The information lattice

Jens Hjörleifur Bárðarson

KTH Royal Institute of Technology, Stockholm

Claudia Artiaco
Cologne

Yuliya Bilinskaya
KTH

Matthias Flór
KTH

Soumi Ghosh
KTH

Loïc Herviou
Grenoble

Thomas
Klein Kvorning
KTH

Miguel Martínez
KTH

+ Christoph Fleckenstein (Parity QC) & David Aceituno (KTH)

SciPost Physics **13**, 080 (2022); PRX Quantum **5**, 020352 (2024); PRL **134**, 190401 (2025); arXiv:2510.26696; arXiv:2512:20793

Overview

The **information lattice** provides a decomposition of the total information $I(\rho)$ in a quantum state ρ into scale- and position-resolved **local information** i_n^ℓ

$$\boxed{I(\rho) = \sum_{n,\ell} i_n^\ell}$$

The local information i_n^ℓ is **locally conserved** on the information lattice (spanning the values of n and ℓ)

its structure provides a **universal characterisation** of states and

its dynamics the basis for **time evolution** algorithms like LITE (local information time evolution)

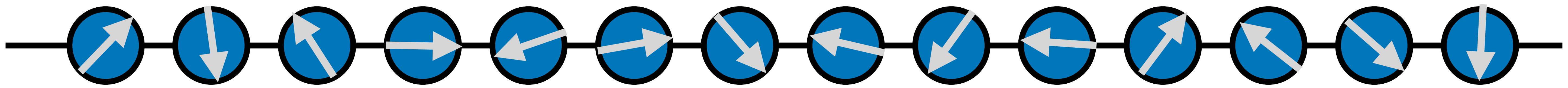
[Klein Kvorning, Herviou, JHB SciPost Physics (2022); Artiaco, Fleckenstein, Aceituno Chávez, Klein Kvorning, JHB PRX Quantum 2024]

How much information is there in the state ρ of L qubits?

$$I(\rho) = \log_2(\dim\rho) - S(\rho) = L + \mathrm{tr}\,\rho\log\rho$$

$$= L + \min_{\{\Lambda_y\}} \sum_y \mathrm{Tr}\,(\Lambda_y\rho)\,\log\left[\mathrm{Tr}\,(\Lambda_y\rho)\right]$$

Example: product state has L bits corresponding to measuring the direction of each qubit



Example: infinite temperature state $\rho_\infty = \mathbb{1}/\dim(\rho)$ has no information

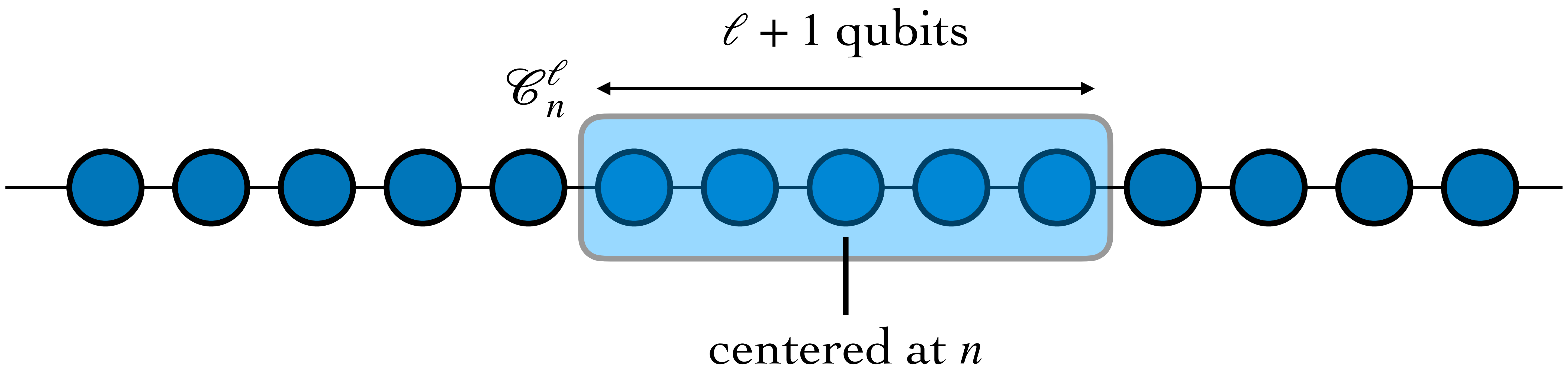
Example: singlet $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$ has two bits corresponding to, for example, measuring $-XX$ and $-ZZ$

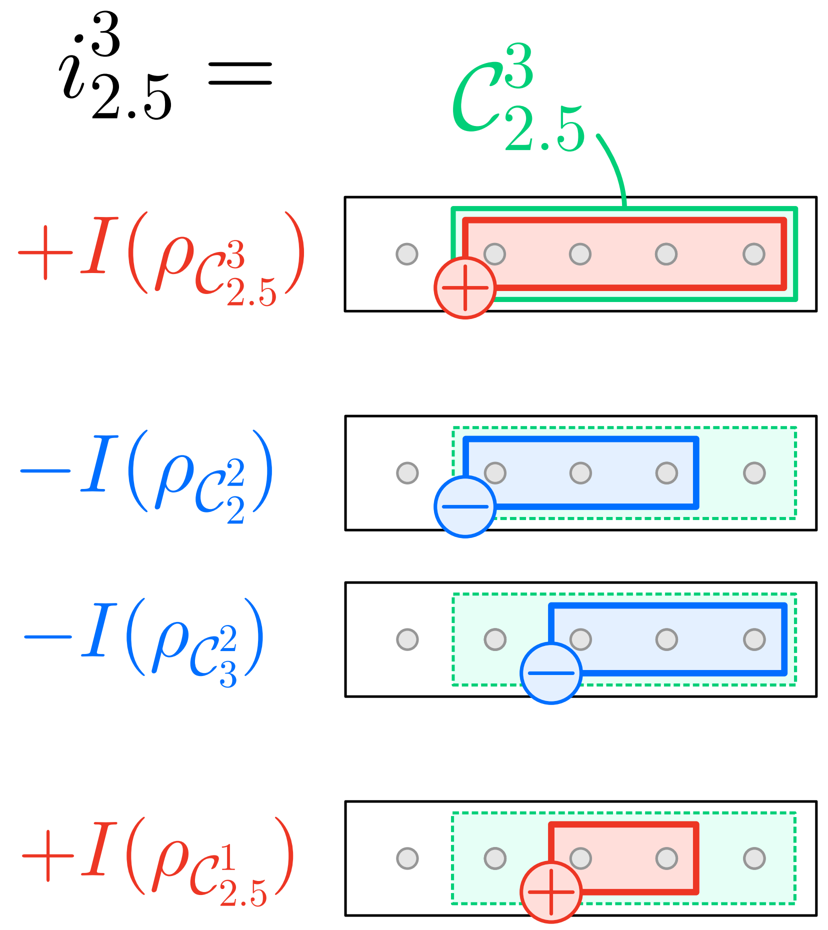
Example: stabiliser state $Q_i|\psi\rangle = |\psi\rangle$ with L commuting stabilisers $[Q_i, Q_j] = 0$

[Klein Kvorning, Herviou, JHB SciPost Physics (2022); Artiaco, Fleckenstein, Aceituno Chávez, Klein Kvorning, JHB PRX Quantum 2024]

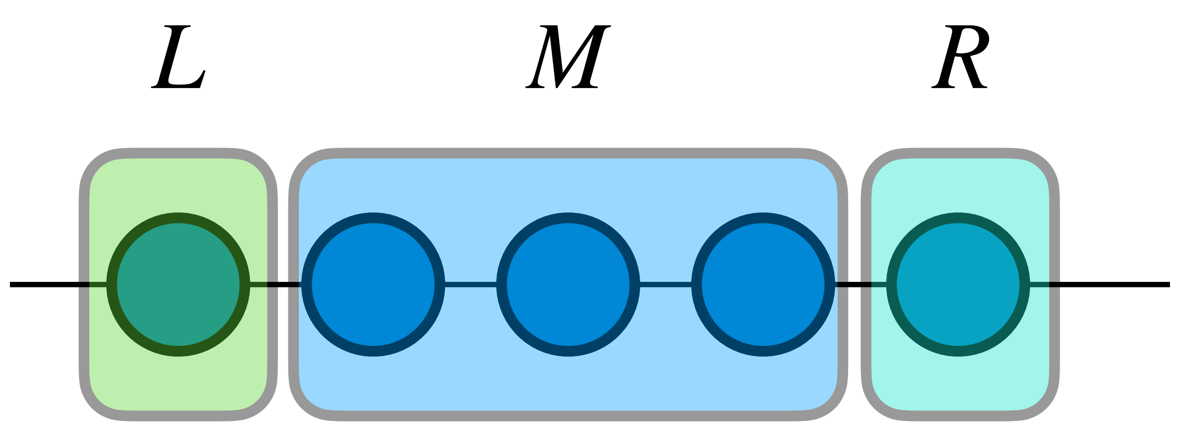
Local information

The local information i_n^ℓ is the information in a subsystem $\mathscr{C}_n^\ell$ that is not contained in any smaller subsystem





$$\boxed{i_n^\ell = I_n^\ell - I_{n-1/2}^{\ell-1} - I_{n+1/2}^{\ell-1} + I_n^{\ell-2} \geq 0}$$

