

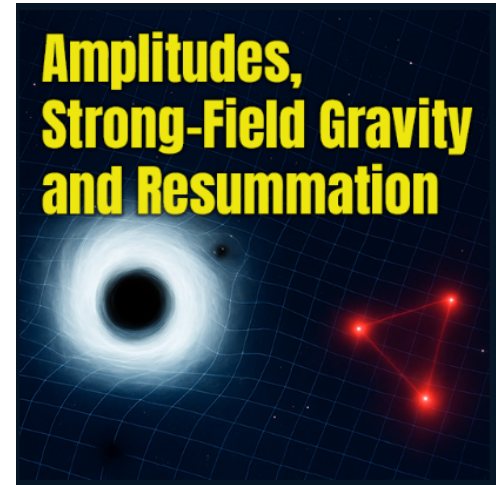
NON LINEAR MEMORY FROM Reverse Unitarity

A. GEORGOUDIS, 16/04/26

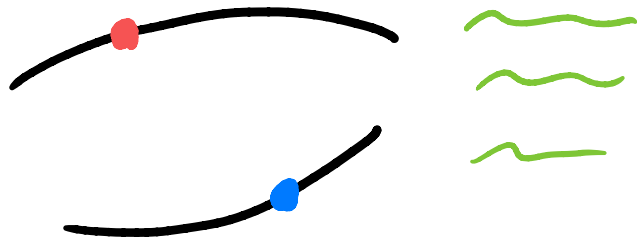
Amplitudes, Strong-Field Gravity and Resummation

Based on [Arxiv: 2506.20733](#)

with: [V. Gonçalves](#), [C. Heissenberg](#), [J. Parric-Martinez](#)

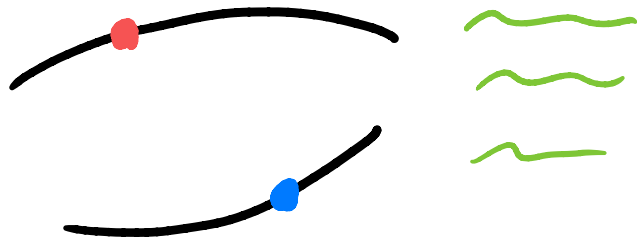


PM Wave Forms



→ Gravitational Radiation
measured at Future null infinity

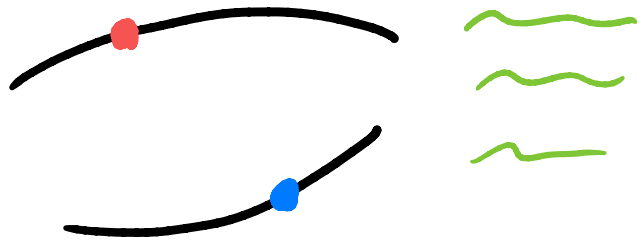
PM Wave Forms



Gravitational Radiation
measured at Future null infinity

- $\Delta_{\oplus} = \langle \text{out} | \oplus | \text{out} \rangle - \langle \text{in} | \oplus | \text{in} \rangle$
 $= \langle \text{in} | S^{\oplus} S - \oplus | \text{in} \rangle$

PM Wave Forms



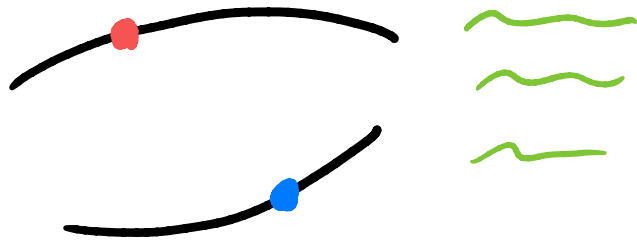
Gravitational Radiation
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$$| \text{out} \rangle = S | \text{in} \rangle \quad S = 1 + iT = e^{i\delta}$$

