



RTG 2575:

Rethinking Quantum Field Theory



Self-Force From Response Theory

Based on [Bohnenblust, Eriksen, JH, Jakobsen, Plefka; 2604.xxxxx]

[JH, Jakobsen, Plefka; 2506.14626]

[Lara's talk on Thursday], [Carl's poster]

Jitze Hoogeveen, Humboldt Universität zu Berlin

Nordita, 17/4 2026

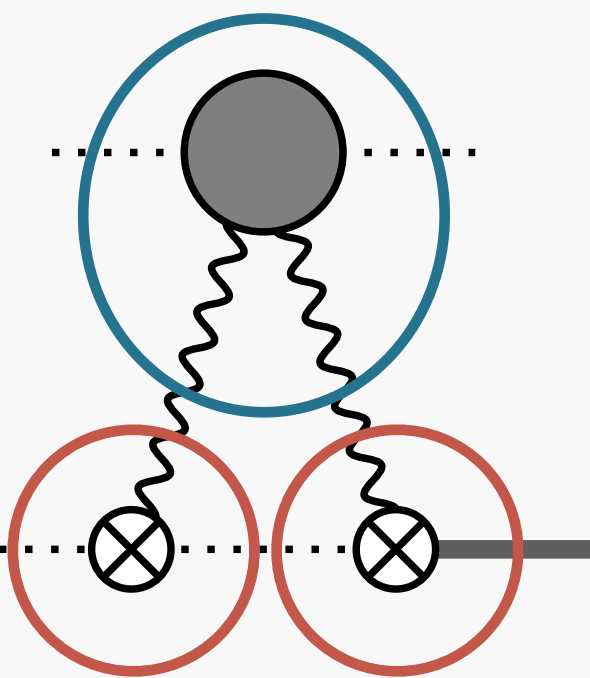
Outline

- Motivation
- Response theory

- SF dynamics
- Shockwaves
- OSF - geodesics on shockwave
- Towards 1SF

Response Theory

2-pt response

$$\langle z^\rho(\omega) \rangle = \text{Diagram} + \mathcal{O}\left(\frac{m^2}{M^2}\right)$$


Where does this come from?

Geodesics

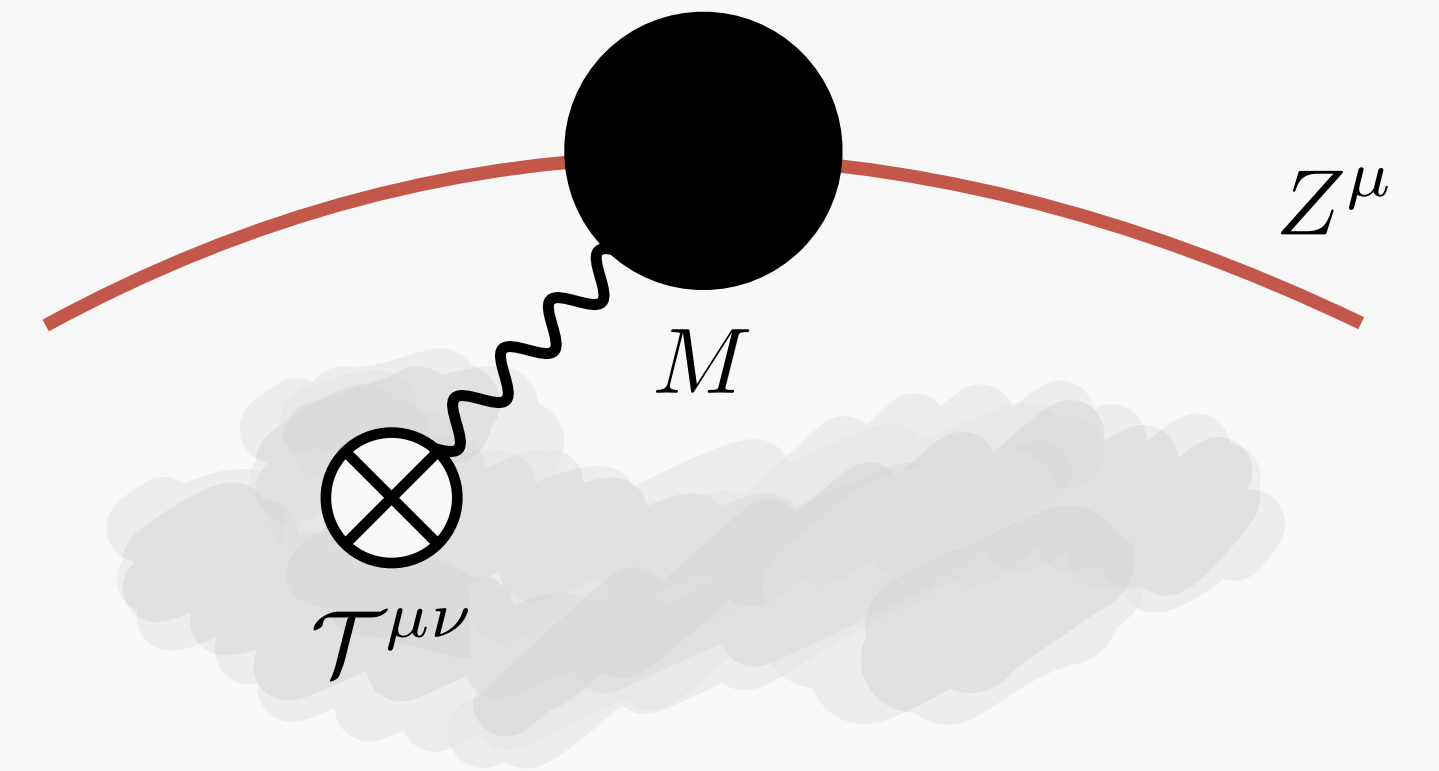
Brief Reminder

Response theory in a nutshell

Black hole coupling to source $\mathcal{T}^{\mu\nu}$

$$S_{\text{BH}}[X, g, \mathcal{T}] = S_{\text{EH}}[g] - S_{\text{M}}[X, g] - \int d^D x h_{\mu\nu}(x) \underline{\mathcal{T}^{\mu\nu}}(x)$$

$$\mathcal{T}^{\mu\nu}(x) = \int d\tau \frac{m}{2} \dot{y}^\mu(\tau) \dot{y}^\nu(\tau) \delta^D(x - y(\tau))$$

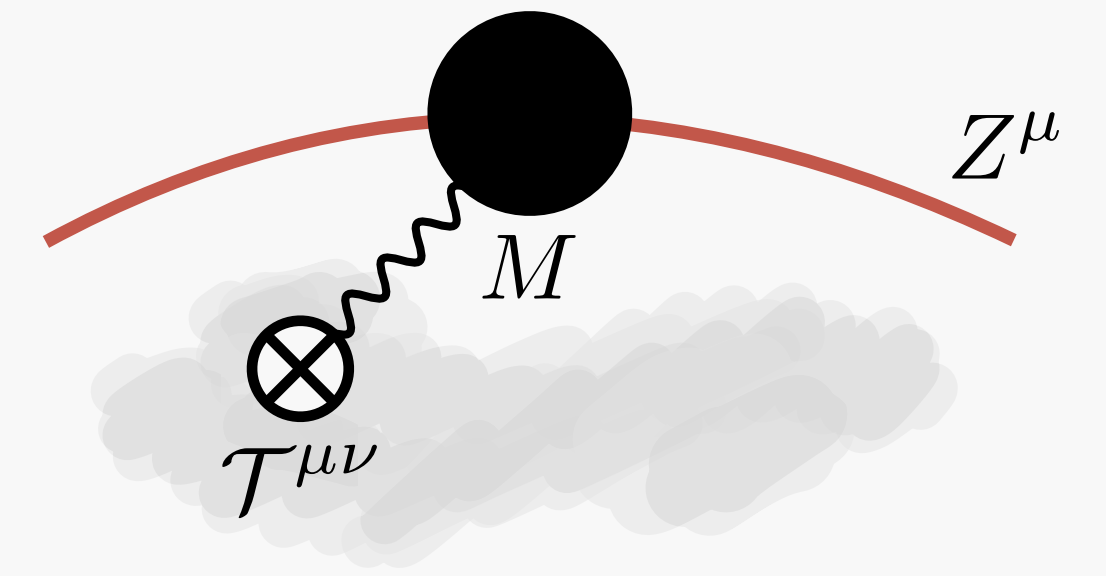


(secondary black hole)

Effective action for environment $W[\mathcal{T}]$

$$\begin{aligned} Z[\mathcal{T}] &= \int \mathcal{D}[h, Z] \exp \left[\frac{i}{\hbar} \left(S_{\text{BH}}[X, g] - \int d^D x h_{\mu\nu}(x) \mathcal{T}^{\mu\nu}(x) \right) \right] \Big|_{\text{tree}} \\ &= \exp \left[\frac{i}{\hbar} W[\mathcal{T}] \right] \end{aligned}$$

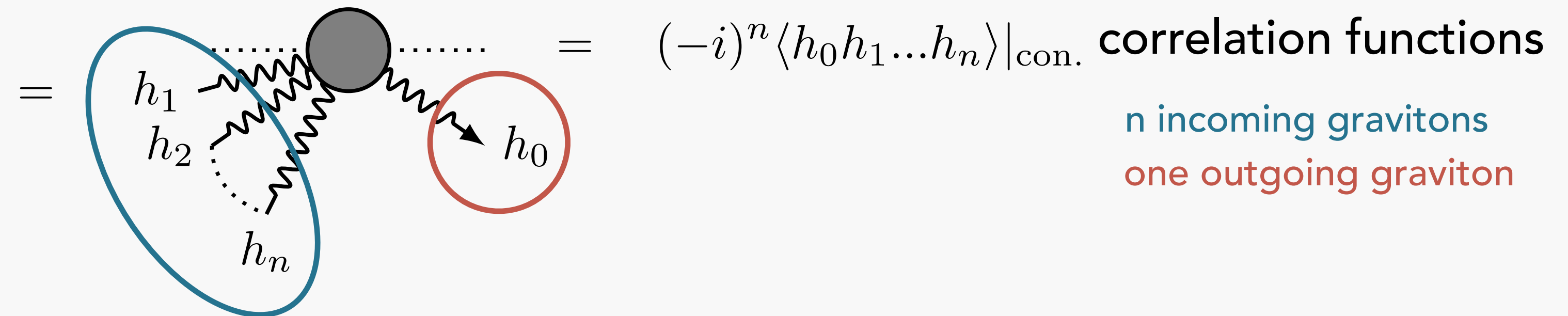
Response functions \mathcal{R}



$$iW[\mathcal{T}] = \int_{k_1} \mathcal{T}^{\alpha\beta}(k_1) \underline{\mathcal{R}_{\alpha\beta}(k_1)} + \frac{1}{2} \int_{k_1, k_2} \mathcal{T}^{\alpha\beta}(k_1) \mathcal{T}^{\mu\nu}(k_2) \underline{\mathcal{R}_{\alpha\beta\mu\nu}(k_1, k_2)}$$

$$+ \frac{1}{3!} \int_{k_1, k_2, k_3} \mathcal{T}^{\alpha\beta}(k_1) \mathcal{T}^{\mu\nu}(k_2) \mathcal{T}^{\rho\sigma}(k_3) \underline{\mathcal{R}_{\alpha\beta\mu\nu\rho\sigma}(k_1, k_2, k_3)} + \dots$$

