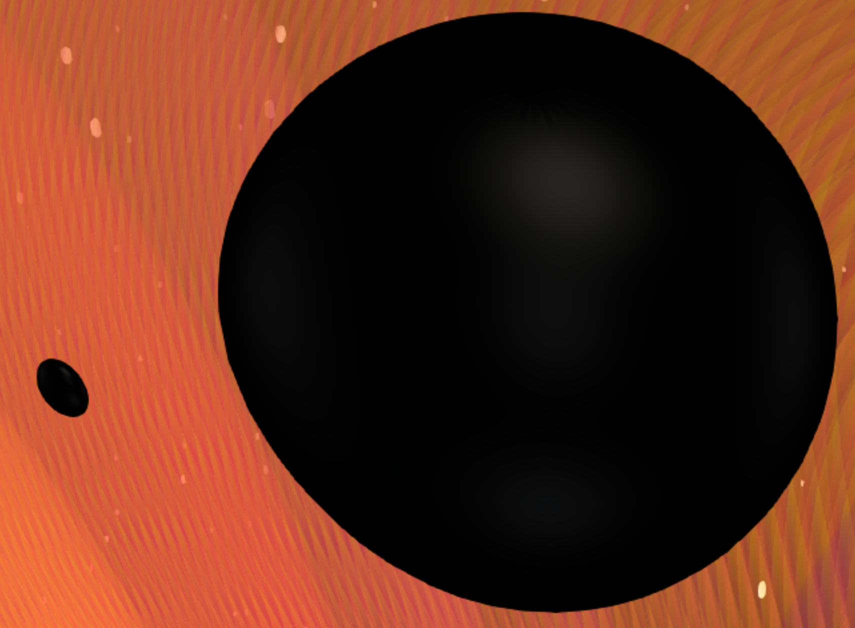
Numerical Relativity Simulations of Binary Black Hole Encounters



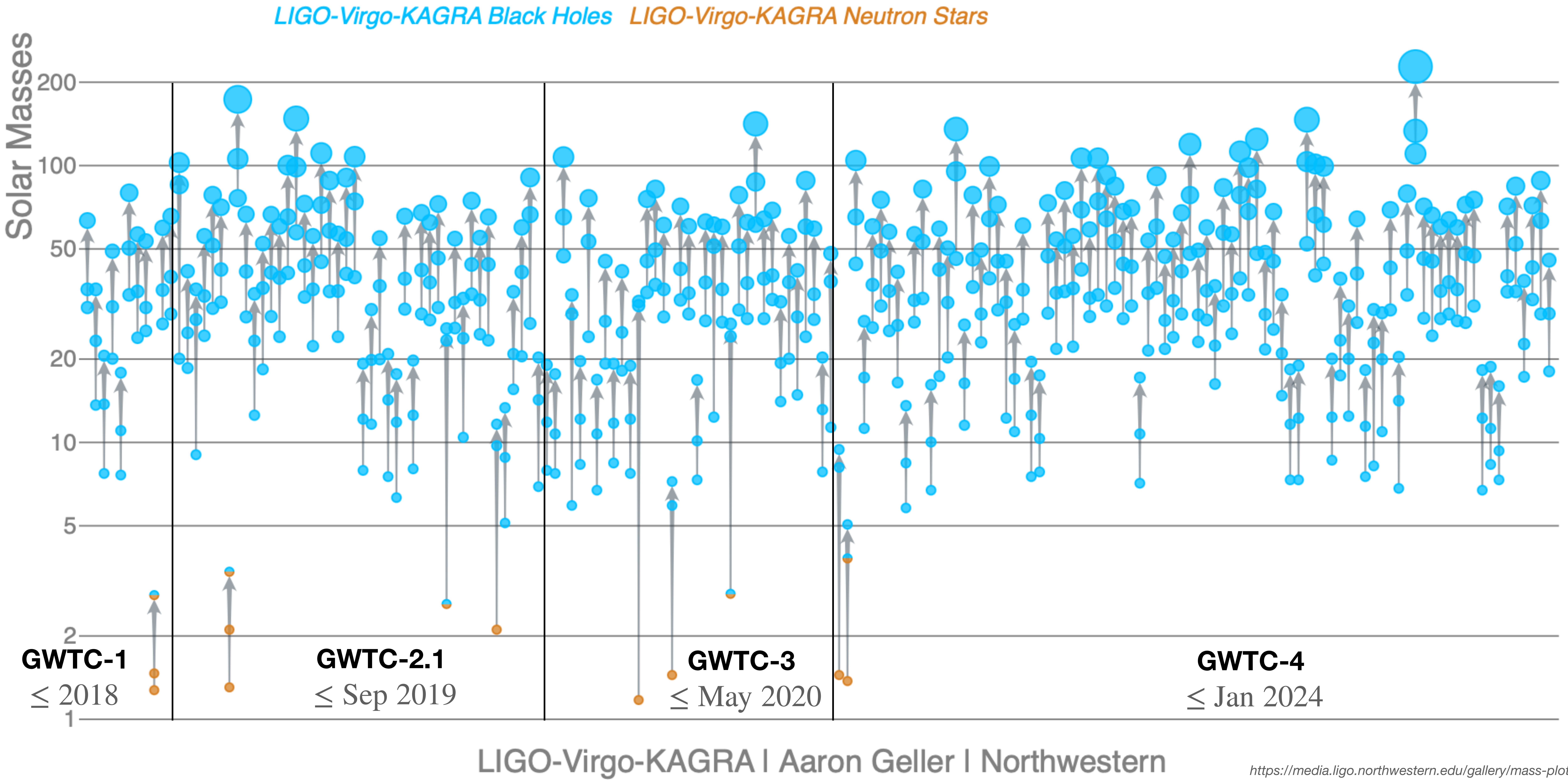
Harald Pfeiffer
MPI for Gravitational Physics

Amplitudes, Strong-field gravity and resummation
NORDITA, Stockholm
Apr 13, 2026

Funded by the European Union
European Research Council
Established by the European Commission

Image: Nils Vu

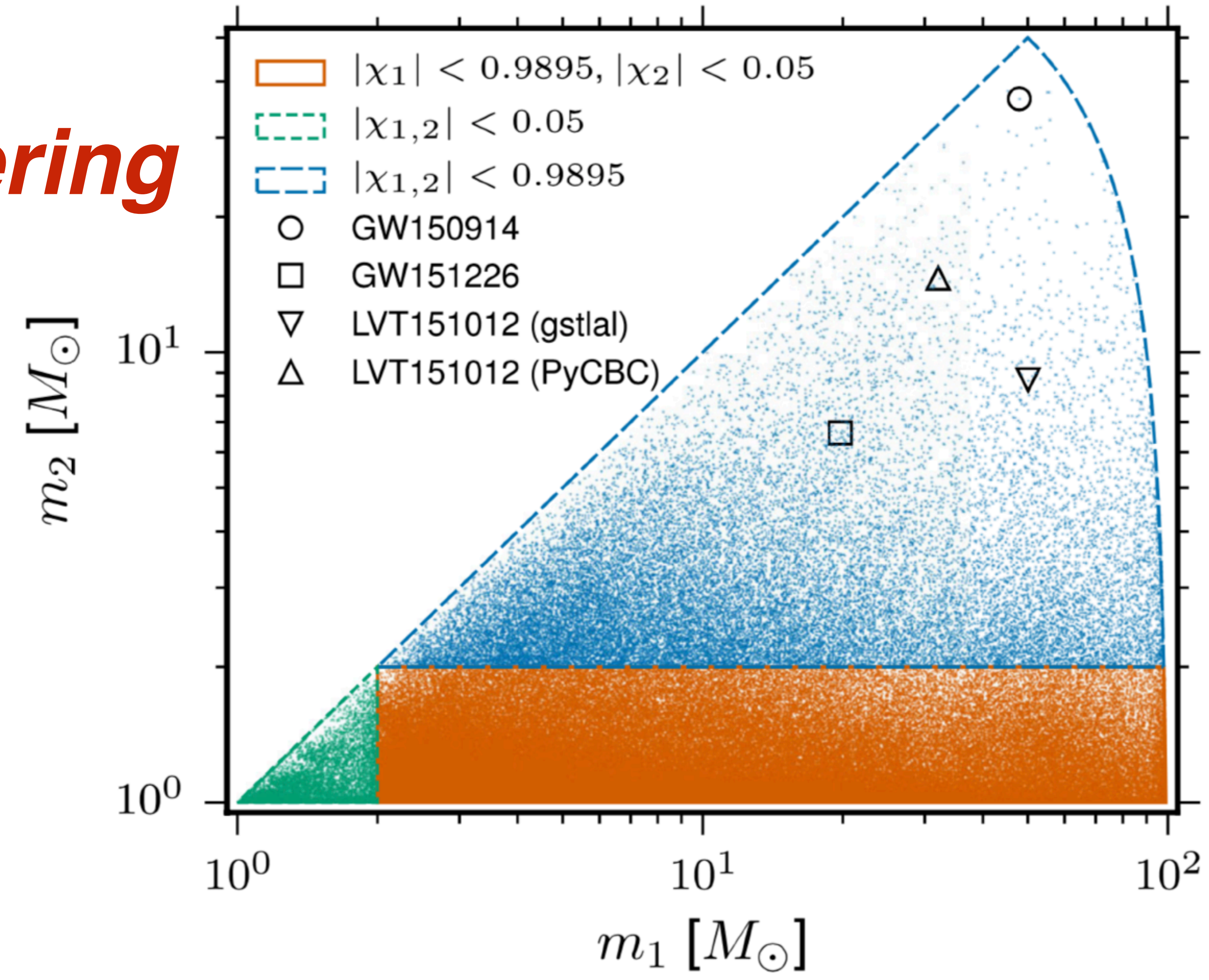
GW Observations



https://media.ligo.northwestern.edu/gallery/mass-plot

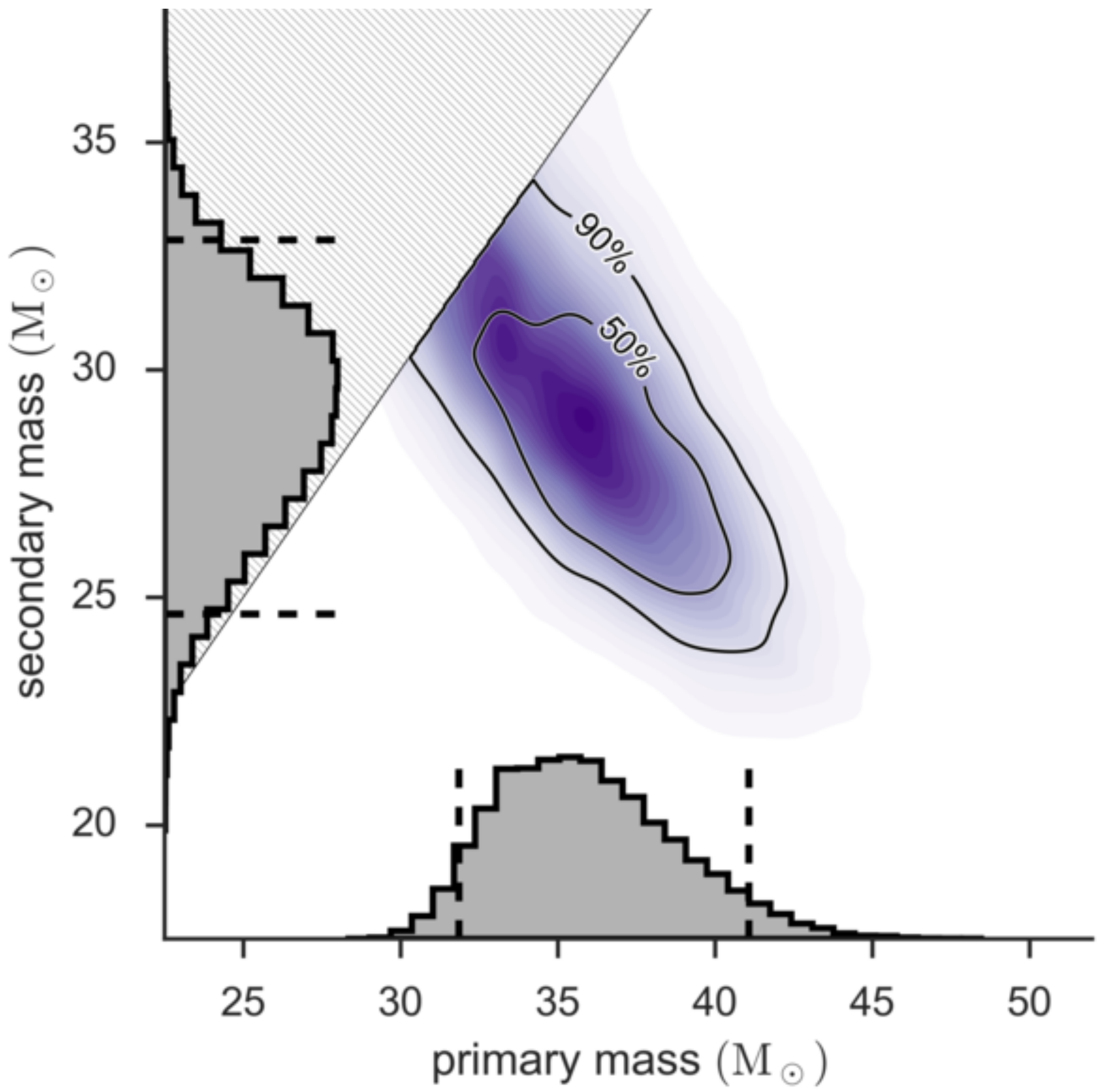
Waveform knowledge ***essential for GW astronomy***

Detection by matched filtering



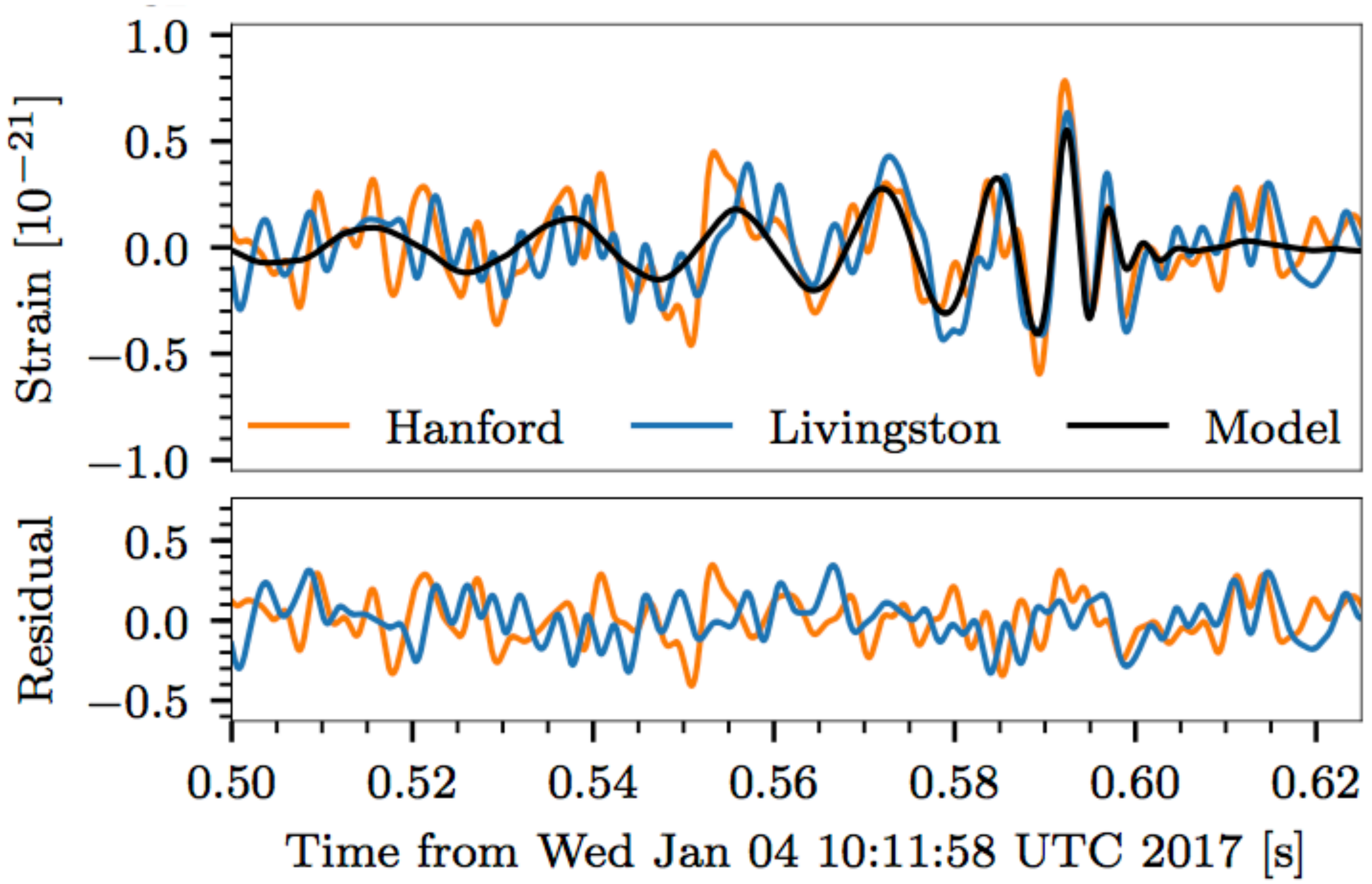
LIGO+Virgo, PRX 2016 (1606.04856)

Parameter estimation



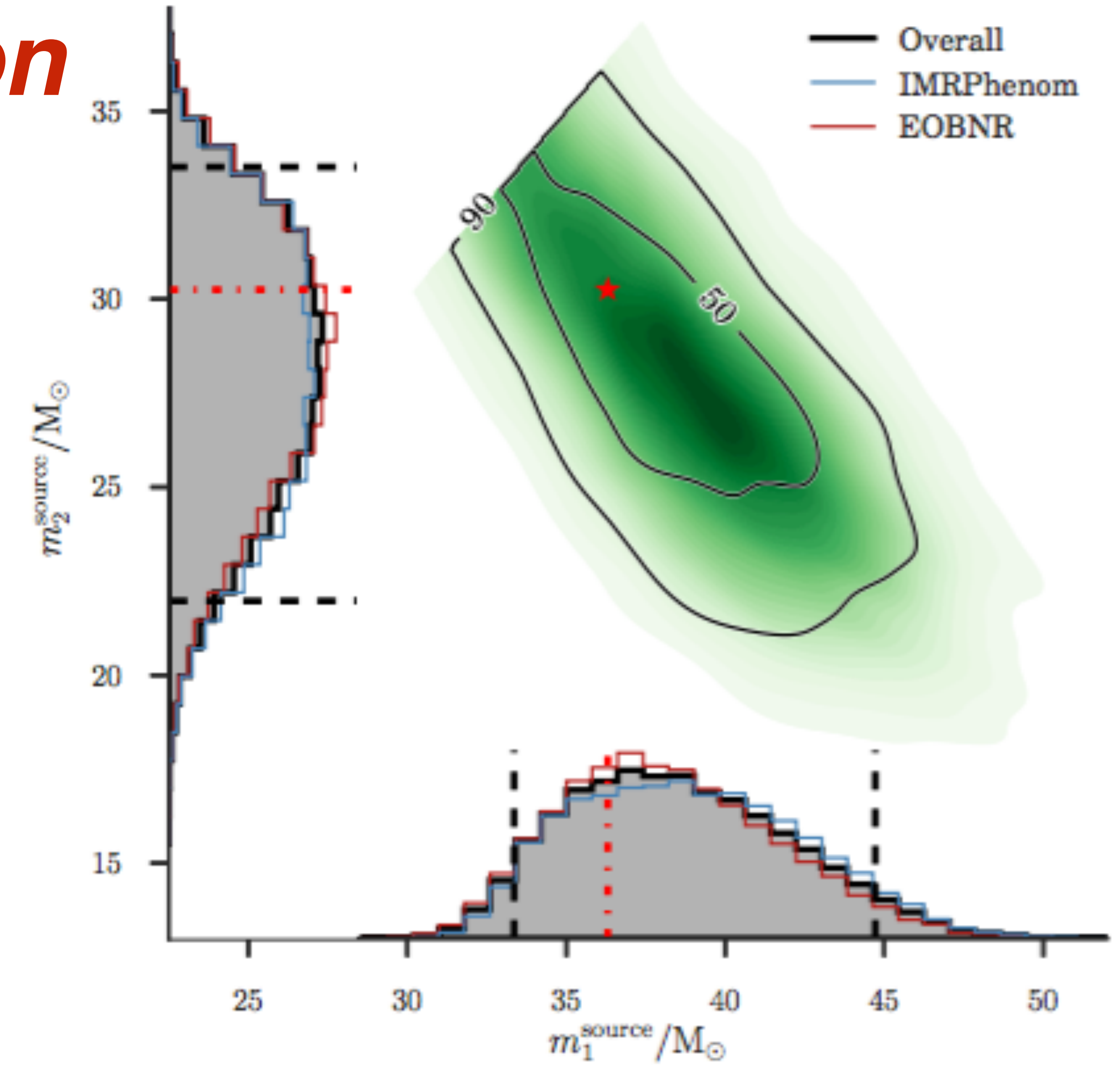
"GW150914" Abbott+ PRL 12016

Testing GR



LIGO+Virgo, PRL 2017 (1706.01812)

Validation

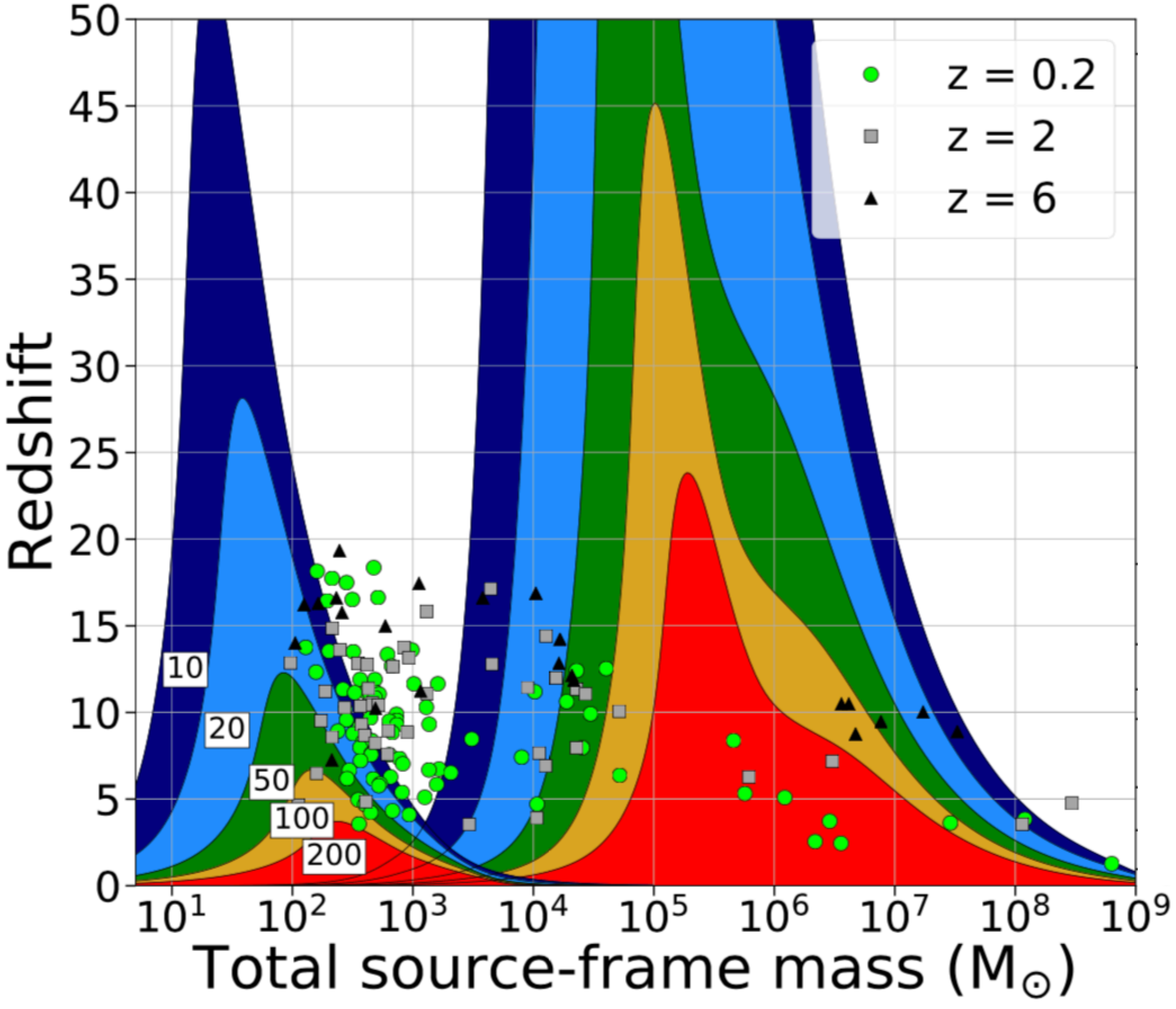


LIGO & Virgo: CQG 2017 (1611.07531)

In future, need ***higher accuracy*** for ***more diverse*** systems

XG & LISA: SNRs of 1000's (instead of 10's)

needed accuracy ~ 1/SNR



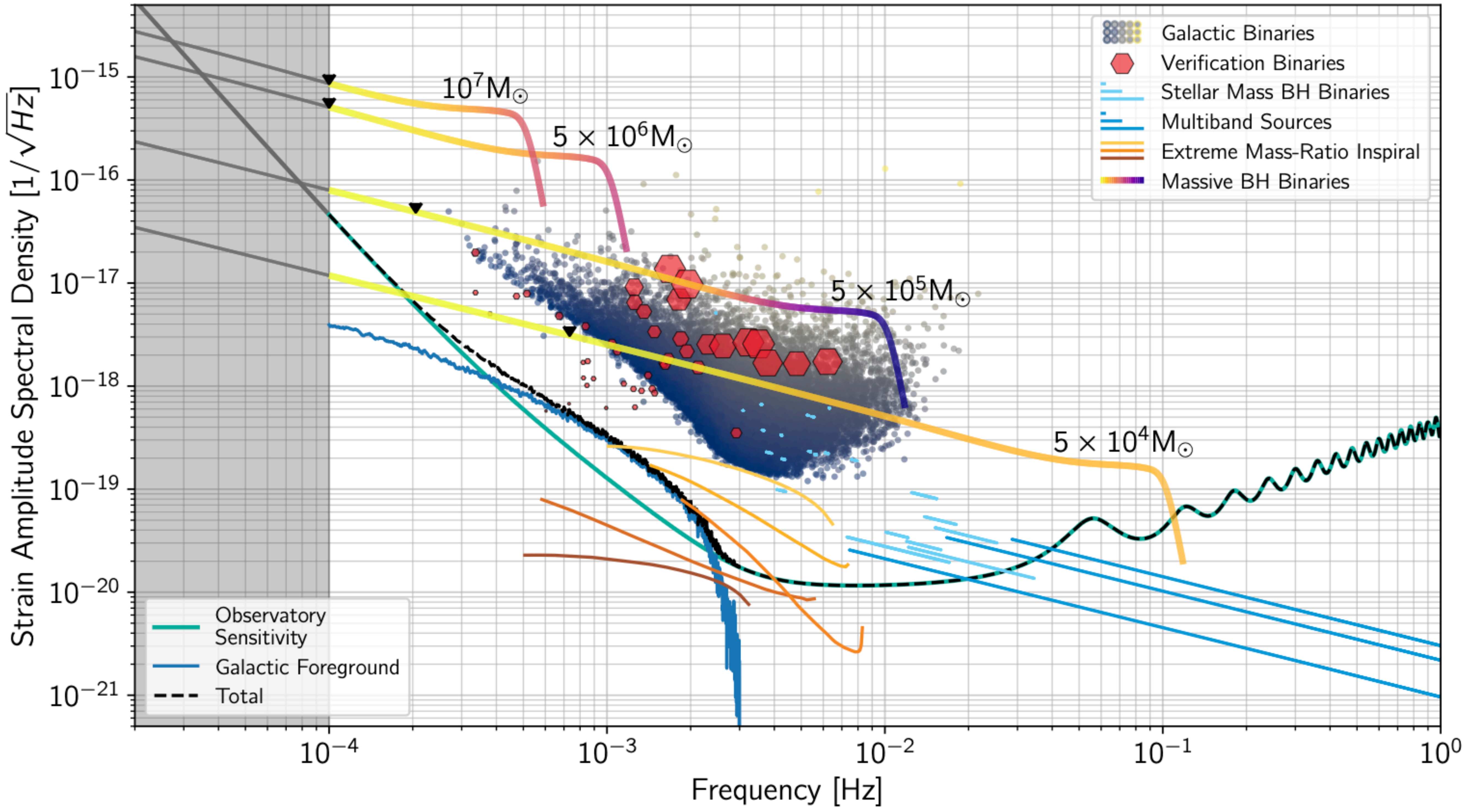
GWIC, 3G Science Case

LISA

among sources: BBH $q = 1 \ldots 10^{-6}$

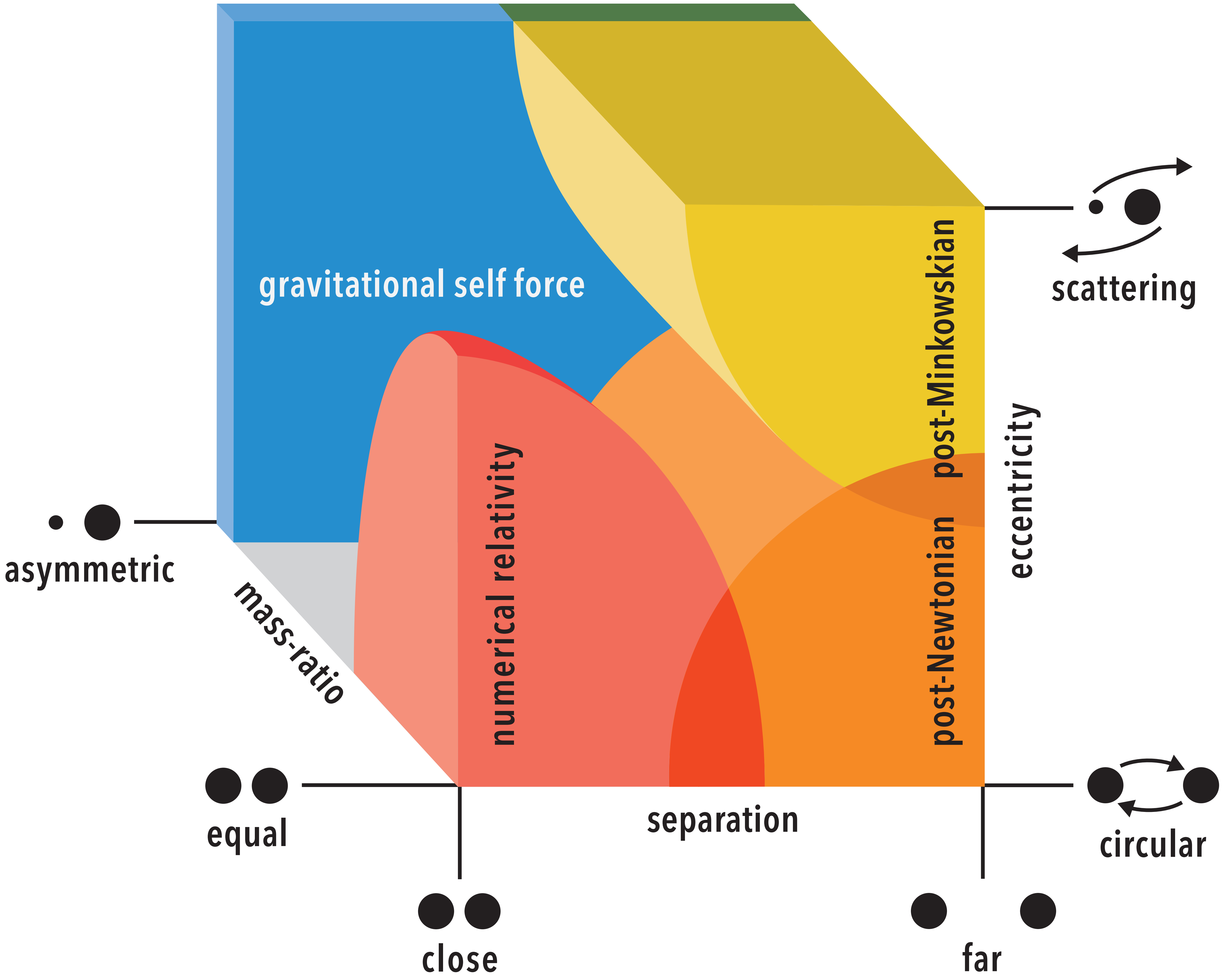
among science targets:

eccentricity measurement to $\delta e < 0.001$



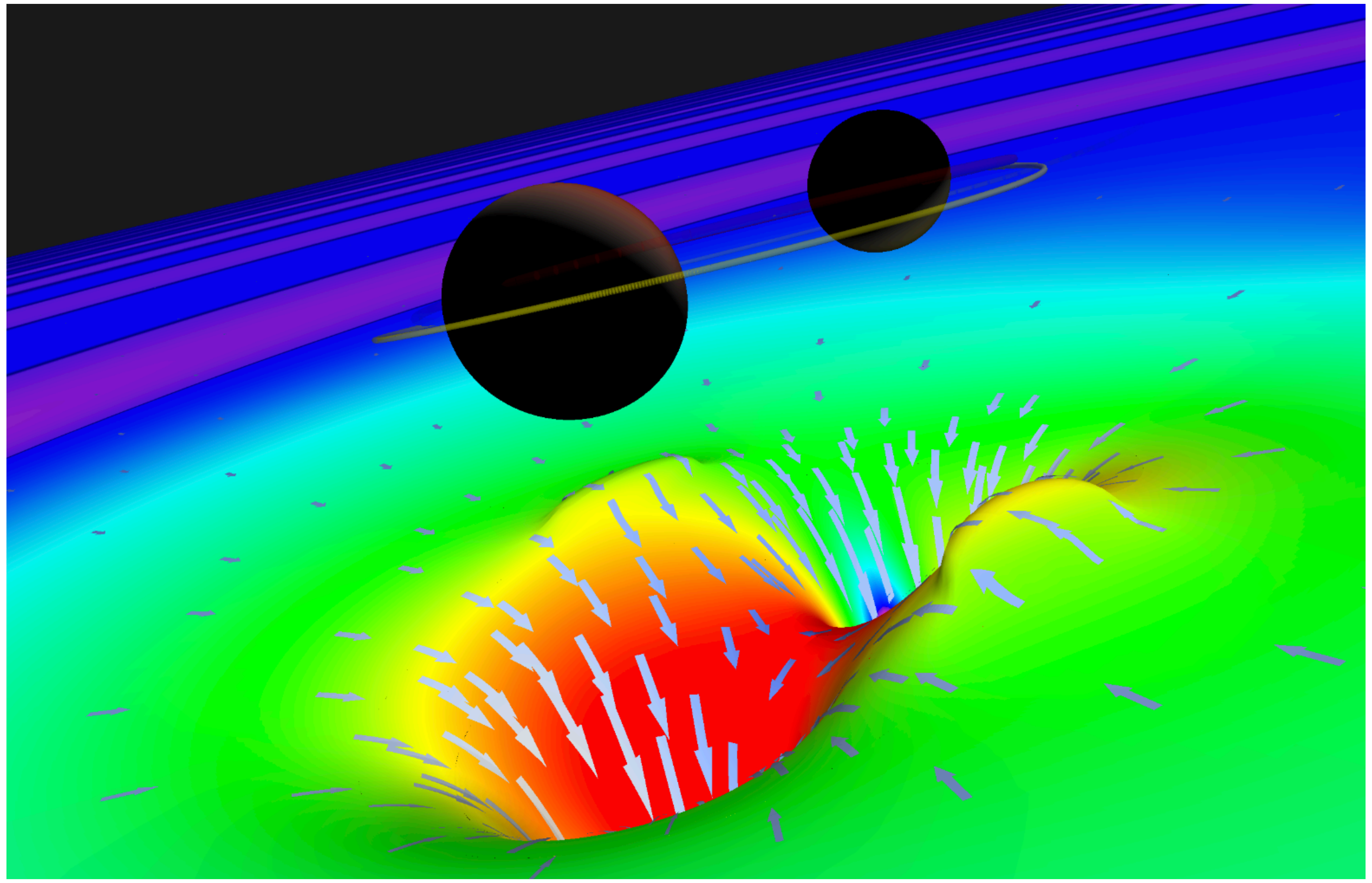
LISA Red Book, 2402.07571

Methods for waveform modeling



- **GW astronomy requires waveforms spanning all regimes & methods**
- **Overlap of methods:**
  - transfer of information
  - validation
  - understand capabilities & limitations

