RTG 2575:
Rethinking
Quantum Field Theory

Self-Force From Response Theory

Based on [Bohnenblust, Eriksen, JH, Jakobsen, Plefka; 2604.xxxxx]
[JH, Jakobsen, Plefka; 2506.14626]
[Lara's talk on Thursday], [Carl's poster]

Jitze Hoogeveen, Humboldt Universität zu Berlin

Nordita, 17/4 2026

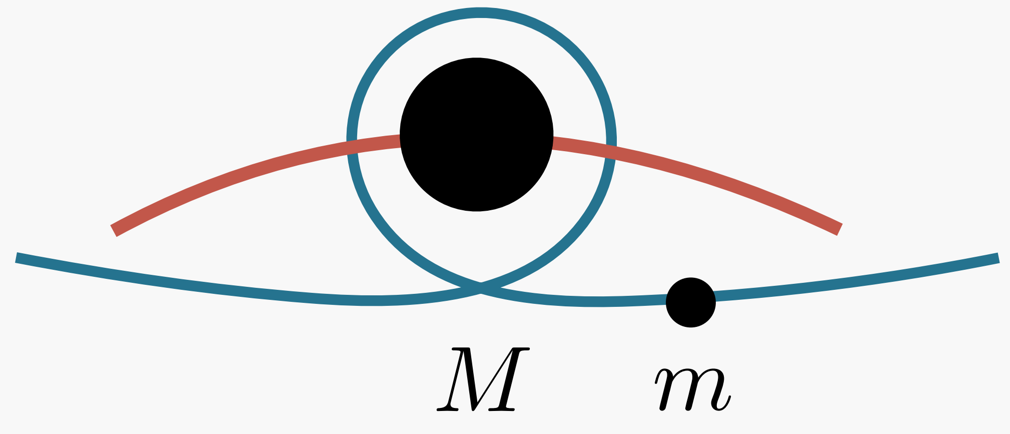
Outline

- Motivation
- Response theory

- SF dynamics
- Shockwaves
- 0SF - geodesics on shockwave
- Towards 1SF

Effective field theory

$m \ll M$



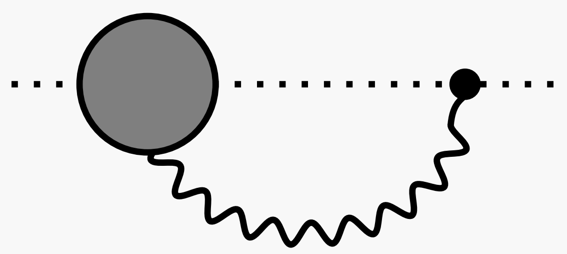
Self-Force from Effective Field Theory

[many people in this room!]

- Effective action

  $$S[Z, z, h] \to S_{\text{eff}}[z]$$

- Metric perturbation vanishes at particle (dimensional regularisation)

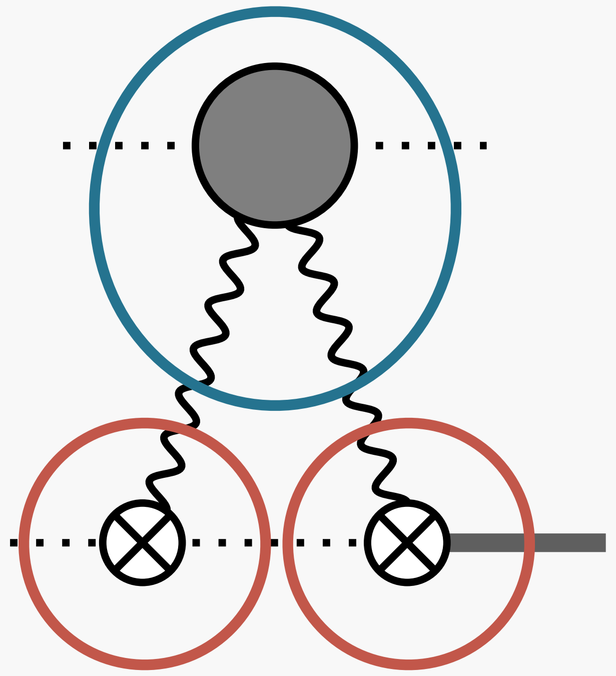
  $\langle h_{\mu\nu}(X(\tau))\rangle =$  $= 0$ [Cheung, Parra-Martinez, Rothstein, Shah, Wilson-Gerow; 240614770]

- QFT integration techniques

Efficient organisation of expansion!

Response Theory

2-pt response

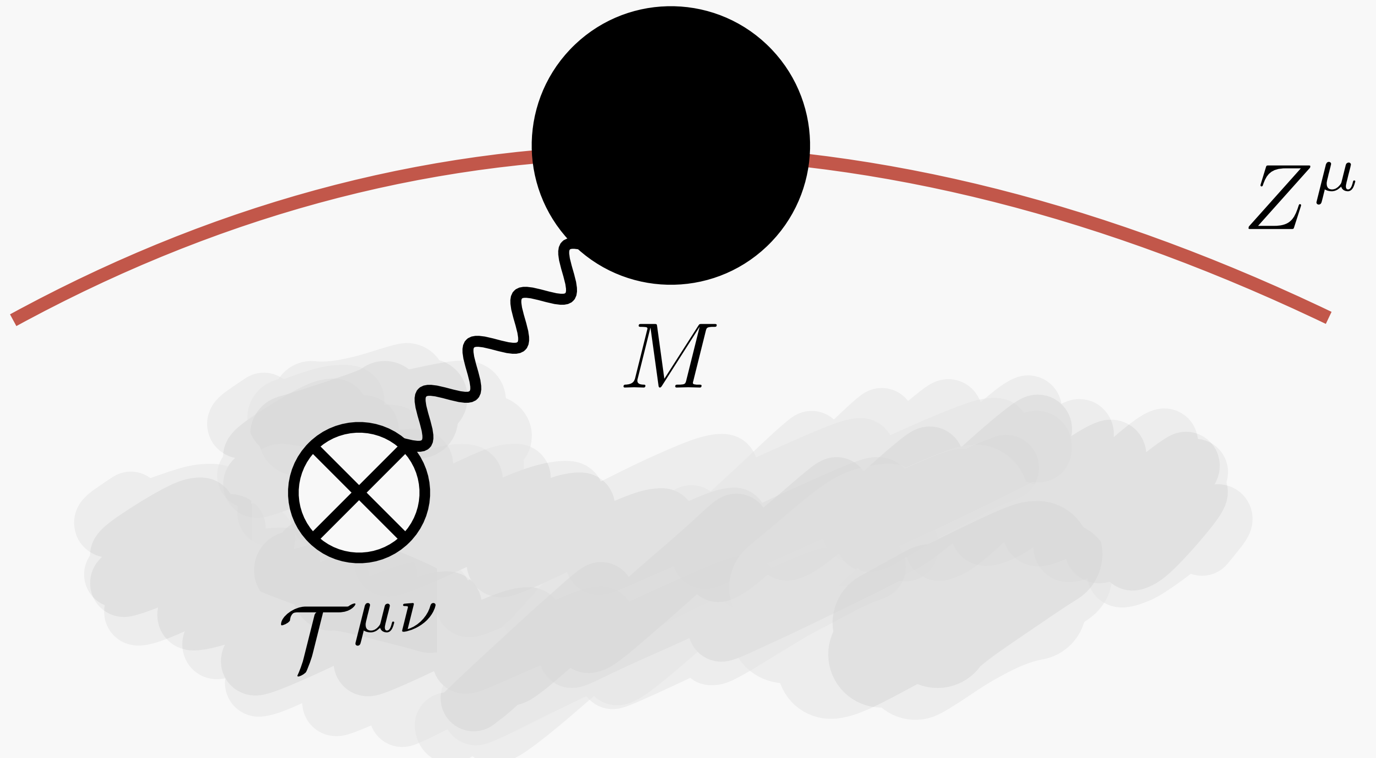
$$\langle z^\rho(\omega)\rangle \;=\;$$  $$+\;\mathcal{O}\Big(\frac{m^2}{M^2}\Big)$$

Where does this come from?

Geodesics

Brief Reminder

Response theory in a nutshell



Black hole coupling to source $\mathcal{T}^{\mu\nu}$

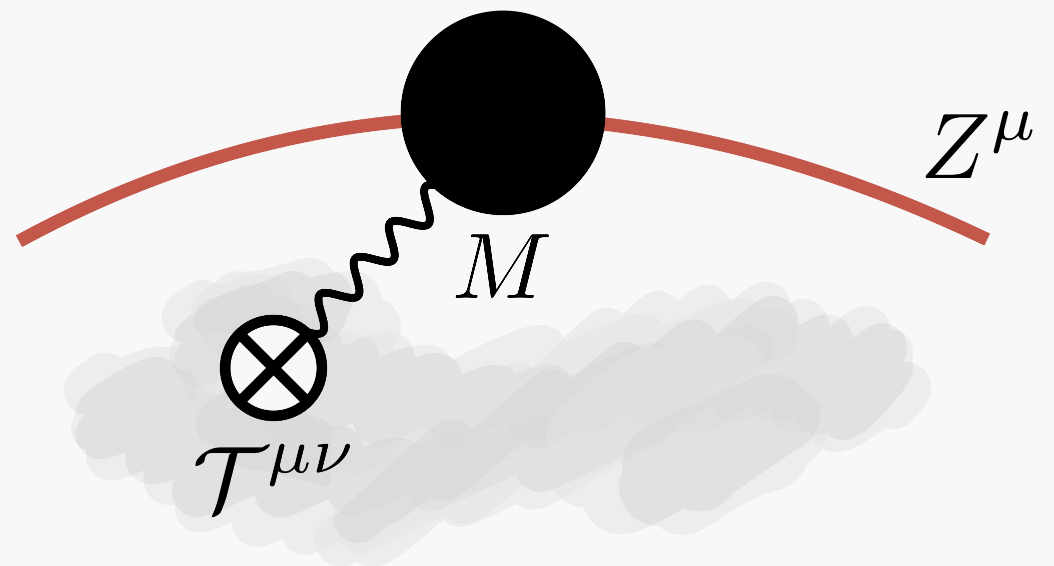
$$S_{\mathrm{BH}}[X, g, \mathcal{T}] = S_{\mathrm{EH}}[g] - S_{\mathrm{M}}[X, g] - \int d^D x \, h_{\mu\nu}(x) \underline{\mathcal{T}^{\mu\nu}(x)}$$

$$\mathcal{T}^{\mu\nu}(x) = \int d\tau \, \frac{m}{2} \dot{y}^\mu(\tau) \dot{y}^\nu(\tau) \delta^D(x - y(\tau))$$

(secondary black hole)

Effective action for environment $W[\mathcal{T}]$

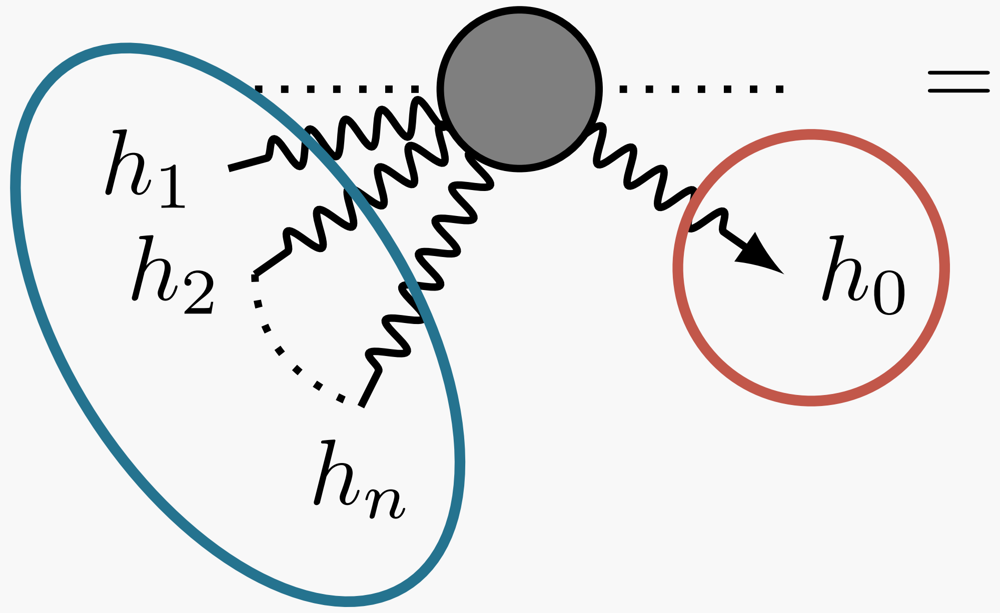
$$Z[\mathcal{T}] = \int \mathcal{D}[h, Z] \exp\Big[\frac{i}{\hbar}\Big(S_{BH}[X, g] - \int d^D x \, h_{\mu\nu}(x) \mathcal{T}^{\mu\nu}(x)\Big)\Big]\Big|_{\mathrm{tree}}$$
$$= \exp\Big[\frac{i}{\hbar} W[\mathcal{T}]\Big]$$



Response functions $\mathcal{R}$

$$iW[\mathcal{T}] = \int_{k_1} \mathcal{T}^{\alpha\beta}(k_1)\mathcal{R}_{\alpha\beta}(k_1) + \frac{1}{2}\int_{k_1,k_2} \mathcal{T}^{\alpha\beta}(k_1)\mathcal{T}^{\mu\nu}(k_2)\mathcal{R}_{\alpha\beta\mu\nu}(k_1,k_2) + \frac{1}{3!}\int_{k_1,k_2,k_3} \mathcal{T}^{\alpha\beta}(k_1)\mathcal{T}^{\mu\nu}(k_2)\mathcal{T}^{\rho\sigma}(k_3)\mathcal{R}_{\alpha\beta\mu\nu\rho\sigma}(k_1,k_2,k_3) + \dots$$

$$\mathcal{R}_{\alpha_0\beta_0\alpha_1\beta_1\dots\alpha_n\beta_n}(k_0,k_1\dots k_n) \equiv \frac{\delta}{\delta\mathcal{T}_0^{(-)}}\frac{\delta}{\delta\mathcal{T}_1^{(+)}}\frac{\delta}{\delta\mathcal{T}_2^{(+)}}\frac{\delta}{\delta\mathcal{T}_n^{(+)}} iW[\mathcal{T}]\Big|_{\mathcal{T}=0}$$