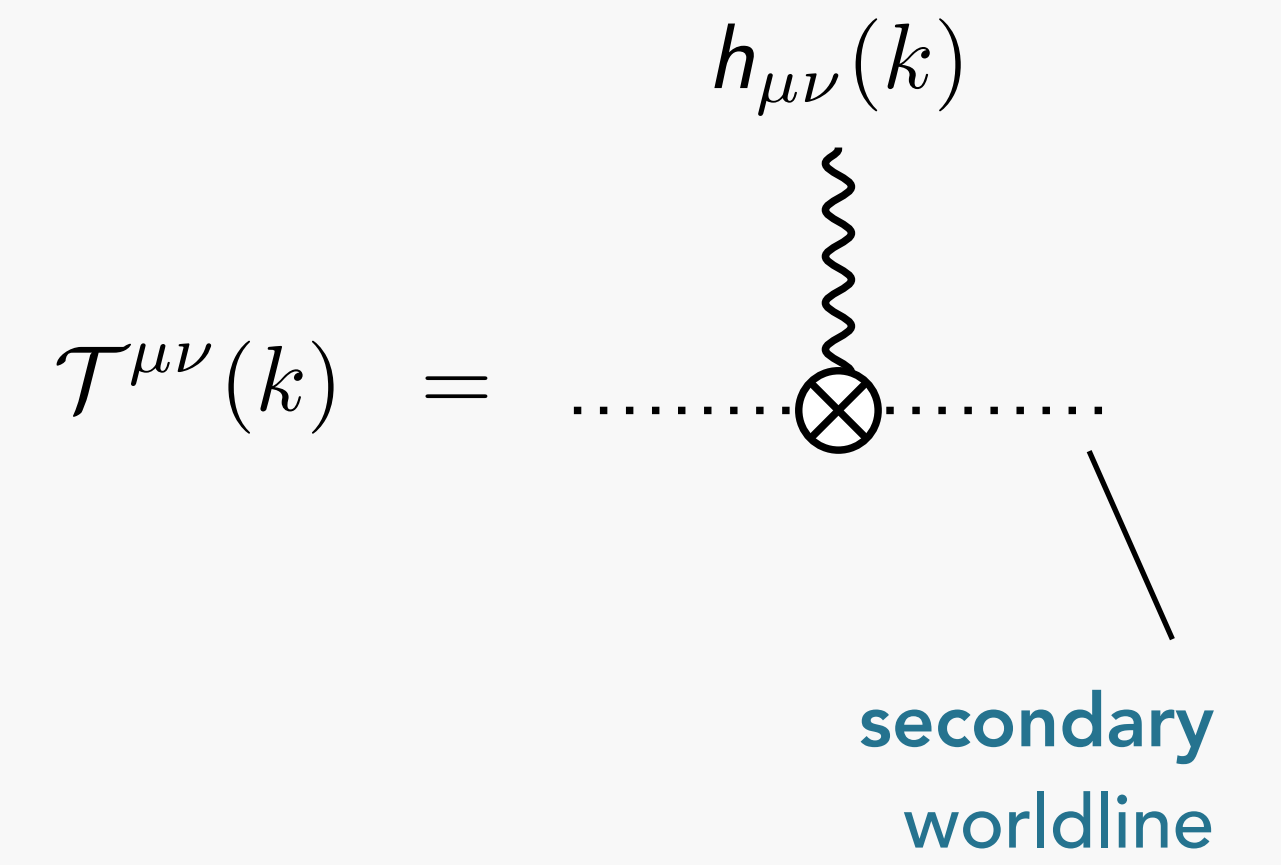
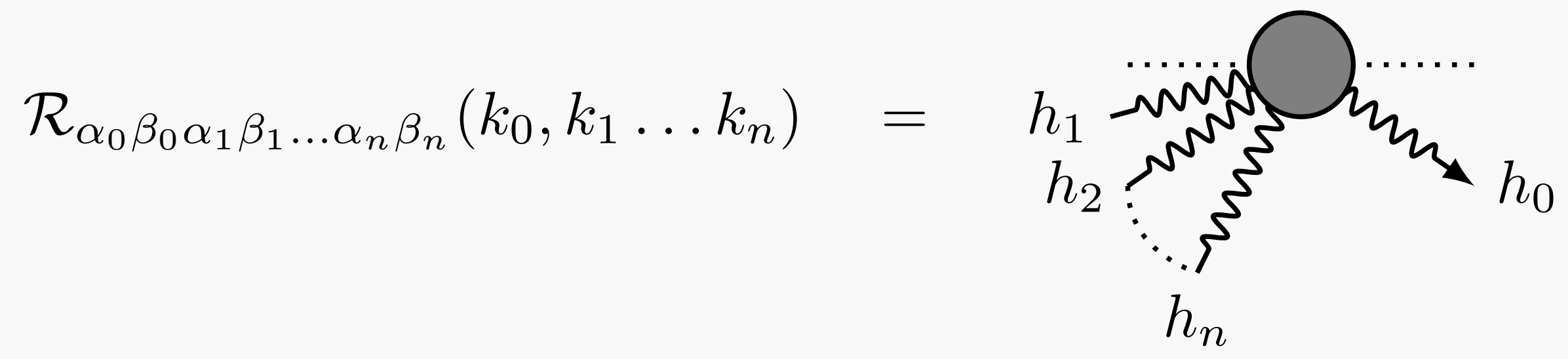
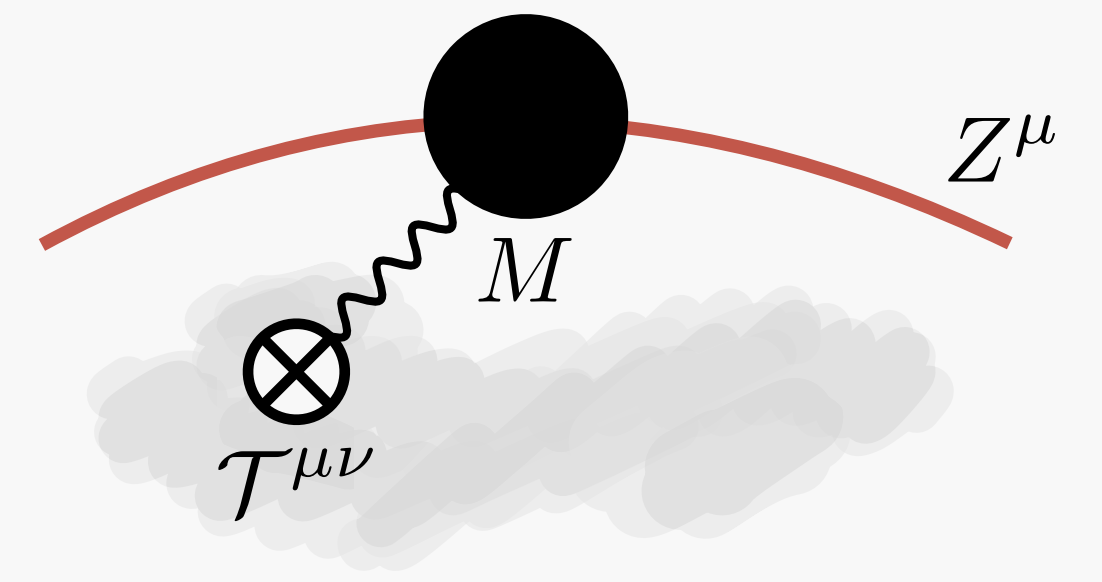
$$\mathcal{R}_{\alpha_0\beta_0\alpha_1\beta_1\dots\alpha_n\beta_n}(k_0, k_1 \dots k_n) \equiv \frac{\delta}{\delta\mathcal{T}_0^{(-)}} \frac{\delta}{\delta\mathcal{T}_1^{(+)}} \frac{\delta}{\delta\mathcal{T}_2^{(+)}} \frac{\delta}{\delta\mathcal{T}_n^{(+)}} iW[\mathcal{T}] \Big|_{\mathcal{T}=0}$$



Response functions \mathcal{R}

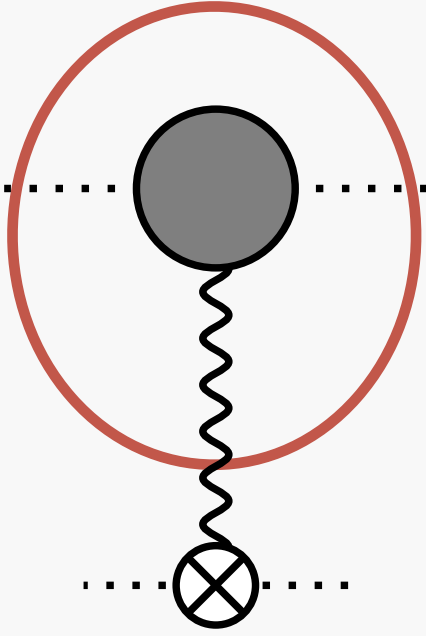
$$iW[\mathcal{T}] = \int_{k_1} \mathcal{T}^{\alpha\beta}(k_1) \underline{\mathcal{R}_{\alpha\beta}(k_1)} + \frac{1}{2} \int_{k_1, k_2} \mathcal{T}^{\alpha\beta}(k_1) \mathcal{T}^{\mu\nu}(k_2) \underline{\mathcal{R}_{\alpha\beta\mu\nu}(k_1, k_2)}$$

$$+ \frac{1}{3!} \int_{k_1, k_2, k_3} \mathcal{T}^{\alpha\beta}(k_1) \mathcal{T}^{\mu\nu}(k_2) \mathcal{T}^{\rho\sigma}(k_3) \underline{\mathcal{R}_{\alpha\beta\mu\nu\rho\sigma}(k_1, k_2, k_3)} + \dots$$



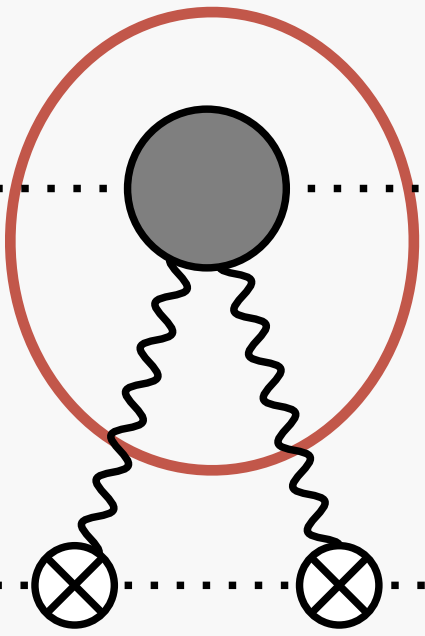
$$iW[\mathcal{T}] =$$

metric

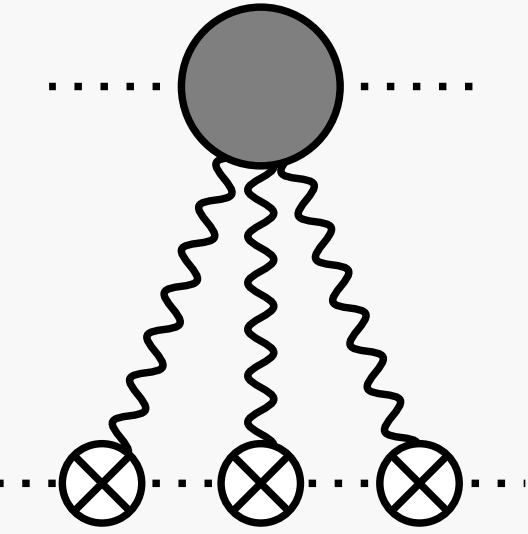


$\langle h_{\mu\nu} \rangle$

“Compton”



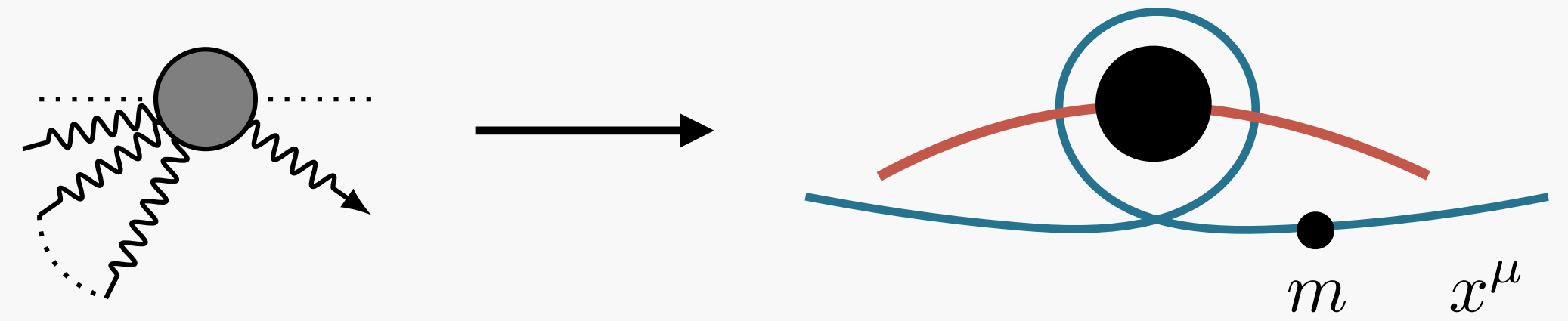
$\langle h_{\mu\nu} h_{\alpha\beta} \rangle |_{\text{con.}}$



