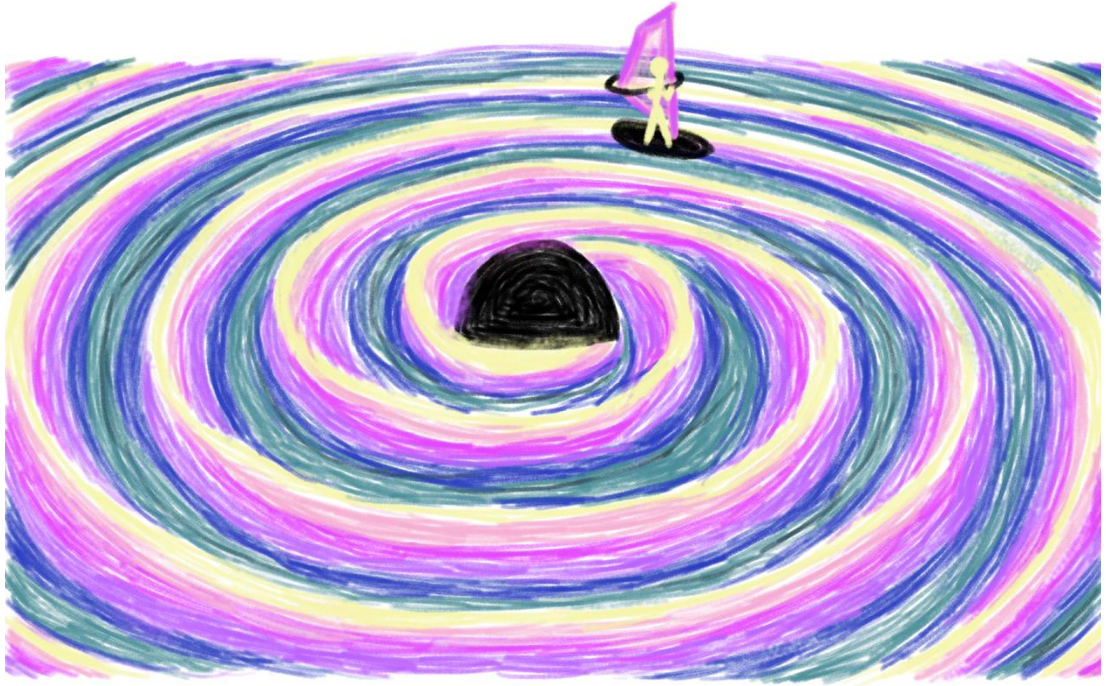


Surfing the Shockwave with Black-Hole Response Theory

Based on [2604.xxxxx] with Carl Jordan Eriksen, Jitze Hoogeveen, Gustav Uhre Jakobsen and Jan Plefka

HUMBOLDT-UNIVERSITÄT ZU BERLIN

RTG 2575: Rethinking Quantum Field Theory

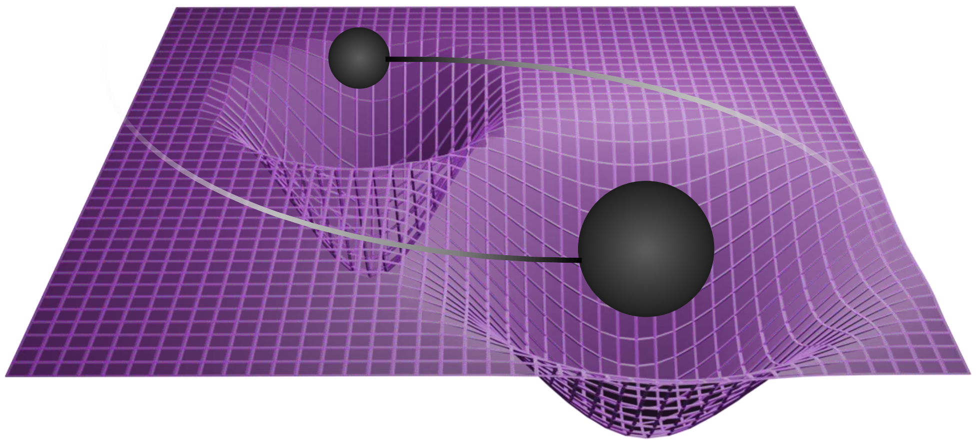
Amplitudes, Strong-Field Gravity and Resummation, Nordita

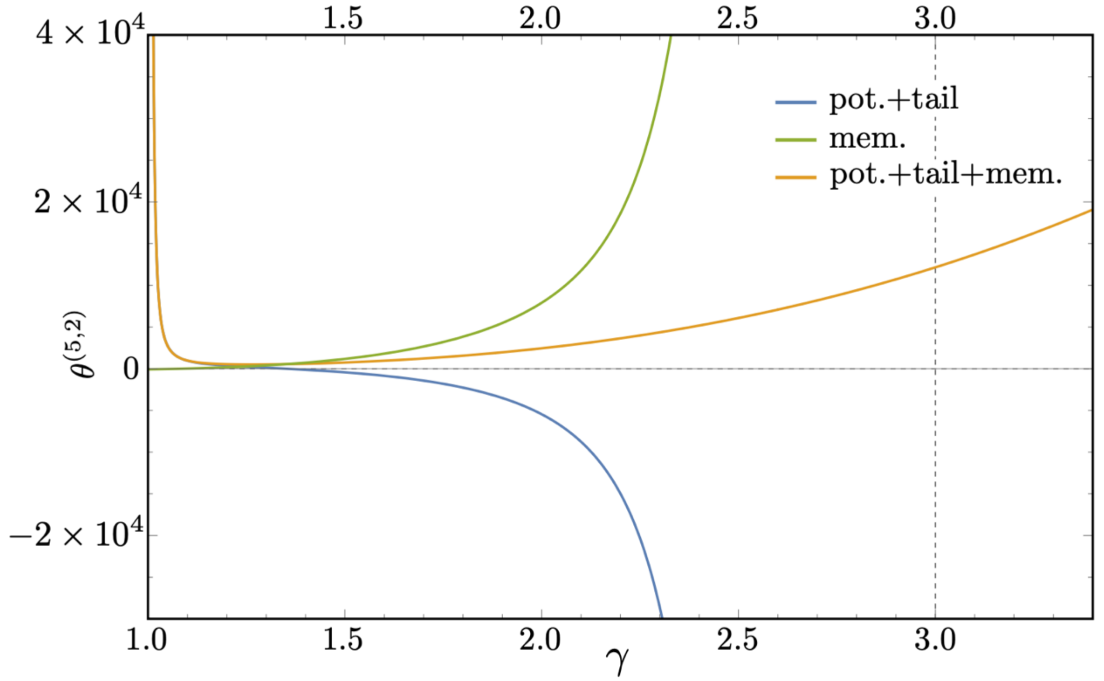
Lara Bohnenblust

16. April 2026

Astrophysical and Cosmological Relativity

Motivation to use worldline quantum field theory (WQFT)





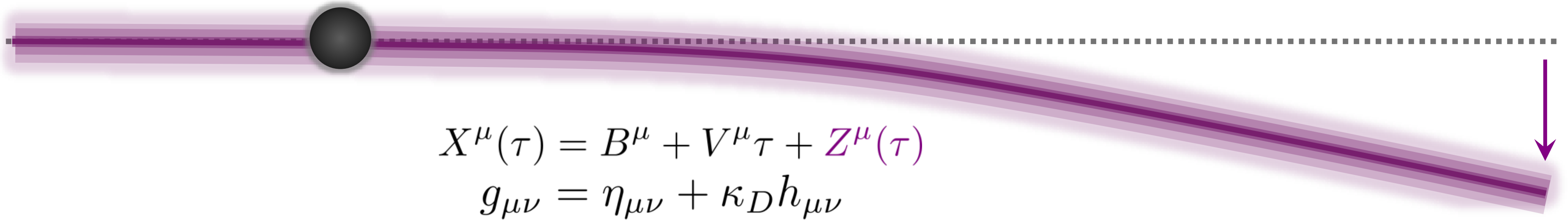
Driesse, Jakobsen, Mogull, Nega, Plefka, Sauer, Usovitsch [2601.16256]

Recent 5PM 2SF computation of the conservative potential with WQFT

By quantizing worldlines:

- **Systematic expansion** in G in terms of Feynman diagrams
- Solve Feynman-like **integrals** with QFT tools
- **Tree-level** captures exactly classically relevant information

WQFT of a single black hole



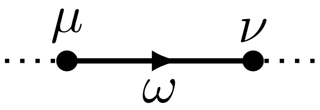
Mogull, Plefka, Steinhoff [2010.02865]

$$X^\mu(\tau) = B^\mu + V^\mu\tau + Z^\mu(\tau)$$
$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa_D h_{\mu\nu}$$

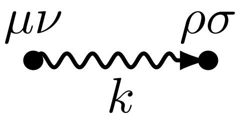
$$S_{\rm BH}[X,g] = -\frac{M}{2}\int \mathrm{d}\tau\, g_{\mu\nu}(X)\dot{X}^\mu\dot{X}^\nu - \int \mathrm{d}^D x\, \frac{2}{\kappa_D^2}\sqrt{|g|}R$$

$$\kappa_D^2 = 32\pi G(4\pi e^{\gamma_E}L^2)^{-\epsilon}$$

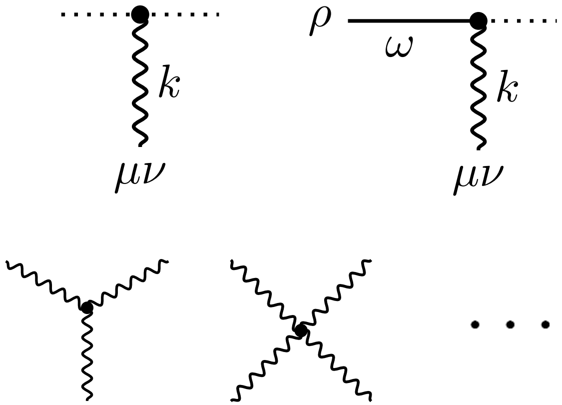
Feynman rules for "in-in" prescription:



$$= \frac{-\mathrm{i}\eta^{\mu\nu}}{M(\omega+\mathrm{i}0^+)^2}$$

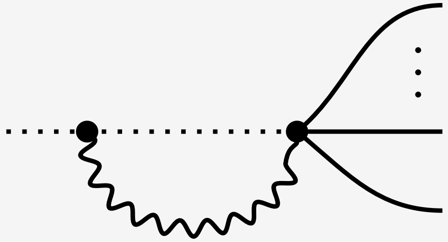


$$= \frac{\mathrm{i}\mathcal{P}_{\mu\nu\rho\sigma}}{k^2+\mathrm{i}0^+(W\cdot k)}$$



Dim.-regularise self-energy:

$$g_{\mu\nu}(X) = \eta_{\mu\nu}$$



$$= 0$$

Cheung, Parra-Martinez, Rothstein, Shah, Wilson-Gerow [2406.14770]