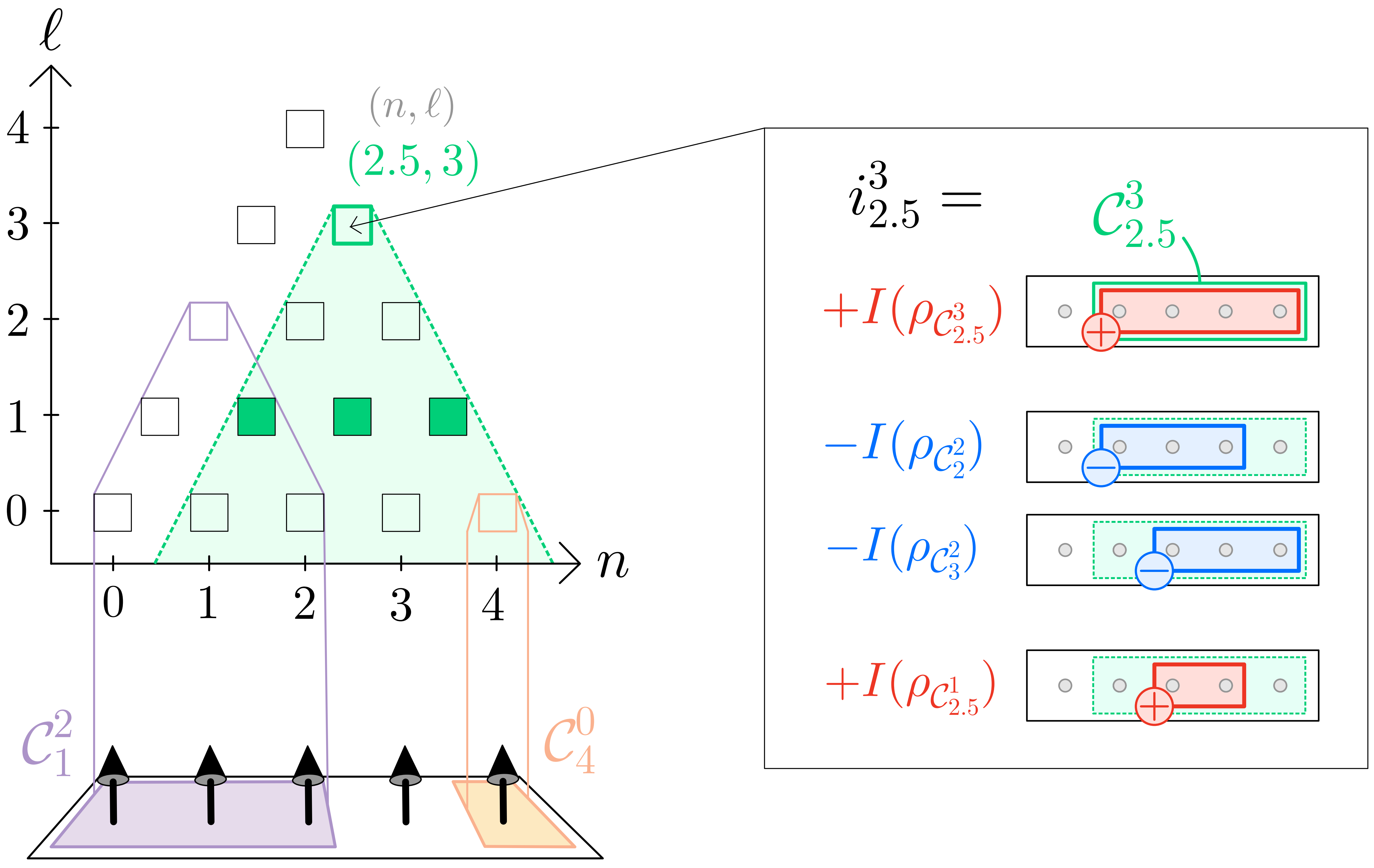


$$i(LMR) = I(LMR) - I(LM) - I(MR) + I(M) = I(L : R \,|\, M)$$

[Klein Kvorning, Herviou, JHB SciPost Physics (2022); Artiaco, Fleckenstein, Aceituno Chávez, Klein Kvorning, JHB PRX Quantum 2024]

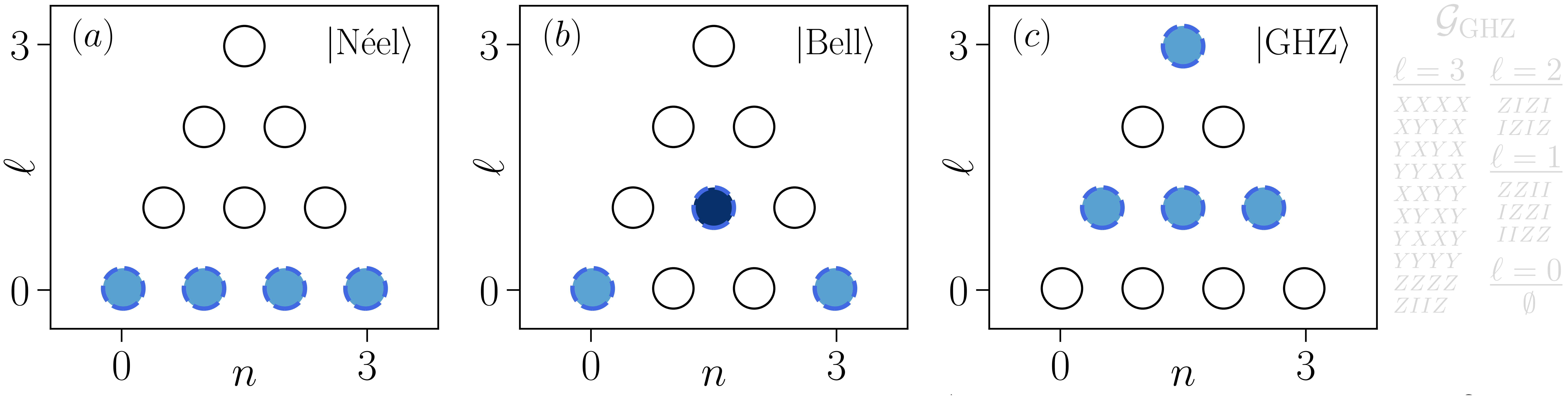
The information lattice is pictorially visualised as a triangle

Each point (n, ℓ) corresponds to a subsystem $\mathscr{C}_n^\ell$ and the value at the point is the corresponding local information i_n^ℓ



[Klein Kvorning, Herviou, JHB SciPost Physics (2022); Artiaco, Fleckenstein, Aceituno Chávez, Klein Kvorning, JHB PRX Quantum 2024]

Information lattice for stabiliser states has integer information



$$|\text{Néel}\rangle = |0101\rangle \qquad |\text{Bell}\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes (|01\rangle + |10\rangle) \otimes |0\rangle \qquad |\text{GHZ}\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$$

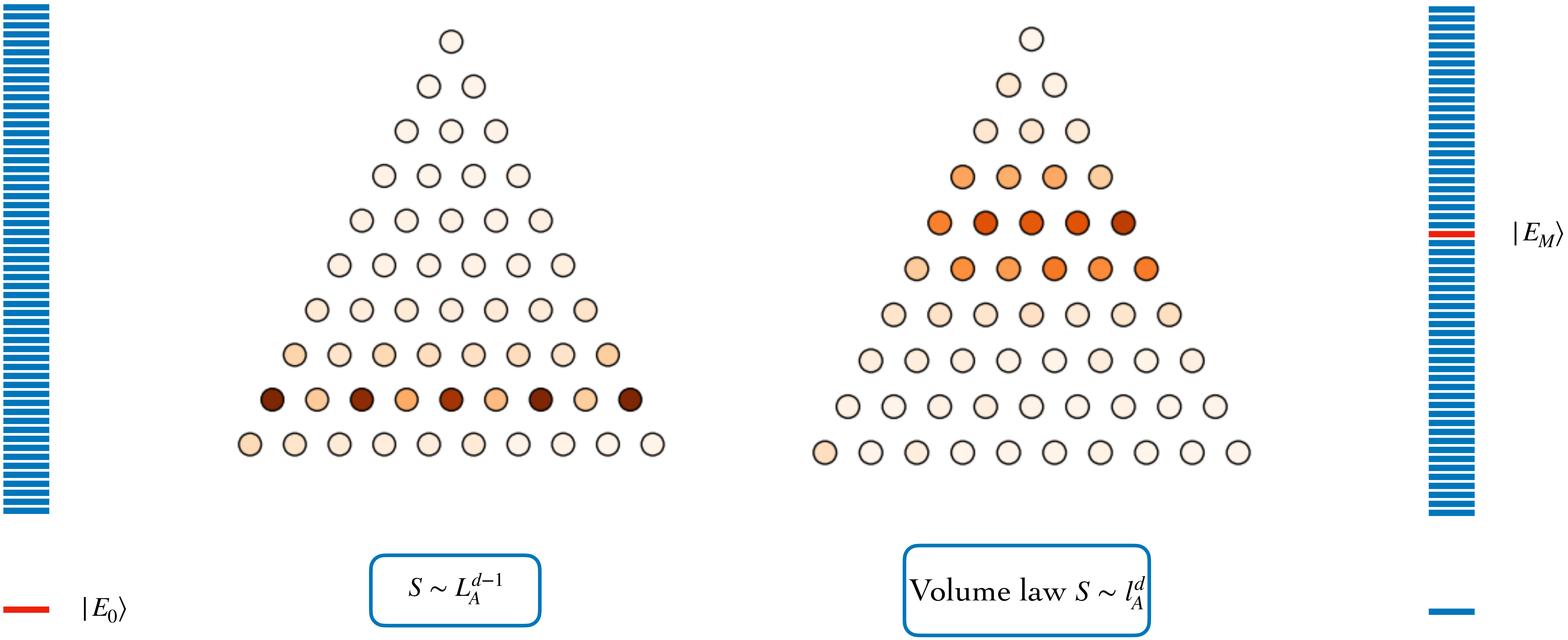
$$IXXI|\text{Bell}\rangle = |\text{Bell}\rangle$$

$$IZZI|\text{Bell}\rangle = |\text{Bell}\rangle$$

$$i_n^\ell = |\mathcal{G}_n^\ell| - |\mathcal{G}_{n-1/2}^{\ell-1}| - |\mathcal{G}_{n+1/2}^{\ell-1}| + |\mathcal{G}_n^{\ell-2}| = \in \{0,1,2\}$$ where $\mathcal{G}_n^\ell$ is the stabilizer subgroup of subsystem $\mathcal{C}_n^\ell$

