^4)$

Zvi Bern,[1] Julio Parra-Martinez,[2] Radu Roiban,[3] Michael S. Ruf,[1] Chia-Hsien Shen,[4] Mikhail P. Solon,[1] and Mao Zeng[5]

WEFT approach to GW physics 5PM

PHYSICAL REVIEW LETTERS 135, 251401 (2025)

Local-in-Time Conservative Binary Dynamics at Fifth Post-Minkowskian and First Self-Force Orders

Christoph Dlapa[1,*] Gregor Kälin[1,†] Zhengwen Liu[2,3,‡] and Rafael A. Porto[1,§]

MOST ACCURATE
BOUND FROM SCATTERING!

All orders in velocity!

Known to 6PN order

$$\hat{H}^{\rm ell}_{5\rm PM} = \sum_{i=1}^{i=5} \frac{\hat{c}_{i(\rm loc)}}{\hat{r}^i} + \sum_{i=1}^{i=5} \frac{\hat{c}_{i(\rm nloc)}}{\hat{r}^i} - \sum_{i=4}^{i=5} G\left(\hat{H}\frac{dE_{\rm src}}{dt}\right)\bigg|_{(i-1)\rm PM} \log\left(\frac{\hat{r}}{e^{2\gamma_E}}\right)$$

$$G\frac{dE_{\rm src}}{dt}\bigg|_{G^4}(\hat{r},\boldsymbol{p}^2) = \frac{\nu^3}{\Gamma^3 x^4 (x^2-1)^2 \xi \hat{r}^5}\left[\frac{\log(2) d_1(x)}{256\Gamma^7 x^4 \xi^2} + \frac{d_2(x)\log^2(x)}{(x^2-1)^4} + \frac{d_3(x){\rm Li}_2(x)}{256(x^2-1)} + \frac{d_4(x){\rm Li}_2(-x)}{256(x^2-1)} + \frac{d_6(x)\log(x+1)}{256\Gamma^7 x^4 \xi^2} + \frac{d_7(x)\log^2(x^2+1)}{1024(x^2-1)} + \frac{d_7(x){\rm Li}_2\left(\frac{1}{x^2+1}\right)}{512(x^2-1)} + \frac{\pi^2 d_8(x)}{2048(x^2-1)} + \frac{d_9(x)}{15052800\Gamma^9 x^5 (x^2-1)^2 (x^2+1)^8 \xi^2} + \log(x)\left(\frac{d_3(x)\log(1-x)}{256(x^2-1)} + \frac{d_4(x)\log(x+1)}{256(x^2-1)}\right) + \frac{d_5(x)}{71680\Gamma^9 x^5 (x^2-1)^3 (x^2+1)^9 \xi^2}\right],$$

$$\frac{\hat{c}^{\rm 6PN(e^8,1SF)}_{5(\rm nloc)}}{\nu} = \left(-383 + \frac{5421492}{5}\log(2) + \frac{631071}{20}\log(3) - \frac{1953125}{4}\log(5)\right) + \hat{\boldsymbol{p}}^2\left(-\frac{237374547}{28000} + \frac{274846146629\log(2)}{9450} + \frac{268234748343\log(3)}{71680} - \frac{5343159203125\log(5)}{387072} - \frac{96889010407\log(7)}{92160}\right) + \hat{\boldsymbol{p}}^4\left(-\frac{12514611952561}{197568000} + \frac{345079750666673\log(2)}{1190700} + \frac{2379761187413283\log(3)}{40140800} - \frac{7512099894456875\log(5)}{55738368} - \frac{334452591435529\log(7)}{13271040}\right).$$

$$x = \gamma - \sqrt{\gamma^2-1}, \qquad \gamma = \nu(\hat{E}_1\hat{E}_2 + \hat{\boldsymbol{p}}^2)$$

