

Surfing the Shockwave with Black-Hole Response Theory

Based on [2604.xxxxx] with Carl Jordan Eriksen, Jitze Hoogeveen, Gustav Uhre Jakobsen and Jan Plefka

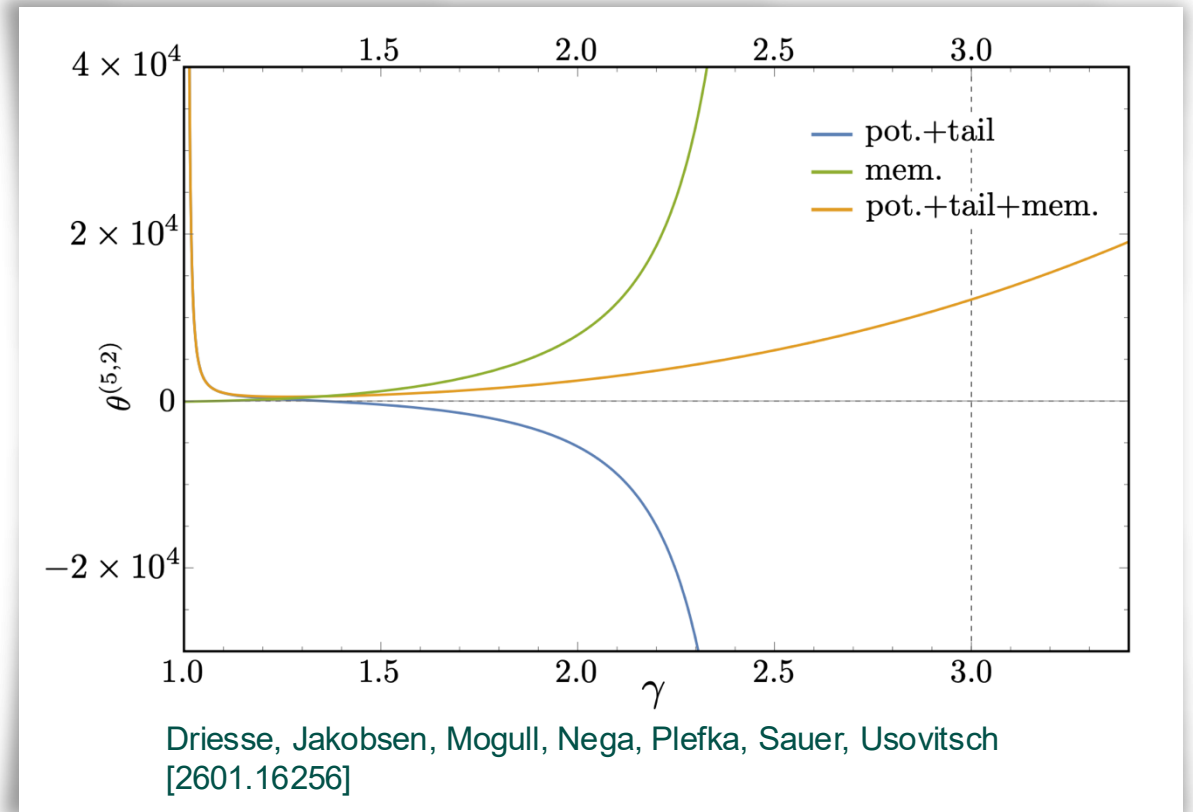
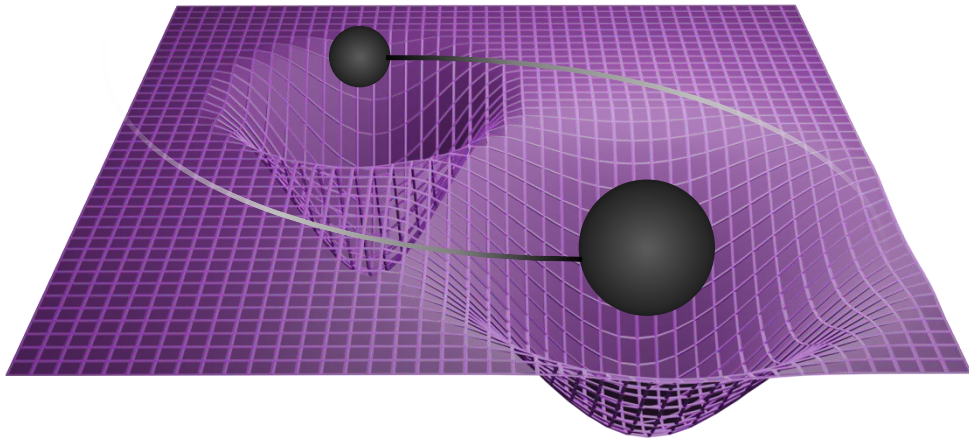
Amplitudes, Strong-Field Gravity and Resummation, Nordita

Lara Bohnenblust

16. April 2026



Motivation to use worldline quantum field theory (WQFT)

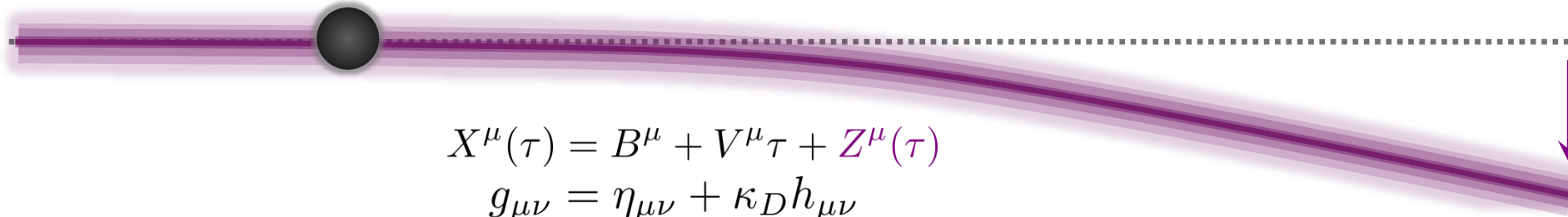


By quantizing worldlines:

- **Systematic expansion** in G in terms of Feynman diagrams
- Solve Feynman-like **integrals** with QFT tools
- **Tree-level** captures exactly classically relevant information

Recent 5PM 2SF computation of the conservative potential with WQFT

WQFT of a single black hole



Mogull, Plefka, Steinhoff [2010.02865]

$$X^\mu(\tau) = B^\mu + V^\mu \tau + Z^\mu(\tau)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa_D h_{\mu\nu}$$

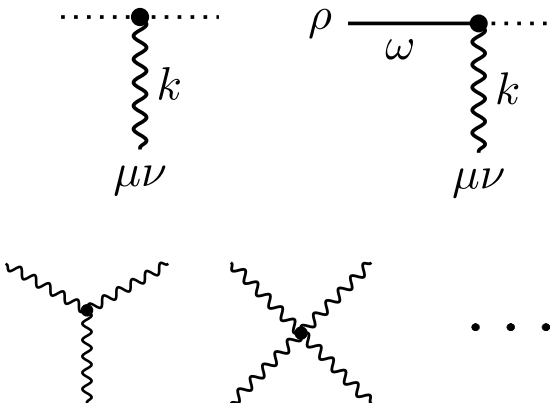
$$S_{\text{BH}}[X, g] = -\frac{M}{2} \int d\tau g_{\mu\nu}(X) \dot{X}^\mu \dot{X}^\nu - \int d^D x \frac{2}{\kappa_D^2} \sqrt{|g|} R$$

$$\kappa_D^2 = 32\pi G (4\pi e^{\gamma_E} L^2)^{-\epsilon}$$

Feynman rules for “in-in” prescription:

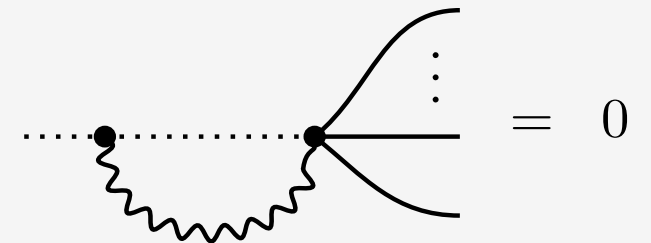
$$\begin{array}{c} \mu \qquad \nu \\ \bullet \xrightarrow{\omega} \bullet \\ \dots \end{array} = \frac{-i\eta^{\mu\nu}}{M(\omega + i0^+)^2}$$

$$\begin{array}{c} \mu\nu \qquad \rho\sigma \\ \bullet \xrightarrow{k} \bullet \\ \dots \end{array} = \frac{i\mathcal{P}_{\mu\nu\rho\sigma}}{k^2 + i0^+(W \cdot k)}$$



Dim.-regularise self-energy:

$$g_{\mu\nu}(X) = \eta_{\mu\nu}$$



Cheung, Parra-Martinez, Rothstein, Shah, Wilson-Gerow [2406.14770]

Black-hole response theory

- Partition Function for single black hole

$$Z[\mathcal{T}] = \int \mathcal{D}[h, Z] e^{\frac{i}{\hbar} (S_{\text{BH}}[X, g] - \int d^D x \mathcal{T}^{\mu\nu} h_{\mu\nu})} \Big|_{\text{tree}} = e^{\frac{i}{\hbar} W[\mathcal{T}]}$$

- **Black hole response effective action**

$$\begin{aligned} iW[\mathcal{T}] = & -i \int_{k_1} \mathcal{T}^{\alpha\beta}(k_1) \mathcal{R}_{\alpha\beta}(k_1) - \frac{1}{2} \int_{k_1, k_2} \mathcal{T}^{\alpha\beta}(k_1) \mathcal{T}^{\mu\nu}(k_2) \mathcal{R}_{\alpha\beta\mu\nu}(k_1, k_2) \\ & + \frac{i}{6} \int_{k_1, k_2, k_3} \mathcal{T}^{\alpha\beta}(k_1) \mathcal{T}^{\mu\nu}(k_2) \mathcal{T}^{\rho\sigma}(k_3) \mathcal{R}_{\alpha\beta\mu\nu\rho\sigma}(k_1, k_2, k_3) + \dots \end{aligned}$$

Black-hole response theory

- Partition Function for single black hole

$$Z[\mathcal{T}] = \int \mathcal{D}[h, Z] e^{\frac{i}{\hbar} (S_{\text{BH}}[X, g] - \int d^D x \mathcal{T}^{\mu\nu} h_{\mu\nu})} \Big|_{\text{tree}} = e^{\frac{i}{\hbar} W[\mathcal{T}]}$$

- **Black hole response effective action**

$$iW[\mathcal{T}] = - \int_{k_1} \dots \text{[Diagram: Grey circle connected to } \mathcal{T}_1 \text{]} + \frac{i}{2} \int_{k_1, k_2} \dots \text{[Diagram: Grey circle connected to } \mathcal{T}_1, \mathcal{T}_2 \text{]} + \frac{1}{6} \int_{k_1, k_2, k_3} \dots \text{[Diagram: Grey circle connected to } \mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3 \text{]} + \dots$$

- Coupling to a secondary black hole

$$\mathcal{T}^{\mu\nu}(x) = \frac{m}{2} \int d\tau \delta^D(x - x(\tau)) x^\mu(\tau) x^\nu(\tau)$$

- each interaction with the secondary gives an extra power of m
- n - point response function contributes to $(n - 1)$ self-force (SF) order observable

➤ See [Jitze Hoogeveen's talk](#)

Black-hole response theory

- Partition Function for single black hole

$$Z[\mathcal{T}] = \int \mathcal{D}[h, Z] e^{\frac{i}{\hbar} (S_{\text{BH}}[X, g] - \int d^D x \mathcal{T}^{\mu\nu} h_{\mu\nu})} \Big|_{\text{tree}} = e^{\frac{i}{\hbar} W[\mathcal{T}]}$$

- **Black hole response effective action**

$$iW[\mathcal{T}] = - \int_{k_1} \text{[Diagram 1]} + \frac{i}{2} \int_{k_1, k_2} \text{[Diagram 2]} + \frac{1}{6} \int_{k_1, k_2, k_3} \text{[Diagram 3]} + \dots$$

The diagrams show a grey circle representing a black hole inside a yellow rounded rectangle. Wavy lines connect the black hole to external sources represented by circles labeled $\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3$. Diagram 1 has one wavy line to \mathcal{T}_1 . Diagram 2 has two wavy lines to \mathcal{T}_1 and \mathcal{T}_2 . Diagram 3 has three wavy lines to $\mathcal{T}_1, \mathcal{T}_2$, and \mathcal{T}_3 .

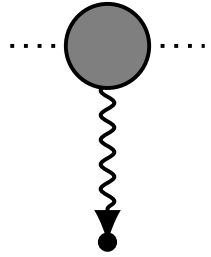
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One-point response function



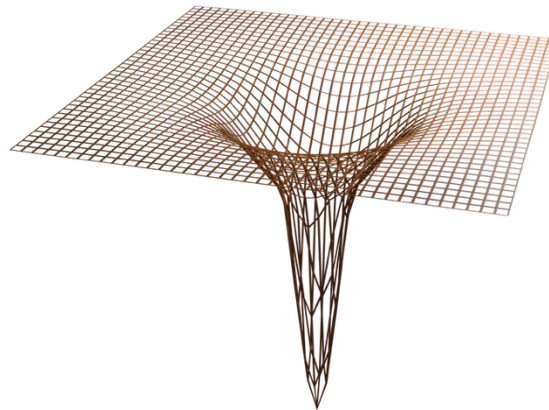
$$= -\frac{\delta W[\mathcal{T}]}{\delta \mathcal{T}^{\mu\nu}(x)} \Big|_{\mathcal{T}=0}$$

$$Z[\mathcal{T}] = \int \mathcal{D}[h, Z] e^{\frac{i}{\hbar} (S_{\text{BH}}[X, g] - \int d^D x \mathcal{T}^{\mu\nu} h_{\mu\nu})} \Big|_{\text{tree}} = e^{\frac{i}{\hbar} W[\mathcal{T}]}$$

Duff, 1974

$$= \text{[tree diagram]} + \frac{1}{2} \text{[1-loop diagram]} + \frac{1}{3!} \text{[2-loop diagram]} + \frac{1}{2} \text{[3-loop diagram]} + \dots =$$

$$= \langle h_{\mu\nu}(x) \rangle_{\mathcal{T}=0} =$$



Background metric

Two-point response function

- Generalized effective action to include a source for the deflection $Z^\mu(\tau)$

$$e^{\frac{i}{\hbar}W[\mathcal{T},f]} = Z[\mathcal{T},f] = \int \mathcal{D}[h, Z] e^{\frac{i}{\hbar}(S_{\text{BH}}[X,g] - \int d^Dx \mathcal{T}^{\mu\nu}(x)h_{\mu\nu}(x) - \int d\tau f_\mu(\tau)Z^\mu(\tau))} \Big|_{\text{tree}}$$

- De Witt notation

$$\langle \phi_A \rangle_J = \{ \langle h_{\mu\nu}(x) \rangle_{\{\mathcal{T},f\}}, \langle Z^\mu(\tau) \rangle_{\{\mathcal{T},f\}} \}, \quad J_A = \{ \mathcal{T}^{\mu\nu}(x), f_\mu(\tau) \},$$

$$\langle \phi_A \rangle_J J_A = \int d^Dx \mathcal{T}^{\mu\nu}(x) \langle h_{\mu\nu}(x) \rangle_{\{\mathcal{T},f\}} + \int d\tau f_\mu(\tau) \langle Z^\mu(\tau) \rangle_{\{\mathcal{T},f\}}$$

- Schwinger-Dyson like equation Response Function

$$\frac{\delta^2 S_{\text{BH}}[\langle \phi \rangle_J]}{\delta \langle \phi_A \rangle_J \delta \langle \phi_B \rangle_J} \frac{\delta^2 W[J]}{\delta J_B \delta J_C} \Big|_{J=0} = -\delta_{AC}$$

Feynman rules in presence of a source

Bautista, Driesse, Haddad, Jakobsen
[2602.06125]

