Post-Minkowskian theory, Numerical Relativity and Effective-One-Body models

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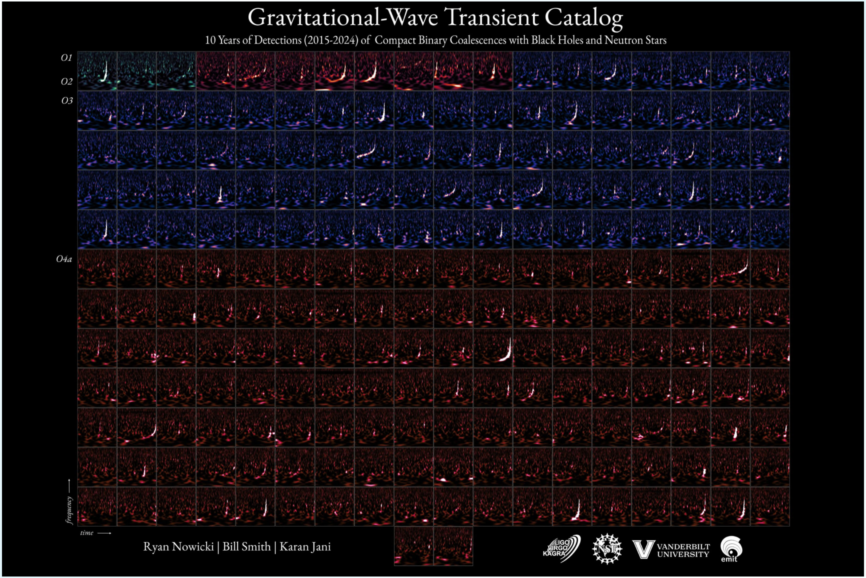
Amplitudes, Strong-Field Gravity and Resummation
Nordita - April 17, 2026

INTRODUCTION


Gravitational-Wave Transient Catalog
10 Years of Detections (2015-2024) of Compact Binary Coalescences with Black Holes and Neutron Stars
O1
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Ryan Nowicki | Bill Smith | Karan Jani
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Theoretical models

The analysis of gravitational-wave (GW) events relies on comparisons with **theoretical predictions** through Bayesian inference.

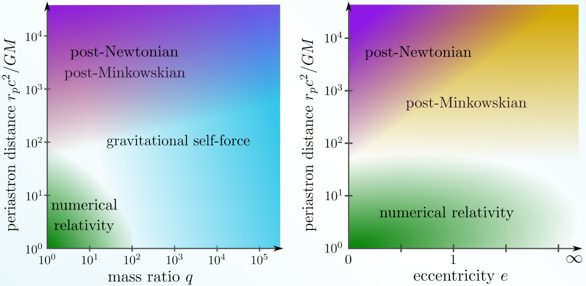
Given the sensitivity of current and future detectors, GW models need to many criteria:

- **high accuracy** to match the improved sensitivities;
- **great speed** to generate the required $\sim 10^7 - 10^8$ waveforms in a reasonable time;
- **physical completeness** (eccentricity, spin-precession, beyond-GR terms) to avoid possible biases.

Perturbation series

We can solve Einstein's equations analytically using approximations, such as:

- ✧ **Post-Newtonian (PN)**, assuming small velocities, $\frac{v}{c} \ll 1$;
- ✧ **Post-Minkowskian (PM)**, assuming weak fields, $\frac{GM}{rc^2} \ll 1$;
- ✧ **Self-Force (SF)**, assuming large mass ratios, $\frac{m_2}{m_1} \ll 1$.

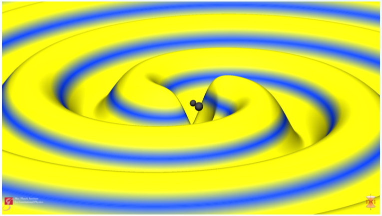


credit: Khalil *et al.* [1]

Numerical Relativity

We can perform **Numerical Relativity (NR)** simulations to get a very accurate solution of Einstein's equations.

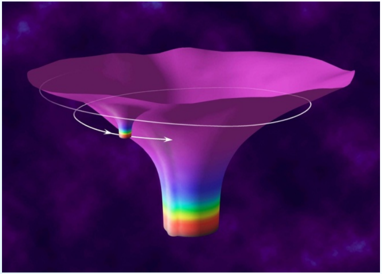
NR simulations are computationally heavy, but we can interpolate waveforms (where available) and build **NR surrogates**.



They are also directly employed to build **EOB-based models** and **phenomenological approximants**, which combine analytical and numerical GR solutions.

Effective-one-body

The **effective-one-body (EOB)** approach, first introduced by Buonanno and Damour [2, 3], maps the two-body dynamics into an effective one-body system.



EOB is a powerful tool, as it:

- includes the **full**, nonperturbative, **test-mass limit**.
- resums the residual perturbative series.