1SF

↓

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cLoc[5] = -((9+72 x^2+241 x^4+408 x^6+1670 x^8-4288 x^10+1670 x^12+408 x^14+241 x^16+72 x^18+9 x^20)/(16 x (1+x^2)^9))+nu (1/(403200 (-1+x)^6 x^4 (1+x)^8 (1+x^2)^8) (1772544+3545088 x-28675032 x^2+47485008 x^3+1589453499 x^4+3580425510
x^5+3154123329 x^6+3068444508 x^7-46763258641 x^8-97678763390 x^9-279682880819 x^10-463854601448 x^11-728997957289 x^12-994574833770 x^13-1139393156099 x^14-1281827114908 x^15-1225573580653 x^16-1166502162238 x^17-1124629435079
x^18-1081208419920 x^19-1124629435079 x^20-1166502162238 x^21-1225573580653 x^22-1281827114908 x^23-1139393156099 x^24-994574833770 x^25-728997957289 x^26-463854601448 x^27-279682880819 x^28-97678763390 x^29-46763258641 x^30+3068444508
x^31+3154123329 x^32+3580425510 x^33+1589453499 x^34+47485008 x^35-28675032 x^36+3545088 x^37+1772544 x^38)-1/(2880 (-1+x)^6 x^4 (1+x)^8 (1+x^2)^10) (45000+50040 x+917483 x^2+809890 x^3+10562067 x^4-5340320 x^5+48299403 x^6-38745070
x^7+85495907 x^8-28667880 x^9-87519692 x^10+203732520 x^11-610962452 x^12+638380160 x^13-820805828 x^14+916869320 x^15-343886292 x^16+38494320 x^17+1044694922 x^18-174337700 x^19+1182767418 x^20-1970028480 x^21+1182767418
x^22-174337700 x^23+1044694922 x^24+38494320 x^25-343886292 x^26+916869320 x^27-820805828 x^28+638380160 x^29-610962452 x^30+203732520 x^31-87519692 x^32-28667880 x^33+85495907 x^34-38745070 x^35+48299403 x^36-5340320 x^37+10562067
x^38+809890 x^39+917483 x^40+50040 x^41+45000 x^42)-((3025+7400 x^2+6455 x^4-5304 x^6+6455 x^8+7400 x^10+3025 x^12) EllipticE[1-x^2]^2)/(2 (-1+x)^4 (1+x)^4 (1+x^2)^3)+(2 (375+2638 x^2+1721 x^4-1782 x^6+1721 x^8+2638 x^10+375 x^12) EllipticE[1-x^2]
EllipticK[1-x^2])/((-1+x)^4 (1+x)^4 (1+x^2)^2)-(x^2 (6495+17088 x^2+11961 x^4-5024 x^6+11961 x^8+17088 x^10+6495 x^12) EllipticK[1-x^2]^2)/(4 (-1+x)^4 (1+x)^4 (1+x^2)^3)-1/(134400 (-1+x)^7 x^4 (1+x)^7 (1+x^2)^9) (386304-10100111 x^2-102870446 x^4+10336462061
x^6+33669530456 x^8-101134766400 x^10-763774550136 x^12-1911666112056 x^14-2675671656024 x^16-2538593976294 x^18-2233047721908 x^20-2538593976294 x^22-2675671656024 x^24-1911666112056 x^26-763774550136 x^28-101134766400
x^30+33669530456 x^32+10336462061 x^34-102870446 x^36-10100111 x^38+386304 x^40) G[{1},1-x]+1/(4480 (-1+x)^8 x^4 (1+x)^8 (1+x^2)^9) (32256+766465 x^2-4619223 x^4+4254775 x^6+519193479 x^8+1112703676 x^10-8590667940 x^12-51812877788