Two-point response function

- Generalized effective action to include a source for the deflection $Z^\mu(\tau)$

$$e^{\frac{i}{\hbar}W[\mathcal{T},f]} = Z[\mathcal{T},f] = \int \mathcal{D}[h, Z] e^{\frac{i}{\hbar}(S_{\text{BH}}[X,g] - \int d^D x \mathcal{T}^{\mu\nu}(x)h_{\mu\nu}(x) - \int d\tau f_\mu(\tau)Z^\mu(\tau))} \Big|_{\text{tree}}$$

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$$\langle \phi_A \rangle_J J_A = \int d^D x \mathcal{T}^{\mu\nu}(x) \langle h_{\mu\nu}(x) \rangle_{\{\mathcal{T},f\}} + \int d\tau f_\mu(\tau) \langle Z^\mu(\tau) \rangle_{\{\mathcal{T},f\}}$$

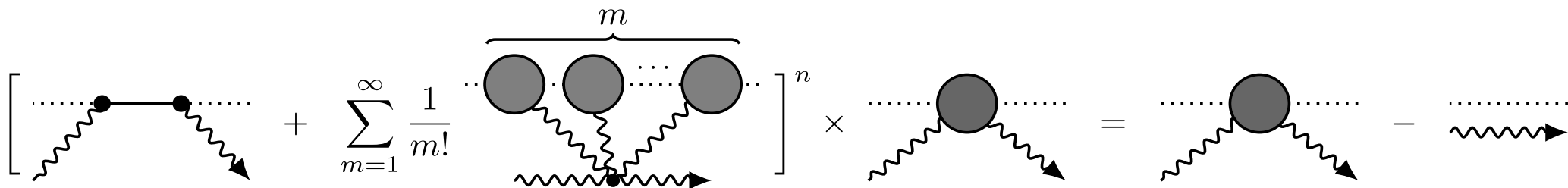
- Schwinger-Dyson like equation

Response Function

$$\frac{\delta^2 S_{\text{BH}}[\langle \phi \rangle_J]}{\delta \langle \phi_A \rangle_J \delta \langle \phi_B \rangle_J} \frac{\delta^2 W[J]}{\delta J_B \delta J_C} \Big|_{J=0} = -\delta_{AC}$$

Bautista, Driesse, Haddad, Jakobsen
[2602.06125]

Feynman rules in presence of a source



Two-point response function

- Generalized effective action to include a source for the deflection $Z^\mu(\tau)$

$$e^{\frac{i}{\hbar}W[\mathcal{T},f]} = Z[\mathcal{T},f] = \int \mathcal{D}[h,Z] e^{\frac{i}{\hbar}(S_{\text{BH}}[X,g] - \int d^Dx \mathcal{T}^{\mu\nu}(x)h_{\mu\nu}(x) - \int d\tau f_\mu(\tau)Z^\mu(\tau))} \Big|_{\text{tree}}$$

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$$\langle \phi_A \rangle_J = \{ \langle h_{\mu\nu}(x) \rangle_{\{\mathcal{T},f\}}, \langle Z^\mu(\tau) \rangle_{\{\mathcal{T},f\}} \}, \quad J_A = \{ \mathcal{T}^{\mu\nu}(x), f_\mu(\tau) \},$$

$$\langle \phi_A \rangle_J J_A = \int d^Dx \mathcal{T}^{\mu\nu}(x) \langle h_{\mu\nu}(x) \rangle_{\{\mathcal{T},f\}} + \int d\tau f_\mu(\tau) \langle Z^\mu(\tau) \rangle_{\{\mathcal{T},f\}}$$

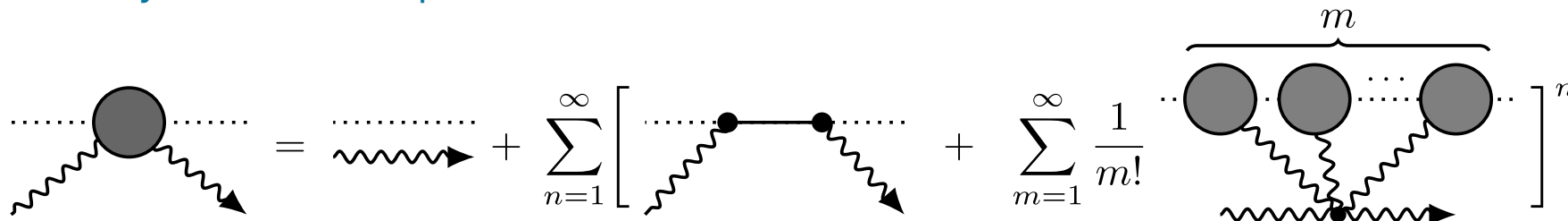
- Schwinger-Dyson like equation

Response Function

$$\frac{\delta^2 S_{\text{BH}}[\langle \phi \rangle_J]}{\delta \langle \phi_A \rangle_J \delta \langle \phi_B \rangle_J} \frac{\delta^2 W[J]}{\delta J_B \delta J_C} \Big|_{J=0} = -\delta_{AC}$$

Bautista, Driesse, Haddad, Jakobsen
[2602.06125]

Feynman rules in presence of a source



Three-point response function (and higher)

- Resumming Feynman rules in presence of a source

$$\frac{\delta^n iS_{\text{BH}}[\langle\phi\rangle_J]}{\delta\langle h_{\mu_1\nu_1}(x_1)\rangle_J \delta\langle h_{\mu_2\nu_2}(x_2)\rangle_J \cdots \delta\langle h_{\mu_n\nu_n}(x_n)\rangle_J} \Big|_{J=0} = \underbrace{\text{Diagram with } n \text{ external wavy lines and one internal circle}}_n = \sum_{m=0}^{\infty} \frac{1}{m!} \underbrace{\text{Diagram with } m \text{ internal circles and } n \text{ external wavy lines}}_m$$

- Apply functional derivative to Schwinger-Dyson equation

$$\frac{\delta}{\delta J_D} \left[\frac{\delta^2 S_{\text{BH}}[\langle\phi\rangle_J]}{\delta\langle\phi_A\rangle_J \delta\langle\phi_B\rangle_J} \frac{\delta^2 W[J]}{\delta J_B \delta J_C} \Big|_{J=0} \right] = 0$$

- Obtain recursion relation for higher-point response functions

$$\text{Diagram with 1 internal circle} = \text{Diagram with 2 internal circles} + \left[\text{Diagram with 3 internal circles and a line} + (\text{two permutations}) \right]$$

Three-point response function (and higher)

