Black hole scattering in the strong-field regime: Merging post-Minkowskian theory with numerical methods

Simulating eXtreme Spacetimes

Black holes, neutron stars, and beyond...

Oliver Long

Amplitudes, Strong-Field Gravity and Resummation

15th April 2026

Funded by the European Union

European Research Council
Established by the European Commission

MAX PLANCK INSTITUTE
FOR GRAVITATIONAL PHYSICS
(ALBERT EINSTEIN INSTITUTE)

The curious case of GW231123

To do all the exciting physics with gravitational waves, we need waveform models that cover the required parameter space at the required accuracy.

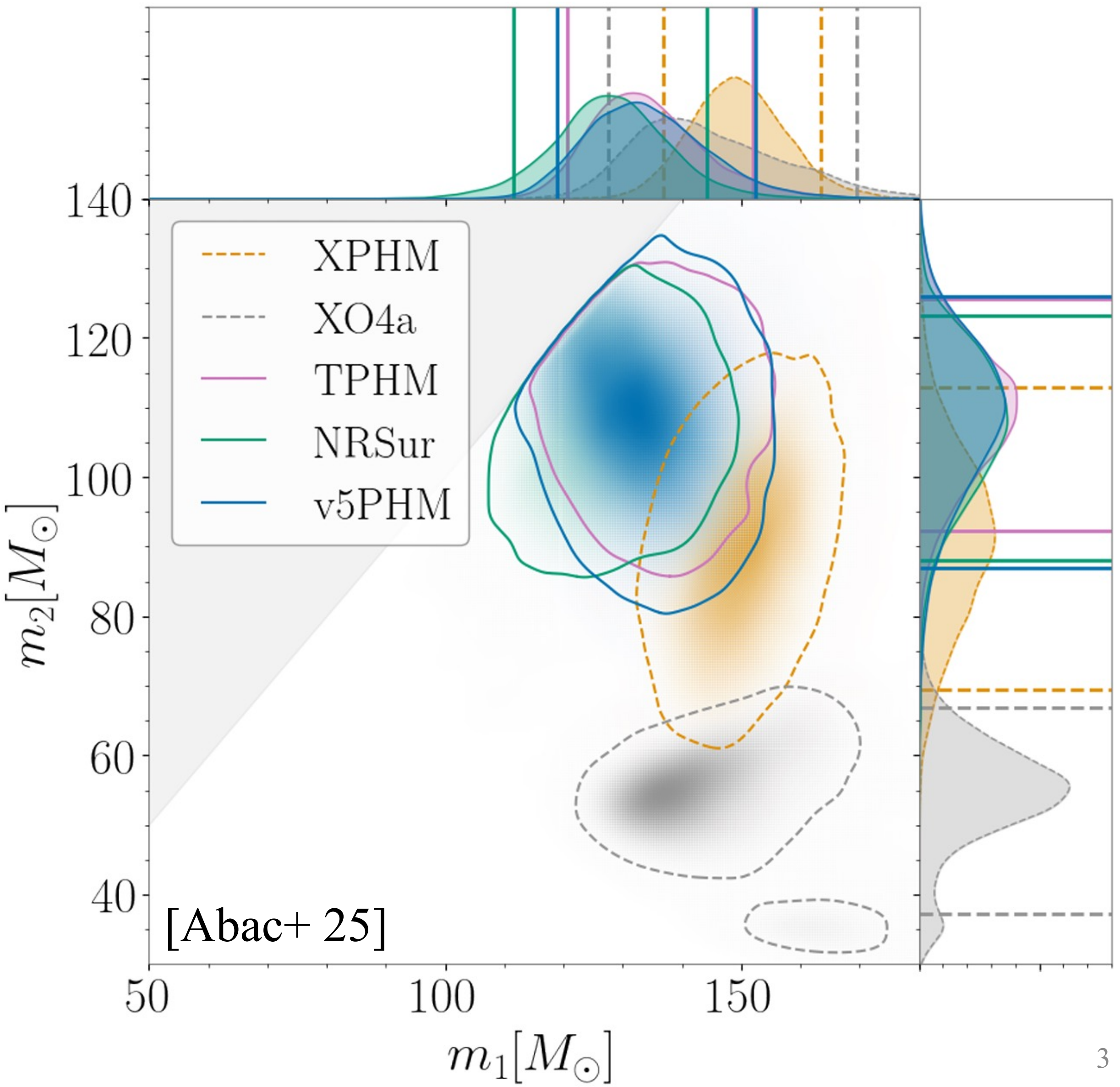
We are already running into problems with our waveform models: GW231123.

Current waveform models are not constructed/calibrated/validated up to spins of ~0.8, but the spins are estimated to be:

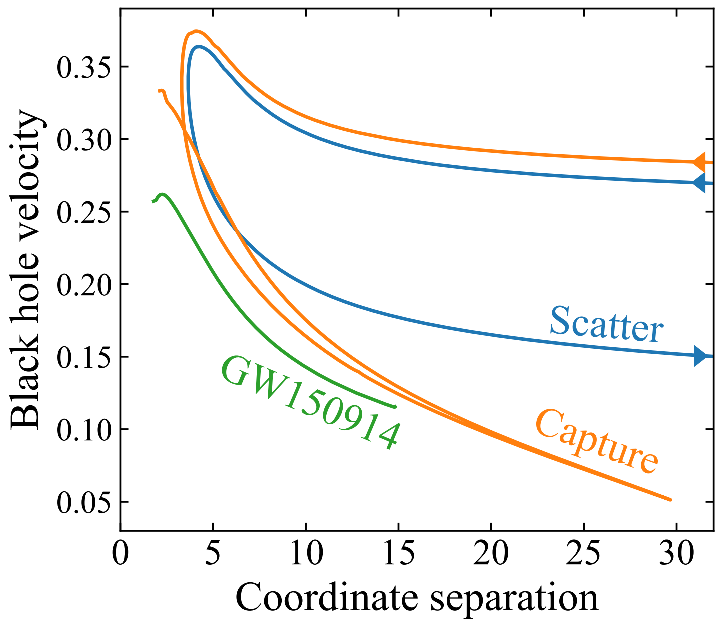
$$a_1 = 0.9^{+0.1}_{-0.19} \qquad a_2 = 0.8^{+0.2}_{-0.52}$$

Need to extend parameter space of models.

To do this, we need to understand the underlying physics of the two-body problem.



Scattering motivation: Model comparisons and strong-field



High-energy scattering systems can reach small separations and return to infinity.

Can parameterise systems in terms of asymptotic quantities e.g. initial energy & angular momentum etc.

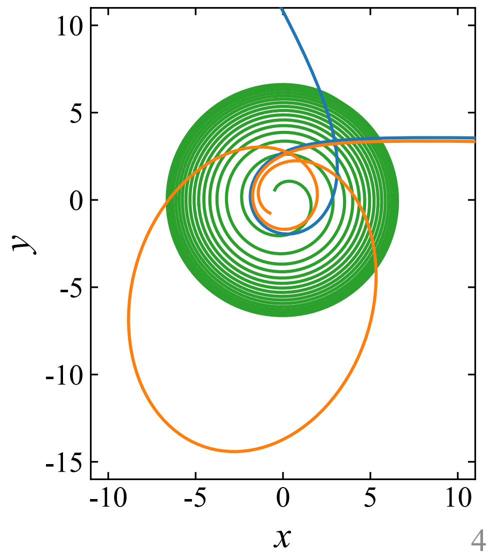
Can define asymptotic observables e.g. scattering angle.

Removes coordinate ambiguities that complicate comparisons for bound systems.

> Scattering systems can act as a clean probe of the ultra-strong gravitational potential.

Information can be translated to from scattering to bound orbits through "Boundary-To-Bound" mappings e.g. scattering angle to periastron advance [Kälin & Porto 20]:

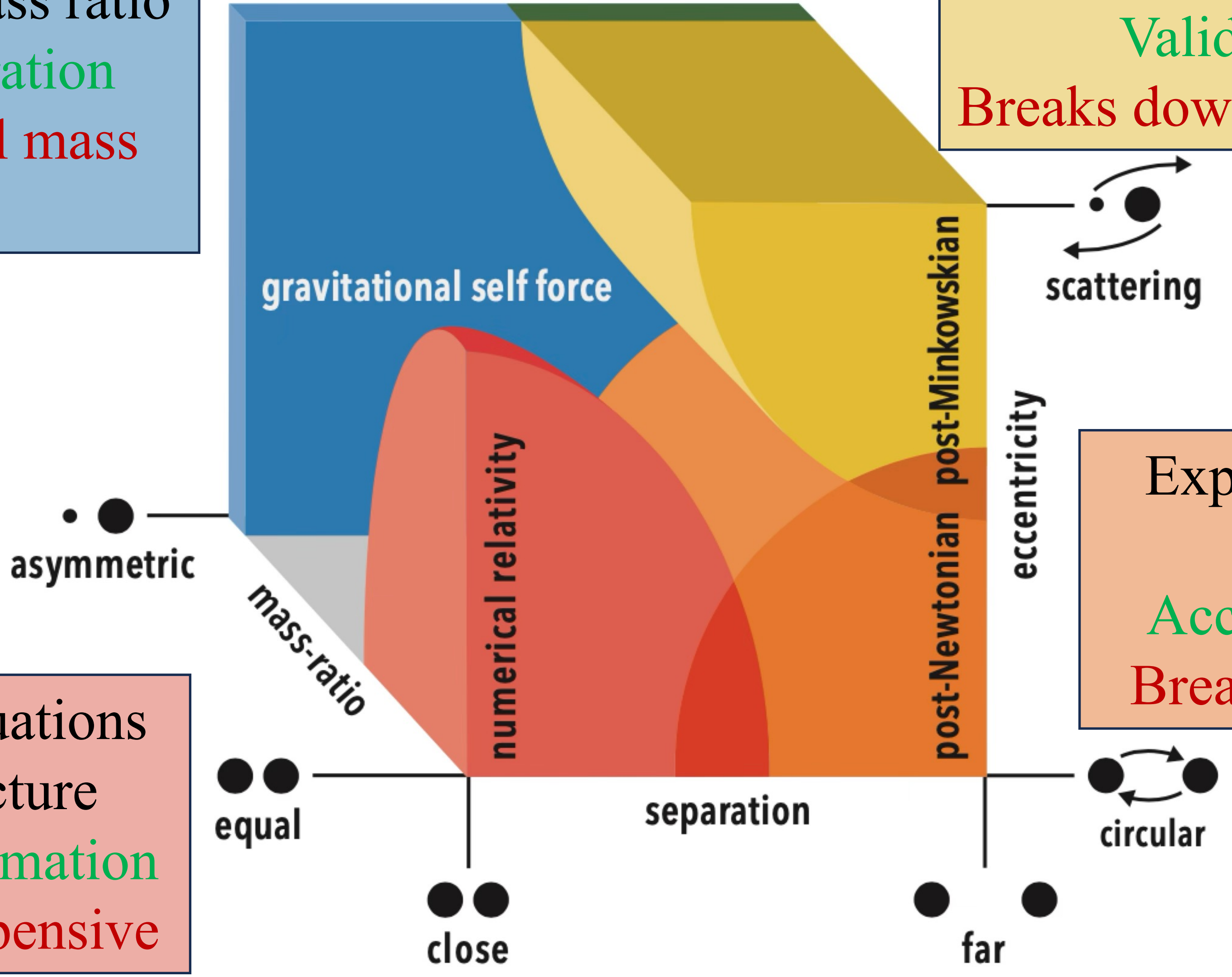
$$\Delta\Phi(E,J) = \theta(E,J) + \theta(E,-J)$$



The two-body parameter space of General Relativity

Expansion in the mass ratio
Valid at any separation
Only valid in small mass ratio limit

Credit: A. Carvalho

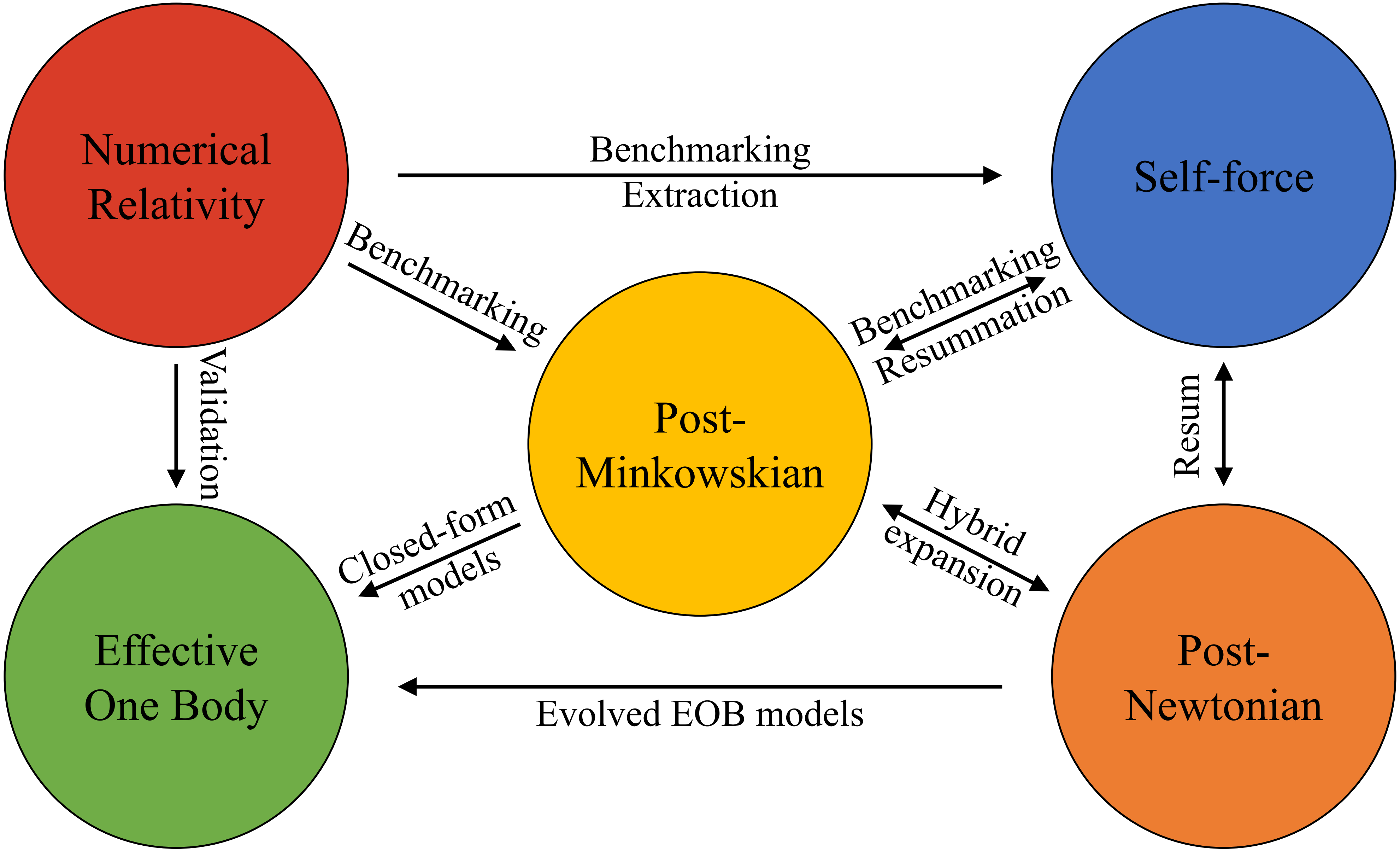


Expansion around flat space
Valid at any velocity
Breaks down for small separations