\dots

Self-force from responses

Self force expansion - action



Point particle source

$$\mathcal{T}_{\text{sec}}^{\mu\nu}(x; x(\tau)) = \int d\tau \frac{m}{2} \dot{x}^\mu(\tau) \dot{x}^\nu(\tau) \delta^D(x - x(\tau))$$

BH response action

$$iS_{\text{BHR}}[z] = iS_{\text{kin}}[z] + iW[\mathcal{T}_{\text{sec}}^{\mu\nu}] = iS_{\text{kin}}[z] + \text{[Diagram 1]} + \frac{1}{2} \text{[Diagram 2]} + \frac{1}{3!} \text{[Diagram 3]} + \dots$$

$$S_{\text{kin}}[z] = - \int d\tau \frac{m}{2} \eta_{\mu\nu} \dot{x}^\mu \dot{x}^\nu$$

Expand around OSF trajectory

$$x^\mu(\tau) = \bar{x}^\mu(\tau) + z^\mu(\tau)$$

Self force expansion - Feynman rules

$$iS_{\text{BHR}}[z] = \underbrace{iS_{\text{kin}}[z]} + \text{[1-loop diagram]} + \frac{1}{2} \text{[2-loop diagram]} + \frac{1}{3!} \text{[3-loop diagram]} + \dots$$

Propagator

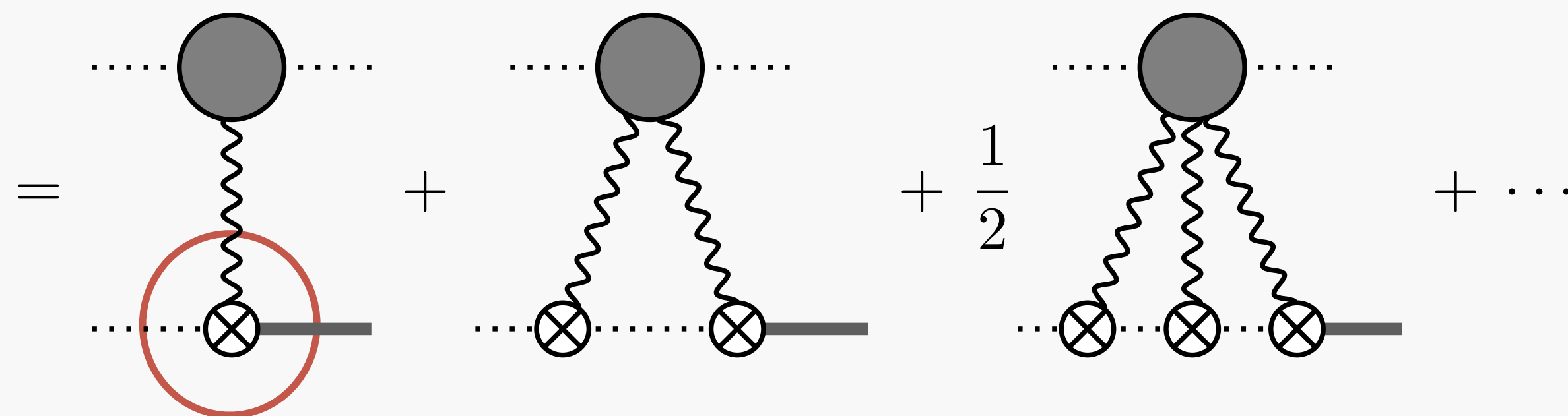
$$\langle z^\mu(\omega_1) z^\nu(\omega_2) \rangle = \dots \overset{\mu}{\otimes} \text{---} \overset{\nu}{\otimes} \dots = \left[\frac{\delta^2 S_{\text{BHR}}^{\text{OSF}}}{\delta z^\mu(\omega_1) \delta z^\mu(\omega_2)} \Big|_{x=\bar{x}} \right]^{-1}$$

Self force expansion - Feynman rules

$$iS_{\text{BHR}}[z] = iS_{\text{kin}}[z] + \text{[diagram: 1 vertex]} + \frac{1}{2} \text{[diagram: 2 vertices]} + \frac{1}{3!} \text{[diagram: 3 vertices]} + \dots$$

Vertices

$$V[z] \equiv \left. \frac{\delta iW[\mathcal{T}]}{\delta x^\mu(\omega)} \right|_{x=\bar{x}} = \int_k \frac{\delta iW[\mathcal{T}]}{\delta \mathcal{T}^{\alpha\beta}(k; x)} \frac{\delta \mathcal{T}^{\alpha\beta}(k; x)}{\delta x^\mu(\omega)} \Big|_{x=\bar{x}}$$



$$V[z^2] \equiv \left. \frac{\delta iW[\mathcal{T}]}{\delta x^\mu(\omega) \delta x^\nu(\omega')} \right|_{x=\bar{x}} \quad \text{etc...}$$

Evaluated @ OSF trajectory $\bar{x}^\mu(\omega)$

$$\mathcal{T}^{\mu\nu}(k) = \text{[diagram: shaded circle vertex connected to cross vertex by wavy line, labeled } h_{\mu\nu}(k)\text{]}$$

$$\left. \frac{\delta^n \mathcal{T}^{\mu\nu}(k)}{[\delta x(\omega)]^n} \right|_{x^\mu = \bar{x}^\mu} = \text{[diagram: shaded circle vertex connected to } n \text{ cross vertices by } n \text{ wavy lines, labeled } h_{\mu\nu}(k)\text{, with external legs } z^{\rho_1}(\omega_1), z^{\rho_2}(\omega_2), \dots, z^{\rho_n}(\omega_n)\text{]}$$

Self force diagrams

$$\langle z^\rho(\omega) \rangle = \frac{-i\eta^{\rho\sigma}}{m\omega^2} \frac{\delta iW[\mathcal{T}_{\text{sec.}}]}{\delta z^\sigma(\omega)} \Big|_{x=x^{\text{SF}}} = \text{Need 0SF!}$$

The diagrammatic expansion of the self-force is shown below, categorized by the number of self-force (SF) diagrams:

- 1SF:** A single particle (grey circle) is connected to two interaction points (crosses in circles) on a surface. The interaction points are circled in red. This diagram is enclosed in a blue rounded rectangle.
- 2SF:** Two particles are connected to two interaction points on a surface. A red horizontal line is drawn above this diagram.
- 3SF:** Three particles are connected to three interaction points on a surface. A red horizontal line is drawn above this diagram.
- 4SF:** Four particles are connected to four interaction points on a surface. A red horizontal line is drawn above this diagram.

The expansion is further detailed with the following terms:

- $\mathcal{O}(m)$: A diagram with two particles and two interaction points. The left interaction point is circled in blue.
- $\mathcal{O}(1/m)$: A diagram with two particles and two interaction points. A blue horizontal line is drawn below the surface.
- $\frac{1}{2}$: A diagram with three particles and three interaction points.
- $\mathcal{O}\left(\frac{m^3}{M^3}\right)$: A diagram with four particles and four interaction points.

A red horizontal line is drawn below the $\mathcal{O}(1/m)$ and $\frac{1}{2}$ terms, with the label **2SF** centered below it.

Self force diagrams

$$-i\langle h_{\mu\nu}(k) \rangle = \left. \frac{\delta iW[\mathcal{T}]}{\delta \mathcal{T}} \right|_{\mathcal{T}=\mathcal{T}_{\text{sec.}}} = \text{1SF} + \frac{1}{2} \text{2SF} + \mathcal{O}\left(\frac{m^3}{M^3}\right)$$

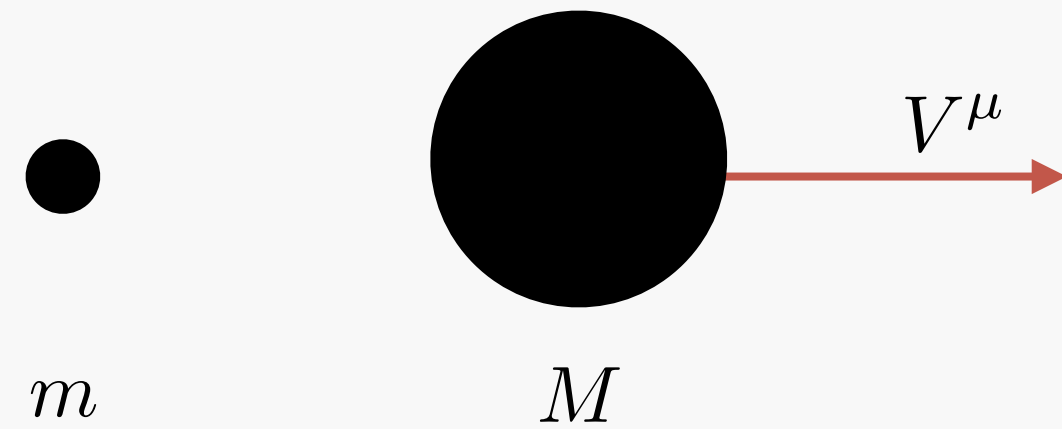
1SF
2SF

Self-force on shockwave

The shockwave

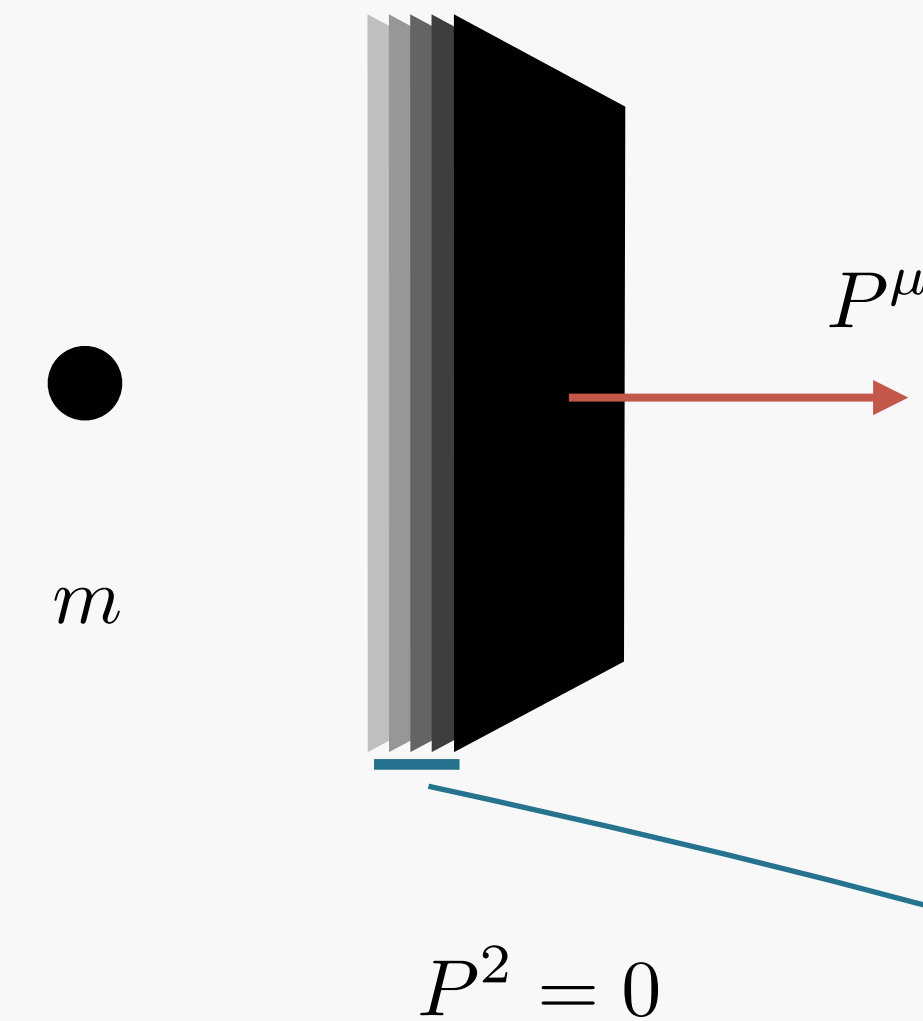
[Aichelburg, Sexl; 1971]