- Resumming Feynman rules in presence of a source

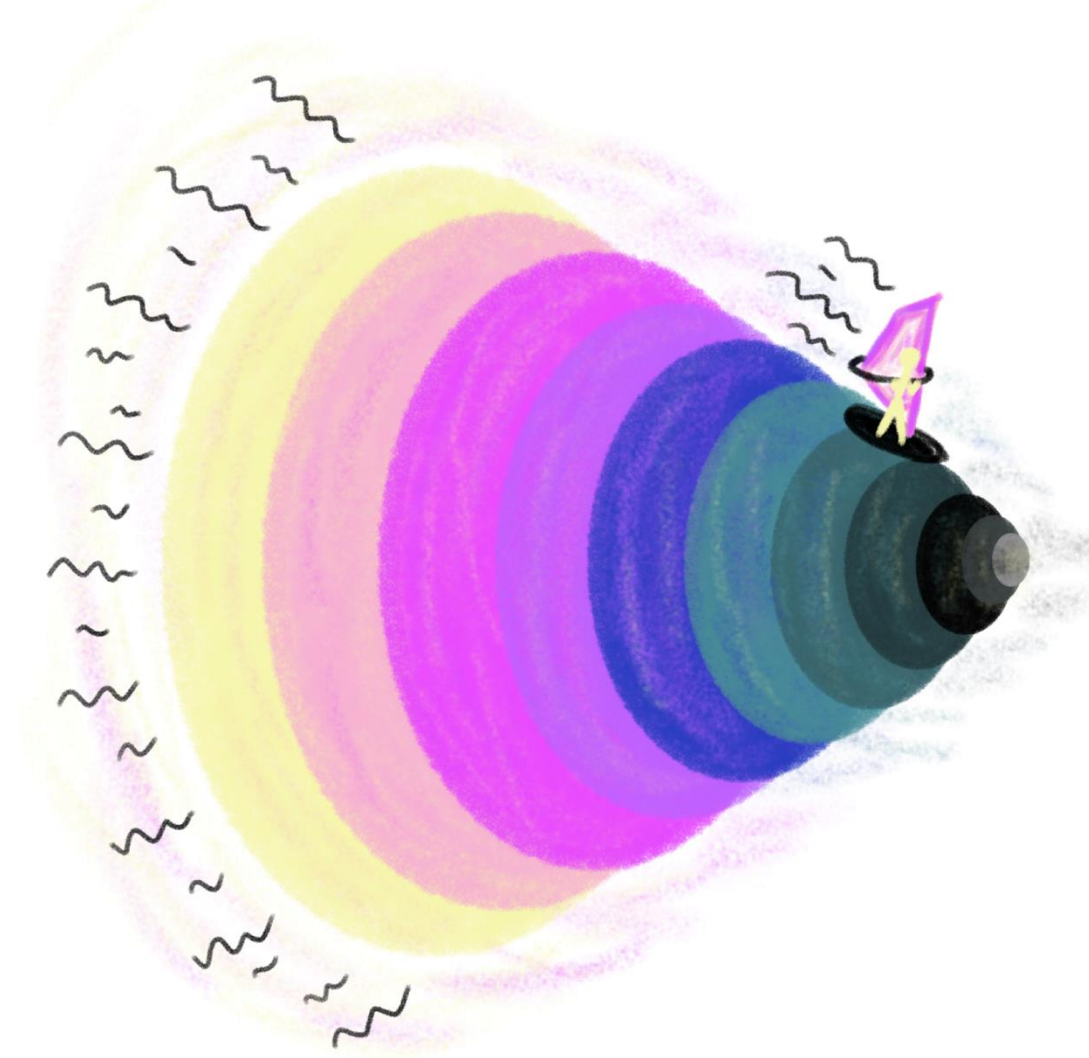
$$\frac{\delta^n i S_{\text{BH}}[\langle \phi \rangle_J]}{\delta \langle h_{\mu_1 \nu_1}(x_1) \rangle_J \delta \langle h_{\mu_2 \nu_2}(x_2) \rangle_J \cdots \delta \langle h_{\mu_n \nu_n}(x_n) \rangle_J} \Big|_{J=0} = \underbrace{\text{Diagram with } n \text{ external wavy lines and one internal circle}}_n = \sum_{m=0}^{\infty} \frac{1}{m!} \underbrace{\text{Diagram with } m \text{ internal circles and } n \text{ external wavy lines}}_m$$

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$$\frac{\delta}{\delta J_D} \left[\frac{\delta^2 S_{\text{BH}}[\langle \phi \rangle_J]}{\delta \langle \phi_A \rangle_J \delta \langle \phi_B \rangle_J} \frac{\delta^2 W[J]}{\delta J_B \delta J_C} \Big|_{J=0} \right] = 0$$

- Obtain recursion relation for higher-point response functions

$$\text{Diagram with one internal circle} = \text{Diagram with three internal circles} + \left[\text{Diagram with two internal circles and a propagator} + (\text{two permutations}) \right]$$



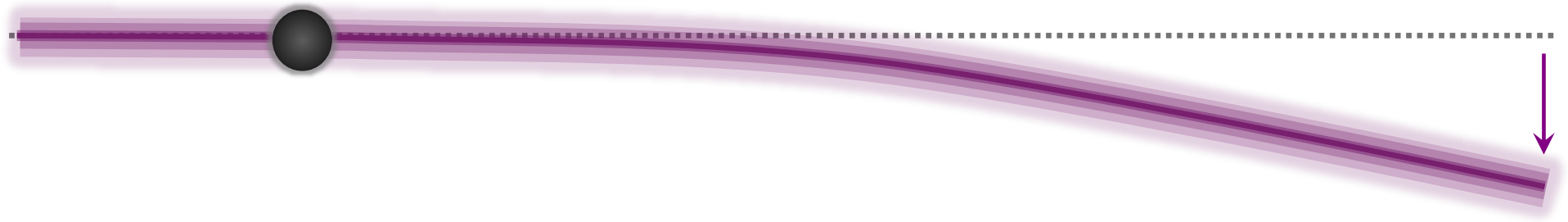
Application of black-hole response theory
to the **shockwave background**

Massless WQFT

Brink-Di Vecchia-Howe form (formerly known as Polyakov form) for massless limit

$$S = -\frac{1}{2} \int d\tau \left(e^{-1} g_{\mu\nu}(X) \dot{X}^\mu \dot{X}^\nu + e M^2 \right) \xrightarrow{M \rightarrow 0, e^{-1} \rightarrow E} -\frac{E}{2} \int d\tau g_{\mu\nu}(X) \dot{X}^\mu \dot{X}^\nu$$

Infinitely boosted black hole with energy E

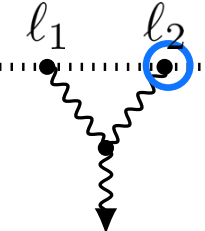


One-point response: The shockwave metric

$$\langle h_{\mu\nu}(x) \rangle_{1\text{PM}} = \int_k e^{-ik \cdot x} \text{---} \downarrow \text{---} = \frac{\kappa_D}{2} P_\mu P_\nu \int_k e^{-ik \cdot x} \frac{\delta(k \cdot P)}{k^2}$$

$h_{\mu\nu}(k)$

- 2PM contribution:



$$= \int_{l_1} \frac{\delta(P \cdot l_1) \delta(P \cdot l_2) P^{\mu_1} P^{\nu_1} P^{\mu_2} P^{\nu_2} \Omega_{\mu_1 \nu_1 \mu_2 \nu_2; \mu\nu}(l_1, l_2)}{l_1^2 l_2^2} \sim P^2, P \cdot l_1, P \cdot l_2 = 0$$

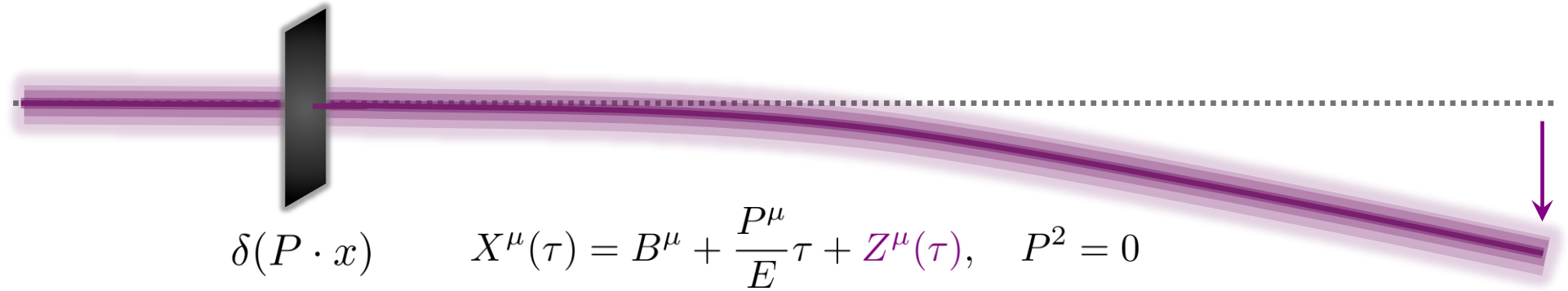
Light-cone decomposition

$$l^\mu = \frac{l^+ e^\mu + l^- \bar{e}^\mu}{2} + l^\mu_\perp$$

$$\sim \int d^D l_1 \frac{\delta(l_1^-)}{l_1^2} = \underbrace{\left(\int \frac{d l_1^+}{2} \right)}_{\text{Non-regularised divergence}} \int d^{D-2} l_{1\perp} \frac{1}{l_{1\perp}^2}$$