Black-hole response theory

- Partition Function for single black hole

$$Z[\mathcal{T}] = \int \mathcal{D}[h, Z]\, \mathrm{e}^{\frac{\mathrm{i}}{\hbar}\left(S_{\mathrm{BH}}[X,g] - \int \mathrm{d}^D x\, \mathcal{T}^{\mu\nu} h_{\mu\nu}\right)}\Big|_{\mathrm{tree}} = \mathrm{e}^{\frac{\mathrm{i}}{\hbar} W[\mathcal{T}]}$$

- **Black hole response effective action**

$$\mathrm{i}W[\mathcal{T}] = -\mathrm{i}\int_{k_1} \mathcal{T}^{\alpha\beta}(k_1)\mathcal{R}_{\alpha\beta}(k_1) - \frac{1}{2}\int_{k_1,k_2} \mathcal{T}^{\alpha\beta}(k_1)\mathcal{T}^{\mu\nu}(k_2)\mathcal{R}_{\alpha\beta\mu\nu}(k_1,k_2)$$

$$+ \frac{\mathrm{i}}{6}\int_{k_1,k_2,k_3} \mathcal{T}^{\alpha\beta}(k_1)\mathcal{T}^{\mu\nu}(k_2)\mathcal{T}^{\rho\sigma}(k_3)\mathcal{R}_{\alpha\beta\mu\nu\rho\sigma}(k_1,k_2,k_3) + \cdots$$

Black-hole response theory

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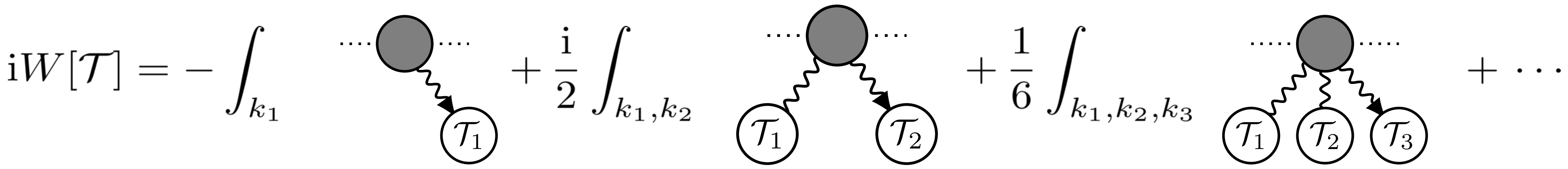
- **Black hole response effective action**



- Coupling to a secondary black hole

$$\mathcal{T}^{\mu\nu}(x) = \frac{m}{2} \int \mathrm{d}\tau\, \delta^D(x - x(\tau)) x^\mu(\tau) x^\nu(\tau)$$

  - ➢ each interaction with the secondary gives an extra power of m
  - ➢ n - point response function contributes to $(n-1)$ self-force (SF) order observable

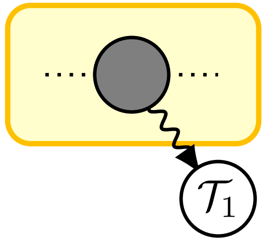
➢ **See Jitze Hoogeveen's talk**

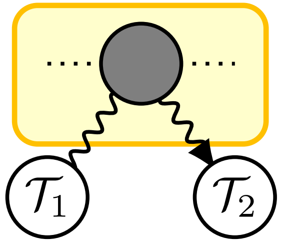
Black-hole response theory

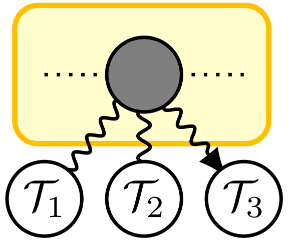
- Partition Function for single black hole

$$Z[\mathcal{T}] = \int \mathcal{D}[h, Z]\, \mathrm{e}^{\frac{\mathrm{i}}{\hbar}\left(S_{\mathrm{BH}}[X,g] - \int \mathrm{d}^D x\, \mathcal{T}^{\mu\nu} h_{\mu\nu}\right)}\Big|_{\mathrm{tree}} = \mathrm{e}^{\frac{\mathrm{i}}{\hbar} W[\mathcal{T}]}$$

- **Black hole response effective action**

$$\mathrm{i}W[\mathcal{T}] = -\int_{k_1}$$


$$+ \frac{\mathrm{i}}{2}\int_{k_1,k_2}$$


$$+ \frac{1}{6}\int_{k_1,k_2,k_3}$$


$$+ \cdots$$

- Coupling to a secondary black hole

$$\mathcal{T}^{\mu\nu}(x) = \frac{m}{2}\int \mathrm{d}\tau\, \delta^D(x - x(\tau))\dot{x}^\mu(\tau)\dot{x}^\nu(\tau)$$

  - ➢ each interaction with the secondary gives an extra power of m
  - ➢ n - point response function contributes to $(n-1)$ self-force (SF) order observable

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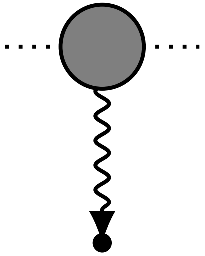
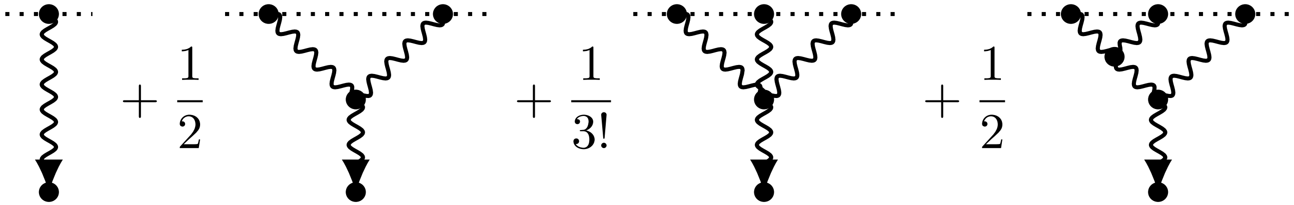
One-point response function

$$Z[\mathcal{T}] = \int \mathcal{D}[h, Z]\, \mathrm{e}^{\frac{\mathrm{i}}{\hbar}\left(S_{\mathrm{BH}}[X,g] - \int \mathrm{d}^D x\, \mathcal{T}^{\mu\nu} h_{\mu\nu}\right)}\Big|_{\mathrm{tree}} = \mathrm{e}^{\frac{\mathrm{i}}{\hbar} W[\mathcal{T}]}$$



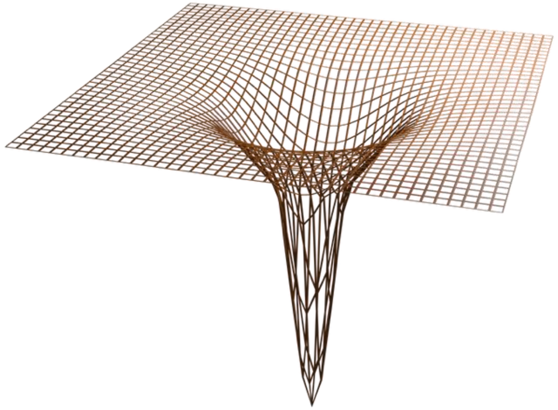
$$= -\frac{\delta W[\mathcal{T}]}{\delta \mathcal{T}^{\mu\nu}(x)}\Big|_{\mathcal{T}=0}$$

Duff, 1974



$$= \ \cdots + \frac{1}{2}\ \cdots + \frac{1}{3!}\ \cdots + \frac{1}{2}\ \cdots + \cdots =$$

$$= \langle h_{\mu\nu}(x) \rangle_{\mathcal{T}=0} \quad =$$



Background metric

Two-point response function

- Generalized effective action to include a source for the deflection $Z^\mu(\tau)$

$$\mathrm{e}^{\frac{\mathrm{i}}{\hbar}W[\mathcal{T},f]} = Z[\mathcal{T},f] = \int \mathcal{D}[h,Z]\,\mathrm{e}^{\frac{\mathrm{i}}{\hbar}\left(S_{\mathrm{BH}}[X,g] - \int \mathrm{d}^D x\,\mathcal{T}^{\mu\nu}(x)h_{\mu\nu}(x) - \int \mathrm{d}\tau\, f_\mu(\tau)Z^\mu(\tau)\right)}\Big|_{\mathrm{tree}}$$

- De Witt notation

$$\langle\phi_A\rangle_J = \{\,\langle h_{\mu\nu}(x)\rangle_{\{\mathcal{T},f\}}\,,\,\langle Z^\mu(\tau)\rangle_{\{\mathcal{T},f\}}\,\}, \qquad J_A = \{\,\mathcal{T}^{\mu\nu}(x)\,,\,f_\mu(\tau)\,\},$$

$$\langle\phi_A\rangle_J J_A = \int \mathrm{d}^D x\,\mathcal{T}^{\mu\nu}(x)\langle h_{\mu\nu}(x)\rangle_{\{\mathcal{T},f\}} + \int \mathrm{d}\tau\, f_\mu(\tau)\langle Z^\mu(\tau)\rangle_{\{\mathcal{T},f\}}$$

- Schwinger-Dyson like equation

Response Function

$$\frac{\delta^2 S_{\mathrm{BH}}[\langle\phi\rangle_J]}{\delta\langle\phi_A\rangle_J\delta\langle\phi_B\rangle_J}\,\frac{\delta^2 W[J]}{\delta J_B \delta J_C}\bigg|_{J=0} = -\delta_{AC}$$

Feynman rules in presence of a source

Bautista, Driesse, Haddad, Jakobsen [2602.06125]

Two-point response function

- Generalized effective action to include a source for the deflection $Z^\mu(\tau)$

$$e^{\frac{i}{\hbar}W[\mathcal{T},f]} = Z[\mathcal{T},f] = \int \mathcal{D}[h,Z]\, e^{\frac{i}{\hbar}\left(S_{\rm BH}[X,g] - \int d^D x\, \mathcal{T}^{\mu\nu}(x) h_{\mu\nu}(x) - \int d\tau\, f_\mu(\tau) Z^\mu(\tau)\right)}\Big|_{\rm tree}$$

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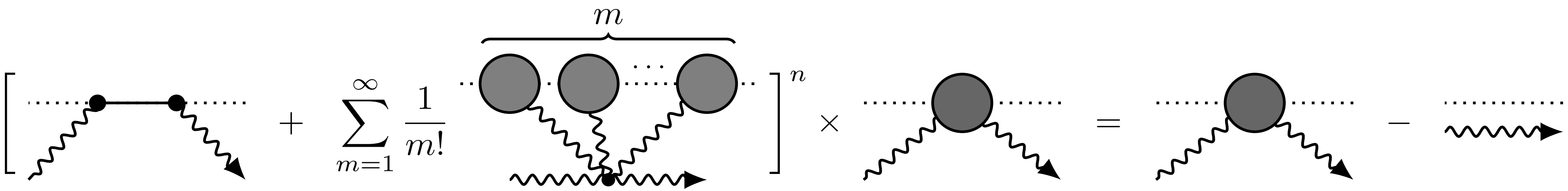