PM Wave Forms

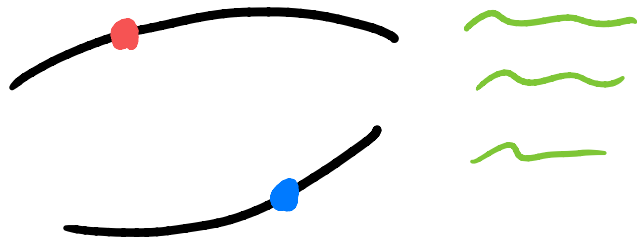


Gravitational Radiation
measured at Future null infinity

$$\bullet \Delta^{\oplus} = \langle \text{out} | \oplus | \text{out} \rangle - \langle \text{in} | \oplus | \text{in} \rangle \\ = \langle \text{in} | S^{\oplus} S - \oplus | \text{in} \rangle$$

$$| \text{out} \rangle = S | \text{in} \rangle \quad S = 1 + iT = e^{iN}$$

PM Wave Forms



Gravitational Radiation
measured at Future null infinity

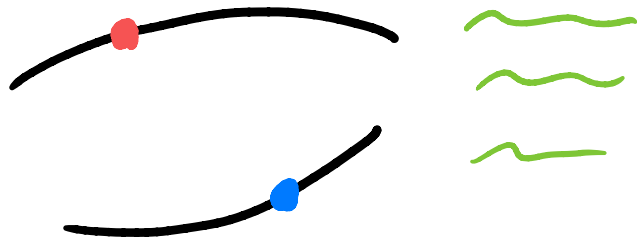
$$- \Delta^{\oplus} = \langle \text{out} | \oplus | \text{out} \rangle - \langle \text{in} | \oplus | \text{in} \rangle$$

$$= \langle \text{in} | S^{\oplus} \oplus S - \oplus | \text{in} \rangle$$

$$| \text{out} \rangle = S | \text{in} \rangle \quad S = 1 + iT = e^{iN}$$

$$\bullet h_{\mu\nu} \approx \langle \text{out} | \hat{H}_{\mu\nu} | \text{out} \rangle \sim \int \frac{d\omega}{2\pi} e^{-i\omega u} \tilde{W}_{\mu\nu}(\omega) + \text{c.c.}$$

PM Wave Forms



Gravitational Radiation
measured at Future null infinity

$$\begin{aligned}
 -\Delta \oplus &= \langle \text{out} | \oplus | \text{out} \rangle - \langle \text{in} | \oplus | \text{in} \rangle \\
 &= \langle \text{in} | S^\dagger \oplus S - \oplus | \text{in} \rangle
 \end{aligned}$$

$$|\text{out}\rangle = S |\text{in}\rangle \quad S = 1 + iT = e^{iN}$$

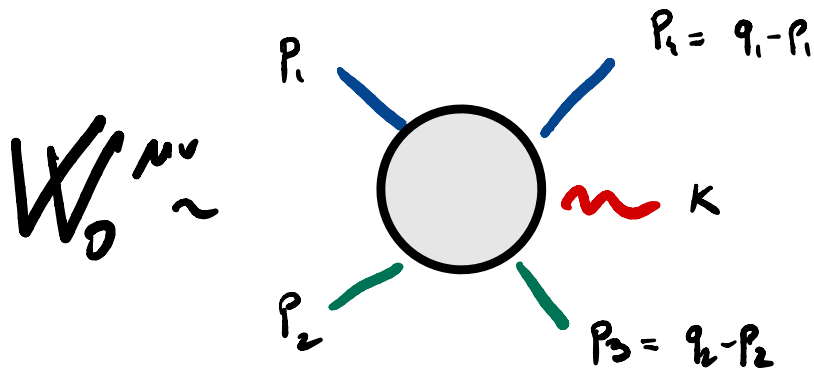
$$h_{\mu\nu} = \langle \text{out} | \hat{H}_{\mu\nu} | \text{out} \rangle \sim \int \frac{d\omega}{2\pi} e^{-i\omega u} \tilde{W}_{\mu\nu}(\omega) + \text{c.c.}$$

$$\uparrow \\
 \text{FT}[W_{\mu\nu}]$$

$$W_{\mu\nu} = \text{[Diagram of a shaded oval with four lines]} - i \sum_x \text{[Diagram of two shaded ovals with a vertical line between them]}$$