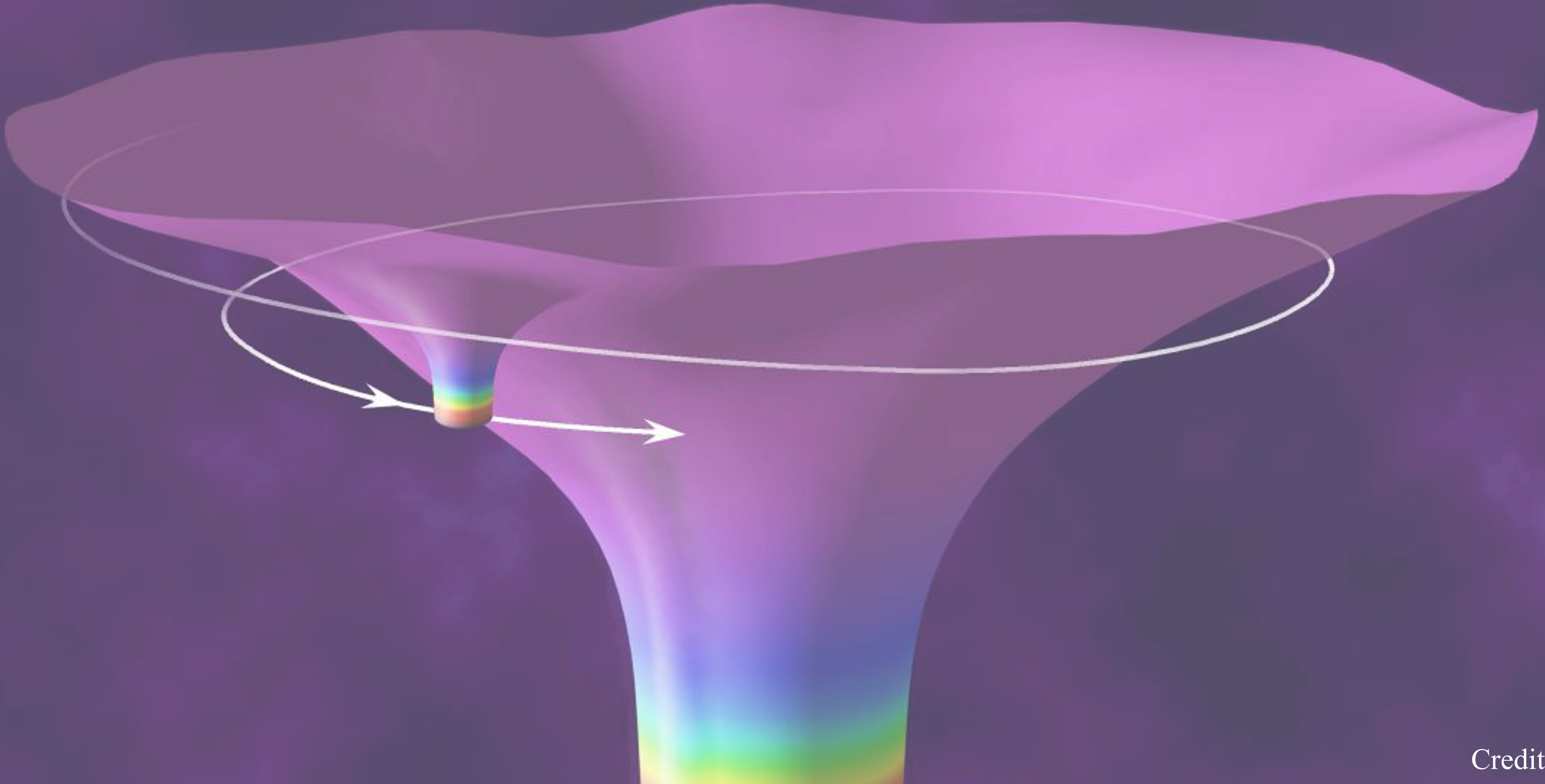
Expansion in weak-field and low velocity
Accurate during inspiral
Breaks down near merger

Solve Einstein's equations on HPC infrastructure
Full non-linear information
Computationally expensive

Contents


Numerical Relativity
Self-force
Post-Minkowskian
Effective One Body
Post-Newtonian
Benchmarking
Extraction
Benchmarking
Benchmarking
Resummation
Validation
Resum
Closed-form models
Hybrid expansion
Evolved EOB models

Self-force



Credit: NASA

Dissipative GSF and the mixed PM/PN expansion [Barack+(inc. **OL**) 26]

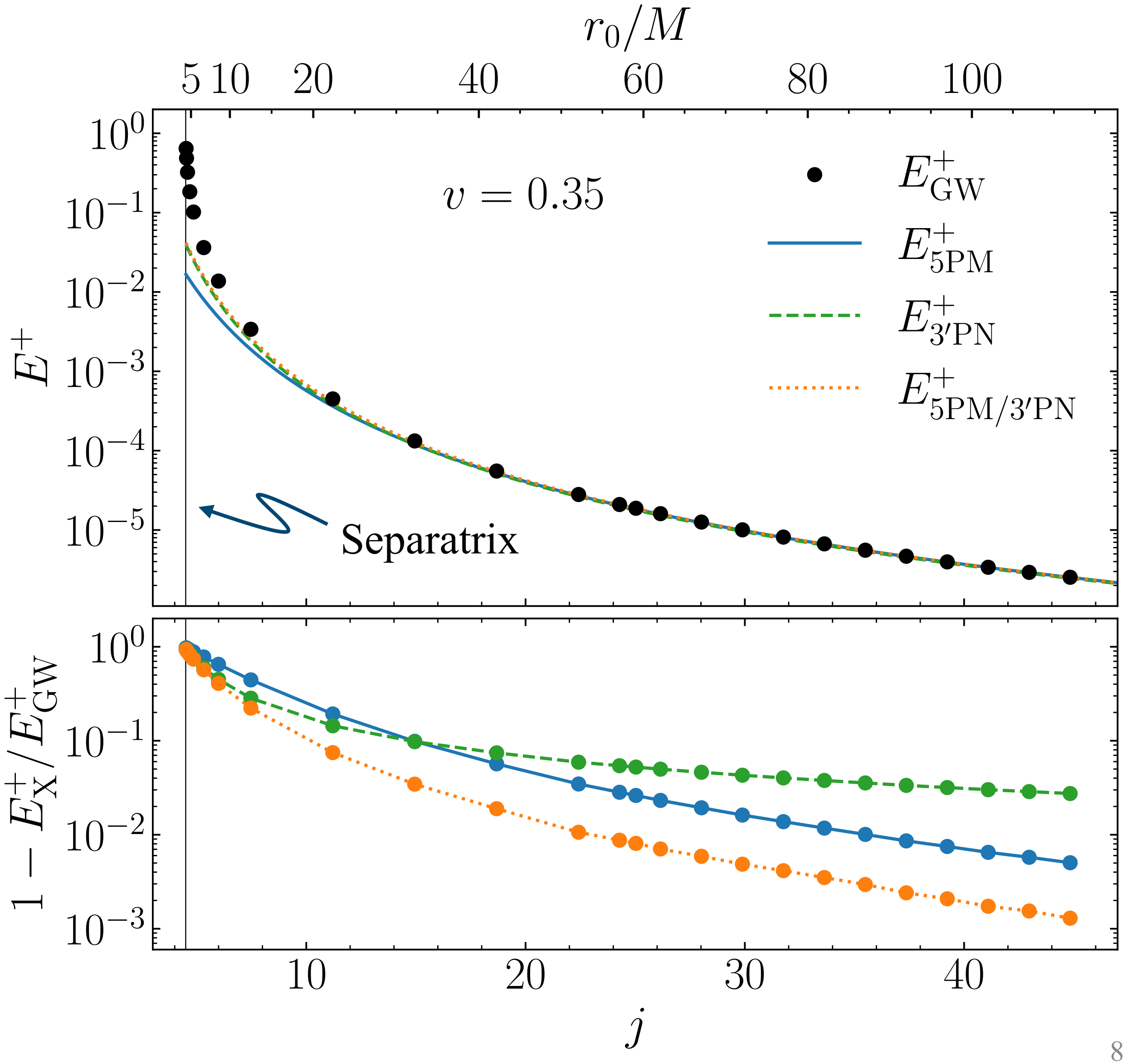
Consider the total radiated energy along the scattering orbit:

$$\nu^2 E^+ + \ldots \qquad \nu = \frac{mM}{m+M}$$

Can compare GSF numerics with analytic expressions at a variety of separations.

Combining data from 5PM and 3PN to form a mixed 5PM/3PN expansion gives more accurate results across most separations.

All of the analytic expressions break down when approaching the scatter-capture separatrix.



Divergence at the scatter-capture separatrix [Damour & Rettegno 23]

Near-separatrix geodesics undergo many revolutions in the very strong field.

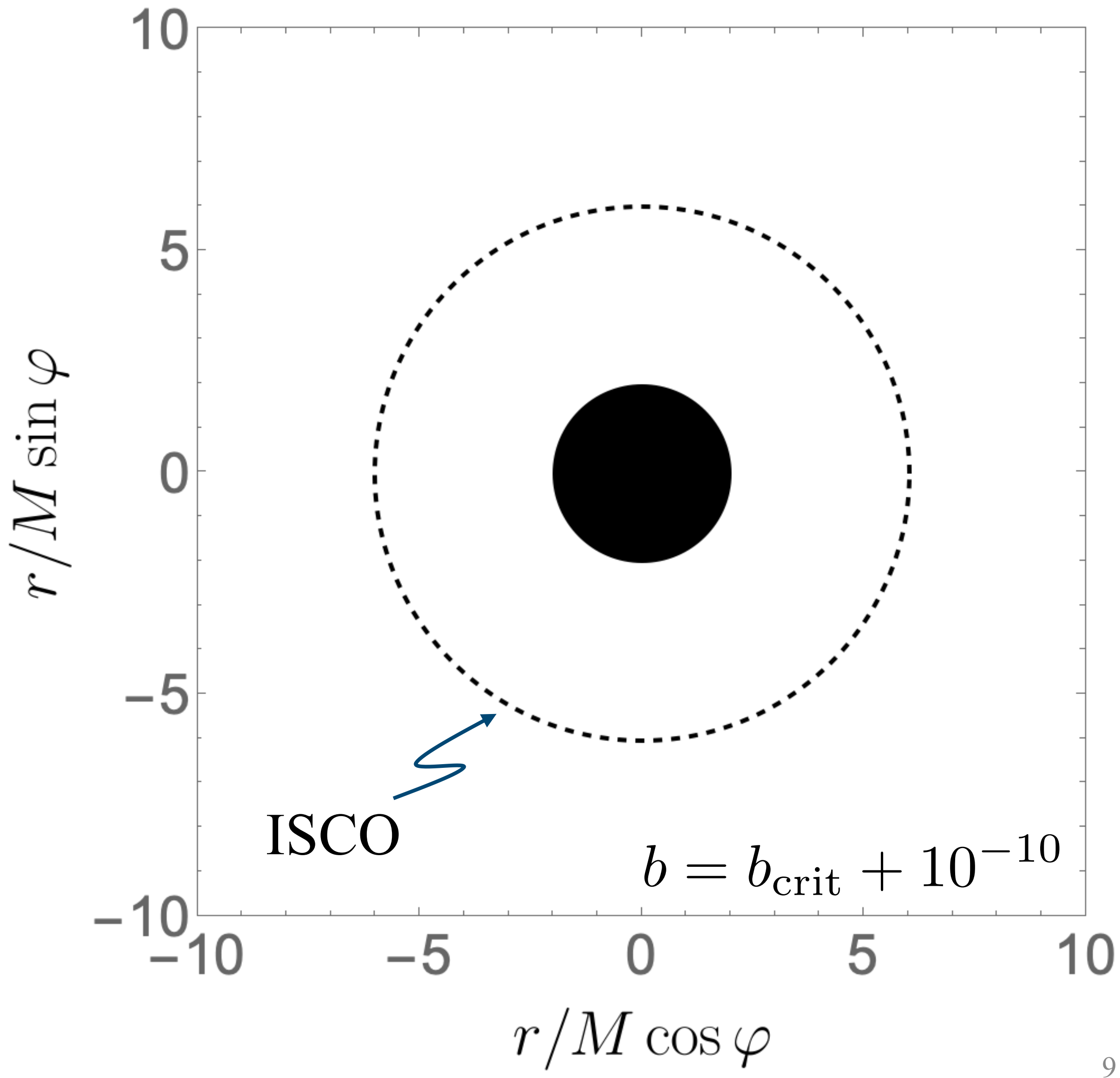
Number of orbits increases logarithmically as approaching the critical orbit:

$$N_{\text{orbits}} \propto \log(b - b_{\text{crit}})$$

The whirl behaviour quickly dominates the value of the angle:

$$\theta_{\text{geo}} \sim \log(b - b_{\text{crit}}) + \text{const}$$

Can use this behaviour to resum PM.