x^14-131786066844 x^16-203095659146 x^18-222735831130 x^20-206430590630 x^22-179150192326 x^24-131654865124 x^26-64809223108 x^28-14166809020 x^30+3043331716 x^32+2463172529 x^34+407633305 x^36-11603193 x^38-3414185 x^40+32256
x^42+38400 x^44) G[{1},1-x]^2-(16 (3-9 x^2-342 x^4+1071 x^6+2709 x^8-6126 x^10-22540 x^12-34998 x^14+10821 x^16+8039 x^18-2334 x^20-105 x^22+35 x^24) G[{1},1-x]^3)/((-1+x)^9 x^2 (1+x)^9 (1+x^2))+\[Pi]^2 ((256-768 x^2+185856 x^4-182057 x^6-16372784
x^8-182057 x^10+185856 x^12-768 x^14+256 x^16)/(896 (-1+x)^2 x^6 (1+x)^2)-1/(192 (-1+x)^3 x^5 (1+x)^3 (1+x^2)^3) (-125-95 x^2+34543 x^4+73605 x^6-12960 x^7+67666 x^8-67968 x^9+76254 x^10-140640 x^11+195746 x^12-89856 x^13-10706 x^14+45216 x^15-139525
x^16+59520 x^17-70063 x^18+10080 x^19-2145 x^20+125 x^22)-((5120+78015 x^2-974436 x^4-2891490 x^6-974436 x^8+78015 x^10+5120 x^12) G[{1},1-x])/(64 (-1+x)^3 x^2 (1+x)^3 (1+x^2))-1/(3584 (-1+x)^5 x^6 (1+x)^5 (1+x^2)^9) (18432+55296 x^2+136815 x^3+1888042
x^4+2827965 x^5-12280320 x^6+9622834 x^7-255574928 x^8-67186338 x^9-783666432 x^10-525752714 x^11-93751784 x^12-1393491270 x^13+2950361856 x^14-1512534566 x^15+4322430224 x^16+376545174 x^17+287695872 x^18+2845590948 x^19-2969349508
x^20+2845590948 x^21+287695872 x^22+376545174 x^23+4322430224 x^24-1512534566 x^25+2950361856 x^26-1393491270 x^27-93751784 x^28-525752714 x^29-783666432 x^30-67186338 x^31-255574928 x^32+9622834 x^33-12280320 x^34+2827965 x^35+1888042
x^36+136815 x^37+55296 x^38+18432 x^40) G[{0,1},1-x]-1/(64 (-1+x)^6 x^3 (1+x)^6 (1+x^2)) 3 (525-10240 x+5655 x^2-122880 x^3-31403 x^4+2383128 x^5+8124 x^6-505496 x^7+95358 x^8-9495144 x^9-97718 x^10+9014952 x^11+41310 x^12+352008 x^13+9948
x^14-2202888 x^15-9803 x^16+127560 x^17+2055 x^18+248 x^19+525 x^20) G[{1},1-x] G[{0,1},1-x]-1/(3584 (-1+x)^5 x^6 (1+x)^5 (1+x^2)^9) (18432+55296 x^2-136815 x^3+1888042 x^4-2827965 x^5-12280320 x^6-9622834 x^7-255574928 x^8+67186338 x^9-783666432
x^10+525752714 x^11-93751784 x^12+1393491270 x^13+2950361856 x^14+1512534566 x^15+4322430224 x^16-376545174 x^17+287695872 x^18-2845590948 x^19-2969349508 x^20-2845590948 x^21+287695872 x^22-376545174 x^23+4322430224 x^24+1512534566
x^25+2950361856 x^26+1393491270 x^27-93751784 x^28+525752714 x^29-783666432 x^30+67186338 x^31-255574928 x^32-9622834 x^33-12280320 x^34-2827965 x^35+1888042 x^36-136815 x^37+55296 x^38+18432 x^40) (G[{1},1-x] G[{2},1-x]-G[{1,2},1-x])+1/(64
(-1+x)^6 x^3 (1+x)^6 (1+x^2)) 3 (525+10240 x+5655 x^2+122880 x^3-31403 x^4-2383128 x^5+8124 x^6+505496 x^7+95358 x^8+9495144 x^9-97718 x^10-9014952 x^11+41310 x^12-352008 x^13+9948 x^14+2202888 x^15-9803 x^16-127560 x^17+2055 x^18-248 x^19+525