Expand in $\frac{GM}{rc^2} \ll 1$ Post Minkowskian regime

Wave Form @ LO



Useful Quantities

$$\cdot \sigma = -v_1 \cdot v_2$$

$$\cdot \omega_1 = -v_1 \cdot k$$

$$\cdot \omega_2 = -v_2 \cdot k$$

Several results available in Time/Frequency domain,
FT analytically known.

$$L \rightarrow \int d^3q_1 d^3q_2 (2\pi)^0 \delta(q_1 + q_2 + k) 2\pi \delta(2p_1 - q_1) 2\pi \delta(2p_2 - q_2) e^{i b_1 \cdot q_1 + i b_2 \cdot q_2} f(q_1, q_2, k)$$

Wave Form @ NLO

$$W_i^{\mu\nu} \sim \overset{N}{\text{Diagram}} + \frac{i}{2} \left(\text{Diagram 1} - \text{Diagram 2} \right) + \frac{i}{2} \left(\text{Diagram 3} + \text{Diagram 4} \right)$$

The diagram shows the wave function $W_i^{\mu\nu}$ at Next-to-Leading Order (NLO). It is expressed as a sum of terms:

- A tree-level term (NLO) represented by a grey circle with a central 'O', four external lines (two blue, two green), and a red wavy line.
- A one-loop correction term with a coefficient $\frac{i}{2}$, enclosed in large parentheses. It contains two diagrams:
 - Diagram 1: Two grey circles connected by a blue loop. The left circle has two blue external lines, and the right circle has two green external lines. A red wavy line connects the two circles.
 - Diagram 2: Two grey circles connected by a green loop. The left circle has two blue external lines, and the right circle has two green external lines. A red wavy line connects the two circles.
- A second one-loop correction term with a coefficient $\frac{i}{2}$, enclosed in large parentheses. It contains two diagrams:
 - Diagram 3: A grey circle with two blue external lines and one green external line. A red wavy line connects it to a smaller grey circle with two blue external lines.
 - Diagram 4: A grey circle with two green external lines and one blue external line. A red wavy line connects it to a smaller grey circle with two green external lines.

Wave Form @ NLO

N

$$W_i^{\mu\nu} \sim \text{tree} + \frac{i}{2} \left(\text{loop}_1 - \text{loop}_2 \right) + \frac{i}{2} \left(\text{loop}_3 + \text{loop}_4 \right)$$

$$B_i^{\mu\nu} = B_{iE}^{\mu\nu} + \underbrace{B_{iO}^{\mu\nu}}_{f(\omega, \omega_1, \omega_2) A_0^{\mu\nu}} \quad (\text{even/odd } \omega_{1,2} \rightarrow -\omega_{1,2})$$

Wave Form @ NLO

$$W_1^{\mu\nu} \sim \overset{N}{\text{tree}} + \frac{i}{2} \left(\text{loop}_1 - \text{loop}_2 \right) + \frac{i}{2} \left(\text{loop}_3 + \text{loop}_4 \right)$$

IR divergence: $\zeta_1^{\mu\nu} \approx \left(-\frac{1}{2\epsilon} + \log \frac{\omega_1}{\mu} \right) A_0^{\mu\nu} + \zeta_1^{(reg)}$

$$W_1^{\mu\nu} = e^{-\frac{i}{\epsilon} GEW} \left(A_0^{\mu\nu} + B_1^{\mu\nu} + \frac{i}{2} C \right)$$

↳ divergence can be absorbed by shift of retarded Time

Wave Form @ NNLO

Soft Wave Form @NNLO

- What is the simplest interesting effect?

$$\left(\lim_{T \rightarrow \infty} w_{\mu\nu}(T, n) \right) - \left(\lim_{T \rightarrow -\infty} w_{\mu\nu}(T, n) \right) = \boxed{-F_{\mu\nu}}$$

→ Permanent displacement
caused by gravitational
radiation.