SF: $m/M \ll 1$



massless
 $V^\mu \rightarrow P^\mu / E$

SF: $m/E \ll 1$



$$g_{\mu\nu}(x) = \eta_{\mu\nu} + 4G f(\mathbf{x}_\perp^2) \delta(P \cdot x) P_\mu P_\nu \quad f(\mathbf{x}_\perp^2) = -\Gamma(-\epsilon) \left(\frac{\mathbf{x}_\perp^2}{4e^{\gamma_E} L^2} \right)^\epsilon$$

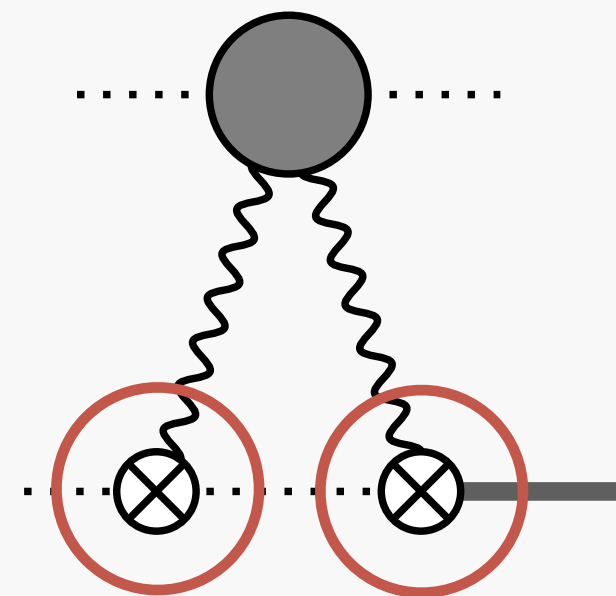
Simplicity of shockwave

- $P^2 = 0 \Rightarrow$ cancellations
- Localisation on shockwave

Finite-width UV regulator $1/\Lambda$ [Steinbauer; 1997]

- $\delta(P \cdot x) \rightarrow \rho_\Lambda(P \cdot x) = \Lambda/2 \exp(-\Lambda|\sigma|)$
- $\Lambda \rightarrow \infty$ recovers single shockwave limit

0SF dynamics



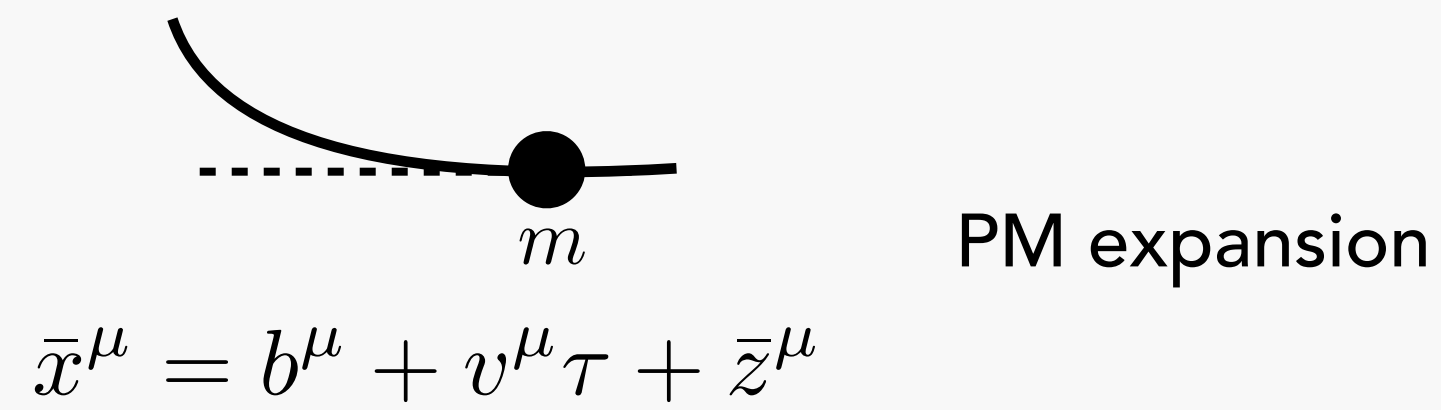
OSF - geodesics on shockwave

$$iS_{\text{BHR}}[z] = iS_{\text{kin}}[z] + \text{[diagram: 1 wavy line]} + \frac{1}{2} \text{[diagram: 2 wavy lines]} + \frac{1}{3!} \text{[diagram: 3 wavy lines]} + \dots$$

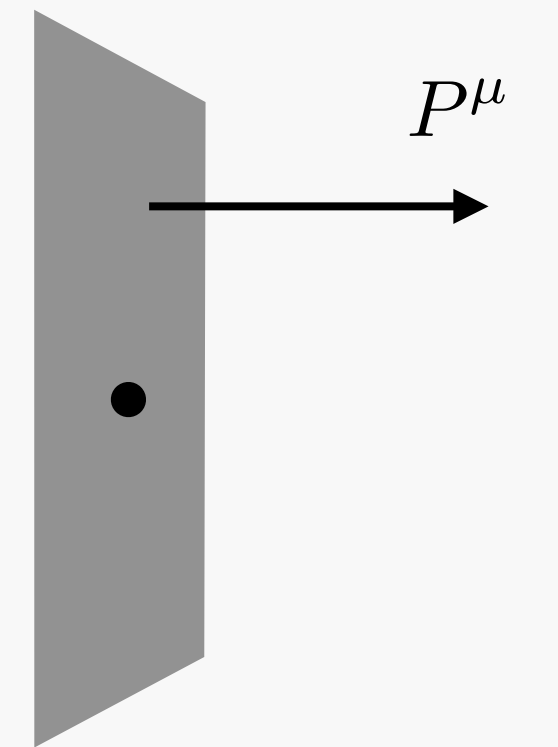
$$iS^{\text{OSF}}[\bar{x}] = -i\frac{m}{2} \int d\tau \bar{g}_{\mu\nu} \dot{\bar{x}}^\mu \dot{\bar{x}}^\nu = iS_{\text{kin}}[\bar{x}] + \text{[diagram: 1 wavy line]}$$

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + 4Gf(\mathbf{x}_\perp^2)\delta(P \cdot x)P_\mu P_\nu$$

$$f(\mathbf{x}_\perp^2) = -\Gamma(-\epsilon) \left(\frac{\mathbf{x}_\perp^2}{4e^{\gamma_E} L^2} \right)^\epsilon$$



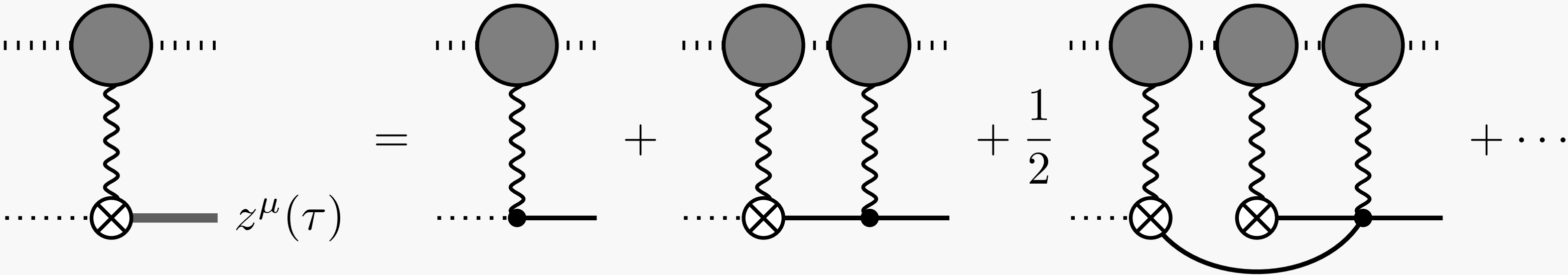
$$-m \langle \ddot{z}^\mu(\tau) \rangle = \eta^{\mu\nu} \frac{\delta S_{\text{int}}[x]}{\delta x^\nu(\tau)} = \text{[diagram: 1 wavy line with external leg } \bar{z}^\mu(\tau)\text{]}$$



Berends-Giele recursion

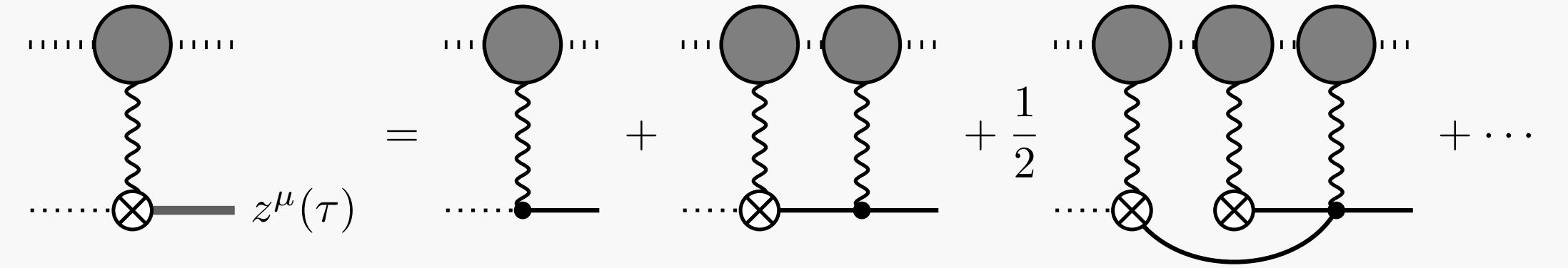
7PM @ $\mathcal{O}(s^4)$ in Kerr

[JH, Jakobsen, Plefka; 2506.14626]



Trajectory truncates at 2PM

Obtaining shockwave geodesics



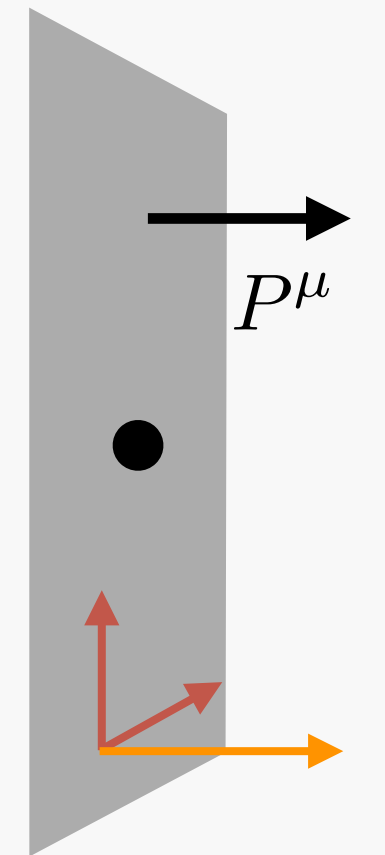
$$S_{\text{int}}^{\text{OSF}}[x] = -\frac{m}{2} \int d\tau \bar{h}_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = -\frac{m}{2} \int d\tau 4G\delta(P \cdot x) f(\mathbf{x}_\perp) (\dot{x}^-)^2$$

$$\ddot{z}^-(\tau) = -\frac{1}{m} \eta^{-+} \frac{\delta S_{\text{int}}[x]}{\delta x^+} = 0$$

$$\eta^{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \mathbf{1} \end{pmatrix}$$