Numerical Relativity



The first 50 Years of numerical relativity for BBH

1962 *ADM*
3+1 formulation

1992,3
Choptuik; Abrahams+Evans
critical phenomena

1999-00
AEI/PSU
grazing collisions

2000-04
AEI/UTB-NASA
revive crashing codes (Lazarus)

2005 *Pretorius*
inspiral-merger-ringdown (IMR) w/ harmonic

2007-
Ajith, AEI, Jena
phenom GW models

2011
Lousto ea
q=100

1964
Hahn-Lindquist
2 wormholes

1997
Brandt-Brügmann
puncture data

~2000 *Choptuik; Schnetter;Brügmann*
mesh refinement

2005-06
Campanelli+; Baker+
IMR w/ BSSN & moving punctures

2009-
UMD, SXS
EOB GW models

2014-
precessing GW models

1984
Unruh
excision

1994-98
BBH Grand Challenge

2005
Gundlach ea
constraint damping

2006-08
Scheel..HP+ *SXS*
IMR w/ spectral

2011
Schmidt ea; Boyle ea
Radiation aligned frame

2015
Szilagyi *ea*
175 orbits

1964 — **~1999** — **~2005** — **2015**

1975-77
Smarr-Eppley
head-on collision

1994 *Cook*
Bowen-York initial data

1999
BSSN
evolution system

2000-02
Alcubierre
gauge conditions

2006,07
Baker ea; Gonzalez ea
non-spinning BBH kicks

2008
all of NR
NINJA

2011
Lovelace ea
$S/M^2=0.97$

1979 *York*
kinematics and dynamics of GR

1994-95
NCSA-WashU
improved head-on collision

1999 *York*
conformal thin sandwich ID

2004
Brügmann ea
one orbit

2011-
Le Tiec ea
self-force studies

2007 *SXS*
PN-NR comparison

2009-11
Bishop, ...
Cauchy characteristic extraction

1989-95
Bona-Masso
modified ADM, (hyperbolicity)

2003-08
Cook, Pfeiffer ea
improved ID

2013
GaTech; SXS
Precessing parameter studies

1999-2005 *York, Cornell, Caltech, LSU*
hyperbolic formulations

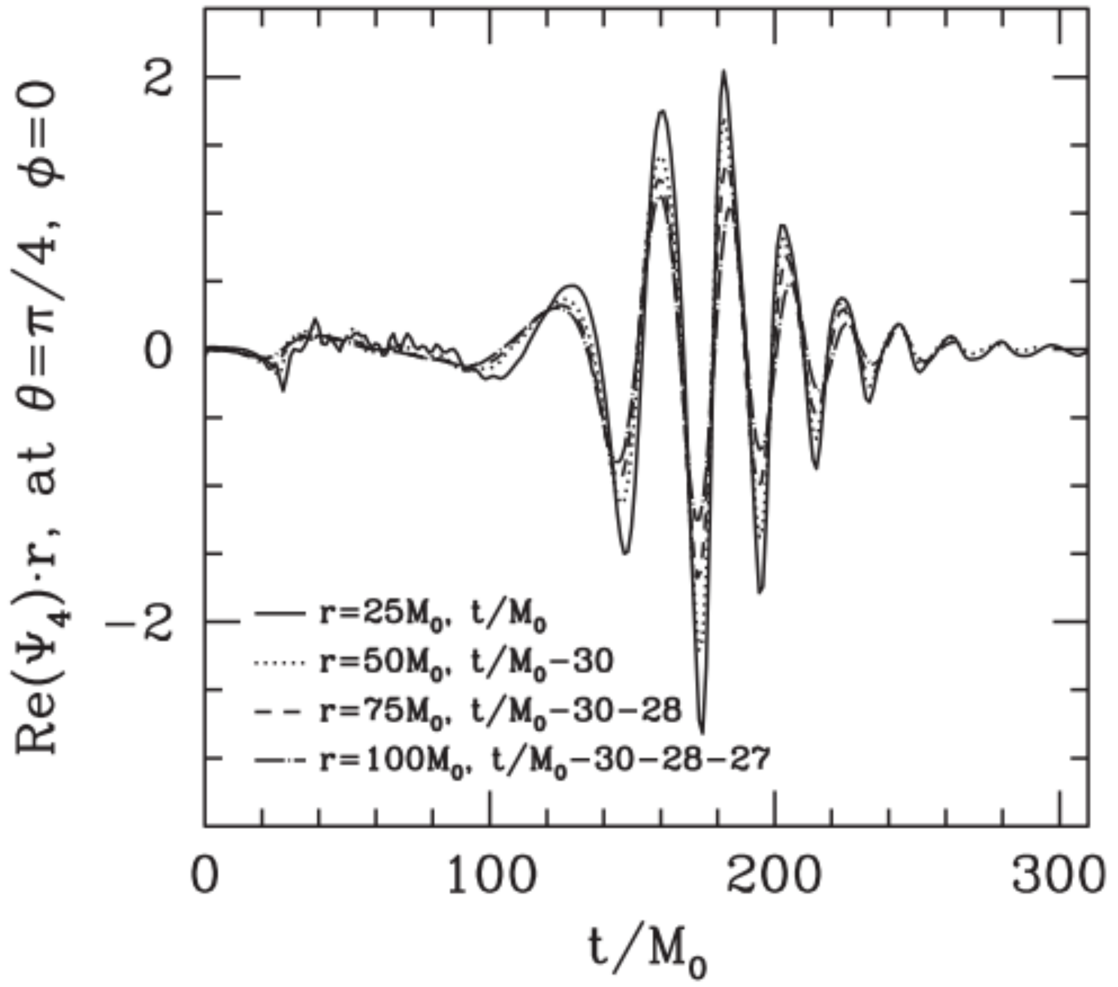
2000 *Ashtekar*
isolated horizons

2007-11
RIT; Jena; AEI;…
BBH superkicks

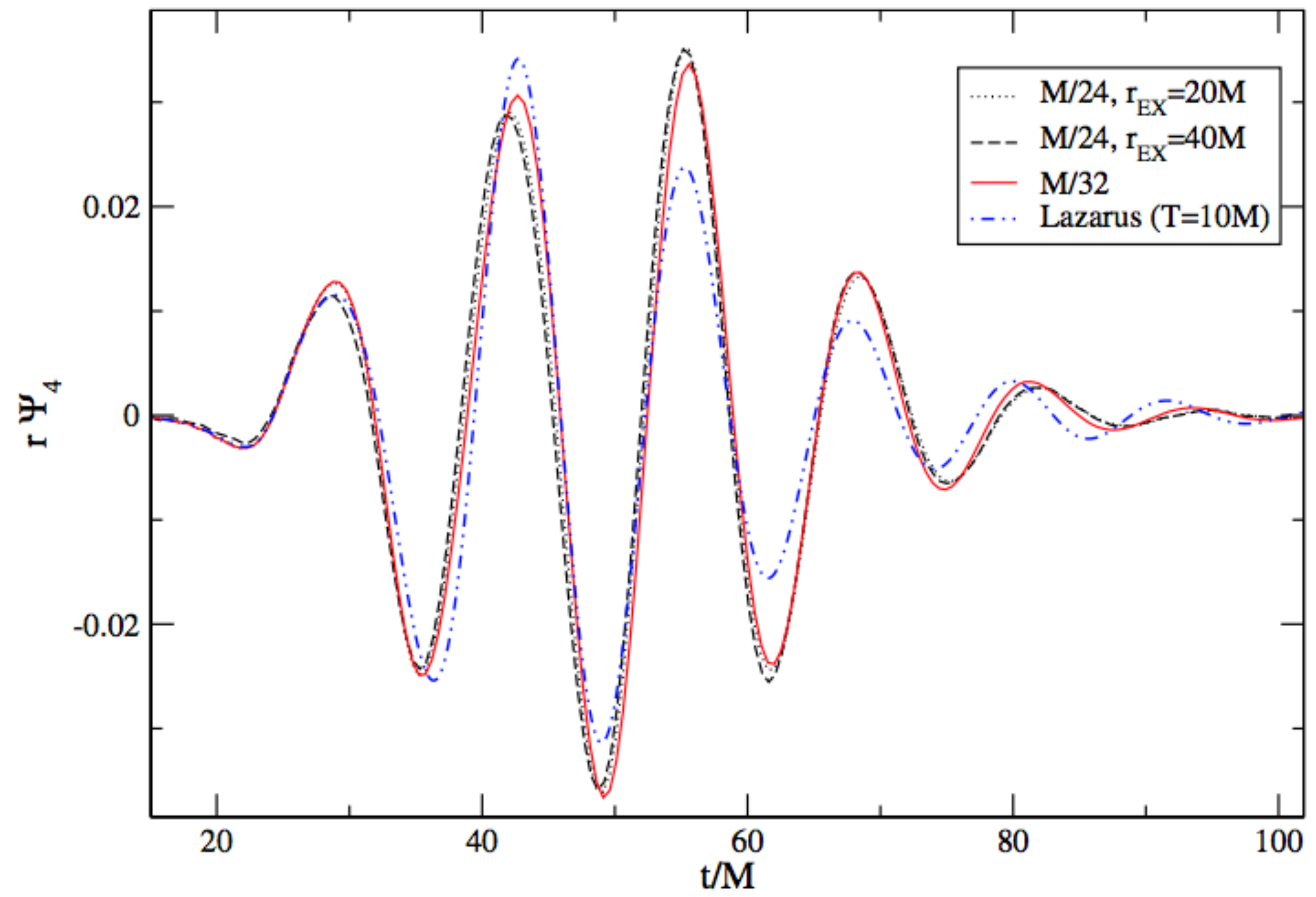
2010
Bernuzzi ea
C4z

Courtesy Carlos Lousto, updated by HP

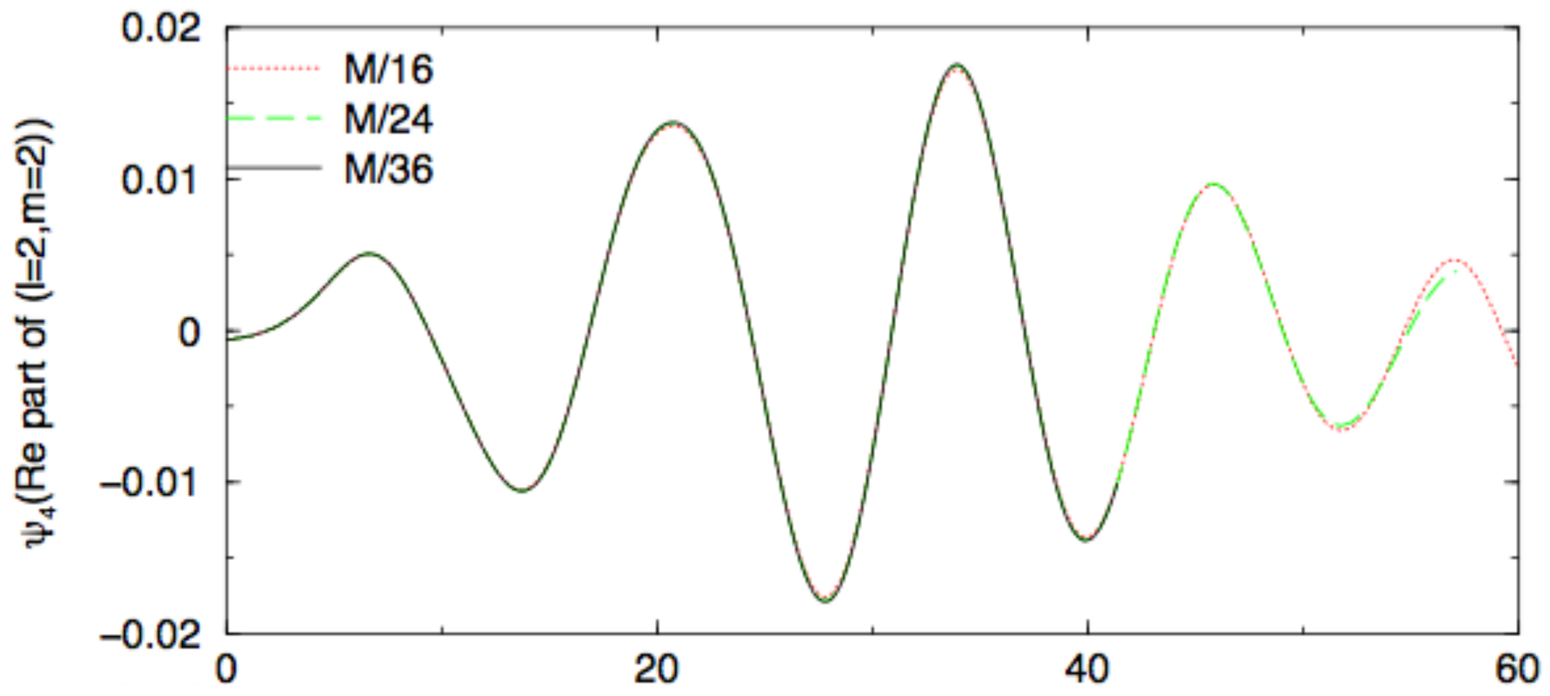
2005: First working BBH inspirals



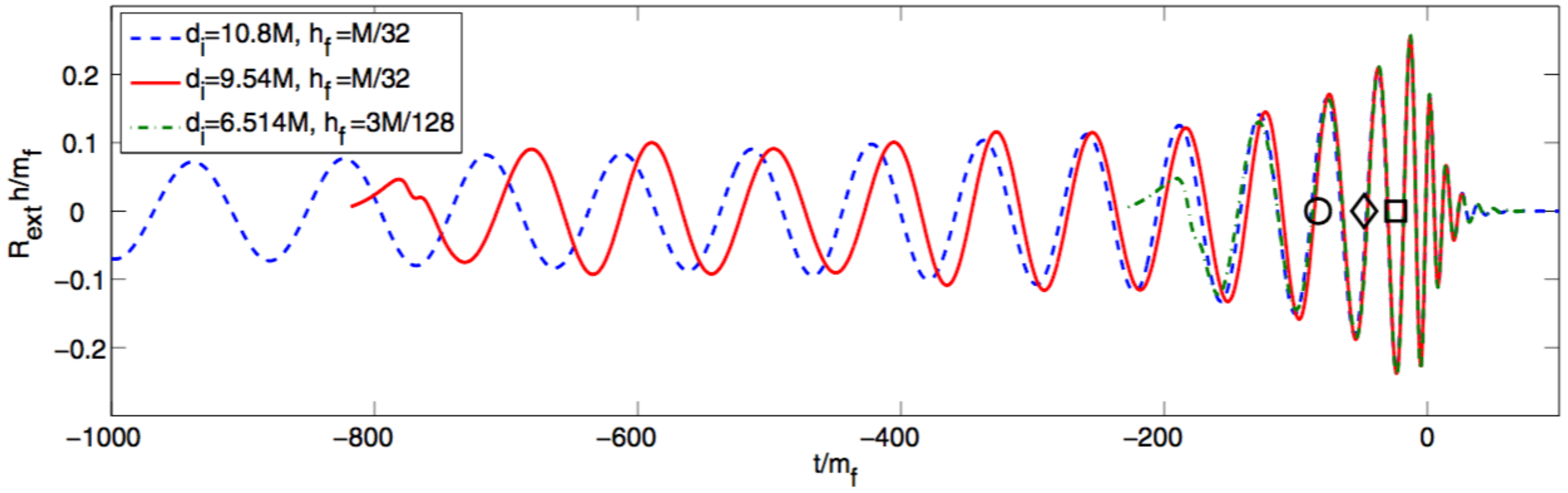
Pretorius 05



Baker+06



Campanelli+06



Baker+07

Important early result:
Simplicity of merger
Continuous transition
inspiral → ringdown

Major approaches towards BBH simulations

"BSSN & Moving punctures"

Puncture initial-data
$\chi \lesssim 0.9$ (but see Zlochower+ 17)

BSSN or CC4z

Moving puncture
mergers "easy"

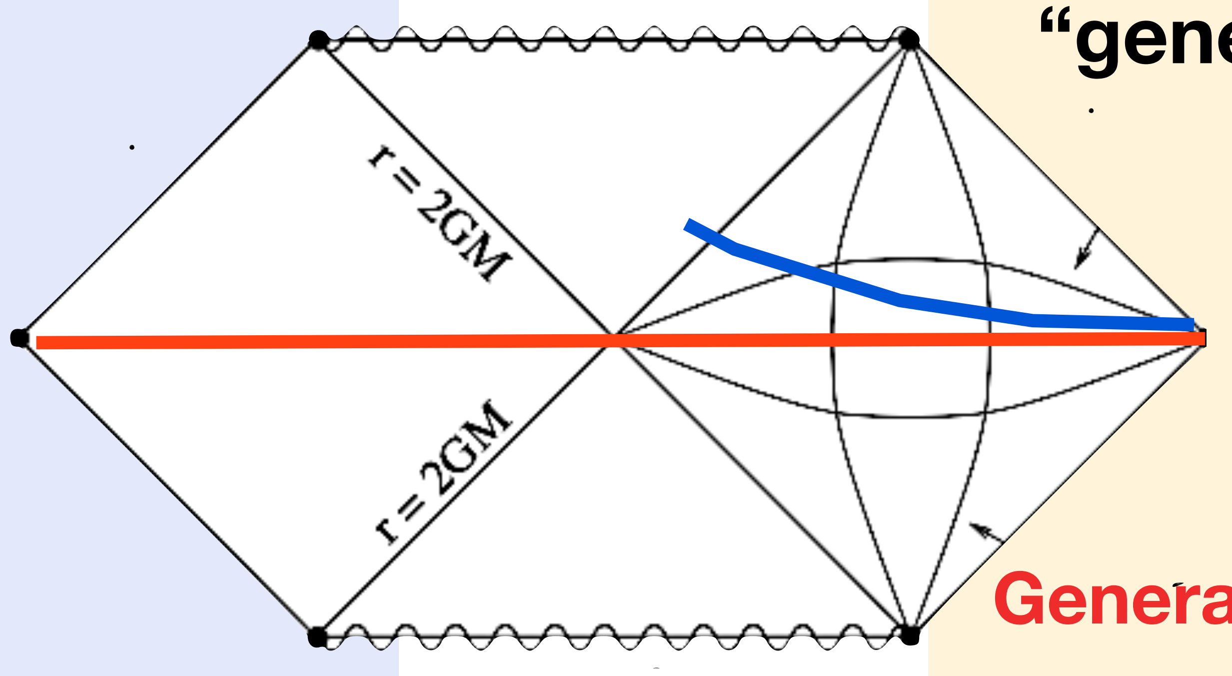
Sommerfeld outer BC

4th to 8th order finite-difference

BHs advect through static grids

GW extrapolation
(Healy,Lousto '20 for LazEv COM correction)

LazEv, Maya, ETK, BAM,
Goddard, GRchombo, …



"generalized harmonic & spectral"

Quasi-equilibrium excision data
$\chi \lesssim 0.999$

Generalized-Harmonic Evolution System

BH excision
mergers more difficult

Constraint preserving,
minimally reflective outer BC

Spectral methods

Moving grid

long, phase-accurate inspirals

GW extrapolation & COM control correction

Cauchy-characteristic extraction & BMS control
accurate m=0 modes, GW memory

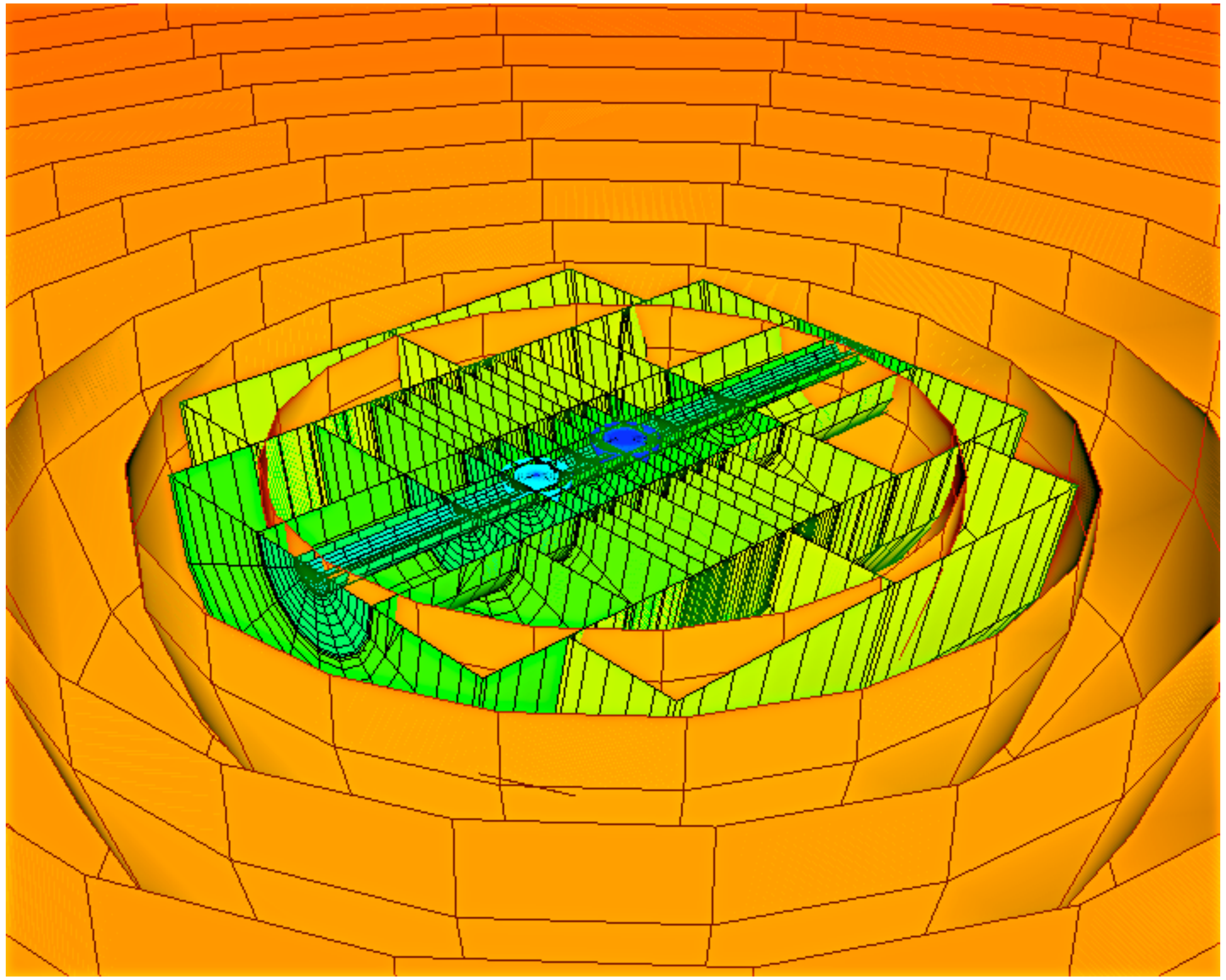
SpEC (SXS collaboration)

Spectral Einstein Code (SpEC)

Simulating eXtreme Spacetimes collaboration

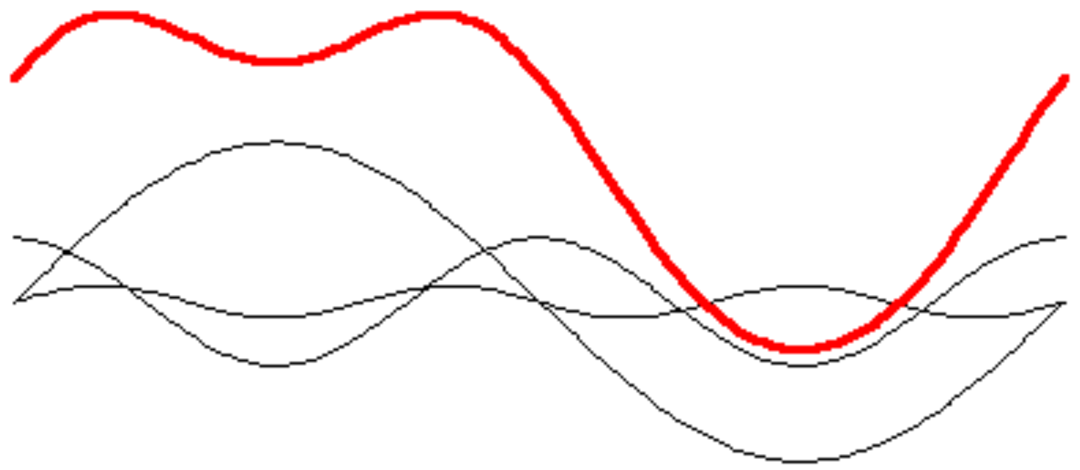
http://www.black-holes.org/SpEC.html

Combine w/ domain-decomposition



- **Expand in basis-functions**

$$u(x,t) = \sum_{k=1}^{N} \tilde{u}_k(t)\Phi_k(x)$$



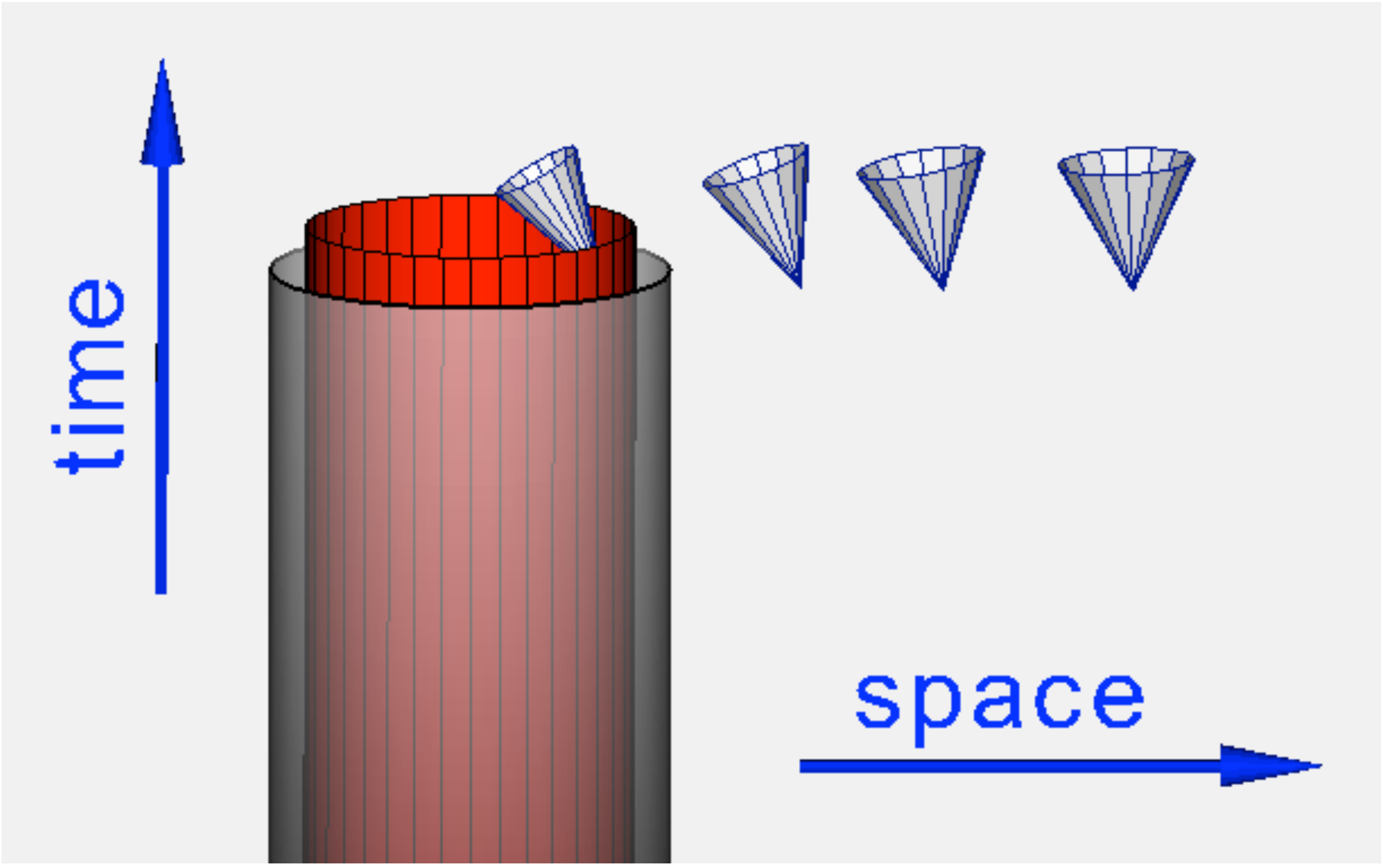
- **Compute derivatives analytically**

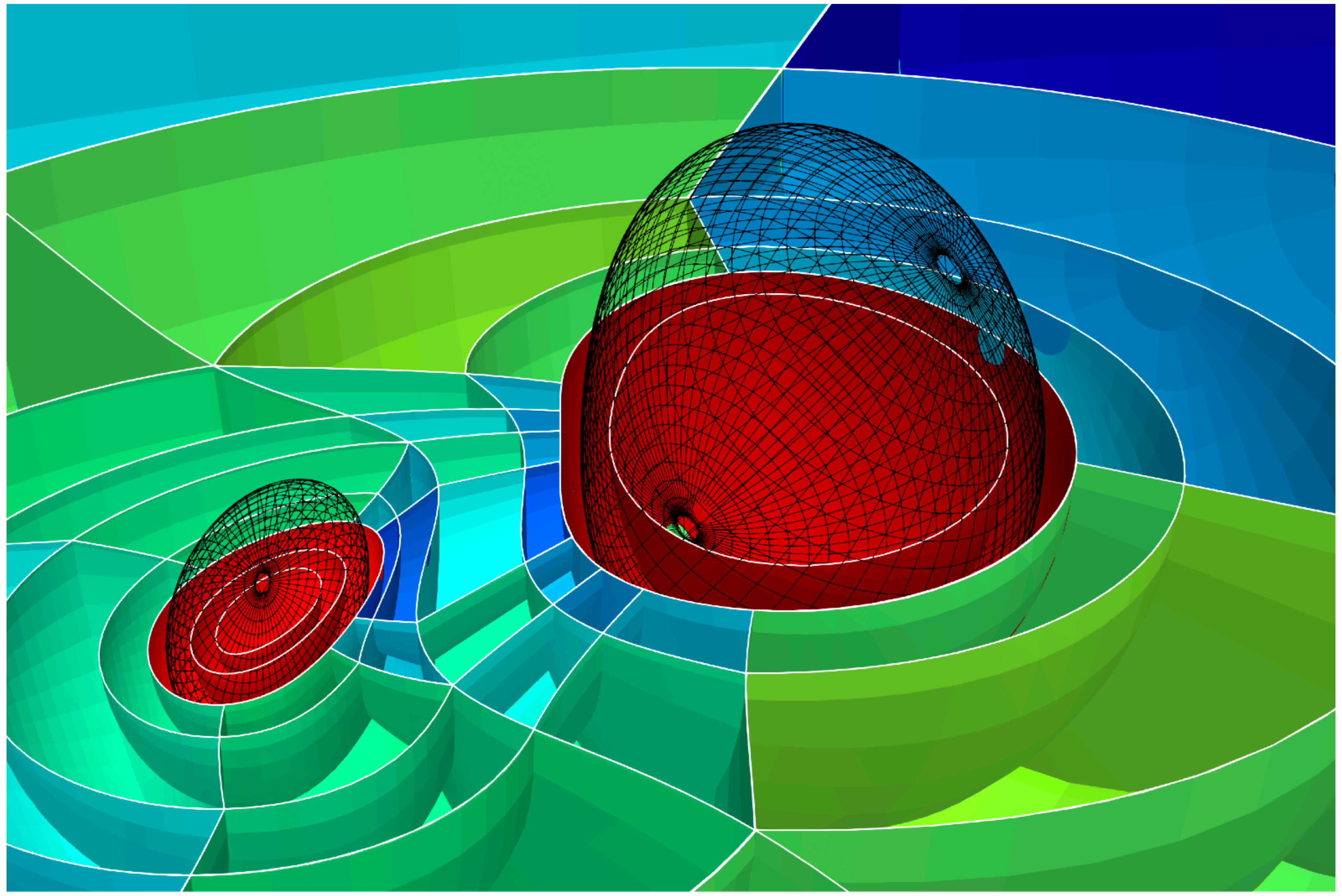
$$u'(x,t) = \sum_{k=1}^{N} \tilde{u}_k(t)\Phi'_k(x)$$

- ***Exponentially fast convergence***
  - for smooth problems

BH Excision

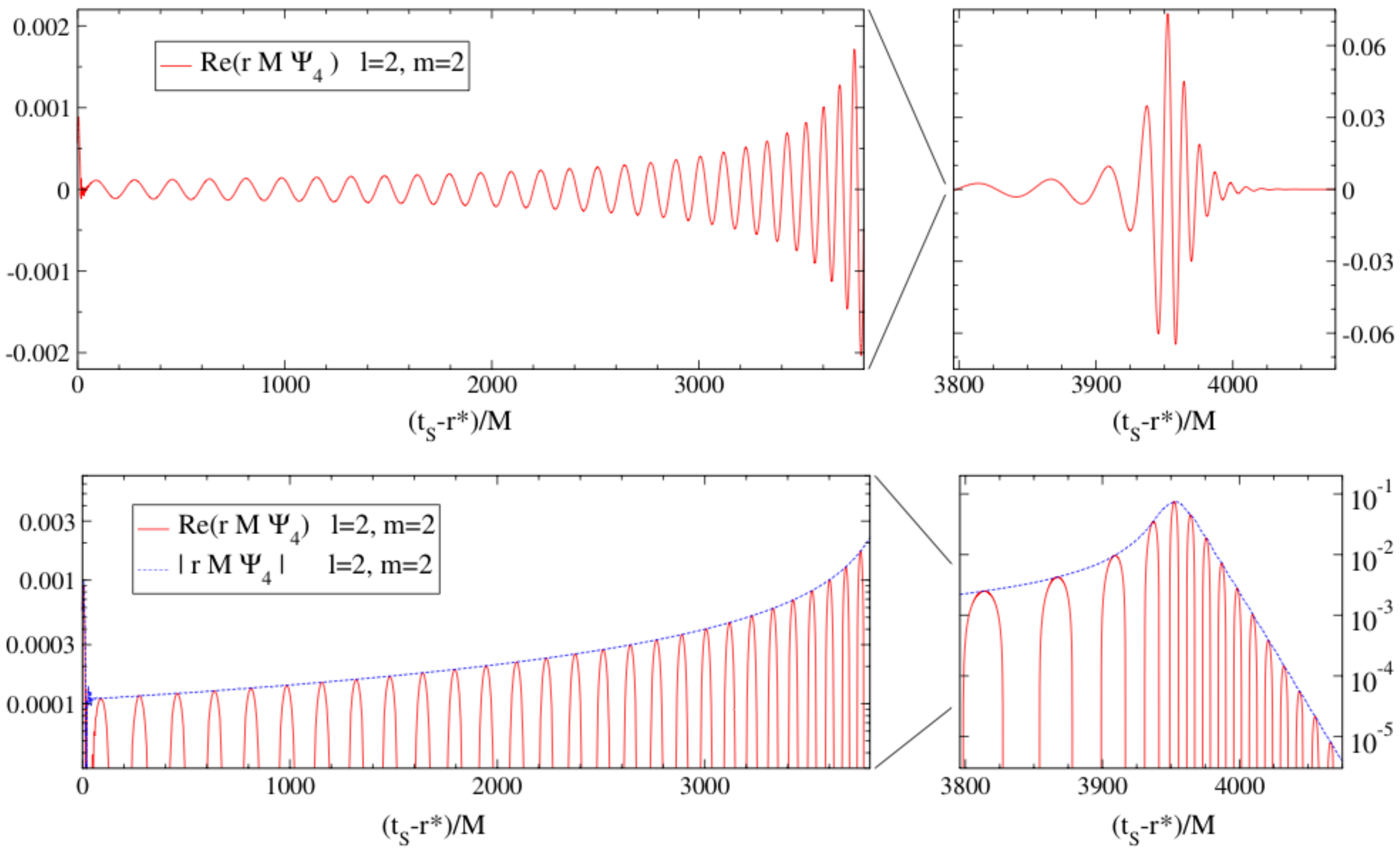
- Excise inside BH horizons
- Excision boundaries:
  - **follow BHs continuously**
  - conform to shape of AH