Geodesic resummation of the scattering angle [**OL**, Whittall & Barack 24]

Consider a resummation of the form:

$$\tilde{\theta}^{0\mathrm{SF}}_{\mathrm{resum}} = \theta^{0\mathrm{SF}}_{\mathrm{sing}} + \theta^{0\mathrm{SF}}_{4\mathrm{PM}} - \mathrm{CT}$$

Strong-field log behaviour

Weak-field analytics

Weak-field expansion of $\theta^{0\mathrm{SF}}_{\mathrm{sing}}$

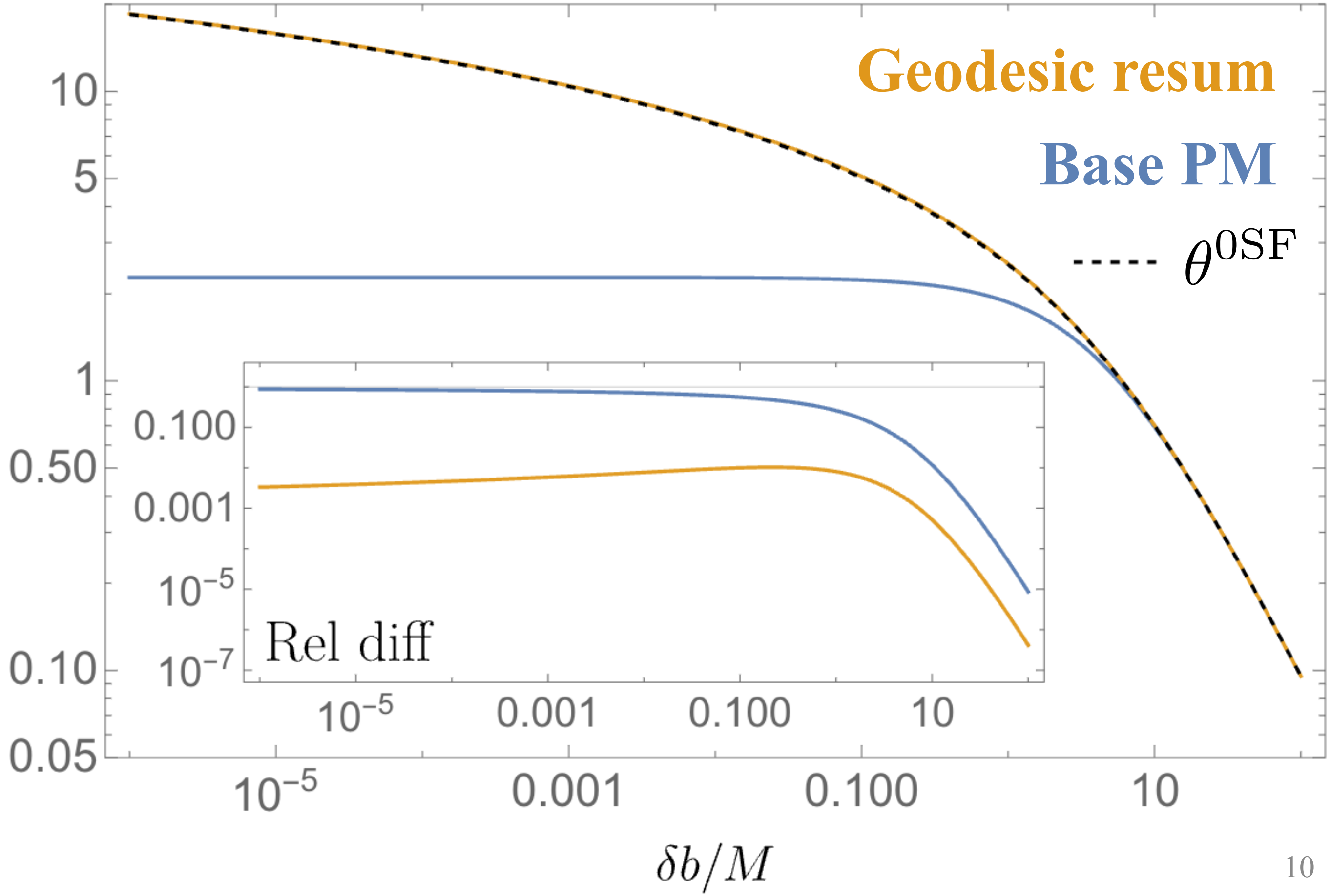
$$\theta^{0\mathrm{SF}}_{\mathrm{sing}} = A_0 \ln\left(1 - \frac{b_{\mathrm{crit}}}{b}\right)$$

$$\mathrm{CT} = A_0 \sum_{k=1}^{4} \frac{1}{k}\left(\frac{b_{\mathrm{crit}}}{b}\right)^k$$

Resummation properties:

- Diverges logarithmically when $b \to b_{\mathrm{crit}}$.
- Recovers 4PM at large impact parameter.

Resummation improves accuracy across all space.



Resummation of radiated energy [Barack+ (inc. **OL**) 26]

Use similar methodology to approximate the singular behaviour of the radiated energy as:

$$E^+_{\rm sing} \sim \mathcal{F}_\circ \times N_{\rm orbits} \sim \mathcal{F}_\circ \log\left(1 - \frac{j}{j_{\rm crit}}\right)$$

Numerical 1SF flux on critical circular orbit

Critical angular momentum

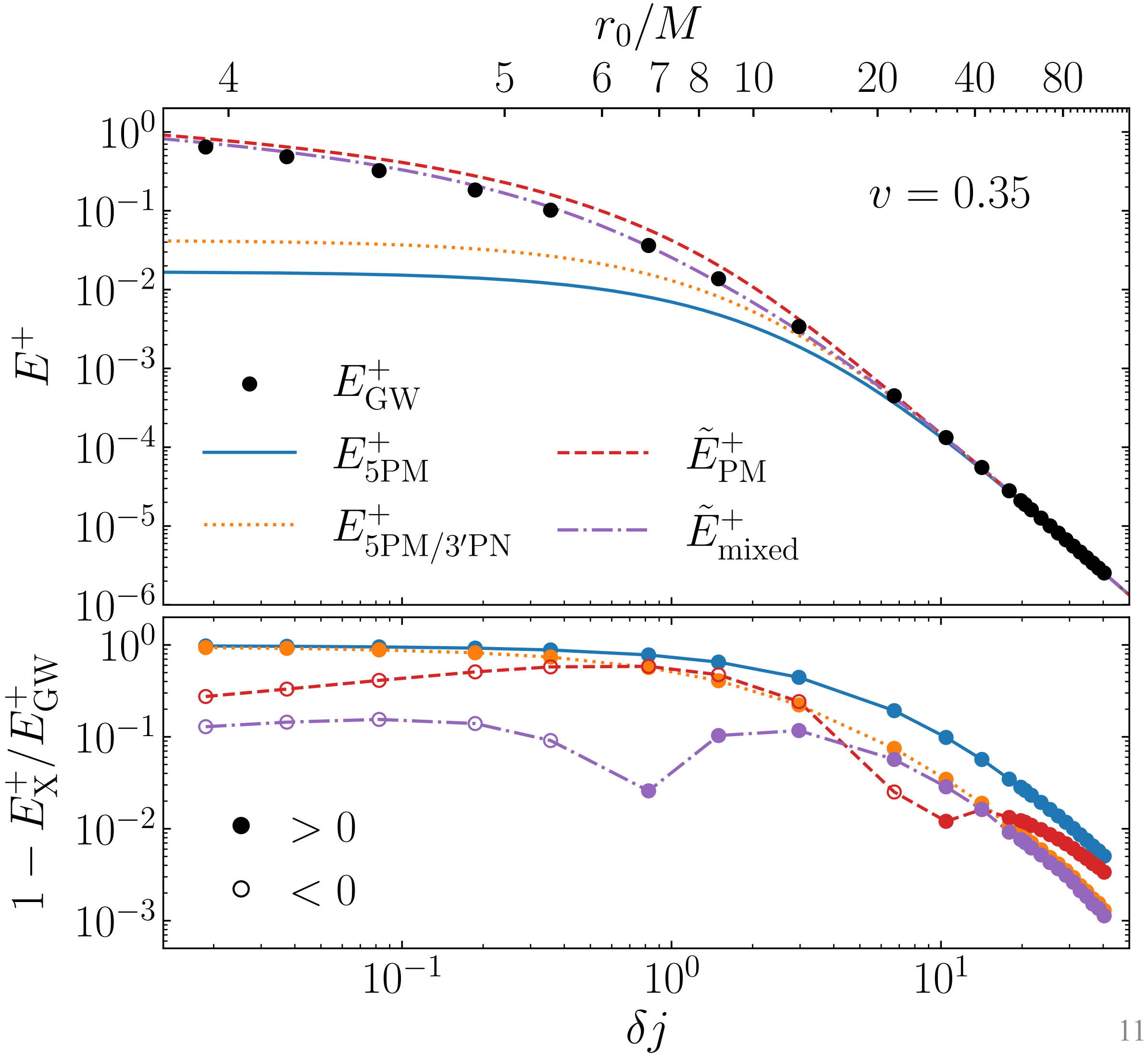
Construct a resummation:

$$\tilde{E}^+ = E^+_{\rm sing} + E^+_{\rm PM/PN} - {\rm CT}^+$$

Strong-field log behaviour

Weak-field analytics

Weak-field expansion of $E^+_{\rm sing}$



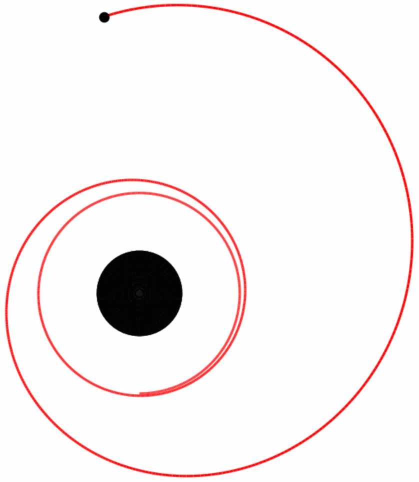
Resummation of bound orbit fluxes

[Barack, Gonzo, Leather, **OL**, Nasipak, Pound, Trestini, & Warburton]

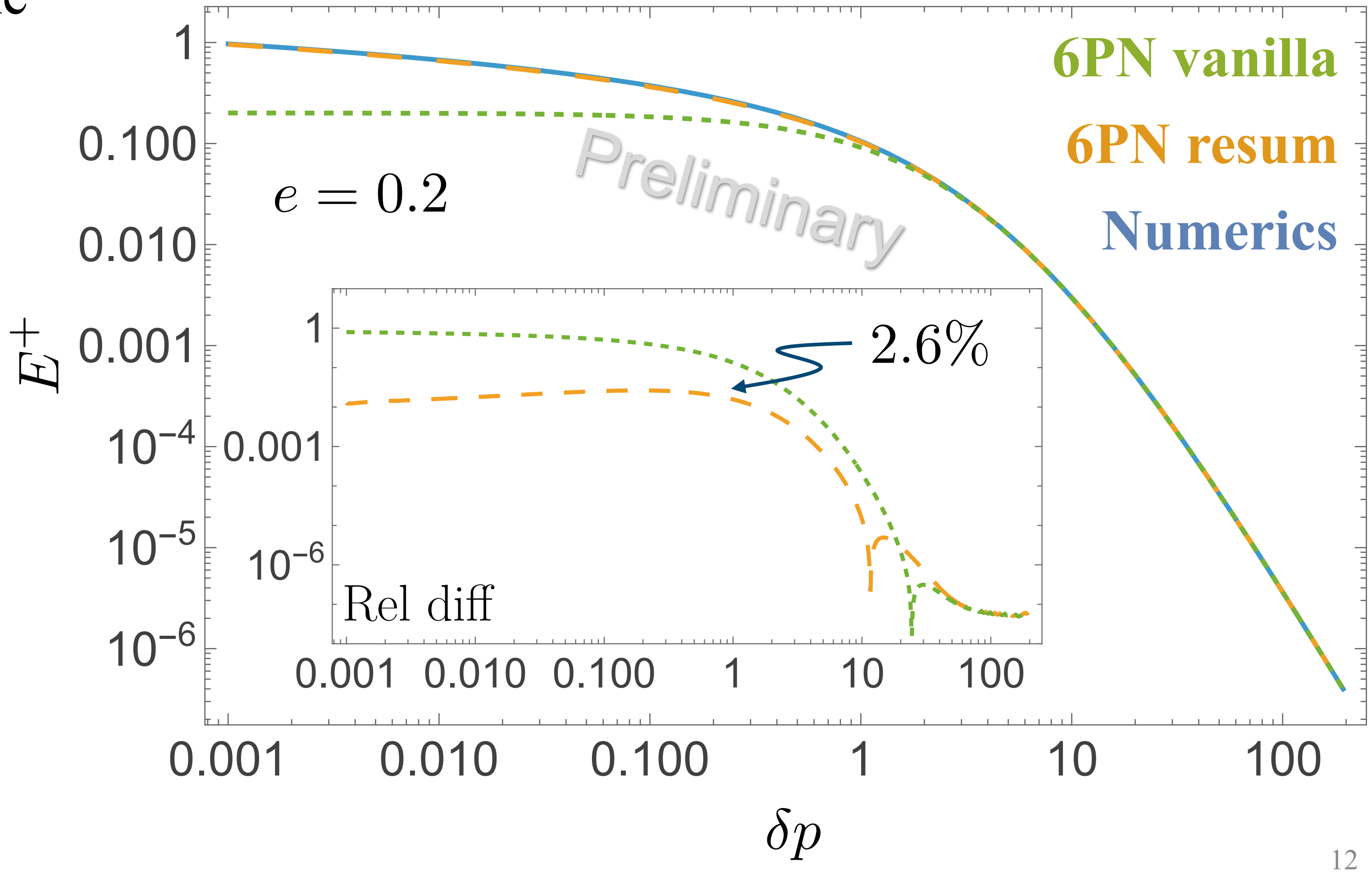
Eccentric bound geodesics show the same near-separatrix zoom-whirl behaviour during each orbit.

Can use the same resummation for the radiated energy per orbit:

$$\tilde{E}^{+} = E^{+}_{\rm sing} + E^{+}_{\rm PN} - {\rm CT}^{+}$$



$(e = 0.5,\ \delta p = 0.01)$



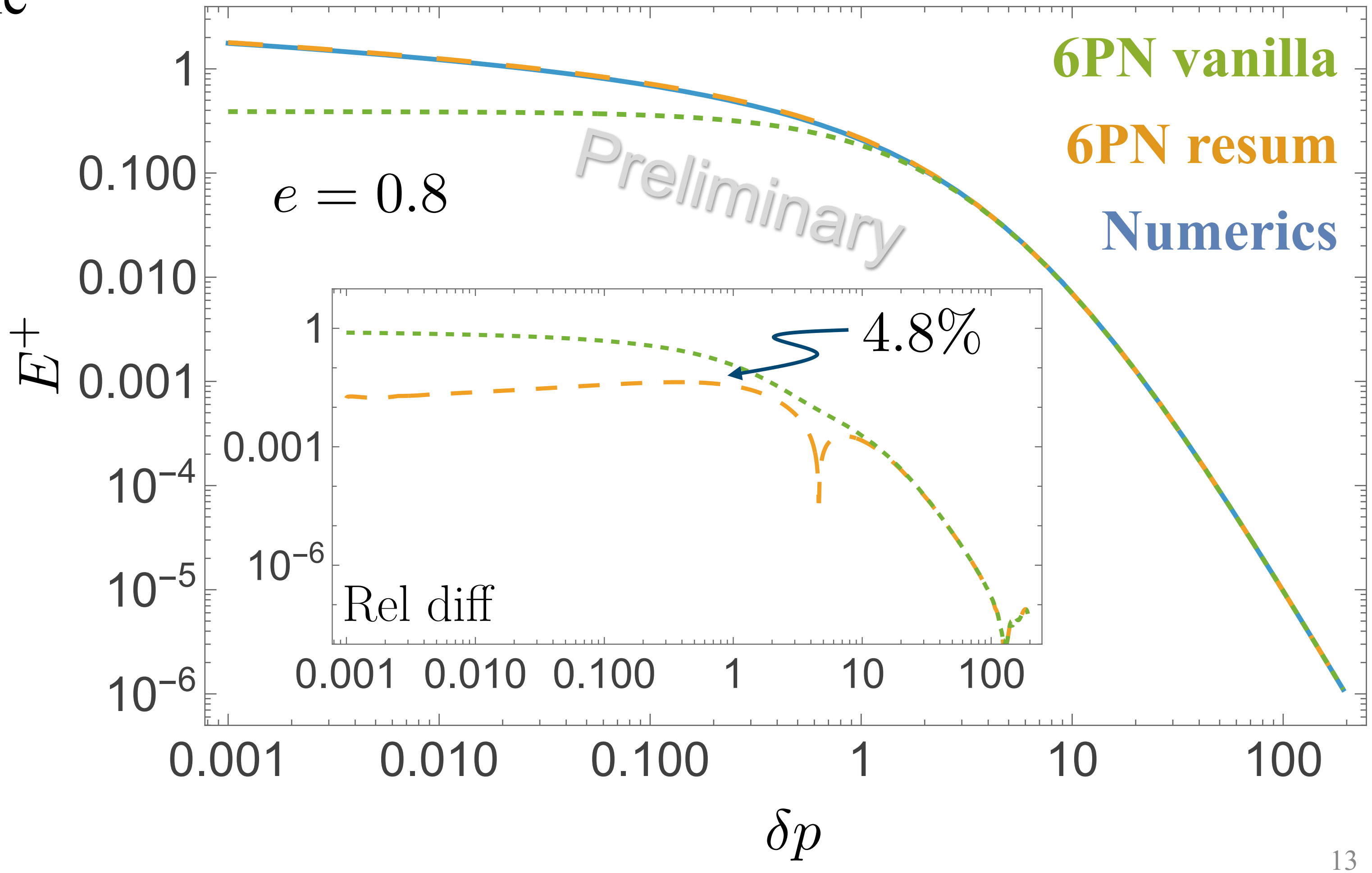
Resummation of bound orbit fluxes

[Barack, Gonzo, Leather, **OL**, Nasipak, Pound, Trestini, & Warburton]

Eccentric bound geodesics show the same near-separatrix zoom-whirl behaviour during each orbit.