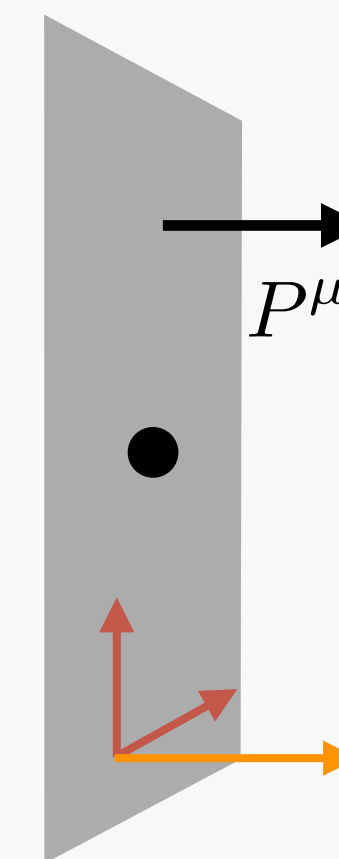
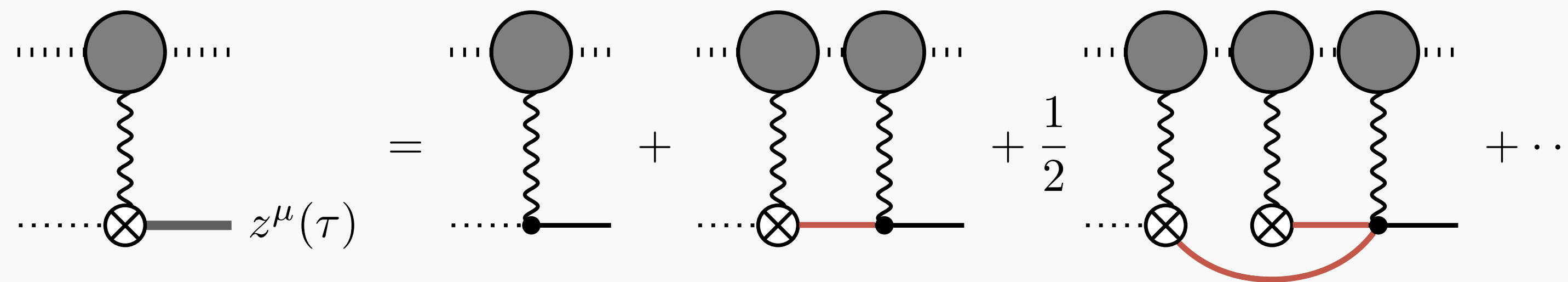
Structure from Berends-Giele

$$\sim \frac{\delta S_{\text{int}}[x]}{\delta x^+} (\tau_2 - \tau_1) \theta(\tau_2 - \tau_1) \frac{\delta S_{\text{int}}[x]}{\delta x^-} = 0$$



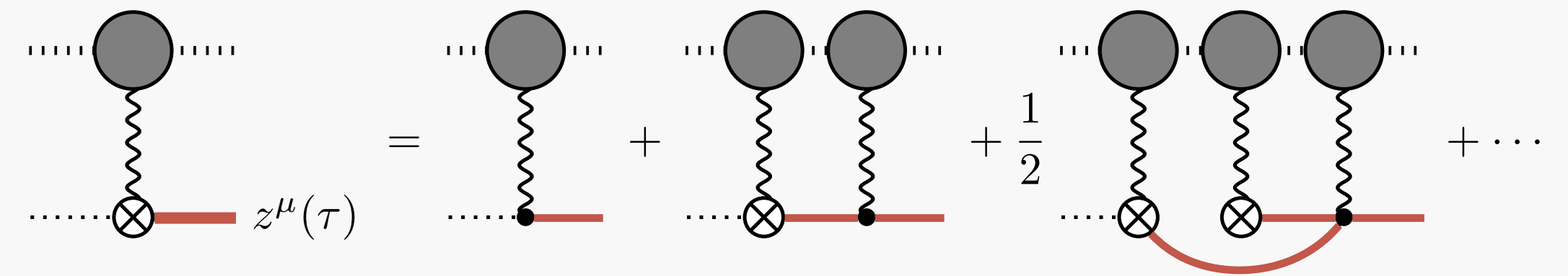
$$x^\mu = \frac{1}{2E} (x^- \bar{P}^\mu + x^+ P^\mu) + \mathbf{x}_\perp^\mu$$

Obtaining shockwave geodesics

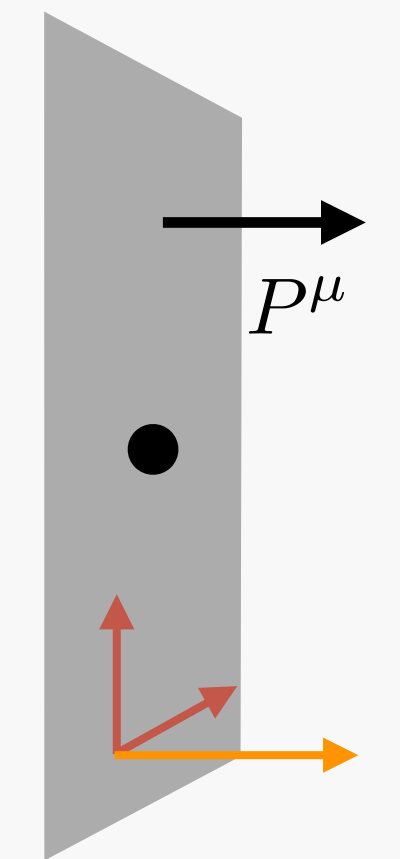
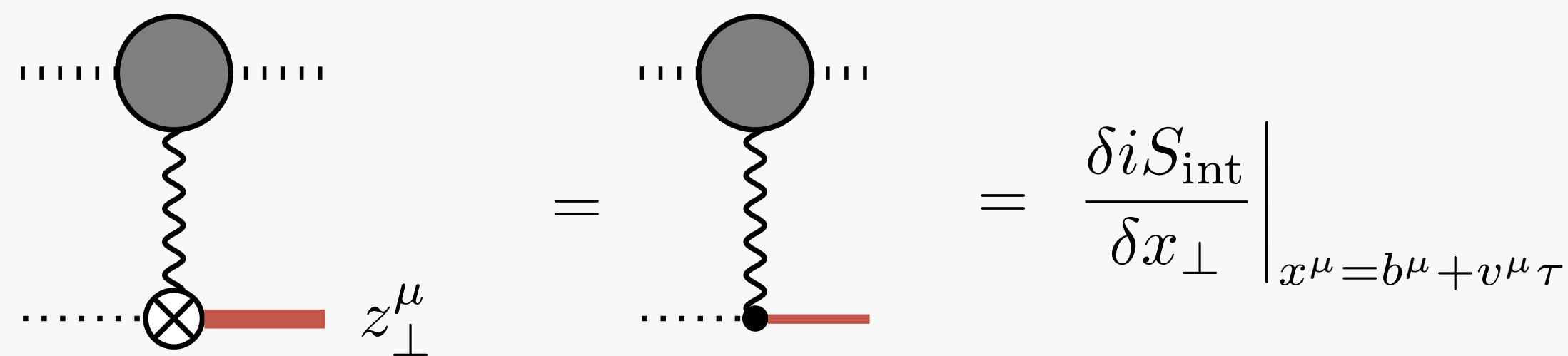
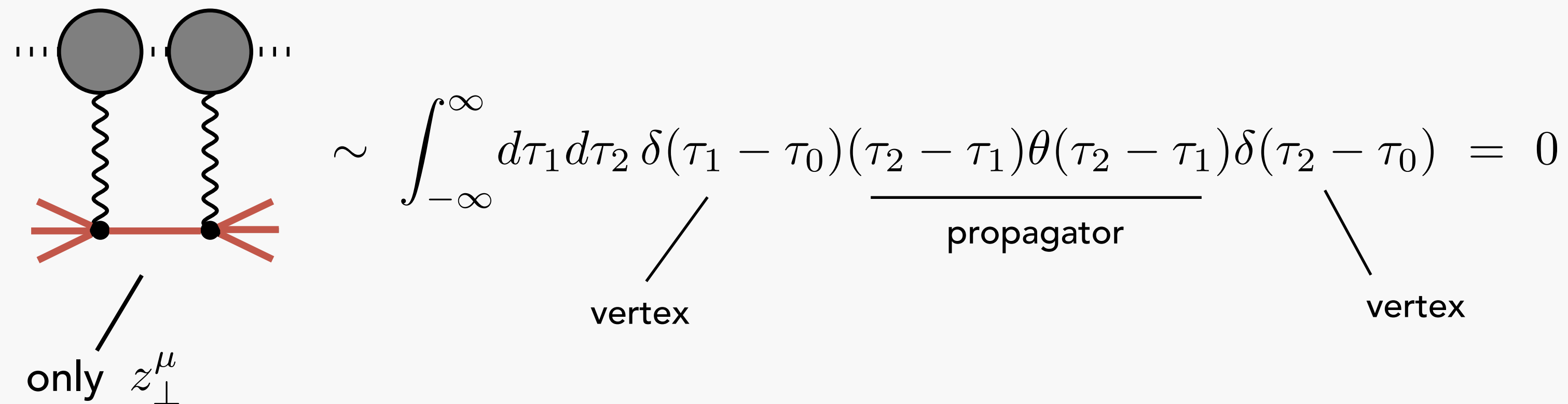


$$x^\mu = \frac{1}{2E} \left(\underbrace{x^- \bar{P}^\mu}_{\text{blue}} + \underbrace{x^+ P^\mu}_{\text{orange}} \right) + \underbrace{\mathbf{x}_\perp^\mu}_{\text{red}}$$

Obtaining shockwave geodesics

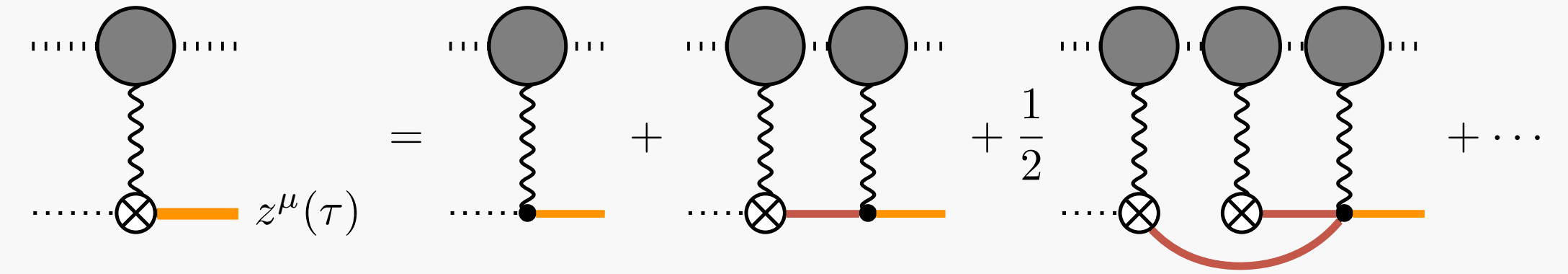


External component z^μ_\perp

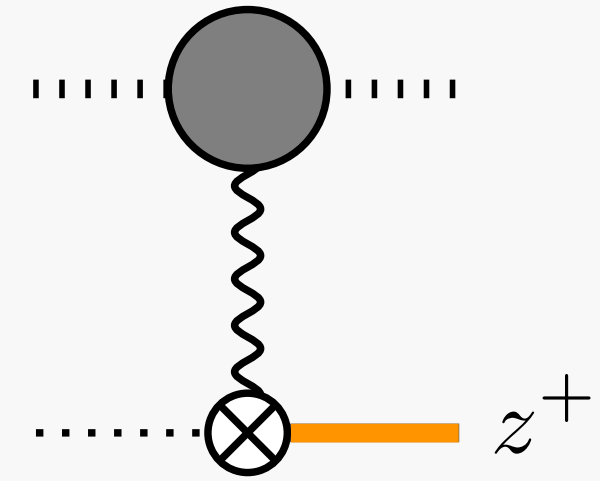


$$x^\mu = \frac{1}{2E} \left(\underbrace{x^- \bar{P}^\mu}_{\text{blue}} + \underbrace{x^+ P^\mu}_{\text{orange}} \right) + \underbrace{\mathbf{x}^\mu_\perp}_{\text{red}}$$