x^20) G[{1},1-x] (G[{1},1-x] G[{2},1-x]-G[{1,2},1-x])-1/(1792 (-1+x)^3 x^6 (1+x)^3 (1+x^2)^7) (6144+18432 x^2-2511833 x^4-1479744 x^6+265925142 x^8+1617339584 x^10+4601782857 x^12+8049870720 x^14+9611415284 x^16+8049870720 x^18+4601782857
x^20+1617339584 x^22+265925142 x^24-1479744 x^26-2511833 x^28+18432 x^30+6144 x^32) (G[{1-I,1},1-x]+G[{1+I,1},1-x])+(9 (175-750 x+1005 x^2-460 x^3-4065 x^4+9870 x^5-16571 x^6+17496 x^7-16571 x^8+9870 x^9-4065 x^10-460 x^11+1005 x^12-750 x^13+175
x^14) (G[{0,0,1},1-x]+1/2 G[{0,1,1},1-x]))/(128 (-1+x)^3 x^3 (1+x)^3 (1+x^2))-1/(16 (-1+x)^3 x^2 (1+x)^3 (1+x^2)) 3 (799+1800 x+2982 x^2-5400 x^3-101295 x^4-22512 x^5-271916 x^6-22512 x^7-101295 x^8-5400 x^9+2982 x^10+1800 x^11+799 x^12) (-G[{0,1,1},1-x]+G[{0,1-
I,1},1-x]+G[{0,1+I,1},1-x])-1/(128 (-1+x)^3 x^3 (1+x)^3 (1+x^2)) 3 (525+11342 x+10215 x^2+25356 x^3-33795 x^4-95214 x^5-139761 x^6-237272 x^7-139761 x^8-95214 x^9-33795 x^10+25356 x^11+10215 x^12+11342 x^13+525 x^14) (G[{2},1-x] G[{0,1},1-x]+G[{0,1,1},1-x]-
G[{0,1,2},1-x]-G[{0,2,1},1-x]+1/4 (-G[{1},1-x]^2 G[{2},1-x]+2 G[{1},1-x] G[{1,2},1-x]-2 G[{1,1,2},1-x]))+1/(128 (-1+x)^3 x^3 (1+x)^3 (1+x^2)) 3 (525-11342 x+10215 x^2-25356 x^3-33795 x^4+95214 x^5-139761 x^6+237272 x^7-139761 x^8+95214 x^9-33795 x^10-25356
x^11+10215 x^12-11342 x^13+525 x^14) (1/2 G[{1},1-x]^2 G[{2},1-x]-G[{1},1-x] G[{1,2},1-x]-1/2 G[{0,1,1},1-x]+G[{0,2,1},1-x]+G[{1,1,2},1-x])+1/(64 (-1+x)^6 x^2 (1+x)^6 (1+x^2)) 3 (1-3 x^2-9 x^4+3 x^6) (1823+6054 x^2-202671 x^4-544300 x^6-202671 x^8+6054 x^10+1823 x^12)
(G[{1},1-x]^2 G[{2},1-x]+G[{1},1-x] (-2 G[{1,2},1-x]+G[{1-I,1},1-x]+G[{1+I,1},1-x])+2 G[{0,1,1},1-x]+2 G[{1,1,2},1-x]-G[{1,1-I,1},1-x]-G[{1,1+I,1},1-x])+(3 (-1536-18432 x^2+357027 x^4+652147 x^6-639754 x^8-367794 x^10+20615 x^12+2335 x^14) (G[{1,1-I,1},1-x]+G[{1,1+I,1},1-
x]))/(8 (-1+x)^4 x^2 (1+x)^4 (1+x^2))-1/(512 (-1+x)^3 x^3 (1+x)^3 (1+x^2)) 9 (175+750 x+1005 x^2+460 x^3-4065 x^4-9870 x^5-16571 x^6-17496 x^7-16571 x^8-9870 x^9-4065 x^10+460 x^11+1005 x^12+750 x^13+175 x^14) (G[{1},1-x]^2 G[{2},1-x]+2 G[{1},1-x] (G[{2},1-x]^2-
G[{1,2},1-x])-4 G[{2},1-x] G[{1,2},1-x]+2 G[{1,1,2},1-x]+4 G[{1,2,2},1-x])-1/((-1+x)^3 x^2 (1+x)^3 (1+x^2)) 576 (1+13 x^2-189 x^4-554 x^6-189 x^8+13 x^10+x^12) (1/2 G[{1},1-x]^2 G[{2},1-x]-G[{1},1-x] G[{1,2},1-x]+G[{0,1,1},1-x]+G[{1,1,2},1-x]+G[{1-I,1-I,1},1-x]+G[{1-I,1+I,1},1-x]