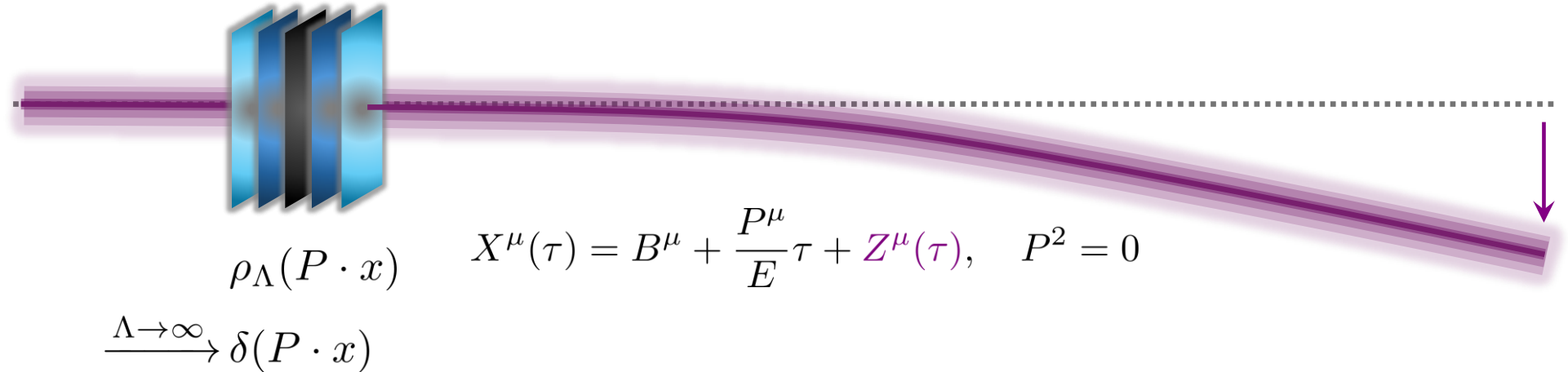
Non-regularised divergence

Regularizing massless WQFT

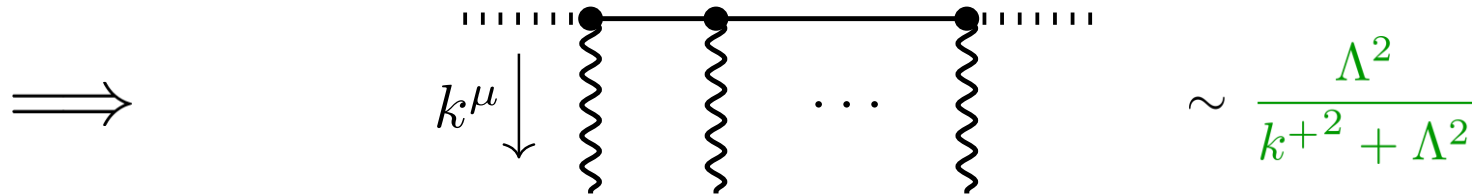


$$\delta(P \cdot x) \quad X^\mu(\tau) = B^\mu + \frac{P^\mu}{E}\tau + Z^\mu(\tau), \quad P^2 = 0$$

Regularizing massless WQFT



Choose $\rho_\Lambda(\sigma) = \frac{\Lambda}{2} \exp(-\Lambda|\sigma|)$ with Fourier transformation $\tilde{\rho}_\Lambda(\omega) = \frac{\Lambda^2}{\omega^2 + \Lambda^2}$



One-point response: The shockwave metric

$$\langle h_{\mu\nu}(x) \rangle_{1\text{PM}} = \int_k e^{-ik \cdot x} \text{ [Diagram: a wavy line with a downward arrow labeled } h_{\mu\nu}(k) \text{ and a dotted line above it]} = \frac{\kappa_D}{2} P_\mu P_\nu \int_k e^{-ik \cdot x} \frac{\delta(k \cdot P)}{k^2}$$

- 2PM contribution:

$$\text{[Diagram: a V-shaped wavy line with two incoming lines labeled } l_1 \text{ and } l_2 \text{ and a downward arrow labeled } h_{\mu\nu}(k) \text{ and a dotted line above it]} = \int_{l_1} \frac{\Lambda^4 \delta(P \cdot l_1) \delta(P \cdot l_2) P^{\mu_1} P^{\nu_1} P^{\mu_2} P^{\nu_2} \Omega_{\mu_1 \nu_1 \mu_2 \nu_2; \mu\nu}(l_1, l_2)}{(\ell_1^{+2} + \Lambda^2)(\ell_2^{+2} + \Lambda^2) \ell_1^2 \ell_2^2} \sim P^2, P \cdot l_1, P \cdot l_2 = 0$$

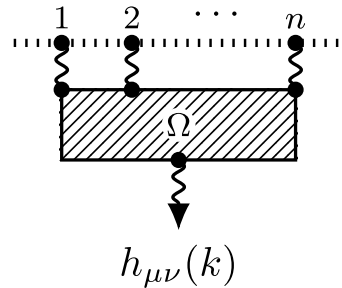
Light-cone decomposition

$$l^\mu = \frac{l^+ e^\mu + l^- \bar{e}^\mu}{2} + l_\perp^\mu$$

$$\sim \int d^D l_1 \frac{\delta(l_1^-)}{l_1^2} \frac{\Lambda^2}{l_1^{+2} + \Lambda^2} = \underbrace{\left(\int \frac{dl_1^+}{2} \frac{\Lambda^2}{l_1^{+2} + \Lambda^2} \right)}_{\text{Regularised!}} \int d^{D-2} l_{1\perp} \frac{1}{l_{1\perp}^2}$$

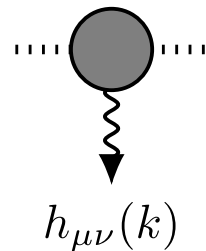
One-point response: The shockwave metric

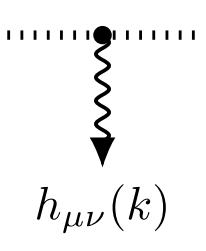
- Higher-point contributions vanish



$$= \int_{\ell_1, \dots, \ell_{n-1}} \left(\prod_{i=1}^n \frac{\delta(P \cdot \ell_i) P^{\mu_i} P^{\nu_i}}{\ell_i^2} \right) \Omega_{\mu_1 \nu_1 \dots \mu_n \nu_n; \mu \nu}(\ell_1, \dots, \ell_n) \sim P^2, P \cdot \ell_i = 0$$

- Truncation at 1PM:



$$\langle h_{\mu\nu}(x) \rangle = \int_k e^{-ik \cdot x} \dots = \int_k e^{-ik \cdot x} \dots = \frac{\kappa_D}{2} P_\mu P_\nu \int_k e^{-ik \cdot x} \frac{\delta(k \cdot P)}{k^2}$$


- 4D limit recovers **Aichelburg-Sexl shockwave background**

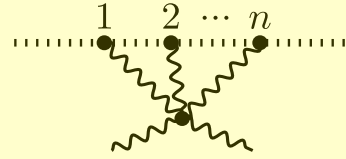
$$ds^2 = dx^- dx^+ - dx_\perp^2 + 4G \delta(P \cdot x) \log\left(\frac{x_\perp^2}{4L^2}\right) (P \cdot dx)^2$$

Aichelburg, Sexl, 1971

Two-point shockwave response

$$\mathcal{R}_{\mu\nu\rho\sigma}(k_1, k_2) = \begin{array}{c} \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \\ h_{\mu\nu}(k_1) \quad h_{\rho\sigma}(k_2) \end{array} = \begin{array}{c} \text{---} \\ \diagdown \\ \text{wavy} \\ h_{\mu\nu}(k_1) \end{array} + \begin{array}{c} \text{---} \circ \text{---} \\ \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \\ h_{\rho\sigma}(k_2) \end{array}$$

$$\begin{array}{c} \text{---} \circ \text{---} \\ \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \end{array} = \sum_{n=1}^{\infty} \left[\begin{array}{c} \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \end{array} + \sum_{m=1}^{\infty} \frac{1}{m!} \left[\begin{array}{c} \overbrace{\text{---} \bullet \text{---}}^m \\ \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \end{array} \right]^n \right]$$