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$$\mathrm{e}^{\frac{\mathrm{i}}{\hbar}W[\mathcal{T},f]} = Z[\mathcal{T},f] = \int \mathcal{D}[h,Z]\, \mathrm{e}^{\frac{\mathrm{i}}{\hbar}\left(S_{\mathrm{BH}}[X,g] - \int \mathrm{d}^D x\, \mathcal{T}^{\mu\nu}(x) h_{\mu\nu}(x) - \int \mathrm{d}\tau\, f_\mu(\tau) Z^\mu(\tau)\right)}\Big|_{\mathrm{tree}}$$

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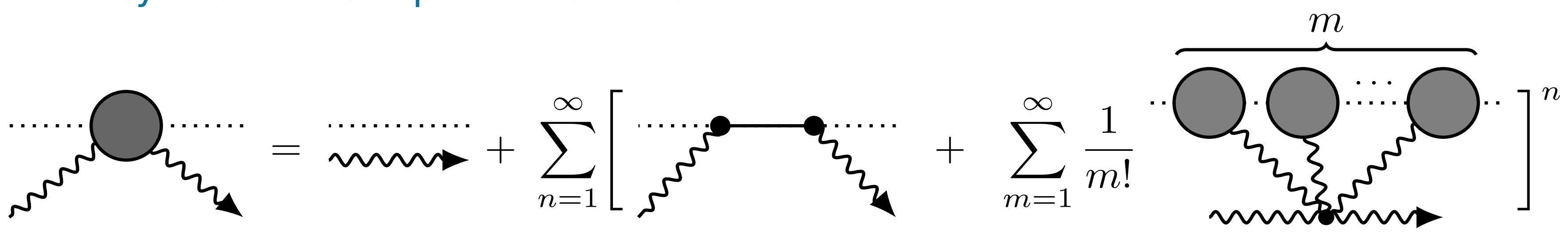
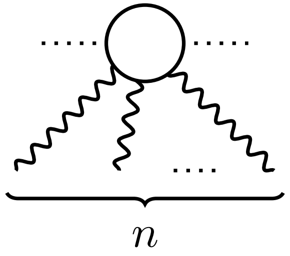


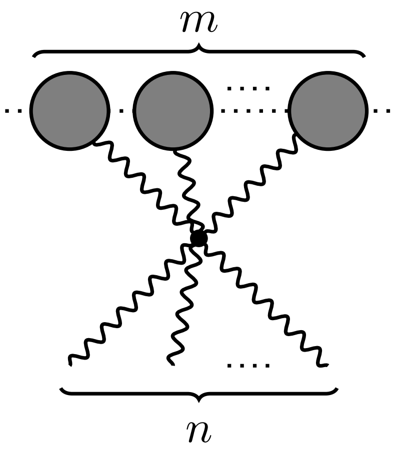
Three-point response function (and higher)

- Resumming Feynman rules in presence of a source

$$\left.\frac{\delta^n \mathrm{i}S_{\mathrm{BH}}[\langle\phi\rangle_J]}{\delta\langle h_{\mu_1\nu_1}(x_1)\rangle_J\delta\langle h_{\mu_2\nu_2}(x_2)\rangle_J\cdots\delta\langle h_{\mu_n\nu_n}(x_n)\rangle_J}\right|_{J=0} =$$



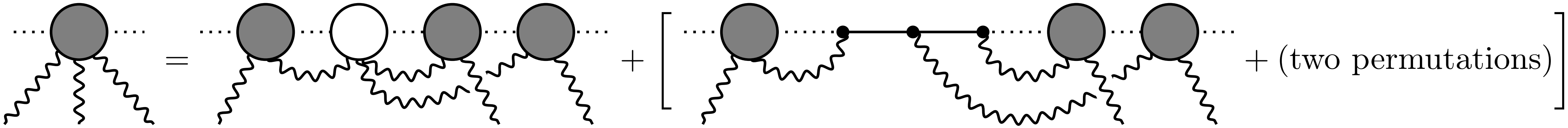
$$= \sum_{m=0}^{\infty}\frac{1}{m!}$$



- Apply functional derivative to Schwinger-Dyson equation

$$\frac{\delta}{\delta J_D}\left[\frac{\delta^2 S_{\mathrm{BH}}[\langle\phi\rangle_J]}{\delta\langle\phi_A\rangle_J\delta\langle\phi_B\rangle_J}\left.\frac{\delta^2 W[J]}{\delta J_B\delta J_C}\right|_{J=0}\right] = 0$$

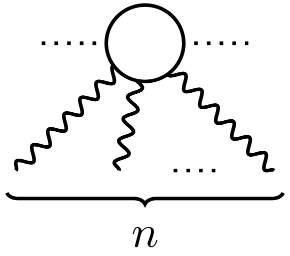
- Obtain recursion relation for higher-point response functions



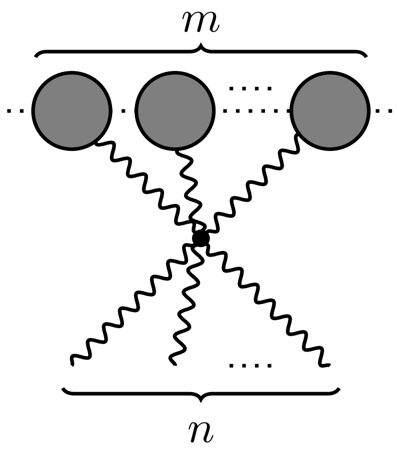
Three-point response function (and higher)

- Resumming Feynman rules in presence of a source

$$\left.\frac{\delta^n \mathrm{i}S_{\mathrm{BH}}[\langle\phi\rangle_J]}{\delta\langle h_{\mu_1\nu_1}(x_1)\rangle_J\,\delta\langle h_{\mu_2\nu_2}(x_2)\rangle_J\cdots\delta\langle h_{\mu_n\nu_n}(x_n)\rangle_J}\right|_{J=0} =$$



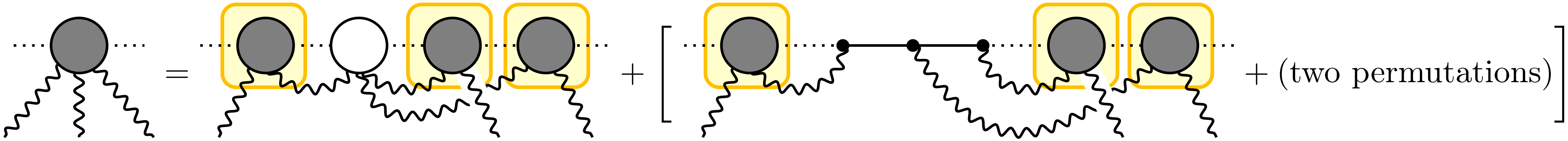
$$= \sum_{m=0}^{\infty}\frac{1}{m!}$$

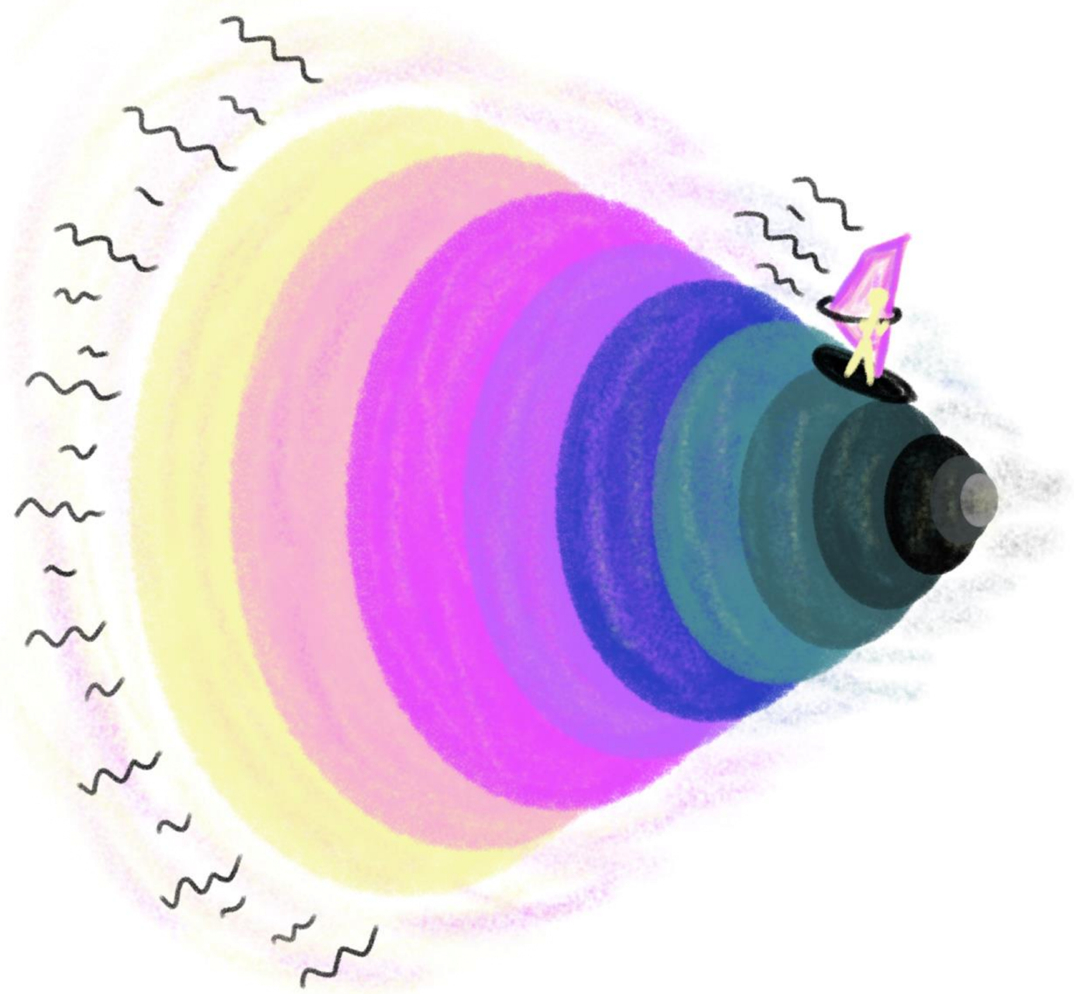


- Apply functional derivative to Schwinger-Dyson equation

$$\frac{\delta}{\delta J_D}\left[\left.\frac{\delta^2 S_{\mathrm{BH}}[\langle\phi\rangle_J]}{\delta\langle\phi_A\rangle_J\,\delta\langle\phi_B\rangle_J}\,\frac{\delta^2 W[J]}{\delta J_B\,\delta J_C}\right|_{J=0}\right] = 0$$

- Obtain recursion relation for higher-point response functions





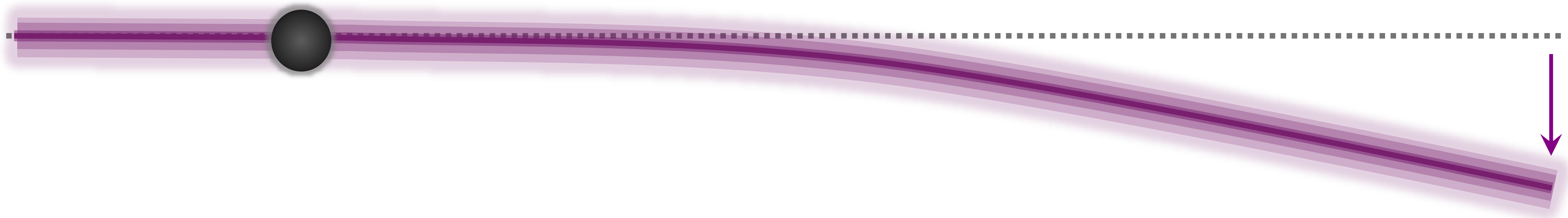
Application of black-hole response theory to the **shockwave background**