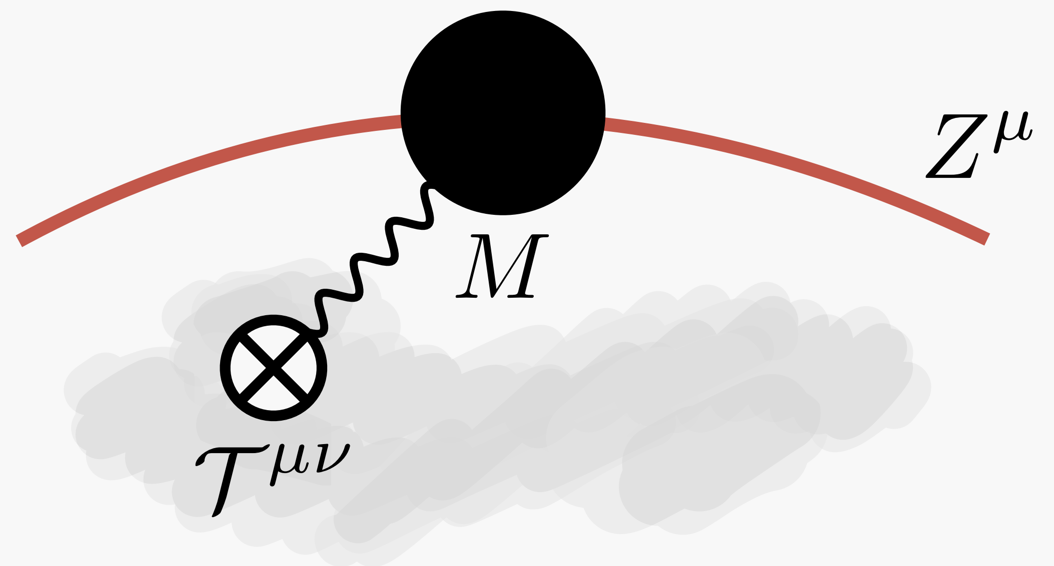
$=$ 

$= (-i)^n\langle h_0 h_1 \dots h_n\rangle|_{\text{con.}}$ correlation functions

n incoming gravitons

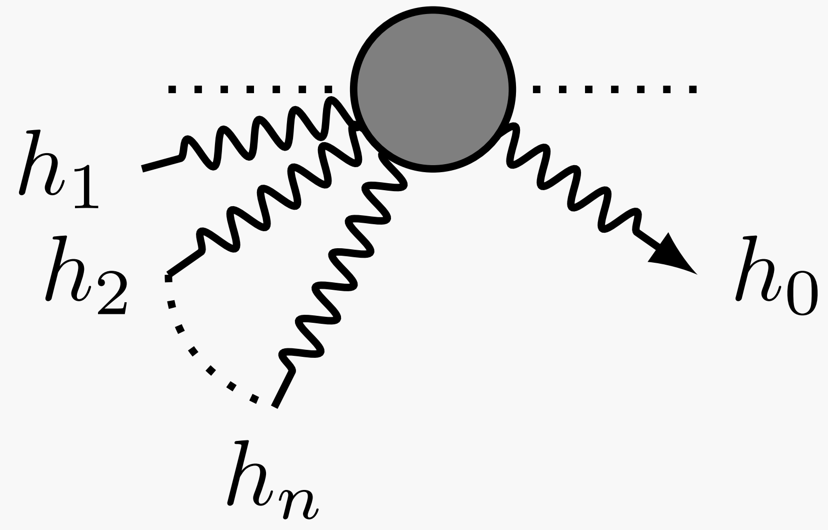
one outgoing graviton

Response functions $\mathcal{R}$

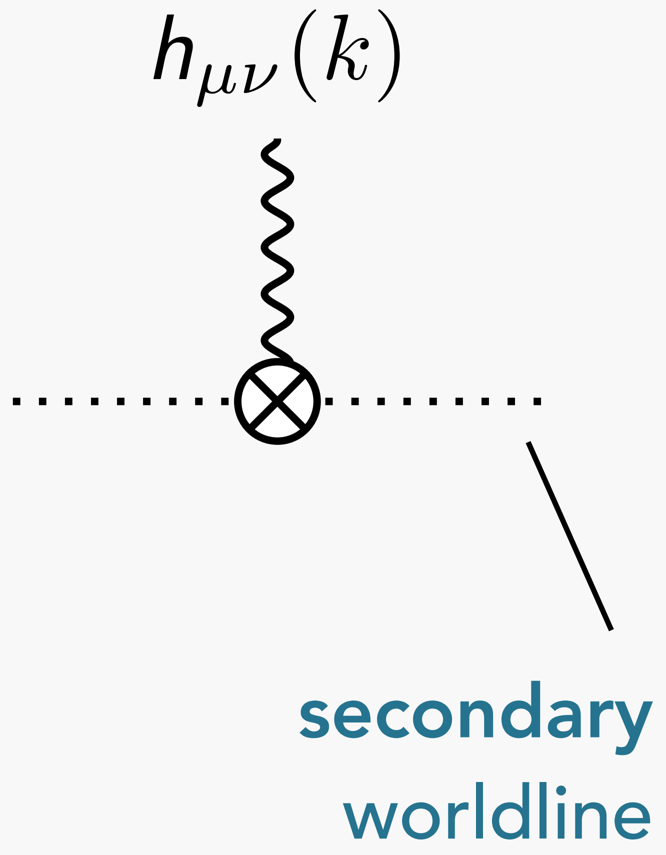


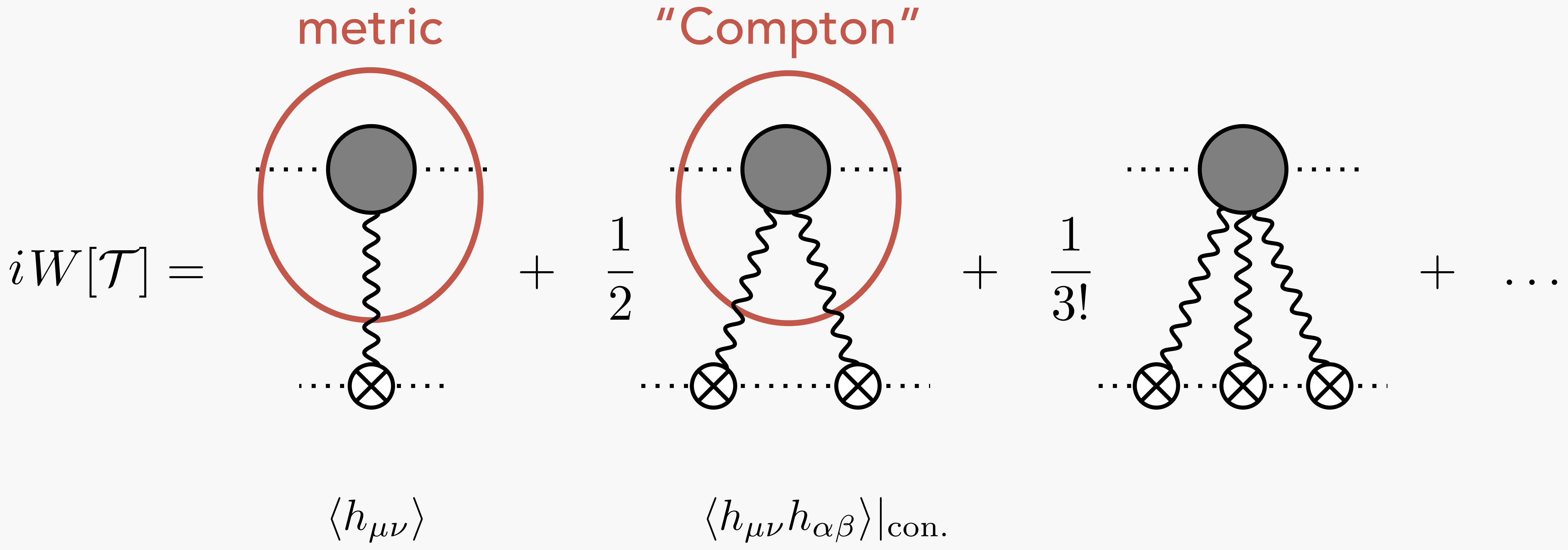
$$iW[\mathcal{T}] = \int_{k_1} \mathcal{T}^{\alpha\beta}(k_1)\mathcal{R}_{\alpha\beta}(k_1) + \frac{1}{2}\int_{k_1,k_2} \mathcal{T}^{\alpha\beta}(k_1)\mathcal{T}^{\mu\nu}(k_2)\mathcal{R}_{\alpha\beta\mu\nu}(k_1,k_2)$$
$$+ \frac{1}{3!}\int_{k_1,k_2,k_3} \mathcal{T}^{\alpha\beta}(k_1)\mathcal{T}^{\mu\nu}(k_2)\mathcal{T}^{\rho\sigma}(k_3)\mathcal{R}_{\alpha\beta\mu\nu\rho\sigma}(k_1,k_2,k_3) + \dots$$

$$\mathcal{R}_{\alpha_0\beta_0\alpha_1\beta_1\dots\alpha_n\beta_n}(k_0,k_1\dots k_n) \quad = $$



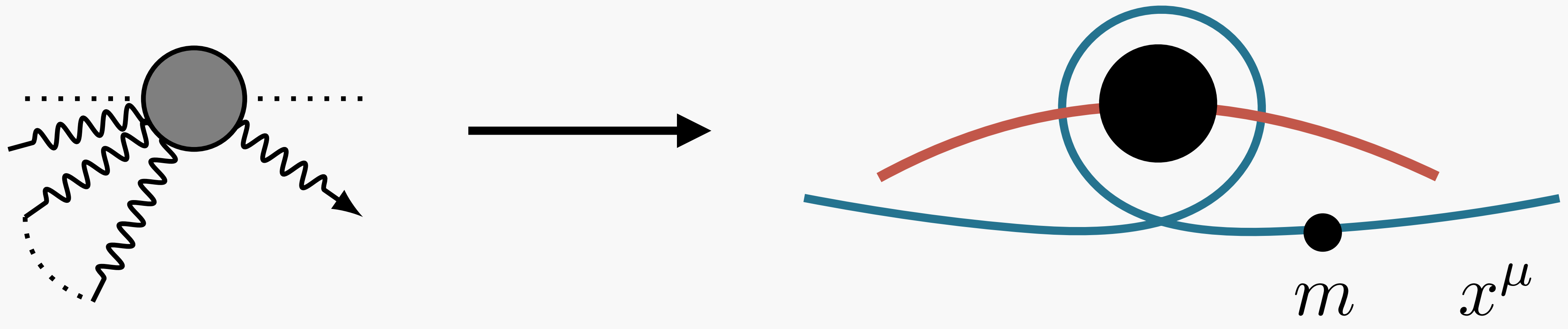
$$\mathcal{T}^{\mu\nu}(k) \quad =$$




metric
"Compton"
$iW[\mathcal{T}] =$
$+ \frac{1}{2}$
$+ \frac{1}{3!}$
$+ \cdots$
$\langle h_{\mu\nu} \rangle$
$\langle h_{\mu\nu} h_{\alpha\beta} \rangle|_{\text{con.}}$

Self-force from responses

Self force expansion - action

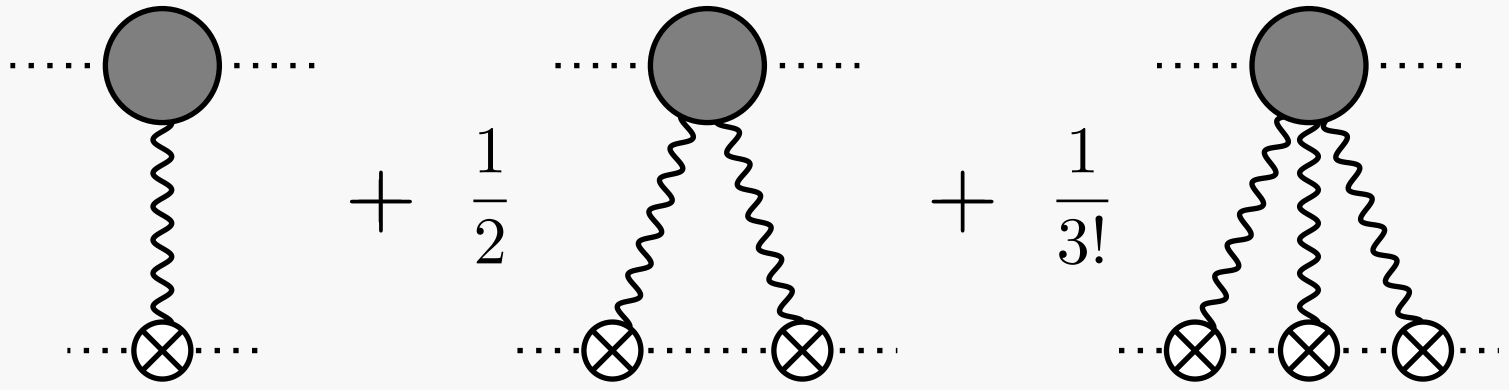


Point particle source

$$\mathcal{T}^{\mu\nu}_{\text{sec}}(x; x(\tau)) = \int d\tau\, \frac{m}{2} \dot{x}^\mu(\tau)\dot{x}^\nu(\tau)\delta^D(x - x(\tau))$$

BH response action

$$iS_{\text{BHR}}[z] = iS_{\text{kin}}[z] + iW[\mathcal{T}^{\mu\nu}_{\text{sec}}] \;=\; iS_{\text{kin}}[z] + \ldots + \frac{1}{2}\ldots + \frac{1}{3!}\ldots + \cdots$$



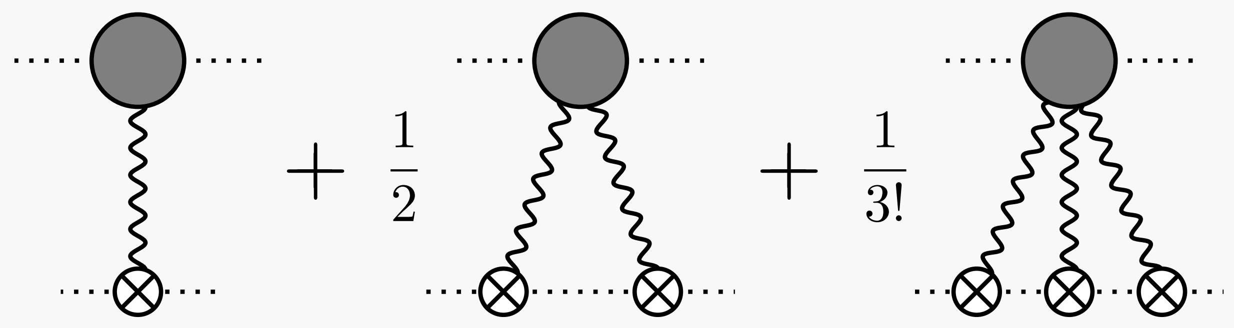
$$S_{\text{kin}}[z] = -\int d\tau\, \frac{m}{2}\eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu$$

Expand around 0SF trajectory

$$x^\mu(\tau) = \bar{x}^\mu(\tau) + z^\mu(\tau)$$

Self force expansion - Feynman rules

$$iS_{\rm BHR}[z] = iS_{\rm kin}[z] + \ldots + \frac{1}{2}\ldots + \frac{1}{3!}\ldots +$$