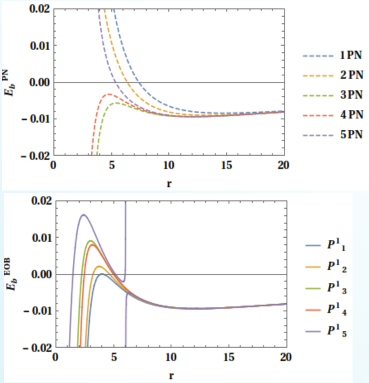
The EOB framework imposes a relation between the energies of the real and effective systems:

$$\frac{E_{\text{eff}}}{\mu} = \frac{(E_{\text{real}})^2 - m_1^2 - m_2^2}{2\, m_1\, m_2}.$$

EOB resummations

The EOB energy relation implies that

$$\hat{H}_{\text{EOB}} = \frac{1}{\nu}\sqrt{1+2\nu\left(\hat{H}_{\text{eff}}-1\right)}, \quad \text{where} \quad \hat{H}_{\text{eff}} = \sqrt{A\left(1+\frac{p_\varphi^2}{r^2}\right)},$$

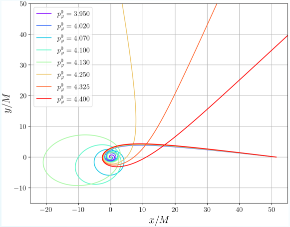


And the A metric potential reads

$$A(u) = 1 - 2u + 2\nu u^3 + \dots, \qquad u \equiv \frac{GM}{rc^2}.$$

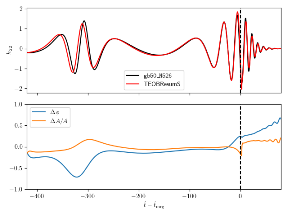
Imposing both the EOB relations and arbitrary
resummations, we obtain a stable result.

PM approximation

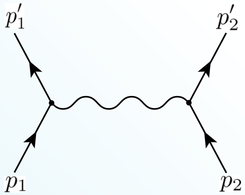


The PM approximation [4, 5, 6], which only assumes weak fields $[GM/(rc^2) \ll 1]$ and **allows for arbitrarily large velocities**, is particularly suitable for describing scattering systems.

Hopefully, it would help improve GW models for eccentric and hyperbolic binaries signals.



PM results



PM results have been computed through various approaches, such as: scattering amplitudes, eikonalization, effective field theory, worldline quantum field theory, . . .

For nonspinning black-hole binaries, **4PM results** are available [6, 7, 8, 9, 10, 11, 12], together with **partial 5PM** contributions [13, 14].

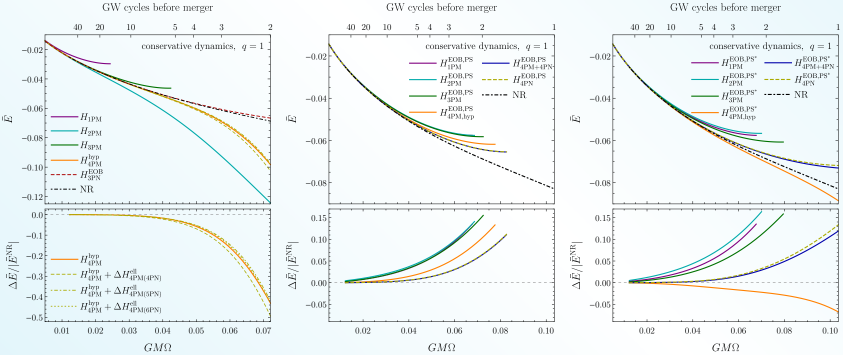
Results are also available for:

- **spinning systems** [15, 16, 17, 18, 19, 20];
- **tidally-interacting bodies** [21, 22, 23, 24];
- **beyond-GR theories** [4, 25];

et cetera.

Energy comparisons

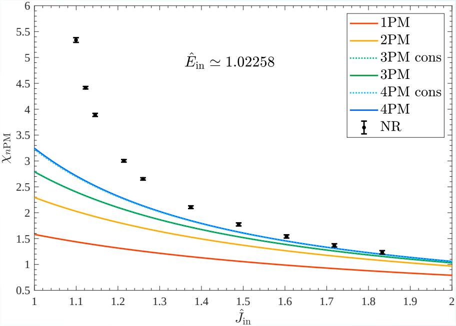
First attempts to use PM results to build EOB-based models for bound orbits were a little disappointing (see, e.g. [26, 1]).



credit: Khalil *et al.* [1]

Scattering angle comparisons

The comparison against NR simulations of nonspinning BBH scatterings [27] again showed poor agreement [28, 1].



The PM expansion of scattering angles

$$\chi_{n\mathrm{PM}}(\gamma, j) = 2\frac{\chi_1(\gamma)}{j} + 2\frac{\chi_2(\gamma)}{j^2} + 2\frac{\chi_3(\gamma)}{j^3} \dots$$

holds for large angular momenta but loses accuracy in strong-field systems.