+G[{1+I,1-I,1},1-x]+G[{1+I,1+I,1},1-x])-1/(16 (-1+x)^3 x^2 (1+x)^3 (1+x^2)) 3 (799-1800 x+2982 x^2+5400 x^3-101295 x^4+22512 x^5-271916 x^6+22512 x^7-101295 x^8+5400 x^9+2982 x^10-1800 x^11+799 x^12) (-(1/2) G[{1},1-x]^2 G[{2},1-x]+G[{1},1-x] G[{1,2},1-x]+G[{2},1-x]
(G[{1-I,1},1-x]+G[{1+I,1},1-x])-G[{1,1,2},1-x]-G[{1-I,1,2},1-x]-G[{1-I,2,1},1-x]-G[{1+I,1,2},1-x]-G[{1+I,2,1},1-x])+1/(32 (-1+x)^3 x^2 (1+x)^3 (1+x^2)) 3 (4321+73818 x^2-958545 x^4-2859476 x^6-958545 x^8+73818 x^10+4321 x^12) (3/2 G[{1},1-x]^2 G[{2},1-x]-3 G[{1},1-x] G[{1,2},1-
x]+3 G[{0,1,1},1-x]+3 G[{1,1,2},1-x]+G[{1-I,0,1},1-x]+G[{1-I,2,1},1-x]+G[{1+I,0,1},1-x]+G[{1+I,2,1},1-x])+1/(16 (-1+x)^2 x^5 (1+x)^2 (1+x^2)^3) (125-1460 x^2-3360 x^3-2163 x^4-23200 x^5-4584 x^6-42592 x^7-9546 x^8-35296 x^9-1608 x^10-35296 x^11-9546 x^12-42592
x^13-4584 x^14-23200 x^15-2163 x^16-3360 x^17-1460 x^18+125 x^20) Log[2]^2-((1+x^4) (25-110 x^2+3799 x^4+8956 x^6+3799 x^8-110 x^10+25 x^12) Log[1+1/2 (-1+x)])/(16 (-1+x)^2 x^4 (1+x) (1+x^2)^2)-((1+x^4) (-60+240 x-304 x^2-217 x^3+788 x^4-889 x^5+446 x^7-788
x^8+198 x^9+304 x^10-325 x^11+60 x^12+35 x^13) Log[1-(-1+x)/x])/(4 (-1+x)^5 x^2 (1+x)^4 (1+x^2)^2)+1/(2 (-1+x)^8 x^2 (1+x)^8 (1+x^2)^3) (-105-95 x^2-105 x^3+1444 x^4-1190 x^5-1892 x^6+2905 x^7-1525 x^8-16 x^9+5117 x^10-2320 x^11-2944 x^12-21732 x^13-2944
x^14+16454 x^15+5117 x^16+1456 x^17-1525 x^18-11373 x^19-1892 x^20+714 x^21+1444 x^22+4609 x^23-95 x^24-1760 x^25-105 x^26+70 x^27) Log[x]^2+1/(48 (-1+x)^4 x^4 (1+x)^4 (1+x^2)^3) (1275-4437 x-2309 x^2+20139 x^3-55377 x^4-127097 x^5+441039 x^6-516513
x^7+836342 x^8-552786 x^9+7830 x^10+111718 x^11+7830 x^12-548562 x^13+836342 x^14-547530 x^15+441039 x^16-123353 x^17-55377 x^18+32271 x^19-2309 x^20-4917 x^21+1275 x^22+75 x^23) Log[1+x]+1/(16 (-1+x)^2 x^5 (1+x)^2 (1+x^2)^3) (125-1460 x^2-3360
x^3-2163 x^4-23200 x^5-4584 x^6-42592 x^7-9546 x^8-35296 x^9-1608 x^10-35296 x^11-9546 x^12-42592 x^13-4584 x^14-23200 x^15-2163 x^16-3360 x^17-1460 x^18+125 x^20) Log[1+x]^2+Log[2] (-(1/(48 (-1+x)^4 x^4 (1+x)^4 (1+x^2)^3))(-75-4437 x-2419 x^2+20139
x^3+3033 x^4-127097 x^5+912705 x^6-516513 x^7+1502218 x^8-552786 x^9+42138 x^10+111718 x^11+42138 x^12-548562 x^13+1502218 x^14-547530 x^15+912705 x^16-123353 x^17+3033 x^18+32271 x^19-2419 x^20-4917 x^21-75 x^22+75 x^23)-(2 (-35-105 x-15
x^2-620 x^3-43 x^4-606 x^5-813 x^6+228 x^7-2014 x^8-3058 x^10-228 x^11-131 x^12+606 x^13+1499 x^14+620 x^15+735 x^16+105 x^17+35 x^18) Log[x])/((-1+x)^3 x^3 (1+x)^3 (1+x^2)^3)-1/(8 (-1+x)^2 x^5 (1+x)^2 (1+x^2)^3) (125-1460 x^2-3360 x^3-2163 x^4-23200