Can use the same resummation for the radiated energy per orbit:

$$\tilde{E}^{+} = E^{+}_{\rm sing} + E^{+}_{\rm PN} - {\rm CT}^{+}$$



Resummation matches numerics to ~1% precision across whole parameter space

Resummation of bound orbit fluxes

[Barack, Gonzo, Leather, **OL**, Nasipak, Pound, Trestini, & Warburton]

Eccentric bound geodesics show the same near-separatrix zoom-whirl behaviour during each orbit.

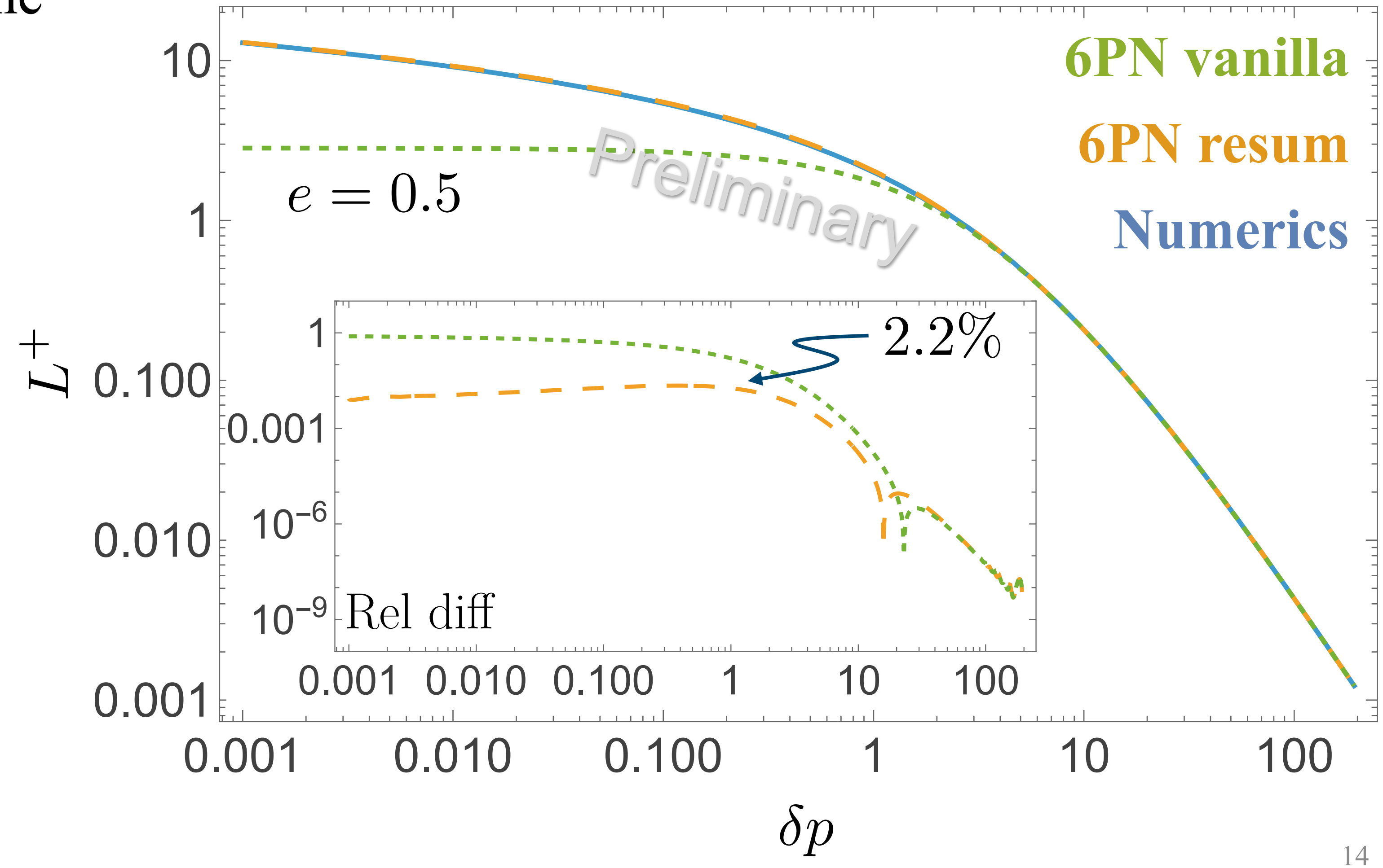
Can use the same resummation for the radiated energy per orbit:

$$\tilde{E}^+ = E^+_{\rm sing} + E^+_{\rm PN} - {\rm CT}^+$$

Or the radiated angular momentum per orbit:

$$\tilde{L}^+ = L^+_{\rm sing} + L^+_{\rm PN} - {\rm CT}^+$$

Resummation matches numerics to ~1% precision across whole parameter space

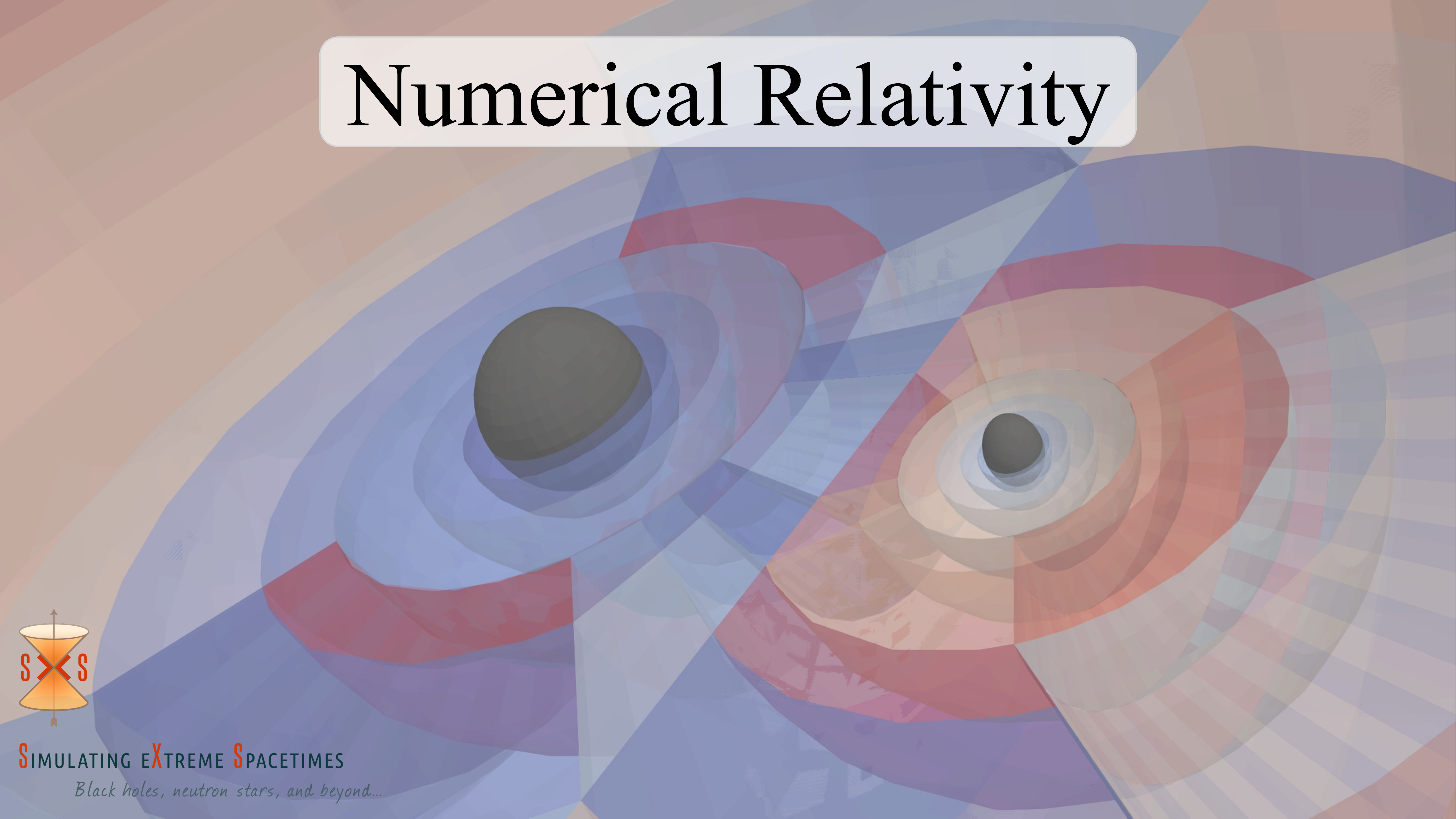


Future work in self-force

Resummation of bound fluxes:

- Include Boundary-to-Bound PM information in flux resummation.
- Extend flux resummation to primary spin and 2SF fluxes.
- Use flux resummations to build a highly eccentric waveform model (WaSABI-E).
- Combine flux models with astrophysics simulations for e.g. more accurate EMRI rates.

Numerical Relativity



SIMULATING EXTREME SPACETIMES

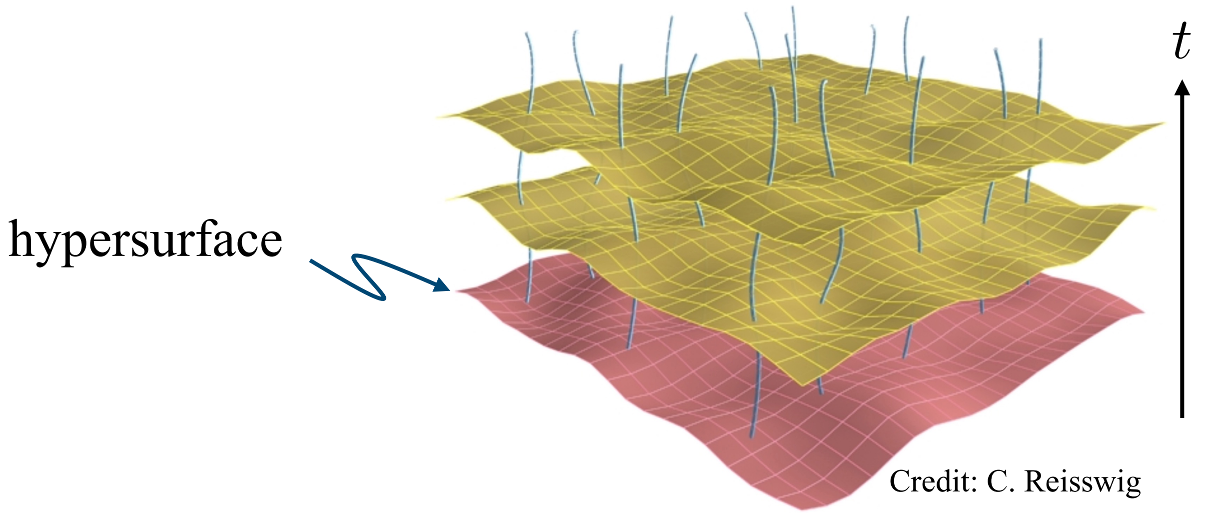
Black holes, neutron stars, and beyond...

The basics of Numerical Relativity

Numerical solving of Einstein's equations:

$$G^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

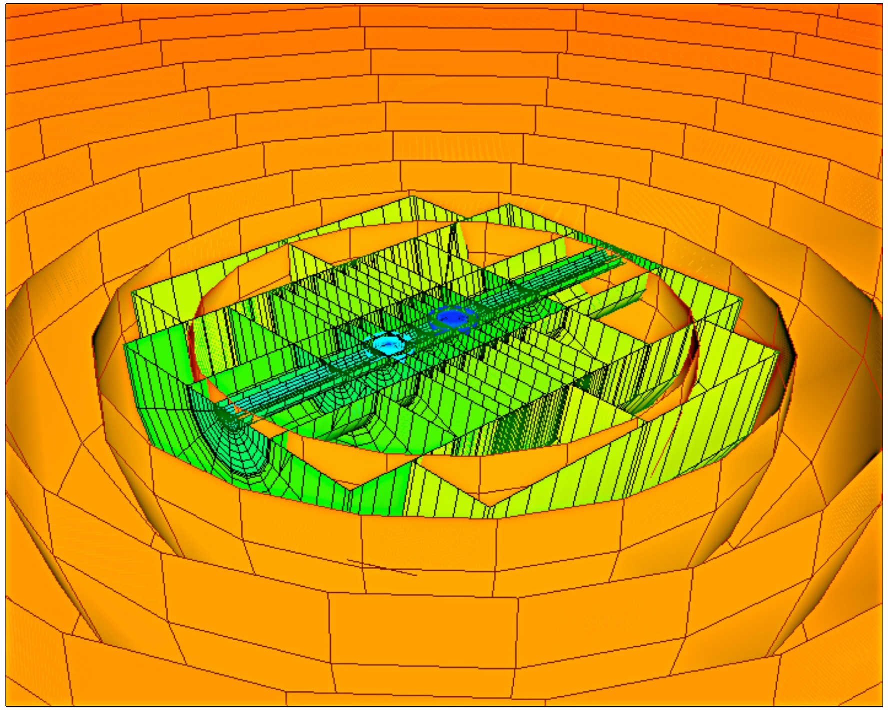
Spacetime split into spatial hypersurfaces and decomposed into numerical domains.



Credit: C. Reisswig

Two types of equations:

- 4 elliptic constraint equations.
- Evolution equations:
  - 40 equations for 10 metric components (generalised harmonic formulation).
- Plus 4 coordinate gauge degrees of freedom.


Credit: H. Pfeiffer

The outputs of Numerical Relativity

Spacetime:

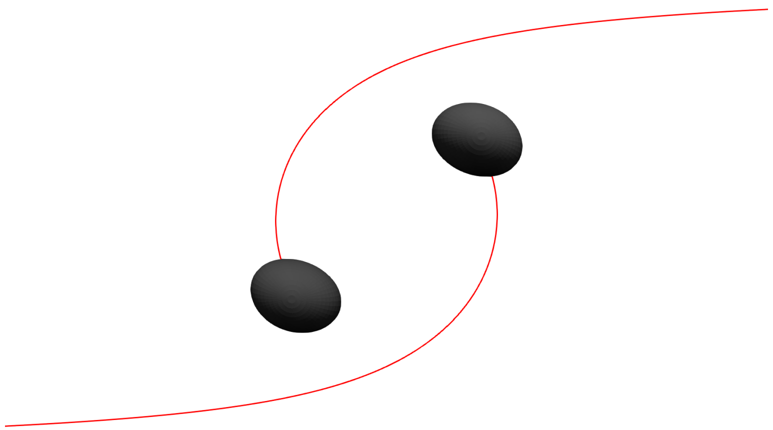
- Curvature scalars.
- Black hole horizons.
- Induced spin.