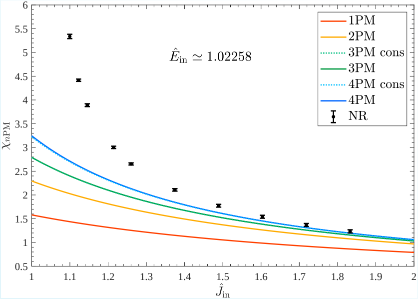
SCATTERING SIMULATIONS

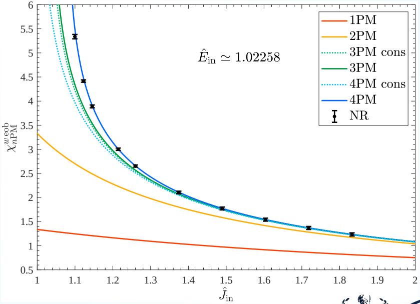
EOB resummation

We first proposed a **resummation** of the PM scattering angles [28], making use of an **EOB gravitational potential** of the form:

$$w_{n\mathrm{PM}}(\bar r,\gamma) = \frac{w_1(\gamma)}{\bar r} + \frac{w_2(\gamma)}{\bar r^2} + \frac{w_2(\gamma)}{\bar r^2} + \dots$$

This reformulation greatly improves the agreement with numerical data (see also [29, 30, 31]):





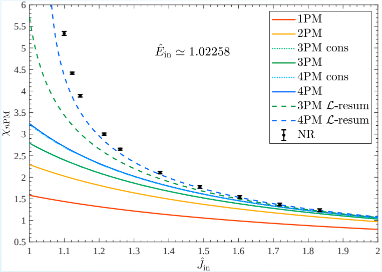
Critical angular momentum

We also introduced [28] a resummation of the PM scattering angles that takes into account the j-singularity due to the boundary between scattering and plunge, such that

$$\chi^{\mathcal{L}}_{n\text{PM}}(\gamma, j) = \mathcal{L}\left(\frac{j_0}{j}\right)\hat{\chi}_{n\text{PM}}(\gamma, j; j_0)\,,$$

with

$$\mathcal{L}(x) \equiv \frac{1}{x}\ln\left[\frac{1}{1-x}\right]\,, \qquad \text{and} \qquad j_0^{n\text{PM}}(\gamma) \equiv \left[n\frac{\chi_n(\gamma)}{\chi_1(\gamma)}\right]^{\frac{1}{n-1}}\,.$$



This procedure already improves the PM-NR agreement.

Inversion formula

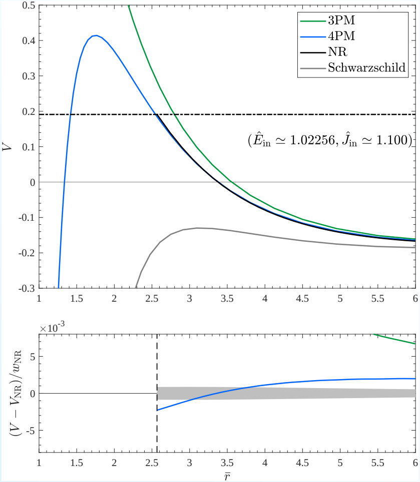
The formula linking the EOB potential and the respective scattering angles is:

$$\pi + \chi\,(\gamma, j) = 2\,j \int_0^{\bar{u}_{\max}(\gamma,j)} \frac{d\bar{u}}{\sqrt{p_\infty^2 + w(\bar{u},\gamma) - j^2 \bar{u}^2}}\,, \quad \text{with} \quad \bar{u} \equiv \frac{1}{\bar{r}}\,.$$

This means we can extract information about the underlying gravitational potential if we know the scattering angles. In particular, we make use of Firsov's inversion formula:

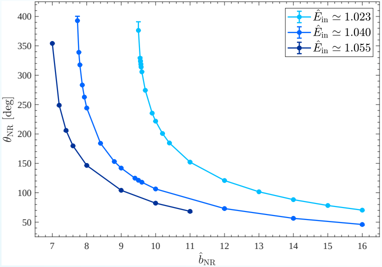
$$\ln\left[1 + \frac{w(\bar{u}, p_\infty)}{p_\infty^2}\right] = \frac{2}{\pi} \int_{\bar{r}\,|p(\bar{r},\gamma)|}^{\infty} dj \frac{\chi(\gamma, j)}{\sqrt{j^2 - \bar{r}^2\, p^2(\bar{r},\gamma)}}\,,$$

NR potential



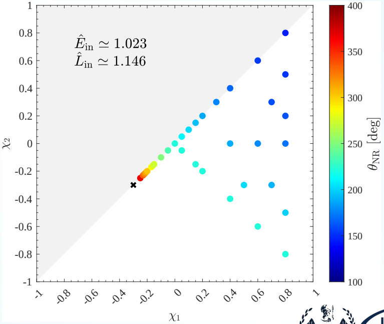
Using Firsov's formula, we can invert the sequence of scattering angles to **obtain an NR gravitational potential**.