Massless WQFT

Brink-Di Vecchia-Howe form (formerly known as Polyakov form) for massless limit

$$S = -\frac{1}{2}\int \mathrm{d}\tau \left(e^{-1}\, g_{\mu\nu}(X)\dot{X}^{\mu}\dot{X}^{\nu} + eM^2\right) \xrightarrow{M\to 0,\, e^{-1}\to E} -\frac{E}{2}\int \mathrm{d}\tau\; g_{\mu\nu}(X)\dot{X}^{\mu}\dot{X}^{\nu}$$

Infinitely boosted black hole with energy E



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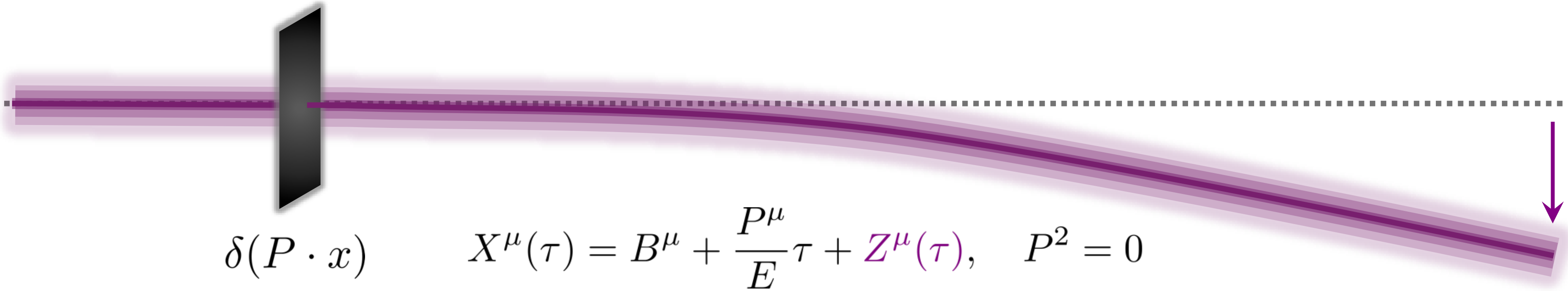
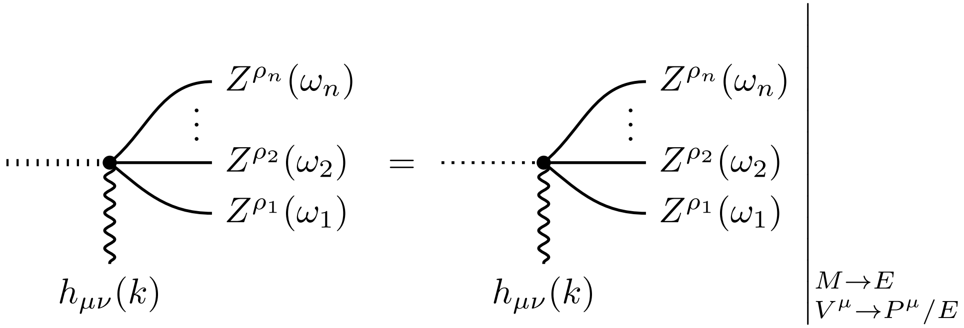
$$S=-\frac{1}{2}\int \mathrm{d}\tau\left(e^{-1}g_{\mu\nu}(X)\dot{X}^{\mu}\dot{X}^{\nu}+eM^{2}\right)\xrightarrow{M\to 0,\,e^{-1}\to E}-\frac{E}{2}\int \mathrm{d}\tau\;g_{\mu\nu}(X)\dot{X}^{\mu}\dot{X}^{\nu}$$

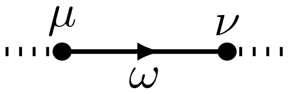
Infinitely boosted black hole with energy E



$\delta(P\cdot x)$

$$X^{\mu}(\tau)=B^{\mu}+\frac{P^{\mu}}{E}\tau+Z^{\mu}(\tau),\qquad P^{2}=0$$

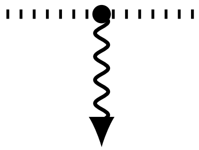




$$=\frac{-\mathrm{i}\eta^{\mu\nu}}{E(\omega+\mathrm{i}0^{+})^{2}}$$

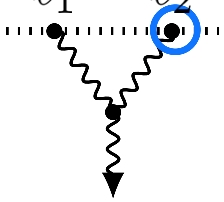
One-point response: The shockwave metric

$$\langle h_{\mu\nu}(x)\rangle_{1\mathrm{PM}} = \int_k e^{-\mathrm{i}k\cdot x}$$



$$= \frac{\kappa_D}{2} P_\mu P_\nu \int_k e^{-\mathrm{i}k\cdot x} \frac{\delta(k\cdot P)}{k^2}$$

- 2PM contribution:



$$= \int_{\ell_1} \frac{\delta(P\cdot\ell_1)\delta(P\cdot\ell_2) P^{\mu_1}P^{\nu_1}P^{\mu_2}P^{\nu_2}\Omega_{\mu_1\nu_1\mu_2\nu_2;\mu\nu}(\ell_1,\ell_2)}{\ell_1^2\ell_2^2} \sim P^2, P\cdot\ell_1, P\cdot\ell_2 = 0$$

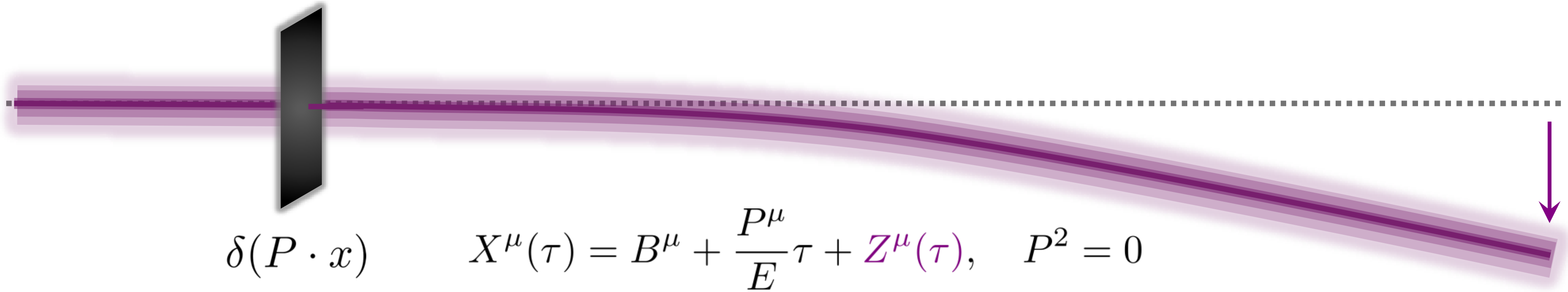
Light-cone decomposition

$$\ell^\mu = \frac{\ell^+ e^\mu + \ell^- \bar{e}^\mu}{2} + \boldsymbol{\ell}_\perp^\mu$$

$$\sim \int \mathrm{d}^D\ell_1 \frac{\delta(\ell_1^-)}{\ell_1^2} = \left(\int \frac{\mathrm{d}\ell_1^+}{2}\right) \int \mathrm{d}^{D-2}\boldsymbol{\ell}_{1\perp} \frac{1}{{\boldsymbol{\ell}_{1\perp}}^2}$$

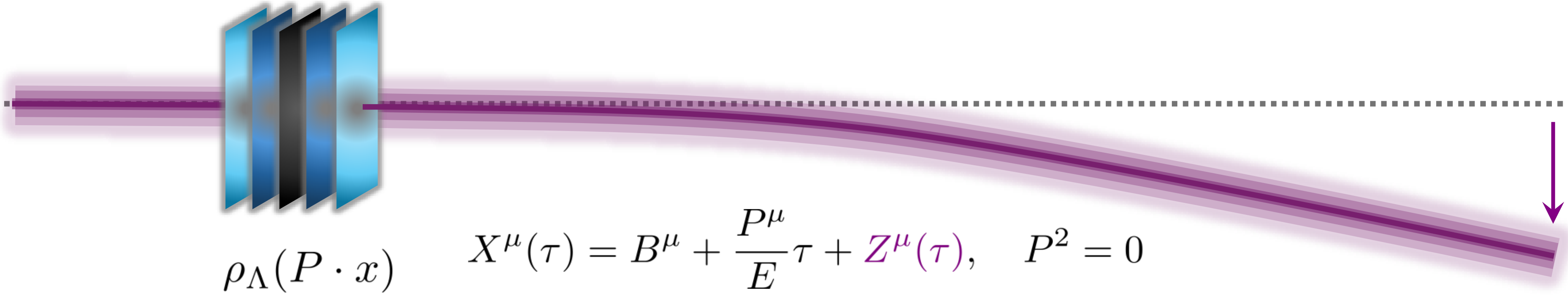
Non-regularised divergence

Regularizing massless WQFT



$$\delta(P \cdot x) \qquad X^\mu(\tau) = B^\mu + \frac{P^\mu}{E}\tau + Z^\mu(\tau), \quad P^2 = 0$$

Regularizing massless WQFT

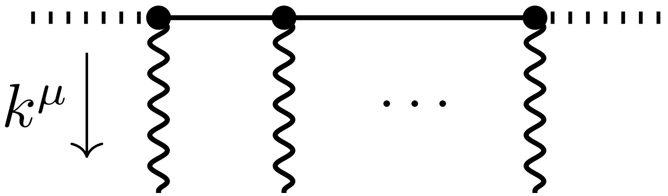


$$\rho_\Lambda(P\cdot x) \xrightarrow{\Lambda\to\infty} \delta(P\cdot x)$$

$$X^\mu(\tau) = B^\mu + \frac{P^\mu}{E}\tau + Z^\mu(\tau), \quad P^2 = 0$$

Choose $\rho_\Lambda(\sigma) = \frac{\Lambda}{2}\exp(-\Lambda|\sigma|)$ with Fourier transformation $\tilde{\rho}_\Lambda(\omega) = \frac{\Lambda^2}{\omega^2+\Lambda^2}$

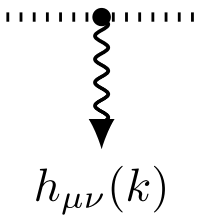
$\Longrightarrow$ $k^\mu \downarrow$



$$\sim \frac{\Lambda^2}{{k^+}^2+\Lambda^2}$$

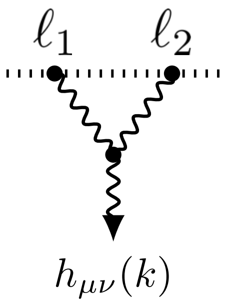
One-point response: The shockwave metric

$$\langle h_{\mu\nu}(x)\rangle_{1\mathrm{PM}} = \int_k e^{-\mathrm{i}k\cdot x}$$



$$= \frac{\kappa_D}{2} P_\mu P_\nu \int_k e^{-\mathrm{i}k\cdot x}\frac{\delta(k\cdot P)}{k^2}$$