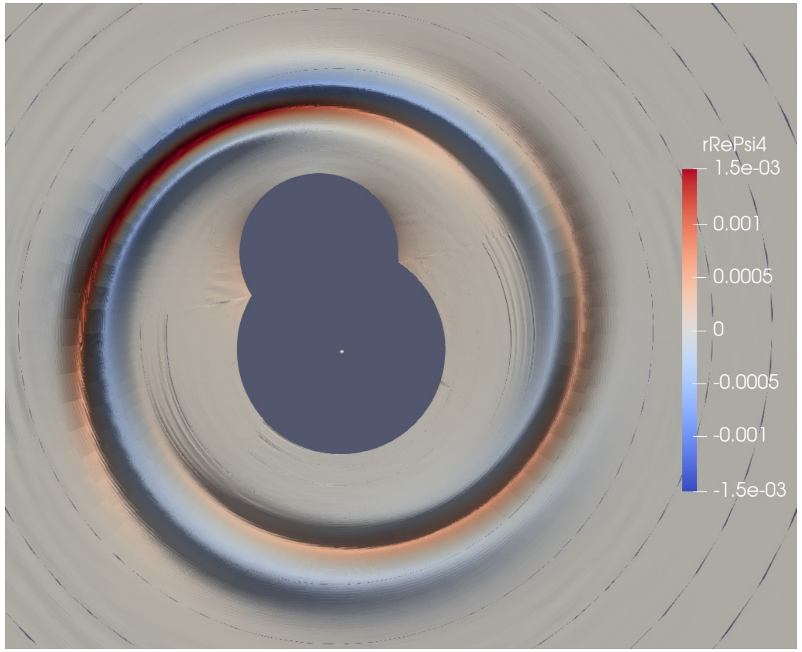
Dynamics:

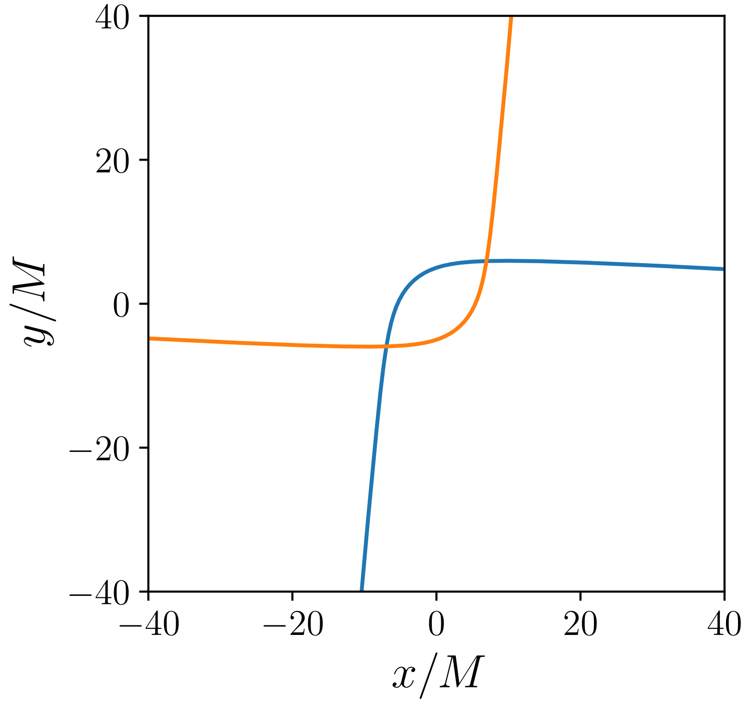
- Scattering angle.
- Centre of mass recoil velocity.

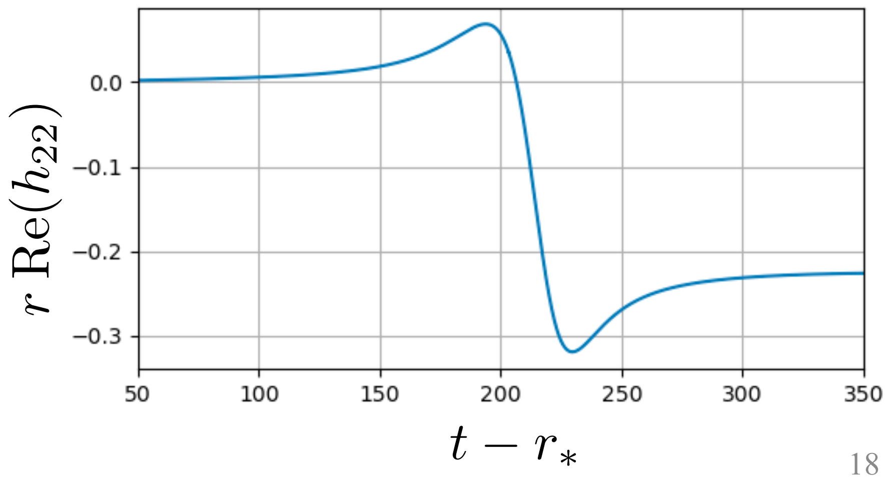
Asymptotics:

- Waveforms.
- Radiated energy and angular momentum.
- Linear/nonlinear memory.









PM/NR comparison: Equal mass [OL+ 25]

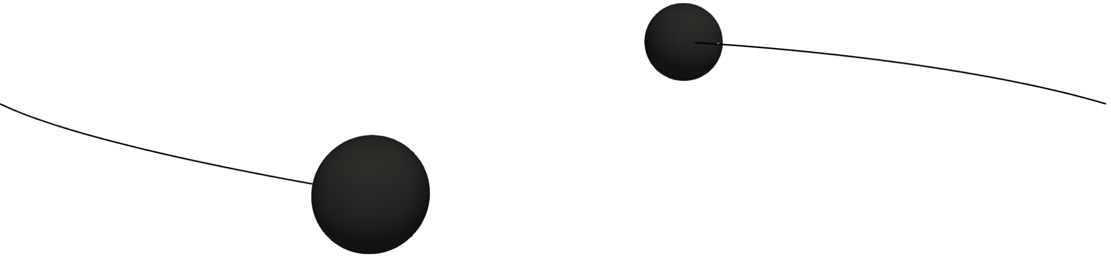
Can compare scattering angle values between NR (SpEC) and PM.

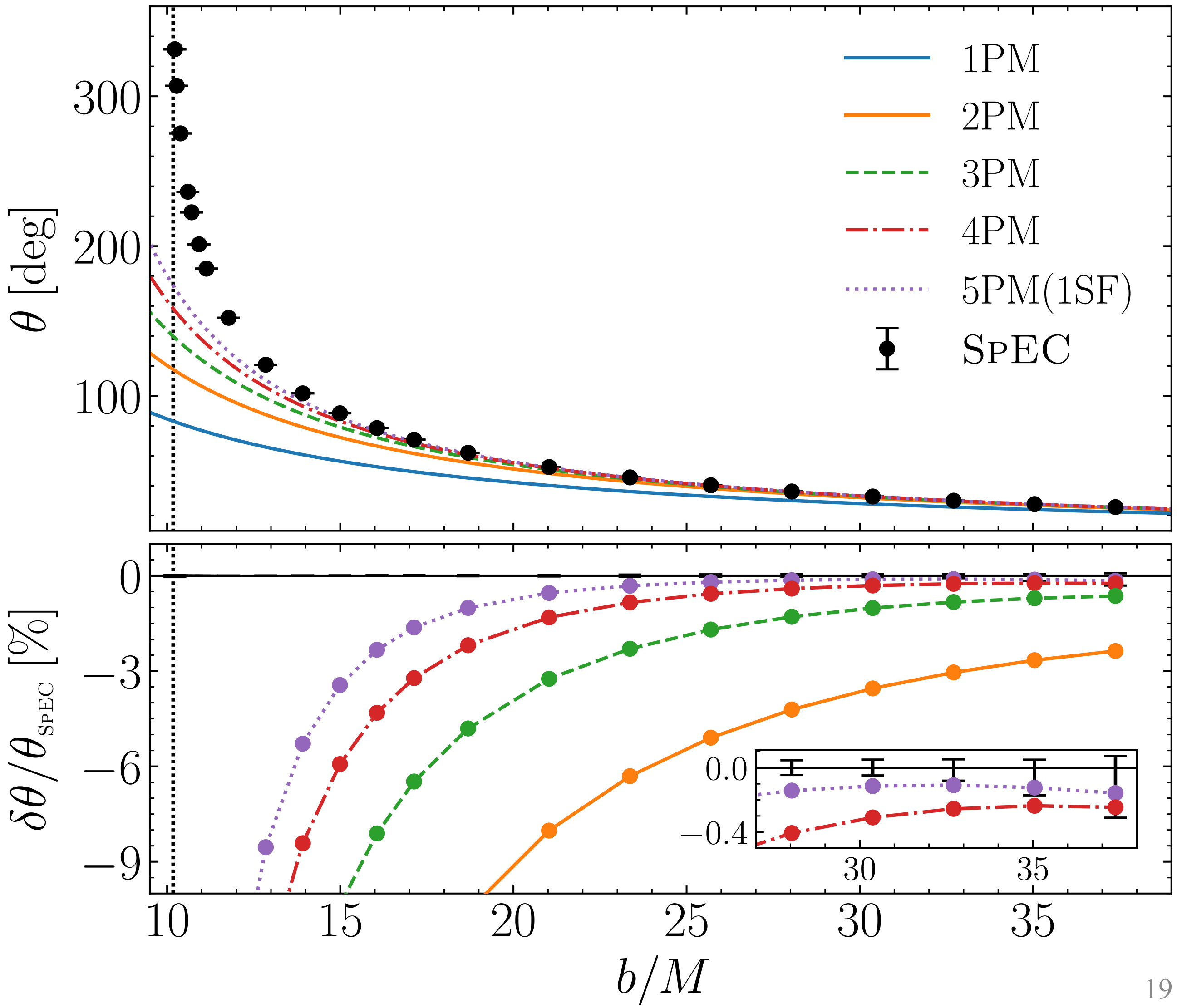
Initial conditions:

- Fixed energy.
- Equal mass.
- Non-spinning.

PM does not capture the divergence at separatrix

5PM(1SF) within NR errors for $b \gtrsim 35M$





Order-by-order PM/NR comparison [OL+ 25]

Can check the convergence order by order.