Spinning simulations



We performed **equal-mass, nonspinning simulations** [29] **at higher energies** using the `Einstein Toolkit` [32].

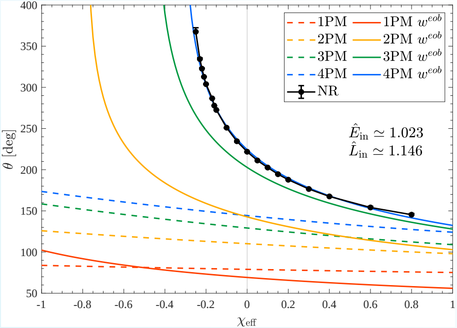
We also computed scattering angles for **equal-mass, unequal-spin BBHs** [29].



Extension to spin

We could extend the EOB potential to take into account (aligned) spin effects:

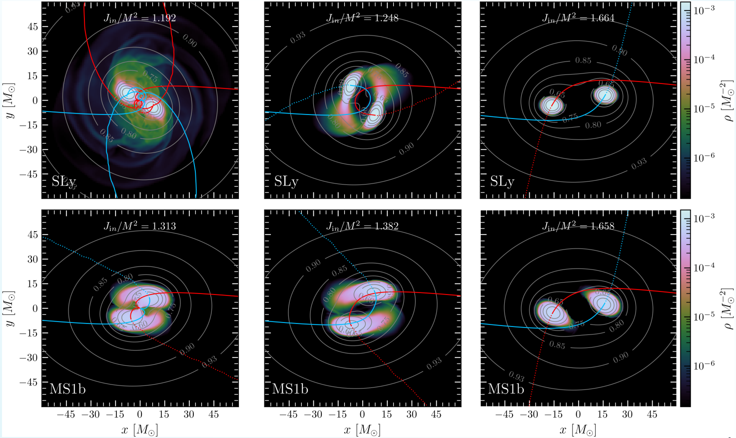
$$w_{n\mathrm{PM}}(\bar{r},\gamma,\ell,S_i) = w^{\mathrm{orb}}(\bar{r},\gamma) + \frac{\ell\, w^{\mathrm{S}}_{n\mathrm{PM}}(\bar{r},\gamma)}{\bar{r}^2} + \frac{w^{\mathrm{S}^2}_{n\mathrm{PM}}(\bar{r},\gamma)}{\bar{r}^2} + \frac{\ell\, w^{\mathrm{S}^3}_{n\mathrm{PM}}(\bar{r},\gamma)}{\bar{r}^4} + \frac{w^{\mathrm{S}^4}_{n\mathrm{PM}}(\bar{r},\gamma)}{\bar{r}^4}.$$



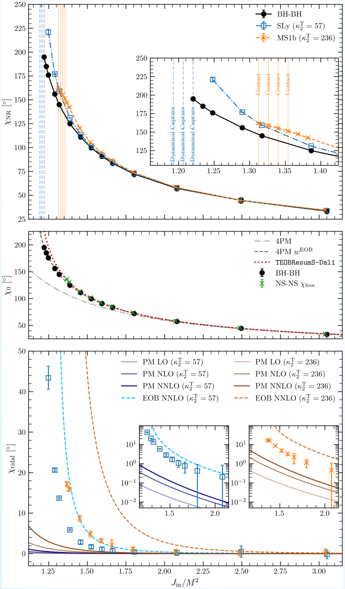
The 4PM EOB-resummed angles are in excellent agreement with numerical data (see also [30]).

Neutron star simulations

We performed **equal-mass, neutron star (NS) scattering simulations** [33] using the `BAM` code [34, 35].



BNS scattering angles



We computed scattering angles for 2 different equations of state.

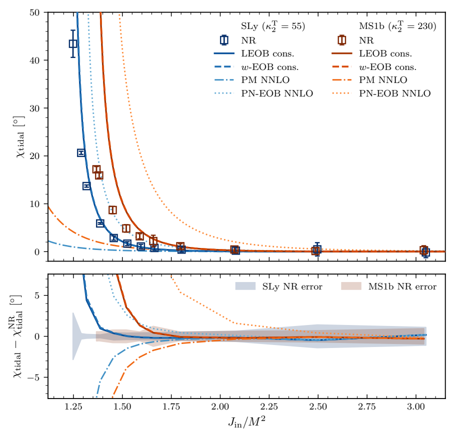
We compared the tidal scattering angle,

$$\chi_{\rm tidal} = \chi - \chi_0^{\rm BH},$$

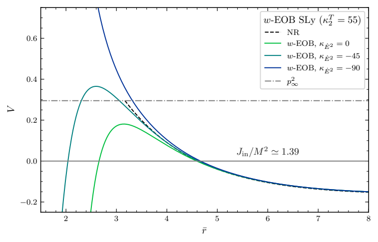
to analytical PM results and found an acceptable agreement only for very high $J_{\rm in}$.