Shockwave self-energy vertices truncate:

 $= 0 \text{ for } n \geq 3$

Two-point shockwave response

$$\mathcal{R}_{\mu\nu\rho\sigma}(k_1, k_2) = \begin{array}{c} \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \\ h_{\mu\nu}(k_1) \quad h_{\rho\sigma}(k_2) \end{array} = \begin{array}{c} \text{---} \\ \diagdown \\ \text{wavy} \\ \end{array} + \begin{array}{c} \text{---} \circ \text{---} \\ \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \\ \end{array}$$

$$\begin{array}{c} \text{---} \circ \text{---} \\ \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \\ \end{array} = \sum_{n=1}^{\infty} \left[\begin{array}{c} \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \\ \end{array} + \begin{array}{c} \text{---} \bullet \\ \text{wavy} \\ \bullet \text{---} \\ \end{array} + \frac{1}{2!} \begin{array}{c} \text{---} \bullet \bullet \text{---} \\ \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \\ \end{array} \right]^n$$

Shockwave self-energy vertices truncate:

$$\begin{array}{c} 1 \quad 2 \quad \dots \quad n \\ \text{---} \bullet \bullet \dots \bullet \text{---} \\ \diagdown \quad \diagup \quad \diagdown \quad \diagup \\ \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \\ \end{array} = 0 \quad \text{for } n \geq 3$$

Computing $\sum_{\text{sum}}^{\mu\nu\alpha\beta}$

$$\mathcal{L}_n^{\mu\nu\alpha\beta} = \begin{array}{c} \ell_1 \quad \ell_2 \quad \dots \quad \ell_{n-1} \quad \ell_n \\ \text{-----} \\ \text{wavy line} \\ \text{-----} \\ h_{\mu\nu}(k_1) \qquad \qquad \qquad h_{\alpha\beta}(k_2) \end{array}$$

$$= \frac{\kappa_D^{2n}}{2^n} (P \cdot k_1)^{2n} \int_{\ell_1, \dots, \ell_{n-1}} \Omega_n^{\mu\nu\alpha\beta}(k_1, k_2, P, \ell_1, \dots, \ell_{n-1}) \mathcal{I}_{n-1},$$

- Evaluate numerator
- Compute generic n - loop integral
- Resummation

Computing $\sum_{\text{sum}}^{\mu\nu\alpha\beta}$: Numerator

$$\mathcal{L}_n^{\mu\nu\alpha\beta} = \text{Diagram} = \frac{\kappa_D^{2n}}{2^n} (P \cdot k_1)^{2n} \int_{\ell_1, \dots, \ell_{n-1}} \Omega_n^{\mu\nu\alpha\beta}(k_1, k_2, P, \ell_1, \dots, \ell_{n-1}) \mathcal{I}_{n-1},$$

The diagram shows a loop with external momenta $l_1, l_2, \dots, l_{n-1}, l_n$. The left side is a wavy line labeled $h_{\mu\nu}(k_1)$ and the right side is a wavy line labeled $h_{\alpha\beta}(k_2)$. A blue circle highlights the loop, with an arrow pointing to the text "231 terms".

Simplification at 4PM and beyond!
(using de Donder gauge)

$$\Omega_{n \geq 4}^{\mu\nu\alpha\beta}(k_1, k_2, P) = -i \Pi^{\mu\nu}_{\kappa\lambda}(k_1) \Pi^{\kappa\lambda\alpha\beta}(k_2)$$

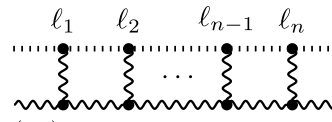
→ Independent of loop momentum

Only scalar integrals

With the TT -projector $\Pi^{\mu\nu\alpha\beta}(k_i) = \pi_i^{\mu(\rho} \pi_i^{\alpha)\beta} - \frac{1}{D-2} \pi_i^{\mu\nu} \pi_i^{\alpha\beta}$

and the *transverse metric* $\pi_i^{\mu\nu} = \eta^{\mu\nu} + \frac{k_i^2 P^\mu P^\nu - 2(P \cdot k_i) k_i^{(\mu} P^{\nu)}}{(P \cdot k_i)^2}$

Computing $\sum_{\text{sum}}^{\mu\nu\alpha\beta}$: Integrals



$$I_n = \prod_{j=1}^n \left[\int_{\ell_j} \frac{\delta(P \cdot \ell_j)}{\ell_j^2 [(k_1 + \sum_{i=1}^j \ell_i)^2 + i0^+(P \cdot k_1)]} \frac{\Lambda^2}{\ell_j^{+2} + \Lambda^2} \right] \frac{\delta(P \cdot q)}{(q - \sum_{i=1}^n \ell_i)^2} \frac{\Lambda^2}{(q^+ - \sum_{i=1}^n \ell_i^+)^2 + \Lambda^2}, \quad q = k_2 - k_1$$

- Perform light-cone decomposition and integrate + component via residues

$$I_n^+ = \prod_{j=1}^n \left[\int_{\ell_j^+} \frac{1}{\sum_i^j \ell_i^+ + m_j + i0^+} \frac{\Lambda^2}{\ell_j^{+2} + \Lambda^2} \right] \frac{\Lambda^2}{(q^+ - \ell_{1\dots n}^+)^2 + \Lambda^2} \xrightarrow{\Lambda \rightarrow \infty} \frac{(-i)^n}{(n+1)!} \quad m_i = k_1^+ - \frac{(\mathbf{k}_{1\perp} + \sum_{j=1}^i \boldsymbol{\ell}_{j\perp})^2}{k_1^-}$$

- Integrate spatial components in method of region where $k_1 \sim k_2 \sim \Lambda^0$ relevant for the on-shell limit

$$I_n = (-1)^{n+1} \frac{\delta(P \cdot q)}{(2P \cdot k_1)^n} \prod_{j=1}^n \left[\int_{\ell_{j\perp}} \frac{1}{\ell_{j\perp}^2} \right] \frac{1}{(\mathbf{q}_\perp - \boldsymbol{\ell}_{1\perp} - \dots - \boldsymbol{\ell}_{n\perp})^2} I_n^+$$

- (a) External scaling: $\ell_{i\perp} \sim \Lambda^0$, (b) Hard scaling: $\ell_{i\perp} \sim \Lambda^{1/2}$.

$$I_n^{\text{external}} = -\frac{i^n}{(n+1)!} \frac{\delta(P \cdot q)}{(2P \cdot k_1)^n} \frac{1}{(\mathbf{q}_\perp^2)^{1+n\epsilon}} \frac{\Gamma(-\epsilon)^{n+1} \Gamma(n\epsilon + 1)}{(4\pi)^{n(1-\epsilon)} \Gamma(-(n+1)\epsilon)}$$

Computing $\sum_{\text{sum}}^{\mu\nu\alpha\beta}$: Resummation

Momentum space

$$\mathcal{L}_n^{\mu\nu\alpha\beta} = 2\kappa_D^{2n} \delta(P \cdot q) \frac{i^n}{2^{2n} n!} \frac{(P \cdot k_1)^{n+1}}{(4\pi)^{(n-1)(1-\epsilon)} (\mathbf{q}_\perp^2)^{1+(n-1)\epsilon}} (\Pi_1 \cdot \Pi_2)^{\mu\nu\alpha\beta} \frac{\Gamma(-\epsilon)^n \Gamma((n-1)\epsilon + 1)}{\Gamma(-n\epsilon)}$$

Position space

$$= \delta(P \cdot q) \int d^{D-2} \mathbf{x}_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{x}_\perp} [\Pi_1 \cdot \Pi_2]^{\mu\nu\alpha\beta} \frac{2i^n}{2^{2n} n!} (P \cdot k_1)^{n+1} \left[8G\Gamma(-\epsilon) \left(\frac{\mathbf{x}_\perp^2}{4e^{\gamma_E} L^2} \right)^\epsilon \right]^n$$

Resumming all ladders beyond 4PM:

$$\sum_{\text{sum}}^{\mu\nu\alpha\beta} (k_1, k_2) = \underbrace{\sum_{n=4}^{\infty} \mathcal{L}_n^{\mu\nu\alpha\beta}}_{\text{What do the low orders contribute?}} \sim \text{exponential}$$

What do the low orders contribute?