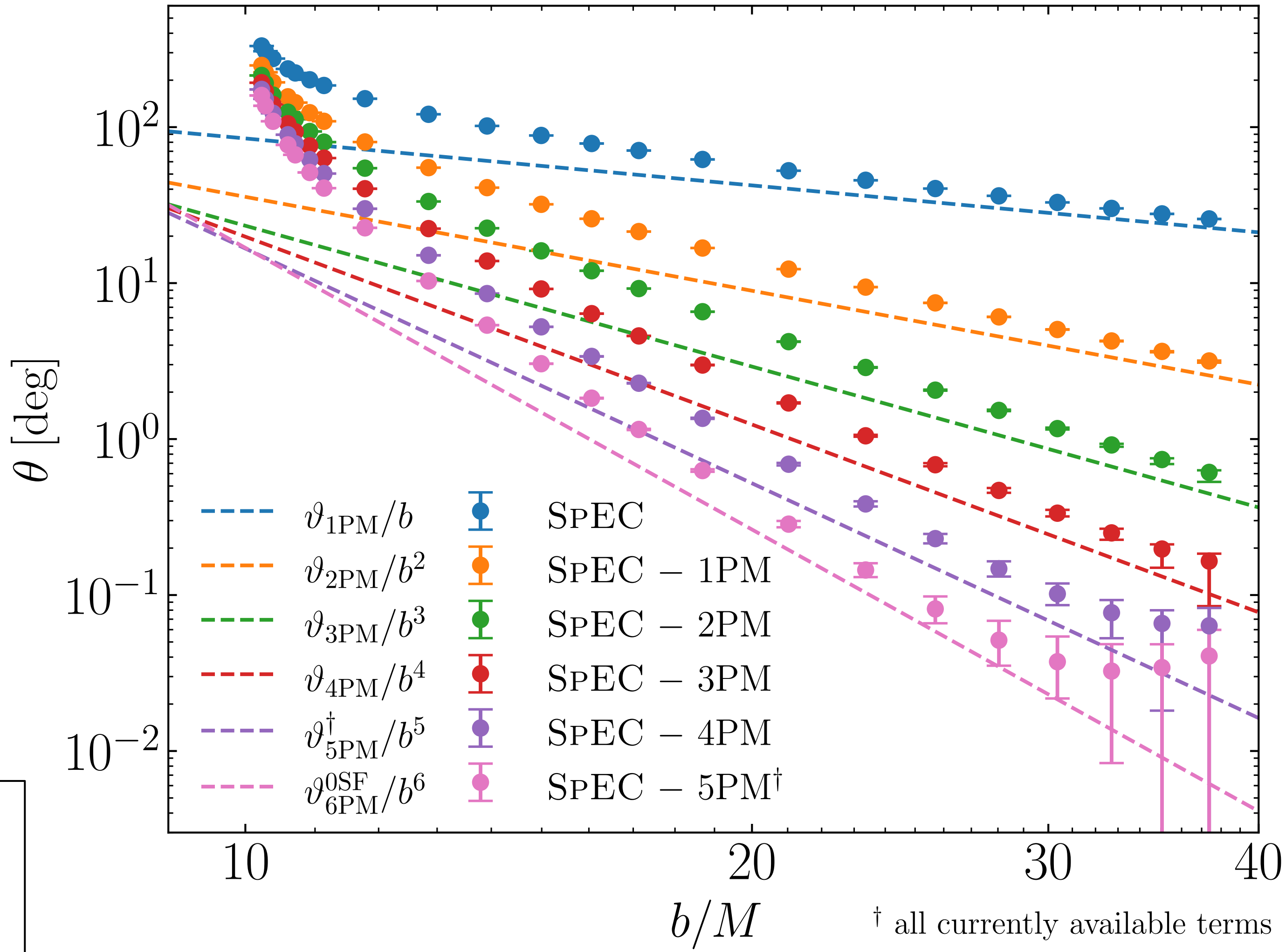
Start by comparing LO with NR

Subtract LO from NR and compare to NLO

Rinse and repeat

NR tends towards next-order PM at all orders

Not able to fit PM coefficients as within errors



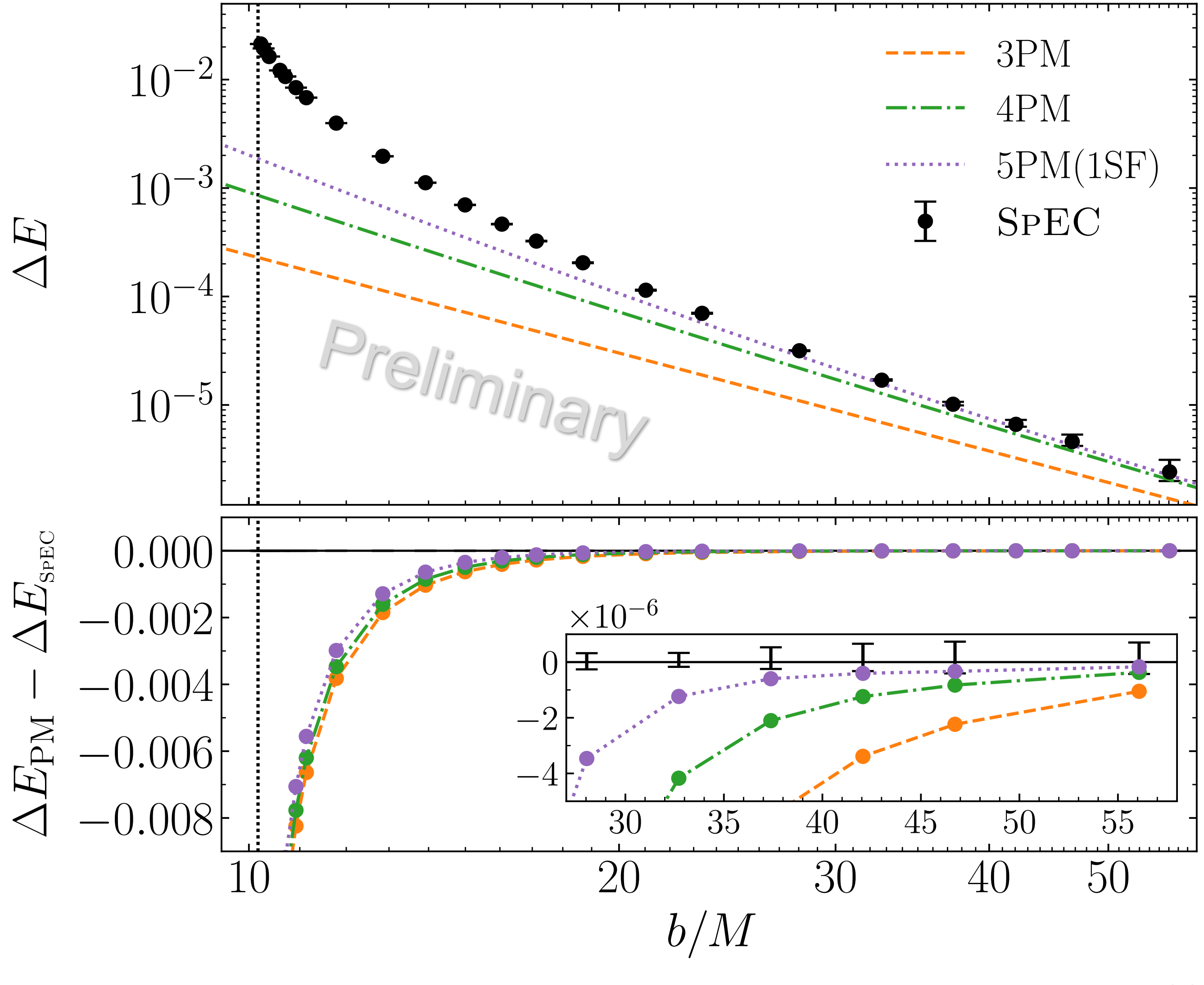
[Driesse+ 26]

PM/NR comparison: Radiated energy

Can also compare radiated energy.

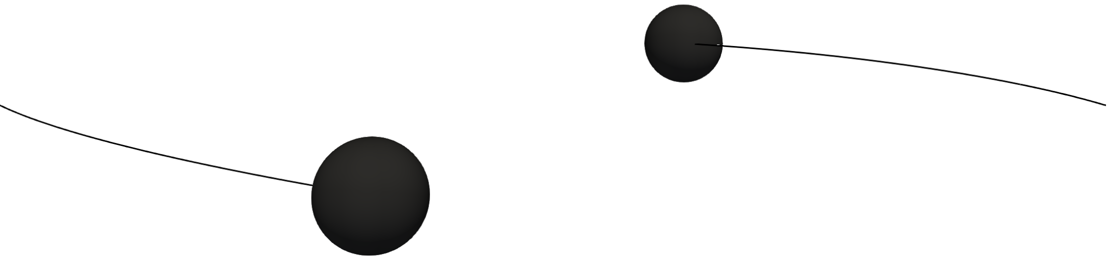
Initial conditions:

- Fixed energy.
- Equal mass.
- Non-spinning.



PM does not capture the divergence at separatrix

5PM(1SF) within NR errors for $b \gtrsim 45M$

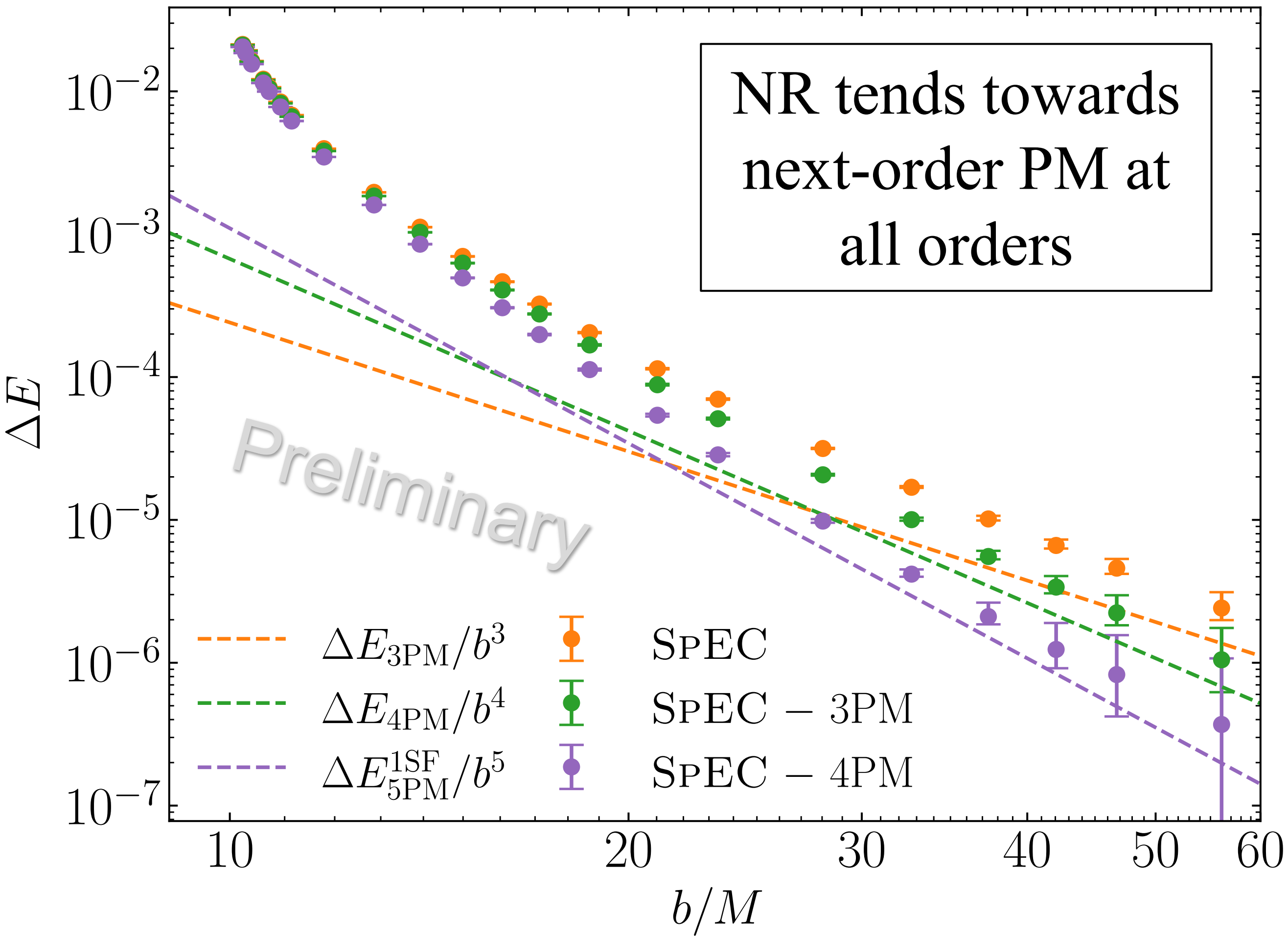


PM/NR comparison: Radiated energy

Can also compare radiated energy.

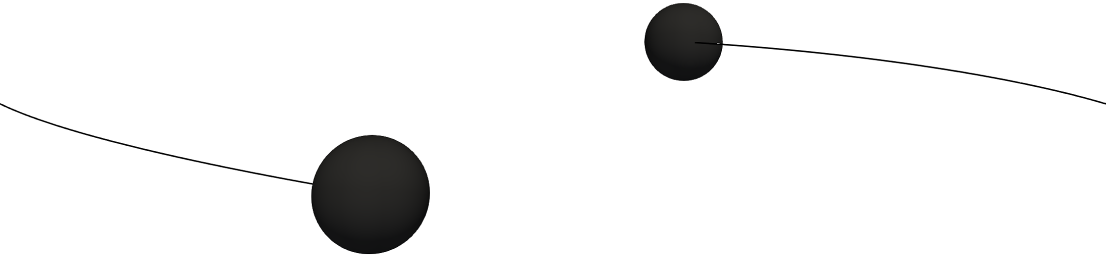
Initial conditions:

- Fixed energy.
- Equal mass.
- Non-spinning.



PM does not capture the divergence at separatrix

5PM(1SF) within NR errors for $b \gtrsim 45M$



Evolved effective one body models

Effective one body (EOB) relies on an expanded Schwarzschild (or Kerr) Hamiltonian:

$$H = \sqrt{p_{r_*}^2 + A(r)\left[\frac{p_\varphi^2}{r^2} + \left(\frac{m_1 m_2}{m_1 + m_2}\right)^2 + Q(r)\right]}$$

A and Q potentials usually expanded via post-Newtonian and calibrated to NR.

System solved numerically using Hamilton's equations with radiation reaction forces:

Dissipative terms

$$\dot{r} = \frac{\partial H_{\mathrm{EOB}}}{\partial p_r} \qquad \dot{\varphi} = \frac{\partial H_{\mathrm{EOB}}}{\partial p_\varphi} \qquad \dot{p}_r = -\frac{\partial H_{\mathrm{EOB}}}{\partial r} + \mathcal{F}_r \qquad \dot{p}_\varphi = \frac{\partial H_{\mathrm{EOB}}}{\partial \varphi} + \mathcal{F}_\varphi$$

Two evolved EOB models for scattering: `SEOBNRv5` and `TEOBResumS-Dalí`.

Quasicircular fluxes (SEOBNRv5) — Eccentric fluxes (TEOBResumS-Dalí)

Closed-form effective one body models

Alternatively can use expanded or resummed post-Minkowskian (PM) potential from mass-shell constraint:

Angular momentum

$$p_r^2 = p_\infty^2(E) - \frac{J^2}{r^2} + w(E, J, r)$$

Fix coefficients of PM expanded w by matching to PM expanded scattering angle:

$$\theta = -\pi - 2\int_{r_{\min}}^{\infty} dr \frac{\partial}{\partial J} p_r$$

The potential w contains both conservative and dissipative information from the scattering angle.

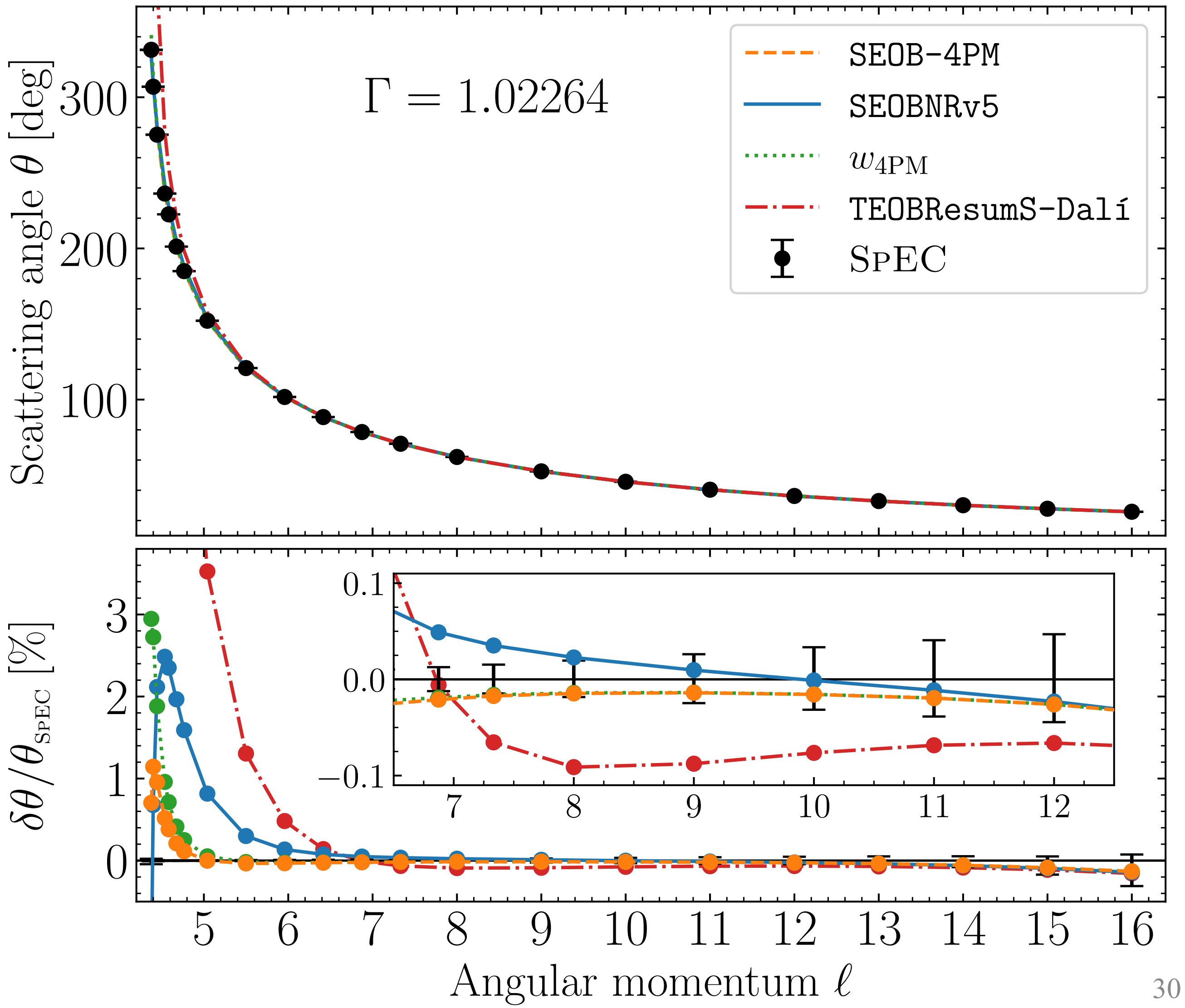
Two models:

w_{EOB}	SEOB-PM
Uses PM-expanded w	Uses PM-resummed w from SEOB-PM Hamiltonian, that reduces to the test-body limit.
[Damour & Rettegno 23, Rettegno+ 24]	[Buonanno, Jakobsen & Mogull 24, **OL**+ 25]

EOB/NR comparison: Equal mass [OL+ 25]

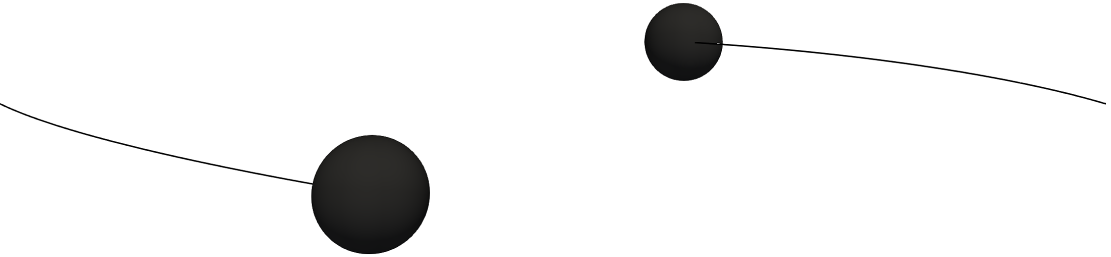
Initial conditions:

- Fixed energy.
- Equal mass.
- Non-spinning.