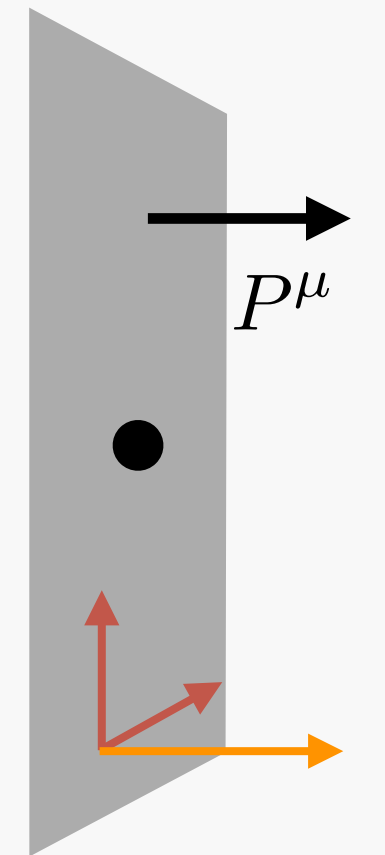
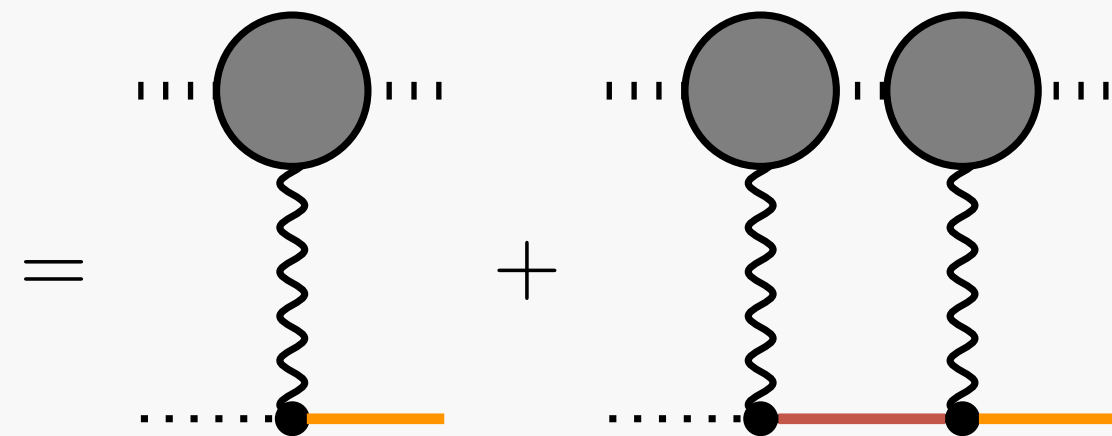
Obtaining shockwave geodesics



External component z^+

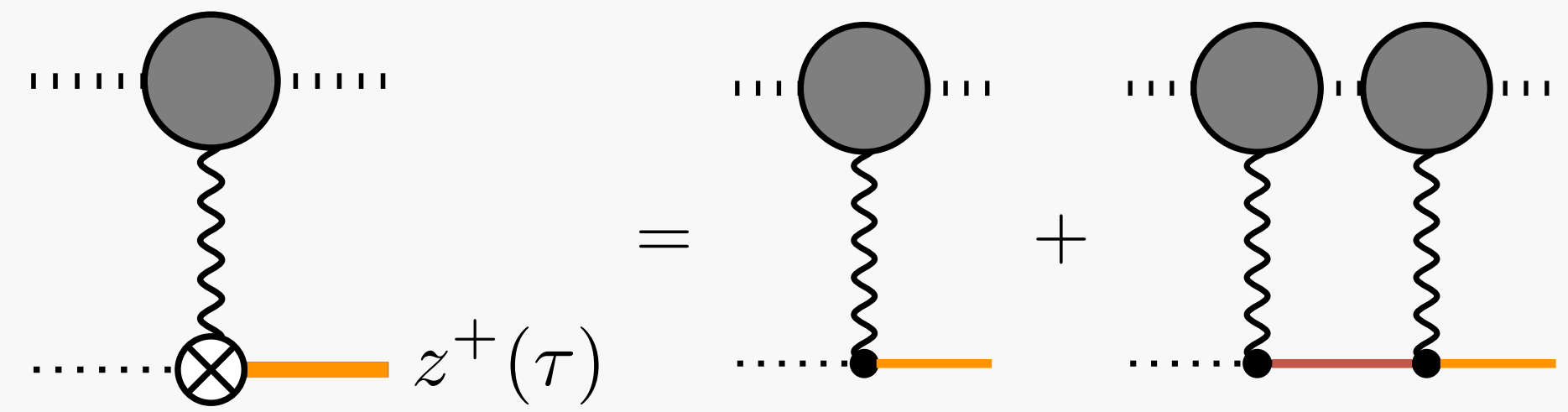
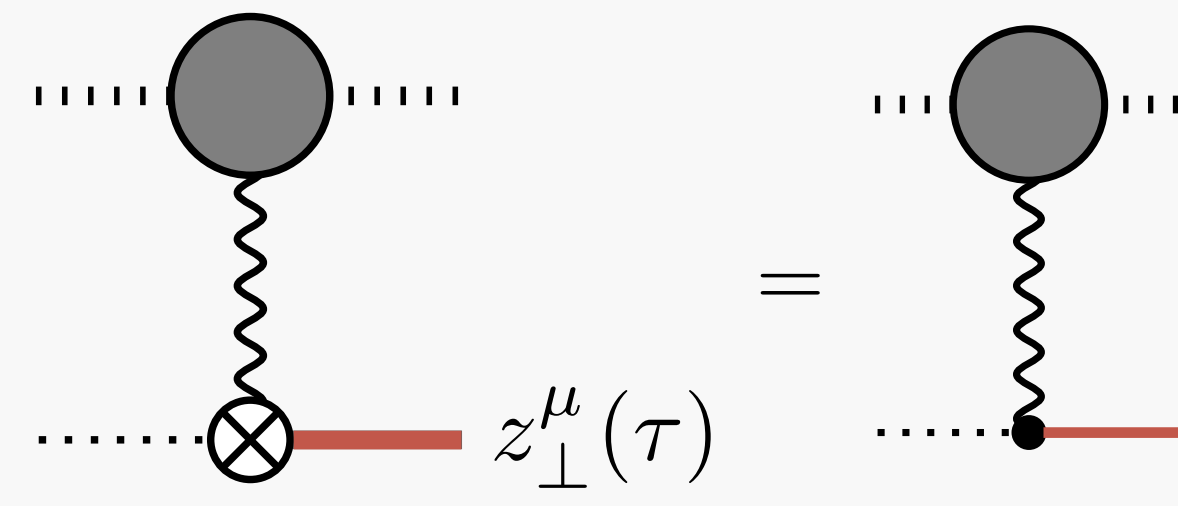
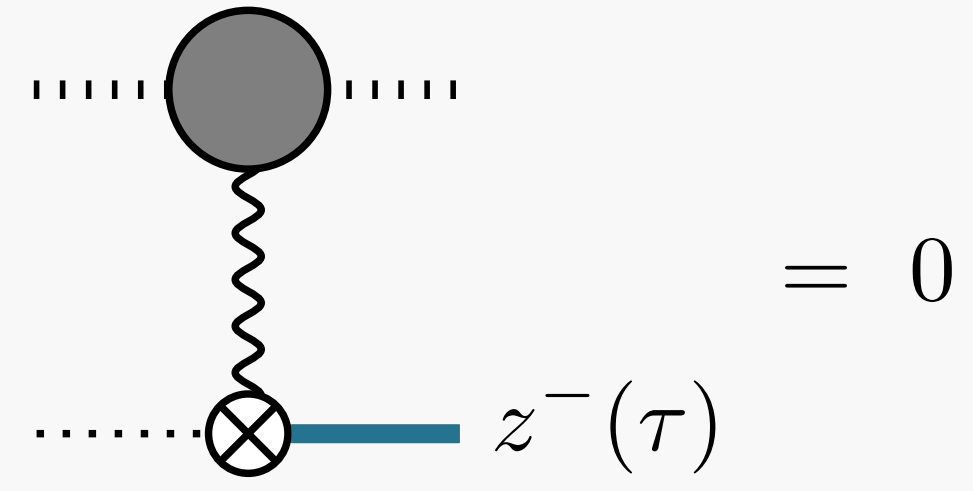
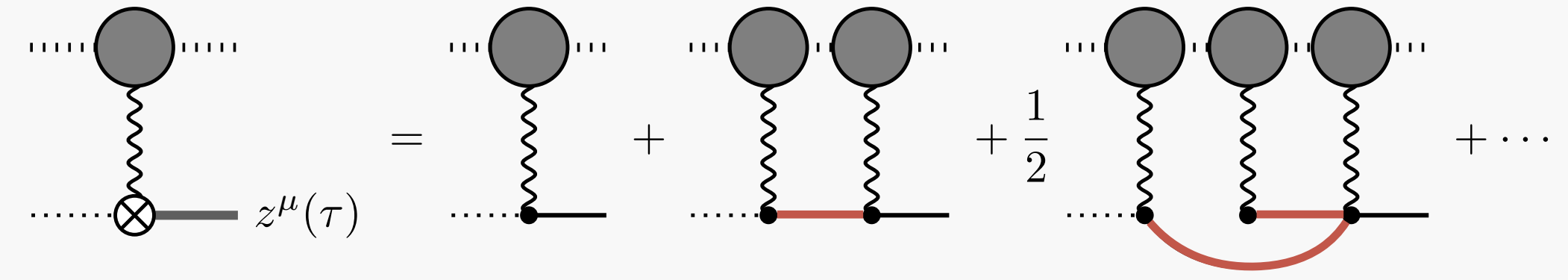


= Taylor expansion **only** in $z^+_{\perp} = \frac{\delta i S_{\text{int}}}{\delta z^-} \Big|_{z^-=0, z^+=0, z^+_{\perp}=1\text{PM}}$
 [JH, Jakobsen, Plefka; 2506.14626]

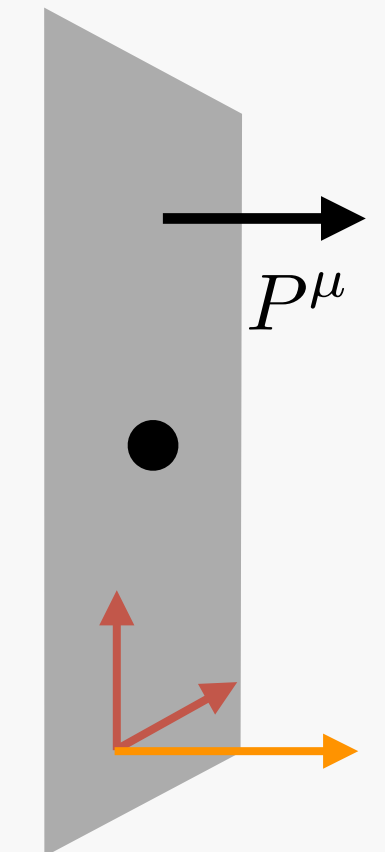


$$x^\mu = \frac{1}{2E} \left(\underbrace{x^-}_{\text{blue}} \bar{P}^\mu + \underbrace{x^+}_{\text{orange}} P^\mu \right) + \underbrace{\mathbf{x}^\mu_{\perp}}_{\text{red}}$$