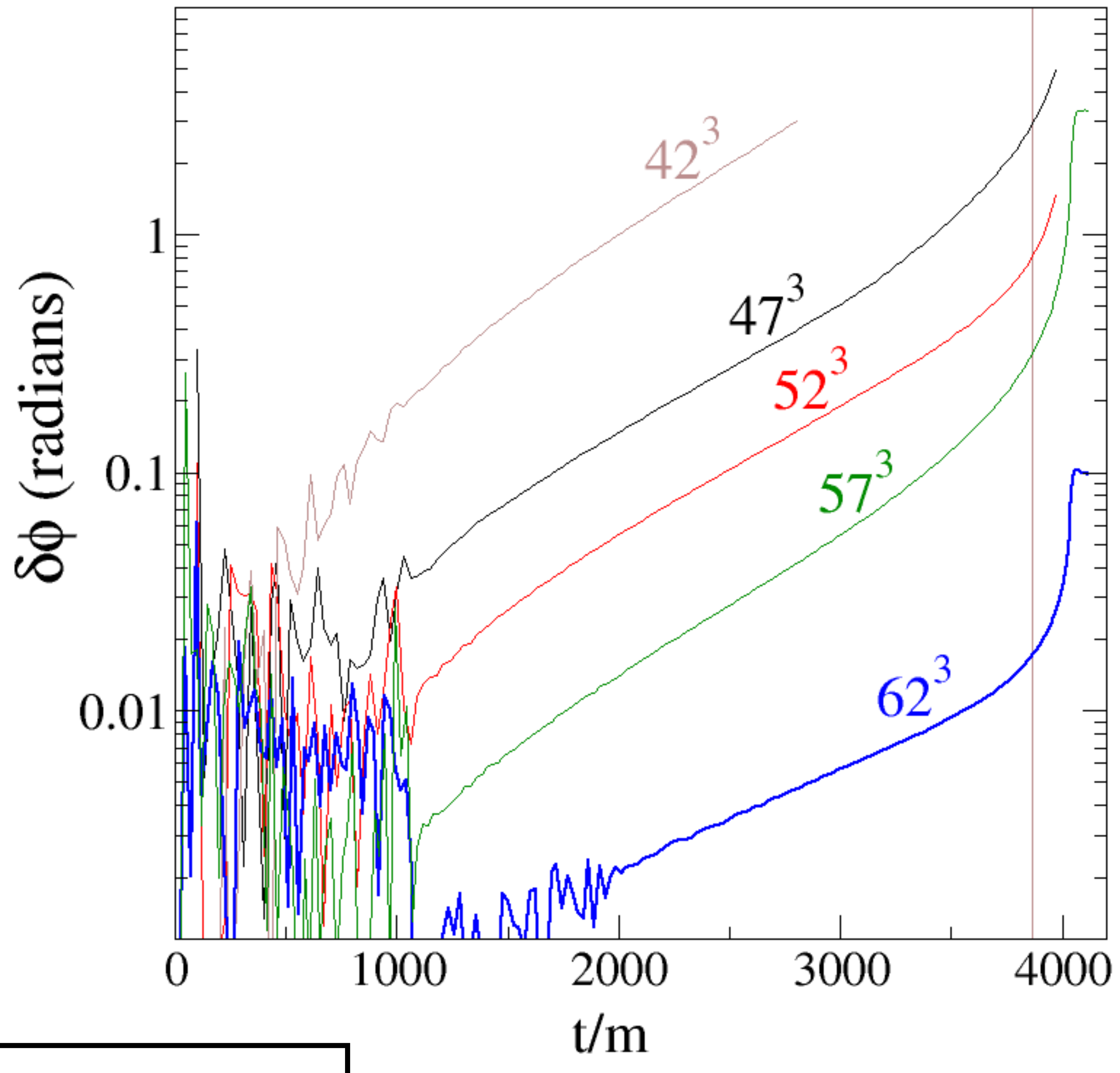


Scheel, HP+ 08, Szilagyi+ 08, Hemberger+ 13

Accuracy of SpEC (circular inspiral)



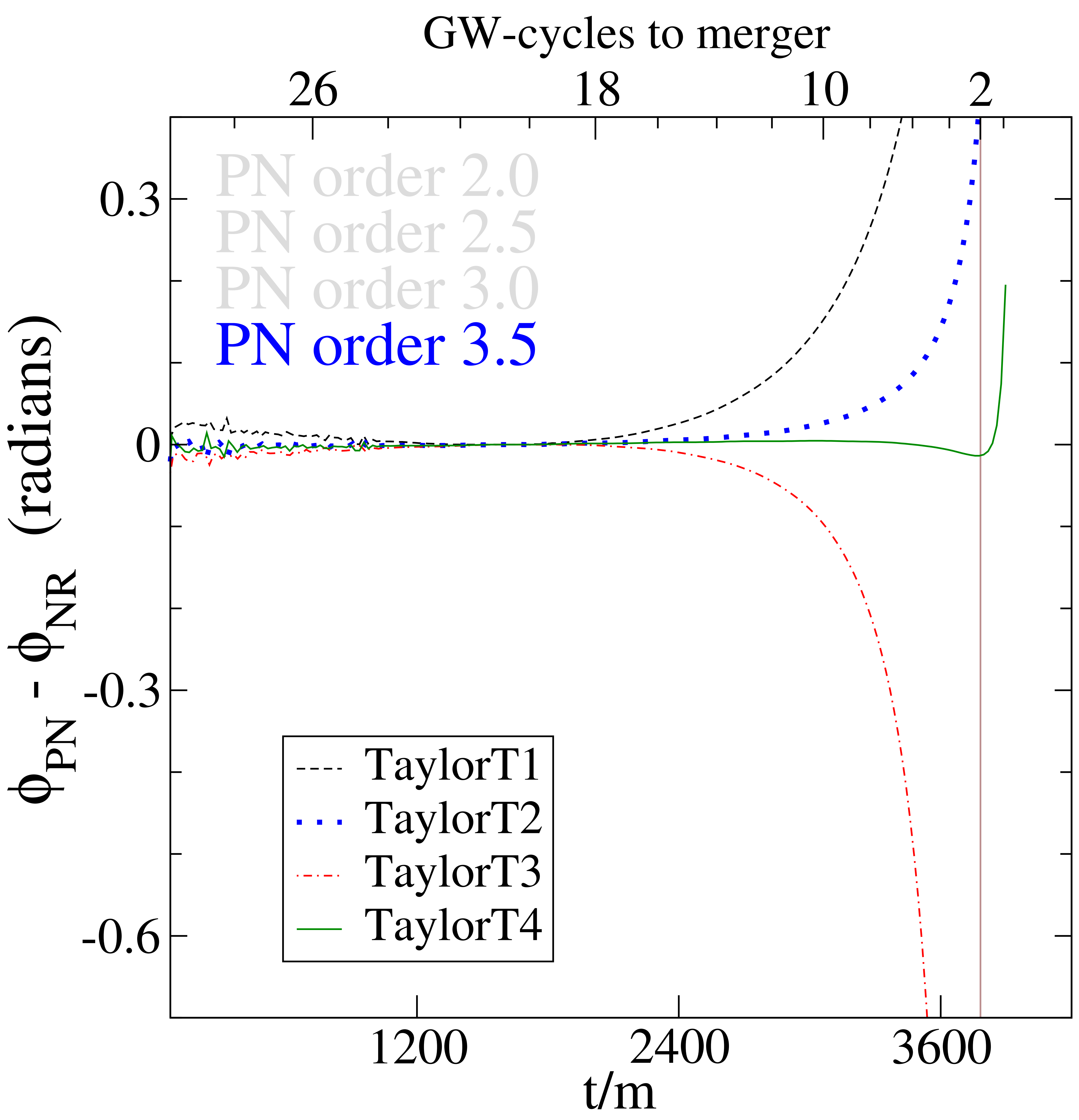
Phase error at different resolutions

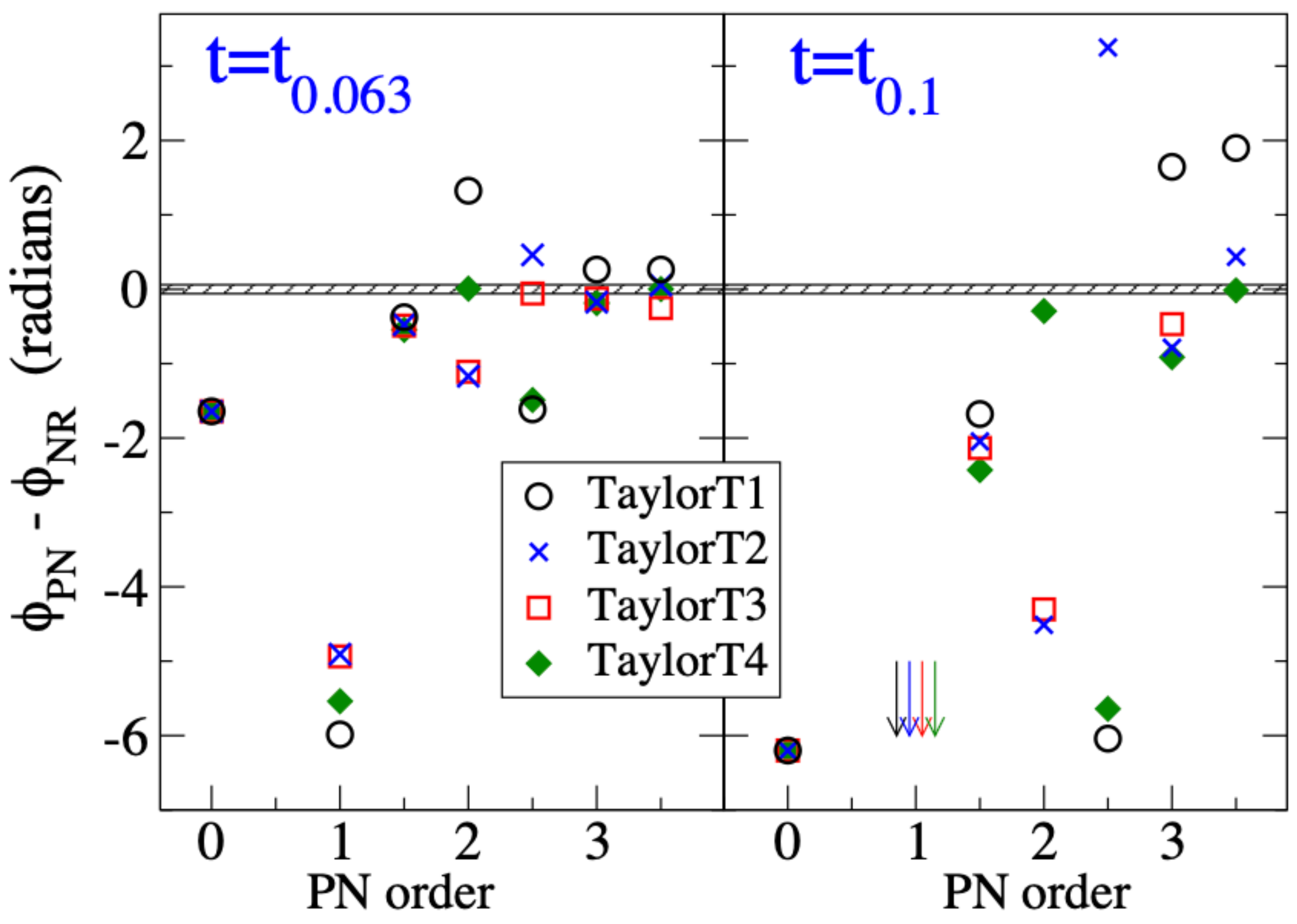


- ***Rapid convergence*** *due to spectral methods*
- *Small errors due to moving grid*
- *Excellent code for long inspirals (but mergers hard)*

Scheel,HP+ 09

post-Newtonian vs. NR





TaylorT1...T4

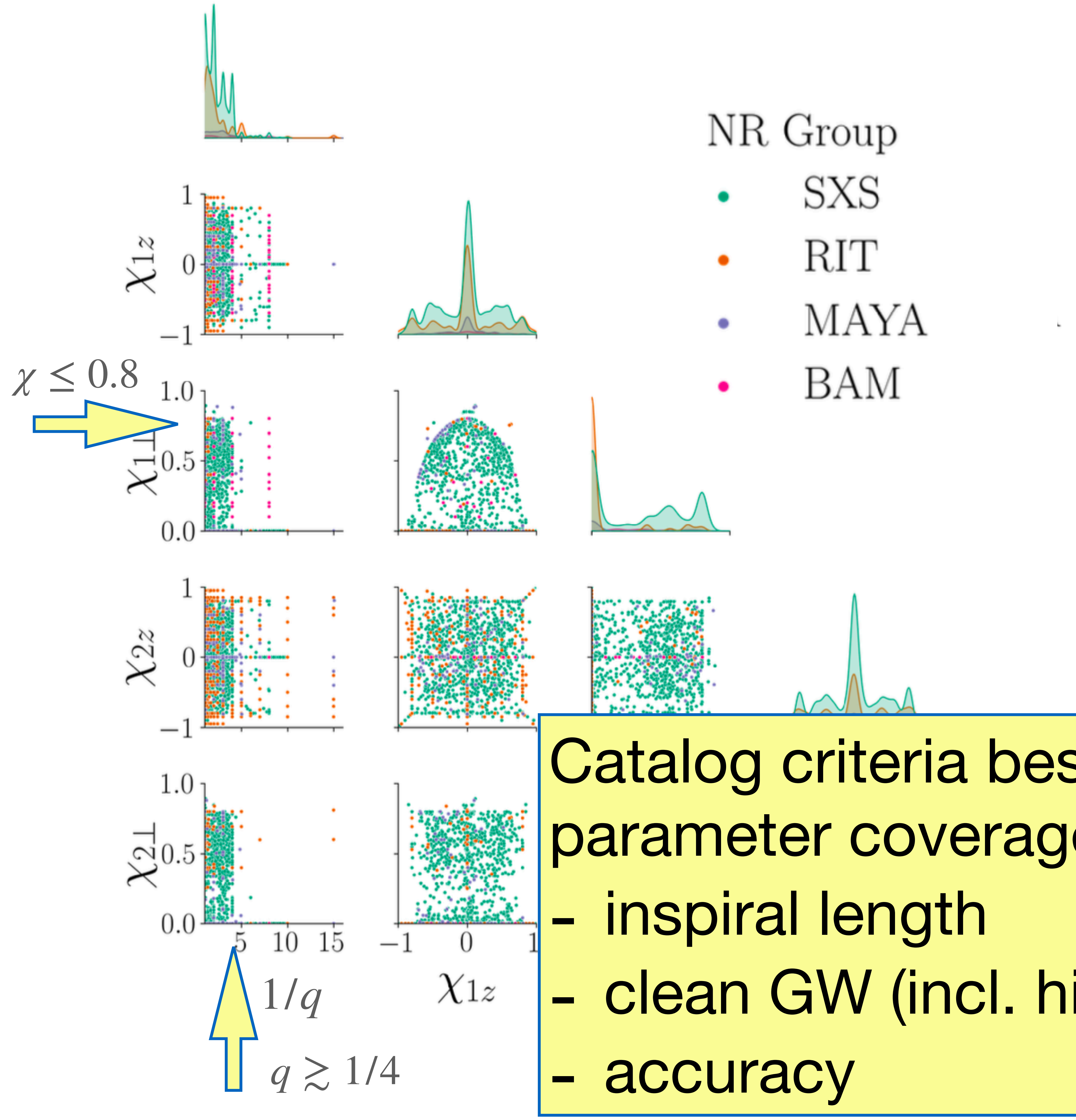
Different choices to truncate energy balance equation

$$\frac{dE}{dt} = -F_{\rm GW}$$

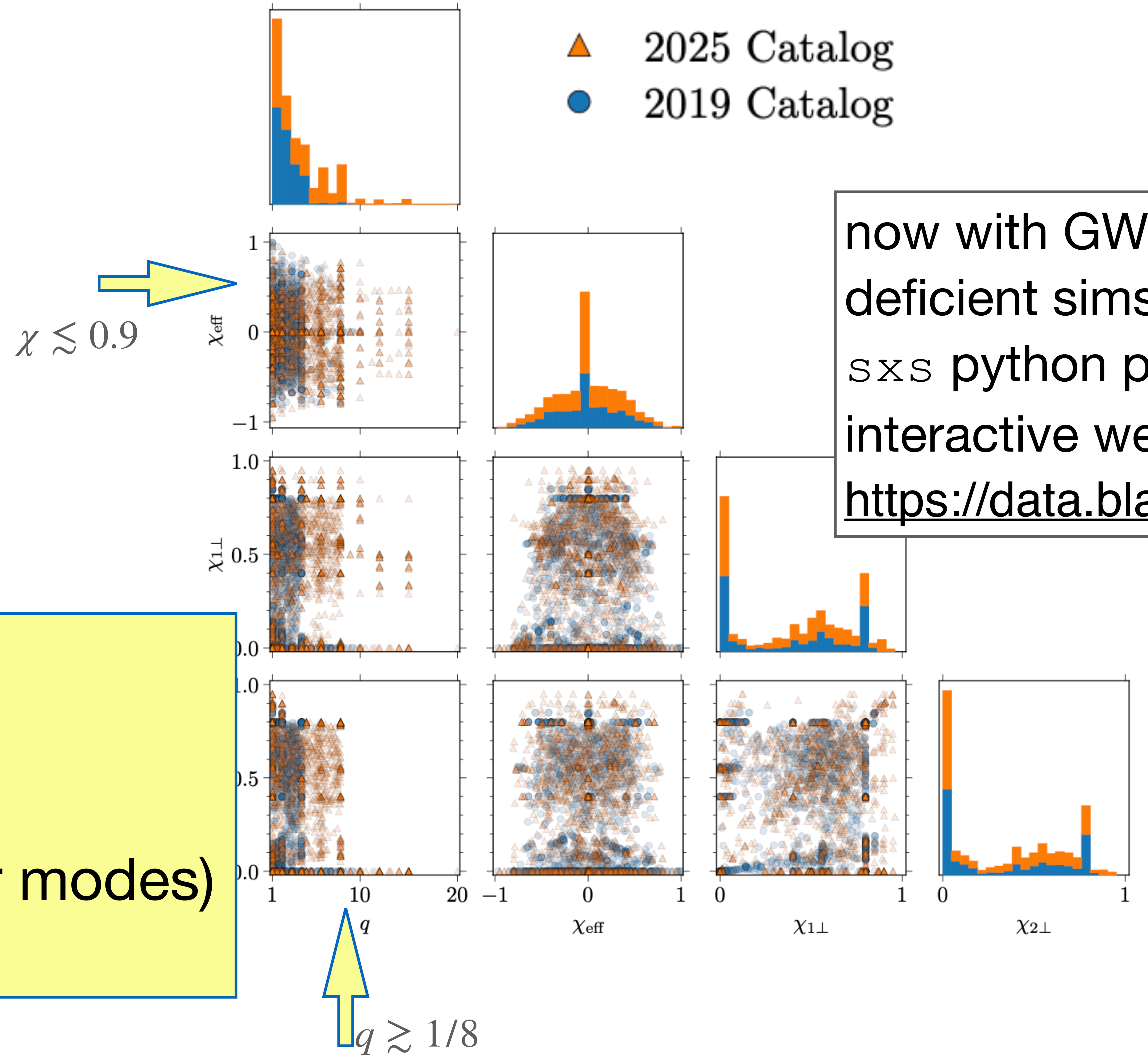
Boyle..HP+ 07

parameter space coverage (bound orbits)

LISA waveform whitepaper
Living Reviews Relativity
(arXiv:2311.01300)



SXS Catalog update
Scheel+ (CQG 2025) arXiv:2505.13378



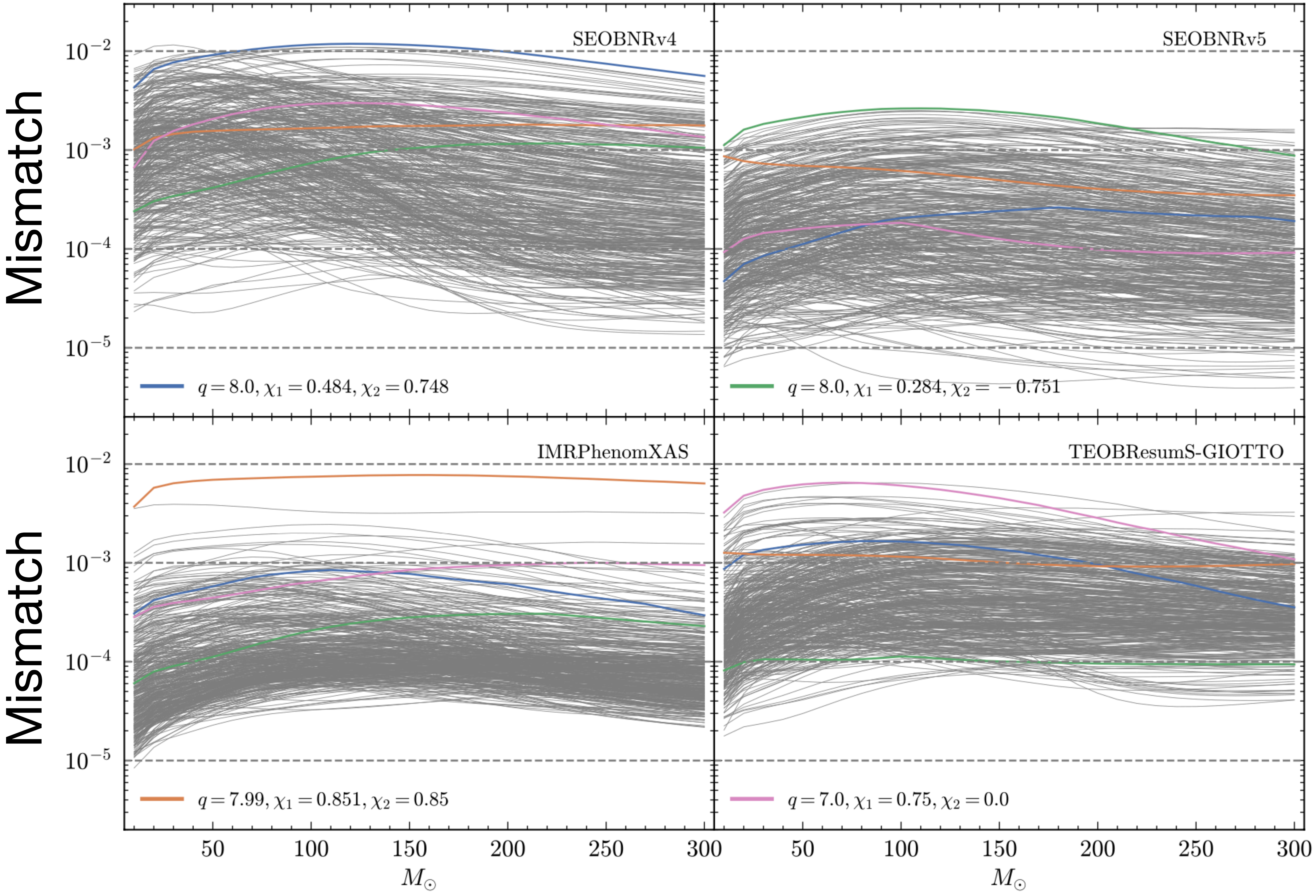
now with GW memory
deficient sims deprecated
`sxs` python package
interactive web-interface
https://data.black-holes.org

Catalog criteria besides parameter coverage:
- inspiral length
- clean GW (incl. higher modes)
- accuracy

Major application of NR: waveform modeling

- **Assess accuracy** of waveform models
- Determine **importance of improvements**
  - higher order
  - additional physics
    - higher modes, precession, memory
- **Calibrate parameters** in model to improve agreement
    - GW modes & fluxes
    - merger dynamics
    - ringdown attachment
    - ...

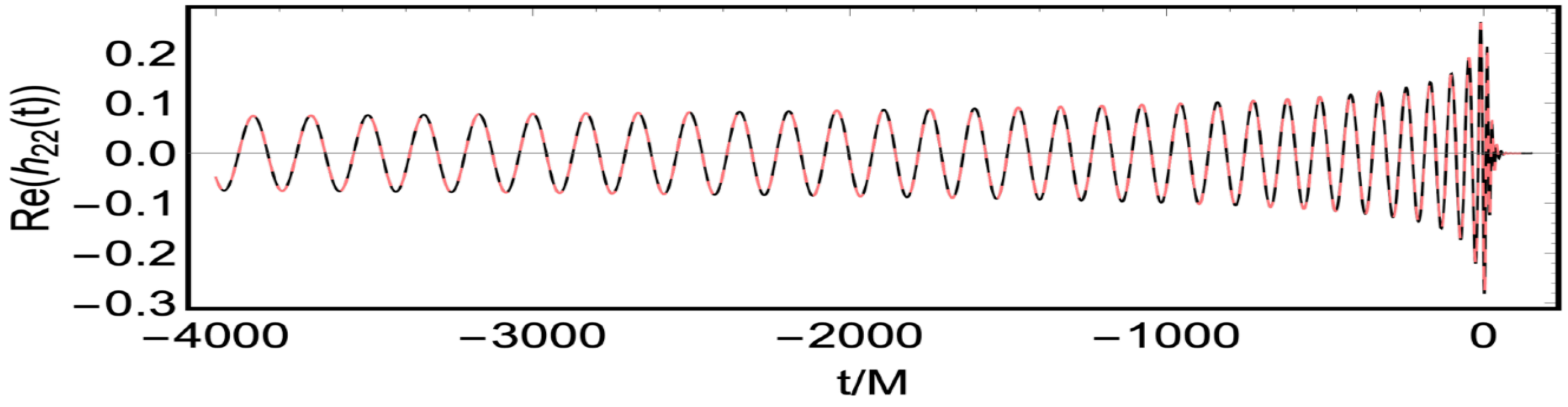
Errors in 4 waveform models as compared to 442 NR simulations



As example: Pompili.+ 2303.18039

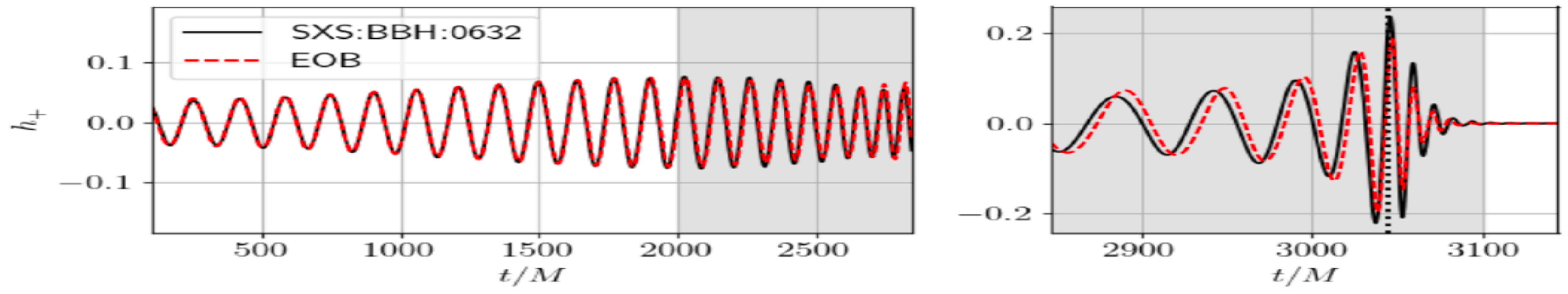
Waveform models: Some example plots

aligned-spin (no precession)

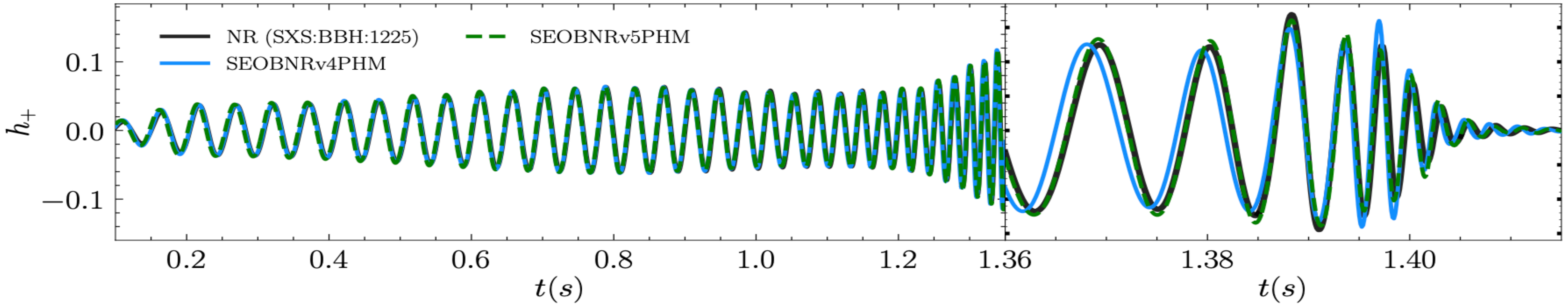


IMRPhenomT
Estelles+ 20

generic spin (orbital plane precession)