[Bilinskaya, Martínez, Ghosh, Klein Kvorning, Artiaco, JHB, arXiv:2510.26696]

The information lattice for eigenstates of the XXZ model with random field



Total information on a given scale is characteristic of the state

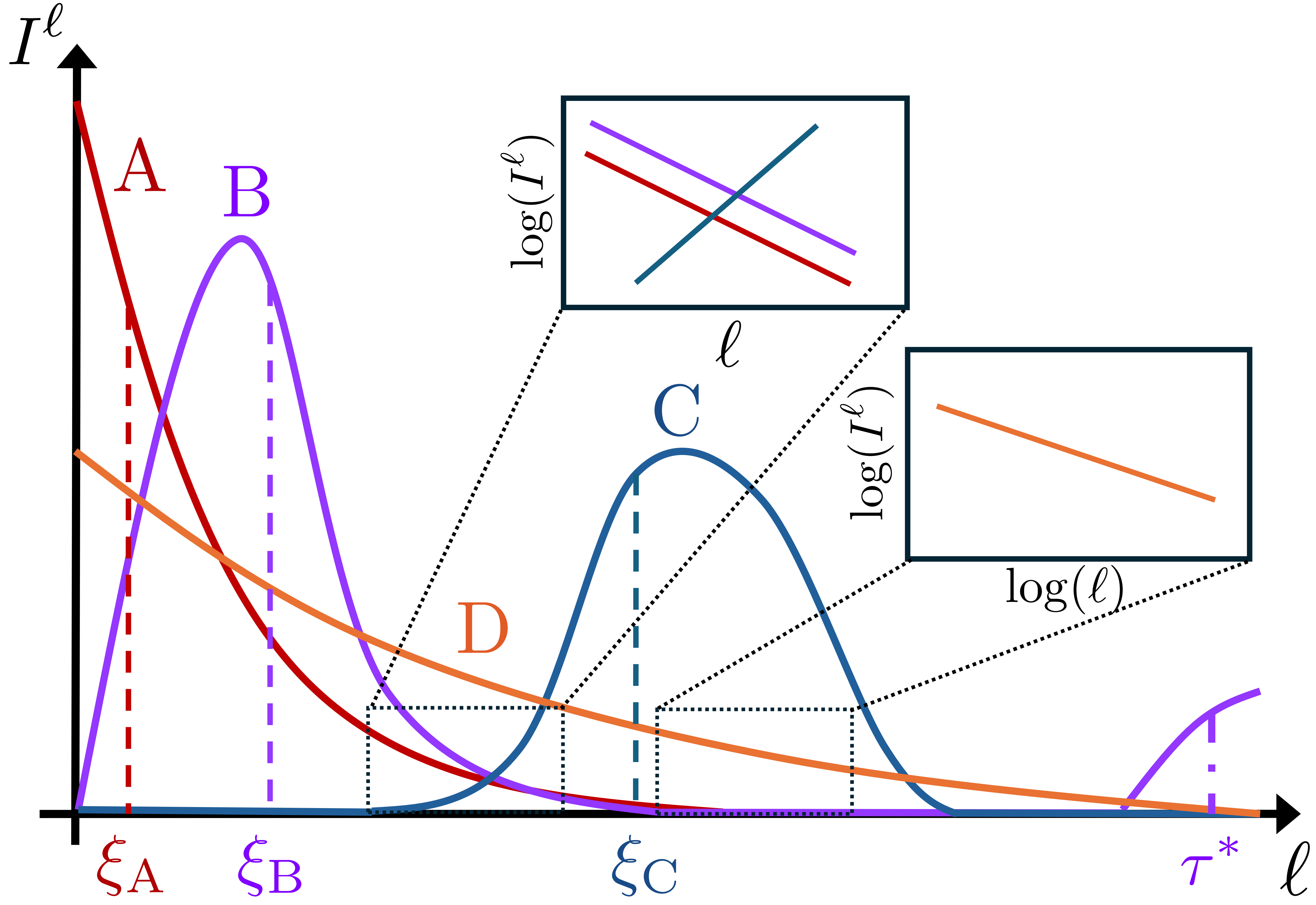
The total information $I^\ell = \sum_n i_n^\ell$ on scale ℓ

A — localized state

B — topological localized state

C — fully ergodic state ($T = \infty$)

D — critical state



[Artiaco, Klein-Kvorning, Aceituno Chávez, Herviou, JHB, PRL 2025 (arXiv:2410.10971)]

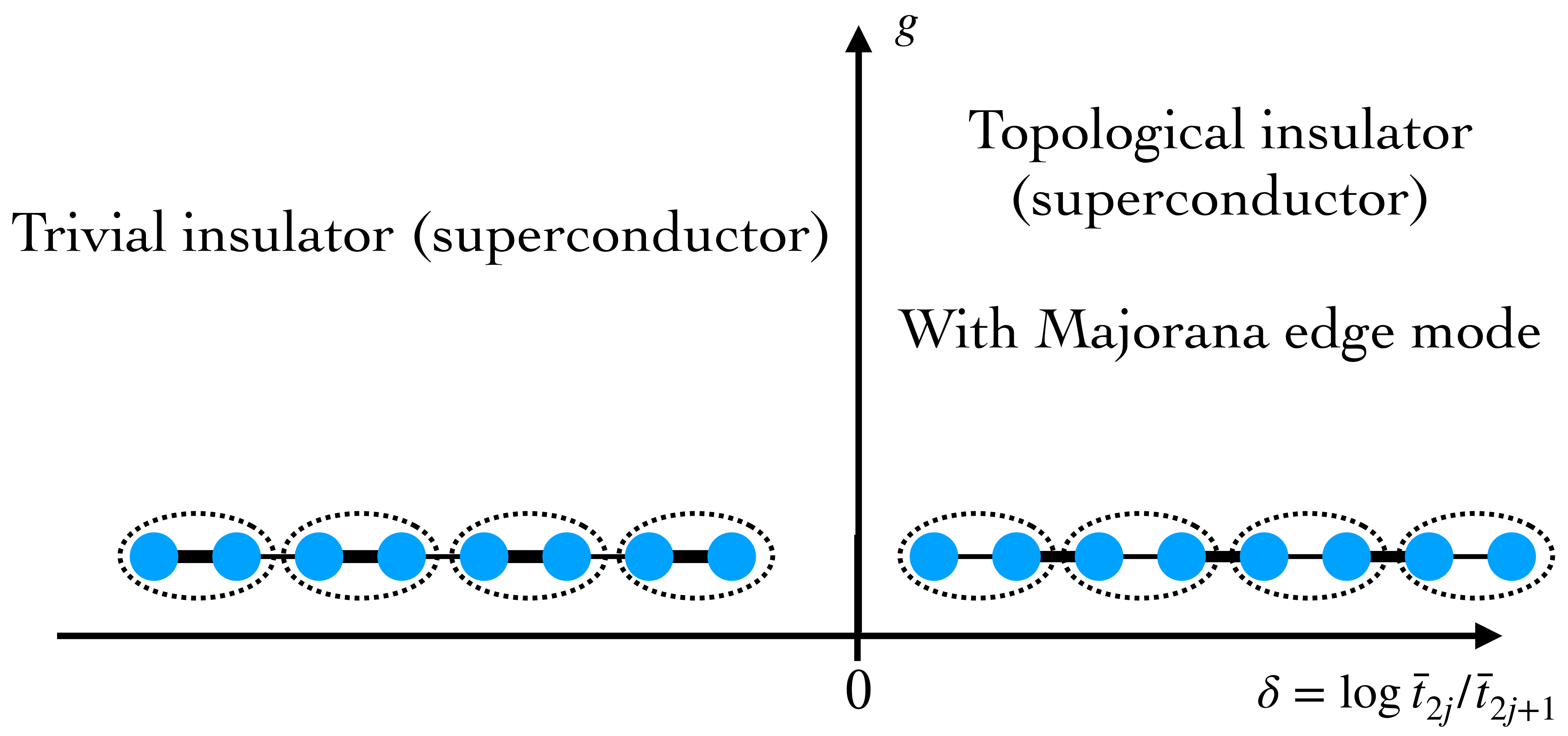
Example: the various phases of the Ising-Majorana model

$$H = -i\sum_{j=1}^{2L-1} t_j\gamma_j\gamma_{j+1} + g\sum_{j=1}^{2L-3}\gamma_j\gamma_{j+1}\gamma_{j+2}\gamma_{j+3} \qquad t_{2j-1}\in[0,e^{-\delta/2}] \qquad t_{2j}\in[0,e^{\delta/2}]$$