Geodesics summary



2PM truncation



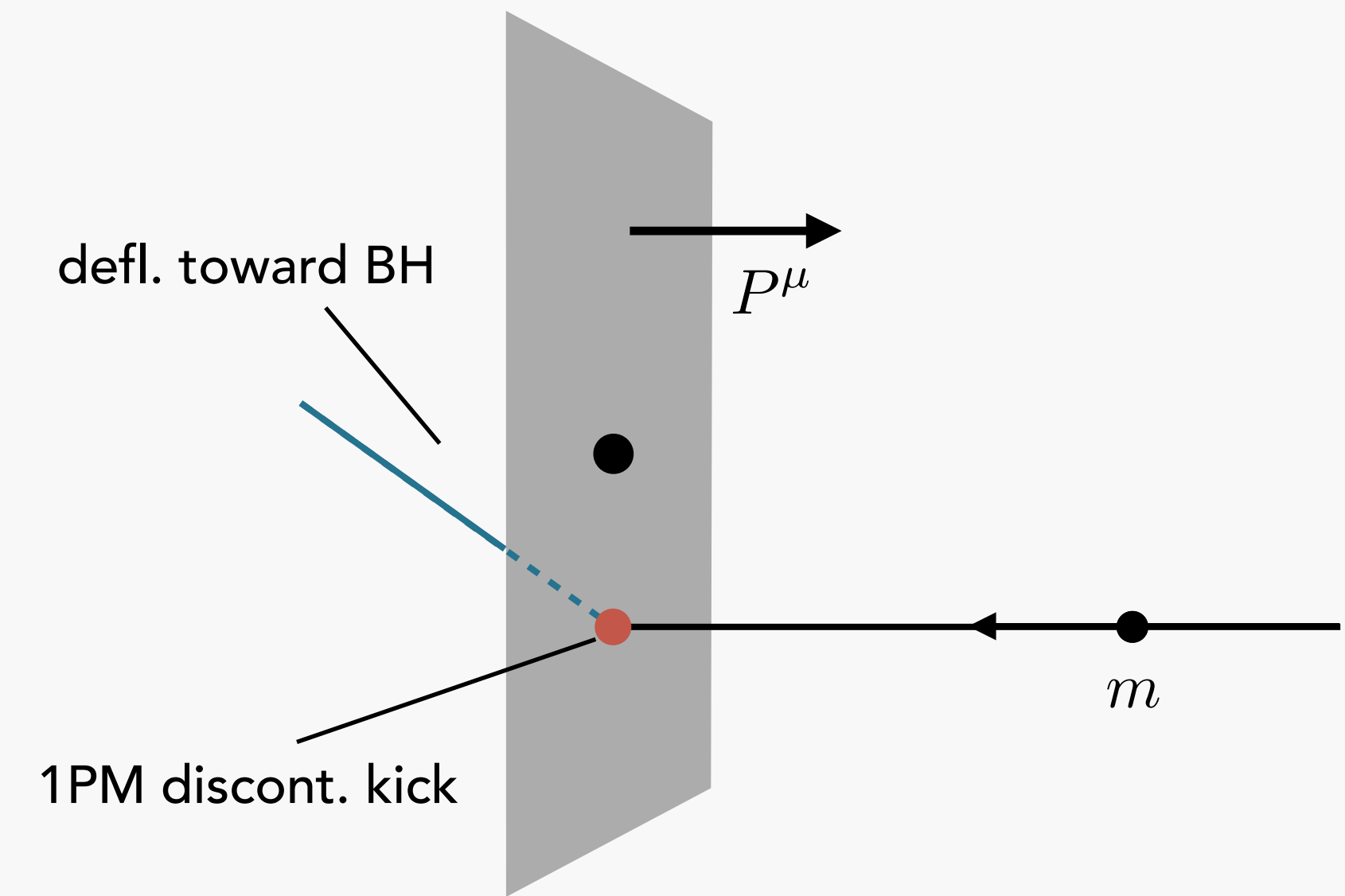
$$x^\mu = \frac{1}{2E} \left(\underline{x^-} \bar{P}^\mu + \underline{x^+} P^\mu \right) + \underline{\mathbf{x}_\perp^\mu}$$

Geodesics summary

$$\bar{x}^\mu(\tau) = (v^\mu \tau + b)\theta(\tau_0 - \tau) + (v_\infty^\mu(\tau - \tau_0) + b_\infty^\mu)\theta(\tau - \tau_0)$$

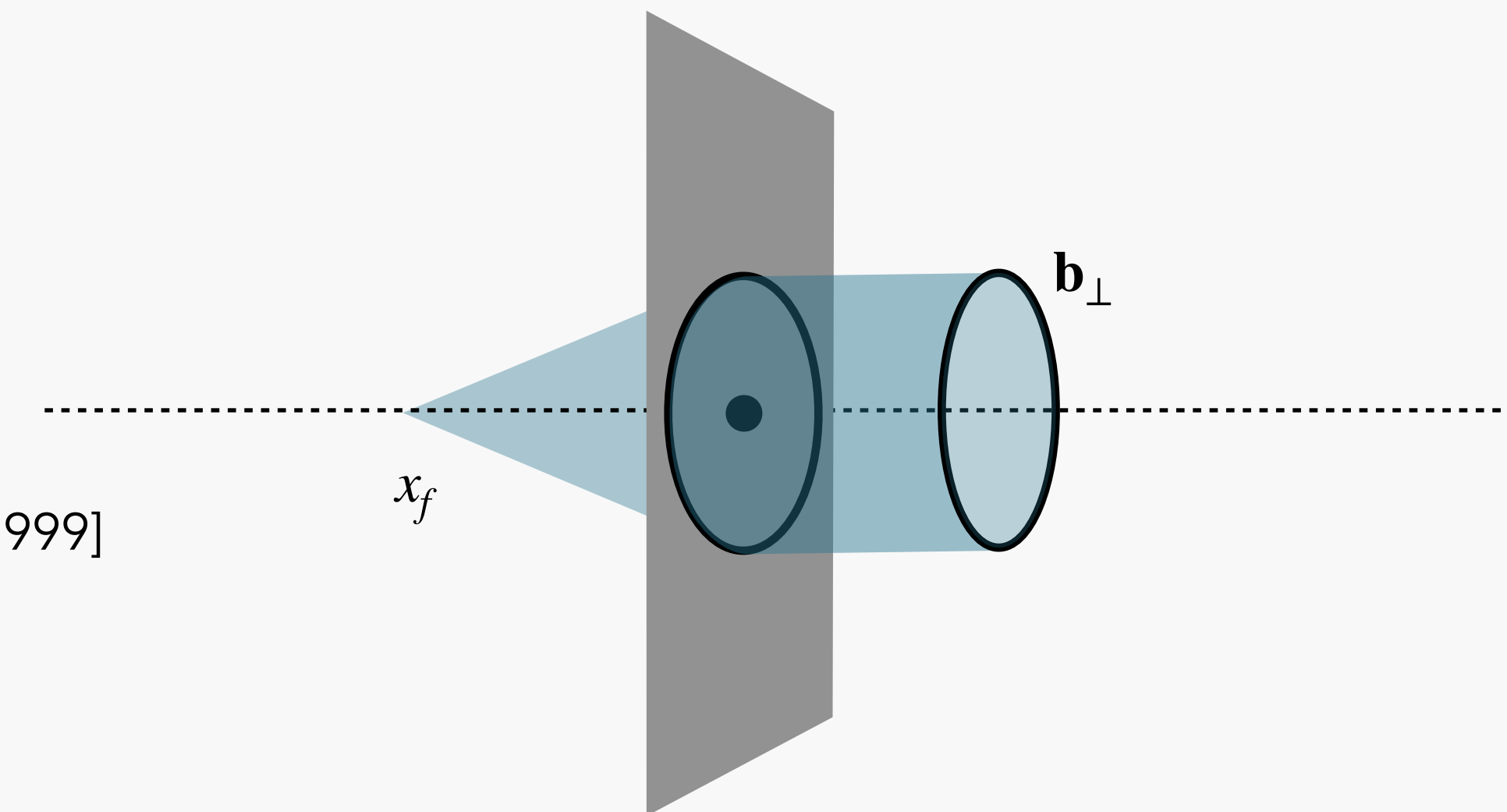
$$v_\infty^\mu = v^\mu + \Delta v^\mu = v^\mu - \frac{4GE\epsilon f(\mathbf{b}_\perp^2)}{|\mathbf{b}_\perp|^2} \left(\mathbf{b}_\perp^\mu - \underline{2G\epsilon f(\mathbf{b}_\perp^2) P^\mu} \right)$$

$$b_\infty^\mu = b^\mu + v^\mu \tau_0 + \Delta b^\mu = b^\mu + v^\mu \tau_0 - \underline{2GP^\mu f(\mathbf{b}_\perp^2)}$$



Focusing lens for null geodesics

[Ferrari, Pendenza, Veneziano; 1988]



$$x_f = \frac{1}{2} (f(\mathbf{b}_\perp^2) + 2b/f'(\mathbf{b}_\perp^2) + bf'(\mathbf{b}_\perp^2))$$

Interpretation

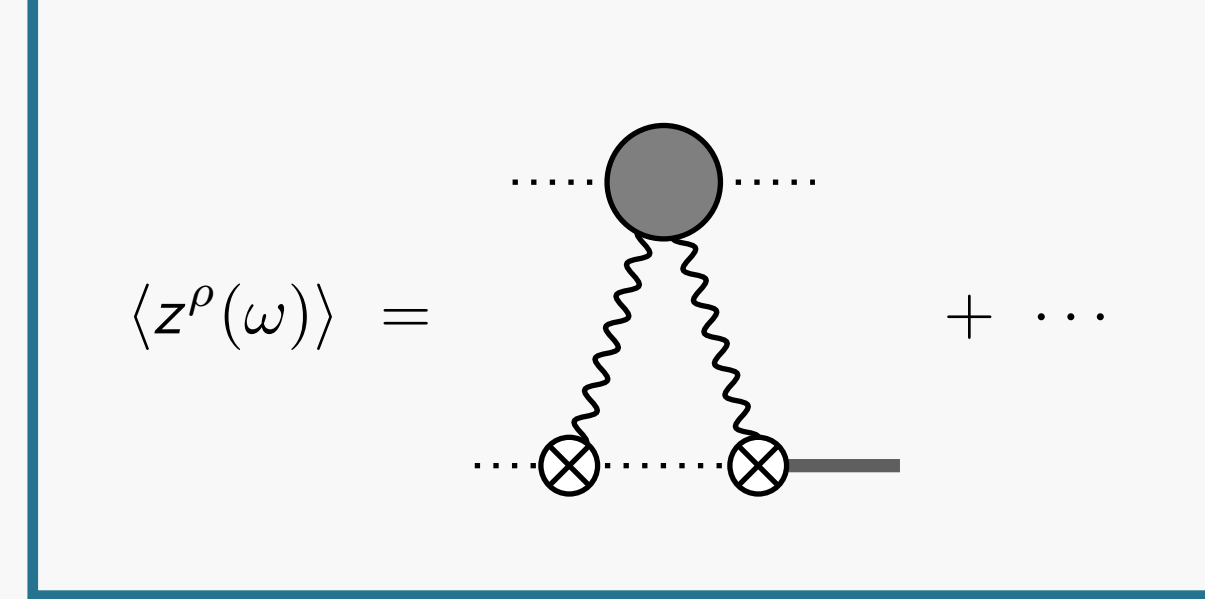
[Dray, 't Hooft; 1985] [Steinbauer; 1998]

● 2 copies of $\eta_{\mu\nu}$ (Penrose junction)

[Penrose; 1972] [Kunzinger, Steinbauer; 1999]

$$b_\infty^\mu \sim \frac{1}{\epsilon} + \log\left(\frac{\mathbf{x}^2}{\tilde{L}}\right) \rightarrow \log\left(\frac{\mathbf{x}^2}{\tilde{L}'}\right)$$