Wave Form @ NNLO $\omega \rightarrow 0$

- What is the simplest interesting effect? \rightarrow Permanent displacement

$$\left(\lim_{T \rightarrow \infty} w_{\mu\nu}(T, n) \right) - \left(\lim_{T \rightarrow -\infty} w_{\mu\nu}(T, n) \right) = -F_{\mu\nu}$$

- Usually discussed in Frequency domain ($\omega \rightarrow 0$ limit)

$$\tilde{w}_{\mu\nu} \sim -\frac{i}{\omega} F_{\mu\nu} + \mathcal{O}(\log \omega)$$

Determined by the soft theorem

$$F_{\mu\nu} = f_{\mu\nu} + \delta f_{\mu\nu} \begin{matrix} \rightarrow \text{non-linear} \\ \leftarrow \text{linear} \end{matrix} \quad (\text{displacement caused by the radiation itself})$$

Soft Theorems and Memory

- According to Weinberg the emission of a **soft graviton** factorizes

$$\mathcal{S}_{s.f.} = \underbrace{e^{iRe 2\mathcal{S}}}_{\text{Hard scattering}} e^{\int_k (F a^\dagger - F^* a)} \leftarrow \text{soft emission}$$

Soft Theorems and Memory

- According to Weinberg the emission of a **soft graviton** factorizes

$$\mathcal{S}_{s.f.} = \underbrace{e^{i\text{Re } 2\mathcal{S}}}_{\text{Hard scattering}} e^{\int_{\mathcal{K}} (F_{\alpha}^{\dagger} - F^*_{\alpha})} \leftarrow \text{soft emission}$$

If we plug this in into the definition of wave form $h_{\mu\nu}$

$$h_{\mu\nu} \approx \int_{\mathcal{K}} d^4k (F_{\mu\nu} e^{ikx} + F_{\mu\nu}^* e^{-ikx}) \longrightarrow F_{\mu\nu} = \sum_a \sqrt{8\pi G} \frac{P_a^{\mu} P_a^{\nu}}{P_a \cdot k - i0}$$

which we can write

$$h_{\mu\nu}^{\text{post infinity}} \sim -\frac{4G}{r} \sum_{\text{in}} \frac{(K_a^{\mu} K_a^{\nu})_{T,T}}{K_a \cdot n}$$

$$h_{\mu\nu}^{\text{future infinity}} \sim -\frac{4G}{r} \sum_{\text{in}} \frac{(K_a^{\mu} K_a^{\nu})_{T,T}}{K_a \cdot n}$$

} Linear memory is sourced by the **IN** and **OUT** particles which differ due to their deflection.

Soft Theorems and Memory II

- What about the graviton? Can it also contribute to the memory effect?

Soft Theorems and Memory II

- What about the graviton? Can it also contribute to the memory effect?

Let us look at Energy momentum emission spectrum

$$K^\mu d\rho(K) = K^\mu |FT(\text{diagram})|^2 2\pi\theta(K^0) \delta(K^2) \frac{d^3K}{(2\pi)^3} + \mathcal{O}(G')$$

Soft Theorems and Memory II

- What about the graviton? Can it also contribute to the memory effect?

Let us look at Total Radiated Energy-momentum

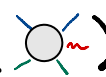
$$P^M = \int K^M d\mu(K) = \int K^M |FT(\text{Sun})|^2 2\pi\theta(K^0) \delta(K^2) \frac{d^0 K}{(2\pi)^0} + \mathcal{O}(G')$$

Soft Theorems and Memory II


- What about the graviton? Can it also contribute to the memory effect?