EOB tidal potentials

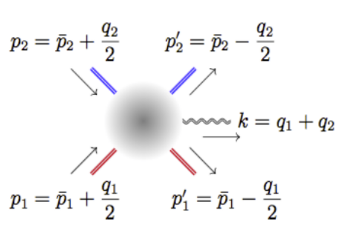
We again mapped the tidal PM angles to EOB potentials (both w-EOB and LEOB) and compared against NR scattering angles [36].



EOB-resumming PM results improves agreement, especially for the LEOB case.



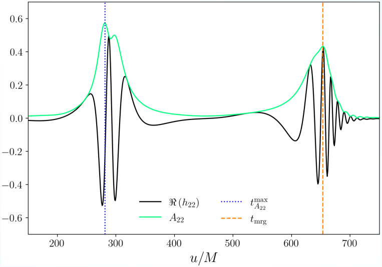
Waveforms



Leading-order PM waveform computed long ago by Kovacs and Thorne [37].
One-loop computations $O\left(G^3\right)$ completed recently [38, 39], but not yet in a form useful for GW modeling.

There are still issues in extracting gravitational waveforms and fluxes from some NR codes [40].

→ maybe SXS?



BOUND ORBITS

EOB-PM Hamiltonian: issue #1

It is possible to extract information about bound orbits from PM results [4, 41], generally by informing a (local) Hamiltonian [26, 1, 42].

While PN terms greatly simplify in the EOB frame [43, 44], PM ones keep their **convoluted dependence on the effective energy** (γ).

$$g^{\mu\nu} p_\mu p_\nu = -\frac{\gamma^2}{A} + \frac{p_r^2}{B} + \frac{j^2 u^2}{C} \,.$$

$$A(u) = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41\pi^2}{32}\right)\nu u^4 + \left(\left(\frac{2275\pi^2}{512} - \frac{4237}{60} + \frac{128}{5}\gamma_E + \frac{256}{5}\ln 2\right)\nu + \left(\frac{41\pi^2}{32} - \frac{221}{6}\right)\nu^2 + \frac{64}{5}\nu \ln u\right)u^5, \tag{2.7a}$$

$$\bar{D}(u) = 1 + 6\nu u^2 + (52\nu - 6\nu^2)u^3 + \left(\left(-\frac{533}{45} - \frac{23761\pi^2}{1536} + \frac{1184}{15}\gamma_E - \frac{6496}{15}\ln 2 + \frac{2916}{5}\ln 3\right)\nu + \left(\frac{123\pi^2}{16} - 260\right)\nu^2 + \frac{592}{15}\nu \ln u\right)u^4, \tag{2.7b}$$

$$\hat{Q}(\mathbf{r}', \mathbf{p}') = \left(2(4-3\nu)\nu u^2 + \left(\left(-\frac{5308}{15} + \frac{496256}{45}\ln 2 - \frac{33048}{5}\ln 3\right)\nu - 83\nu^2 + 10\nu^3\right)u^3\right)(\mathbf{n}'\cdot\mathbf{p}')^4 + \left(\left(-\frac{827}{3} - \frac{2358912}{25}\ln 2 + \frac{1399437}{50}\ln 3 + \frac{390625}{18}\ln 5\right)\nu - \frac{27}{5}\nu^2 + 6\nu^3\right)u^2(\mathbf{n}'\cdot\mathbf{p}')^6 + O[\nu u(\mathbf{n}'\cdot\mathbf{p}')^8]. \tag{2.7c}$$