- 2PM contribution:



$$= \int_{\ell_1} \frac{\Lambda^4 \delta(P\cdot\ell_1)\delta(P\cdot\ell_2) P^{\mu_1}P^{\nu_1}P^{\mu_2}P^{\nu_2}\Omega_{\mu_1\nu_1\mu_2\nu_2;\mu\nu}(\ell_1,\ell_2)}{({\ell_1^+}^2+\Lambda^2)({\ell_2^+}^2+\Lambda^2)\ell_1^2\ell_2^2} \sim P^2, P\cdot\ell_1, P\cdot\ell_2 = 0$$

Light-cone decomposition

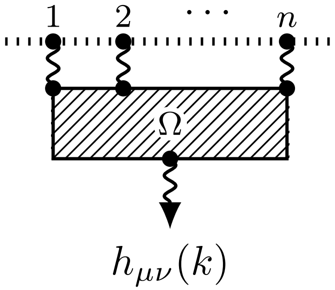
$$\ell^\mu = \frac{\ell^+ e^\mu + \ell^- \bar{e}^\mu}{2} + \boldsymbol{\ell}_\perp^\mu$$

$$\sim \int \mathrm{d}^D\ell_1 \frac{\delta(\ell_1^-)}{\ell_1^2}\frac{\Lambda^2}{{\ell_1^+}^2+\Lambda^2} = \left(\int \frac{\mathrm{d}\ell_1^+}{2}\frac{\Lambda^2}{{\ell_1^+}^2+\Lambda^2}\right)\int \mathrm{d}^{D-2}\boldsymbol{\ell}_{1\perp}\frac{1}{{\boldsymbol{\ell}_{1\perp}}^2}$$

Regularised!

One-point response: The shockwave metric

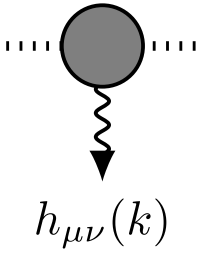
- Higher-point contributions vanish



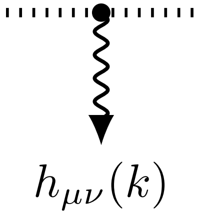
$$= \int_{\ell_1,\ldots,\ell_{n-1}} \left(\prod_{i=1}^{n} \frac{\delta(P\cdot\ell_i) P^{\mu_i} P^{\nu_i}}{\ell_i^2} \right) \Omega_{\mu_1\nu_1\cdots\mu_n\nu_n;\mu\nu}(\ell_1,\ldots,\ell_n) \sim P^2,\ P\cdot\ell_i = 0$$

- Truncation at 1PM:

$$\langle h_{\mu\nu}(x)\rangle = \int_k e^{-\mathrm{i}k\cdot x}$$



$$= \int_k e^{-\mathrm{i}k\cdot x}$$



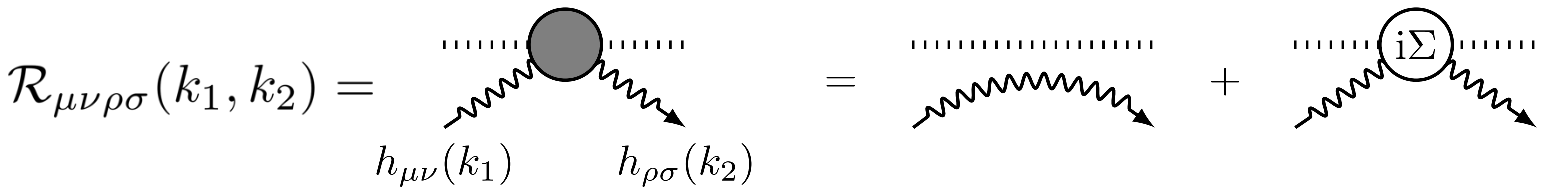
$$= \frac{\kappa_D}{2} P_\mu P_\nu \int_k e^{-\mathrm{i}k\cdot x} \frac{\delta(k\cdot P)}{k^2}$$

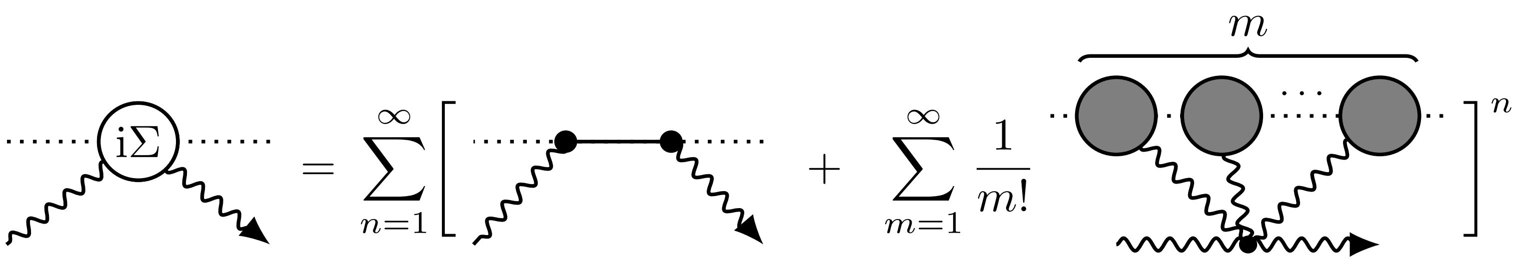
- 4D limit recovers Aichelburg-Sexl shockwave background

$$\mathrm{d}s^2 = \mathrm{d}x^-\,\mathrm{d}x^+ - \mathrm{d}\boldsymbol{x}_\perp^2 + 4G\delta(P\cdot x)\log\left(\frac{\boldsymbol{x}_\perp^2}{4L^2}\right)(P\cdot\mathrm{d}x)^2$$

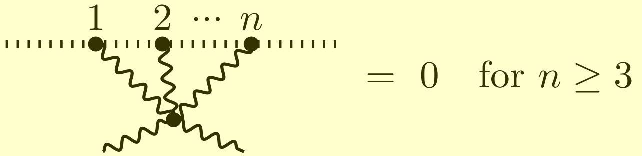
Aichelburg, Sexl, 1971

Two-point shockwave response

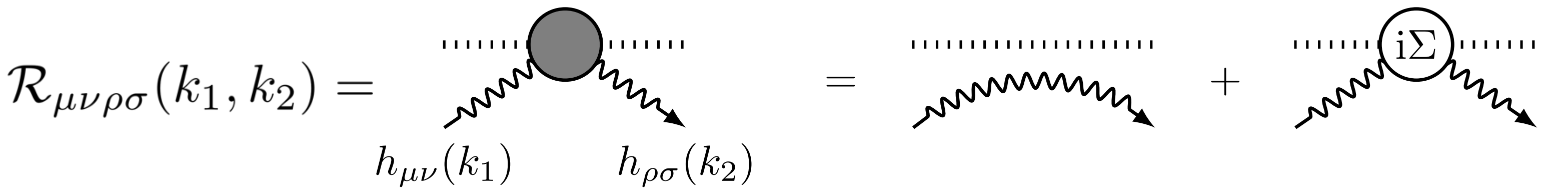


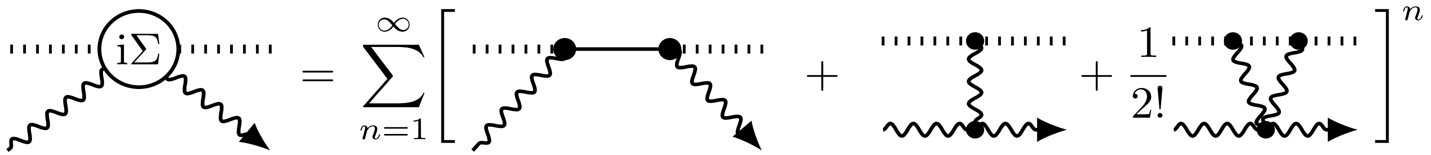


Shockwave self-energy vertices truncate:

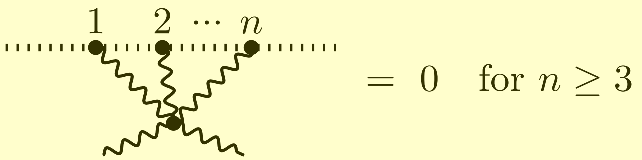


Two-point shockwave response



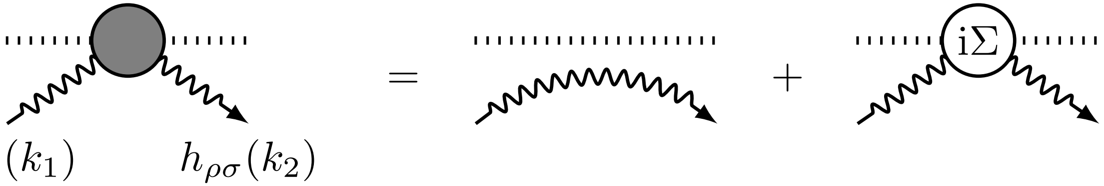


Shockwave self-energy vertices truncate:

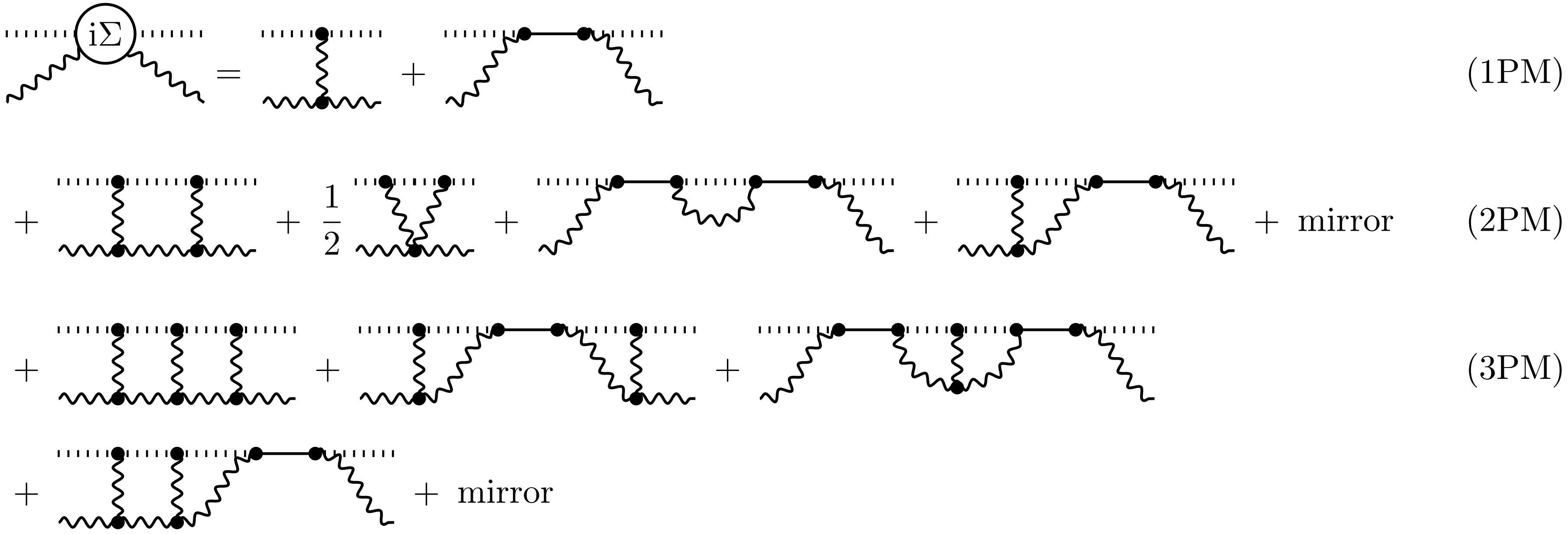


Two-point shockwave response

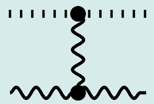
$\mathcal{R}_{\mu\nu\rho\sigma}(k_1, k_2) =$