OSF - resummed vertices



$$h^{\mu\nu}(k) = \mathcal{T}_{\text{sec.}}^{\mu\nu}(k; x)|_{x=\bar{x}} =$$

$$= -\frac{m\kappa_D}{2} e^{ik \cdot (v\tau_0 + b)} \left(\frac{v^\mu v^\nu}{v \cdot k - i0} - \frac{e^{ik \cdot \Delta \bar{b}} v_\infty^\mu v_\infty^\nu}{v_\infty \cdot k + i0} \right) - im\kappa_D e^{ik \cdot \bar{x}(\tau_0)} (v^{(\mu} \Delta \bar{b}^{\nu)}) + \theta(0) \Delta \bar{v}^{(\mu} \Delta \bar{b}^{\nu)})$$

eval @ OSF

$$h_{\mu\nu}(k) = \frac{\delta \mathcal{T}_{\text{sec.}}^{\mu\nu}(k; x)}{\delta x^\rho} \Big|_{x=\bar{x}} =$$

$$= -\frac{im\kappa_D}{2} e^{ik \cdot (v\tau_0 + b)} \left[2\omega \left(\frac{v^{(\mu} \delta_\rho^{\nu)}}{v \cdot k + \omega - i0} - \frac{e^{ik \cdot \Delta \bar{b}} v_\infty^{(\mu} \delta_\rho^{\nu)}}{v_\infty \cdot k + \omega + i0} \right) + k_\rho \left(\frac{v^\mu v^\nu}{v \cdot k + \omega - i0} - \frac{e^{ik \cdot \Delta \bar{b}} v_\infty^\mu v_\infty^\nu}{v_\infty \cdot k + \omega + i0} \right) \right] - m\kappa_D e^{ik \cdot \bar{x}(\tau_0) + i\omega\tau_0} (\omega \Delta \bar{b}^{(\mu} \delta_\rho^{\nu)}) + k_\rho v^{(\mu} \Delta \bar{b}^{\nu)} + \theta(0) k_\rho \Delta \bar{v}^{(\mu} \Delta \bar{b}^{\nu)})$$

- < kick
- > kick
- = kick

[Steinbauer; 1998]

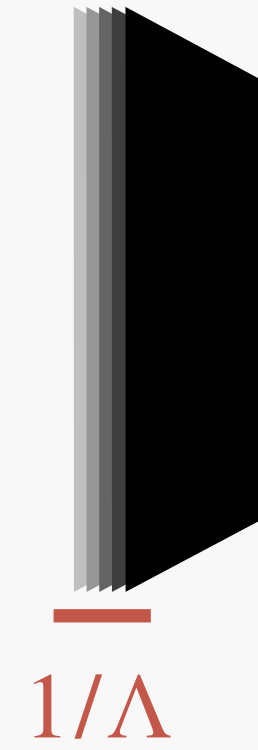
$$\theta(0) = 1/2$$

relation to flat-space

$$e^{ik \cdot b} i\delta(k \cdot v + \Omega) \left\{ \begin{matrix} v^\mu v^\nu \\ v^\mu \end{matrix} \right\} \rightarrow \int d\tau e^{ik \cdot \bar{x}(\tau) + i\Omega\tau} \left\{ \begin{matrix} \dot{\bar{x}}^\mu(\tau) \dot{\bar{x}}^\nu(\tau) \\ \dot{\bar{x}}^\mu(\tau) \end{matrix} \right\}$$

Towards 1SF

Towards 1SF



$$\langle z^\rho(\omega) \rangle = \text{Diagram} + \mathcal{O}\left(\frac{m^2}{E^2}\right)$$

The diagram shows a grey sphere (representing a mass) enclosed in a blue oval. Two wavy lines (representing gravitons) extend downwards from the sphere to two red circles, each containing a cross (representing a detector). A horizontal dashed line passes through the sphere and the detector circles.

$$-i\langle h_{\mu\nu}(k) \rangle = \text{Diagram} + \mathcal{O}\left(\frac{m^2}{E^2}\right)$$

The diagram shows a grey sphere enclosed in a blue oval. A single wavy line extends downwards from the sphere to a red circle containing a cross. A horizontal dashed line passes through the sphere and the detector circle.

Geodesics

2-pt response

$$\mathcal{R}^{\mu\nu\rho\sigma}(k_1, k_2) \sim [\Pi \cdot \Pi]^{\mu\nu\rho\sigma} \int d^{D-2} \mathbf{x}_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{x}_\perp} [e^{-2iG(P \cdot k_1) f(x_\perp)} - 1]$$

Caveat: Method of regions

$k_1 \sim k_2 \sim \Lambda^0$ appropriate on-shell

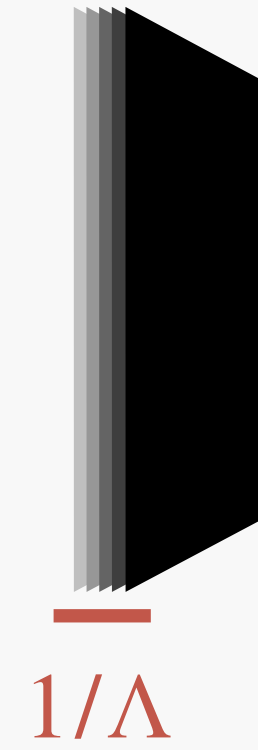
- ▶ Off-shell: $k_1 \sim \Lambda^a, k_2 \sim \Lambda^b$
- ▶ 1SF Waveform: No additional regions!

$$\int_{\ell} \frac{1}{(\ell - \mathbf{q}_\perp)^2 ((\mathbf{k}_1^\perp + \ell)^2 - k_1^- (k_1^+ + i\Lambda))}$$

Towards 1SF

$$\langle z^\rho(\omega) \rangle = \text{Diagram} + \mathcal{O}\left(\frac{m^2}{E^2}\right)$$

$$-i\langle h_{\mu\nu}(k) \rangle = \text{Diagram} + \mathcal{O}\left(\frac{m^2}{E^2}\right)$$



Geodesics

2-pt response

$$\mathcal{R}^{\mu\nu\rho\sigma}(k_1, k_2) \sim [\Pi \cdot \Pi]^{\mu\nu\rho\sigma} \int d^{D-2} \mathbf{x}_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{x}_\perp} [e^{-2iG(P \cdot k_1) f(x_\perp)} - 1]$$

Resummation pending!

Summary

- ▶ **Response Theory** efficient EFT for Self-Force

- 1SF = 2-pt response coupled to **geodesics**

- ▶ **Shockwave**

- 2-pt response **exponentiates @ all PM orders**
 - Geodesics truncate at 2PM

- ▶ **1SF observables** on the way!

$$\langle z^\rho(\omega) \rangle = \text{Diagram} + \mathcal{O}\left(\frac{m^2}{M^2}\right)$$

$$W[\mathcal{T}] = \text{Diagram 1} + \frac{1}{2} \text{Diagram 2} + \frac{1}{3!} \text{Diagram 3} + \dots$$



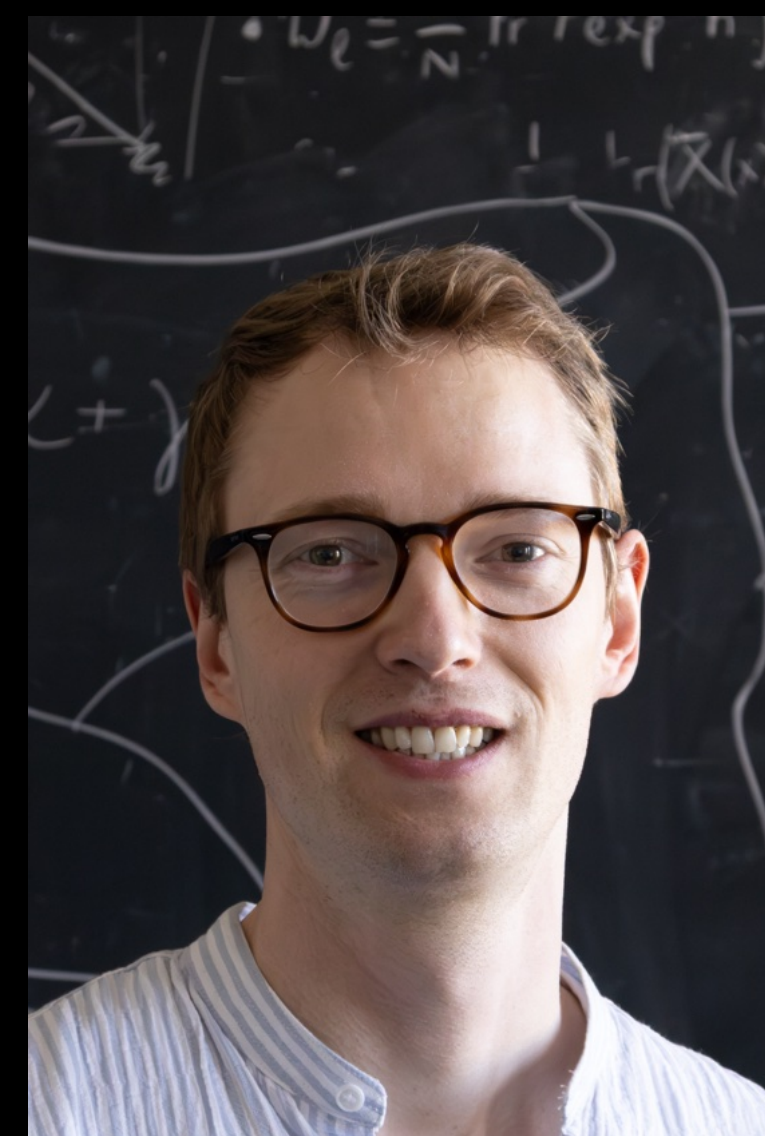
Lara Bohnenblust



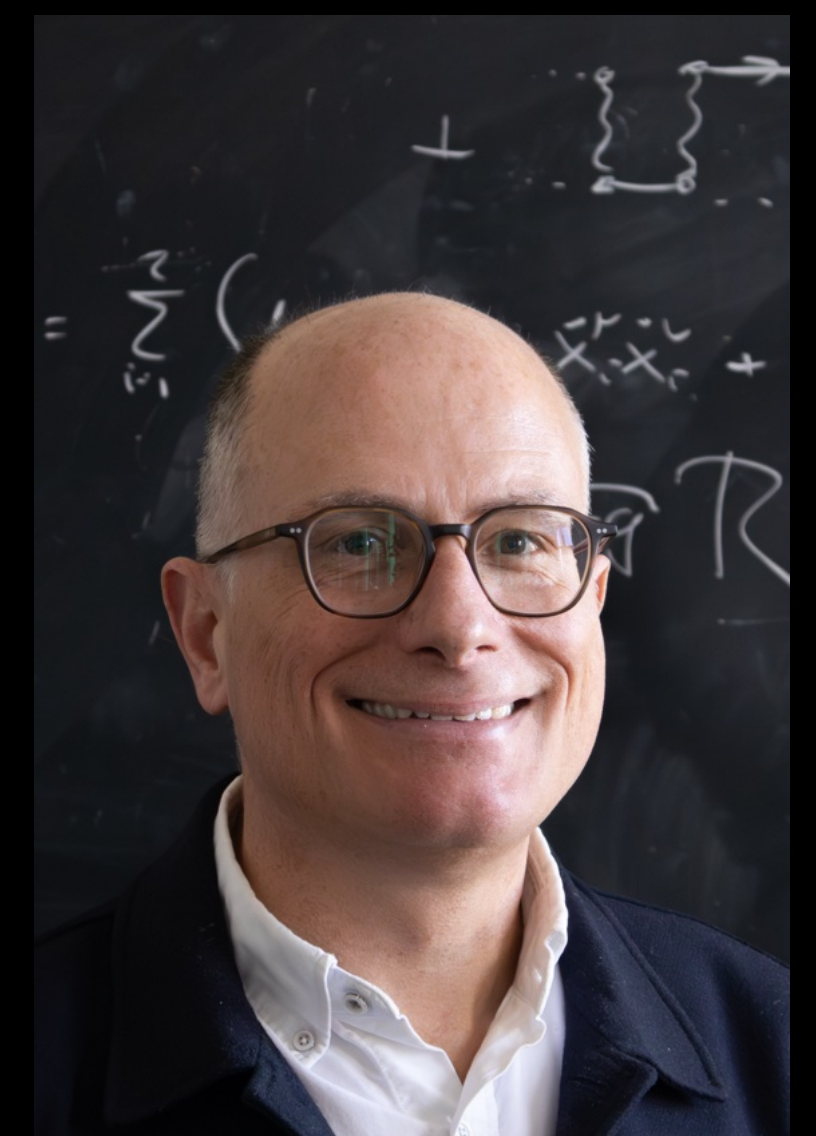
Carl Jordan Eriksen



Jitze Hoogeveen



Gustav Uhre Jakobsen



Jan Plefka