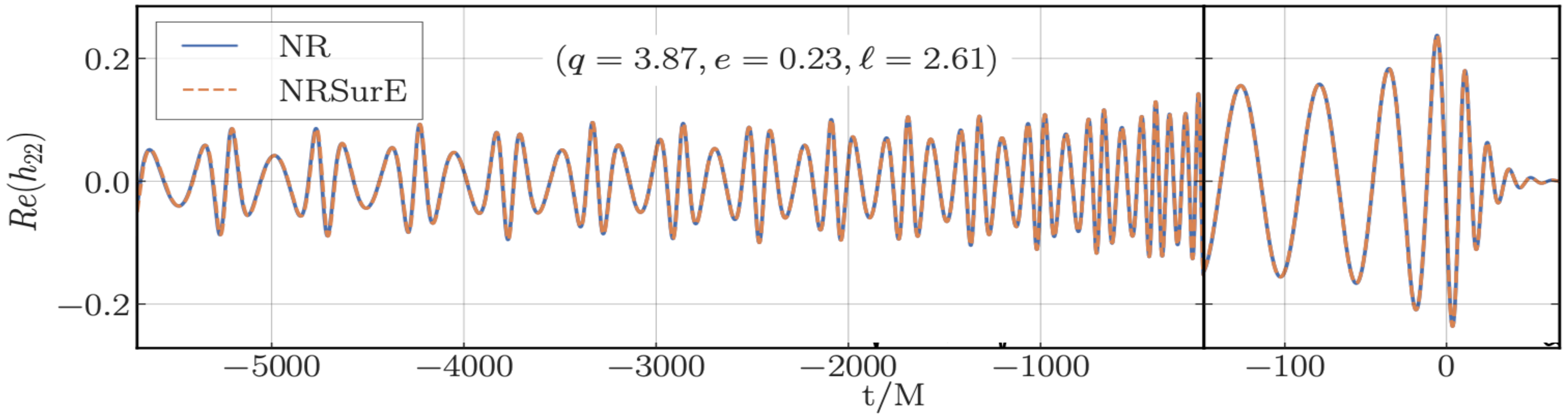


TEOBResumS
Gamba+ 22



SEOBNRv5PHM
Ramos-Buades+ 23

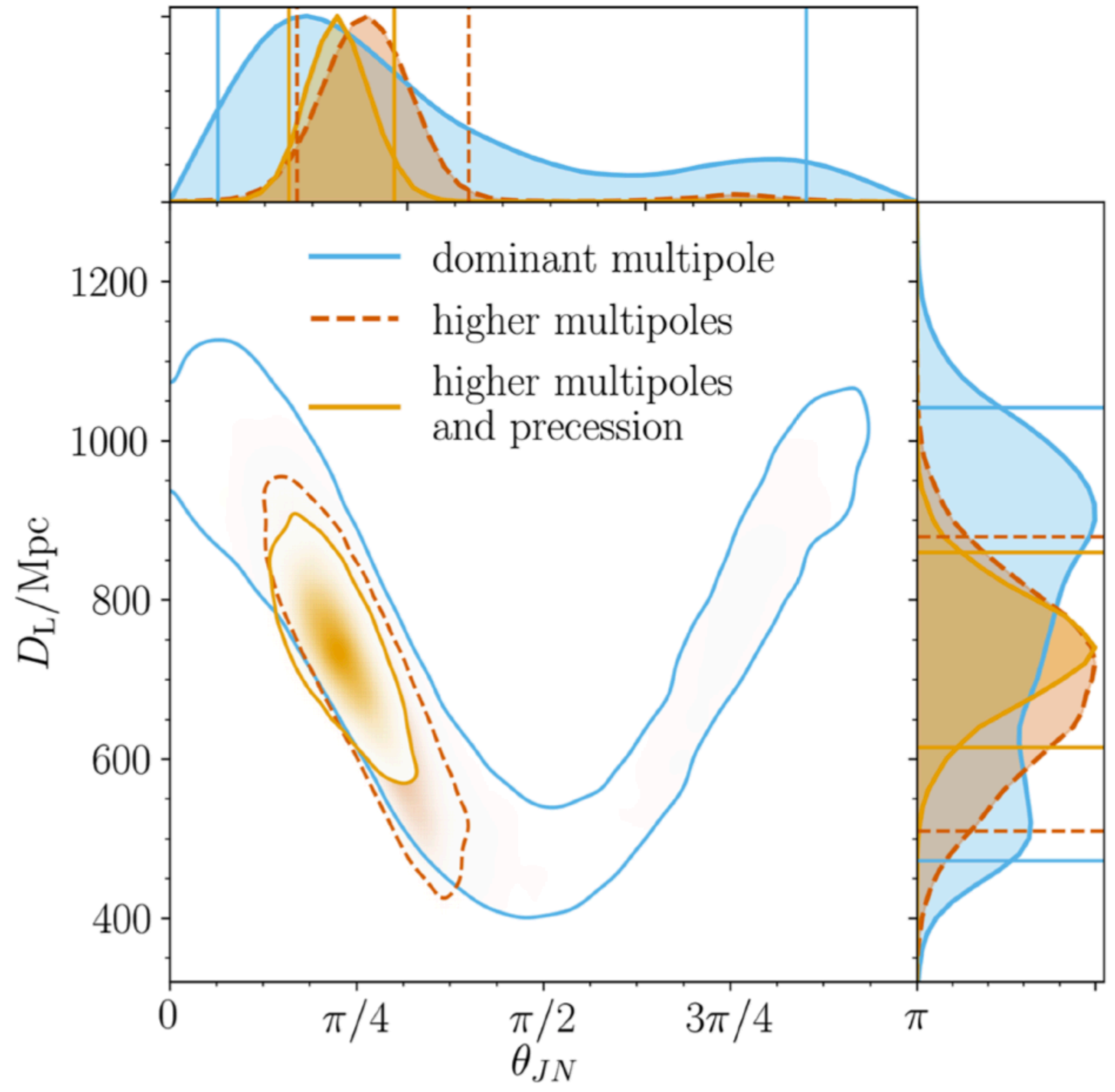
non-precessing eccentric



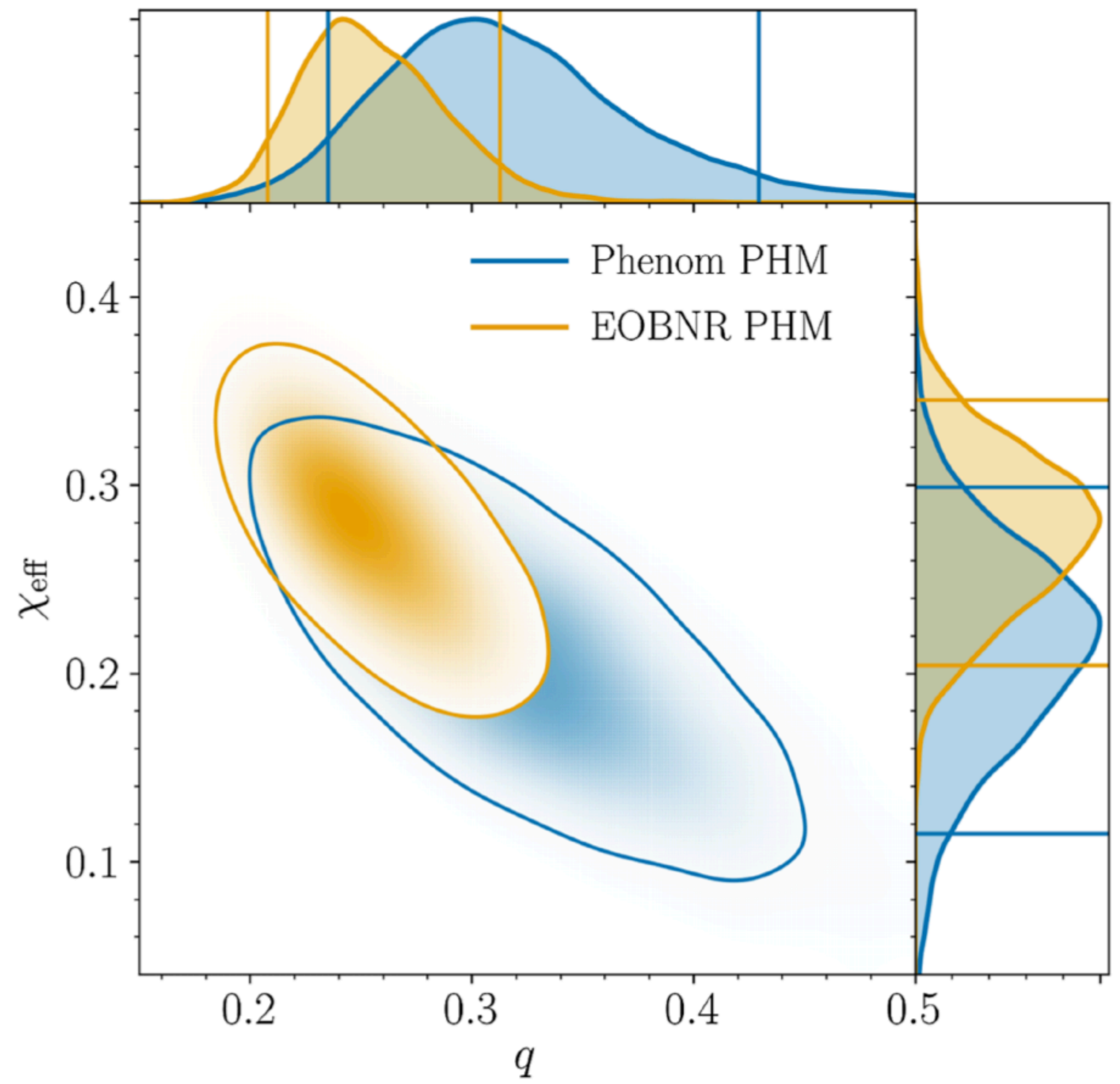
NRsurE
Nee+ 25

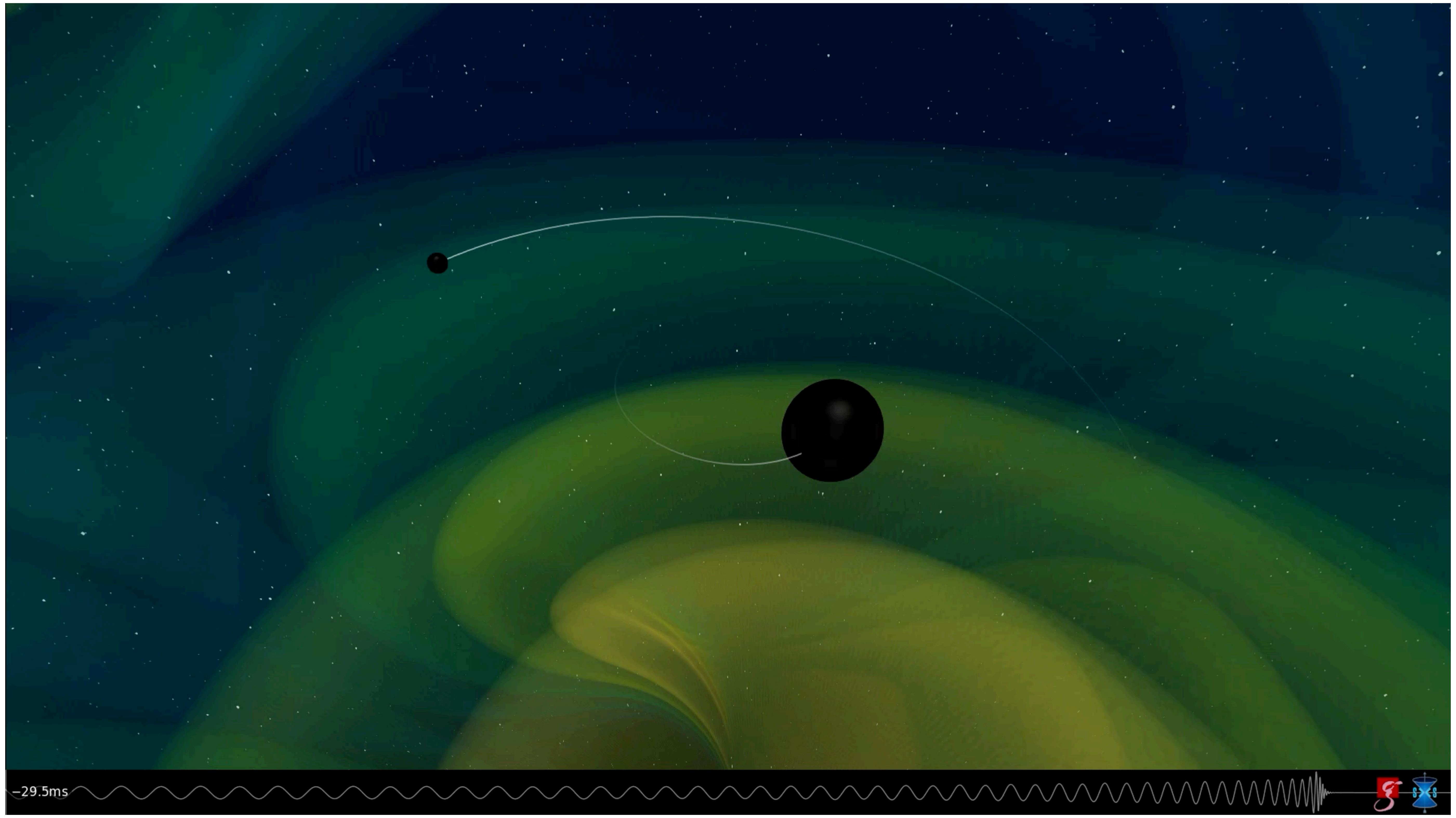
Case study: GW190412 $(30+8)M_\odot$ at SNR=19

higher-modes break degeneracies



Abbott et al, PRD 102 043015 (2020)





N. Vu, HP

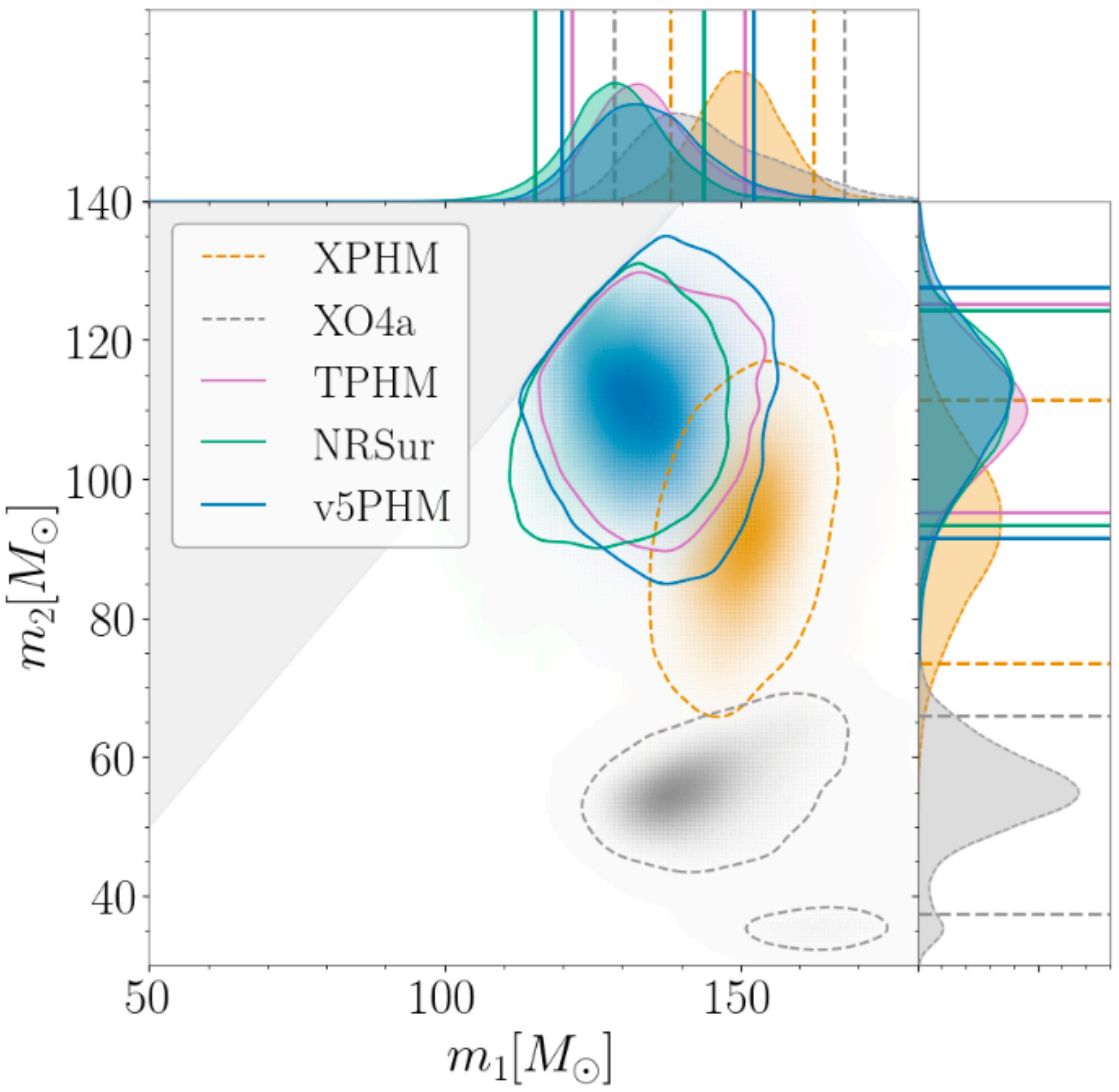
some differences between waveform models remain even at current SNR

IMRPhenomPv3HM Khan et al 19; 20
SEOBNRv4PHM Ossokine+ 20

Scientific payoff depends on accurate waveform models

- **GW231123**:
  - large total mass
  - high spins
  - precession

- **Signal with large systematics**



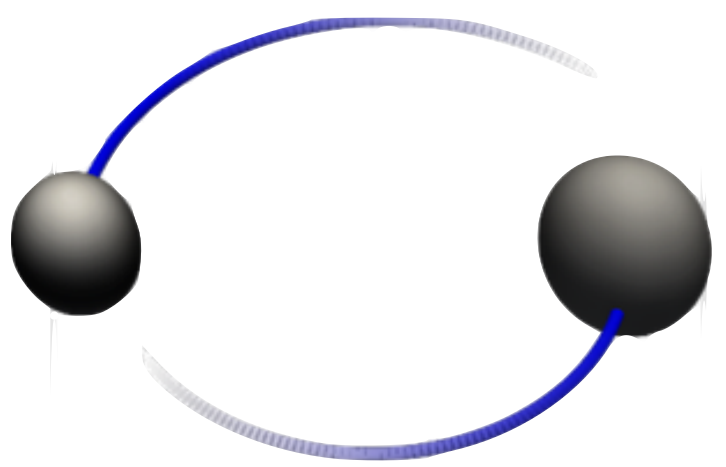
GW231123 arXiv:2507.0821

NR challenges

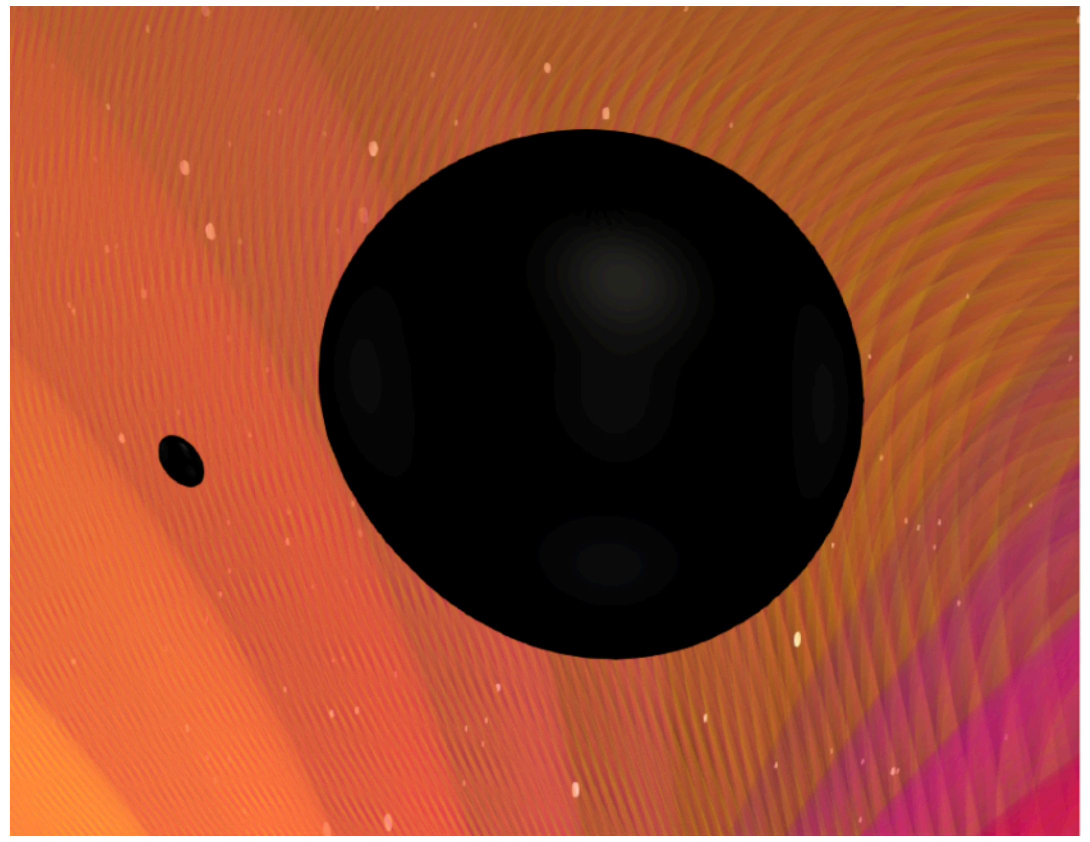
- $q \ll 1$: **scale-separation** leads to cost $\sim 1/q^2$
  - $\Delta t_{\rm CFL} \propto m_2 \sim m\, q$ combined with $T_{\rm inspiral} \propto 1/q$
- eccentric & scattering: **fast dynamics**
- **large spins**
  - BH excision boundary must be $\in [r_-, r_+]$. But $r_- \to r_+$
  - in puncture gauge, $r_{\rm AH}$ becomes very small
  - puncture initial data limited to $\chi_{\rm relaxed} < 0.94$ (Lovelace+ '08)
- **GW extraction for scattering**
  - $R_{\rm GW} \gg |\vec{c}_1 - \vec{c}_2|$ difficult to achieve
  - little time for junk radiation to dissipate, especially at large γ

Binaries at all mass-ratios

q=1

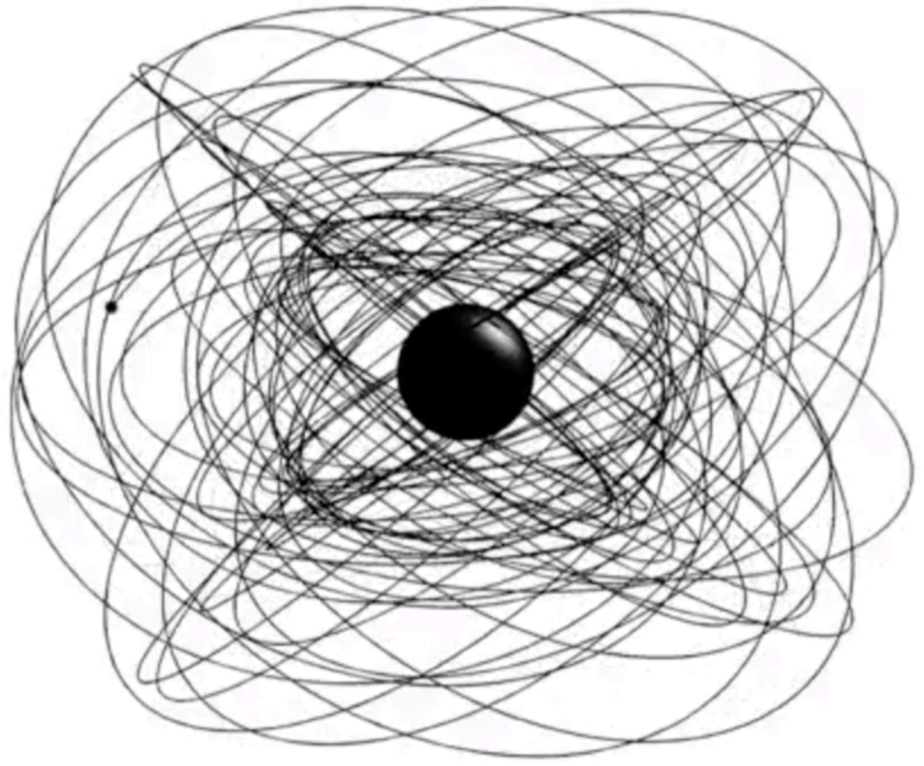


LVC



Vu, HP

NASA



S. Drasco

GW150914

GW190814

Intermediate mass BH
$(10 + 1000)M_\odot$
$(10^3 + 10^6)M_\odot$

EMRI
$(10 + 10^6)M_\odot$

?

NR
$q \gtrsim 1/20$

GSF

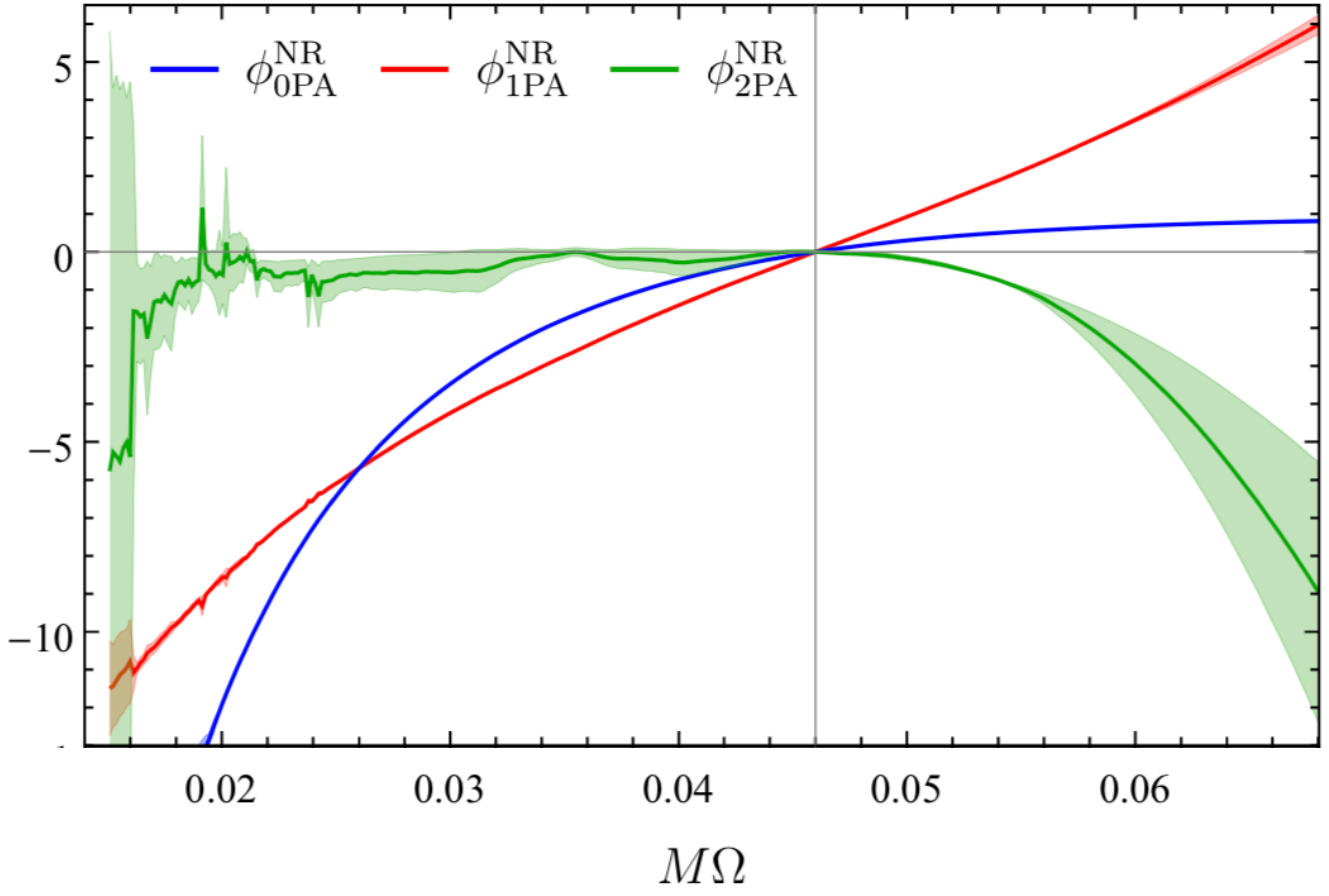
expansion in symmetric mass-ratio $\nu = q/(1+q)^2$

$$\text{Cost} \sim \frac{T}{\Delta t} \sim \frac{q^{-1}(M\Omega_i)^{-8/3}}{q} \sim \mathbf{q^{-2}\,(M\Omega_i)^{-8/3}}$$

Extract GSF information directly from NR

$$\Phi(M\Omega) = \frac{1}{\nu}\Phi_0(M\Omega) + \Phi_1(M\Omega) + \nu\,\Phi_2(M\Omega) + \ldots + \frac{1}{\nu^{1/2}}\Phi_{\rm resonances} + \frac{1}{\nu^{1/5}}\Phi_{\rm plunge}$$

- **Fit $\Phi_{NR}(\Omega_{NR})$ to GSF expansion**
  - 55 NR sims at different ν
  - non-spinning
- Extract different orders in ν
  - $\Phi_0(M\Omega)$ - agrees with 0PA
  - $\Phi_1(M\Omega)$ - first computed here
  - $\Phi_2(M\Omega)$ - remarkably small



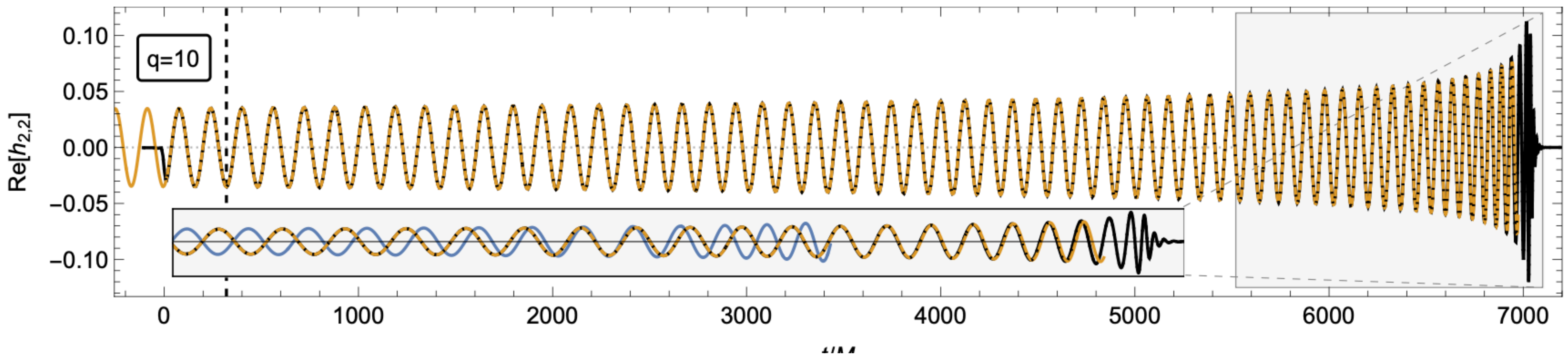
van de Meent, HP 2020

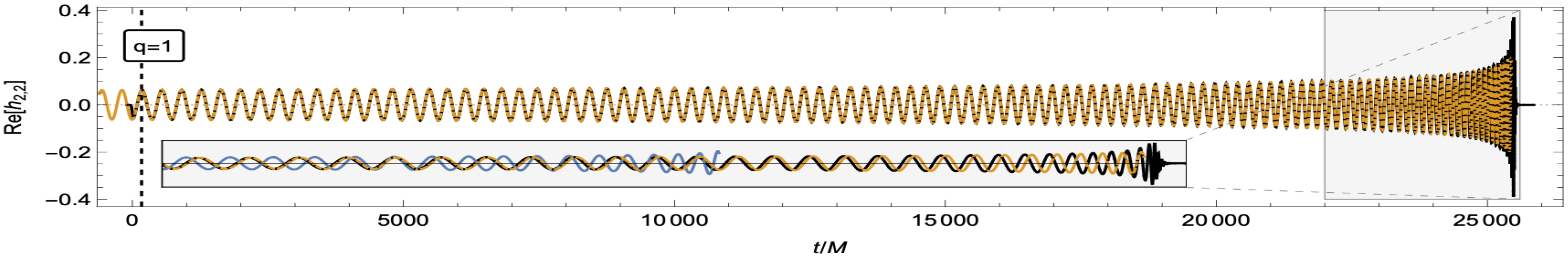
Actual 1PA calculation

- **High accuracy, even at q=1**
  - Zero spin, quasi-circular
  - Compared to SpEC NR

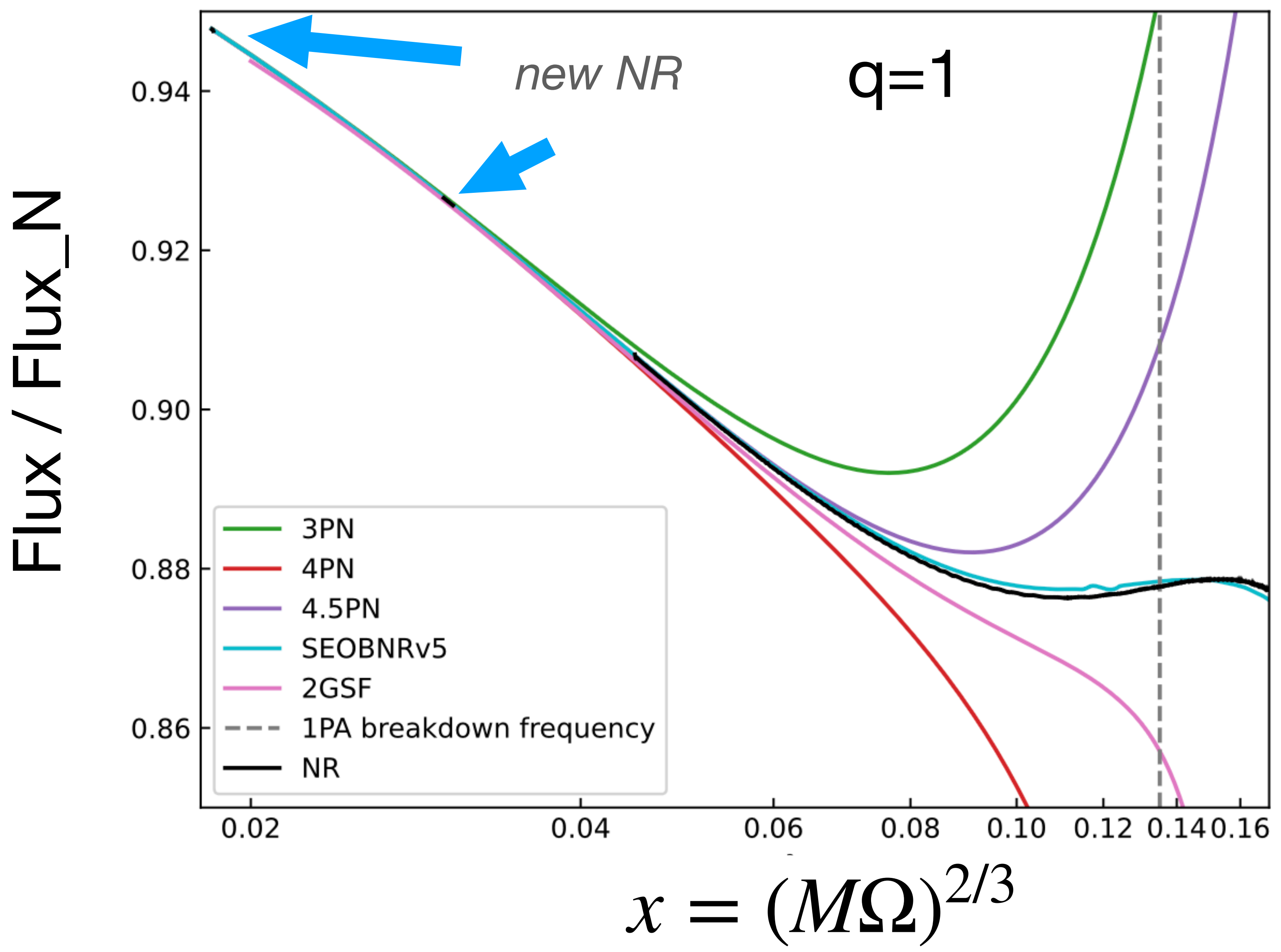
→ Niels Warburton (Tue)

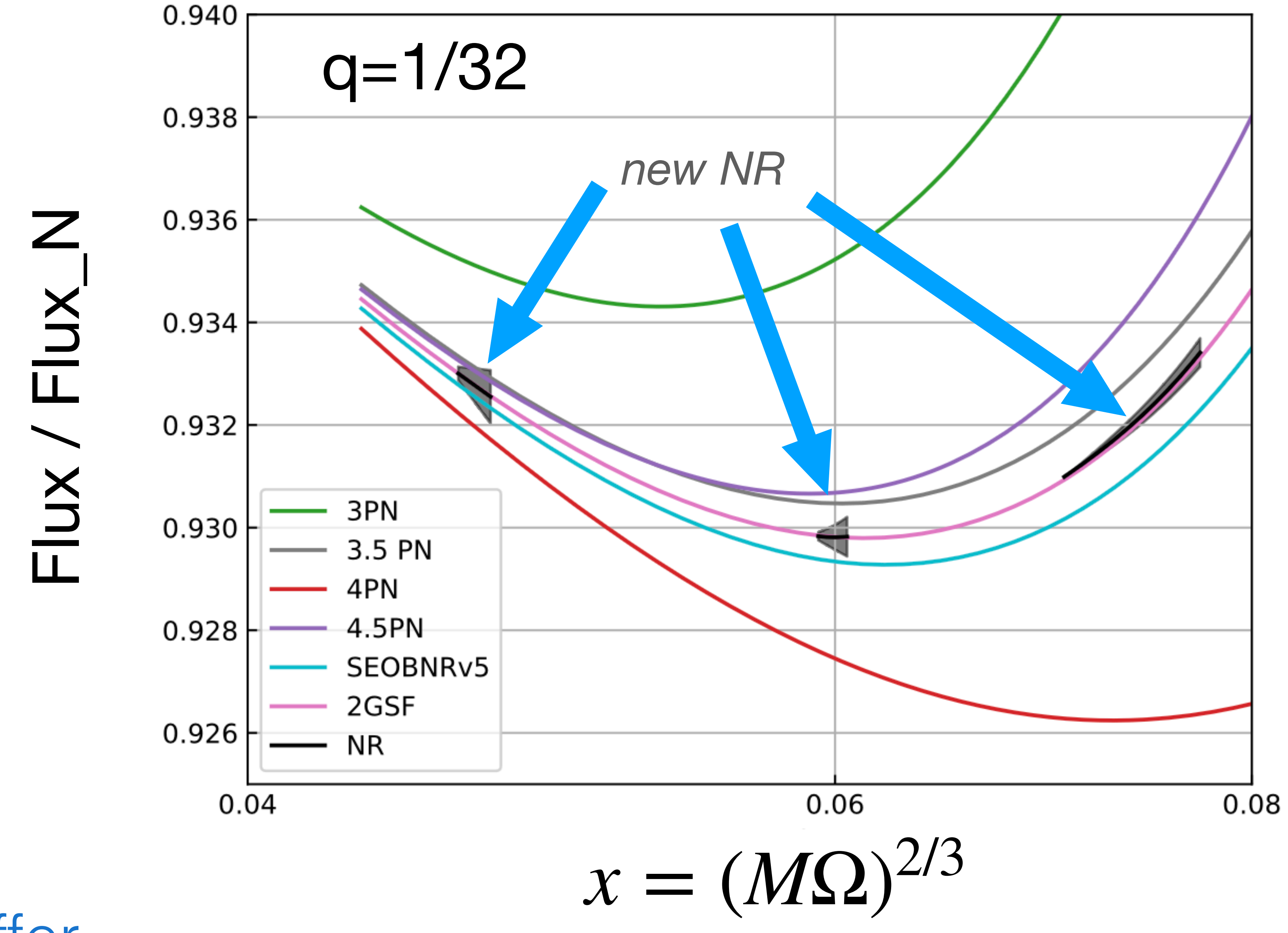
Wardell, Pound, Warburton, Miller, Durkan, Le Tiec 21