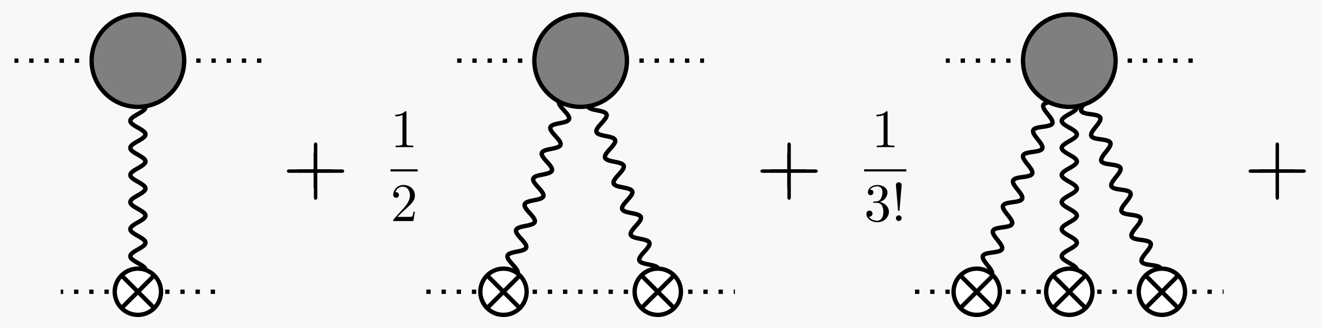
Propagator

$$\langle z^\mu(\omega_1)\, z^\nu(\omega_2)\rangle =$$

$$= \left[\frac{\delta^2 S^{\rm 0SF}_{\rm BHR}}{\delta z^\mu(\omega_1)\delta z^\mu(\omega_2)}\bigg|_{x=\bar{x}} \right]^{-1}$$

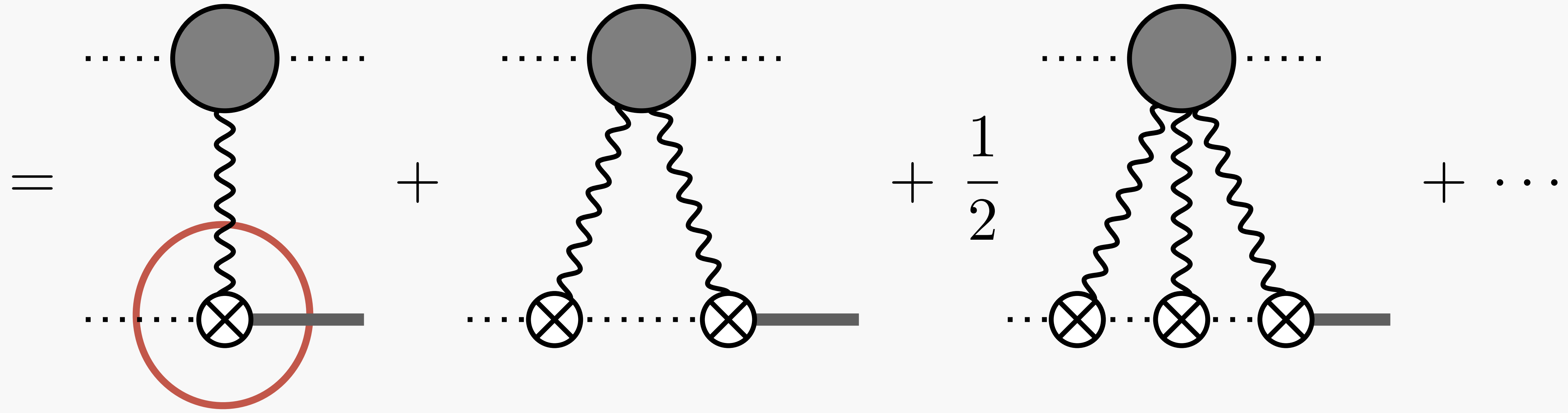
Self force expansion - Feynman rules

$iS_{\mathrm{BHR}}[z] = iS_{\mathrm{kin}}[z] + \ldots$



Vertices

$$V[z] \equiv \left.\frac{\delta iW[\mathcal{T}]}{\delta x^\mu(\omega)}\right|_{x=\bar{x}} = \int_k \frac{\delta iW[\mathcal{T}]}{\delta \mathcal{T}^{\alpha\beta}(k;x)} \left.\frac{\delta \mathcal{T}^{\alpha\beta}(k;x)}{\delta x^\mu(\omega)}\right|_{x=\bar{x}}$$

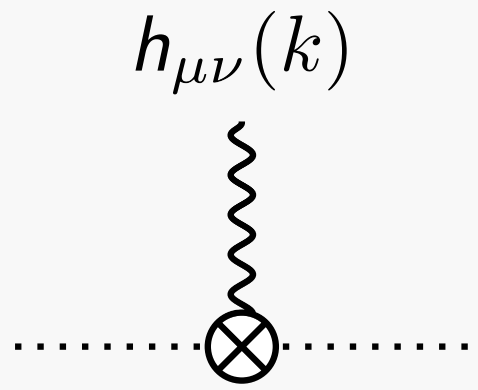


$$V[z^2] \equiv \left.\frac{\delta iW[\mathcal{T}]}{\delta x^\mu(\omega)\delta x^\nu(\omega')}\right|_{x=\bar{x}}$$

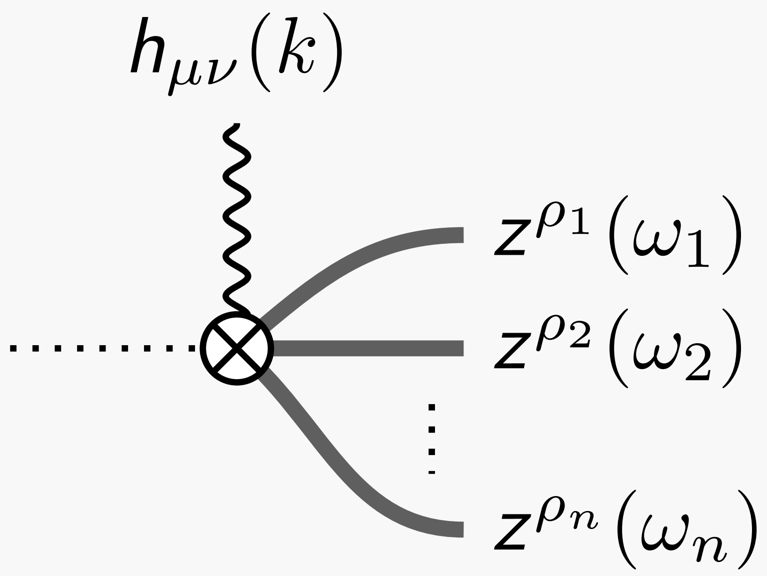
etc…

Evaluated @ 0SF trajectory $\bar{x}^\mu(\omega)$

$\mathcal{T}^{\mu\nu}(k) =$

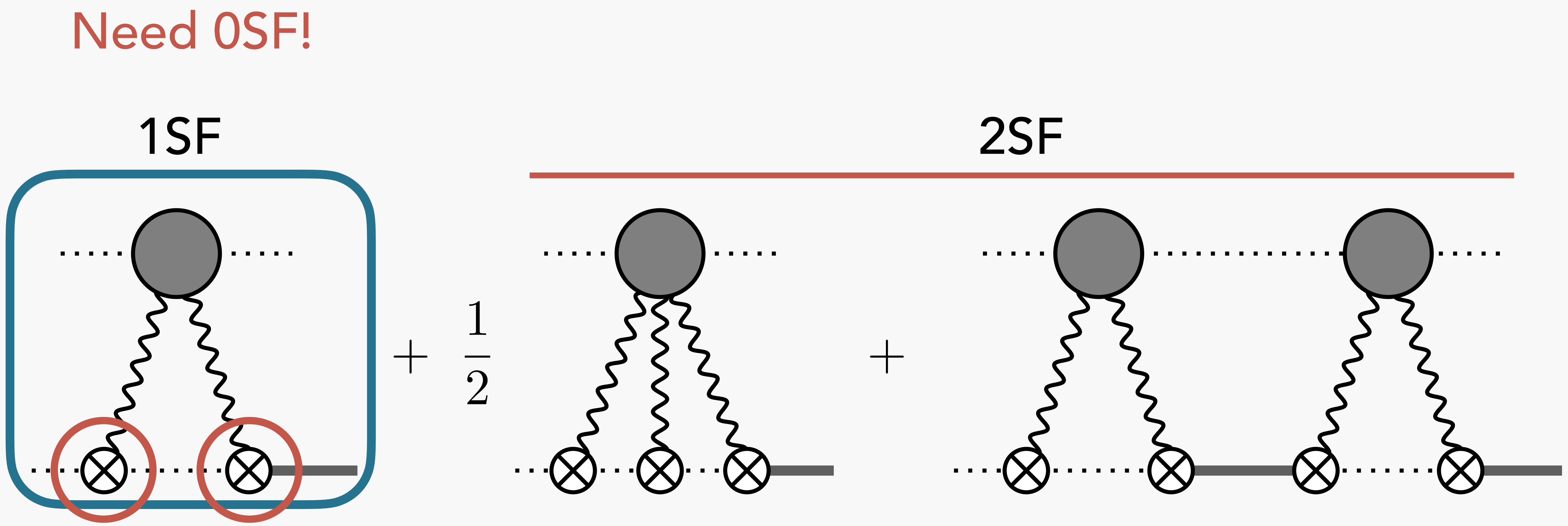


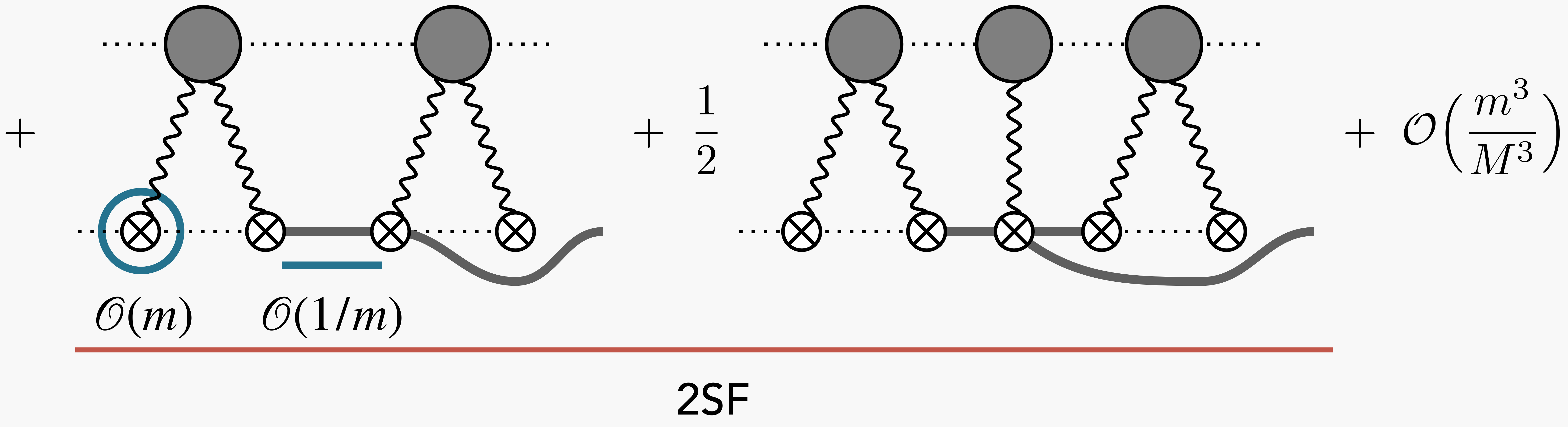
$\left.\frac{\delta^n \mathcal{T}^{\mu\nu}(k)}{[\delta x(\omega)]^n}\right|_{x^\mu=\bar{x}^\mu} =$



Self force diagrams

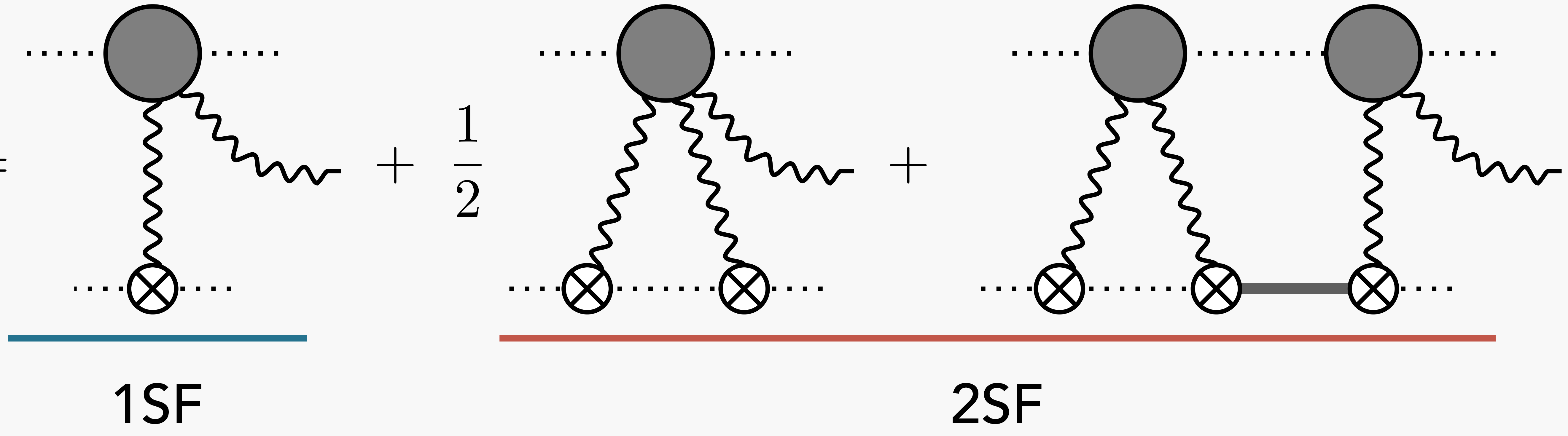
$$\langle z^\rho(\omega)\rangle = \frac{-i\eta^{\rho\sigma}}{m\omega^2}\frac{\delta iW[\mathcal{T}_{\mathrm{sec.}}]}{\delta z^\sigma(\omega)}\bigg|_{x=x^{\mathrm{SF}}} =$$





Self force diagrams

$$-i\langle h_{\mu\nu}(k)\rangle = \left.\frac{\delta iW[\mathcal{T}]}{\delta\mathcal{T}}\right|_{\mathcal{T}=\mathcal{T}_{\text{sec.}}} =$$