x^5-4584 x^6-42592 x^7-9546 x^8-35296 x^9-1608 x^10-35296 x^11-9546 x^12-42592 x^13-4584 x^14-23200 x^15-2163 x^16-3360 x^17-1460 x^18+125 x^20) Log[1+x])+Log[x] (1/(24 (-1+x)^7 x^4 (1+x)^7 (1+x^2)^5) (-375+840 x-2060 x^2+12618 x^3-23674 x^4+72426
x^5+140086 x^6+14562 x^7+251506 x^8-289938 x^9-770440 x^10+1017736 x^11-3248078 x^12+6107036 x^13-8383702 x^14+12687484 x^15-13501496 x^16+13533124 x^17-12162316 x^18+6533018 x^19-2654150 x^20+307618 x^21+1609610 x^22-466646 x^23+37318
x^24+222566 x^25-532448 x^26+153844 x^27-79858 x^28+2040 x^29-1898 x^30+3096 x^31+375 x^32)+1/(16 (-1+x)^3 x^5 (1+x)^3 (1+x^2)^3) (-125-95 x^2+1680 x^3+5743 x^4+9920 x^5+13685 x^6+5376 x^7+20386 x^8-26304 x^9+32414 x^10-46880 x^11+63010
x^12-26304 x^13+542 x^14+5376 x^15-34997 x^16+9920 x^17-17743 x^18+1680 x^19-1025 x^20+125 x^22) Log[1-x]-1/(16 (-1+x)^3 x^5 (1+x)^3 (1+x^2)^3) (-125+1025 x^2+3360 x^3+11983 x^4+19840 x^5+25029 x^6+15072 x^7+45970 x^8-29952 x^9+86046 x^10-46880
x^11+157410 x^12-22656 x^13-658 x^14-4320 x^15-94917 x^16-46543 x^18-1025 x^20+125 x^22) Log[1+x])-((675-1970 x^2-25320 x^4-154638 x^6+208906 x^8-154638 x^10-25320 x^12-1970 x^14+675 x^16) Log[1+x^2])/(24 (-1+x)^4 x^4 (1+x)^4)-((1+x^4) (25+30 x^2-1881
x^4-5052 x^6-1881 x^8+30 x^10+25 x^12) Log[1+(-1+x)^2/(1+x)^2])/(8 (-1+x)^2 x^4 (1+x)^2 (1+x^2)^2)+((1+x^4) (35+120 x-2380 x^2+968 x^3-5934 x^4+968 x^5-2380 x^6+120 x^7+35 x^8) Log[1-(-1+x)/(1+x)])/(2 (-1+x)^2 x^2 (1+x)^2 (1+x^2)^2)+((125-340 x^2+10957
x^4+33208 x^6+39158 x^8+30392 x^10+39158 x^12+33208 x^14+10957 x^16-340 x^18+125 x^20) PolyLog[2,(1-x)/2])/(8 (-1+x)^2 x^5 (1+x)^2 (1+x^2)^3)+((-105+360 x-620 x^2+623 x^3-606 x^4-27 x^5+228 x^6-676 x^7-216 x^9-228 x^10-337 x^11+606 x^12-747 x^13+620
x^14-330 x^15+105 x^16+70 x^17) PolyLog[2,(-1+x)/x])/((-1+x)^3 x^2 (1+x)^3 (1+x^2)^3)-1/(16 (-1+x)^3 x^5 (1+x)^3 (1+x^2)^3) (-125-95 x^2+11503 x^4+23653 x^6-4320 x^7+19954 x^8-22656 x^9+21598 x^10-46880 x^11+59554 x^12-29952 x^13-4850 x^14+15072 x^15-46949
x^16+19840 x^17-23023 x^18+3360 x^19+95 x^20+125 x^22) PolyLog[2,-x]+1/(16 (-1+x)^3 x^5 (1+x)^3 (1+x^2)^3) (-125-95 x^2+1680 x^3+5743 x^4+9920 x^5+13685 x^6+5376 x^7+20386 x^8-26304 x^9+32414 x^10-46880 x^11+63010 x^12-26304 x^13+542 x^14+5376
x^15-34997 x^16+9920 x^17-17743 x^18+1680 x^19-1025 x^20+125 x^22) PolyLog[2,x]+((125+220 x^2-5523 x^4-17528 x^6-21994 x^8-15048 x^10-21994 x^12-17528 x^14-5523 x^16+220 x^18+125 x^20) PolyLog[2,-((-1+x)^2/(1+x)^2)])/(16 (-1+x)^2 x^5 (1+x)^2 (1+x^2)^3)-(2