Two-point shockwave response

$$\mathcal{R}_{\mu\nu\rho\sigma}(k_1, k_2) = \begin{array}{c} \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \\ h_{\mu\nu}(k_1) \quad h_{\rho\sigma}(k_2) \end{array} = \begin{array}{c} \text{---} \\ \diagdown \\ \text{wavy} \\ \end{array} + \begin{array}{c} \text{---} \circ \text{---} \\ \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \\ \end{array}$$

- All-order expansion in PM

$$\begin{array}{c} \text{---} \circ \text{---} \\ \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \end{array} = \begin{array}{c} \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \end{array} + \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \end{array}$$

(1PM)

$$+ \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \end{array} + \frac{1}{2} \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \end{array} + \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \end{array} + \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \end{array} + \text{mirror}$$

(2PM)

$$+ \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \end{array} + \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \end{array} + \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \end{array}$$

(3PM)

$$+ \begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \quad \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \quad \text{wavy} \end{array} + \text{mirror}$$

3PM exact remainder

$$\Sigma_{\text{rem}}^{\mu\nu\alpha\beta}(k_1, k_2)$$

$$+ \sum_{n=4}^{\infty} \left[\begin{array}{c} \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \\ \text{wavy} \quad \text{wavy} \end{array} \right]^n$$

Resum to $\Sigma_{\text{sum}}^{\mu\nu\alpha\beta}(k_1, k_2)$

($\geq 4\text{PM}$)

Two-point shockwave response

$$\mathcal{R}_{\mu\nu\rho\sigma}(k_1, k_2) = \begin{array}{c} \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \\ h_{\mu\nu}(k_1) \quad h_{\rho\sigma}(k_2) \end{array} = \begin{array}{c} \text{---} \text{---} \\ \diagdown \quad \diagup \end{array} + \begin{array}{c} \text{---} \text{---} \\ \diagup \quad \diagdown \\ \text{i}\Sigma \end{array}$$

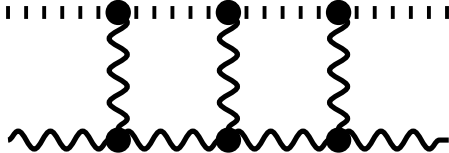
- All-order expansion in PM

$$\begin{array}{l}
 \text{---} \text{---} \\
 \diagup \quad \diagdown \\
 \text{i}\Sigma
 \end{array}
 = \begin{array}{l}
 \text{---} \bullet \text{---} \\
 \diagup \quad \diagdown \\
 \text{---} \quad \text{---}
 \end{array}
 + \begin{array}{l}
 \text{---} \bullet \text{---} \\
 \diagup \quad \diagdown \\
 \text{---} \quad \text{---}
 \end{array}
 + \text{mirror}
 \quad (1\text{PM}) \\
 + \begin{array}{l}
 \text{---} \bullet \text{---} \\
 \diagup \quad \diagdown \\
 \text{---} \quad \text{---}
 \end{array}
 + \frac{1}{2} \begin{array}{l}
 \text{---} \bullet \text{---} \\
 \diagup \quad \diagdown \\
 \text{---} \quad \text{---}
 \end{array}
 + \begin{array}{l}
 \text{---} \bullet \text{---} \\
 \diagup \quad \diagdown \\
 \text{---} \quad \text{---}
 \end{array}
 + \begin{array}{l}
 \text{---} \bullet \text{---} \\
 \diagup \quad \diagdown \\
 \text{---} \quad \text{---}
 \end{array}
 + \text{mirror}
 \quad (2\text{PM}) \\
 + \begin{array}{l}
 \text{---} \bullet \text{---} \\
 \diagup \quad \diagdown \\
 \text{---} \quad \text{---}
 \end{array}
 + \begin{array}{l}
 \text{---} \bullet \text{---} \\
 \diagup \quad \diagdown \\
 \text{---} \quad \text{---}
 \end{array}
 + \begin{array}{l}
 \text{---} \bullet \text{---} \\
 \diagup \quad \diagdown \\
 \text{---} \quad \text{---}
 \end{array}
 + \text{mirror}
 \quad (3\text{PM}) \\
 + \sum_{n=4}^{\infty} \left[\begin{array}{c} \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \\ \text{---} \quad \text{---} \end{array} \right]^n \quad (\geq 4\text{PM})
 \end{array}
 \longrightarrow \text{Resum to } \Sigma_{\text{sum}}^{\mu\nu\alpha\beta}(k_1, k_2)$$

3PM exact remainder $\Sigma_{\text{rem}}^{\mu\nu\alpha\beta}(k_1, k_2)$

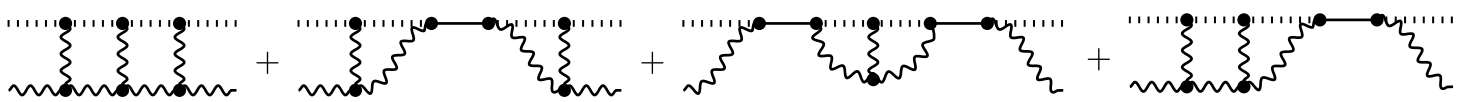
The remainder $\Sigma_{\text{rem}}^{\mu\nu\alpha\beta}$ at 3PM

- 3PM and below, ladders do **not** follow the pattern observed at higher orders ...



$$\neq -i \frac{\kappa_D^{2n} (P \cdot k_1)^{2n}}{2^n} (\Pi_1 \cdot \Pi_2)^{\mu\nu\alpha\beta} I_{n-1} \Big|_{n=3}$$

- But the sum of all 3PM diagrams does!



$$+ \text{mirror} = -i \frac{\kappa_D^{2n} (P \cdot k_1)^{2n}}{2^n} (\Pi_1 \cdot \Pi_2)^{\mu\nu\alpha\beta} I_{n-1} \Big|_{n=3}$$

Same pattern as high-orders!

- 3PM remainder evaluates to 0!

$$\Sigma_{\text{rem}}^{\mu\nu\alpha\beta} \Big|_{3\text{PM}} = \underbrace{\Sigma^{\mu\nu\alpha\beta} \Big|_{3\text{PM}}}_{\text{Full 3PM response}} - \left(-i \frac{\kappa_D^{2n} (P \cdot k_1)^{2n}}{2^n} (\Pi_1 \cdot \Pi_2)^{\mu\nu\alpha\beta} I_{n-1} \Big|_{n=3} \right) = 0.$$

The full remainder $\Sigma_{\text{rem}}^{\mu\nu\alpha\beta}$

- Similar structure at 1PM and 2PM, but with a remainder

$$\Sigma^{\mu\nu\alpha\beta} \Big|_{1\text{PM}+2\text{PM}} = \sum_{n=1}^2 \left[-i \frac{\kappa_D^{2n} (P \cdot k_1)^{2n}}{2^n} (\Pi_1 \cdot \Pi_2)^{\mu\nu\alpha\beta} I_{n-1} \right] + \Sigma_{\text{rem}}^{\mu\nu\alpha\beta}$$

Follows high-PM pattern
Remainder

$$\begin{aligned} \Sigma_{\text{rem}}^{\mu\nu\alpha\beta}(k_1, k_2) = & \frac{\delta(P \cdot q)}{4} \left(\frac{\kappa_D^2}{q^2} \left[P^\alpha P^\beta (2k_1^{(\mu} q^{\nu)} - \eta^{\mu\nu}) + P^{(\mu} q^{\nu)} P^{(\alpha} q^{\beta)} - (P \cdot k_1)^2 \frac{\pi_1^{\mu\nu} \pi_2^{\alpha\beta}}{D-2} \right. \right. \\ & \left. \left. - \frac{(k_1^2 + k_2^2) P^\mu P^\nu}{2} \left(\frac{q^2 P^\alpha P^\beta}{2(P \cdot k_1)^2} - \pi_1^{\alpha\beta} + \pi_2^{\alpha\beta} \right) + (P \cdot k_1)^2 \frac{(\pi_1^{\mu\nu} + \pi_2^{\mu\nu})(\pi_1^{\alpha\beta} + \pi_2^{\alpha\beta})}{4} \right] \right. \\ & \left. + \frac{i\kappa_D^4}{64\pi} (k_1 \cdot P) \pi_1^{\mu\nu} P^\alpha P^\beta \left[2 \log \left(\frac{\Lambda k_1^-}{q_\perp^2} \right) - 3\pi i - 1 \right] + (\mu\nu, k_1 \leftrightarrow \alpha\beta, -k_2) \right) + \mathcal{O}(\epsilon, \Lambda^{-1}) \end{aligned}$$