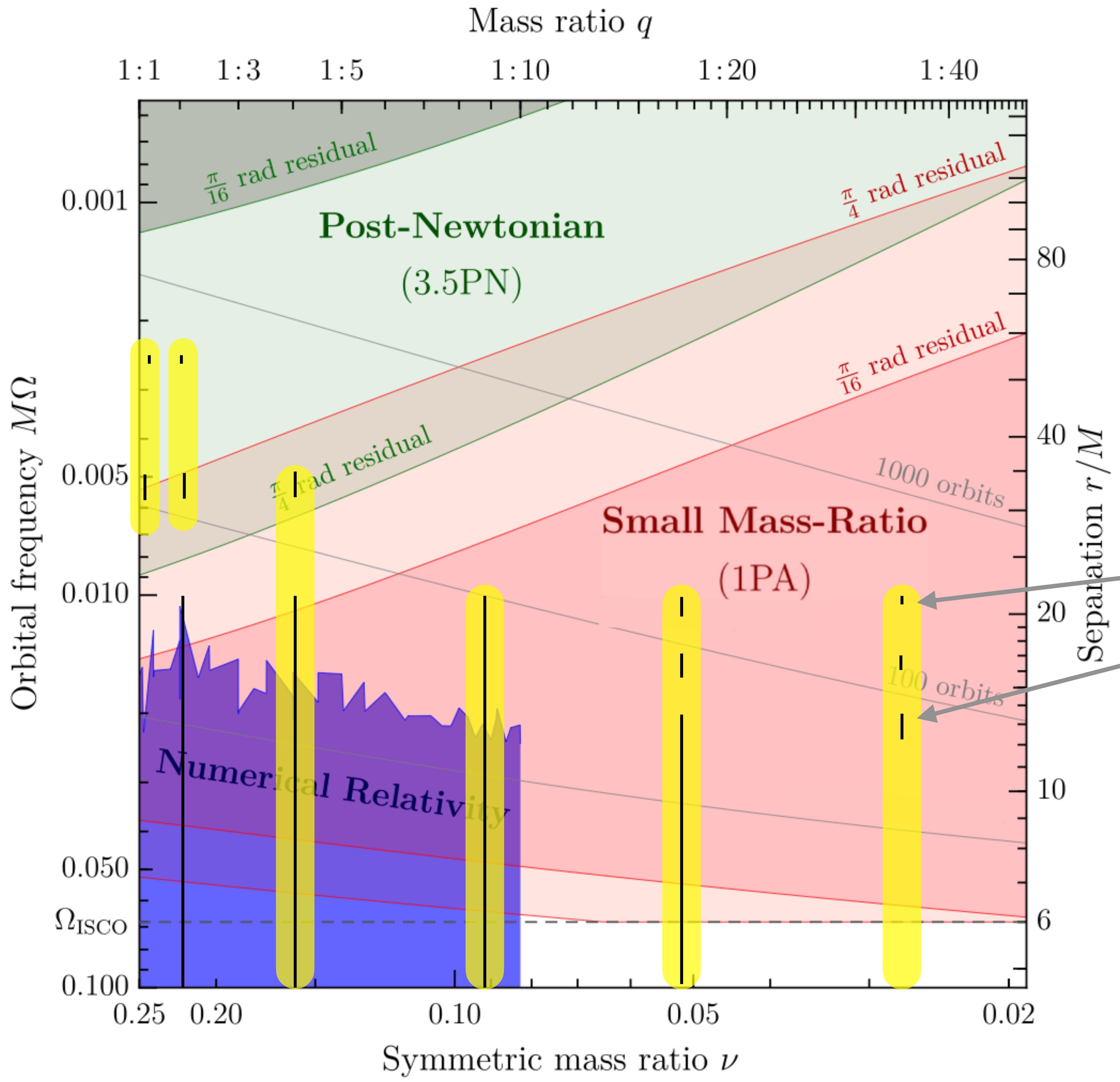




ongoing work: deeper into asymptotic regimes







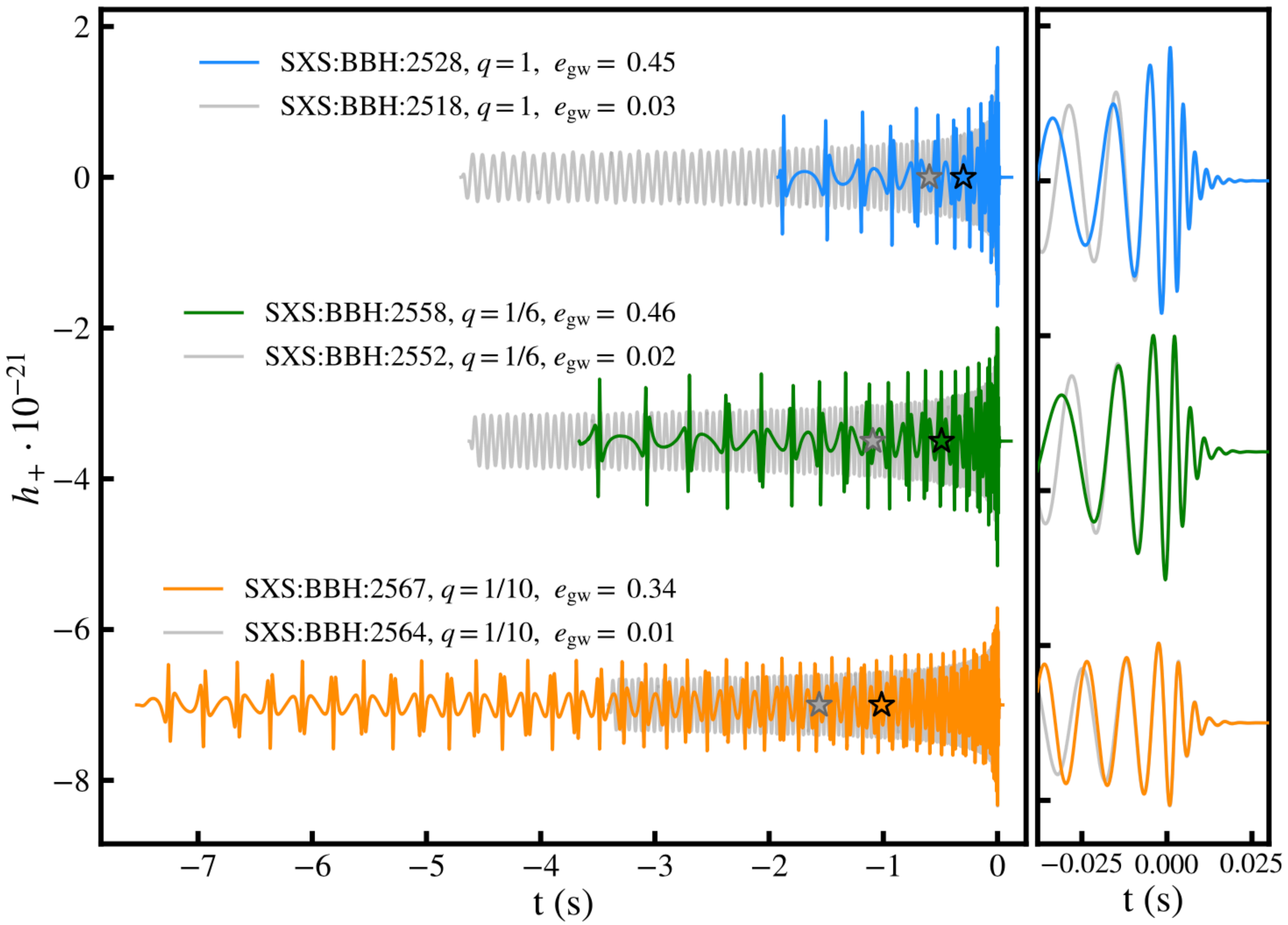
Oliver Markwell

Partial BBH inspirals several months of run-time

plot from van de Meent, HP 2020

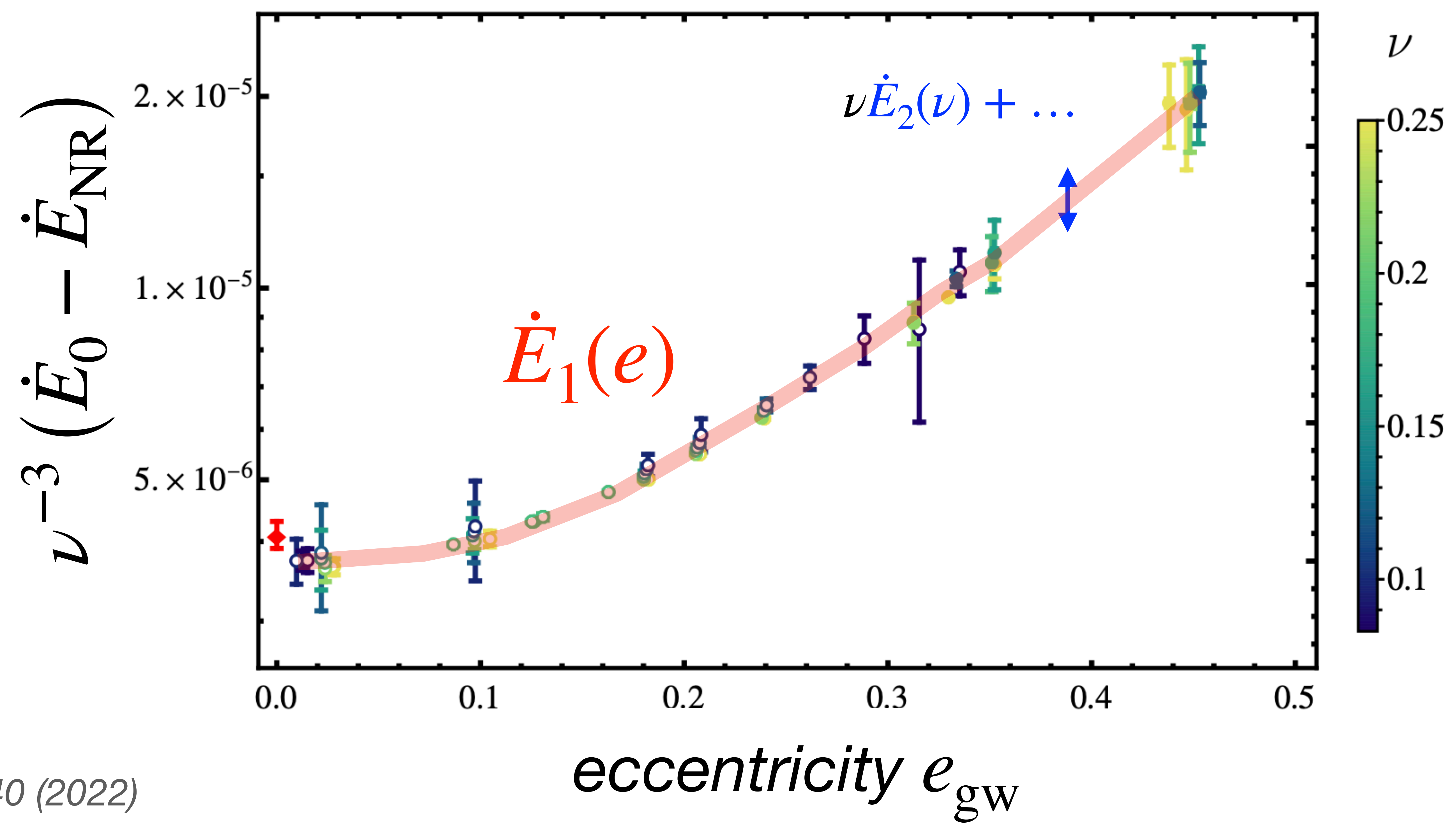
Eccentric BBH & GSF

NR sims: $e \lesssim 0.7, q \geq 1/10$



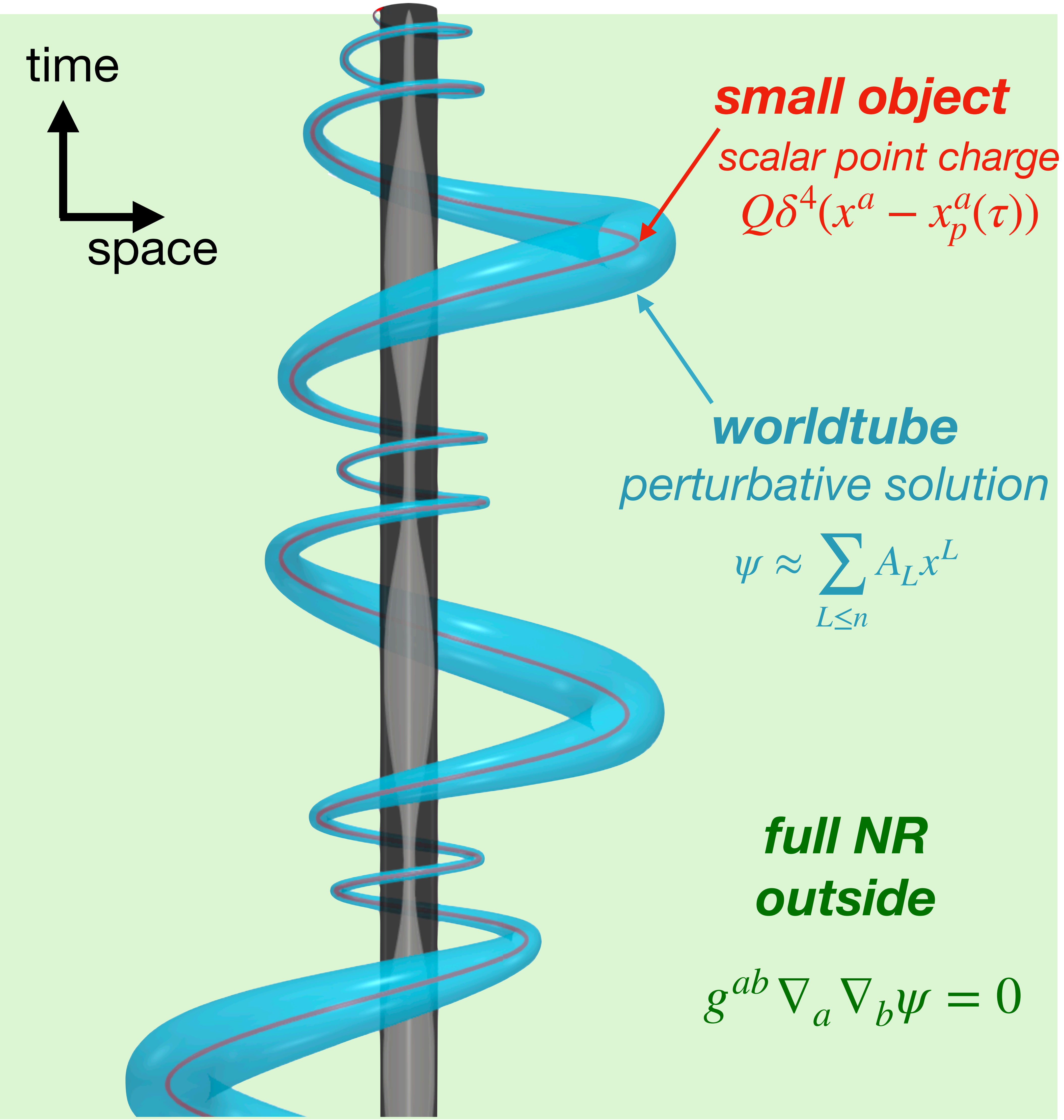
GW fluxes at fixed frequency

$$\dot{E}_{\mathrm{GW}}(\nu, e) = \nu^2 \dot{E}_0(e) + \nu^3 \dot{E}_1(e) + \nu^4 \dot{E}_2(e) + \ldots$$



Ramos-Buades, vdMeent, HP+. PRD 106 124040 (2022)

High-mass ratio: Worldtube method



spectre

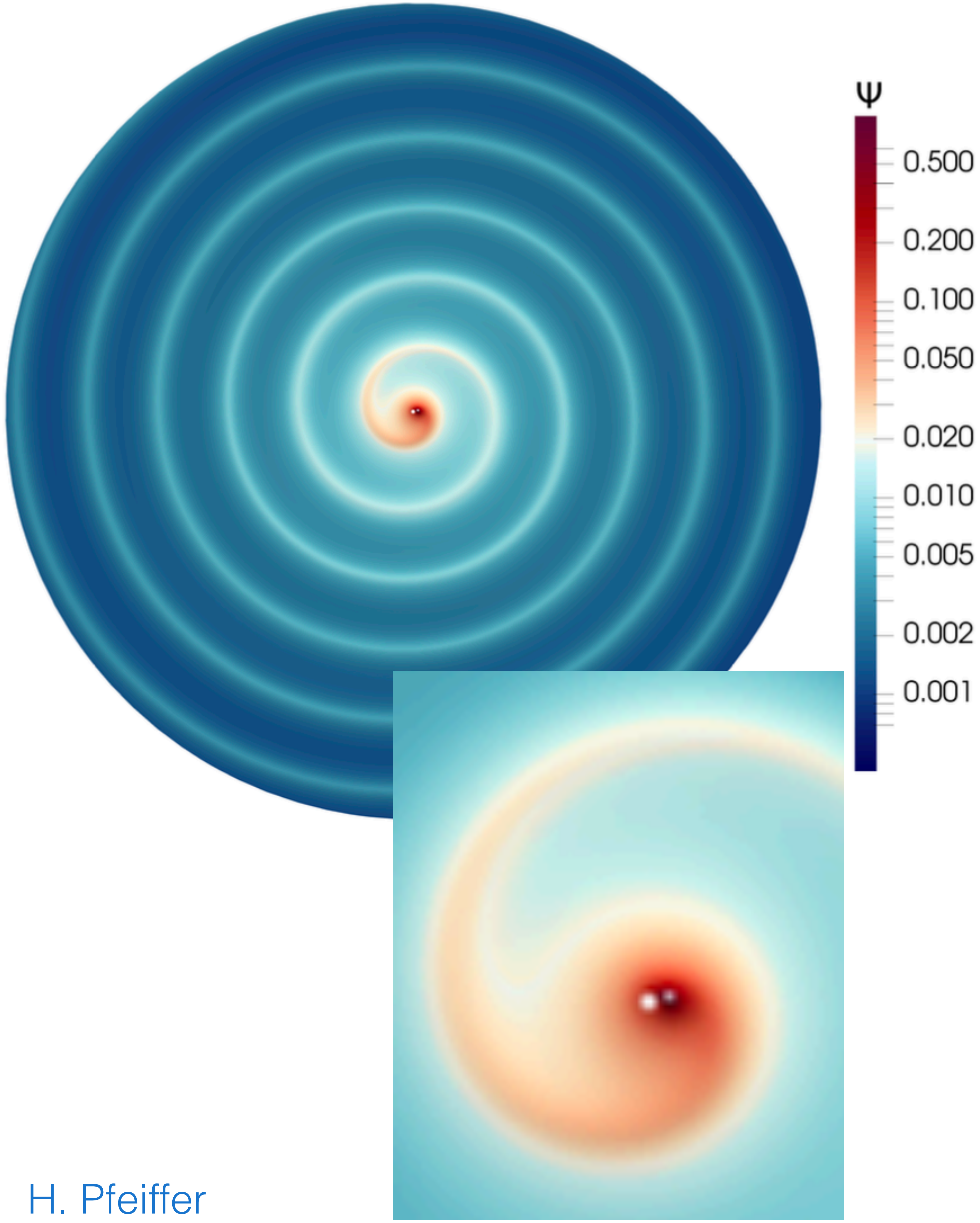
- Alleviates **Courant limit on time-step**
  - $\Delta t \propto R_{\text{worldtube}}$
- So far test-problem: scalar point-charge around BH

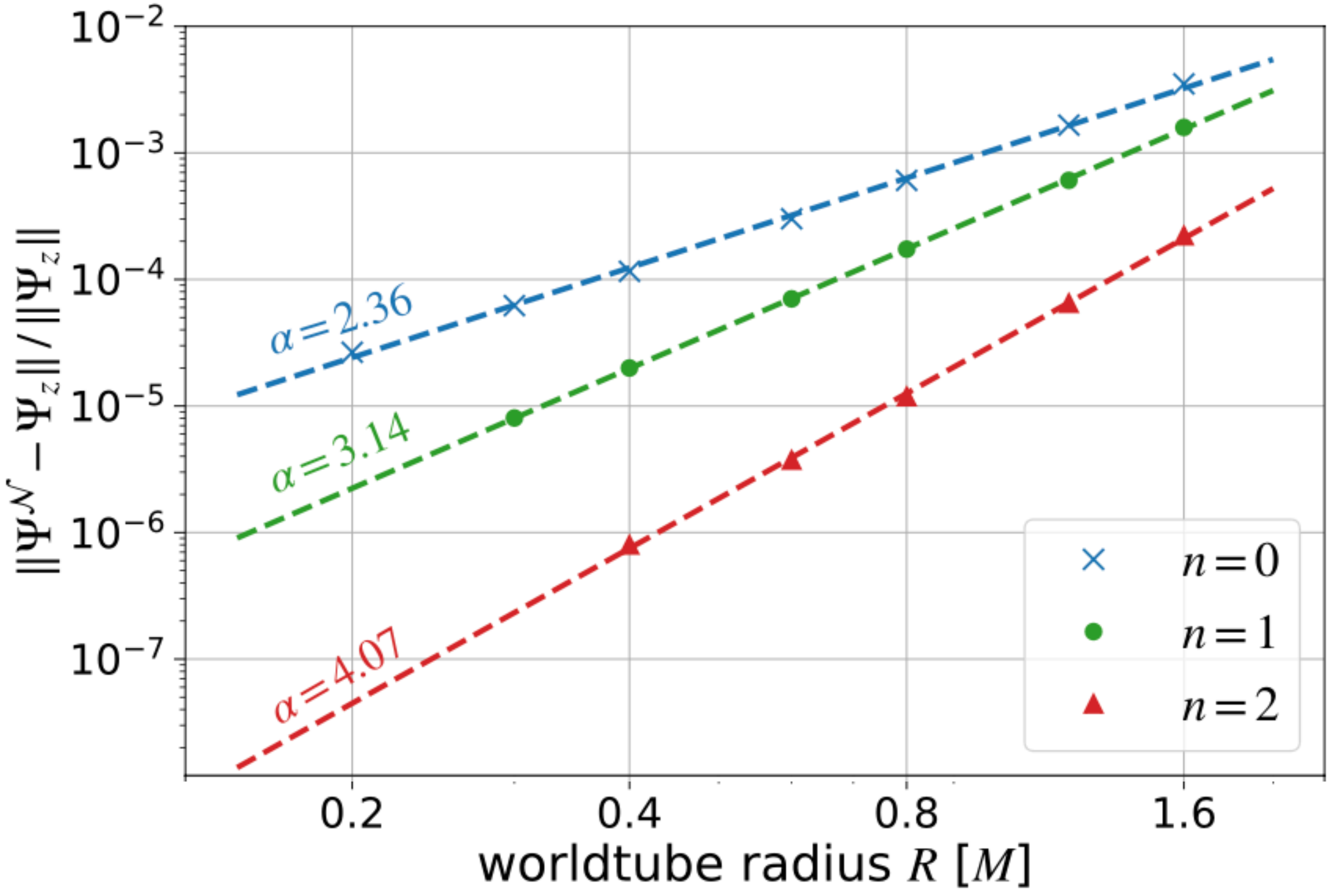
Dhesi, Rüter+ PRD (2021)
Wittek, Dhesi..HP+ PRD (2023)
Wittek, Pound, Barack, HP PRD (2024)
Wittek, Barack..HP+ PRL (2025)

Wittek+ 23,23,24
(also Dhesi, Rüter+ 21)

Circular orbits

spectre



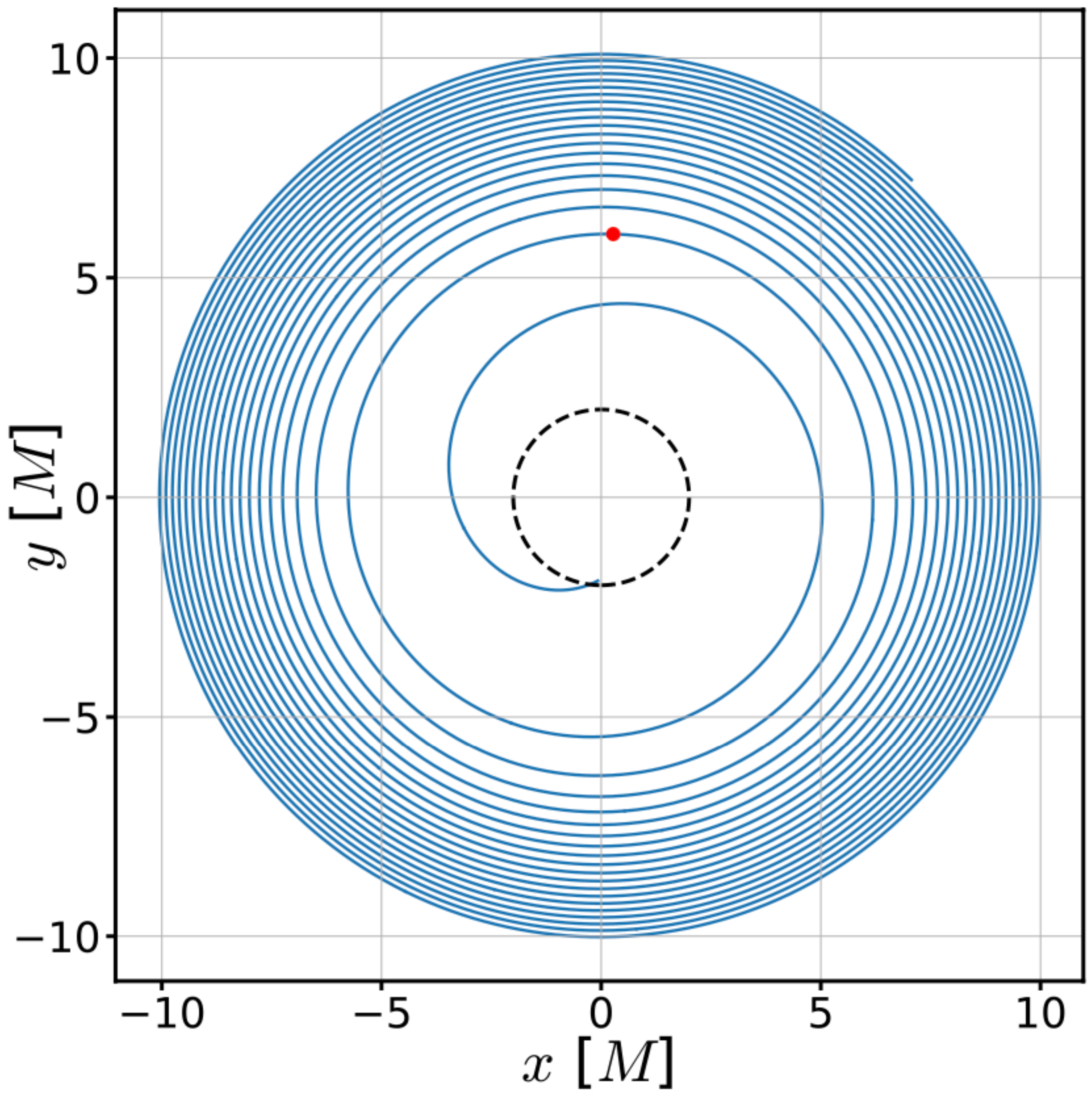


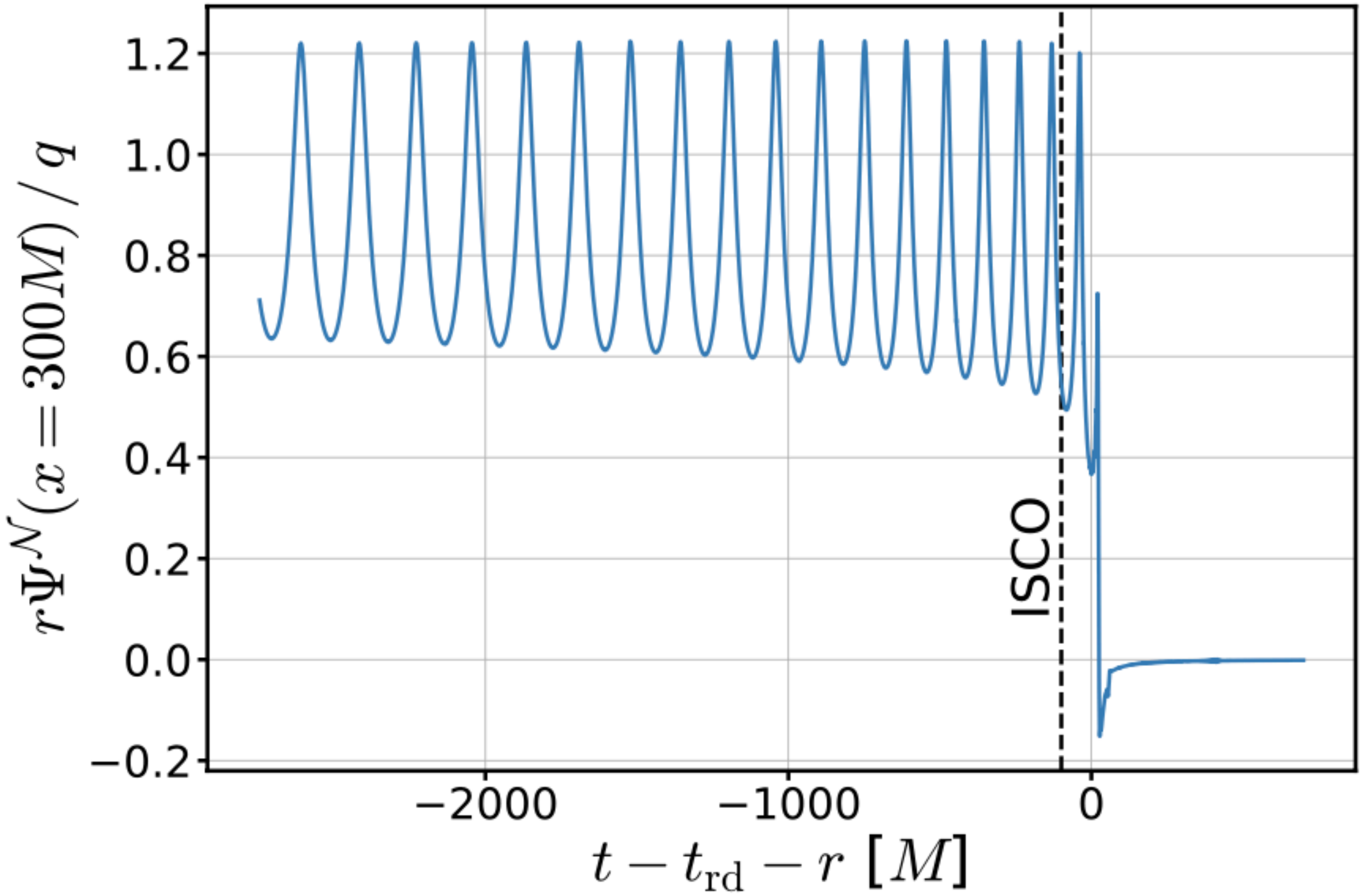
Wittek, Dhesi..HP+ PRD (2023)

Self-force driven inspirals

spectre

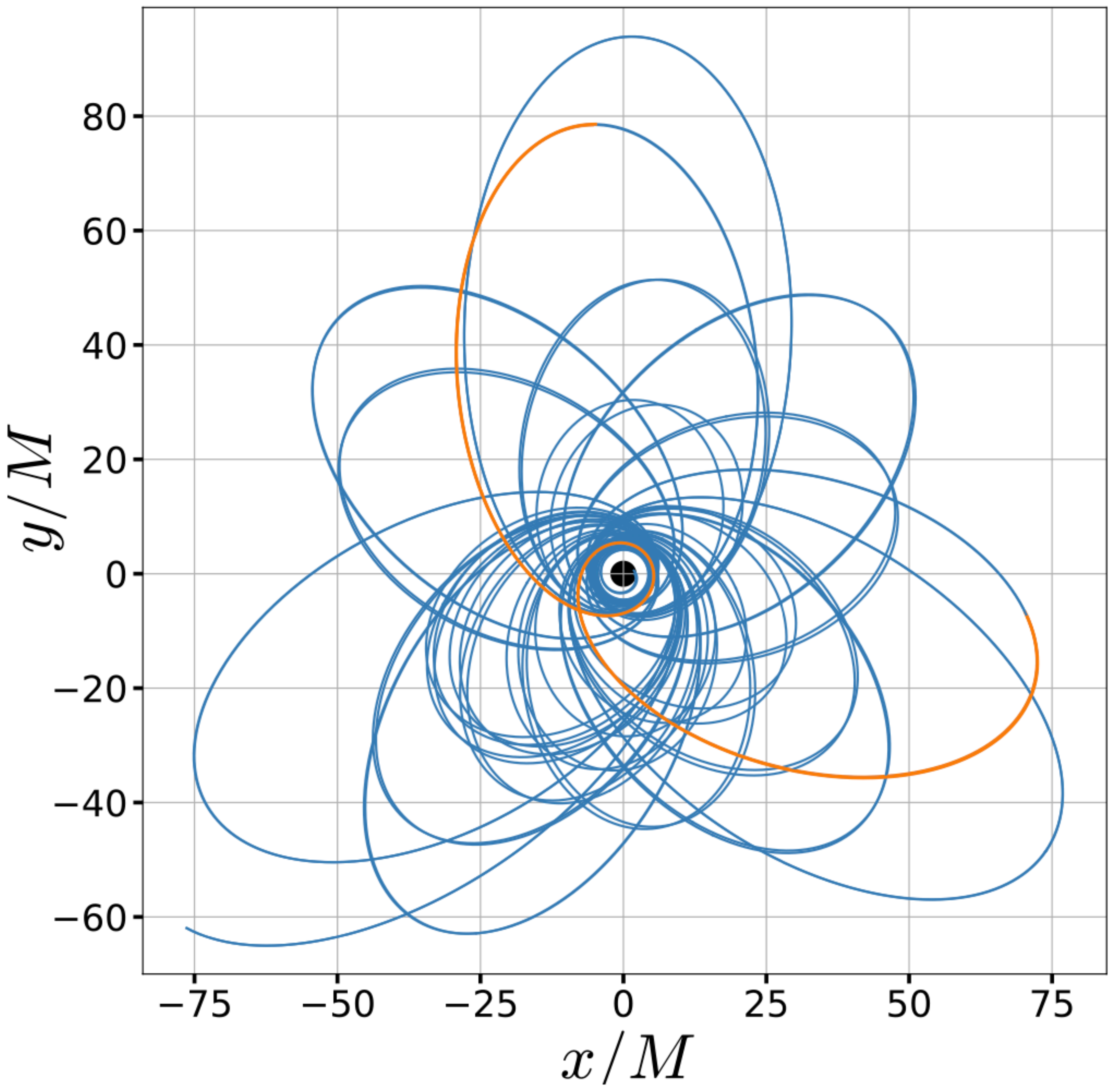
$$u^\beta \nabla_\beta (\mu u^\alpha) = Q \nabla^\alpha \Psi^\mathcal{R}$$