Very good agreement between SpEC and EOBs in weak field.

Good agreement persists into strong field ($\lesssim 3\%$).

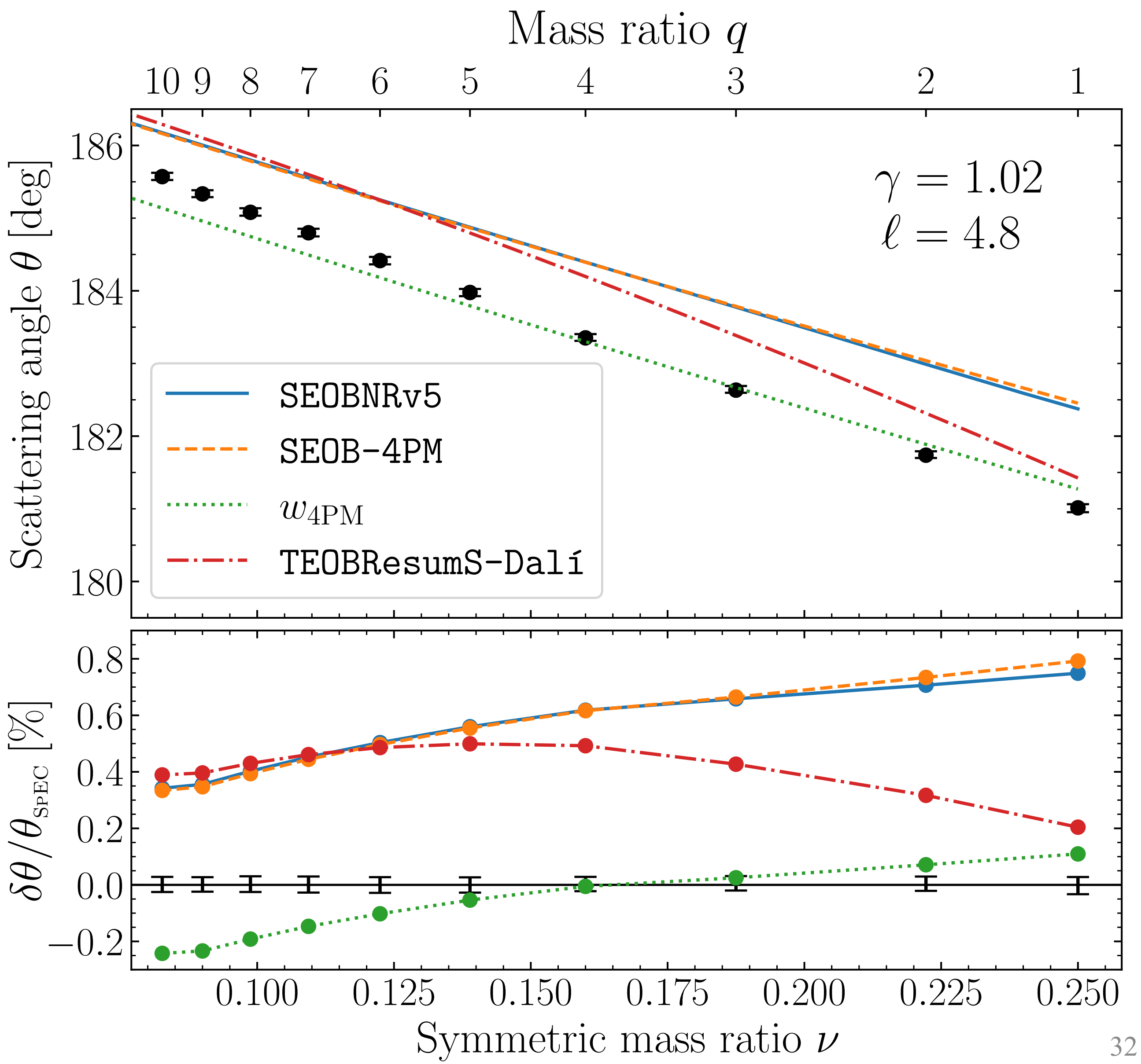


Comparison to EOB: Unequal mass

Initial conditions:

- Fixed relative Lorentz factor.
- Fixed rescaled angular momentum.
- Non-spinning.

Very good agreement between SpEC and EOBs (<1%).



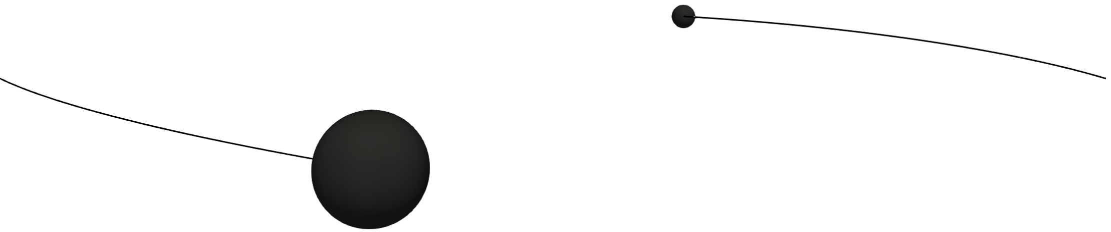
Extracting self-force from NR: Scattering angle [OL+ 25]

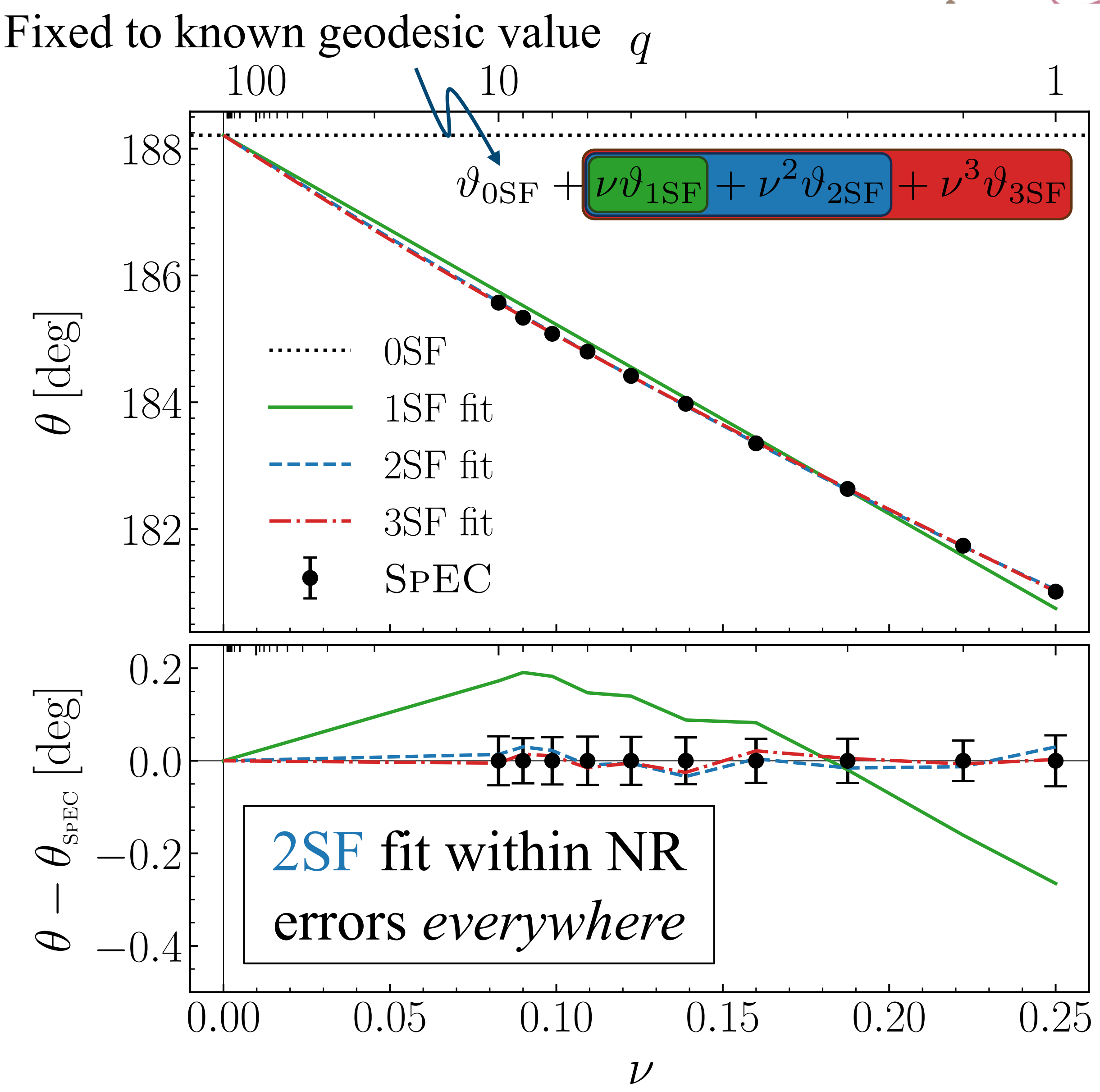
Can use NR data to fit self-force coefficients of the scattering angle:

$$\theta = \vartheta_{0SF} + \nu\vartheta_{1SF} + \nu^2\vartheta_{2SF} + \nu^3\vartheta_{3SF} + \dots$$

ϑ_{0SF}	ϑ_{1SF}	ϑ_{2SF}	ϑ_{3SF}
187.77(3)	−27.2(2)	–	–
188.06(4)	−31.2(7)	12(2)	–
188.1(2)	−32(4)	19(24)	−12(50)
188.211	−29.8(3)	–	–
188.211	−33.3(2)	18.5(9)	–
188.211	−34.4(4)	32(5)	−40(14)

Bold is fixed to known 0SF value





Extracting self-force from NR: Scattering angle [OL+ 25]

Can use NR data to fit self-force coefficients of the scattering angle:

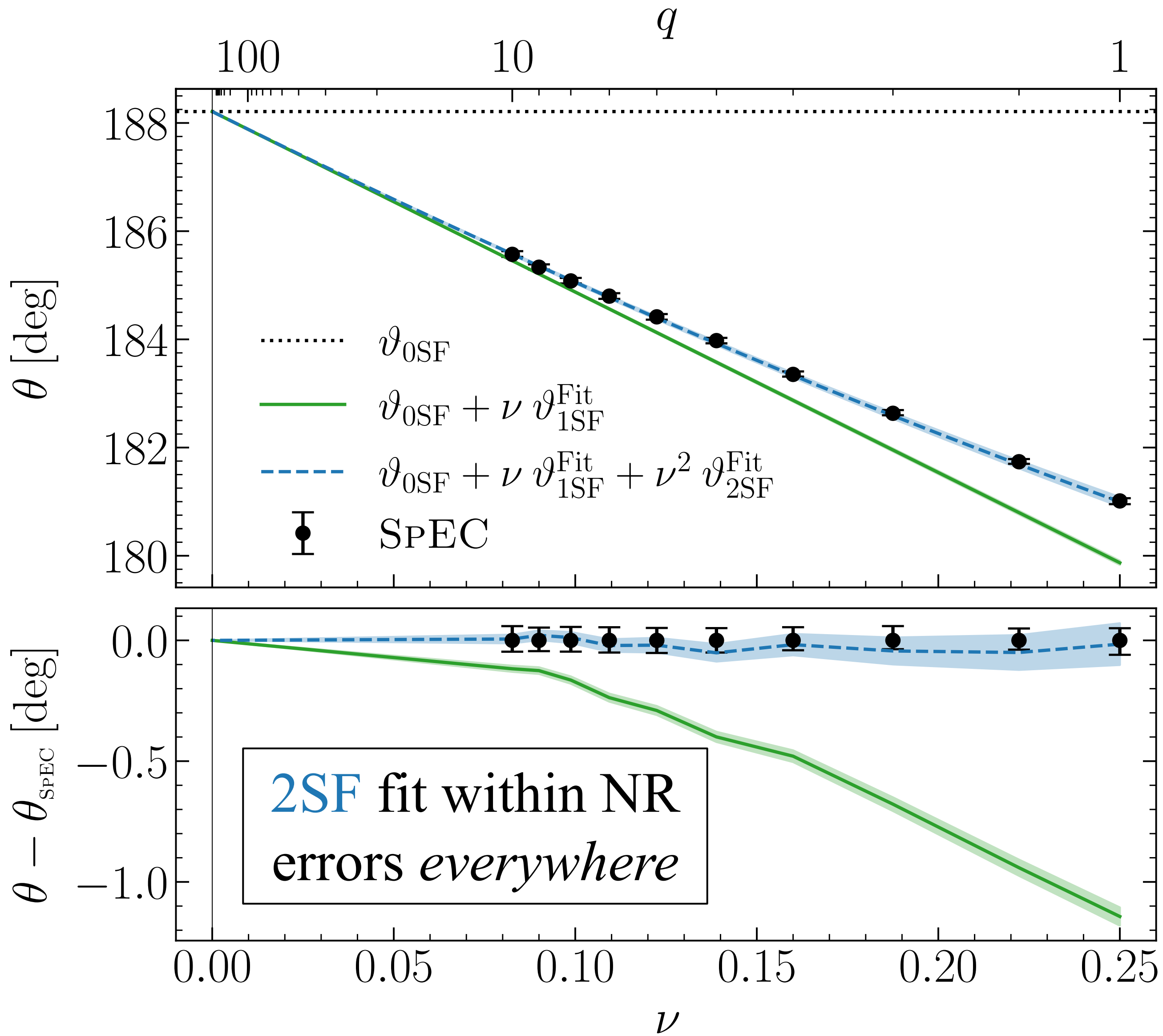
$$\theta = \vartheta_{0SF} + \nu\vartheta_{1SF} + \nu^2\vartheta_{2SF} + \nu^3\vartheta_{3SF} + \dots$$

ϑ_{0SF}	ϑ_{1SF}	ϑ_{2SF}	ϑ_{3SF}
187.77(3)	−27.2(2)	–	–
188.06(4)	−31.2(7)	12(2)	–
188.1(2)	−32(4)	19(24)	−12(50)
188.211	−29.8(3)	–	–
188.211	−33.3(2)	18.5(9)	–
188.211	−34.4(4)	32(5)	−40(14)

Bold is fixed to known 0SF value

Weighted average values

$$\vartheta_{1SF}^{Fit} = -33.36 \pm 0.15\ (0.45\%)$$
$$\vartheta_{2SF}^{Fit} = 18.0 \pm 0.8\ (4.4\%)$$



Extracting self-force from NR: Radiated energy

Can also extract radiated energy SF coefficients.