(35+105 x-1750 x^2+725 x^3-5347 x^4+1331 x^5-6494 x^6+1103 x^7-4760 x^8+1103 x^9-6494 x^10+1331 x^11-5347 x^12+725 x^13-1750 x^14+105 x^15+35 x^16) PolyLog[2,(-1+x)/(1+x)])/((-1+x)^2 x^3 (1+x)^2 (1+x^2)^3));
```

WEFT approach to GW physics *5PM*

PHYSICAL REVIEW LETTERS **135**, 251401 (2025)

Local-in-Time Conservative Binary Dynamics at Fifth Post-Minkowskian and First Self-Force Orders

Christoph Dlapa,[1,*] Gregor Kälin,[1,†] Zhengwen Liu,[2,3,‡] and Rafael A. Porto[1,§]

All orders in velocity!

Known to 6PN order

$$\hat{H}^{\rm ell}_{5\rm PM} = \sum_{i=1}^{i=5} \frac{\hat{c}_{i(\rm loc)}}{\hat{r}^i} + \sum_{i=1}^{i=5} \frac{\hat{c}_{i(\rm nloc)}}{\hat{r}^i} - \sum_{i=4}^{i=5} G\left(\hat{H}\frac{dE_{\rm src}}{dt}\right)\Bigg|_{(i-1)\rm PM} \log\left(\frac{\hat{r}}{e^{2\gamma_E}}\right)$$

$$G\frac{dE_{\rm src}}{dt}\bigg|_{G^4}(\hat{r},\boldsymbol{p}^2) = \frac{\nu^3}{\Gamma^3 x^4 (x^2-1)^2 \xi \hat{r}^5}\Bigg[\frac{\log(2)d_1(x)}{256\Gamma^7 x^4 \xi^2} + \frac{d_2(x)\log^2(x)}{(x^2-1)^4} + \frac{d_3(x){\rm Li}_2(x)}{256(x^2-1)} + \frac{d_4(x){\rm Li}_2(-x)}{256(x^2-1)}$$
$$+ \frac{d_6(x)\log(x+1)}{256\Gamma^7 x^4\xi^2} + \frac{d_7(x)\log^2(x^2+1)}{1024(x^2-1)} + \frac{d_7(x){\rm Li}_2\left(\frac{1}{x^2+1}\right)}{512(x^2-1)} + \frac{\pi^2 d_8(x)}{2048(x^2-1)}$$
$$+ \frac{d_9(x)}{15052800\Gamma^9 x^5 (x^2-1)^2 (x^2+1)^8 \xi^2} + \log(x)\left(\frac{d_3(x)\log(1-x)}{256(x^2-1)} + \frac{d_4(x)\log(x+1)}{256(x^2-1)}\right.$$
$$\left.+ \frac{d_5(x)}{71680\Gamma^9 x^5 (x^2-1)^3 (x^2+1)^9 \xi^2}\right)\Bigg],$$

$$\frac{\hat{c}^{\rm 6PN(e^8,1SF)}_{5(\rm nloc)}}{\nu} = \left(-383 + \frac{5421492}{5}\log(2) + \frac{631071}{20}\log(3) - \frac{1953125}{4}\log(5)\right)$$
$$+ \hat{\boldsymbol{p}}^2\left(-\frac{237374547}{28000} + \frac{274846146629\log(2)}{9450} + \frac{268234748343\log(3)}{71680} - \frac{5343159203125\log(5)}{387072} - \frac{96889010407\log(7)}{92160}\right)$$
$$+ \hat{\boldsymbol{p}}^4\left(-\frac{12514611952561}{197568000} + \frac{345079750666673\log(2)}{1190700} + \frac{2379761187413283\log(3)}{40140800} - \frac{7512099894456875\log(5)}{55738368} - \frac{334452591435529\log(7)}{13271040}\right).$$

$$x = \gamma - \sqrt{\gamma^2-1}, \qquad \gamma = \nu(\hat{E}_1\hat{E}_2 + \hat{\boldsymbol{p}}^2)$$

MOST ACCURATE BOUND FROM SCATTERING!

Dlapa Kalin Liu **RAP** (2026)

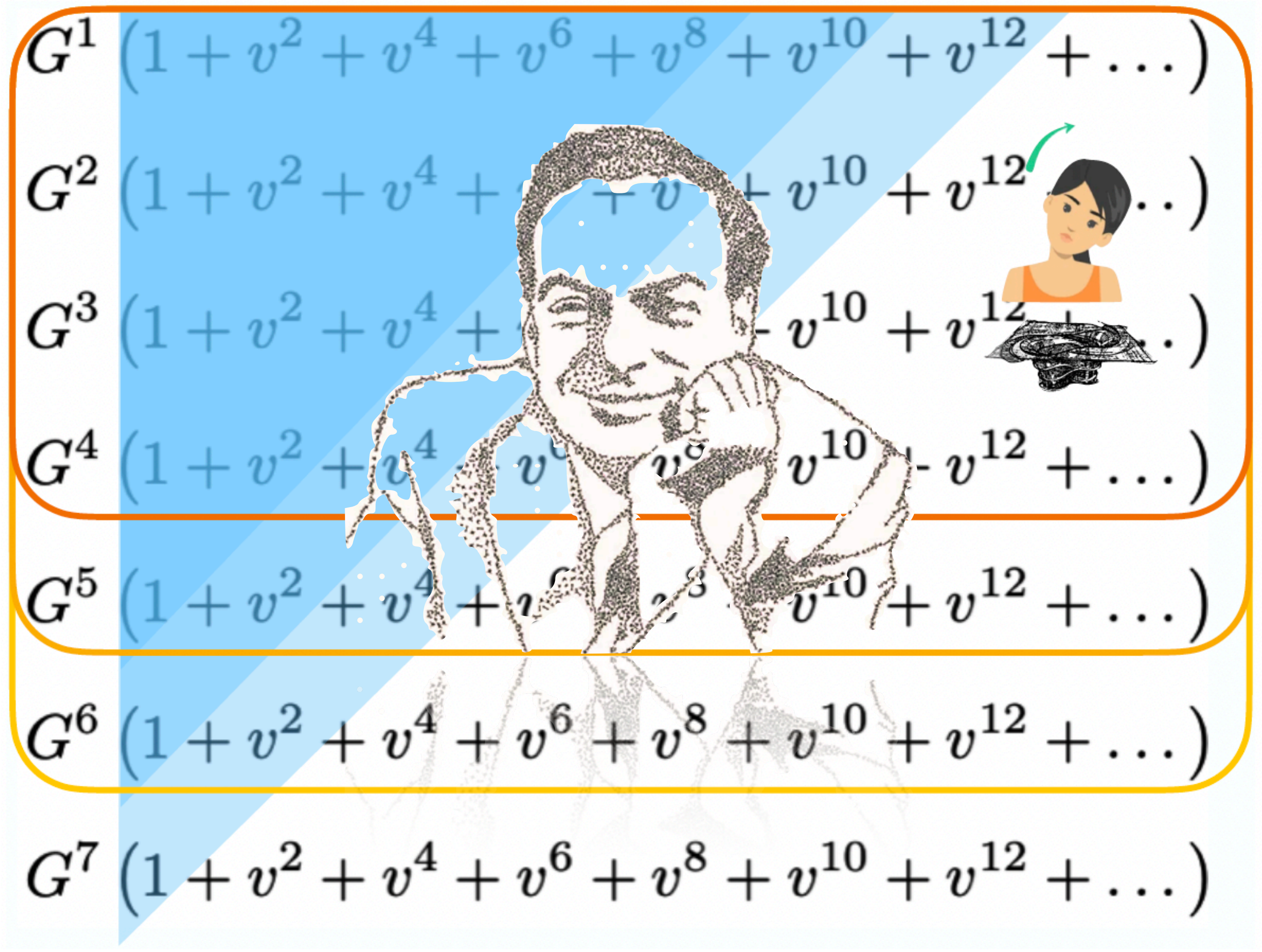
5PM/10SF COMING!!

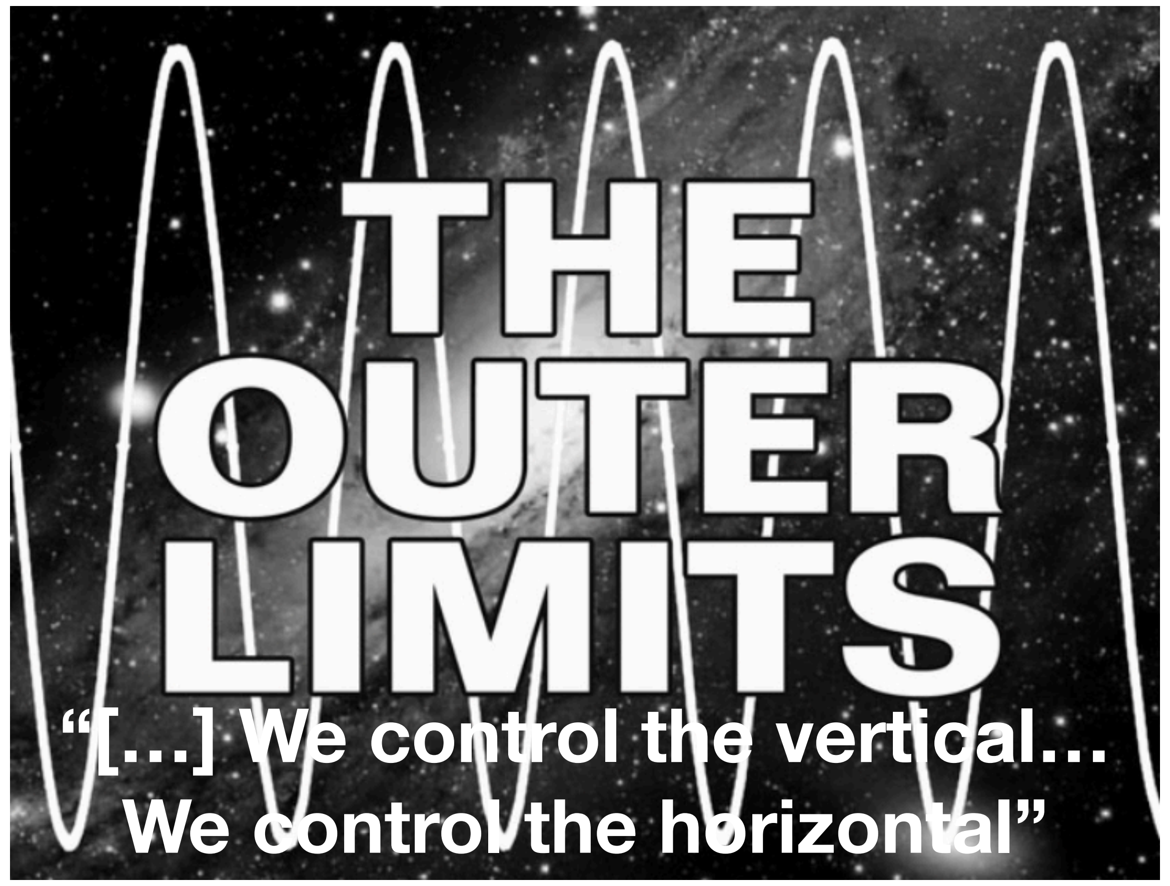
Perfect agreement with state of the art thru 6PN!

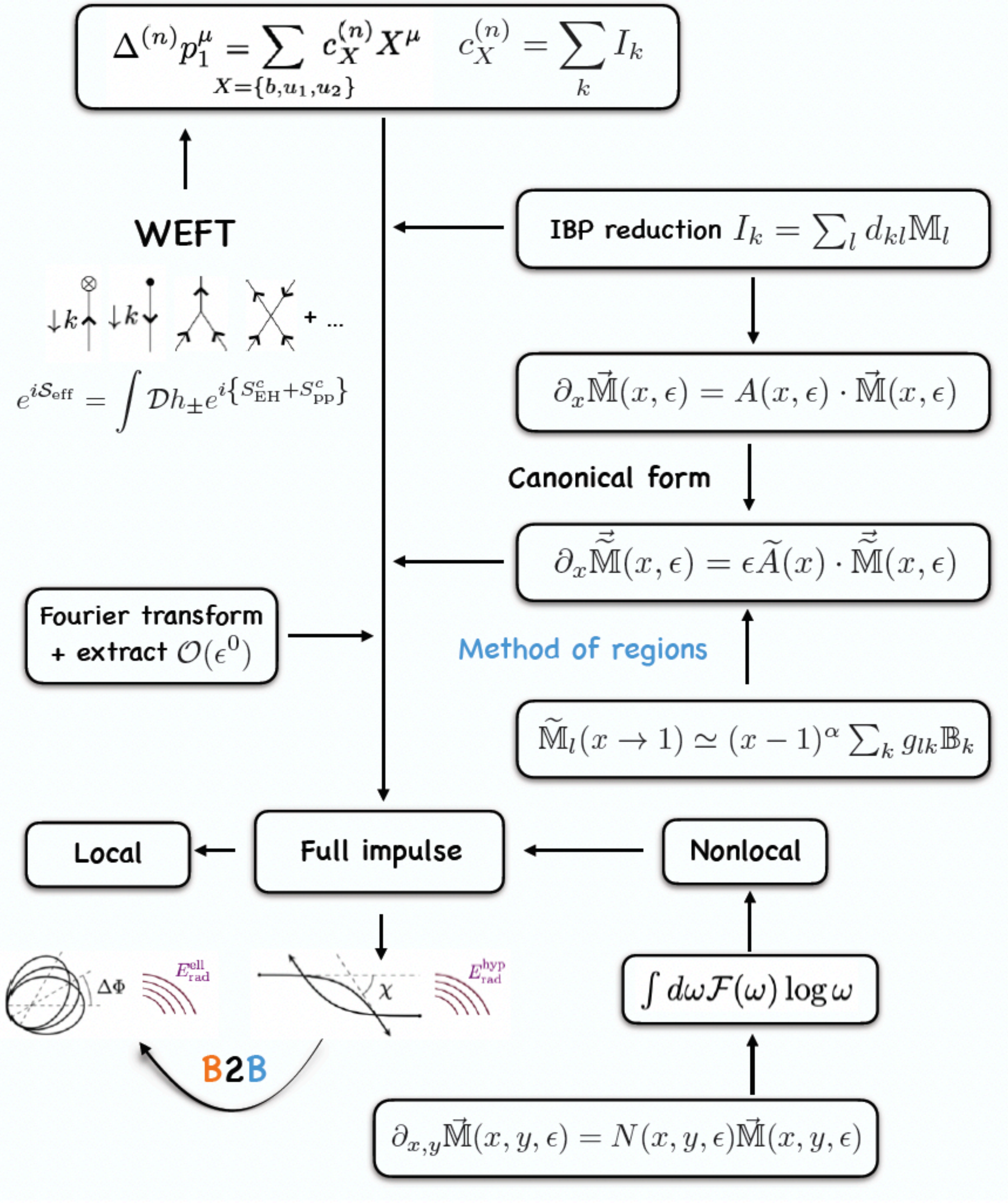
$$\frac{\chi^{(5)}_{j(\rm nloc)}}{\nu v_\infty^3} = -\frac{25088}{45}\log\left(\frac{2v_\infty}{\sqrt{j}}\right) - \frac{5440}{27}$$