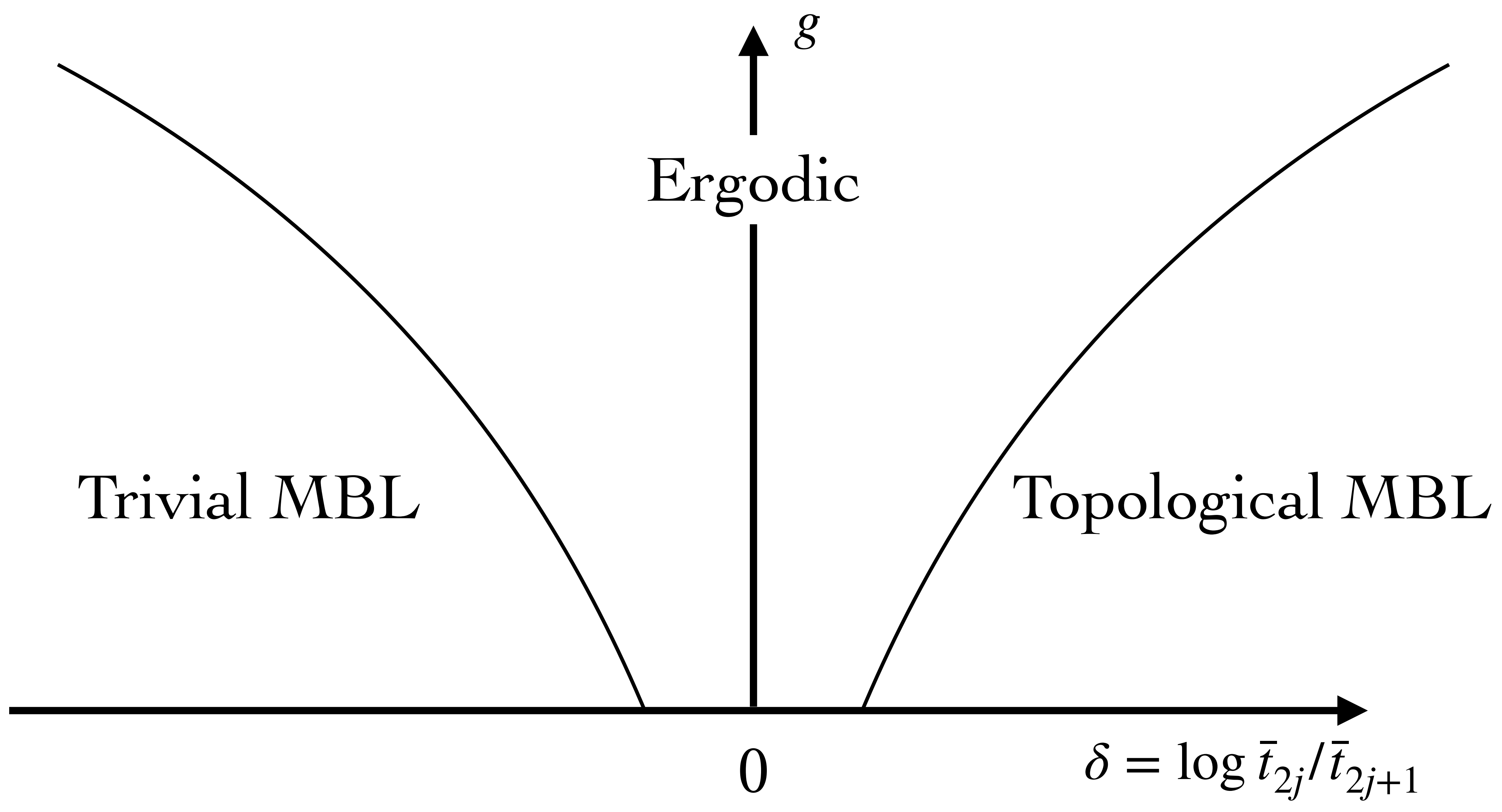
$$c_i = \gamma_{2i-1} + i\gamma_{2i} \qquad \gamma_i^\dagger = \gamma_i$$

$$c_i^\dagger = \gamma_{2i-1} - i\gamma_{2i} \qquad \{\gamma_i,\gamma_j\} = 2\delta_{i,j}$$

Ground state phase diagram



Excited state many-body localisation (MBL) phase diagram



[Artiaco, Klein-Kvorning, Aceituno Chávez, Herviou, JHB, PRL 2025 (arXiv:2410.10971)]

Example: the various phases of the Ising-Majorana model

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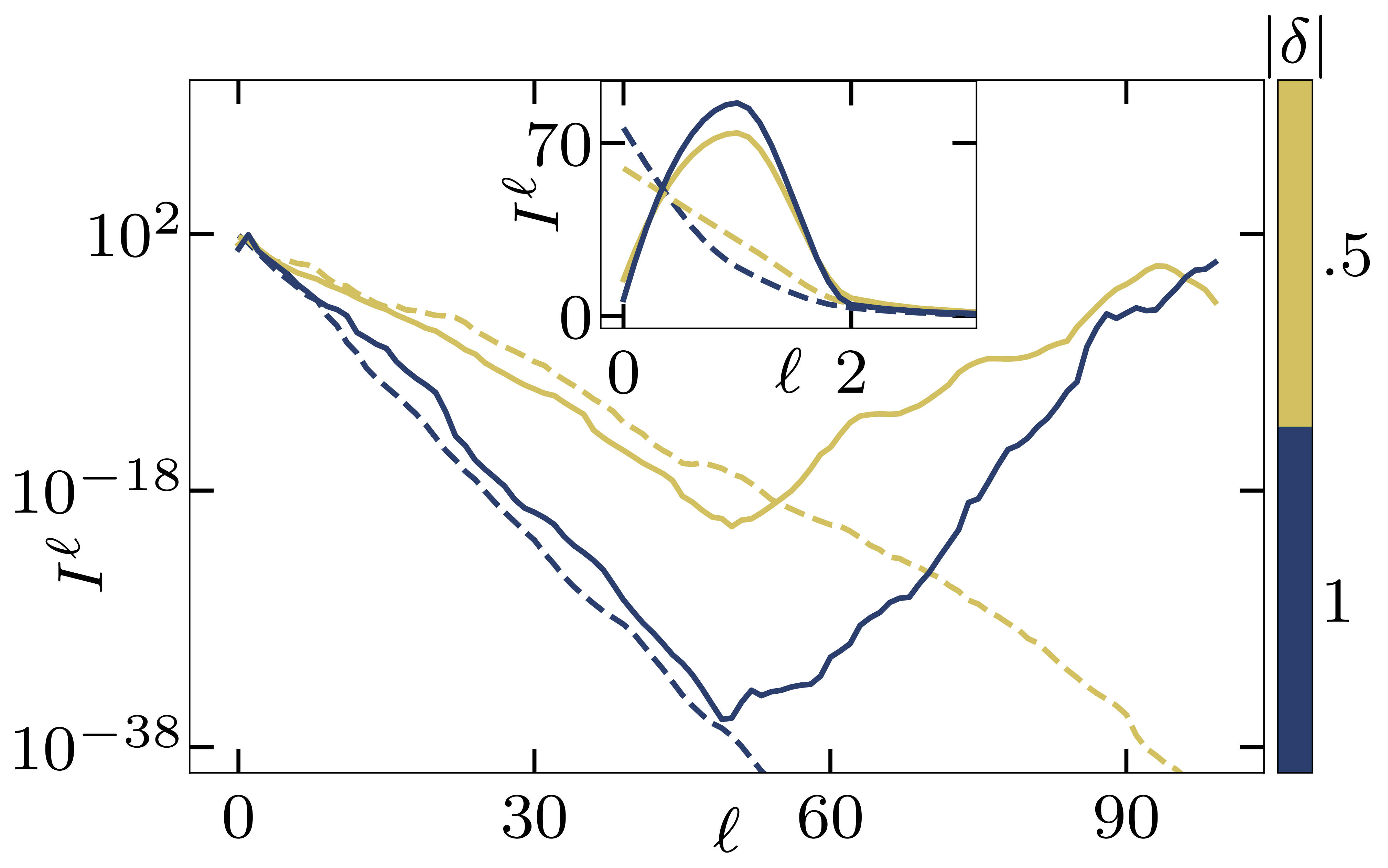
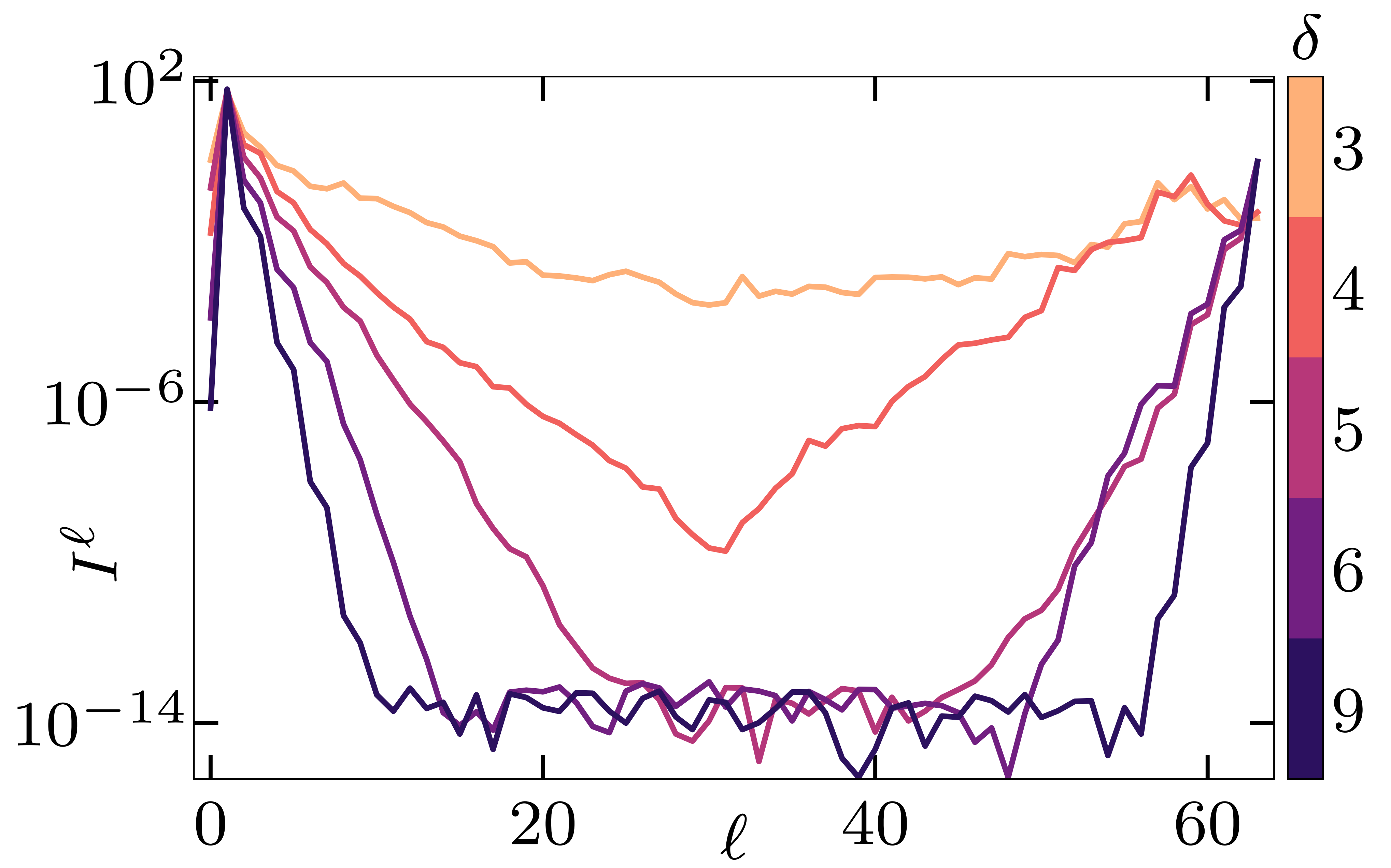
$t_{2j-1} \in [0, e^{-\delta/2}]$ $t_{2j} \in [0, e^{\delta/2}]$