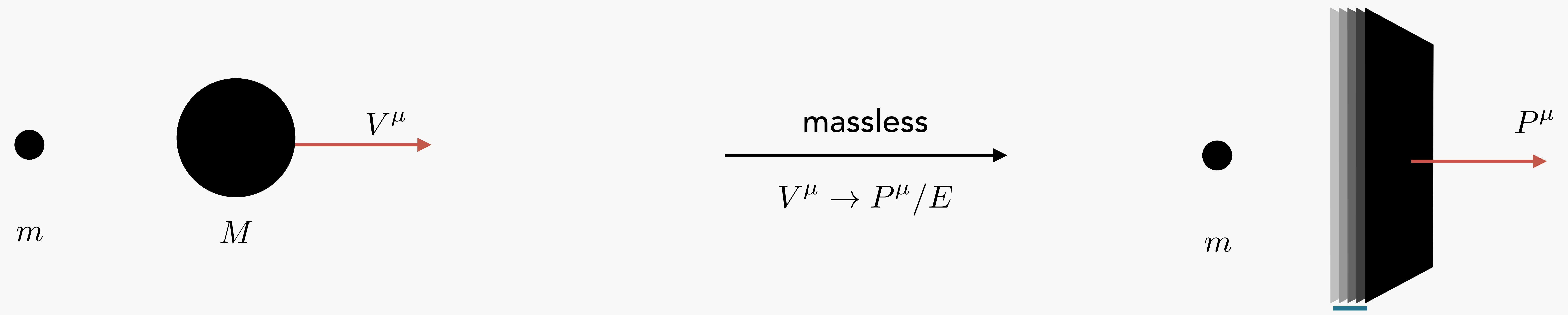
$$+\ \mathcal{O}\left(\frac{m^3}{M^3}\right)$$

Self-force on shockwave

The shockwave

[Aichelburg, Sexl; 1971]

SF: $m/M \ll 1$ SF: $m/E \ll 1$



$P^2 = 0$

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + 4G f(\mathbf{x}_\perp^2)\delta(P\cdot x)P_\mu P_\nu \qquad f(\mathbf{x}_\perp^2) = -\Gamma(-\epsilon)\left(\frac{\mathbf{x}_\perp^2}{4e^{\gamma_E}L^2}\right)^\epsilon$$

Simplicity of shockwave

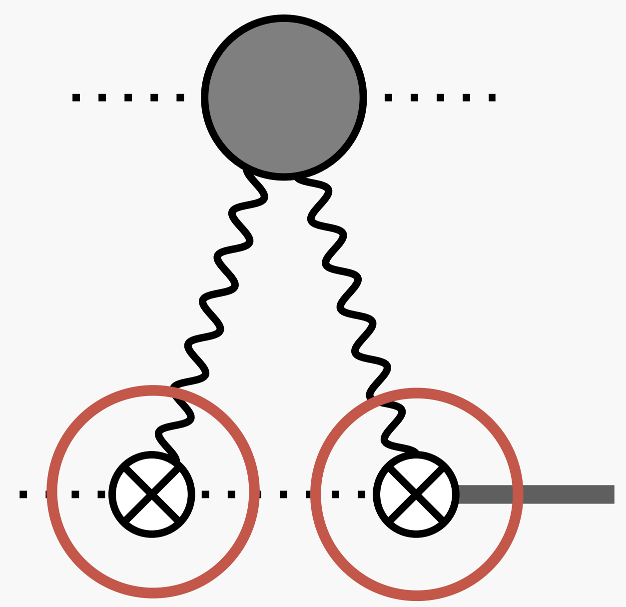
- $P^2 = 0 \quad \Rightarrow$ cancellations
- Localisation on shockwave

Finite-width UV regulator $1/\Lambda$

[Steinbauer; 1997]

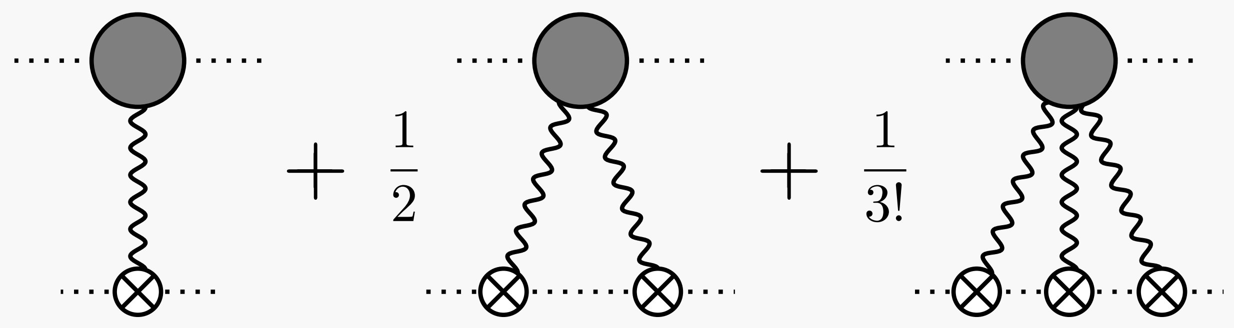
- $\delta(P\cdot x) \quad \to \quad \rho_\Lambda(P\cdot x) = \Lambda/2\,\exp(-\Lambda|\sigma|)$
- $\Lambda \to \infty$ recovers single shockwave limit

0SF dynamics

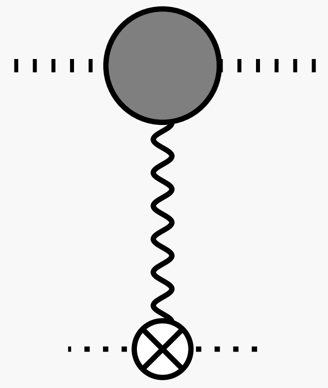


0SF - geodesics on shockwave

$$iS_{\mathrm{BHR}}[z] = iS_{\mathrm{kin}}[z] + \quad + \frac{1}{2} \quad + \frac{1}{3!} \quad +$$

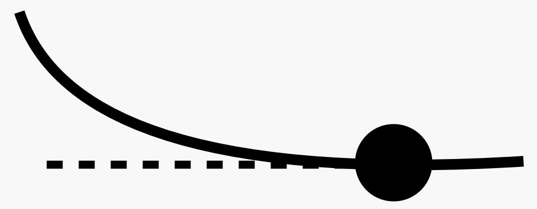


$$iS^{0\mathrm{SF}}[\bar{x}] = -i\frac{m}{2}\int d\tau \bar{g}_{\mu\nu}\dot{\bar{x}}^{\mu}\dot{\bar{x}}^{\nu} = iS_{\mathrm{kin}}[\bar{x}] \quad +$$



$$g_{\mu\nu}(x) = \eta_{\mu\nu} + 4Gf(\mathbf{x}_{\perp}^2)\delta(P\cdot x)P_{\mu}P_{\nu}$$

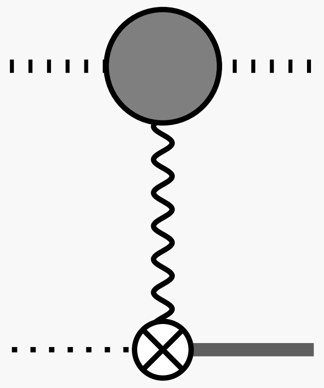
$$f(\mathbf{x}_{\perp}^2) = -\Gamma(-\epsilon)\left(\frac{\mathbf{x}_{\perp}^2}{4e^{\gamma_E}L^2}\right)^{\epsilon}$$

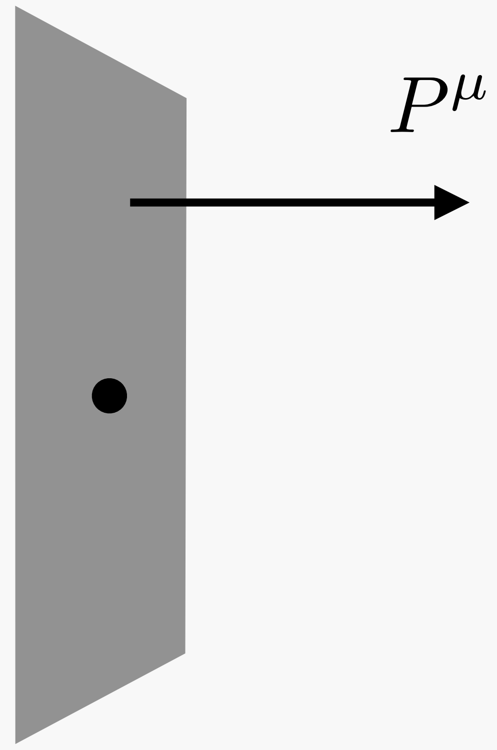


PM expansion

$$\bar{x}^{\mu} = b^{\mu} + v^{\mu}\tau + \bar{z}^{\mu}$$

$$-m\langle\ddot{\bar{z}}^{\mu}(\tau)\rangle = \eta^{\mu\nu}\frac{\delta S_{\mathrm{int}}[x]}{\delta x^{\nu}(\tau)} =$$

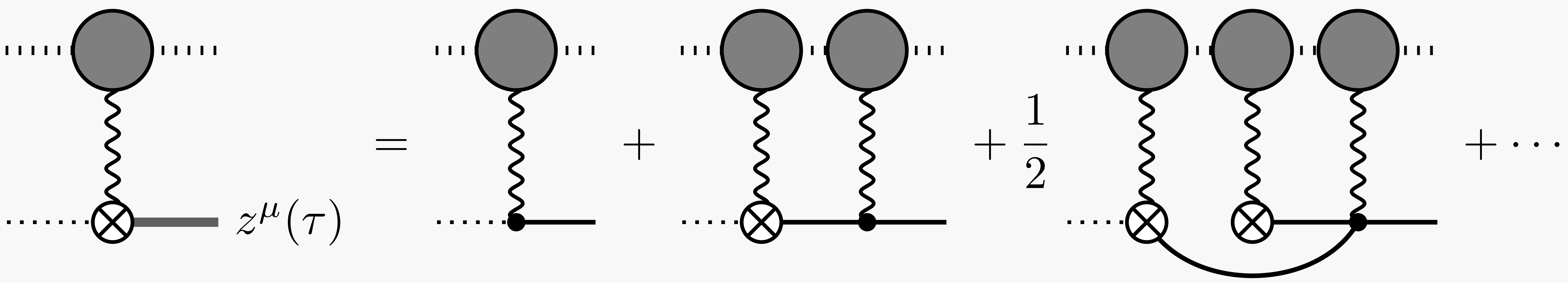




Berends-Giele recursion

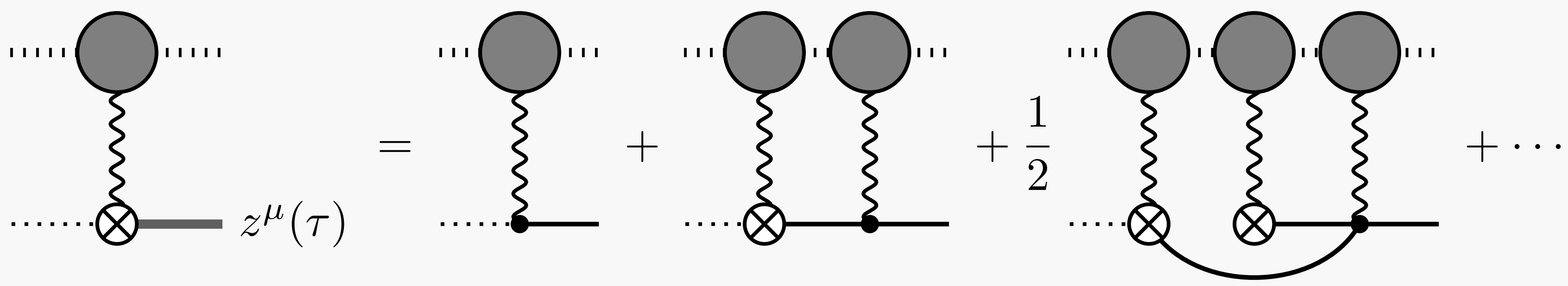
7PM @ $\mathcal{O}(s^4)$ in Kerr

[JH, Jakobsen, Plefka; 2506.14626]



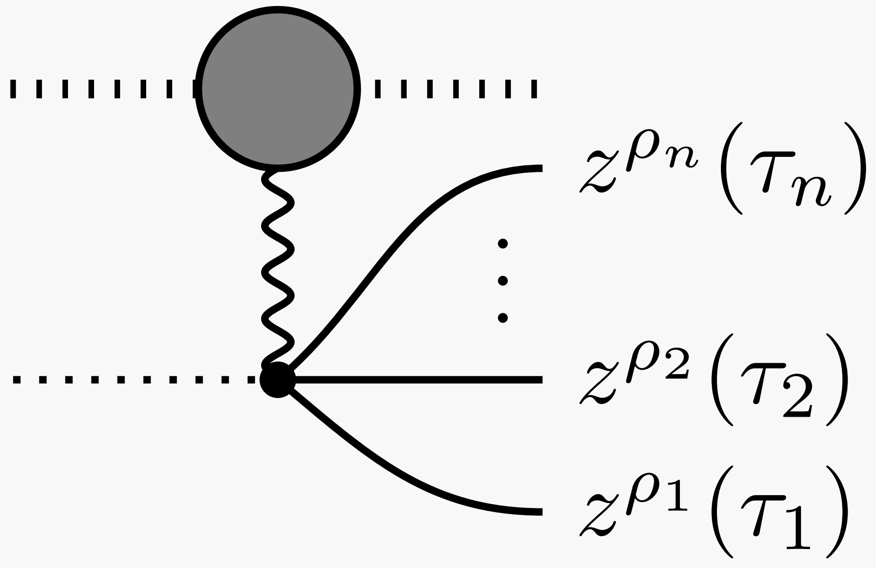
Trajectory truncates at 2PM

Berends-Giele recursion

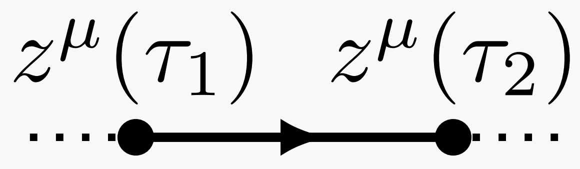


Position space PM diagrammatics

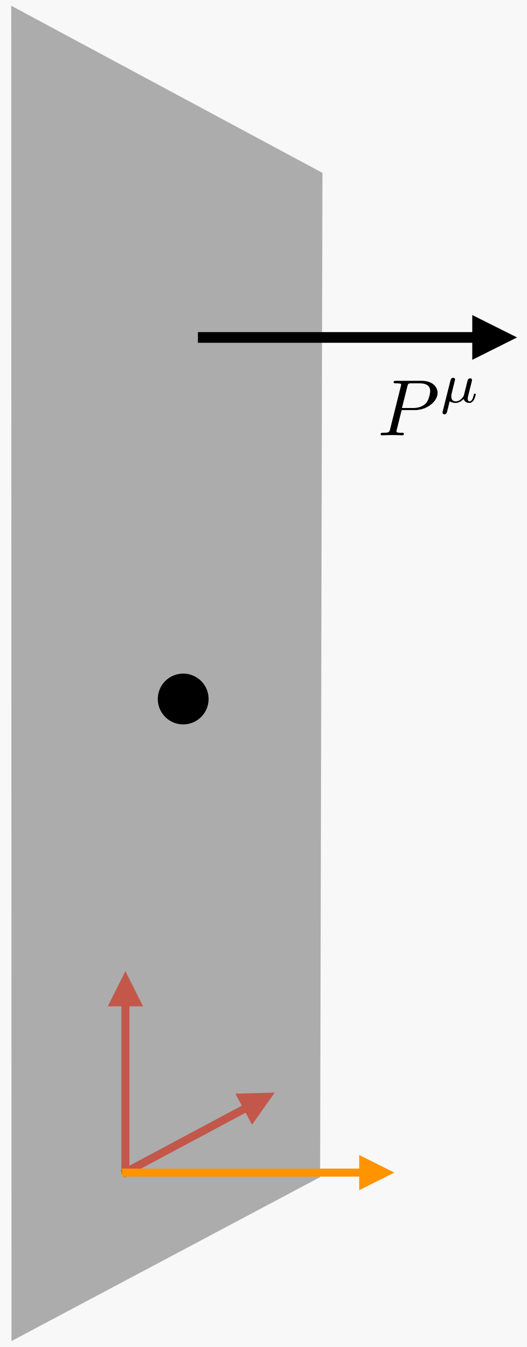
[JH, Jakobsen, Plefka; 2506.14626]



$$= \left.\frac{i\delta^n S_{\text{int}}[x]}{\delta x_1^\rho(\tau_1)\cdots\delta x_n^\rho(\tau_n)}\right|_{x^\mu=b^\mu+v^\mu\tau}$$