$$+ v_\infty^2\left[\left(\frac{55808\nu}{45} - \frac{297728}{525}\right)\log\left(\frac{2v_\infty}{\sqrt{j}}\right) + \frac{155200\nu}{189} + \frac{374368}{2625}\right]$$
$$+ v_\infty^4\left[\left(-\frac{86528\nu^2}{45} + \frac{1152896\nu}{1575} - \frac{3525568}{11025}\right)\log\left(\frac{2v_\infty}{\sqrt{j}}\right) - \frac{7750528\nu^2}{4725} - \frac{41361872\nu}{165375} + \frac{166150088}{385875}\right]$$
$$+ v_\infty^6\left[\left(\frac{117248\nu^3}{45} - \frac{875008\nu^2}{1575} + \frac{3096256\nu}{6615} + \frac{2429664}{13475}\right)\log\left(\frac{2v_\infty}{\sqrt{j}}\right) + \frac{1046272\nu^3}{405} + \frac{21978496\nu^2}{33075} - \frac{22499132024\nu}{38201625} + \frac{1137056876}{7640325}\right]$$
$$+ v_\infty^8\left[\left(-\frac{147968\nu^4}{45} + \frac{3968\nu^3}{105} - \frac{886048\nu^2}{1323} - \frac{279650936\nu}{363825} + \frac{1620350092}{2477475}\right)\log\left(\frac{2v_\infty}{\sqrt{j}}\right) - \frac{80567008\nu^4}{22275} - \frac{2743988288\nu^3}{1819125} + \frac{204678995432\nu^2}{420217875} - \frac{3137954156509\nu}{5462832375} + \frac{38172272288071}{120182312250}\right]$$
$$+ v_\infty^{10}\left[\left(\frac{178688\nu^5}{45} + \frac{1293568\nu^4}{1575} + \frac{1289632\nu^3}{1225} + \frac{16506176\nu^2}{11025} - \frac{307880492\nu}{315315} - \frac{735017339222}{676350675}\right)\log\left(\frac{2v_\infty}{\sqrt{j}}\right) + \frac{1367324512\nu^5}{289575} + \frac{18434632624\nu^4}{6449625} + \frac{75486806288\nu^3}{2630252625} + \frac{88618604658512\nu^2}{71016820875} - \frac{308380410342341\nu}{426100925250} - \frac{815157612347048 29}{121864864621500}\right]$$
$$+ v_\infty^{12}\left[\left(-\frac{209408\nu^6}{45} - \frac{3184256\nu^5}{1575} - \frac{57721472\nu^4}{33075} - \frac{98860192\nu^3}{40425} + \frac{48577741364\nu^2}{52026975} + \frac{525179938397\nu}{225450225} + \frac{108812096826773}{68987768850}\right)\log\left(\frac{2v_\infty}{\sqrt{j}}\right) - \frac{1704352288\nu^6}{289575} - \frac{338314320616\nu^5}{70945875} - \frac{245498965780408\nu^4}{213050462625} - \frac{486269791848056\nu^3}{213050462625} + \frac{2009480696060767\nu^2}{2343555088875} + \frac{25168487123370619421\nu}{12430216191393000} + \frac{27550186358971497707}{24860432382786000}\right]$$

$+\ldots$

WEFT approach state-of-the-art in PN/PM

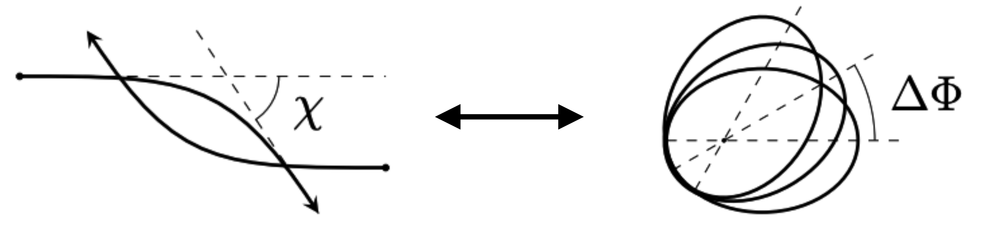






Alternative (non-H) B2B approach to 2-body problem!

$\{\chi, \tau, \Delta E, \Delta \boldsymbol{L}_z\}$



Thank you...