For $g = 0$ noninteracting Kitaev model with two phases:

i) trivial superconductor / insulator

ii) topological superconductor with Majorana edge modes

Interactions $g \neq 0$ have same phases in ground state

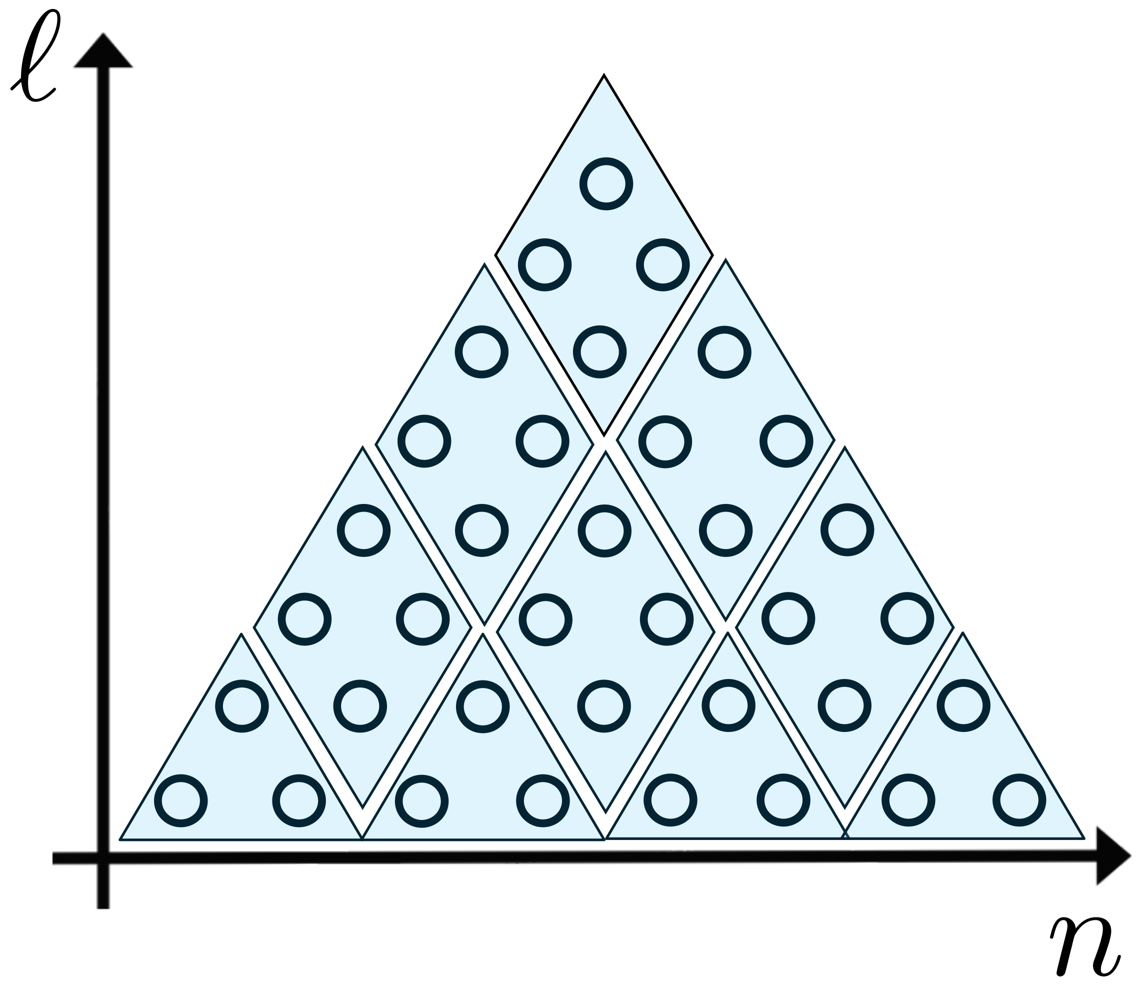
[Artiaco, Klein-Kvorning, Aceituno Chávez, Herviou, JHB, PRL 2025 (arXiv:2410.10971)]

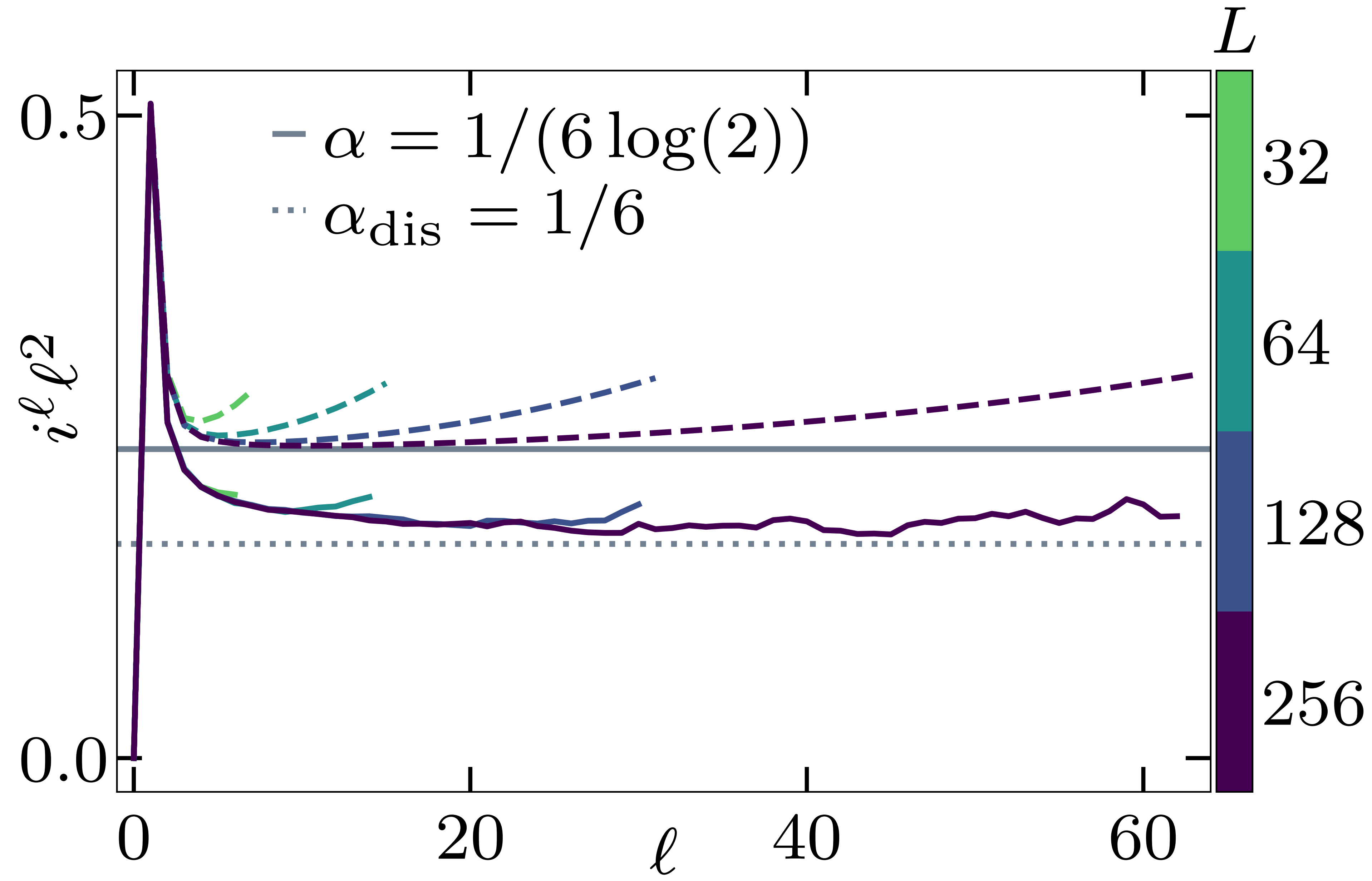
Example: the various phases of the Ising-Majorana model

$$H = -i \sum_{j=1}^{2L-1} t_j \gamma_j \gamma_{j+1} + g \sum_{j=1}^{2L-3} \gamma_j \gamma_{j+1} \gamma_{j+2} \gamma_{j+3}$$