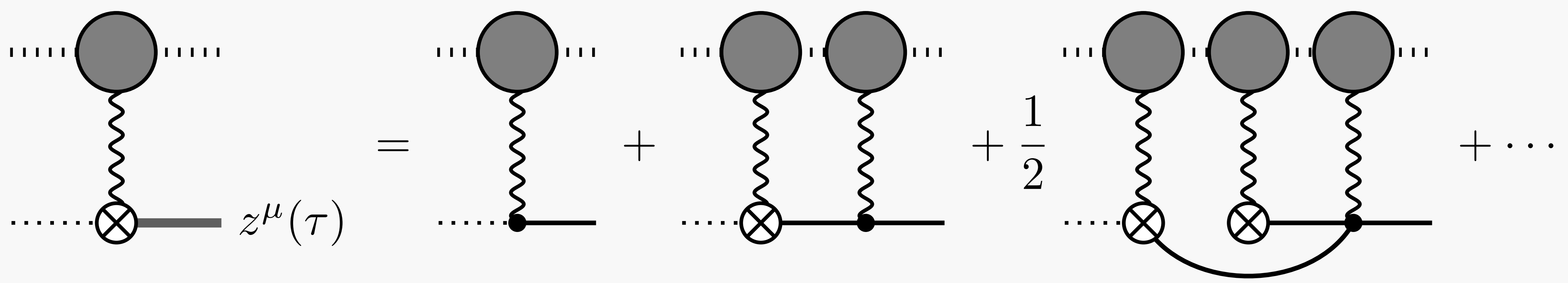
$$= \frac{i\eta^{\mu\nu}}{m}(\tau_2-\tau_1)\theta(\tau_2-\tau_1)$$



$$x^\mu = \frac{1}{2E}\left(x^-\bar{P}^\mu + x^+P^\mu\right) + \mathbf{x}_\perp^\mu$$

$$\eta^{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \mathbf{1} \end{pmatrix}$$

Obtaining shockwave geodesics

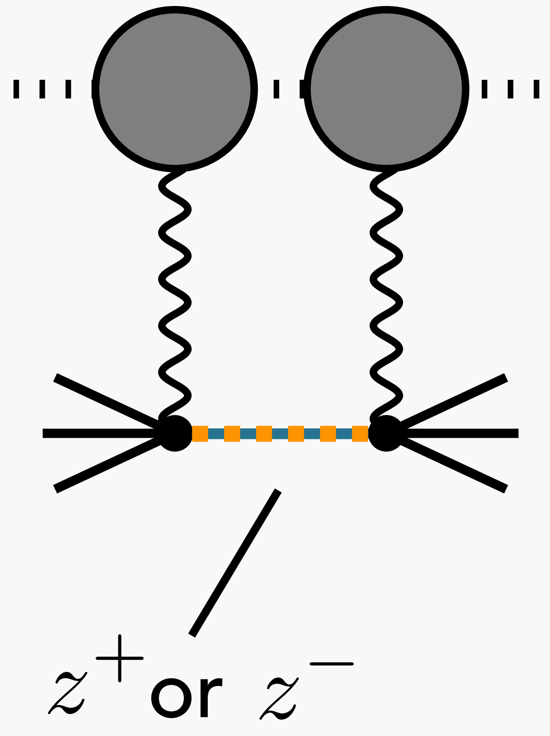


$$S_{\text{int}}^{0\text{SF}}[x] = -\frac{m}{2}\int d\tau \bar{h}_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = -\frac{m}{2}\int d\tau\, 4G\delta(P\cdot x) f(\mathbf{x}_\perp)(\dot{x}^-)^2$$

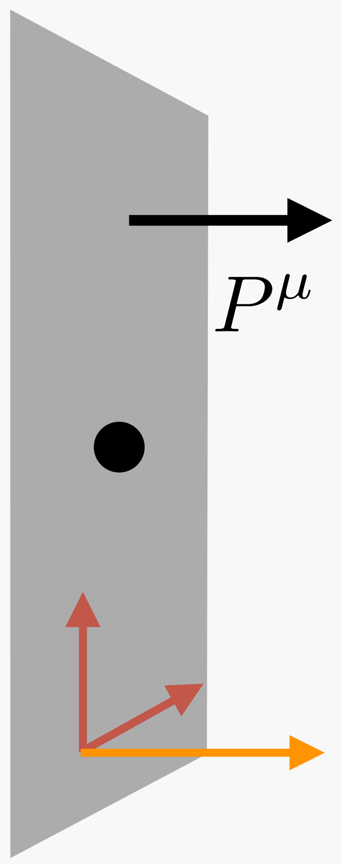
$$\ddot{z}^-(\tau) = -\frac{1}{m}\eta^{-+}\frac{\delta S_{\text{int}}[x]}{\delta x^+} = 0$$

$$\eta^{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \mathbf{1} \end{pmatrix}$$

Structure from Berends-Giele

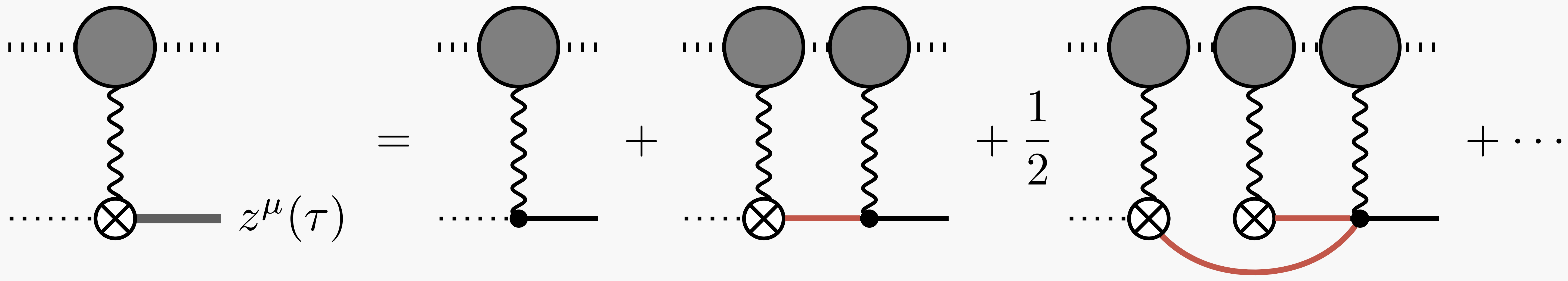


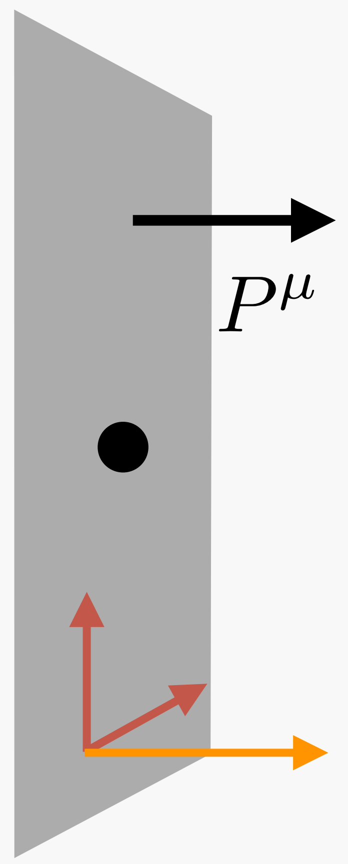
$$\sim \frac{\delta S_{\text{int}}[x]}{\delta x^+}(\tau_2 - \tau_1)\theta(\tau_2-\tau_1)\frac{\delta S_{\text{int}}[x]}{\delta x^-} = 0$$



$$x^\mu = \frac{1}{2E}\left(x^-\bar{P}^\mu + x^+ P^\mu\right) + \mathbf{x}_\perp^\mu$$

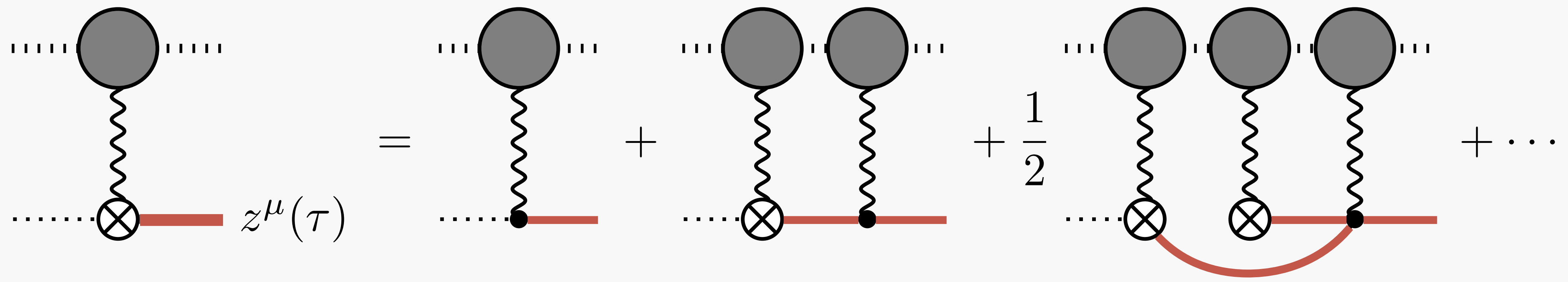
Obtaining shockwave geodesics



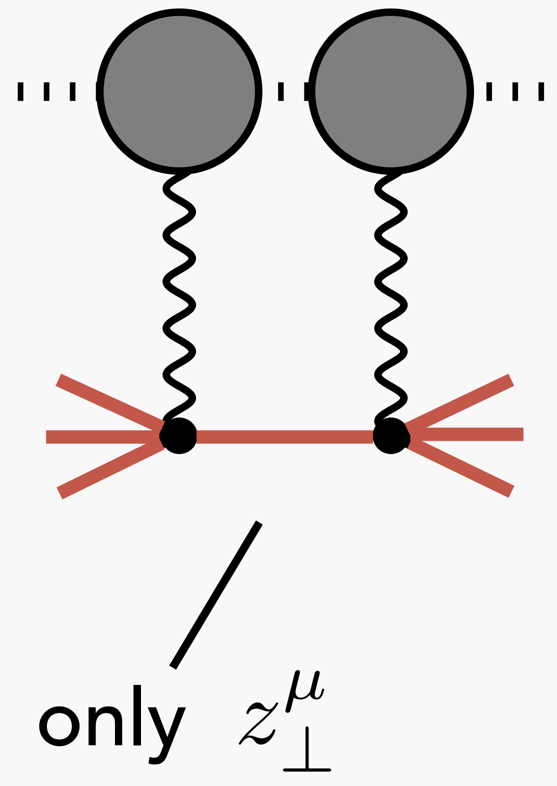


$$x^\mu = \frac{1}{2E}\left(x^- \bar{P}^\mu + x^+ P^\mu\right) + \mathbf{x}_\perp^\mu$$

Obtaining shockwave geodesics

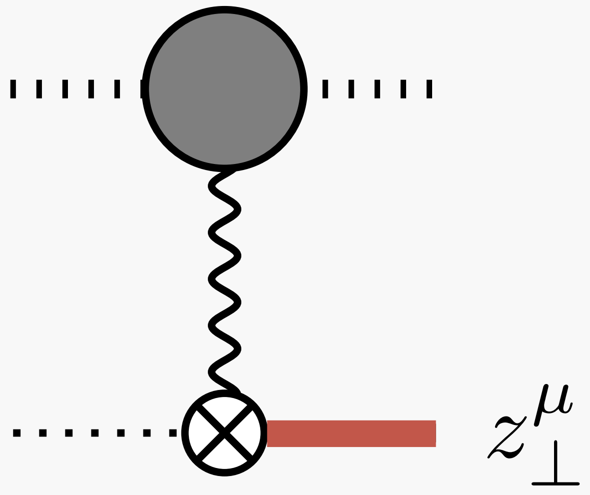


External component $z_\perp^\mu$

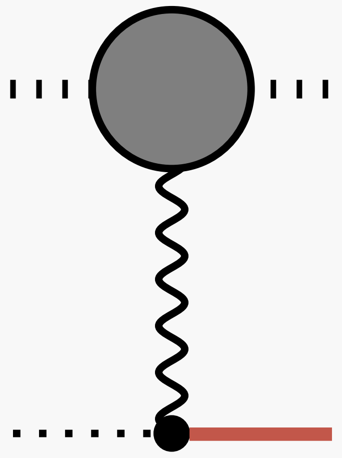


$$\sim \int_{-\infty}^{\infty} d\tau_1 d\tau_2\, \delta(\tau_1-\tau_0)\underbrace{(\tau_2-\tau_1)\theta(\tau_2-\tau_1)}_{\text{propagator}}\delta(\tau_2-\tau_0) \;=\; 0$$