- All-order expansion in PM

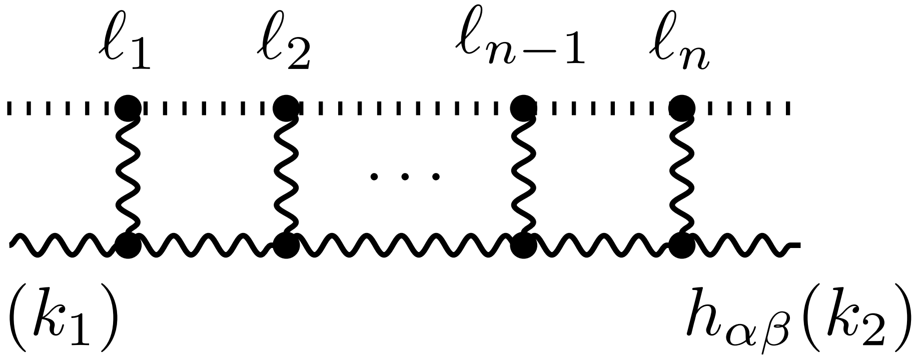


3PM exact remainder $\Sigma^{\mu\nu\alpha\beta}_{\text{rem}}(k_1, k_2)$

$+ \sum_{n=4}^{\infty}$ []^n ⟶ Resum to $\Sigma^{\mu\nu\alpha\beta}_{\text{sum}}(k_1, k_2)$ $(\geq 4\text{PM})$

Computing $\Sigma^{\mu\nu\alpha\beta}_{\text{sum}}$

$$\mathcal{L}^{\mu\nu\alpha\beta}_n =$$

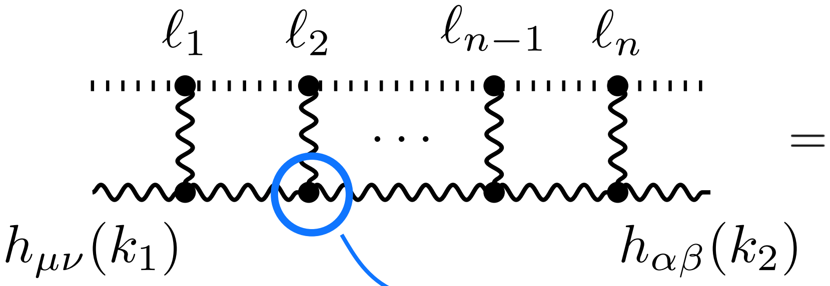


$$= \frac{\kappa_D^{2n}}{2^n}(P\cdot k_1)^{2n}\int_{\ell_1,\ldots,\ell_{n-1}} \Omega^{\mu\nu\alpha\beta}_n(k_1,k_2,P,\ell_1,\cdots,\ell_{n-1})\,\mathcal{I}_{n-1},$$

- Evaluate numerator
- Compute generic n - loop integral
- Resummation

Computing $\Sigma_{\text{sum}}^{\mu\nu\alpha\beta}$: Numerator

$$\mathcal{L}_n^{\mu\nu\alpha\beta} =$$



$$= \frac{\kappa_D^{2n}}{2^n}(P\cdot k_1)^{2n}\int_{\ell_1,\dots,\ell_{n-1}} \Omega_n^{\mu\nu\alpha\beta}(k_1,k_2,P,\ell_1,\cdots,\ell_{n-1})\,\mathcal{I}_{n-1},$$

231 terms

Simplification at 4PM and beyond!
(using de Donder gauge)

$$\Omega_{n\geq 4}^{\mu\nu\alpha\beta}(k_1,k_2,P) = -\mathrm{i}\,\Pi^{\mu\nu}{}_{\kappa\lambda}(k_1)\Pi^{\kappa\lambda\alpha\beta}(k_2)$$

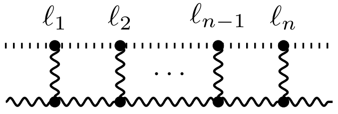
→ Independent of loop momentum

Only scalar integrals

With the *TT-projector* $\Pi^{\mu\nu\alpha\beta}(k_i) = \pi_i^{\mu(\rho}\pi_i^{\alpha)\beta} - \frac{1}{D-2}\pi_i^{\mu\nu}\pi_i^{\alpha\beta}$

and the *transverse metric* $\pi_i^{\mu\nu} = \eta^{\mu\nu} + \frac{k_i^2 P^\mu P^\nu - 2(P\cdot k_i)k_i^{(\mu}P^{\nu)}}{(P\cdot k_i)^2}$

Computing $\Sigma_{\text{sum}}^{\mu\nu\alpha\beta}$: Integrals



$$I_n = \prod_{j=1}^{n} \left[\int_{\ell_j} \frac{\delta(P\cdot \ell_j)}{\ell_j^2[(k_1+\sum_{i=1}^{j}\ell_i)^2 + \mathrm{i}0^+(P\cdot k_1)]} \frac{\Lambda^2}{{\ell_j^+}^2+\Lambda^2} \right] \frac{\delta(P\cdot q)}{(q-\sum_{i=1}^{n}\ell_i)^2} \frac{\Lambda^2}{(q^+ - \sum_{i=1}^{n}\ell_i^+)^2+\Lambda^2}, \qquad q = k_2 - k_1$$

- Perform light-cone decomposition and integrate + component via residues

$$m_i = k_1^+ - \frac{(\boldsymbol{k}_{1\perp} + \sum_{j=1}^{i}\boldsymbol{\ell}_{j\perp})^2}{k_1^-}$$

$$I_n^+ = \prod_{j=1}^{n} \left[\int_{\ell_j^+} \frac{1}{\sum_i^j \ell_i^+ + m_j + \mathrm{i}0^+} \frac{\Lambda^2}{{\ell_j^+}^2+\Lambda^2} \right] \frac{\Lambda^2}{(q^+ - \ell_{1\cdots n}^+)^2+\Lambda^2} \xrightarrow{\Lambda\to\infty} \frac{(-\mathrm{i})^n}{(n+1)!}$$

- Integrate spatial components in method of region where $k_1 \sim k_2 \sim \Lambda^0$ relevant for the on-shell limit

$$I_n = (-1)^{n+1} \frac{\delta(P\cdot q)}{(2P\cdot k_1)^n} \prod_{j=1}^{n} \left[\int_{\boldsymbol{\ell}_{j\perp}} \frac{1}{{\boldsymbol{\ell}_{j\perp}}^2} \right] \frac{1}{(\boldsymbol{q}_\perp - \boldsymbol{\ell}_{1\perp} - \cdots - \boldsymbol{\ell}_{n\perp})^2} I_n^+$$

(a) External scaling: $\boldsymbol{\ell}_{i\perp} \sim \Lambda^0$, (b) Hard scaling: $\boldsymbol{\ell}_{i\perp} \sim \Lambda^{1/2}$.

$$I_n^{\text{external}} = -\frac{\mathrm{i}^n}{(n+1)!} \frac{\delta(P\cdot q)}{(2P\cdot k_1)^n} \frac{1}{(\boldsymbol{q}_\perp^2)^{1+n\epsilon}} \frac{\Gamma(-\epsilon)^{n+1}\Gamma(n\epsilon+1)}{(4\pi)^{n(1-\epsilon)}\Gamma(-(n+1)\epsilon)}$$

Computing $\Sigma_{\rm sum}^{\mu\nu\alpha\beta}$: Resummation

Momentum space

$$\mathcal{L}_n^{\mu\nu\alpha\beta} = 2\kappa_D^{2n}\hat{\delta}(P\cdot q)\frac{\mathrm{i}^n}{2^{2n}n!}\frac{(P\cdot k_1)^{n+1}}{(4\pi)^{(n-1)(1-\epsilon)}(\boldsymbol{q}_\perp^2)^{1+(n-1)\epsilon}}(\Pi_1\cdot\Pi_2)^{\mu\nu\alpha\beta}\frac{\Gamma(-\epsilon)^n\Gamma((n-1)\epsilon+1)}{\Gamma(-n\epsilon)}$$

Position space

$$= \hat{\delta}(P\cdot q)\int \mathrm{d}^{D-2}\boldsymbol{x}_\perp\, \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp\cdot\boldsymbol{x}_\perp}\left[\Pi_1\cdot\Pi_2\right]^{\mu\nu\alpha\beta}\frac{2\mathrm{i}^n}{2^{2n}n!}(P\cdot k_1)^{n+1}\left[8G\Gamma(-\epsilon)\left(\frac{\boldsymbol{x}_\perp^2}{4\mathrm{e}^{\gamma_{\rm E}}L^2}\right)^{\epsilon}\right]^n$$

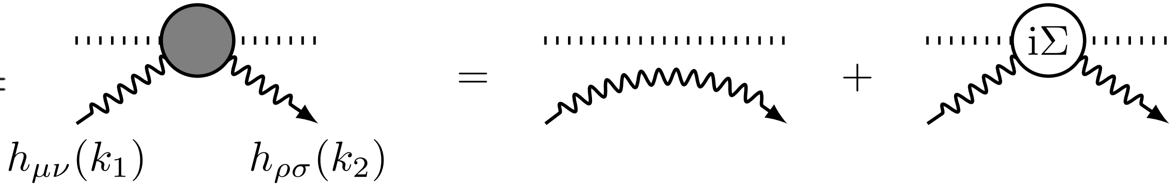
Resumming all ladders beyond 4PM:

$$\Sigma_{\rm sum}^{\mu\nu\alpha\beta}(k_1,k_2) = \sum_{n=4}^{\infty}\mathcal{L}_n^{\mu\nu\alpha\beta} \sim \text{exponential}$$