Let us look at number of emitted gravitons

Number of Emitted gravitons

$$d\rho(k) = |FT(\text{diagram})|^2 2\pi\theta(k^0) \delta(k^2) \frac{d^0k}{(2\pi)^0} + \mathcal{O}(G^4)$$


So the soft gravitons contribution can be written as

$$\tilde{W}_{NL}^{\mu\nu} = \int \sqrt{8\pi G} \frac{(K^\mu K^\nu)_{TT}}{K \cdot K_0 - i0} d\rho(k) = \int \frac{\sqrt{8\pi G}}{K \cdot K_0 - i0} (K^\mu K^\nu)_{TT} |FT(\text{diagram})|^2 2\pi\theta(k^0) \delta(k^2) \frac{d^0k}{(2\pi)^0}$$


Wave Form @ NNLO $\omega \rightarrow 0$

- What is the interesting effect?

→ Permanent displacement

$$\left(\lim_{T \rightarrow \infty} w_{\mu\nu}(T, n) \right) - \left(\lim_{T \rightarrow -\infty} w_{\mu\nu}(T, n) \right) = -F_{\mu\nu}$$

- Usually discussed in Frequency domain ($\omega \rightarrow 0$ limit)

$$\tilde{w}_{\mu\nu} \sim -\frac{i}{\omega} F_{\mu\nu} + \mathcal{O}(\log \omega)$$

Determined by the soft theorem

First appears at NNL

$$F_{\mu\nu} = f_{\mu\nu} + \delta f_{\mu\nu} \rightarrow \int d\tilde{k} \rho(k) \frac{k^\mu k^\nu}{k \cdot n}$$

$$\sum_{a=1}^4 \frac{p_a^\mu p_a^\nu}{p_a \cdot n} \leftarrow$$

Wave Form @ NNLO ω → 0

$$\begin{aligned}
 W_2^{\mu\nu} \sim & \text{Diagram 1} + \frac{i}{2} \left(\text{Diagram 2} - \text{Diagram 3} \right) \\
 & + \frac{i}{2} \left(\text{Diagram 4} + \text{Diagram 5} - \text{Diagram 6} - \text{Diagram 7} \right) \\
 & - \frac{1}{6} \left(\text{Diagram 8} - 2 \text{Diagram 9} + \text{Diagram 10} \right)
 \end{aligned}$$

The diagrams are Feynman diagrams for a process involving two incoming particles (blue lines) and two outgoing particles (green lines). The diagrams are arranged in three rows:

- Row 1:** A tree-level diagram with a central grey circle labeled 'N' containing two white circles. A red wavy line connects the right side of this circle to the first of two diagrams in the second row. The second diagram in the row shows two grey circles connected by a blue arc (top) and a green arc (bottom), with a red wavy line between them. The third diagram is similar but with the red wavy line on the other side.
- Row 2:** Four diagrams showing a grey circle with a white circle inside, connected to another grey circle. The red wavy line is attached to the white circle. The diagrams are separated by '+' and '-' signs.
- Row 3:** Three diagrams showing a chain of three grey circles connected by blue and green arcs. The red wavy line is attached to the first, second, or third circle. The diagrams are separated by '- 2' and '+' signs.

Wave Form

@ NNLO

$\omega \rightarrow 0$

Part of Linear + NON-LINEAR (Gauge inv)

$$\begin{aligned}
 W_2^{\mu\nu} \sim & \text{Diagram 1} + \frac{i}{2} \left(\text{Diagram 2} - \text{Diagram 3} \right) \\
 & + \frac{i}{2} \left(\text{Diagram 4} + \text{Diagram 5} - \text{Diagram 6} - \text{Diagram 7} \right) \\
 & - \frac{1}{6} \left(\text{Diagram 8} - 2 \text{Diagram 9} + \text{Diagram 10} \right)
 \end{aligned}$$

The diagrams consist of grey circles representing interaction vertices. Blue lines represent fermions, green lines represent scalars, and red wavy lines represent photons.