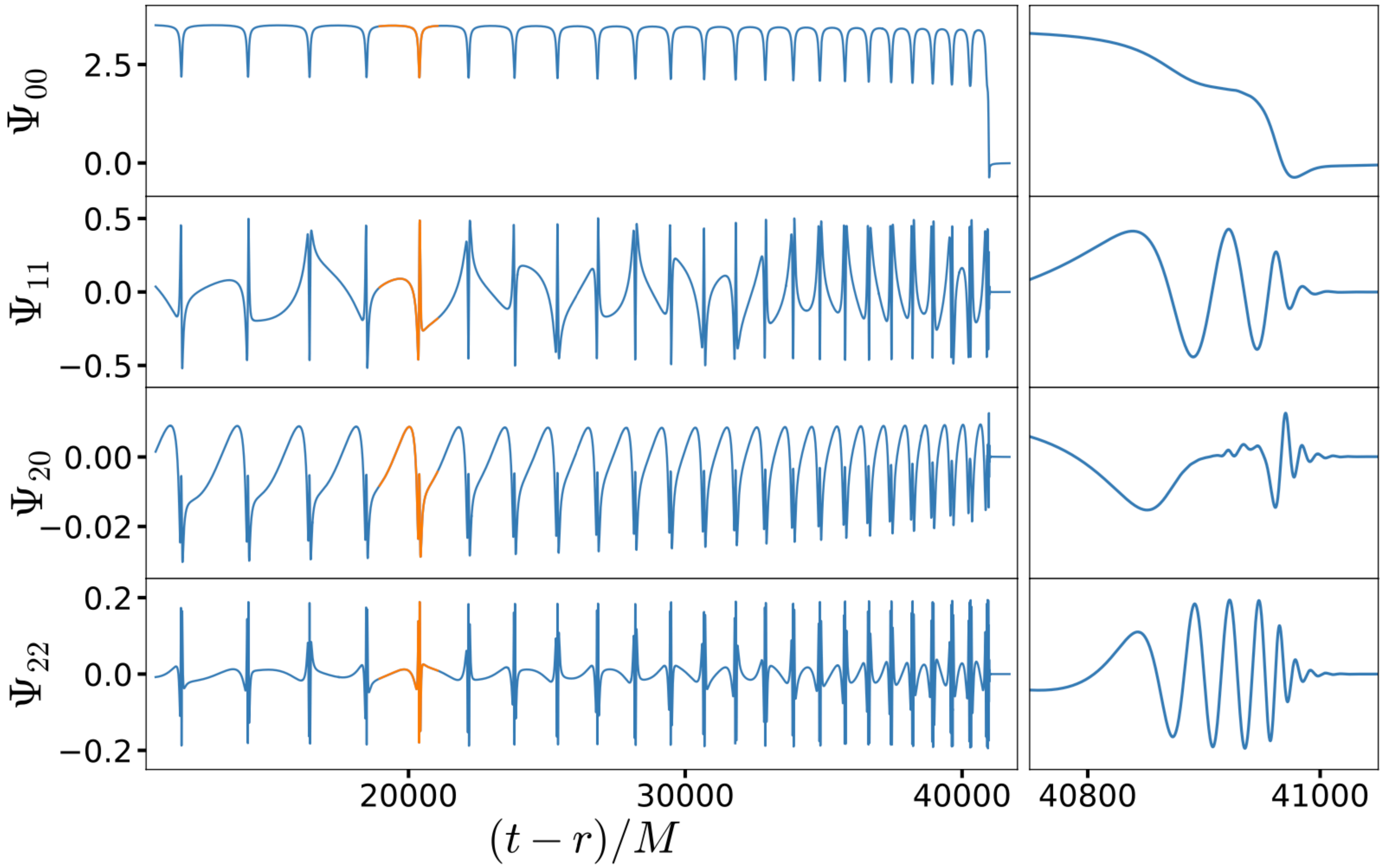




Wittek, Pound, Barack, HP PRD (2024)

Generic orbits

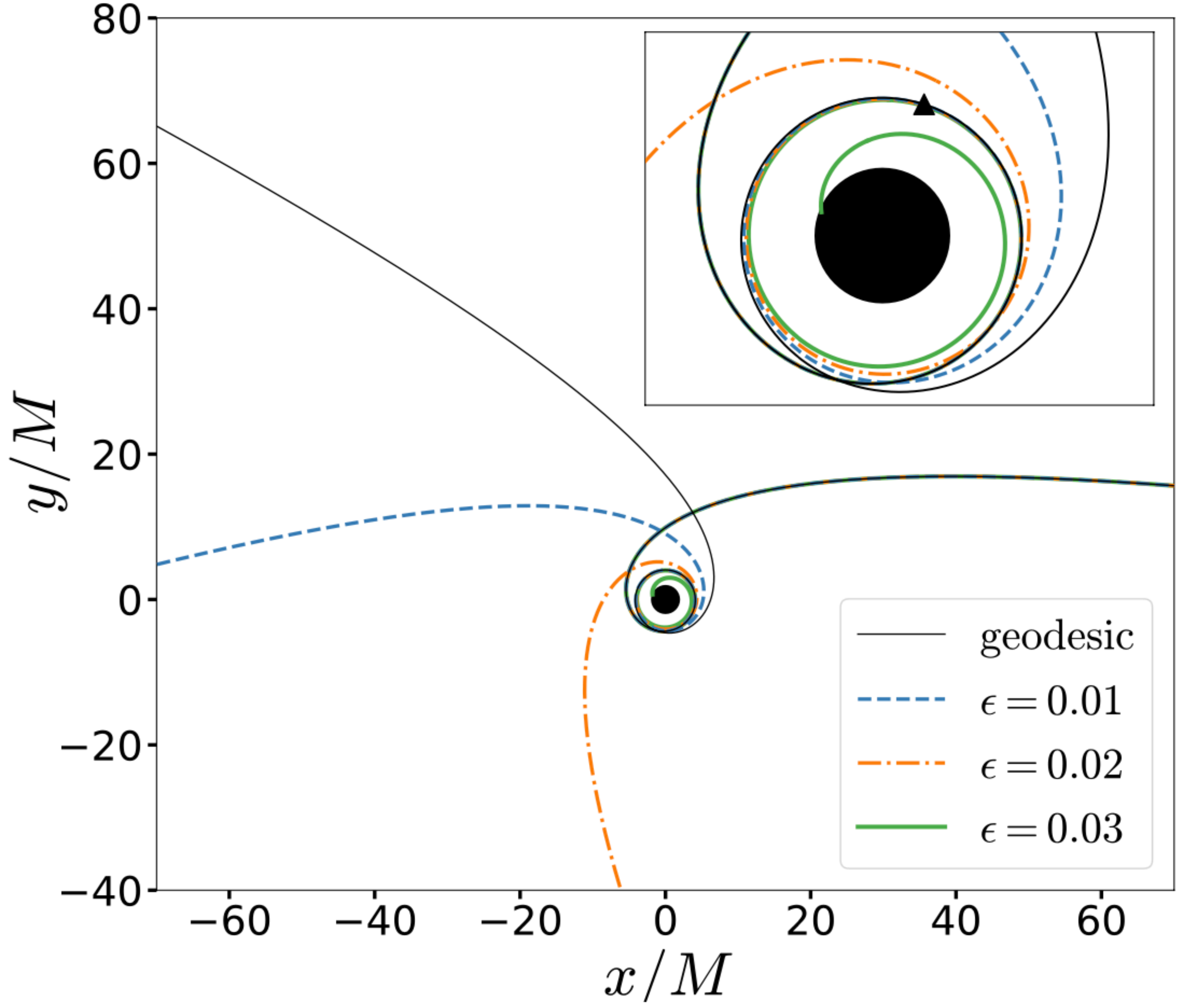


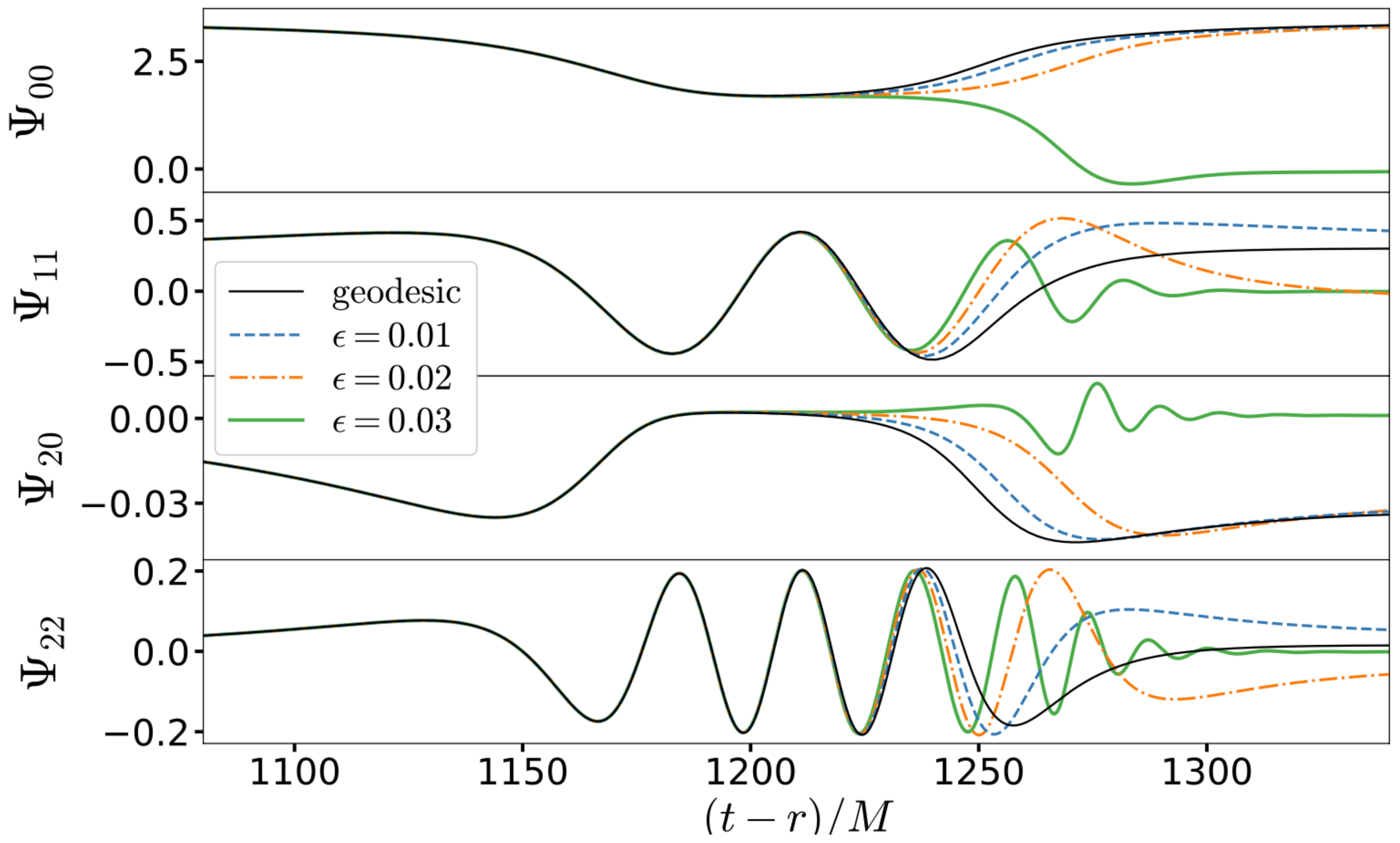


Wittek, Barack..HP+ PRL (2025)

scatter encounters

spectre



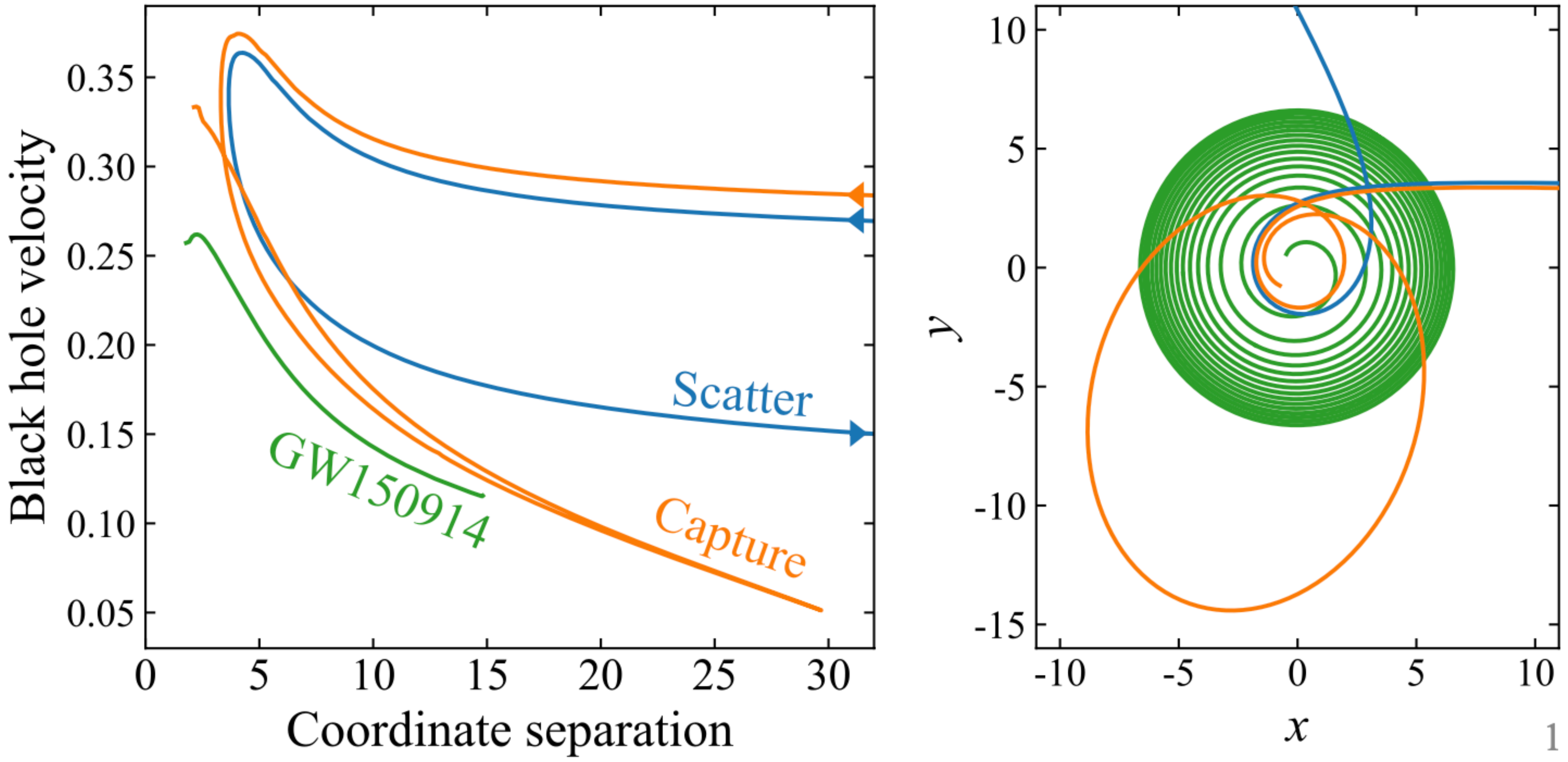


For scalar charge, control over all approximations and perturbative parameters.

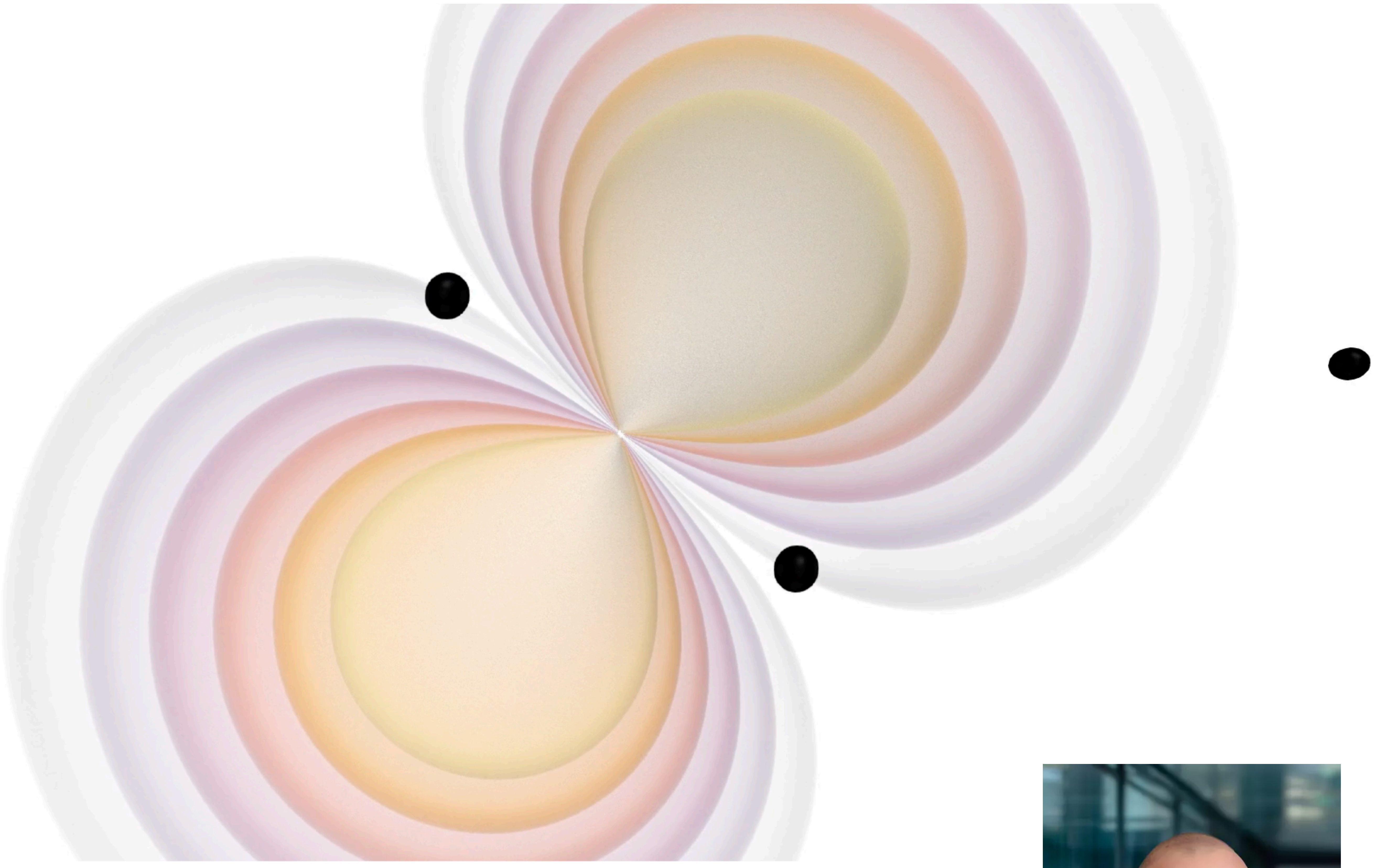
Wittek, Barack..HP+ PRL (2025)

BH Scattering

- **Clean probe** of strong field dynamics
  - **Gauge invariant** quantities
    - scattering angle(s) θ
    - $E_{\rm GW}$, $L_{\rm GW}$, $\Delta v_{\infty,i}$, Δm_i, $\Delta \vec{S}_i$
- **more relativistic** than inspirals

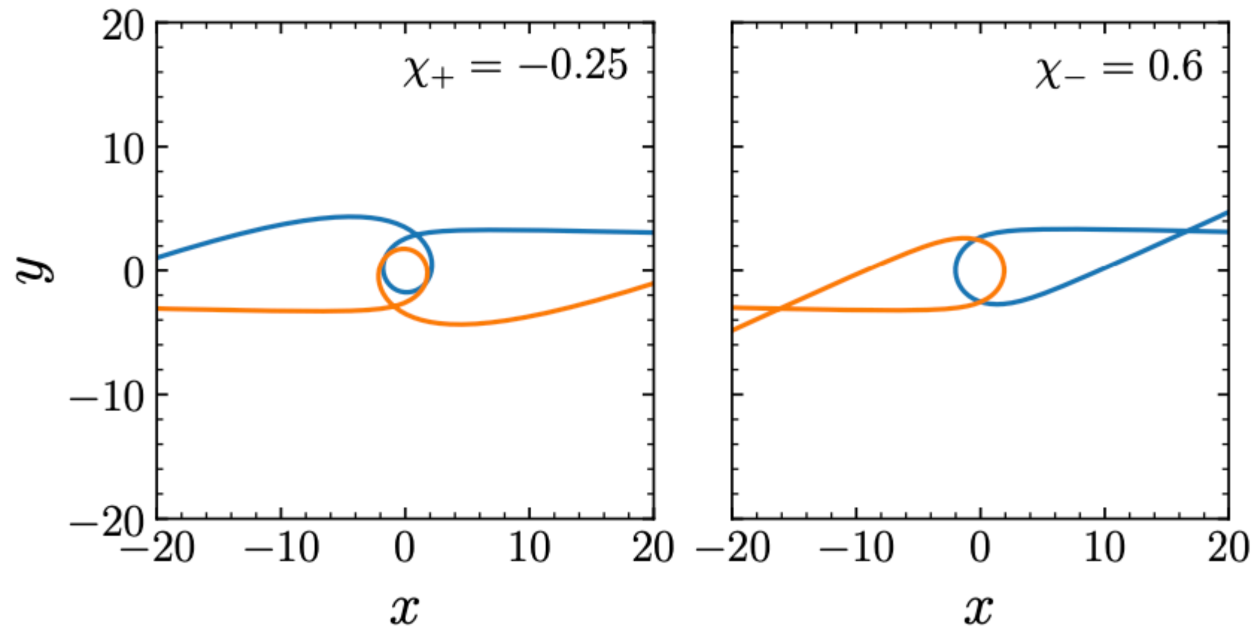


courtesy Oliver Long

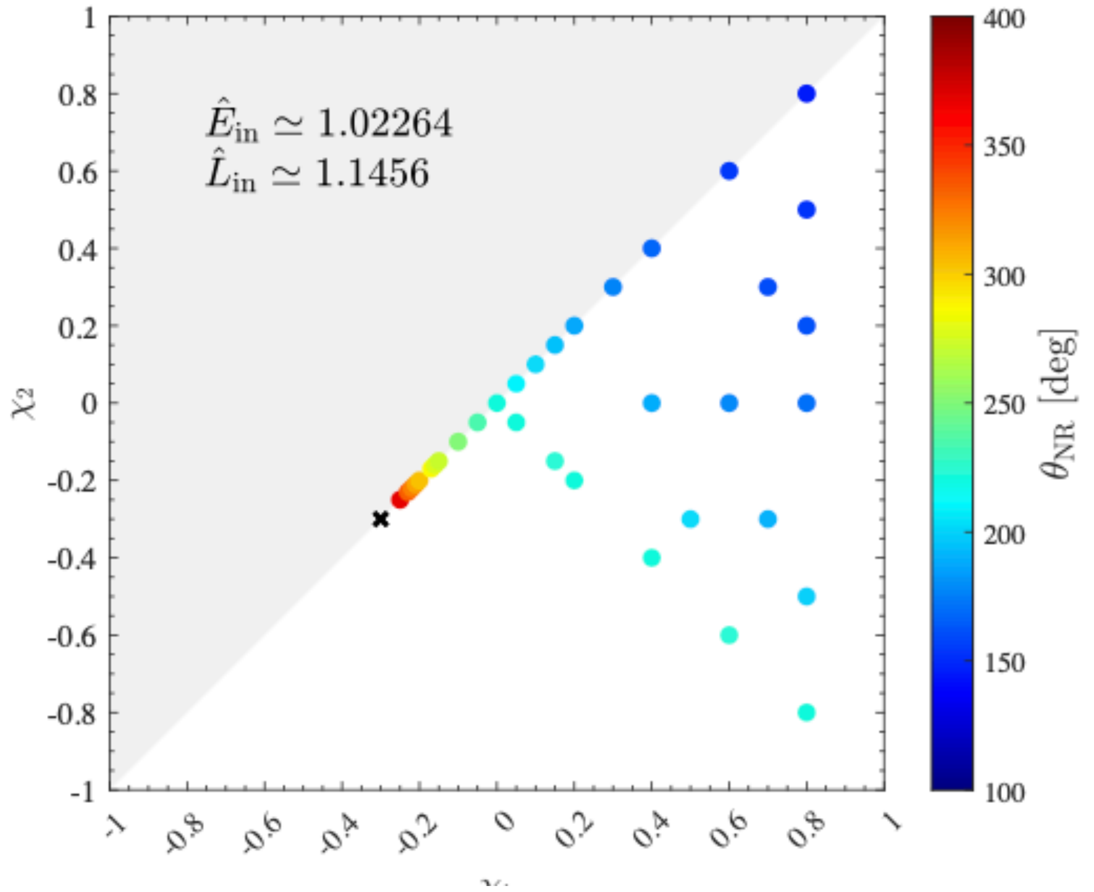


Oliver Long

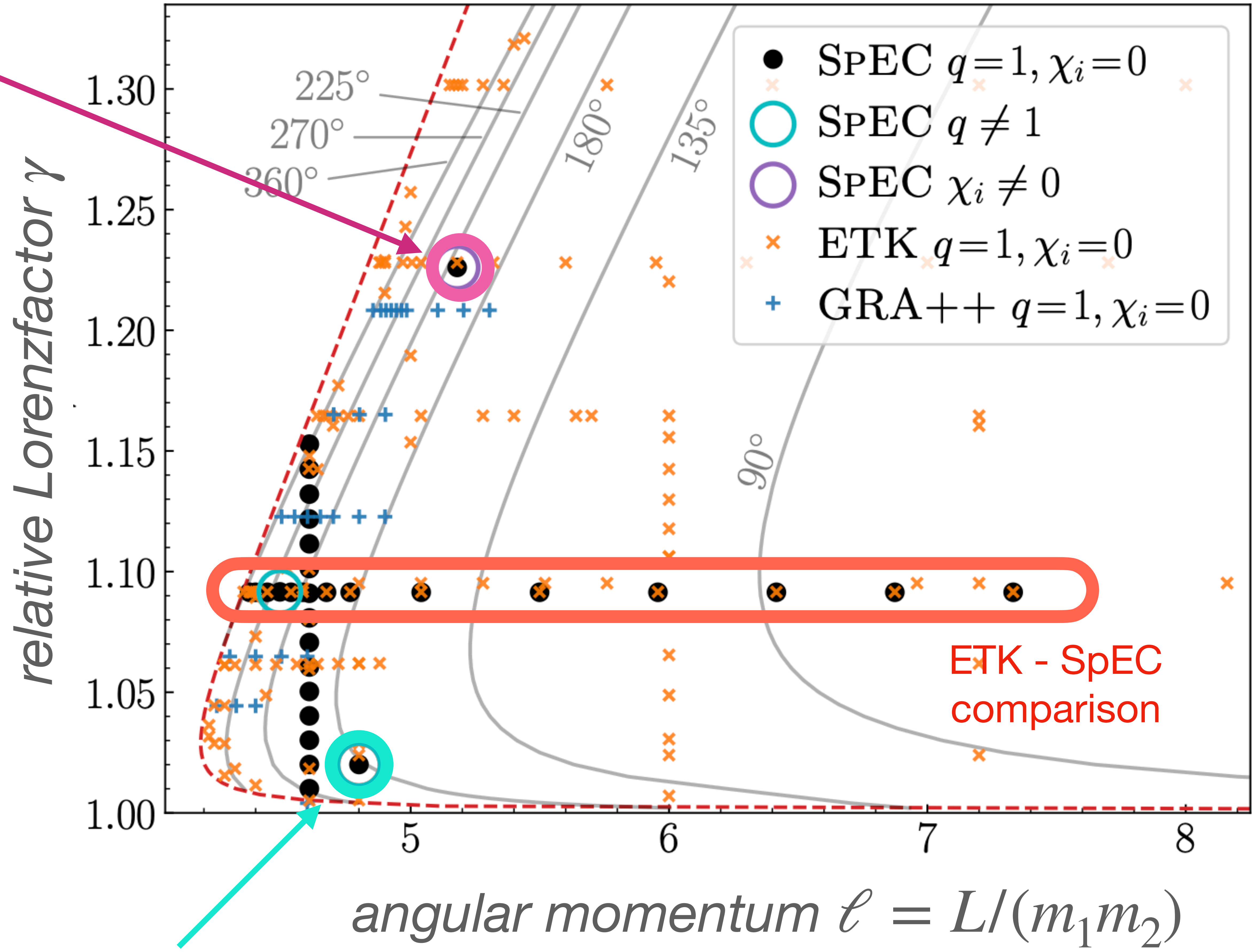
Scattering BBH parameter space



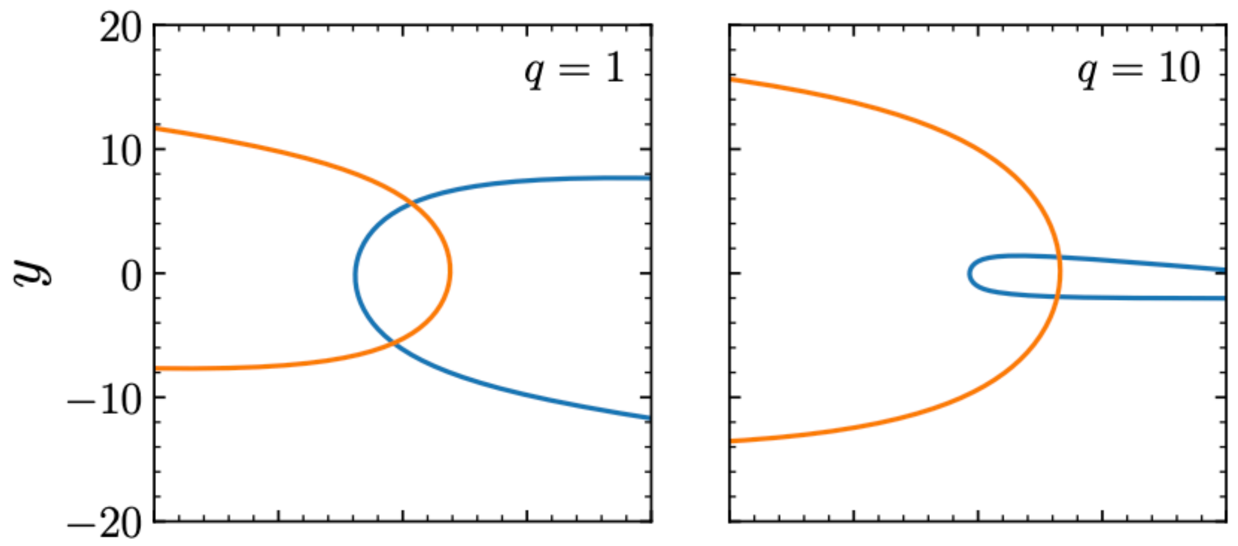
Also Rettegno+ 2023



vary spins



vary mass-ratio



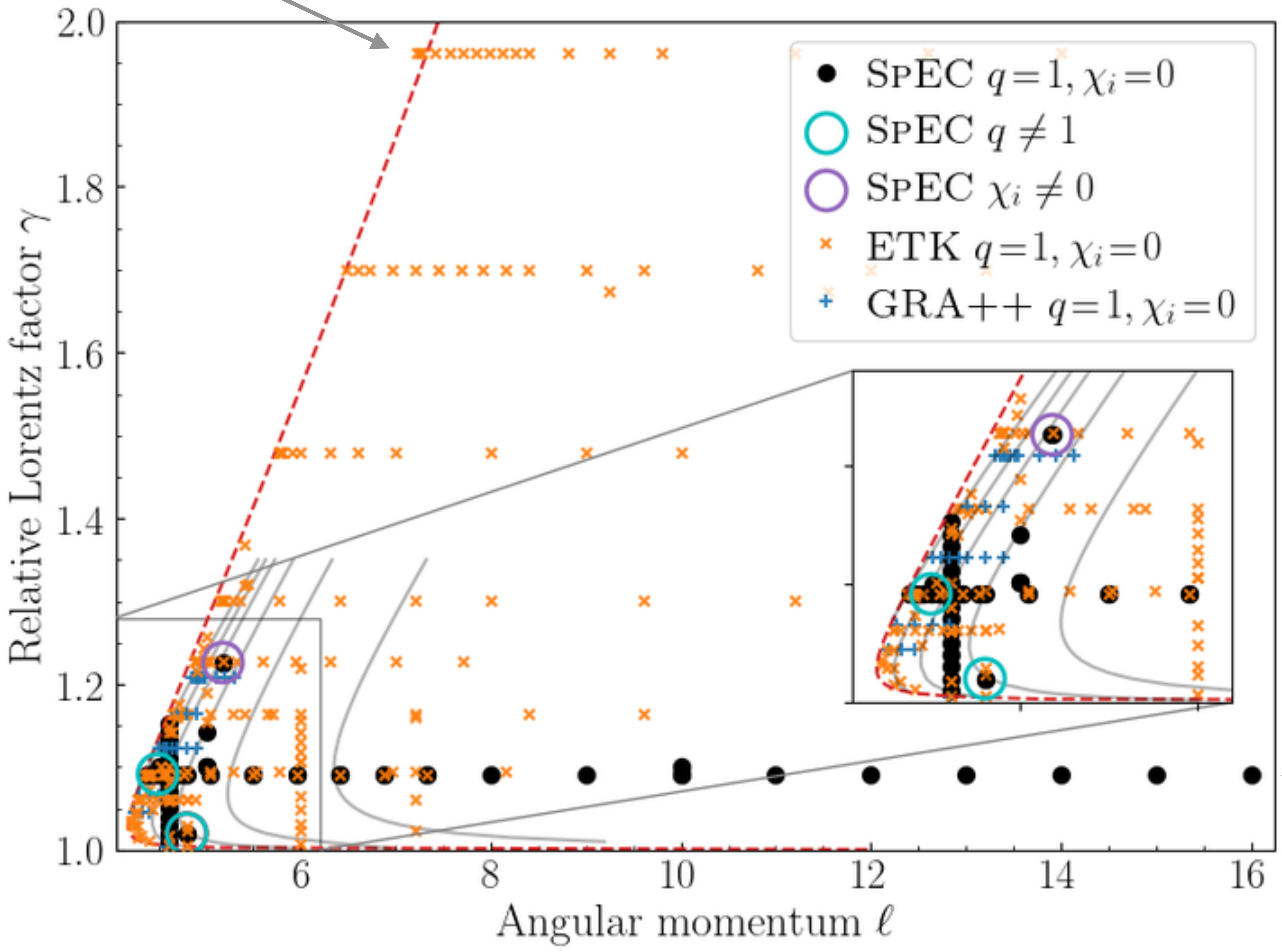
***Long**, HP, Buonanno, Jakobsen, Mogull+ PRD 2025 (2507.08071)*

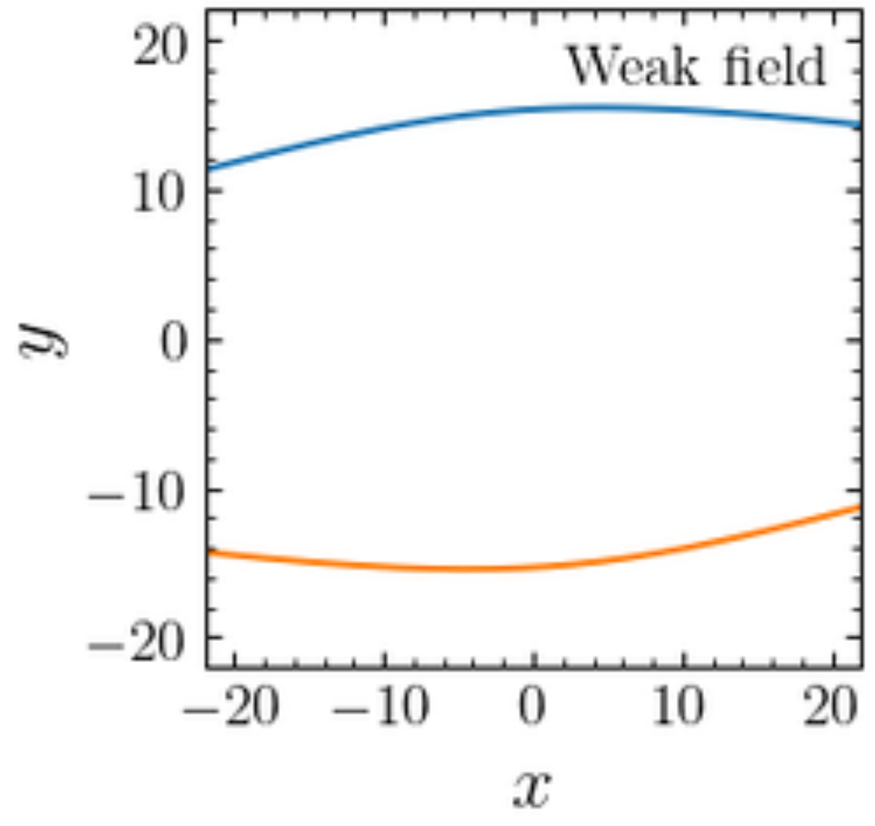
ETK
Damour, Guercilena, Hinder+ PRD (2014)
Hopper, Nagar, Rettegno PRD (2023) 2204.10299
Rettegno, Pratten, Thomas+ PRD (2023) 2307.06999
Swain,Pratten,Schmidt 2411.09652
GR-Athena++
Albanesi, Rashti, Zappa+ PRD (2025) 2405.20398