Hamiltonian potentials up to 4PN

$$\begin{aligned}
\alpha^{(4)}_{(0,0)} = {} & \frac{7\nu\left(380\gamma^2+169\right)}{8(\gamma-1)\gamma^2\Gamma^3}\mathrm{E}^2\left(\frac{\gamma-1}{\gamma+1}\right) + \frac{\left(1200\gamma^2+2095\gamma+834\right)\nu}{4\gamma^2\left(\gamma^2-1\right)\Gamma^3}\mathrm{K}^2\left(\frac{\gamma-1}{\gamma+1}\right) \\
& + \frac{\left(-1200\gamma^3-2660\gamma^2-2929\gamma-1183\right)\nu}{4\gamma^2\left(\gamma^2-1\right)\Gamma^3}\mathrm{E}\left(\frac{\gamma-1}{\gamma+1}\right)\mathrm{K}\left(\frac{\gamma-1}{\gamma+1}\right) \\
& + \frac{\left(-25\gamma^6+30\gamma^4+111\gamma^2+20\right)\nu}{\gamma^2\Gamma^3}\mathrm{Li}_2\left(\frac{1-\gamma}{1+\gamma}\right) + \frac{(\gamma+1)\left(25\gamma^5-25\gamma^4-5\gamma^3+65\gamma^2+64\gamma+12\right)\nu}{2\gamma^2\Gamma^3}\mathrm{Li}_2\left(\frac{\gamma-1}{\gamma+1}\right) \\
& + \frac{\left(35\gamma^4+120\gamma^3+90\gamma^2+152\gamma+27\right)\nu}{2\gamma^2\Gamma^3}\log^2\left(\frac{\gamma+1}{2}\right) - \frac{4\left(2\gamma^2-3\right)\left(15\gamma^2-15\gamma+4\right)\nu}{\gamma(\gamma+1)\Gamma^3}\frac{\mathrm{Ti}_2\left(\sqrt{\frac{1-\gamma}{1+\gamma}}\right)}{\sqrt{1-\gamma^2}} \\
& + \frac{\left(2\gamma^2-3\right)^2\left(35\gamma^4-30\gamma^2+11\right)\nu}{8\left(\gamma^2-1\right)^3\Gamma^3}\arccos^2\gamma + \frac{2\left(75\gamma^6-140\gamma^4-283\gamma^2-852\right)\nu}{3\gamma\left(\gamma^2-1\right)\Gamma^3}\log(\gamma) \\
& + \frac{\left(210\gamma^6-552\gamma^5+339\gamma^4-912\gamma^3+3148\gamma^2-3336\gamma+1151\right)\nu}{12\gamma^2\left(\gamma^2-1\right)\Gamma^3}\log\left(\frac{u}{4}\right) \\
& + \left(\frac{\left(-35\gamma^4-60\gamma^3+150\gamma^2-76\gamma+5\right)\nu}{2\gamma^2\Gamma^3}\log\left(\frac{u}{4}\right)\right. \\
& \qquad \left. + \frac{\left(-75\gamma^7+416\gamma^5+612\gamma^4+739\gamma^3+136\gamma^2+2520\gamma+152\right)\nu}{3\gamma^2\left(\gamma^2-1\right)\Gamma^3}\right)\log\left(\frac{\gamma+1}{2}\right) \\
& + \left(\frac{\left(-420\gamma^9+96\gamma^8-48\gamma^7+5328\gamma^6-5279\gamma^5-1584\gamma^4+7142\gamma^3-9360\gamma^2+3453\gamma+720\right)\nu}{12\gamma^2(\gamma^2-1)^2\Gamma^3}\right. \\
& \qquad - \frac{48\left(7\gamma^2-5\right)\left(4\gamma^4-12\gamma^2-3\right)(\Gamma-1)\nu}{12\gamma^2(\gamma^2-1)\Gamma^3} \\
& \qquad \left. - \frac{\left(2\gamma^2-3\right)\left(35\gamma^4-30\gamma^2+11\right)\nu}{4\gamma(1-\gamma^2)\Gamma^3}\log\left(\frac{u}{4}\right) + \frac{4\left(2\gamma^2-3\right)\left(15\gamma^2+2\right)\nu}{\gamma(1-\gamma^2)\Gamma^3}\log\left(\frac{\gamma+1}{2}\right)\right)\frac{\arccos\gamma}{\sqrt{1-\gamma^2}} \\
& + (\Gamma-1)\left(\frac{5115\gamma^8-9537\gamma^6+5657\gamma^4-1115\gamma^2+72}{16\gamma^4\left(\gamma^2-1\right)^2\Gamma^3}\right. \\
& \qquad \left. + \frac{\left(8159\gamma^8-3136\gamma^7-23601\gamma^6-3360\gamma^5+15409\gamma^4+4000\gamma^3-1995\gamma^2+108\right)\nu}{24(\gamma-1)\gamma^4(\gamma+1)^2\Gamma^3}\right) \\
& + \frac{\nu}{144\gamma^9\left(\gamma^2-1\right)^2\Gamma^3}\left(-600\pi^2\gamma^{17}+3600\gamma^{16}+480\left(9+4\pi^2\right)\gamma^{15}+2\left(720\pi^2-28843\right)\gamma^{14}+\left(36759-5136\pi^2\right)\gamma^{13}\right. \\
& \qquad + \left(44698-1056\pi^2\right)\gamma^{12}+\left(6624\pi^2-43235\right)\gamma^{11}+\left(7702-2208\pi^2\right)\gamma^{10}-5\left(2155+504\pi^2\right)\gamma^9 \\
& \qquad \left. + 2\left(23947+912\pi^2\right)\gamma^8-\left(45605+288\pi^2\right)\gamma^7+12701\gamma^6+648\gamma^5-1471\gamma^4+207\gamma^2-45\right),
\end{aligned}$$

Hamiltonian contribution at 4PM

EOB-PM Hamiltonian: issue #2

PM computations inherently include **open-orbit hereditary contributions**.