 Diagram 1: A single vertex with two blue lines and two green lines, and two internal white circles. A red wavy line is attached to the right.

 Diagram 2: Two vertices connected by a blue fermion loop and a green scalar loop. A red wavy line is attached to the right.

 Diagram 3: Similar to Diagram 2, but with the red wavy line attached to the left.

 Diagram 4: Two vertices connected by a blue fermion loop and a green scalar loop. A red wavy line is attached to the right. The left vertex has a white circle.

 Diagram 5: Similar to Diagram 4, but with the red wavy line attached to the left.

 Diagram 6: Similar to Diagram 4, but with the red wavy line attached to the right and the right vertex having a white circle.

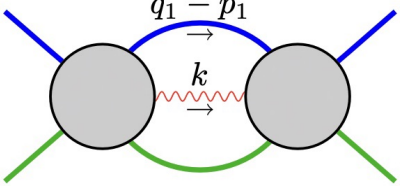
 Diagram 7: Similar to Diagram 4, but with the red wavy line attached to the left and the left vertex having a white circle.

 Diagram 8: Three vertices connected by blue fermion and green scalar loops. A red wavy line is attached to the left.

 Diagram 9: Similar to Diagram 8, but with the red wavy line attached to the middle vertex.

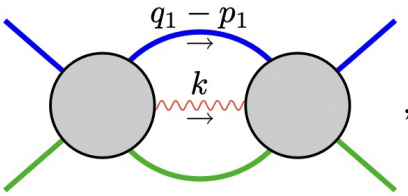
 Diagram 10: Similar to Diagram 8, but with the red wavy line attached to the right.

NON Linear Memory

$$\delta F^{\mu\nu} = \text{FT}_4 \int_{q_1} \int_k \frac{k^\mu k^\nu}{k \cdot n}$$


The diagram shows two gray circular vertices connected by a red wavy line representing a propagator with momentum k and direction \rightarrow . The left vertex has two external lines: a blue line entering from the top-left and a green line exiting from the bottom-left. The right vertex has two external lines: a blue line entering from the top-right and a green line exiting from the bottom-right. A blue arc connects the top of the two vertices, labeled with momentum $q_1 - p_1$ and direction \rightarrow . A green arc connects the bottom of the two vertices, also labeled with momentum $q_1 - p_1$ and direction \rightarrow .

NON Linear Memory

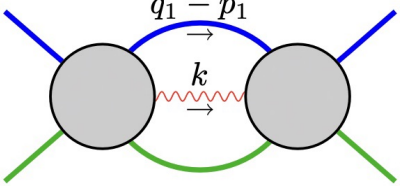
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Known 4-point families
but still a function of 3-variables..



NON Linear Memory

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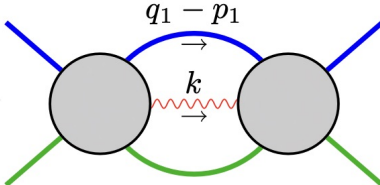
$$\delta F^{\mu\nu} = \text{FT}_4 \int_{q_1} \int_k \frac{k^\mu k^\nu}{k \cdot n}$$




We can further simplify this by studying directly the multipole decomposition of $\delta F^{\mu\nu}$

$$\delta F^{\ell m} = \oint Y_{\pm 2}^{\ell m*} \delta F_{\mu\nu} \varepsilon_{\pm}^{\mu} \varepsilon_{\pm}^{\nu} d\Omega. \quad \rightarrow \quad \delta F^{\ell m} = -\frac{2\pi}{\ell(\ell-1)} \mathcal{N}_2^{\ell m} Y_{\mu_1 \dots \mu_\ell}^{\ell m*} \text{FT}_4[\mathbb{I}^{\mu_1 \dots \mu_\ell}],$$