$t_{2j-1} \in [0, e^{-\delta/2}]$ $\quad t_{2j} \in [0, e^{\delta/2}]$

For $g = 0$ and $\delta = 0$ we are at a critical point in between the normal superconductor and the topological superconductor





Scale invariance implies $i^\ell \to (\ell^*)^2 i^{\ell/\ell^*} \Rightarrow i^\ell = \dfrac{\alpha}{\ell^2}$

In CFT: $\alpha = \dfrac{c}{3\log 2}$

[Artiaco, Klein-Kvorning, Aceituno Chávez, Herviou, JHB, PRL 2025 (arXiv:2410.10971)]

Example: the various phases of the Ising-Majorana model

$$H = -i \sum_{j=1}^{2L-1} t_j \gamma_j \gamma_{j+1} + g \sum_{j=1}^{2L-3} \gamma_j \gamma_{j+1} \gamma_{j+2} \gamma_{j+3}$$

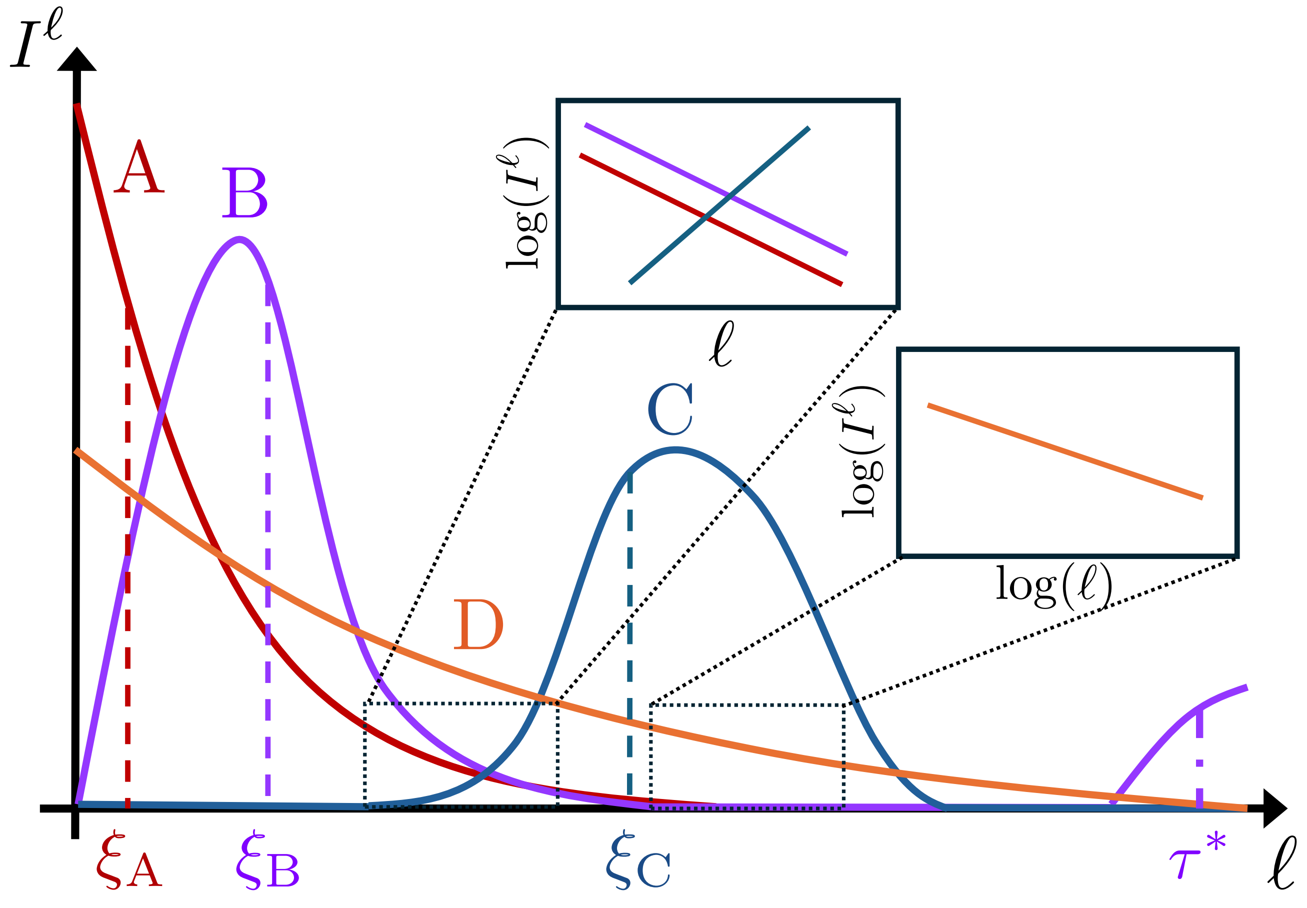
$$t_{2j-1} \in [0, e^{-\delta/2}] \qquad t_{2j} \in [0, e^{\delta/2}]$$

For $g > 0$ and midspectrum states there are three types of states (in finite systems):