Scattering BBH parameter space (zoomed out)

higher energy

Swain, Pratten, Schmidt 2025





Courtesy Oliver Long

NR code-code comparison

- **ETK & SpEC consistent**
  - except extremely tight encounters
- **multiple codes essential for validation**
- SpEC accuracy due to
  - larger D_{initial}, D_{final}
  - improved trajectory fit for θ

ETK
Damour, Guercilena, Hinder+ PRD (2014)
Hopper, Nagar, Rettegno PRD (2023) 2204.10299
Rettegno, Pratten, Thomas+ PRD (2023) 2307.06999
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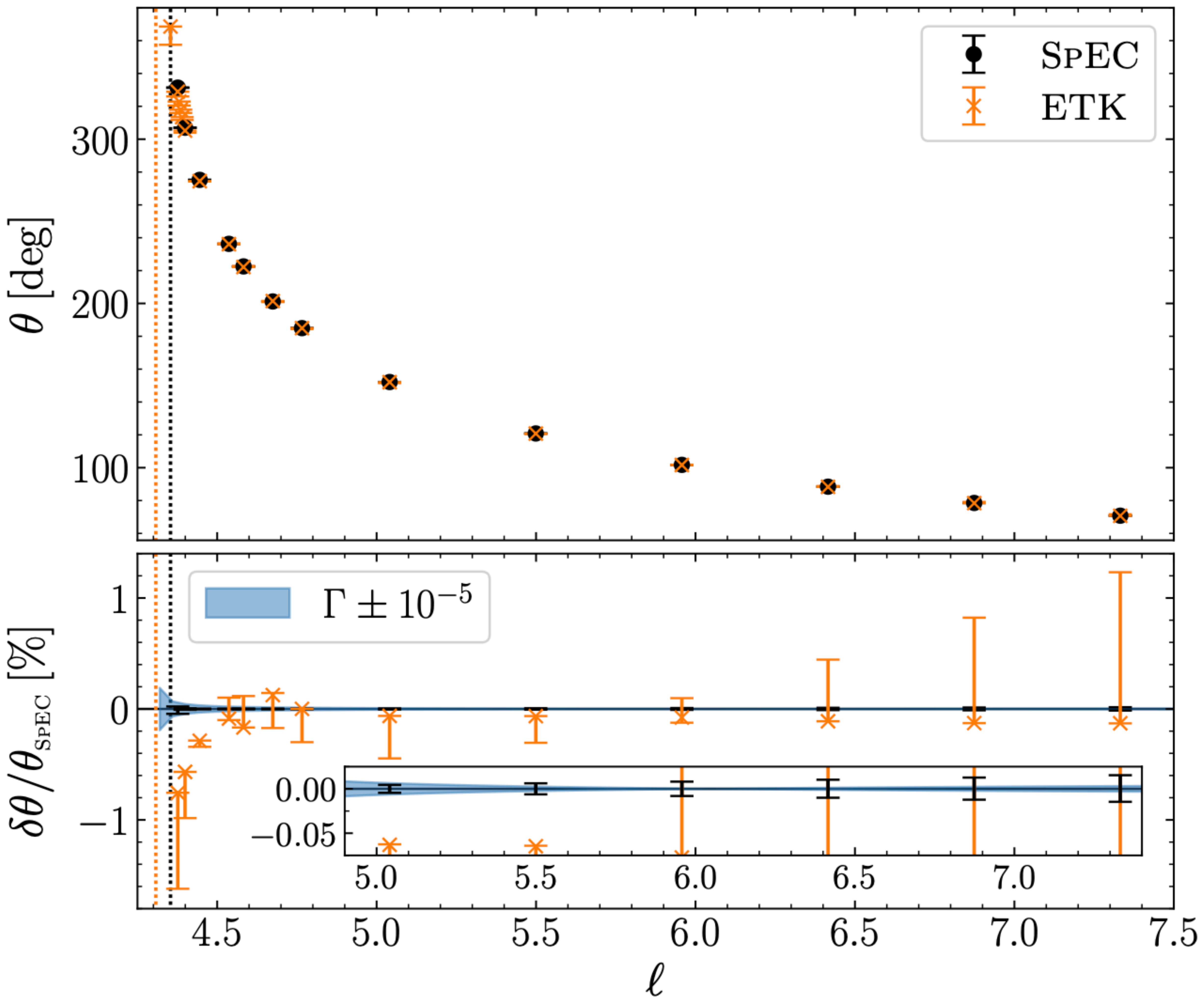
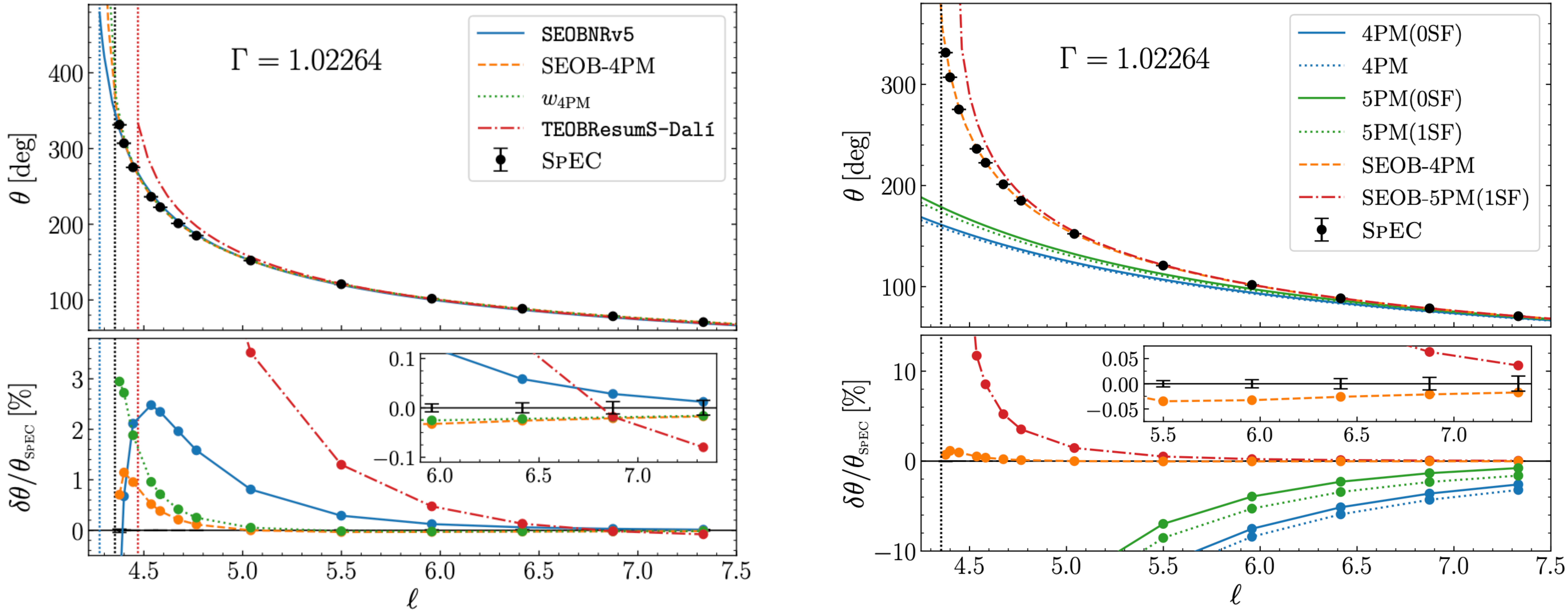


__Long__, HP, Buonanno, Jakobsen, Mogull+ PRD 2025 (2507.08071)

Comparison with post-Minkowski & EOB models



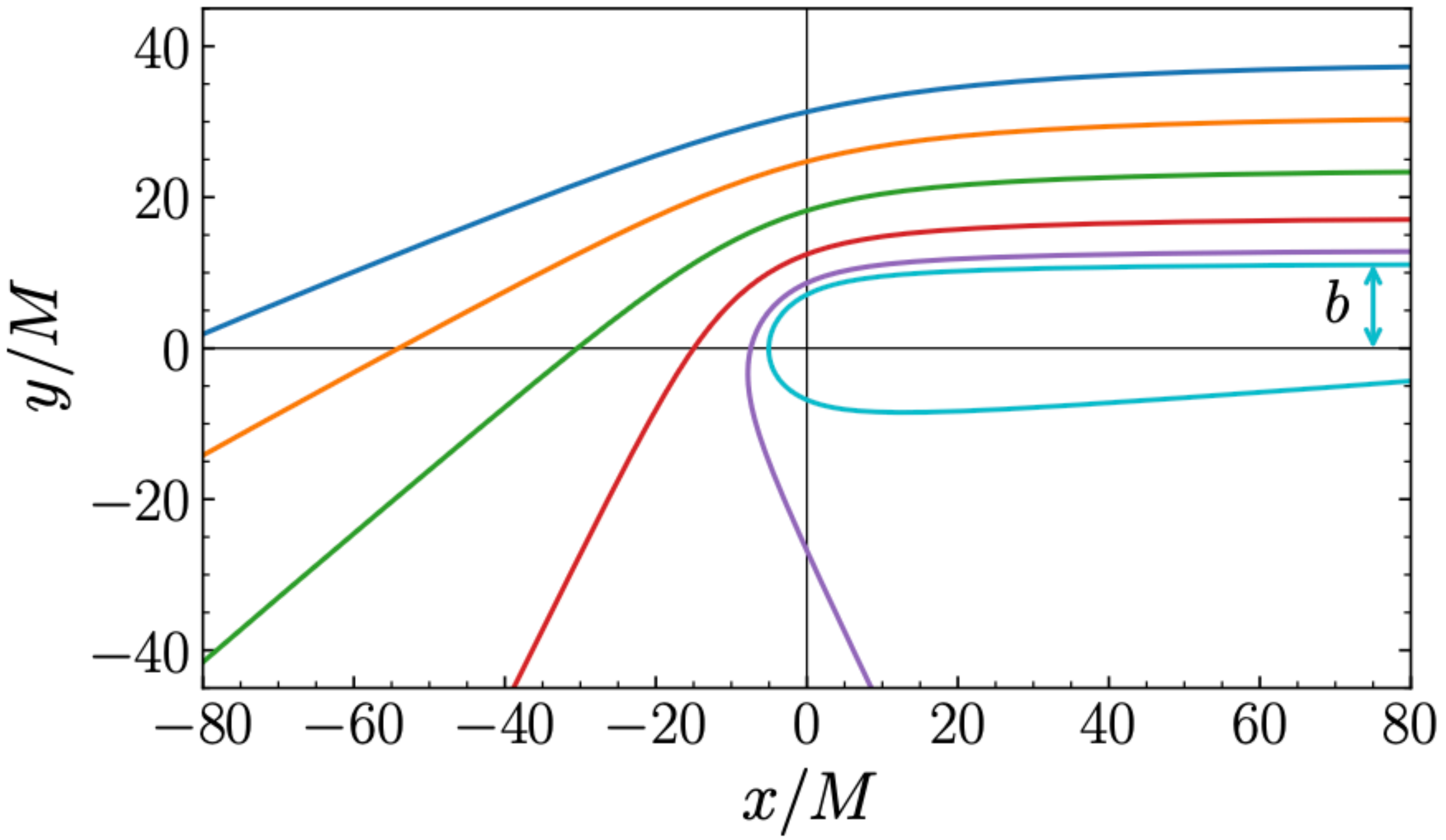
Long, HP, Buonanno, Jakobsen, Mogull+ PRD 2025 (2507.08071)

→ Piero Rettegno, Oliver Long, Alessandro Nagar

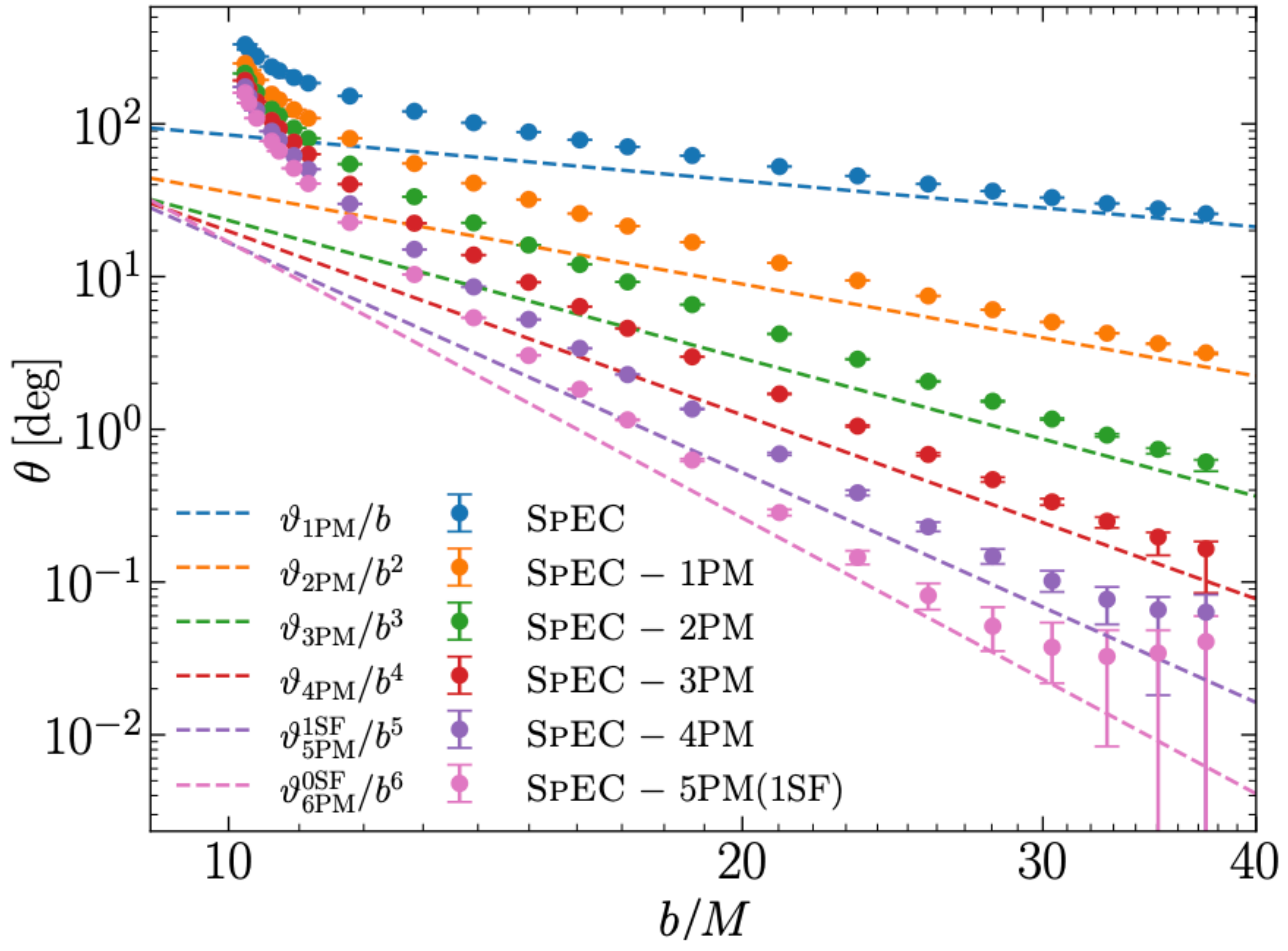
Also Rettegno, Pratten, Thomas+ PRD (2023) 2307.06999

Towards large impact parameter

b>20 new



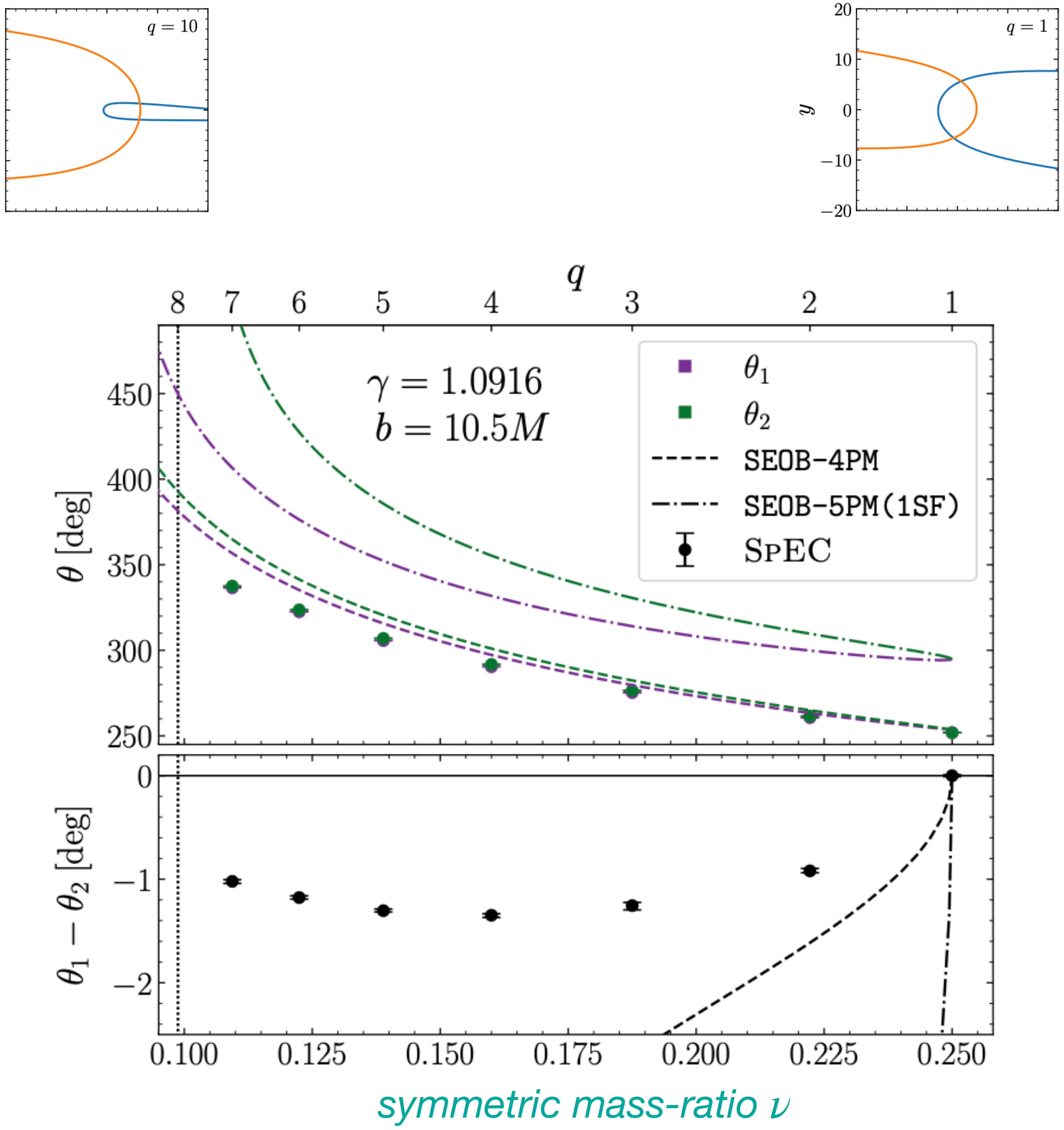
Long, *HP,+ PRD 2025 (2511.10196)*



→ Oliver Long

Unequal masses

- **numerically resolve $\theta_1 - \theta_2$**
- Deviations give insights into how to improve EOB models

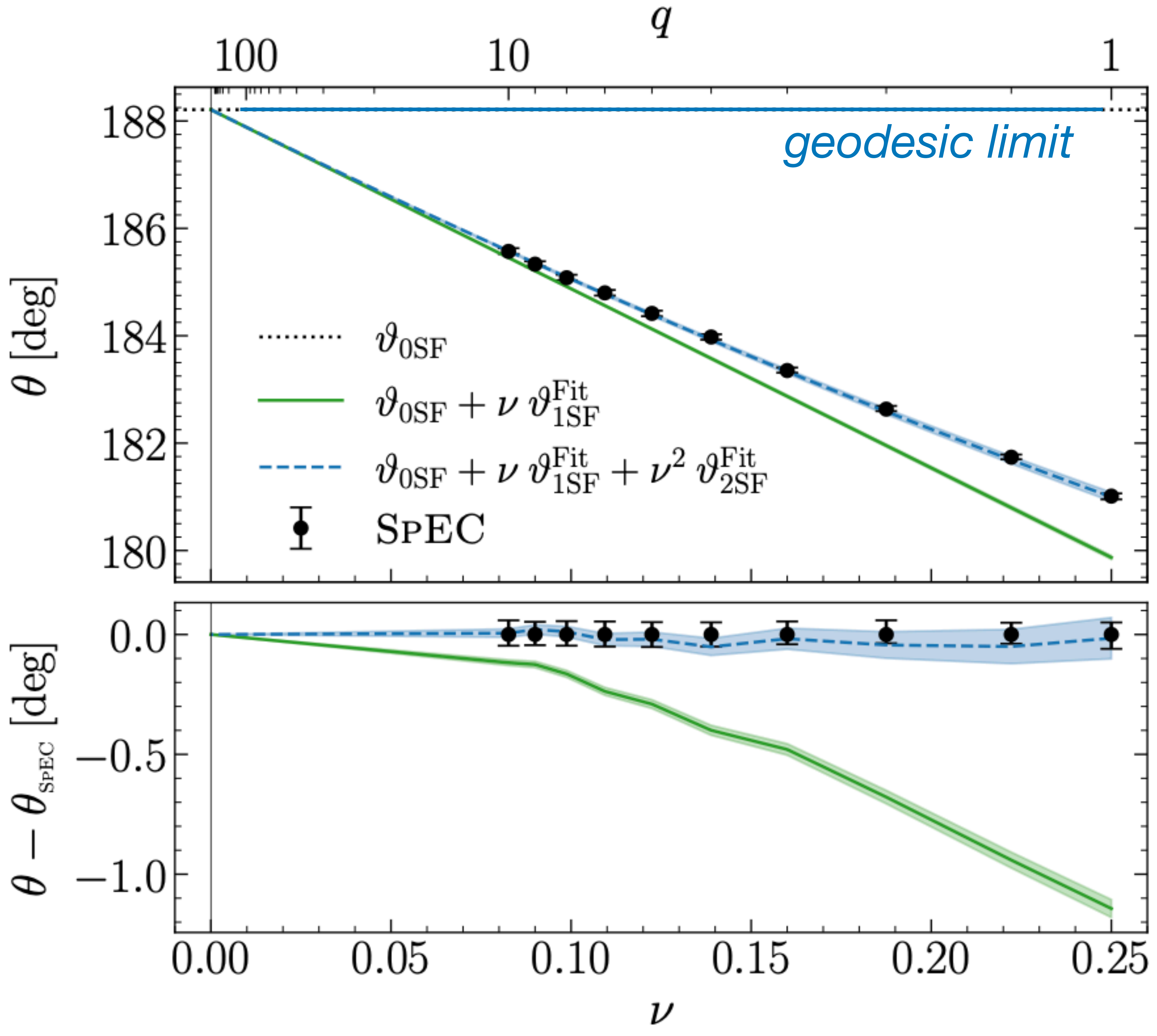


***Long**, HP, Buonanno, Jakobsen, Mogull+ PRD 2025 (2507.08071)*

Extracting GSF information $(\gamma = 1.02, \ell = 4.8)$

- Can determine 1-GSF and 2-GSF contributions to scattering angle
- 2-GSF is accurate even at q=1 !

Long, HP,+ PRD 2025 (2511.10196)



... in other news ...

SpECTRE

spectre

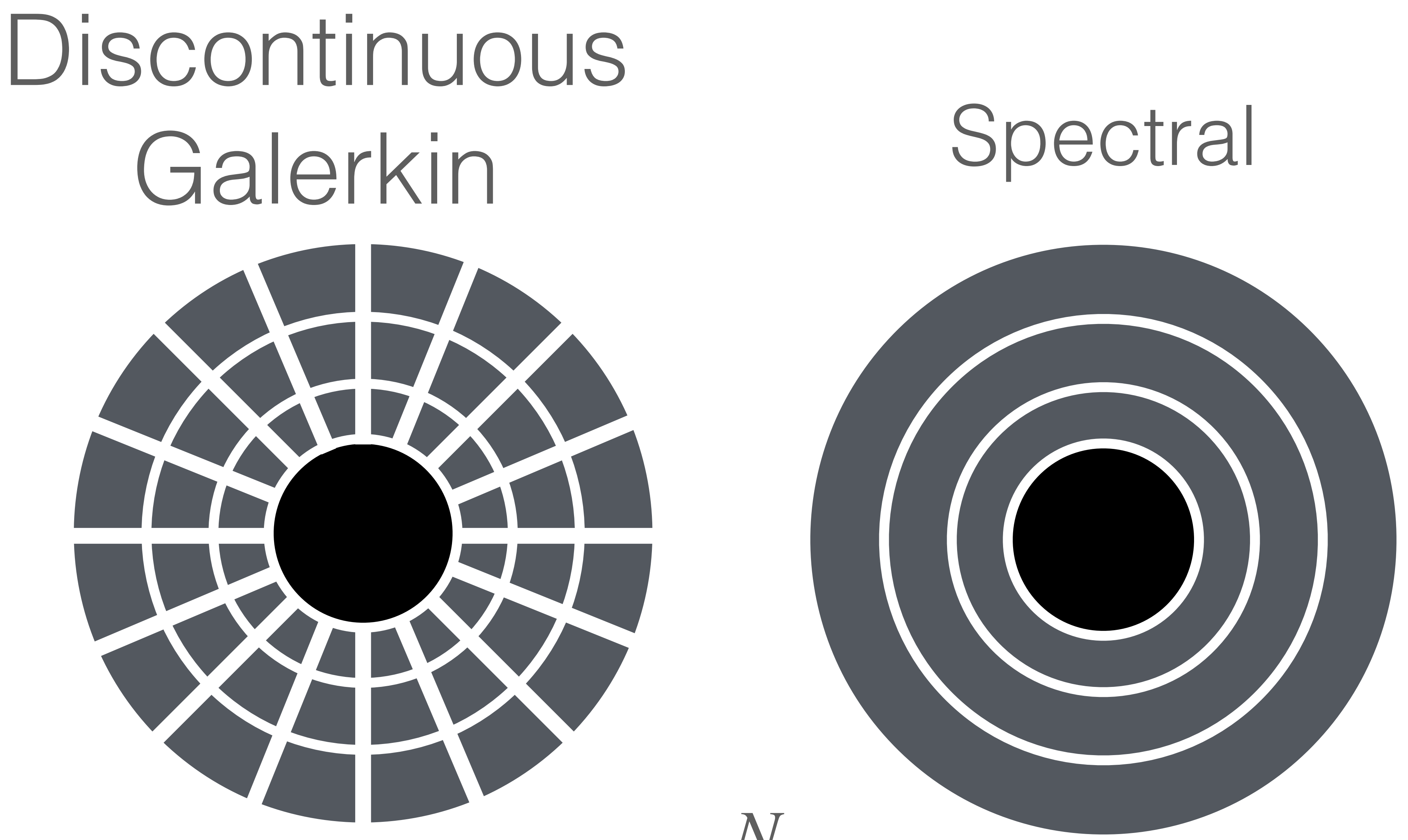
- **New relativistic astrophysics code developed by SXS**

SXS

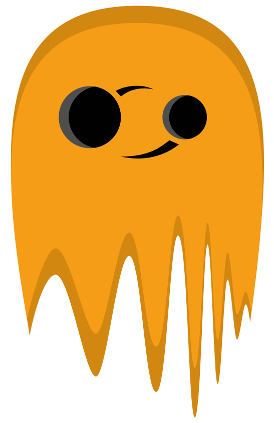
Simulating eXtreme Spacetimes collaboration

- **Algorithms for modern hardware**
  - more, smaller elements
  - task-based load balancing for many cores
  - local time-stepping, better hydro, AMR

- **Complete rewrite from SpEC**
  - efficient on today's hardware
  - to implement 'now-we-know-better' situations
  - open source and maintainable
  - ...but C++ template madness...



$$f(x) = \sum_{n=0}^{N} a_n \phi(x)$$

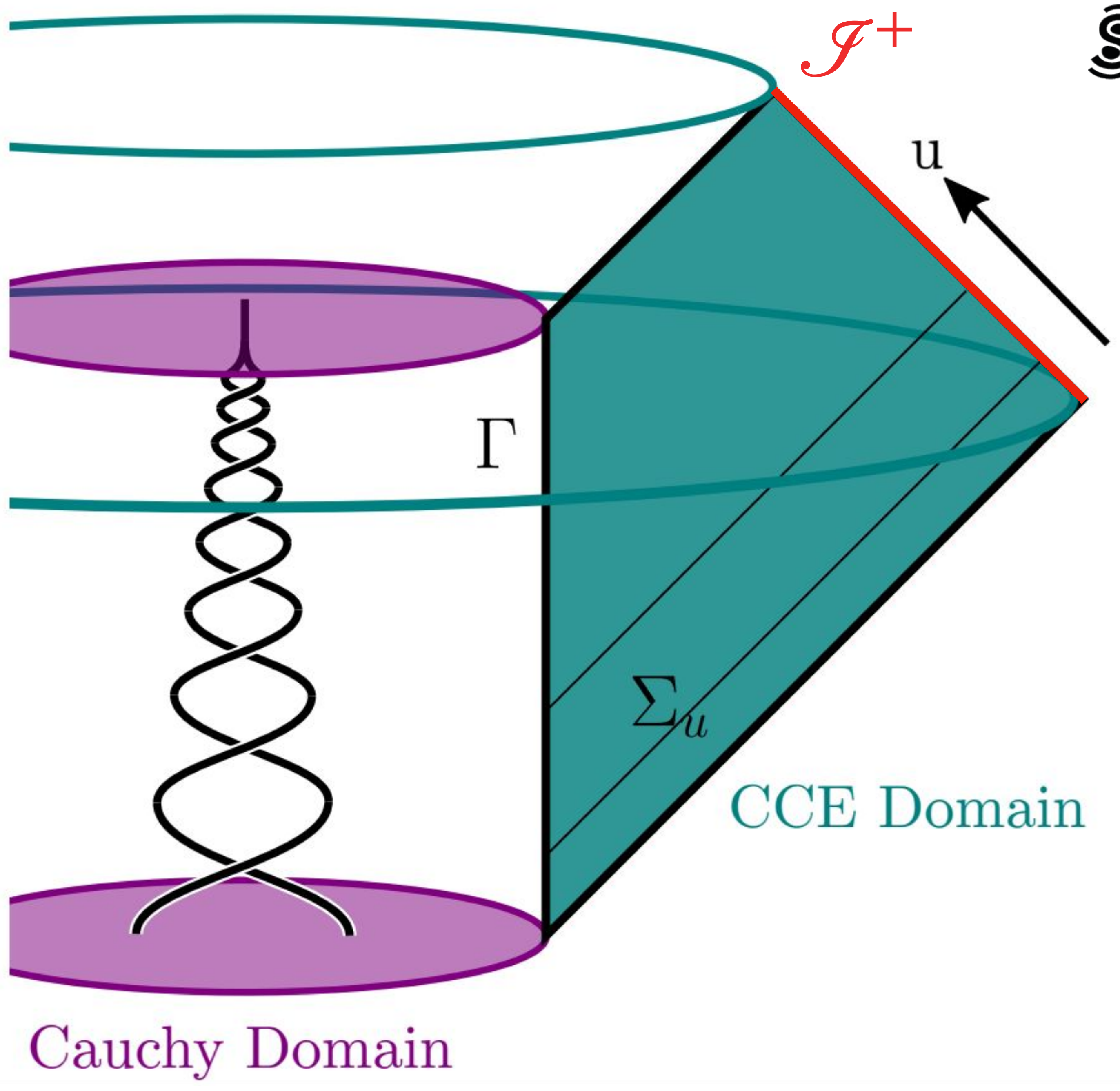


https://spectre-code.org

Improved wave extraction & GW memory

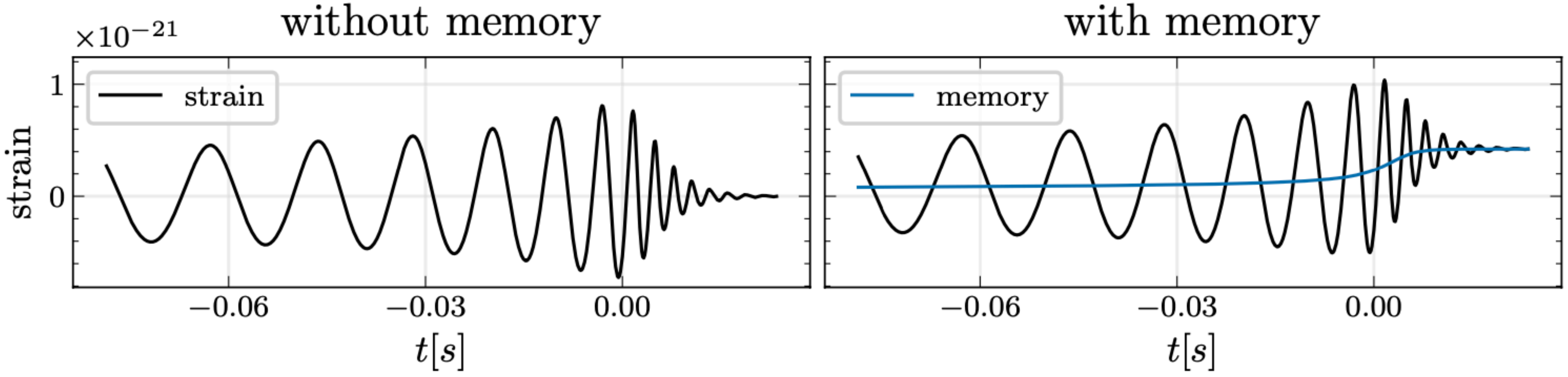
- **Chauchy characteristic extraction**
  - solve Einstein equations **up to $\mathscr{I}^+$** (rather thar
- **⇒ more accurate GWs with GW memory**
- Possible to post-process SpEC

spectre



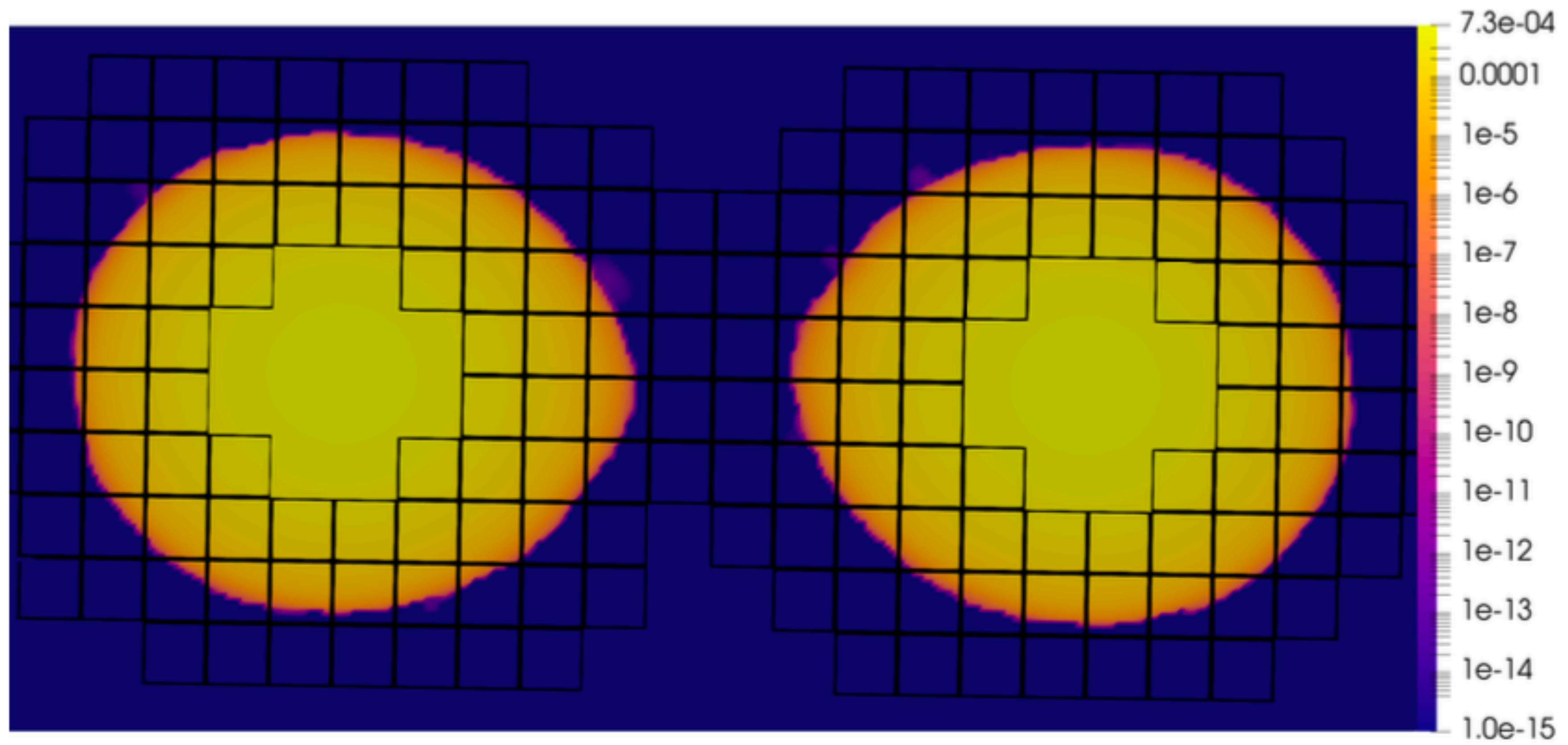
Mitman+, arXiv:2007.11562
Mitman+, arXiv:2024.08868