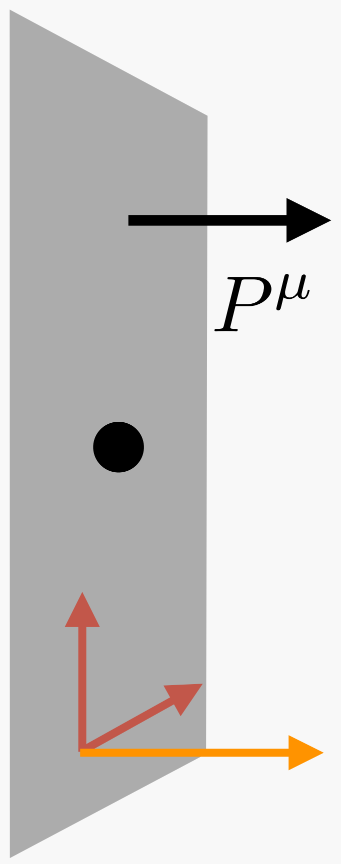
vertex   vertex



$=$

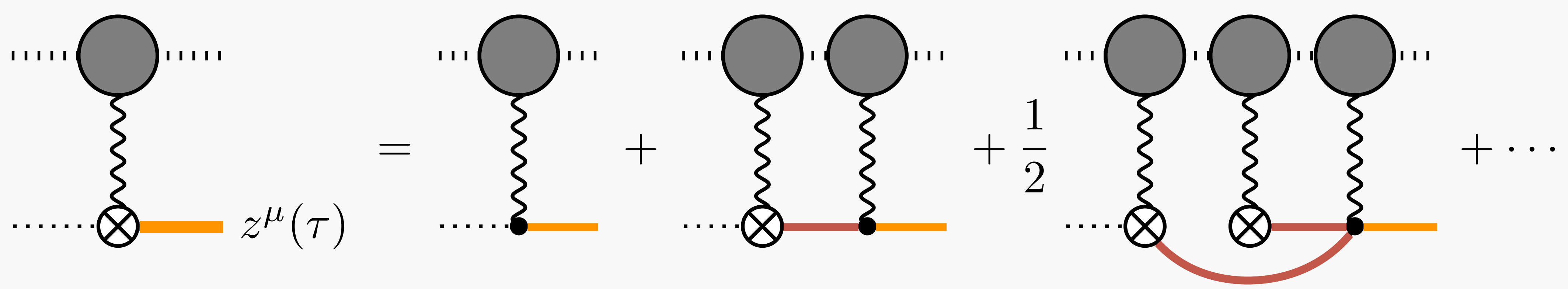


$$= \left.\frac{\delta i S_{\text{int}}}{\delta x_\perp}\right|_{x^\mu=b^\mu+v^\mu\tau}$$

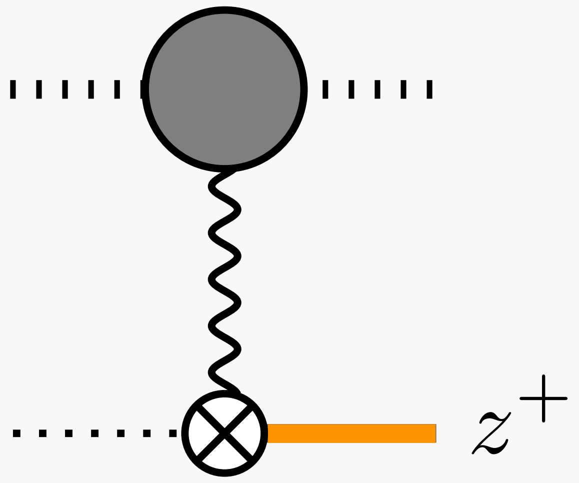


$$x^\mu = \frac{1}{2E}\left(x^-\bar{P}^\mu + x^+P^\mu\right) + \mathbf{x}_\perp^\mu$$

Obtaining shockwave geodesics

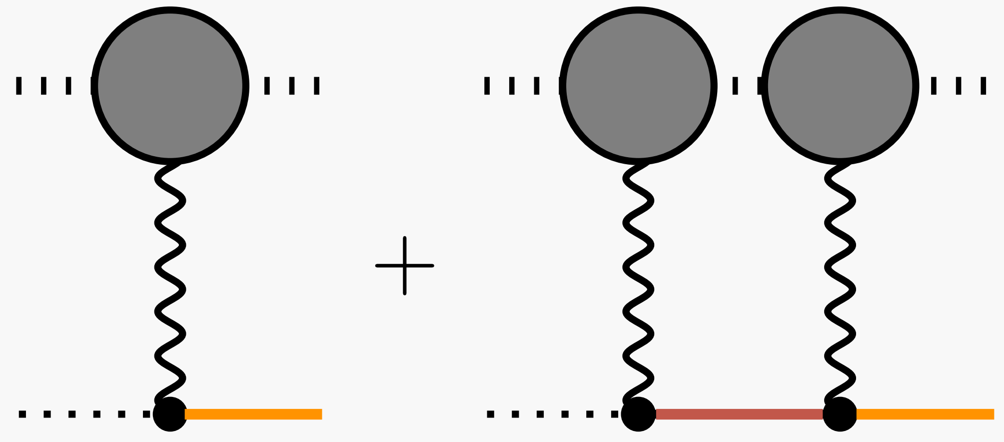


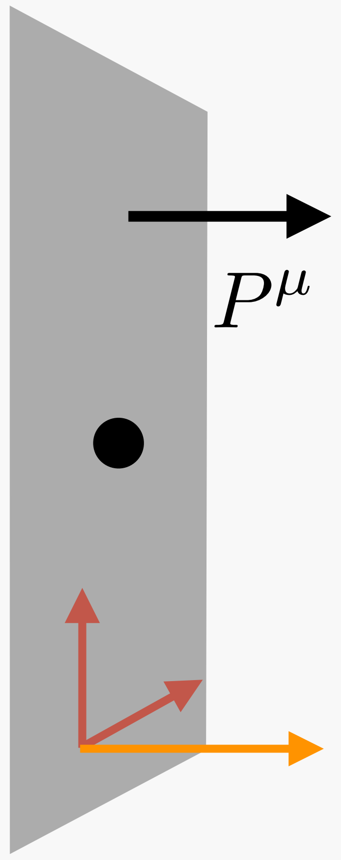
External component z^+



$=$ taylor expansion **only** in $z_\perp^\mu$ $= \left.\dfrac{\delta i S_{\text{int}}}{\delta z^-}\right|_{z^-=0,\ z^+=0,\ z_\perp^\mu=1\text{PM}}$

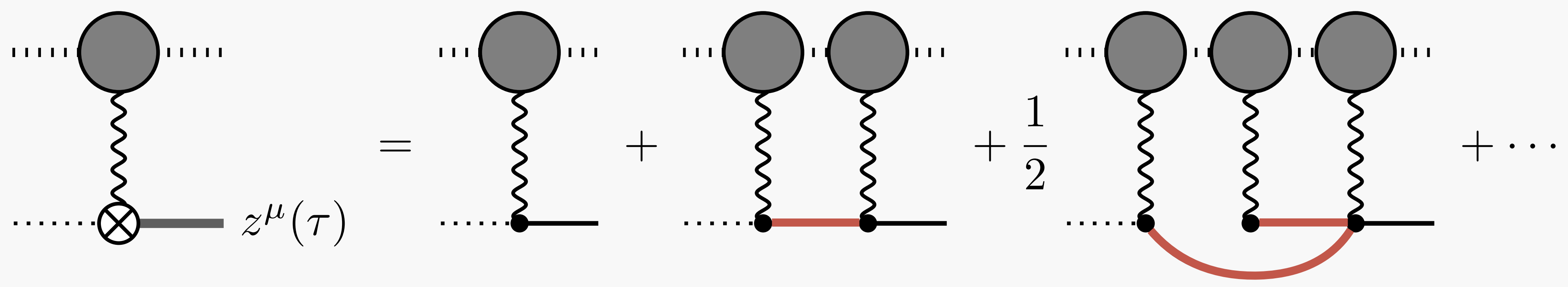
[JH, Jakobsen, Plefka; 2506.14626]

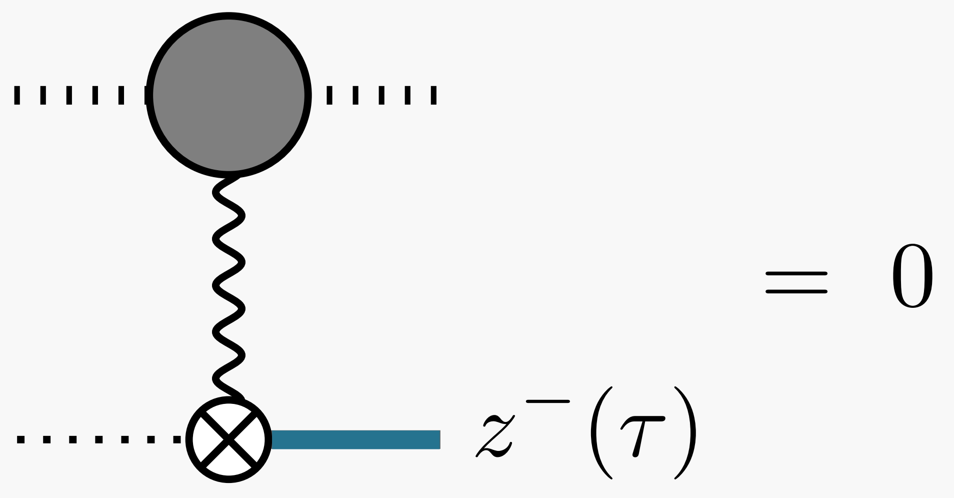
$=$ 

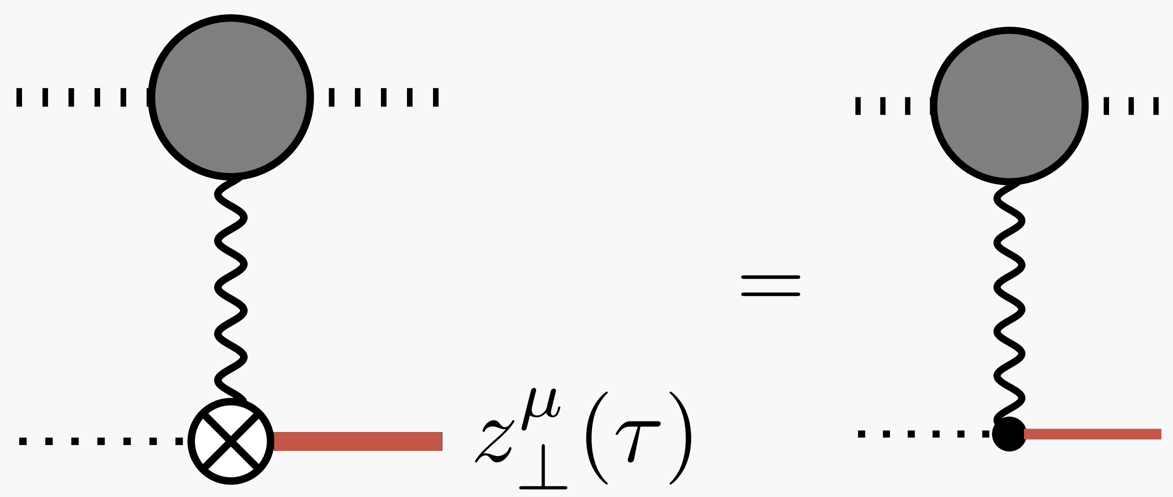


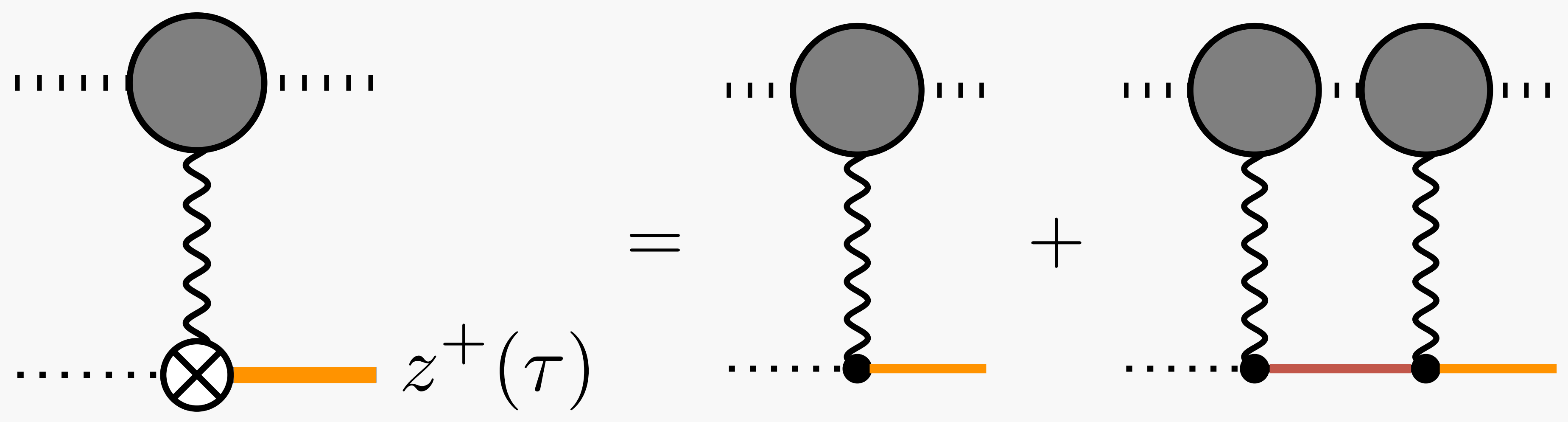
$$x^\mu = \frac{1}{2E}\left(x^-\bar{P}^\mu + x^+ P^\mu\right) + \mathbf{x}_\perp^\mu$$