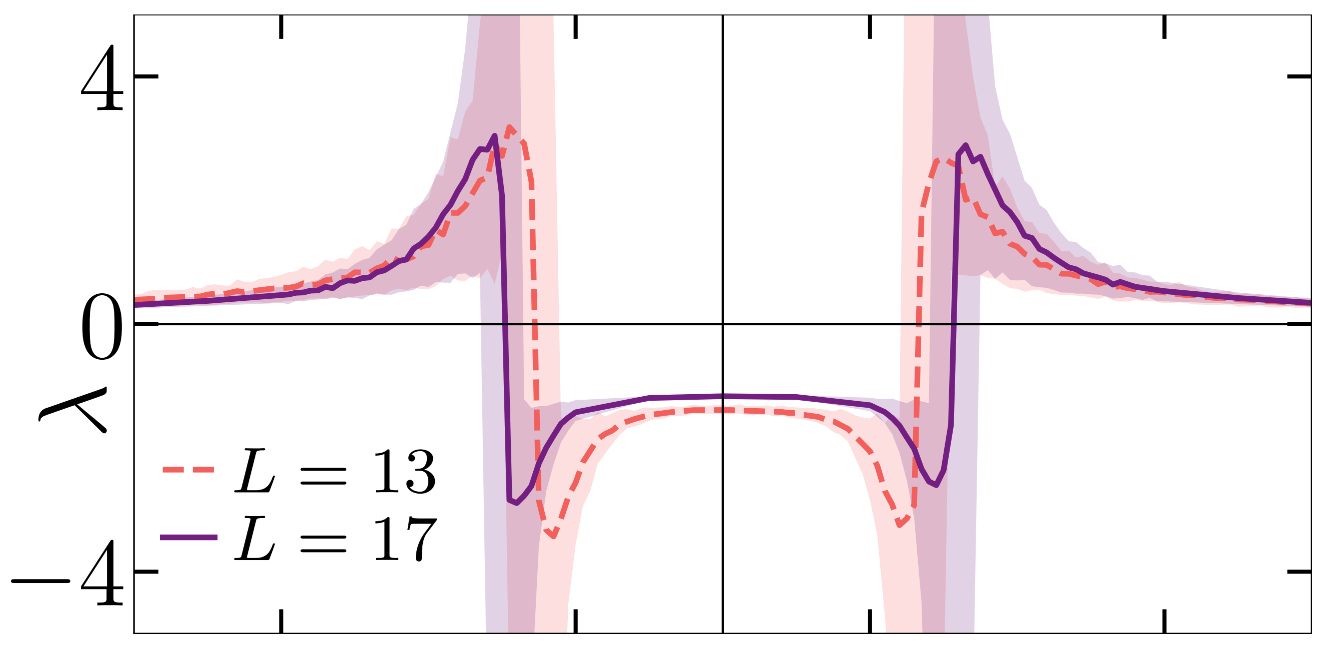
i) Many-body localised states

ii) Ergodic states

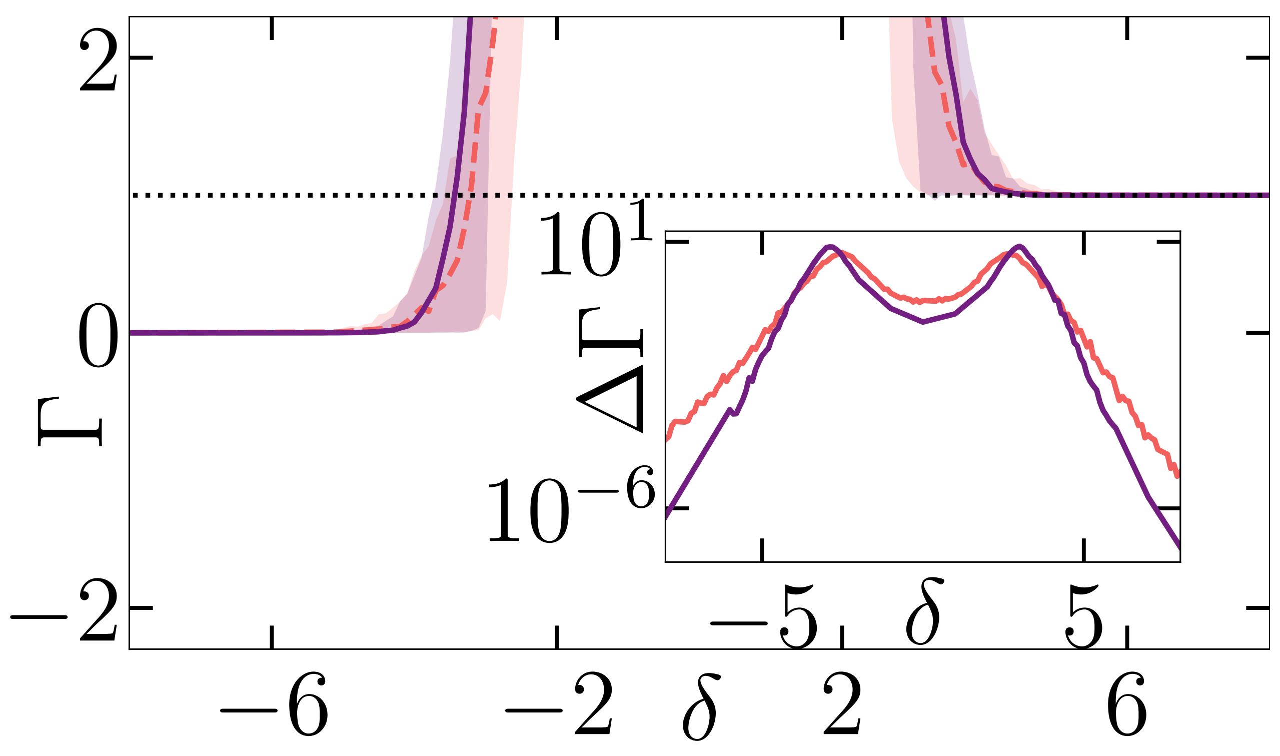
iii) Topological many-body localised states



Correlation decay length (Localisation length)



Total info on large scales

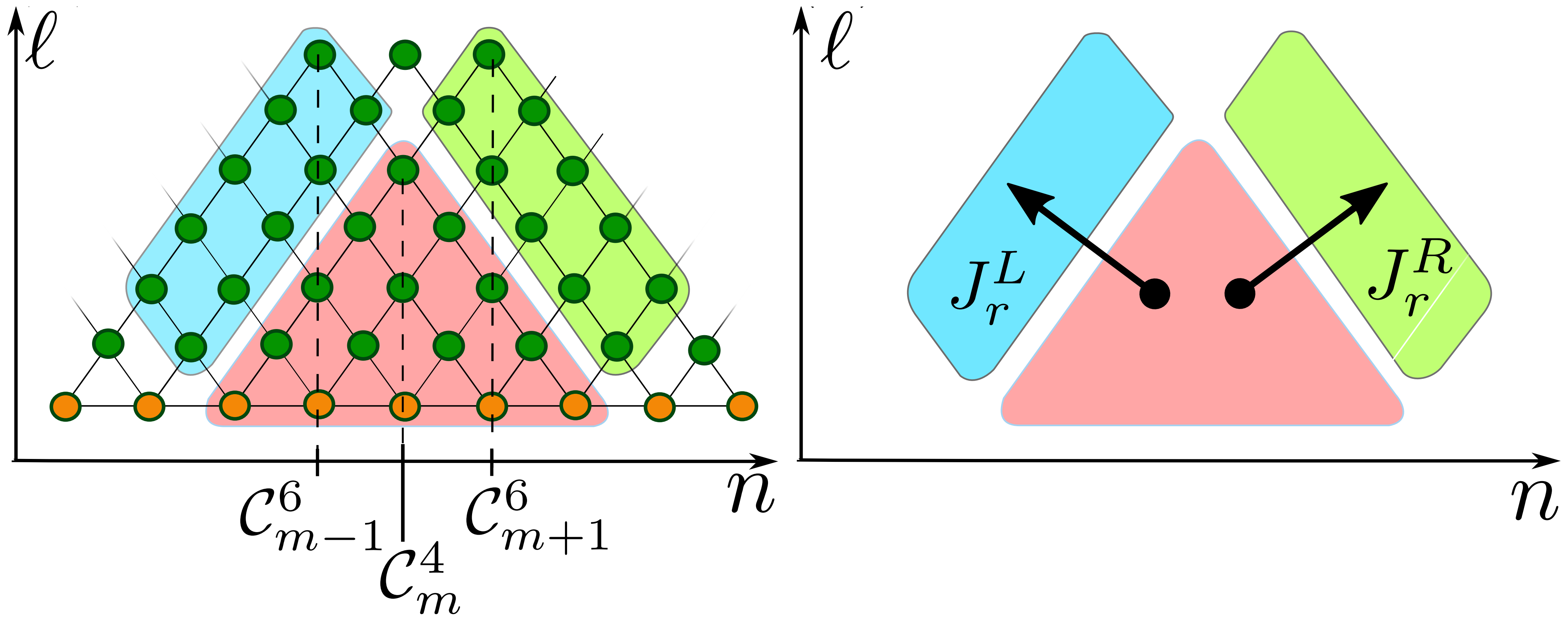


$$\Gamma = \sum_{\ell=\lfloor L/2 \rfloor}^{L-1} I_\ell = N$$

[Artiaco, Klein-Kvorning, Aceituno Chávez, Herviou, JHB, PRL 2025 (arXiv:2410.10971)]

Information locally conserved and moves locally under local unitary transformations

Total information conserved under unitary dynamics, local information i_n^ℓ is a density that is follows a local conservation law with local information currents



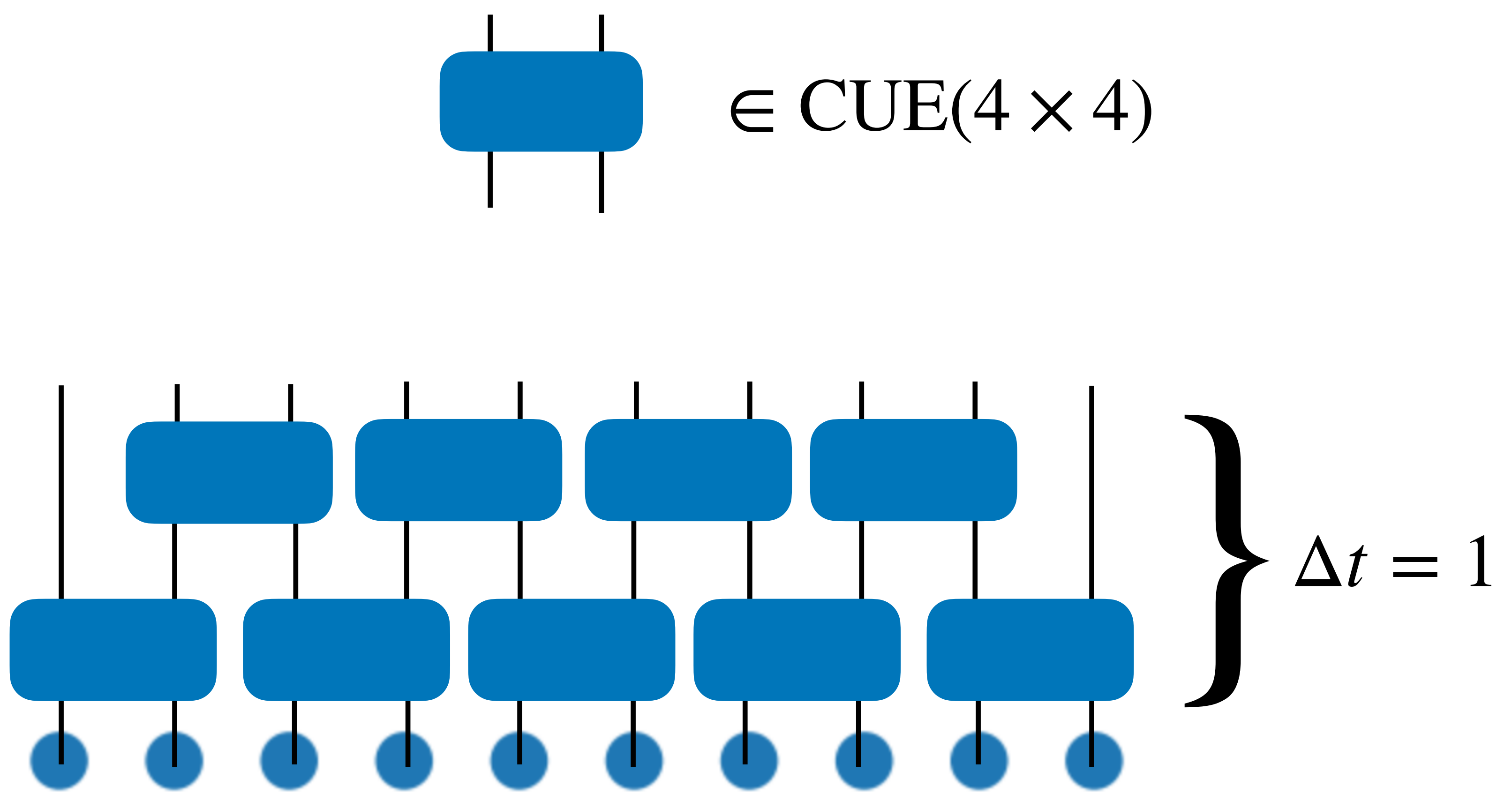
Technical details

$$
\begin{aligned}
\partial_t I(\rho_n^\ell) = &- i\ \mathrm{Tr}\left(\left[\nabla S_n^\ell, H_n^\ell\right]\ \rho_n^\ell\right) \\
&- i\ \mathrm{Tr}\left(\left[1_{2r} \otimes \nabla S_n^\ell, H^{\ell+r}_{n-r/2} - H_n^\ell\right]\ \rho^{\ell+r}_{n-r/2}\right) \\
&- i\ \mathrm{Tr}\left(\left[\nabla S_n^\ell \otimes 1_{2r}, H^{\ell+r}_{n+r/2} - H_n^\ell\right]\ \rho^{\ell+r}_{n+r/2}\right) \\
= &\ J_r^R(\rho^{\ell+r}_{n-r/2}) + J_r^L(\rho^{\ell+r}_{n+r/2})
\end{aligned}
$$