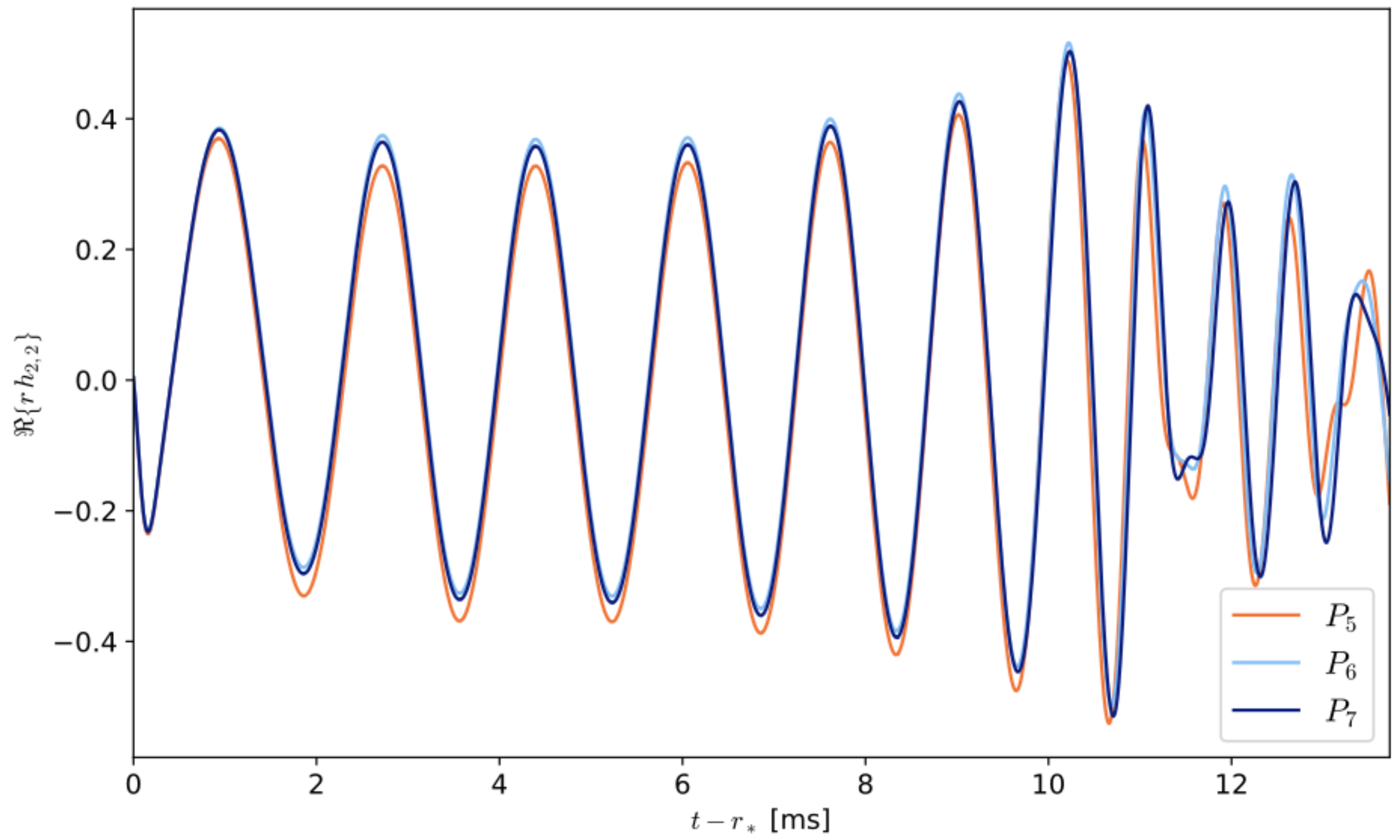
$30 + 30M_\odot$
$\vec{\chi}_1 = \vec{\chi}_2 = 0.6\hat{L}$
$D = 400\text{Mpc}$



Recent news: First binary mergers with Spectre

Binary neutron star

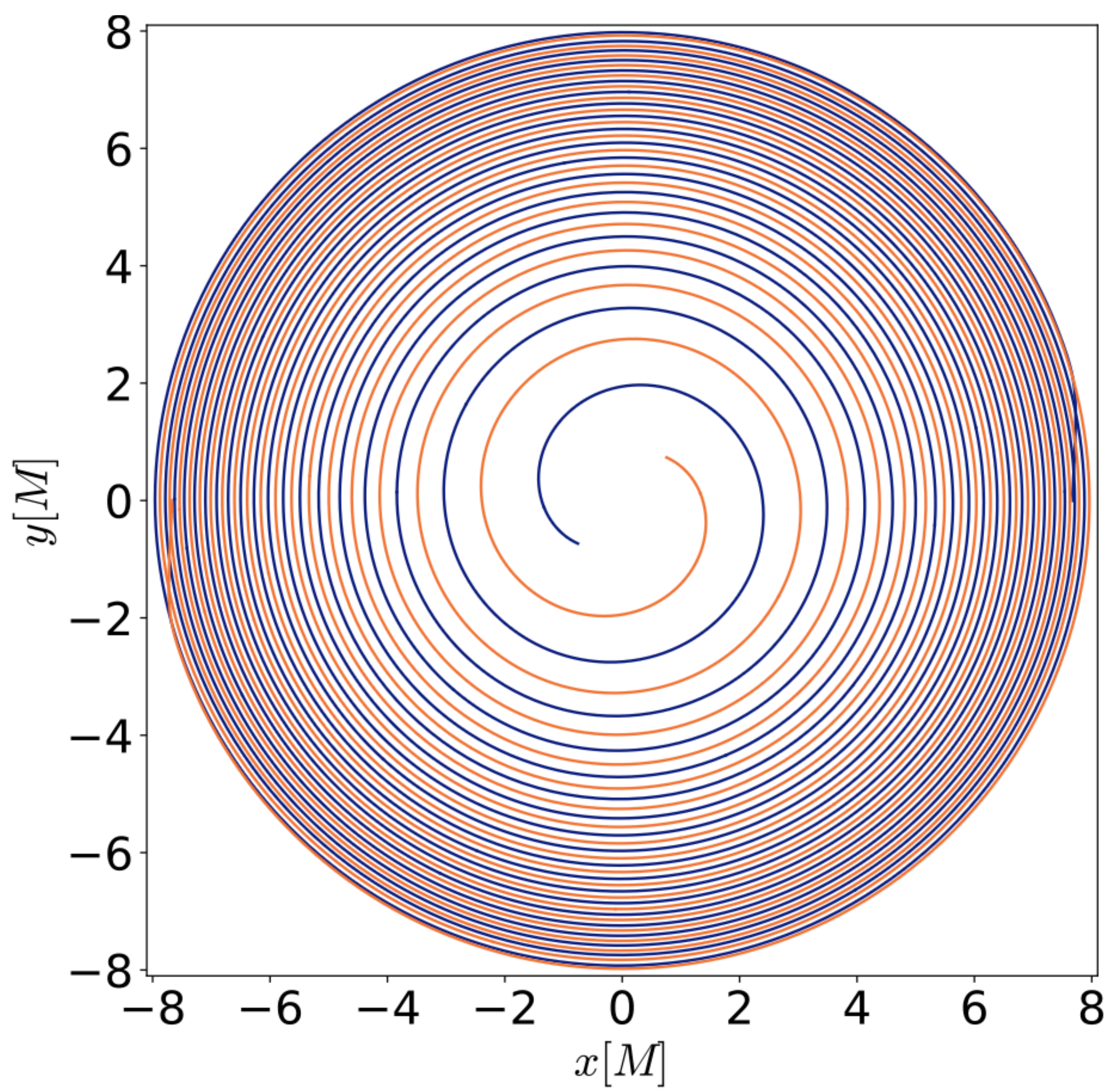


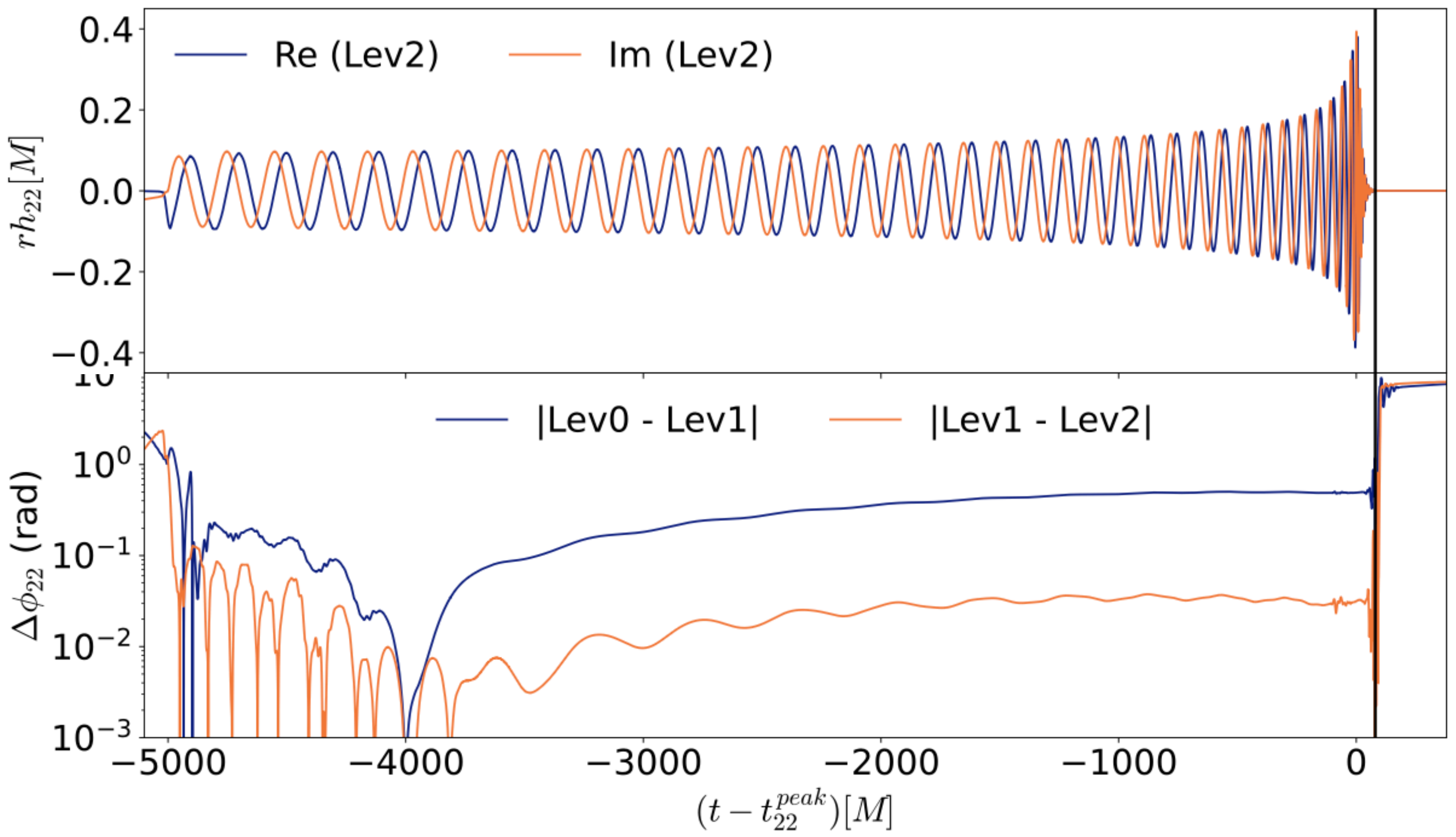


Deppe+ CQG 2024
2024.19038

Binary black hole

spectre





Lovelace+ CQG 2025
2410.00265

beyond GR: scalar Gauss-Bonnet

spectre

- **Why?**
  - explore nonlinear behavior (especially qualitatively new features)
  - falsify or confirm with GW observations

- **sGB: scalar field Ψ, coupled through Gauss-Bonnet invariant**

$$S\left[g_{ab},\Psi\right] \equiv \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa} - \frac{1}{2}(\nabla_a\Psi\nabla^a\Psi) + \ell^2 f(\Psi)\,\mathcal{G}\right]$$

$$\mathcal{G} \equiv R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2$$

$$f(\Psi) \equiv \frac{1}{8}\Psi^2 + \frac{\zeta}{16}\Psi^4$$

- Equations of Motion

$$G_{ab} = \kappa\left(T^{(\Psi)}_{ab} + \ell^2 H_{ab}\right),$$

$$T^{(\Psi)}_{ab} \equiv \nabla_a\Psi\nabla_b\Psi - \frac{1}{2}g_{ab}\nabla_c\Psi\nabla^c\Psi,$$

$$H_{ab} \equiv P_{acbd}\nabla^c\nabla^d f(\Psi)\,,\qquad P_{abcd} \equiv R_{abcd} - 2g_{a[c}R_{d]b} + 2g_{b[c}R_{d]a} + g_{a[c}g_{d]b}R$$

$$\Box\Psi = -\ell^2 f'(\Psi)\,\mathcal{G}$$

2nd derivatives changing principal parts

- Source term $-\ell^2 f'(\Psi)\,\mathcal{G}$ can create scalar hair on BHs

Doneva, Yazadjiev (2018)

Quasi-stationary sGB initial data

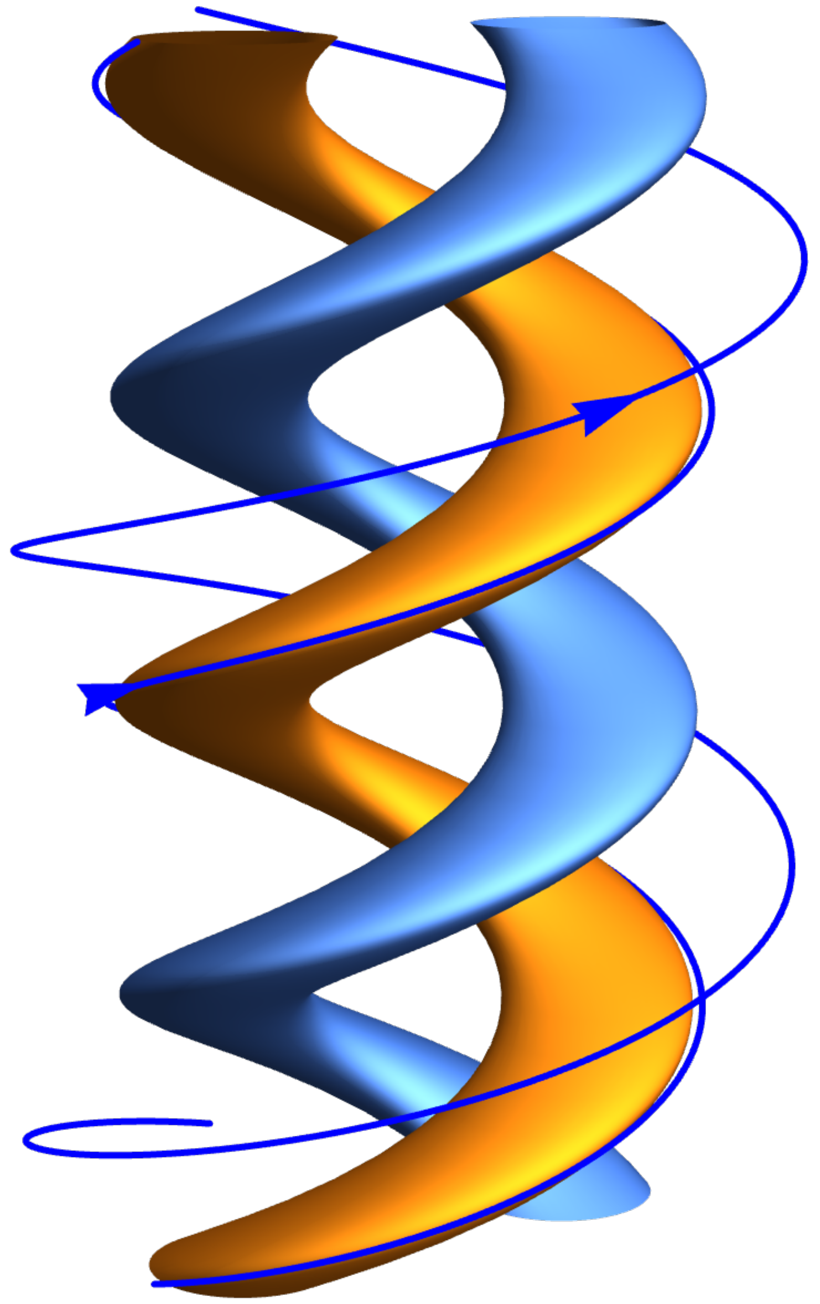
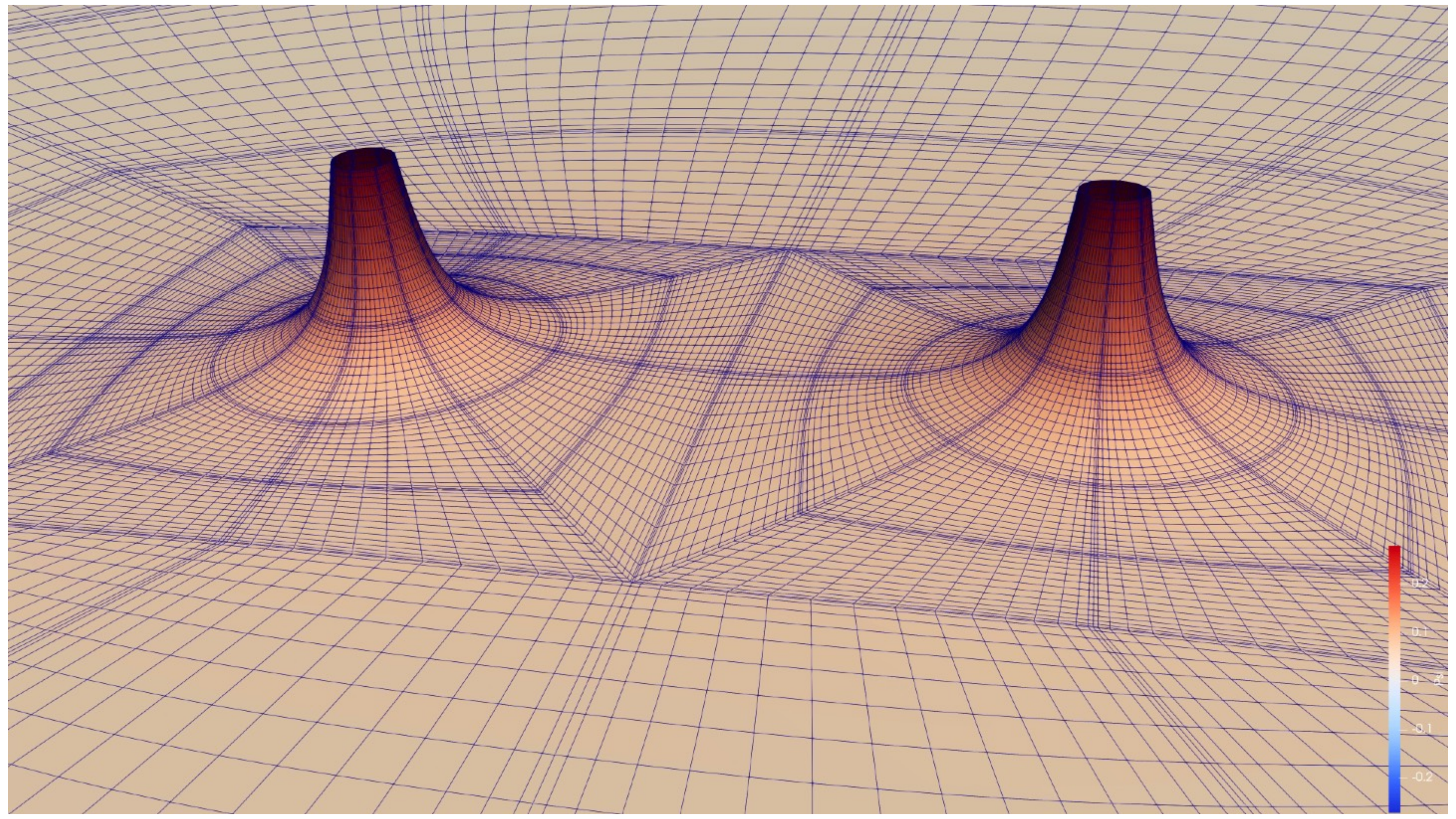


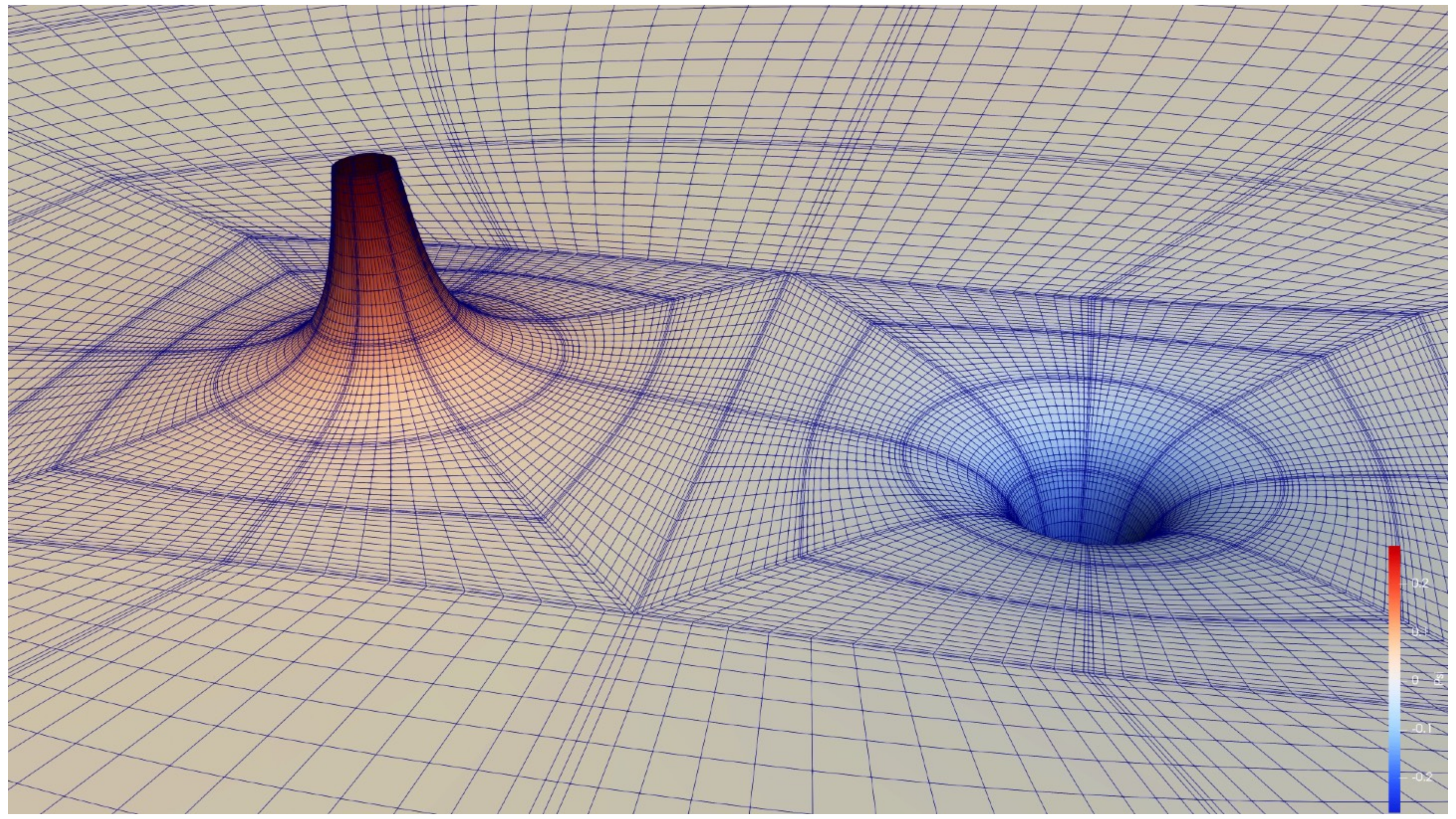
Image credit: Khairnar+ 2410.16373

spectre

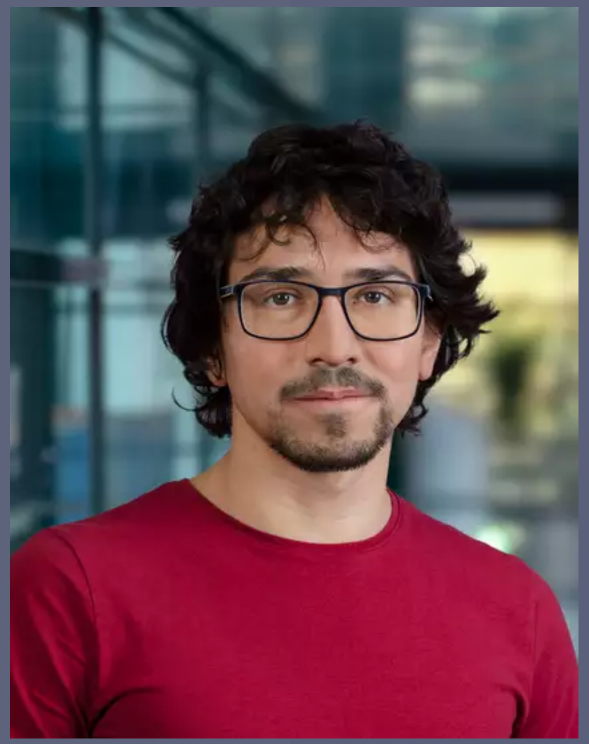
- Scalar field in **equilibrium** (helikal Killing Vector)
- Yields elliptic equation **singular** on horizon
- Solve with **SpECTRE elliptic solver**
- Single and binary **scalarized BH**

Nee*, Lara, HP, + PRD 2024*
(arXiv:2406.8410)



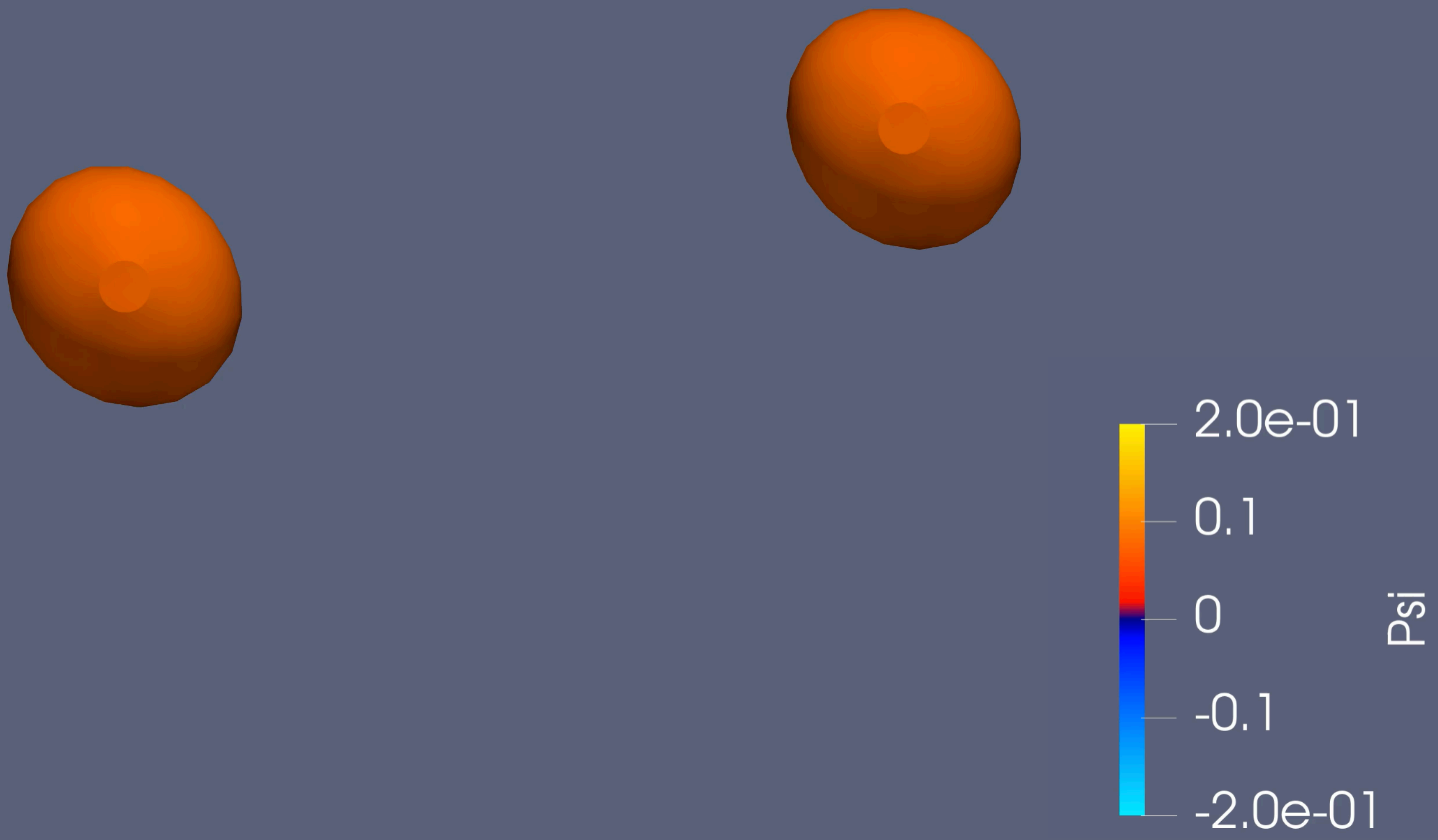


Time: 3800.00



Guillermo Lara

spectre



First sGB BBH merger w/ Spectre

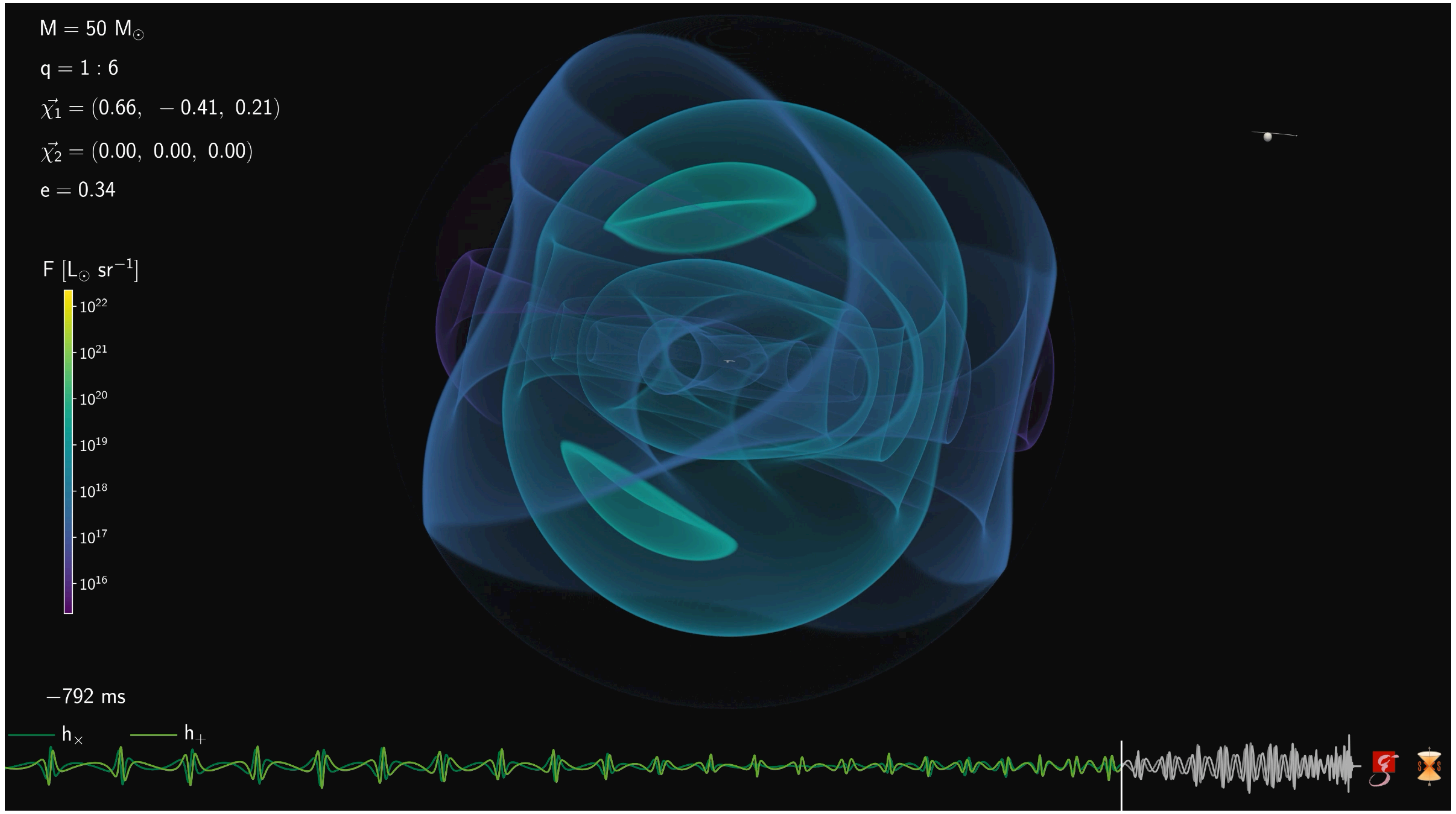
Using "fixing-the-equations" driver
developed in Lara et al PRD 2024

Test-field approximation
(fully coupled evolution ongoing)

More sGB results in
Lara, Corman, Nee+, arXiv:2505.14785
(charge flips in sGB inspirals)

Summary

- **NR for quasi-circular binaries mature pillar of GW astronomy**
    - must keep up with improving detectors!
- **High mass-ratio challenging**
    - continued progress
    - 2GSF accurate at all q
        - but zero spin!
    - worldtube-method promising
- **Progress in NR scattering**
    - E, b, q, spins
    - but large parameter space only scratched so far



Visualization of GW energy flux
Markin, Ramos-Buades, HP