In order to utilize PM results for bound systems, we need to separate local and nonlocal-in-time terms [45, 46].

Bini and Damour [46] have obtained the missing 4PM term in the local action up to order p_∞^{30}.

A new approach to EOB

Recently [47] we proposed a new way of solving the EOB equations of motion.

Instead of solving the usual Hamilton's equations

$$S[x^\mu, p_\mu] = \int \left[p_i \frac{dx^i}{t_{\rm eff}} - \hat{H}_{\rm eff}(x^i, p_i) \right] dt_{\rm eff} \quad \longrightarrow \quad \begin{cases} \dfrac{dx^i}{dt} = \dfrac{\partial \hat{H}_{\rm eff}}{\partial p_i} \\ \dfrac{dp_i}{dt} = -\dfrac{\partial \hat{H}_{\rm eff}}{\partial x^i} + \mathcal{F}_i \end{cases}$$

one can introduce a **Lagrange multiplier** $e_{\rm L}$ and a **constraint** $\mathcal{C}$ such that

$$S[x^\mu, p_\mu, e_{\rm L}] = \int \left[p_\mu \frac{dx^\mu}{d\tau} - e_{\rm L}\, \mathcal{C}\,(x^\mu, p_\mu) \right] d\tau \quad \longrightarrow \quad \begin{cases} \dfrac{dx^\mu}{d\tau} = e_{\rm L}\, \dfrac{\partial \mathcal{C}}{\partial p_\mu} \\ \dfrac{dp_\mu}{d\tau} = -e_{\rm L}\, \dfrac{\partial \mathcal{C}}{\partial x^\mu} + \mathcal{F}_\mu \\ \mathcal{C} = 0 \end{cases}$$

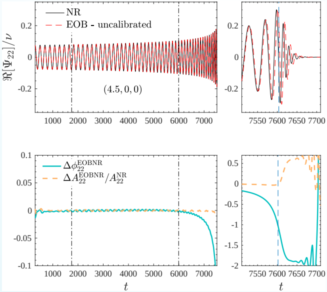
Analytical content

We built an EOB-NR model based (mostly) on PM results, exploiting the newly-presented formalism.

In particular, we included:

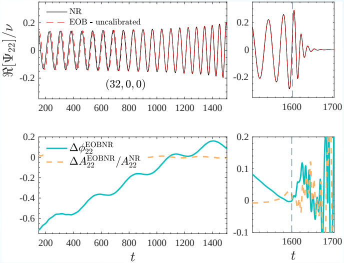
- **analytical information at the (physical) 5PM order and up to S^4** when available;
- when unavailable (either still unknown or containing nonlocal terms), we used the equivalent **leading (local + elliptic) PN term**;
- PN-based formulas for the radiative sector;
- NR information to model the merger and ringdown phases.

Uncalibrated results

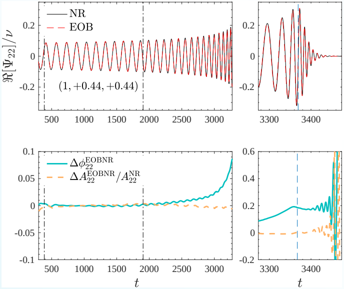


With no NR calibration (up to merger), we are able to reach a reasonable agreement with nonspinning SXS waveforms (mismatches $\sim 10^{-2}$).

The agreement improves as the mass ratio increases.



Calibrated results

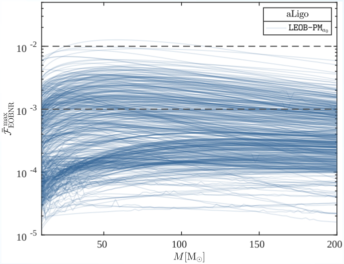


We also calibrated two free parameters to NR simulations:

$$a_5^0(\gamma,\nu) = a_{50}(\nu) + \nu\, a_{52}^{\rm NR}(\nu)\, p_\infty^2\,,$$

$$\hat{g}_3^0(\gamma,\nu,\tilde{a}_i) = \hat{g}_{30}(\nu) + \nu\, \hat{g}_{32}^{\rm NR}(\nu,\tilde{a}_i)\, p_\infty^2\,.$$

Doing so, we are able to reach a mismatch of $10^{-4}/10^{-3}$ against most quasi-circular SXS waveforms.



Conclusions

- **High-order PM results**, once recast in a particular EOB form, **show excellent agreement with NR scattering** simulations.

- The first **EOB-PM models** applied to **bound orbits** show promising signs toward their use in real data analysis.

  However, we can still improve in many ways:

  1. higher-order **analytical information** (PM, PN or SF) will help build a fully analytical model up to merger;
  2. some **physical effects** are still missing: spin-precession, environmental effects, beyond-GR terms, . . .
  3. a **PM-based** description of the **radiative sector** is not yet usable;
  4. additional **NR simulations** (and especially waveforms) are necessary to validate our models.