Geodesics summary

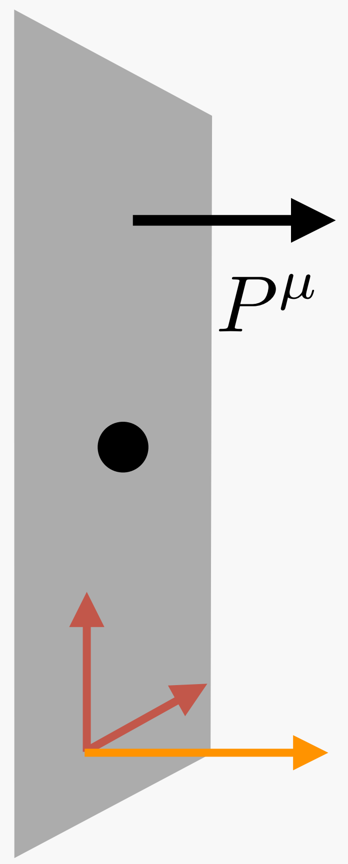








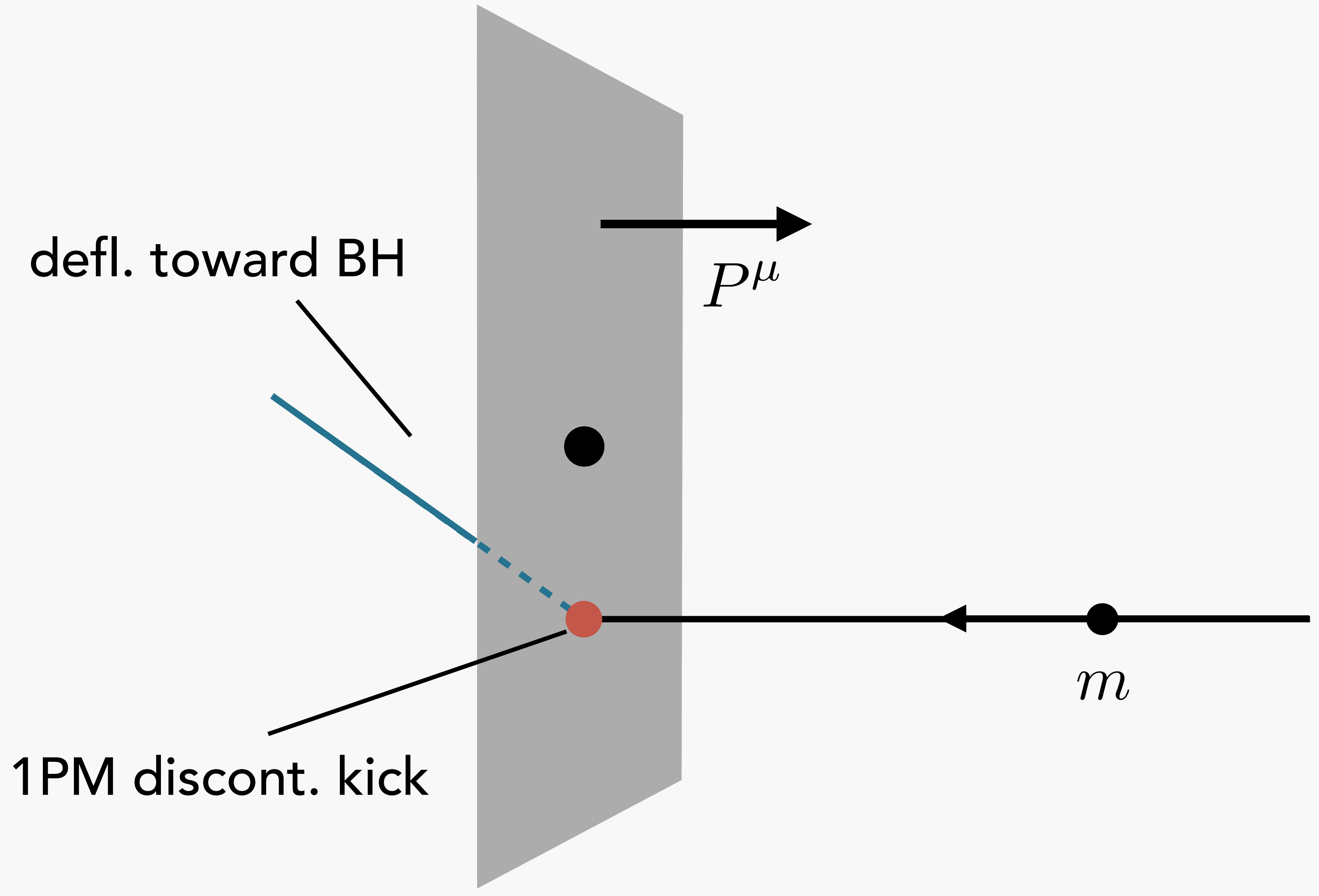
2PM truncation



$$x^\mu = \frac{1}{2E}\left(x^- \bar{P}^\mu + x^+ P^\mu\right) + \mathbf{x}_\perp^\mu$$

Geodesics summary

$$\bar{x}^\mu(\tau) = (v^\mu \tau + b)\theta(\tau_0 - \tau) + \big(v^\mu_\infty(\tau - \tau_0) + b^\mu_\infty\big)\theta(\tau - \tau_0)$$

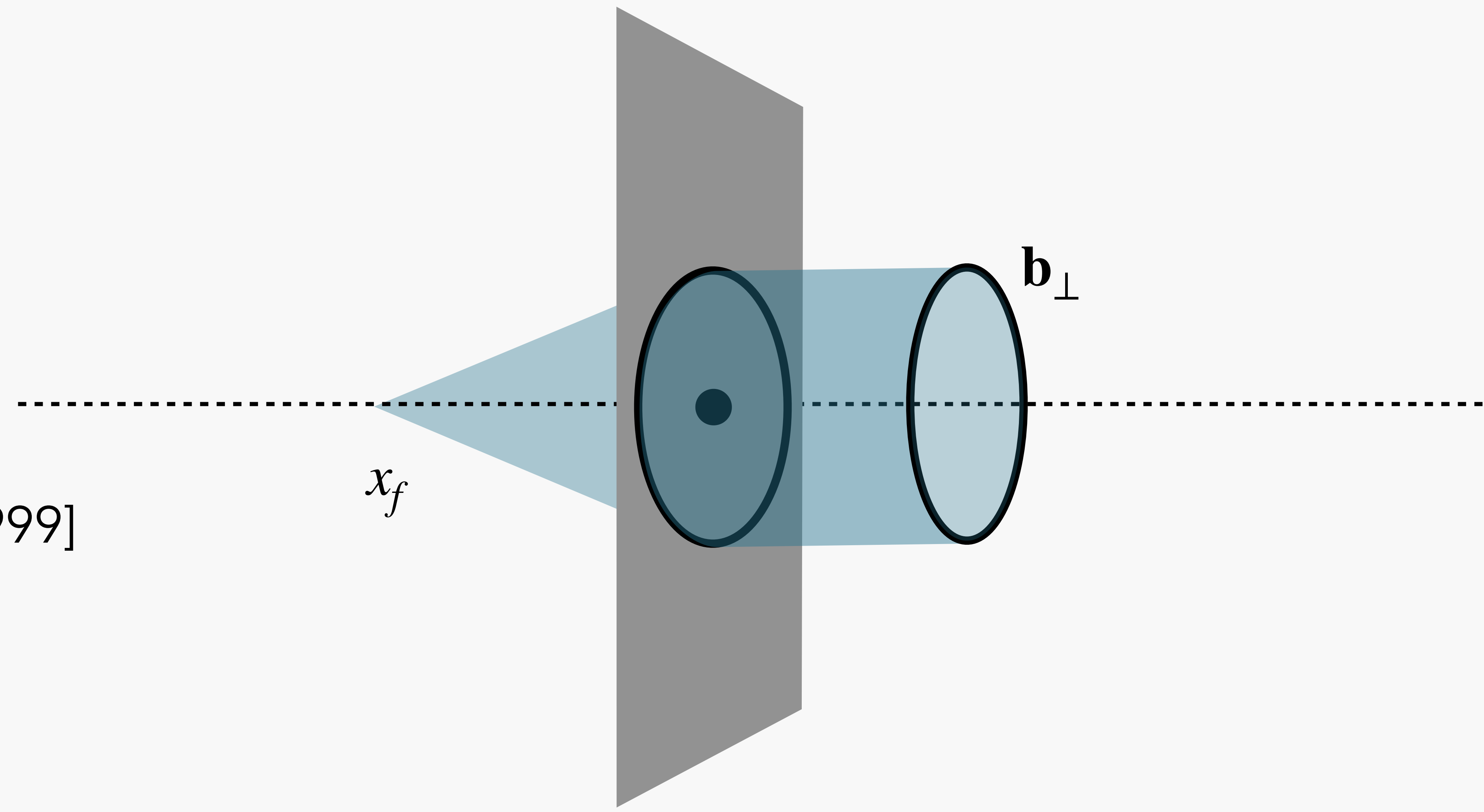


$$v^\mu_\infty = v^\mu + \Delta v^\mu = v^\mu - \frac{4GE\epsilon f(\mathbf{b}_\perp^2)}{|\mathbf{b}_\perp|^2}\left(\mathbf{b}^\mu_\perp - 2G\epsilon f(\mathbf{b}_\perp^2)P^\mu\right)$$

$$b^\mu_\infty = b^\mu + v^\mu\tau_0 + \Delta b^\mu = b^\mu + v^\mu \tau_0 - 2GP^\mu f(\mathbf{b}_\perp^2)$$

Focusing lens for null geodesics

[Ferrari, Pendenza, Veneziano; 1988]



$$x_f = \frac{1}{2}\left(f(\mathbf{b}_\perp^2) + 2b/f'(\mathbf{b}_\perp^2) + bf'(\mathbf{b}_\perp^2)\right)$$

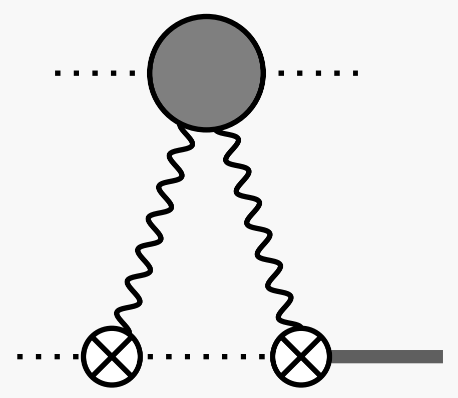
Interpretation

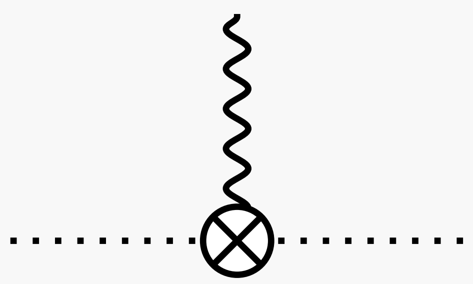
[Dray, 't Hooft; 1985] [Steinbauer; 1998]

- 2 copies of $\eta_{\mu\nu}$ (Penrose junction) [Penrose; 1972] [Kunzinger, Steinbauer; 1999]

$$b^\mu_\infty \sim \frac{1}{\epsilon} + \log\left(\frac{\mathbf{x}^2}{\tilde{L}}\right) \to \log\left(\frac{\mathbf{x}^2}{\tilde{L}'}\right)$$

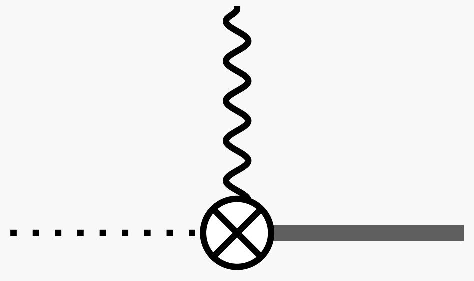
0SF - resummed vertices

$$\langle z^\rho(\omega)\rangle =$$  $$+\cdots$$



$$= \mathcal{T}^{\mu\nu}_{\text{sec.}}(k;x)\big|_{x=\bar{x}} = -\frac{m\kappa_D}{2} e^{ik\cdot(v\tau_0+b)}\left(\frac{v^\mu v^\nu}{v\cdot k - i0} - \frac{e^{ik\cdot\Delta\bar{b}} v_\infty^\mu v_\infty^\nu}{v_\infty\cdot k + i0}\right) - im\kappa_D e^{ik\cdot\bar{x}(\tau_0)}\left(v^{(\mu}\Delta\bar{b}^{\nu)} + \theta(0)\Delta\bar{v}^{(\mu}\Delta\bar{b}^{\nu)}\right)$$

eval @ 0SF



$$= \frac{\delta\mathcal{T}^{\mu\nu}_{\text{sec.}}(k;x)}{\delta x^\rho}\Big|_{x=\bar{x}} = -\frac{im\kappa_D}{2}e^{ik\cdot(v\tau_0+b)}\left[2\omega\left(\frac{v^{(\mu}\delta^{\nu)}_\rho}{v\cdot k+\omega-i0} - \frac{e^{ik\cdot\Delta\bar{b}}v_\infty^{(\mu}\delta^{\nu)}_\rho}{v_\infty\cdot k+\omega+i0}\right) + k_\rho\left(\frac{v^\mu v^\nu}{v\cdot k+\omega-i0} - \frac{e^{ik\cdot\Delta\bar{b}}v^\mu_\infty v^\nu_\infty}{v_\infty\cdot k+\omega+i0}\right)\right] - m\kappa_D e^{ik\cdot\bar{x}(\tau_0)+i\omega\tau_0}\left(\omega\Delta\bar{b}^{(\mu}\delta^{\nu)}_\rho + k_\rho v^{(\mu}\Delta\bar{b}^{\nu)} + \theta(0)k_\rho\Delta\bar{v}^{(\mu}\Delta\bar{b}^{\nu)}\right)$$

< kick

> kick

= kick

[Steinbauer; 1998]

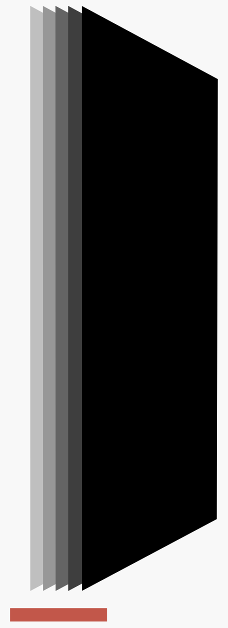
$$\theta(0) = 1/2$$

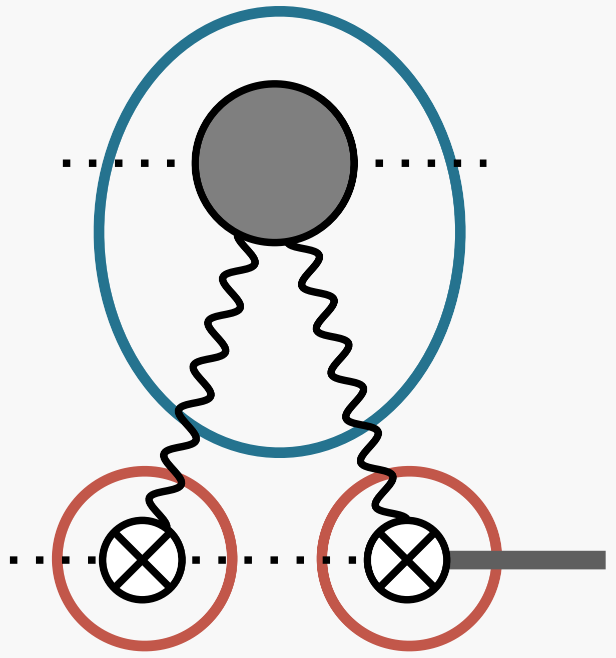
relation to flat-space

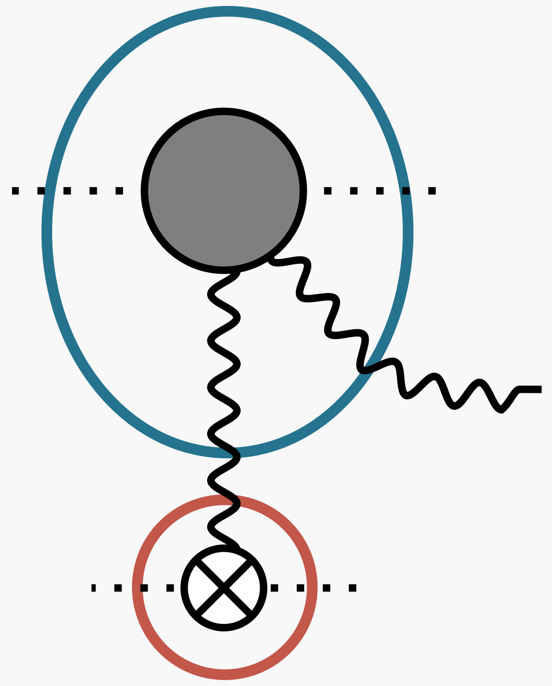
$$e^{ik\cdot b} i\delta(k\cdot v+\Omega)\begin{Bmatrix} v^\mu v^\nu \\ v^\mu\end{Bmatrix} \to \int d\tau\, e^{ik\cdot\bar{x}(\tau)+i\Omega\tau}\begin{Bmatrix}\dot{\bar{x}}^\mu(\tau)\dot{\bar{x}}^\nu(\tau)\\ \dot{\bar{x}}^\mu(\tau)\end{Bmatrix}$$