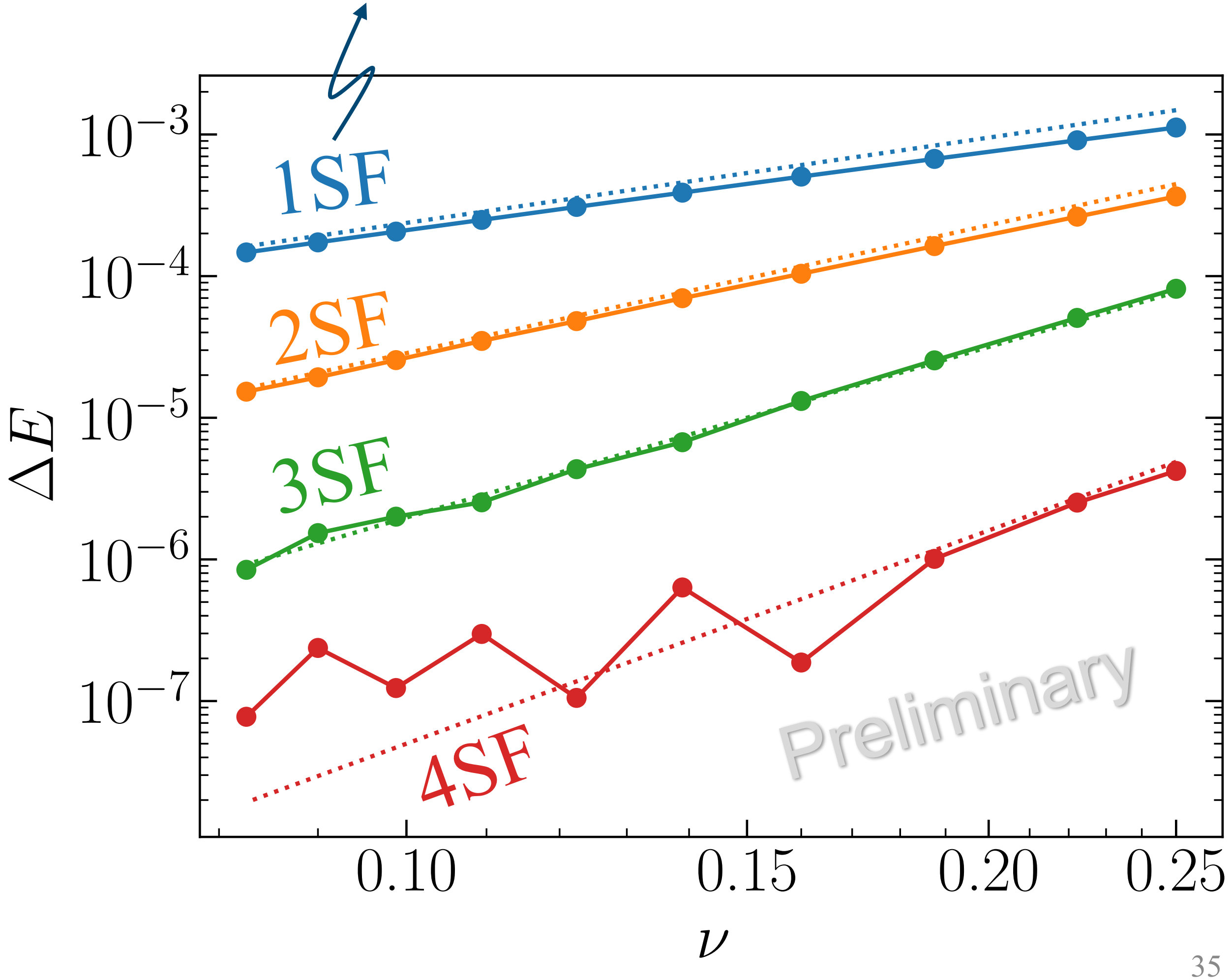
NR value : 0.02355

SF value : 0.02375 (Warburton)

1SF NR fit agrees with direct SF calculation to <1%

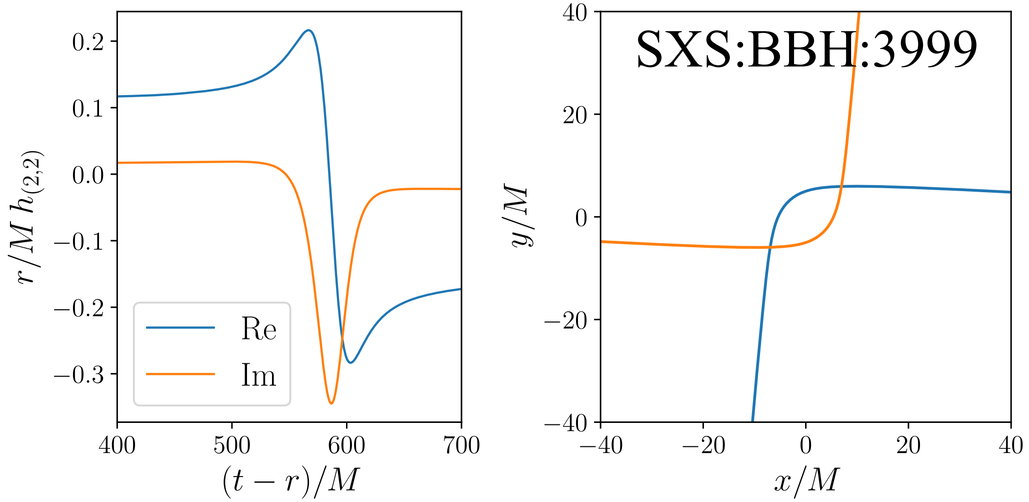
Can extract 2SF and 3SF coefficients

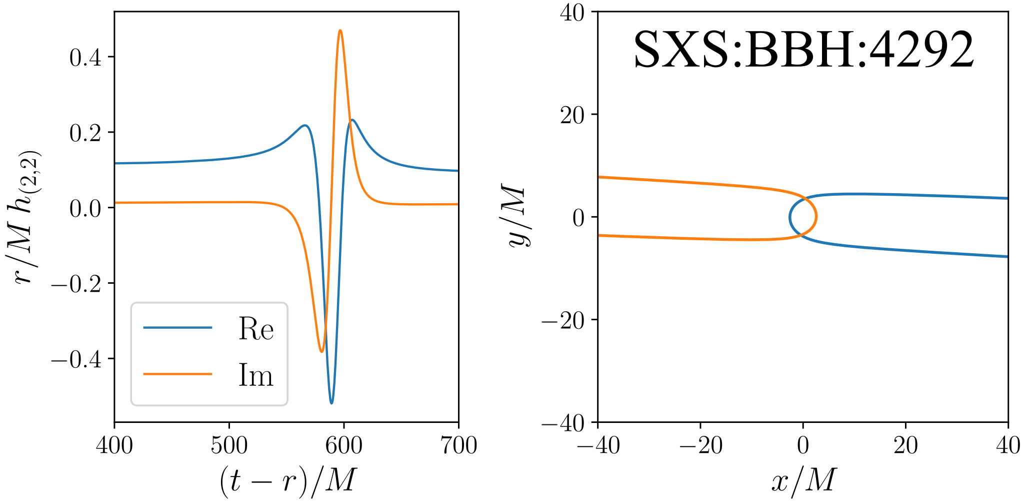
Potential to put bounds on 4SF coefficient

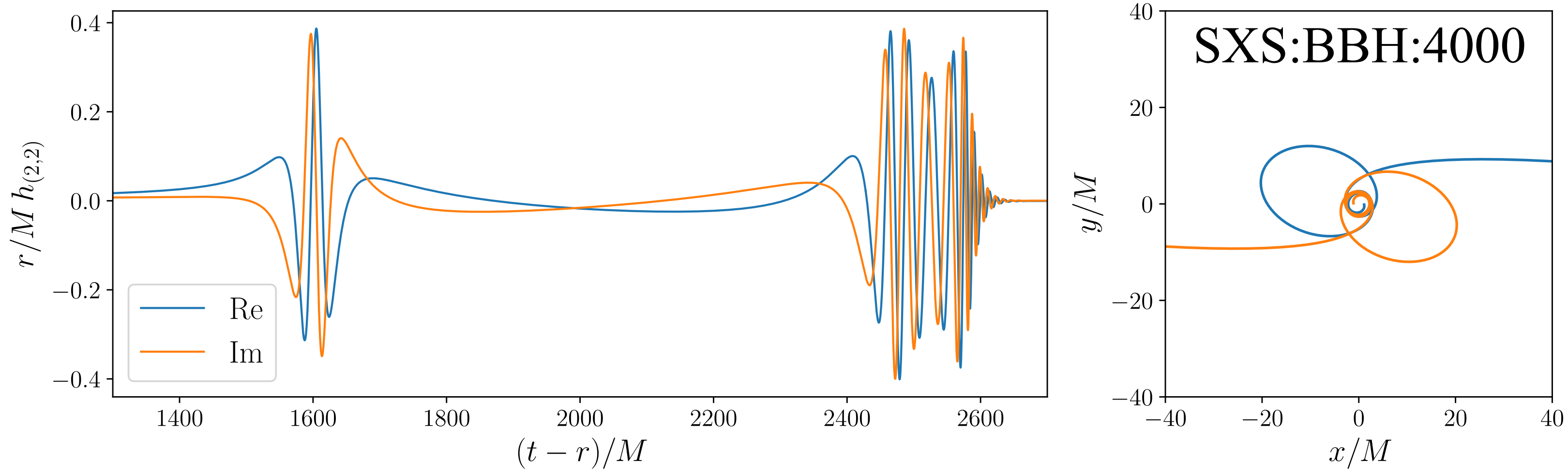


Simulating eXtreme Spacetimes catalog

Unbound simulations now available in the SXS catalog: https://data.black-holes.org



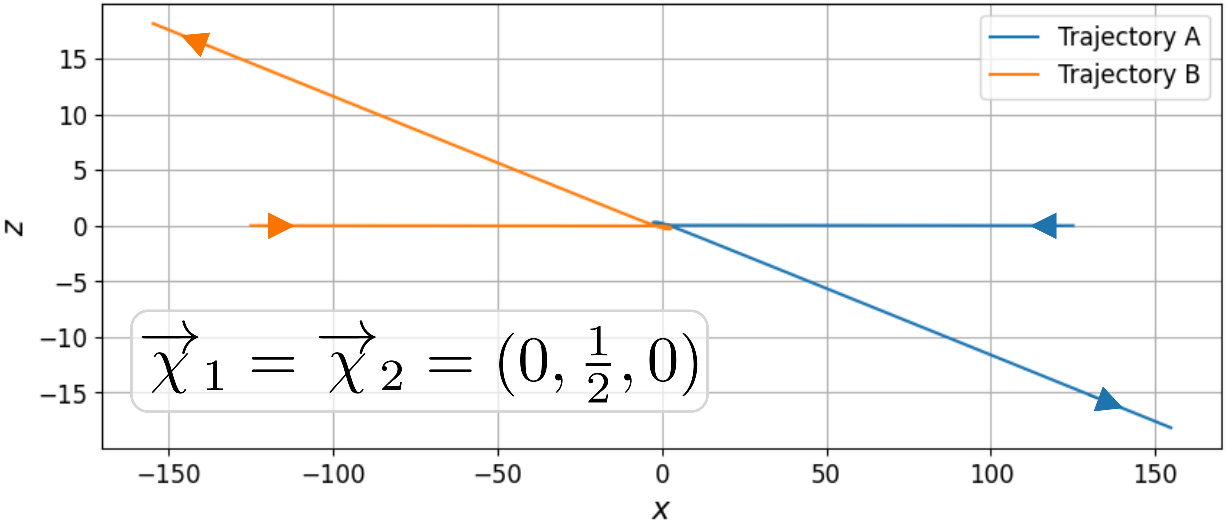




Future work in Numerical Relativity

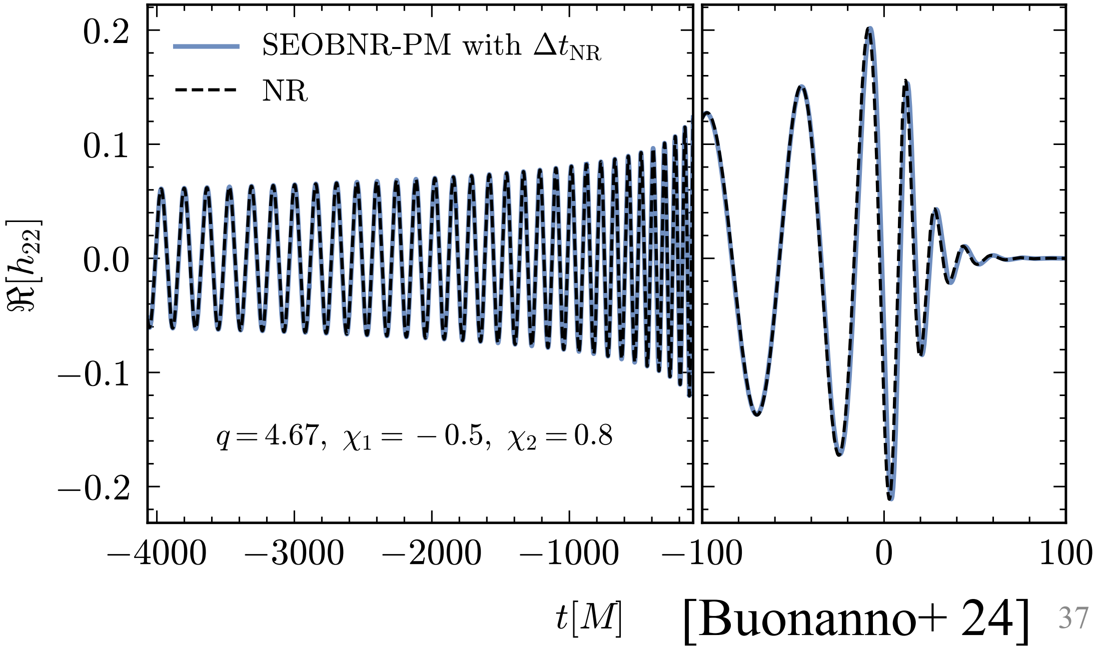
Expand the NR parameter space:

- Higher energies
- More unequal masses.
- More extreme spins.
- Generic spins.



Interfacing with other models:

- Comparison to the GSF results.
- Waveform comparisons to PM/EOB.
- Calibration of EOB with NR scattering data.



[Buonanno+ 24]

Take aways

Circular SF data can be used to accurately resum scattering and bound radiated quantities

Used precision NR simulations to benchmark PM calculations and extract SF coefficients

PM-resummed EOB match NR results across the available parameter space.

Next steps in self-force:
More resummations and gravitational self-force

Next steps in Numerical Relativity:
Extend parameter space and inform waveform models