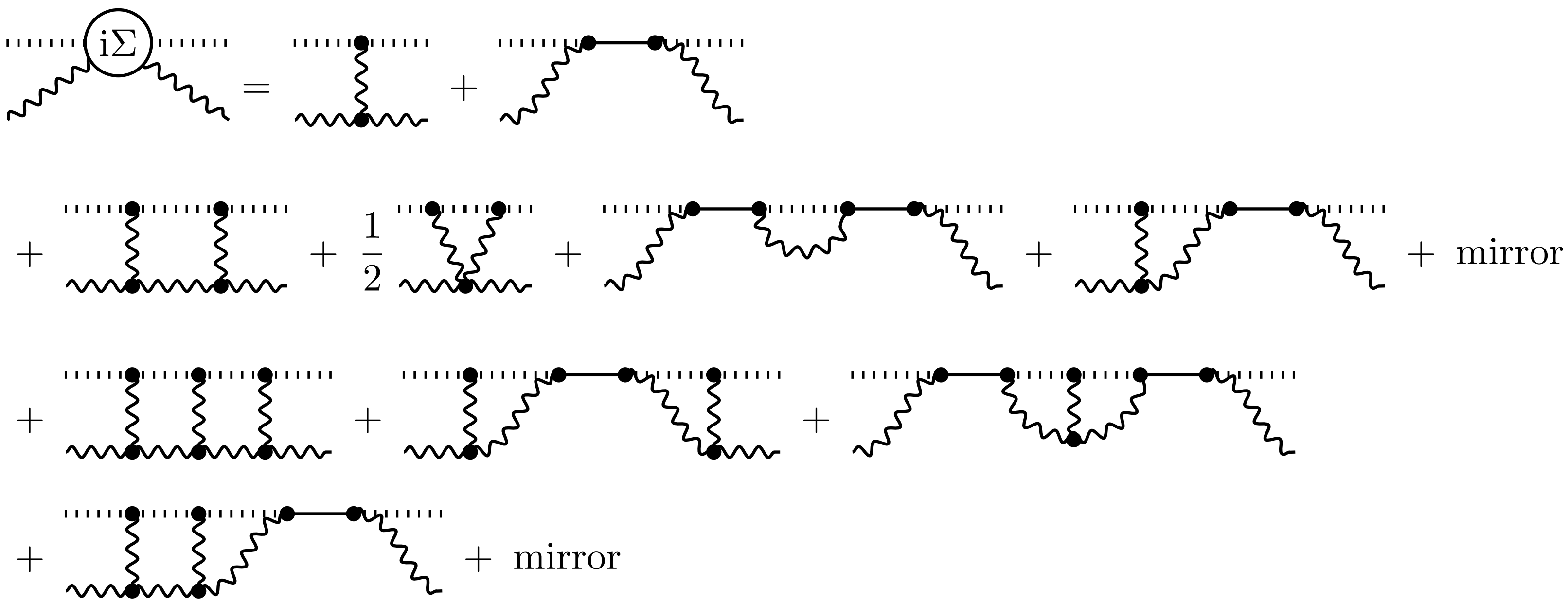
What do the low orders contribute?

Two-point shockwave response

$\mathcal{R}_{\mu\nu\rho\sigma}(k_1, k_2) =$



- All-order expansion in PM

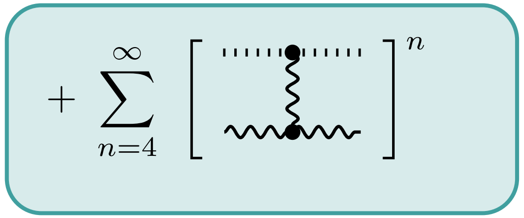


(1PM)

(2PM)

(3PM)

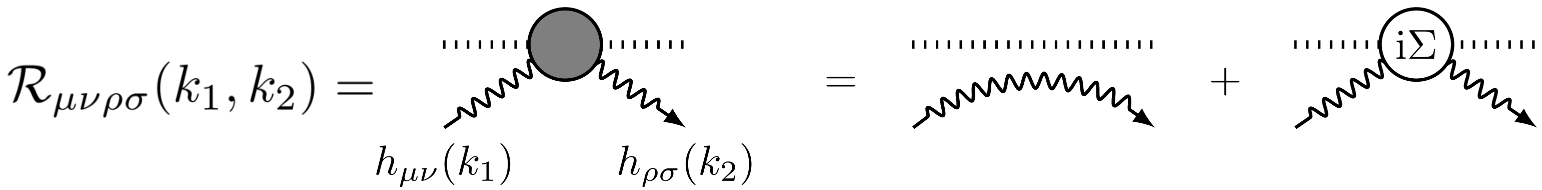
3PM exact remainder $\Sigma^{\mu\nu\alpha\beta}_{\text{rem}}(k_1, k_2)$



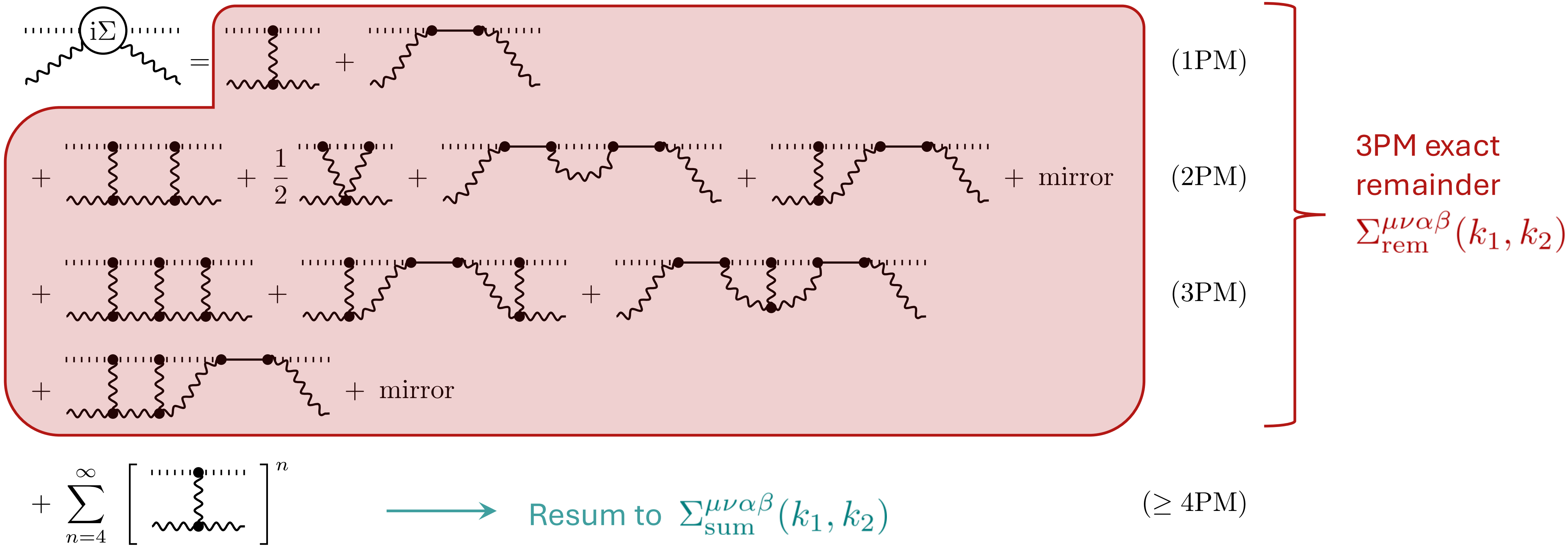
Resum to $\Sigma^{\mu\nu\alpha\beta}_{\text{sum}}(k_1, k_2)$

($\geq$ 4PM)

Two-point shockwave response

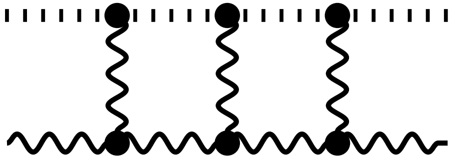


- All-order expansion in PM



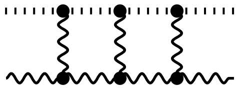
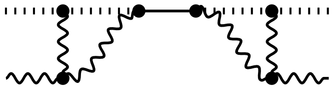
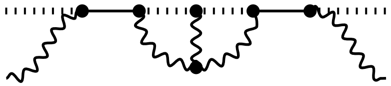
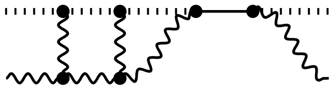
The remainder $\Sigma_{\text{rem}}^{\mu\nu\alpha\beta}$ at 3PM

- 3PM and below, ladders do **not** follow the pattern observed at higher orders …



$$\neq -\mathrm{i}\frac{\kappa_D^{2n}(P\cdot k_1)^{2n}}{2^n}(\Pi_1\cdot\Pi_2)^{\mu\nu\alpha\beta}\, I_{n-1}\Big|_{n=3}$$

- But the sum of all 3PM diagrams does!

 $+$  $+$  $+$ 

$$+\ \text{mirror} = -\mathrm{i}\frac{\kappa_D^{2n}(P\cdot k_1)^{2n}}{2^n}(\Pi_1\cdot\Pi_2)^{\mu\nu\alpha\beta}\, I_{n-1}\Big|_{n=3}$$

Same pattern as high-orders!

- 3PM remainder evalutes to 0!

$$\Sigma_{\text{rem}}^{\mu\nu\alpha\beta}\Big|_{3\text{PM}} = \Sigma^{\mu\nu\alpha\beta}\Big|_{3\text{PM}} - \left(-\mathrm{i}\frac{\kappa_D^{2n}(P\cdot k_1)^{2n}}{2^n}(\Pi_1\cdot\Pi_2)^{\mu\nu\alpha\beta}\, I_{n-1}\Big|_{n=3}\right) = 0\,.$$

Full 3PM response

The full remainder $\Sigma^{\mu\nu\alpha\beta}_{\text{rem}}$

- Similar structure at 1PM and 2PM, but with a remainder

$$\Sigma^{\mu\nu\alpha\beta}\Big|_{\text{1PM+2PM}} = \underbrace{\sum_{n=1}^{2}\left[-\mathrm{i}\frac{\kappa_D^{2n}(P\cdot k_1)^{2n}}{2^n}(\Pi_1\cdot\Pi_2)^{\mu\nu\alpha\beta}\, I_{n-1}\right]}_{\text{Follows high-PM pattern}} + \underbrace{\Sigma^{\mu\nu\alpha\beta}_{\text{rem}}}_{\text{Remainder}}$$

$$\Sigma^{\mu\nu\alpha\beta}_{\text{rem}}(k_1,k_2) = \frac{\delta(P\cdot q)}{4}\Bigg(\frac{\kappa_D^2}{q^2}\Bigg[P^\alpha P^\beta(2k_1^{(\mu}q^{\nu)} - \eta^{\mu\nu}) + P^{(\mu}q^{\nu)}P^{(\alpha}q^{\beta)} - (P\cdot k_1)^2\frac{\pi_1^{\mu\nu}\pi_2^{\alpha\beta}}{D-2}$$
$$- \frac{(k_1^2+k_2^2)P^\mu P^\nu}{2}\left(\frac{q^2P^\alpha P^\beta}{2(P\cdot k_1)^2} - \pi_1^{\alpha\beta} + \pi_2^{\alpha\beta}\right) + (P\cdot k_1)^2\frac{(\pi_1^{\mu\nu}+\pi_2^{\mu\nu})(\pi_1^{\alpha\beta}+\pi_2^{\alpha\beta})}{4}\Bigg]$$
$$+ \frac{\mathrm{i}\kappa_D^4}{64\pi}(k_1\cdot P)\pi_1^{\mu\nu}P^\alpha P^\beta\left[2\log\left(\frac{\Lambda k_1^-}{\boldsymbol{q}_\perp^2}\right) - 3\pi\mathrm{i} - 1\right] + (\mu\nu, k_1 \leftrightarrow \alpha\beta, -k_2)\Bigg) + \mathcal{O}(\epsilon,\Lambda^{-1})$$