Towards 1SF

Towards 1SF



$\langle z^\rho(\omega)\rangle =$  $+ \mathcal{O}\left(\frac{m^2}{E^2}\right)$

$-i\langle h_{\mu\nu}(k)\rangle =$  $+ \mathcal{O}\left(\frac{m^2}{E^2}\right)$

Geodesics

2-pt response

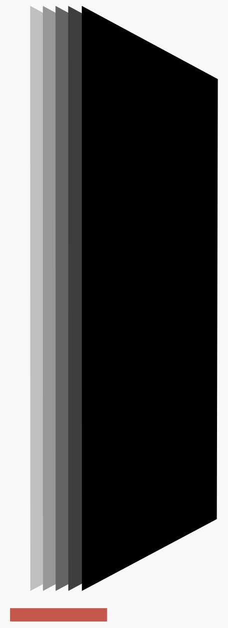
$$\mathcal{R}^{\mu\nu\rho\sigma}(k_1,k_2) \sim [\Pi\cdot\Pi]^{\mu\nu\rho\sigma}\int d^{D-2}\mathbf{x}_\perp\, e^{-i\mathbf{q}_\perp\cdot\mathbf{x}_\perp}\left[e^{-2iG(P\cdot k_1)f(x_\perp)}-1\right]$$

Caveat: Method of regions

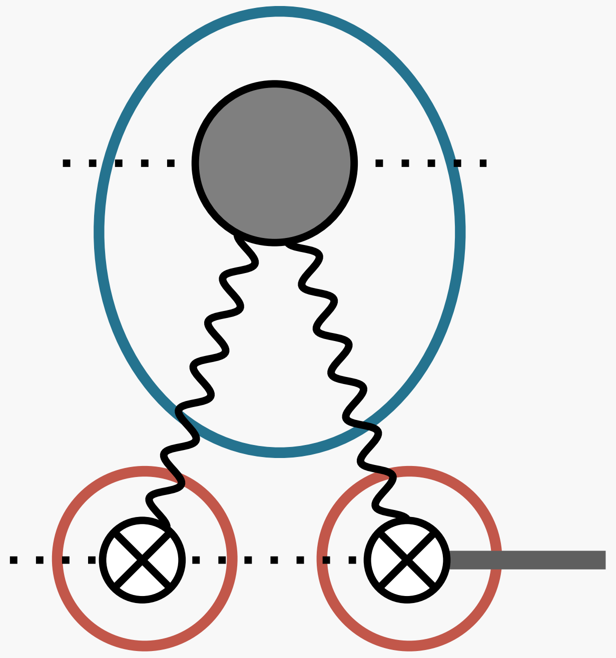
$k_1 \sim k_2 \sim \Lambda^0$ appropriate on-shell

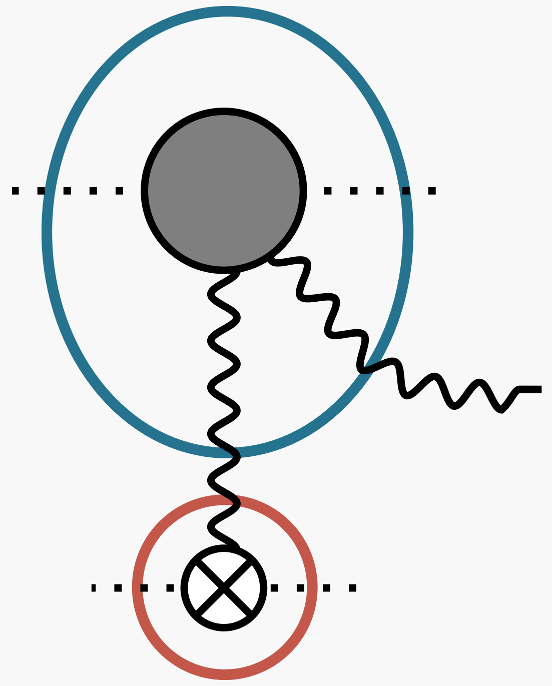
- Off-shell: $k_1 \sim \Lambda^a, \quad k_2 \sim \Lambda^b$
- 1SF Waveform: No additional regions!

$$\int_{\boldsymbol{\ell}} \frac{1}{(\boldsymbol{\ell}-\mathbf{q}_\perp)^2((\mathbf{k}_1^\perp+\boldsymbol{\ell})^2-k_1^-(k_1^++i\Lambda))}$$



Towards 1SF

$\langle z^\rho(\omega)\rangle =$  $+\ \mathcal{O}\Big(\frac{m^2}{E^2}\Big)$

$-i\langle h_{\mu\nu}(k)\rangle =$  $+\ \mathcal{O}\Big(\frac{m^2}{E^2}\Big)$

Geodesics

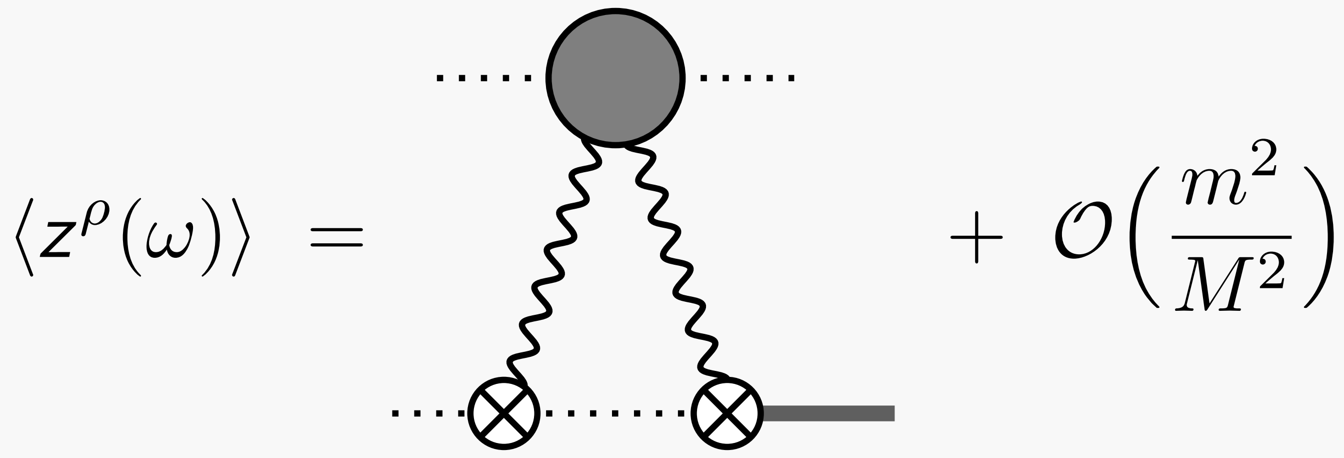
2-pt response

$$\mathcal{R}^{\mu\nu\rho\sigma}(k_1,k_2) \sim [\Pi\cdot\Pi]^{\mu\nu\rho\sigma}\int d^{D-2}\mathbf{x}_\perp\, e^{-i\mathbf{q}_\perp\cdot\mathbf{x}_\perp}\left[e^{-2iG(P\cdot k_1)f(x_\perp)}-1\right]$$

Resummation pending!

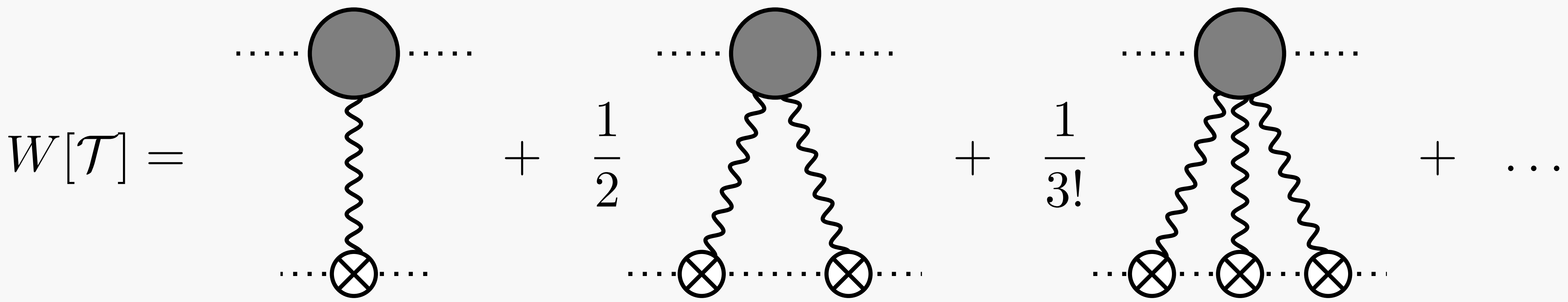
Summary

- **Response Theory** efficient EFT for Self-Force
  - 1SF = 2-pt response coupled to **geodesics**



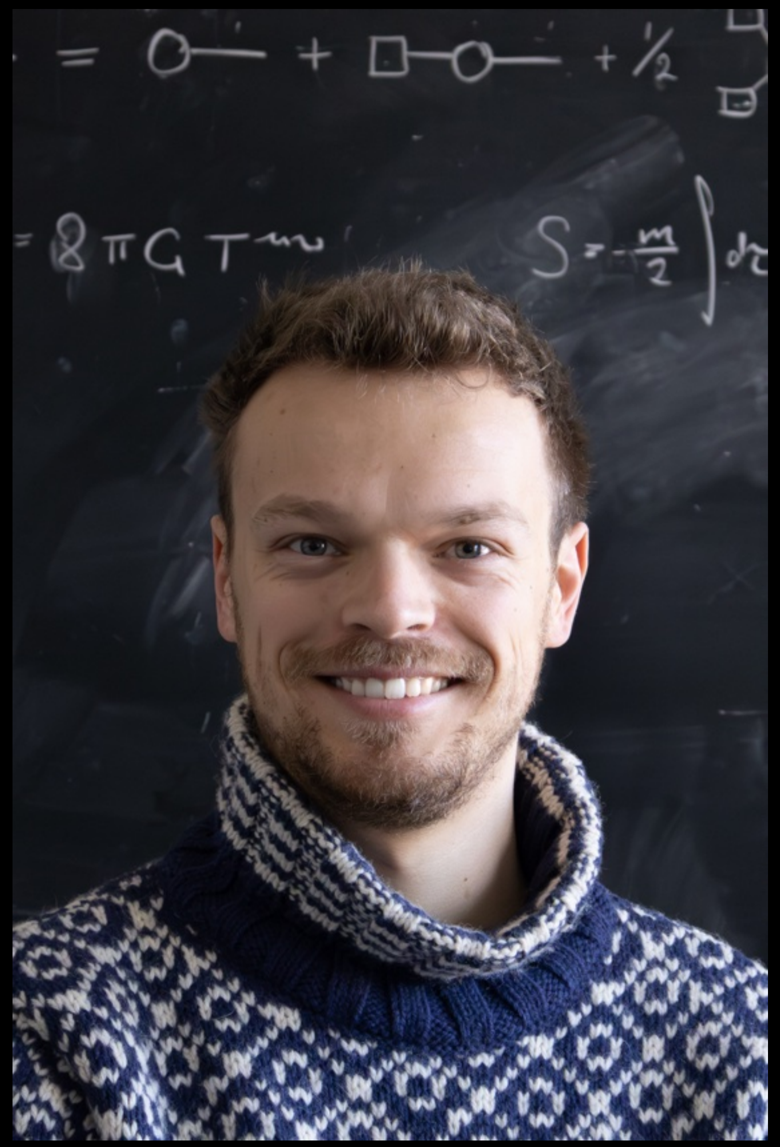
- **Shockwave**
  - 2-pt response **exponentiates @ all PM orders**
  - Geodesics truncate at 2PM

- **1SF observables** on the way!



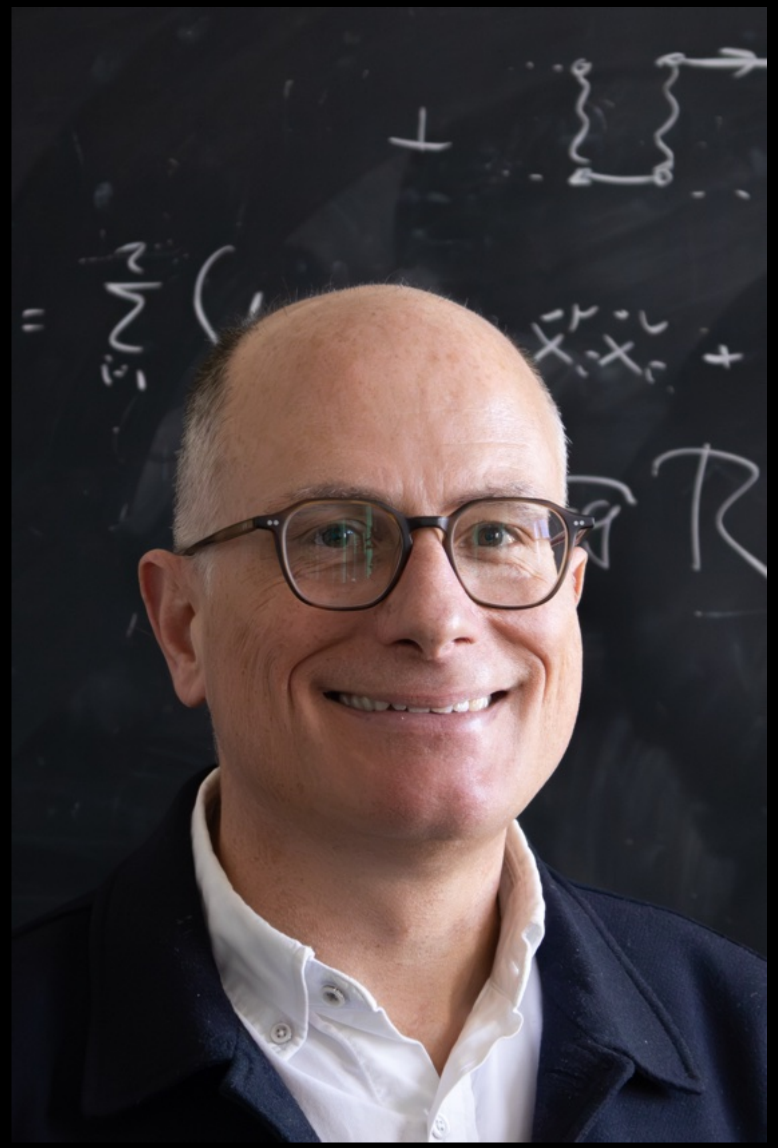
Lara Bohnenblust

Carl Jordan Eriksen



Jitze Hoogeveen

Gustav Uhre Jakobsen



Jan Plefka