Thank you for your attention

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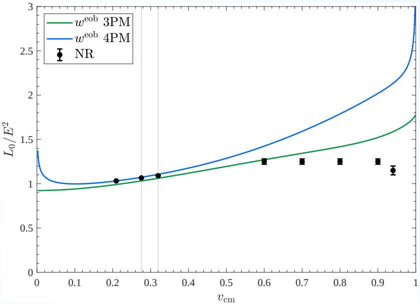
Critical j_0 predictions

We can compare analytical and numerical predictions for the critical angular momentum J_0,

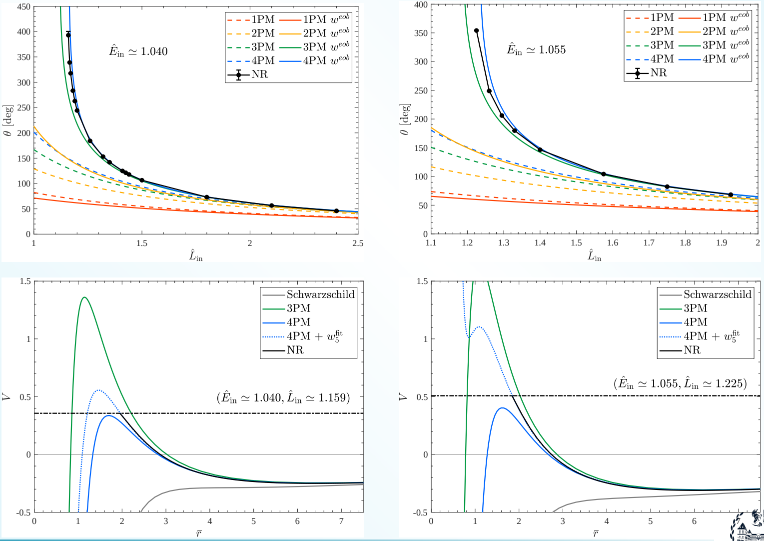
$$\frac{J_0}{E^2} = \frac{\nu\, j_0}{1 + 2\nu\,(\gamma - 1)}\,, \tag{1}$$

determining the boundary between scattering and plunge.

We were able to extend the parameter-space covered by nonspinning numerical simulations.



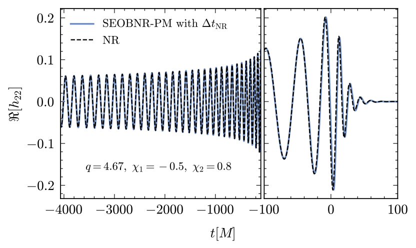
High E results

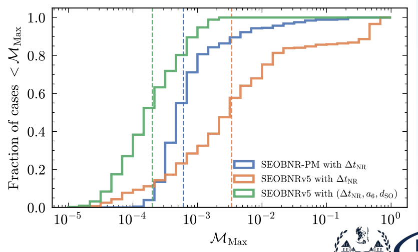


A possible EOB-PM Hamiltonian

Buonanno *et al.* [42] proposed an EOB-PM model for bound orbits:

- the spinning Hamiltonian is obtained by iteratively expressing γ as a function of phase-space variables ($\gamma \to \hat{H}_{\text{Schw}} + \hat{H}_{\text{2PM}} + \dots$);
- problematic factors in nonlocal terms are substituted by well-defined quantities [$\log(\gamma^2 - 1) \to \log(u), \dots$];
- the nonspinning Hamiltonian is completed by 4PN terms, both local and nonlocal (for bound orbits);
- the waveform is calibrated to NR simulations.





Arbitrary LEOB choices

In general, the (nonspinning) EOB mass-shell constraint will look like

$$\mathcal{C} = g^{\mu\nu} p_\mu p_\nu + 1 + Q = -\frac{\gamma^2}{A} + \frac{p_r^2}{B} + \frac{p_\varphi^2 u^2}{C} + 1 + Q = 0.$$

(Schwarzschild is recovered for $A = B^{-1} = 1 - 2u$, $C = 1$, $Q = 0$)

We chose **coordinates and gauges** so to be close to the Kerr metric in Boyer-Lindquist coordinates, i.e. $C = 1$ and

$$A\left(r, \gamma, \tilde{a}_i\right) = \frac{1 + 2u_c(\tilde{a}_0)}{1 + 2u} A_{\rm tot}\left(r, \gamma, a_i\right),$$

$$A\left(r, \gamma, \tilde{a}_i\right) B\left(r, \gamma, \tilde{a}_i\right) = \frac{r^2}{r_c^2}.$$

We also needed to chose **resummations** for our potentials, e.g.

$$A_{\rm tot} = P_4^1 \left[A_{\rm orb}^{\rm 4PM\text{-}4PN} + A_{\rm SS}^{\rm 5PM} \right].$$