- **Vanishing on-shell**

$$\varepsilon_{1\mu}^{(h_1)}\varepsilon_{1\nu}^{(h_1)}\Sigma^{\mu\nu\alpha\beta}_{\text{rem}}(k_1,k_2)\bar{\varepsilon}_{2\alpha}^{(h_2)}\bar{\varepsilon}_{2\beta}^{(h_2)}\Big|_{\text{on-shell}} = 0$$

Resumming ladder pattern for all PM orders

- Extend pattern observed beyond 4PM to all orders and sum up ...

$$\mathrm{i}\Sigma^{\mu\nu\alpha\beta}_{\mathrm{sum}}(k_1,k_2) \equiv \underbrace{\sum_{n=1}^{\infty}\left[-\mathrm{i}\frac{\kappa_D^{2n}(P\cdot k_1)^{2n}}{2^n}(\Pi_1\cdot\Pi_2)^{\mu\nu\alpha\beta}\, I_{n-1}\right]}_{\text{High-order ladder pattern}} = 2\hat{\delta}[P\cdot(k_1-k_2)]\,(P\cdot k_1)\left[\Pi_1\cdot\Pi_2\right]^{\mu\nu\alpha\beta}\mathcal{F}(q_\perp;\epsilon)\,,$$

- ... into an expanded form

$$\mathcal{F}(q_\perp;\epsilon) = \left(\frac{4\pi}{\boldsymbol{q}_\perp^2}\right)^{1-\epsilon}\sum_{n=1}^{\infty}\frac{(\mathrm{i}W)^n}{n!}\frac{\Gamma(-\epsilon)^n}{(\boldsymbol{q}_\perp^2\,\mathrm{e}^{\gamma_{\mathrm{E}}}L^2)^{n\epsilon}}\frac{\Gamma(1+(n-1)\epsilon)}{\Gamma(-n\epsilon)}\,,$$

Weinberg factor

$$W = 2G(P\cdot k_1)$$

- ... into a resummed form!

$$\mathcal{F}(q_\perp;\epsilon) = \int \mathrm{d}^{D-2}\boldsymbol{x}_\perp\,\mathrm{e}^{-\mathrm{i}\boldsymbol{q}_\perp\cdot\boldsymbol{x}_\perp}\left[\mathrm{e}^{\mathrm{i}W\Gamma(-\epsilon)\left(\frac{\boldsymbol{x}_\perp^2}{4\mathrm{e}^{\gamma_{\mathrm{E}}}L^2}\right)^{\epsilon}}-1\right]$$

Same structure as calculation in eikonal approximation
Raj, Venugopalan [2406.10483]

- ... with the 4-dimensional limit

$$\lim_{\epsilon\to 0}\mathrm{e}^{\mathrm{i}W/\epsilon}\mathcal{F}(q_\perp;\epsilon) = 4\pi\mathrm{i}W\frac{L^{2\mathrm{i}W}}{\boldsymbol{q}_\perp^{2(1-\mathrm{i}W)}}\frac{\Gamma(1-\mathrm{i}W)}{\Gamma(1+\mathrm{i}W)}$$

't Hooft, 1987

The on-shell limit and the Magnusian

Magnus, 1954
Damgaard, Planté, Vanhove [2107.12891]

- **On-shell** response function (Compton amplitude)

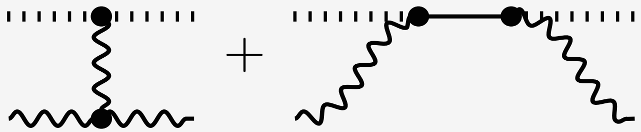
$$\langle k_2, h_2|\mathrm{i}\hat{T}|k_1, h_1\rangle = \mathrm{i}\varepsilon^{(h_1)}_{1\mu}\varepsilon^{(h_1)}_{1\nu}\Sigma^{\mu\nu\alpha\beta}\bar{\varepsilon}^{(h_2)}_{2\alpha}\bar{\varepsilon}^{(h_2)}_{2\beta}\Big|_{\text{on-shell}} = 2\delta\!\!\!{}^{-}[P\cdot(k_1-k_2)]\,\frac{(P\cdot F_1^{(h_1)}\cdot F_2^{(h_2)}\cdot P)^2}{(P\cdot k_1)^3}\mathcal{F}(q_\perp;\epsilon)\,,$$

with the linearised field-strength tensor $F_i^{(h_i)\mu\nu} = 2k_i^{[\mu}\varepsilon_i^{(h_i)\nu]}$

Double copy

- **Mangusian** is 1PM exact and ϵ - finite! $\hat{S} = \mathbb{1} + \mathrm{i}\hat{T} = e^{\mathrm{i}\hat{N}}$

$$\langle k_2, h_2|\mathrm{i}\hat{N}|k_1, h_1\rangle = \langle k_2, h_2|\mathrm{i}\hat{T}|k_1, h_1\rangle\Big|_{1\mathrm{PM}} =$$

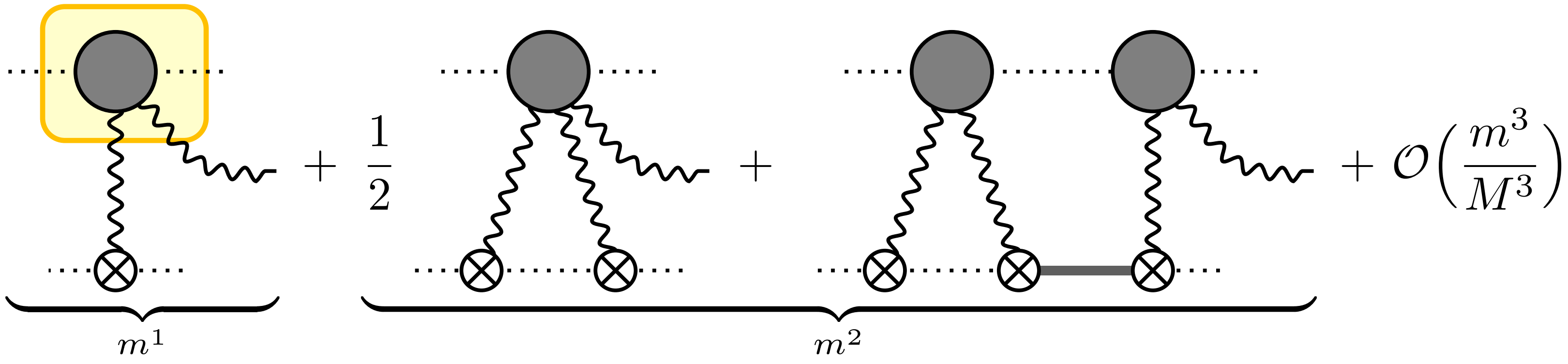


$$= \frac{16\pi \mathrm{i}G}{(4\pi e^{\gamma_E}L^2)^\epsilon}\delta\!\!\!{}^{-}[P\cdot(k_1-k_2)]\,\frac{(P\cdot F_1^{(h_1)}\cdot F_2^{(h_2)}\cdot P)^2}{(P\cdot k_1)}$$

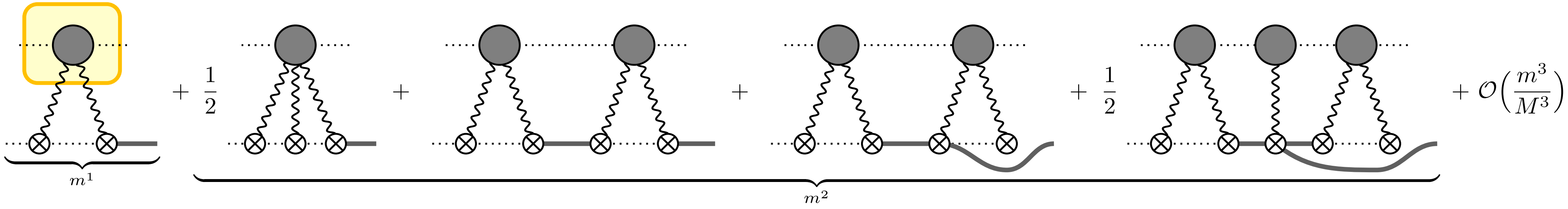
Now what?

- Observables!

$\langle h_{\mu\nu}(k) \rangle =$



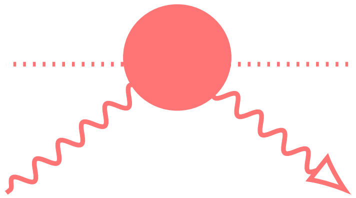
$\langle z^\sigma(\omega) \rangle =$



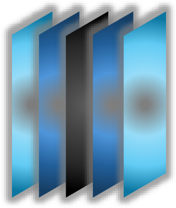
- We need to resum the vertices for the probe
  → **Come back tomorrow for Jitze Hoogeveen's talk**

Summary and Outlook

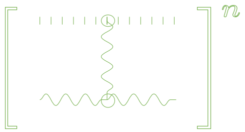
In this talk

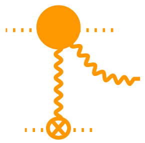


- Novel way of treating **SF expansion with WQFT**!
- Importance of **two-point response** function as propagator
- Proof of concept by applying the formalism to the **shockwave**



- Obtained the **first exact in G** two-point response function





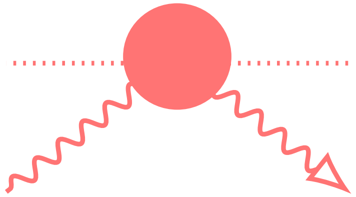
Outlook

- Compute **observables** for the shockwave metric

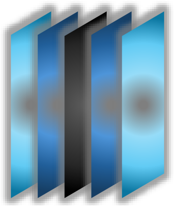
- Study the shockwave in **black-hole perturbation theory** for comparison
- Connection to **high-energy limit** of massive particle?

Summary and Outlook

In this talk



- Novel way of treating **SF expansion with WQFT**!
- Importance of **two-point response** function as propagator
- Proof of concept by applying the formalism to the **shockwave**



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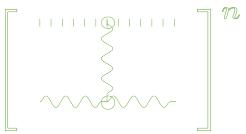

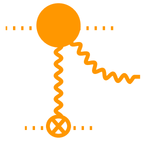

Outlook

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Thanks!

QCD MEETS GRAVITY BERLIN

2026

14–18 DEC

Local organising committee:
Alessandra Buonanno, Olaf Hohm, Gustav Uhre Jakobsen, Peter Marquard, Gustav Mogull, Jan Plefka, Jan Steinhoff, Johann Usovitsch

RTG 2575: Rethinking Quantum Field Theory

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