Where :

$$\mathbb{I}^{\mu_1 \dots \mu_\ell} = \int_{q_1} \int_k \frac{k^{\mu_1} \dots k^{\mu_\ell}}{(-t \cdot k)^{\ell-1}}$$


and we are free to choose a particular R.F. (ex. sitting on p_i)

NON Linear Memory: Multipoles

For each l, m simple dependence on σ $N_2^{l,m}$ is a normalization

$$\delta F^{l,m} = \frac{G^3 \pi^2 m_1 m_2^2}{b^3 (\sigma^2 - 1)} ; \quad N_2^{l,m} F(\sigma) + \dots$$

$$\hookrightarrow (\sigma^2 - 1)^{-\frac{l}{2}} \left[f_1^{l,m} + f_2^{l,m} \log\left(\frac{\sigma+1}{2}\right) + f_3^{l,m} \frac{\operatorname{arccosh} \sigma}{\sqrt{\sigma^2 - 1}} \right]$$

f_i polynomials in σ ($f_2^{2,2} = \frac{(\sigma^2 - 1)^2}{8} (35\sigma^4 + 420\sigma^3 - 510\sigma^2 + 232\sigma + 67)$)

Match with PN expansion! Also:

$$\delta F^{l,m} = 0 \quad \text{if } l+m \text{ is Odd}$$

$$\delta F^{l,-m} = (-1)^l \delta F^{l,m} \quad \text{sym of } p(k) \text{ for } b \rightarrow -b$$

NON Linear Memory II

What happens if we stay in c.o.m frame?

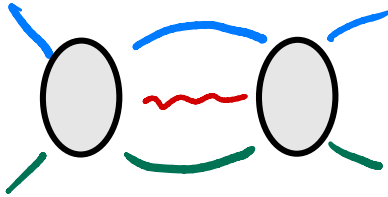
Extra propagator add new scale ($\lambda = \frac{m_2}{m_1}$) and complexity... ($\lambda \ell \cdot u_2 + \ell \cdot u_1$)

NON Linear Memory II

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Can we go higher?



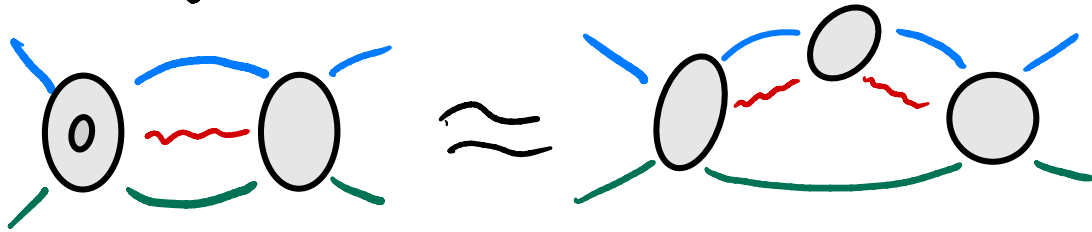
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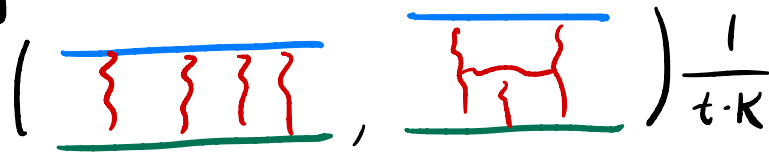
Extra propagator add new scale ($\lambda = \frac{m_2}{m_1}$) and complexity... ($\lambda \ell \cdot u_2 + \ell \cdot u_1$)

Can we go higher?

Real part difficult (Except B_{10} !)



New topologies



We expect
at most L_{12}

Conclusion & Outlook

- Presented computation of the non-linear memory from amplitudes and PM expansion.
- Needed to add higher point cuts. Does this indicates correlation between classical observables?

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-
- Compute full NNLO waveform?
 - Understand Memory in detector formalism?
 - Squeezed / Non-squeezed Classical gravitational Radiation?