- **Vanishing on-shell**

$$\epsilon_{1\mu}^{(h_1)} \epsilon_{1\nu}^{(h_1)} \Sigma_{\text{rem}}^{\mu\nu\alpha\beta}(k_1, k_2) \bar{\epsilon}_{2\alpha}^{(h_2)} \bar{\epsilon}_{2\beta}^{(h_2)} \Big|_{\text{on-shell}} = 0$$

Resumming ladder pattern for all PM orders

- Extend pattern observed beyond 4PM to all orders and sum up ...

$$i\Sigma_{\text{sum}}^{\mu\nu\alpha\beta}(k_1, k_2) \equiv \sum_{n=1}^{\infty} \underbrace{\left[-i \frac{\kappa_D^{2n} (P \cdot k_1)^{2n}}{2^n} (\Pi_1 \cdot \Pi_2)^{\mu\nu\alpha\beta} I_{n-1} \right]}_{\text{High-order ladder pattern}} = 2\delta[P \cdot (k_1 - k_2)] (P \cdot k_1) [\Pi_1 \cdot \Pi_2]^{\mu\nu\alpha\beta} \mathcal{F}(q_{\perp}; \epsilon),$$

- ... into an expanded form

$$\mathcal{F}(q_{\perp}; \epsilon) = \left(\frac{4\pi}{\mathbf{q}_{\perp}^2} \right)^{1-\epsilon} \sum_{n=1}^{\infty} \frac{(iW)^n}{n!} \frac{\Gamma(-\epsilon)^n}{(\mathbf{q}_{\perp}^2 e^{\gamma_E} L^2)^{n\epsilon}} \frac{\Gamma(1 + (n-1)\epsilon)}{\Gamma(-n\epsilon)}, \quad \begin{array}{l} \text{Weinberg factor} \\ W = 2G(P \cdot k_1) \end{array}$$

- ... into a resummed form!

$$\mathcal{F}(q_{\perp}; \epsilon) = \int d^{D-2} \mathbf{x}_{\perp} e^{-i\mathbf{q}_{\perp} \cdot \mathbf{x}_{\perp}} \left[e^{iW\Gamma(-\epsilon) \left(\frac{\mathbf{x}_{\perp}^2}{4e^{\gamma_E} L^2} \right)^{\epsilon}} - 1 \right]$$

Same structure as calculation in eikonal approximation
Raj, Venugopalan [2406.10483]

- ... with the 4-dimensional limit

$$\lim_{\epsilon \rightarrow 0} e^{iW/\epsilon} \mathcal{F}(q_{\perp}; \epsilon) = 4\pi iW \frac{L^{2iW}}{\mathbf{q}_{\perp}^{2(1-iW)}} \frac{\Gamma(1 - iW)}{\Gamma(1 + iW)}$$

't Hooft, 1987

- **On-shell** response function (Compton amplitude)

$$\langle k_2, h_2 | i\hat{T} | k_1, h_1 \rangle = i\epsilon_{1\mu}^{(h_1)} \epsilon_{1\nu}^{(h_1)} \Sigma^{\mu\nu\alpha\beta} \bar{\epsilon}_{2\alpha}^{(h_2)} \bar{\epsilon}_{2\beta}^{(h_2)} \Big|_{\text{on-shell}} = 2\delta[P \cdot (k_1 - k_2)] \frac{(P \cdot F_1^{(h_1)} \cdot F_2^{(h_2)} \cdot P)^2}{(P \cdot k_1)^3} \mathcal{F}(q_\perp; \epsilon),$$

with the linearised field-strength tensor $F_i^{(h_i)\mu\nu} = 2k_i^{[\mu} \epsilon_i^{(h_i)\nu]}$

Double copy

- **Mangusian** is 1PM exact and ϵ - finite! $\hat{S} = \mathbb{1} + i\hat{T} = e^{i\hat{N}}$

$$\begin{aligned} \langle k_2, h_2 | i\hat{N} | k_1, h_1 \rangle &= \langle k_2, h_2 | i\hat{T} | k_1, h_1 \rangle \Big|_{1\text{PM}} = \text{[Diagram 1]} + \text{[Diagram 2]} \\ &= \frac{16\pi i G}{(4\pi e^{\gamma_E} L^2)^\epsilon} \delta[P \cdot (k_1 - k_2)] \frac{(P \cdot F_1^{(h_1)} \cdot F_2^{(h_2)} \cdot P)^2}{(P \cdot k_1)} \end{aligned}$$

Now what?

- Observables!

$$\langle h_{\mu\nu}(k) \rangle = \underbrace{\text{[Diagram: 1 vertex, 1 wavy line]}_{m^1} + \frac{1}{2} \underbrace{\text{[Diagram: 1 vertex, 2 wavy lines]} + \text{[Diagram: 2 vertices, 2 wavy lines]}_{m^2} + \mathcal{O}\left(\frac{m^3}{M^3}\right)$$

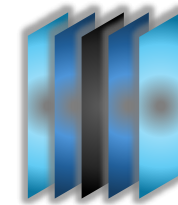
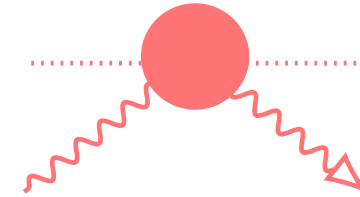
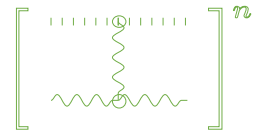
$$\langle z^\sigma(\omega) \rangle = \underbrace{\text{[Diagram: 1 vertex, 2 wavy lines]}_{m^1} + \frac{1}{2} \underbrace{\text{[Diagram: 1 vertex, 3 wavy lines]} + \text{[Diagram: 2 vertices, 3 wavy lines]} + \text{[Diagram: 2 vertices, 4 wavy lines]}_{m^2} + \frac{1}{2} \underbrace{\text{[Diagram: 3 vertices, 3 wavy lines]} + \text{[Diagram: 3 vertices, 4 wavy lines]}_{m^2} + \mathcal{O}\left(\frac{m^3}{M^3}\right)$$

- We need to resum the vertices for the probe
 → **Come back tomorrow for Jitze Hoogeveen's talk**

Summary and Outlook

In this talk

- ▶ Novel way of treating **SF expansion with WQFT!**
- ▶ Importance of **two-point response** function as propagator
- ▶ Proof of concept by applying the formalism to the **shockwave**
- ▶ Obtained the **first exact in G** two-point response function



Outlook

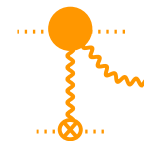
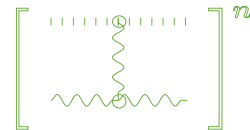
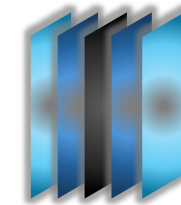
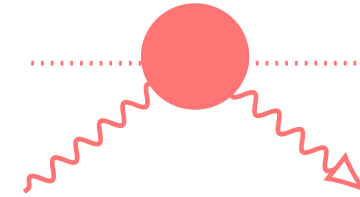
- ▶ Compute **observables** for the shockwave metric
- ▶ Study the shockwave in **black-hole perturbation theory** for comparison
- ▶ Connection to **high-energy limit** of massive particle?



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Thanks!

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Alessandra Buonanno, Olaf Hohm, Gustav Uhré
Jakobsen, Peter Marquard, Gustav Mogull, Jan Plefka,
Jan Steinhoff, Johann Usovitsch



RTG 2575

Rethinking
Quantum Field Theory



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