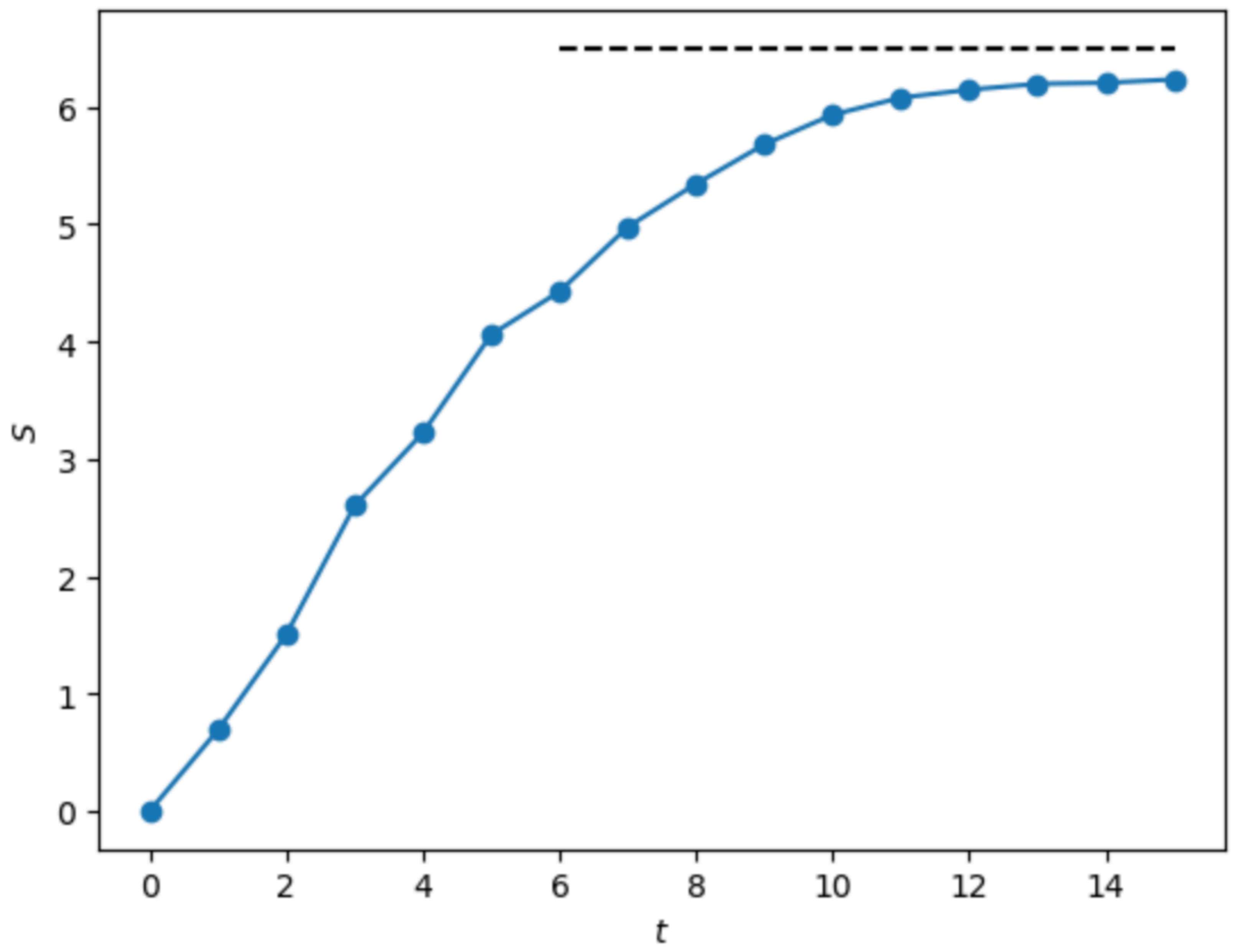
[Klein Kvorning, Herviou, JHB SciPost Physics (2022); Artiaco, Fleckenstein, Aceituno Chávez, Klein Kvorning, JHB PRX Quantum 2024]

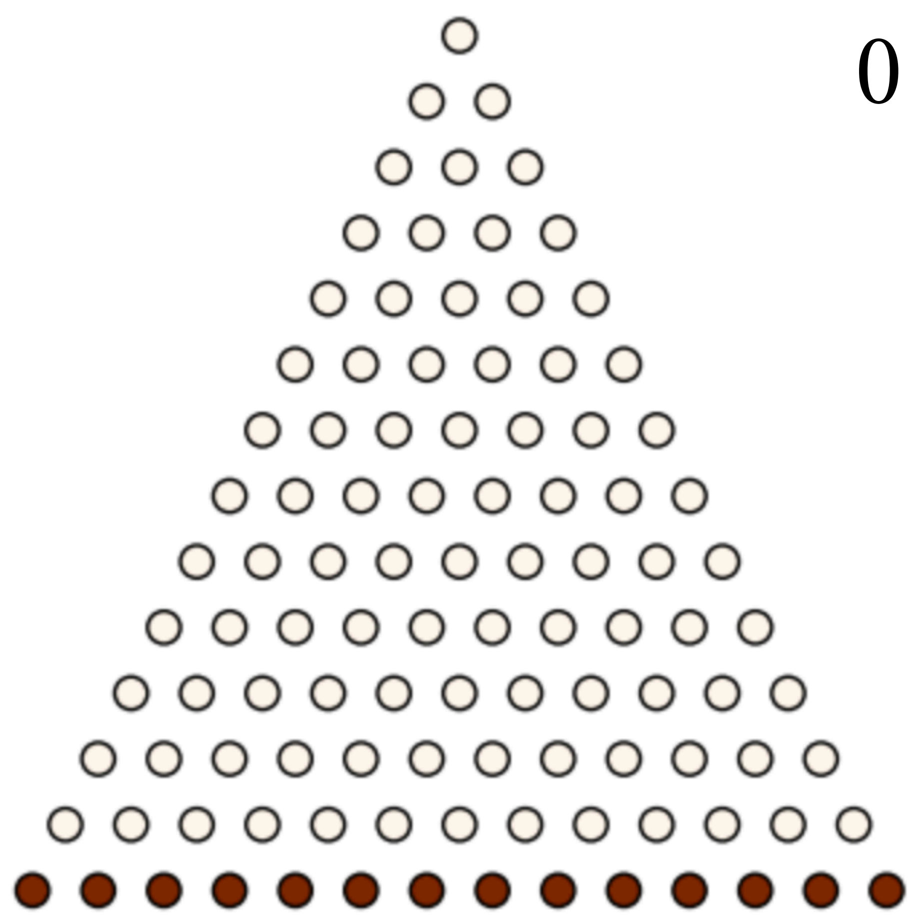
Dynamics on the information lattice – ballistic growth of entanglement in a random unitary circuit

Random brickwork of independent random unitaries

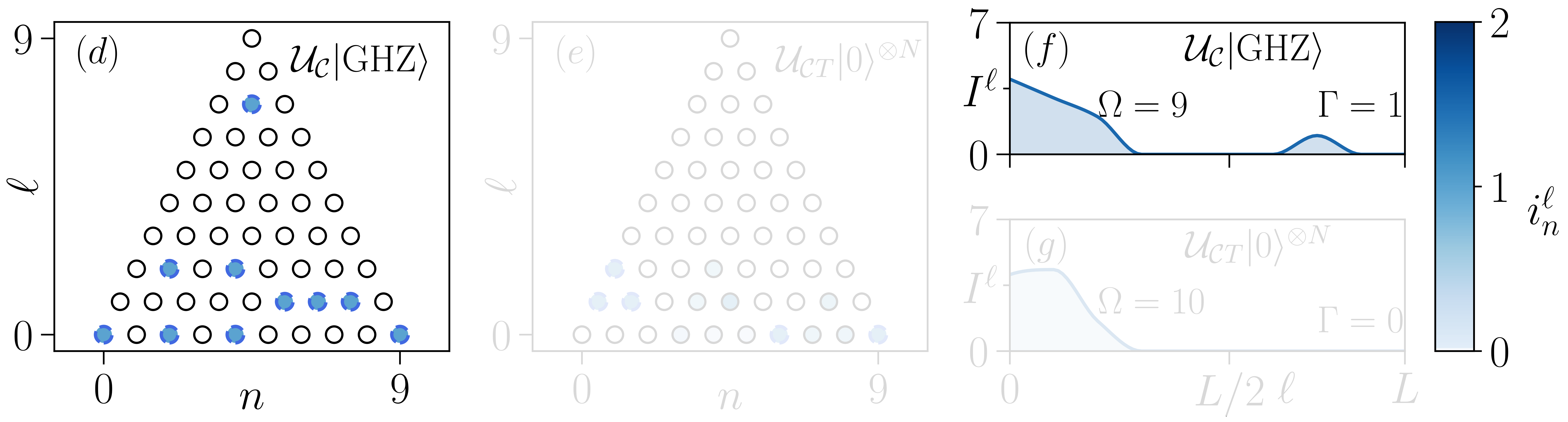


Entanglement entropy grows ballistically before saturating at the Page value





Stabilizer states with information gap and long-range magic



$\mathcal{U}_C$ Local Clifford circuit

$\mathcal{U}_{CT}$ Local Clifford+Tgate circuit

Information per scale $I^\ell = \sum_n i_n^\ell$

Small scale information $\Omega = \sum_{\ell=0}^{\lfloor L/2-1 \rfloor} i_n^\ell$

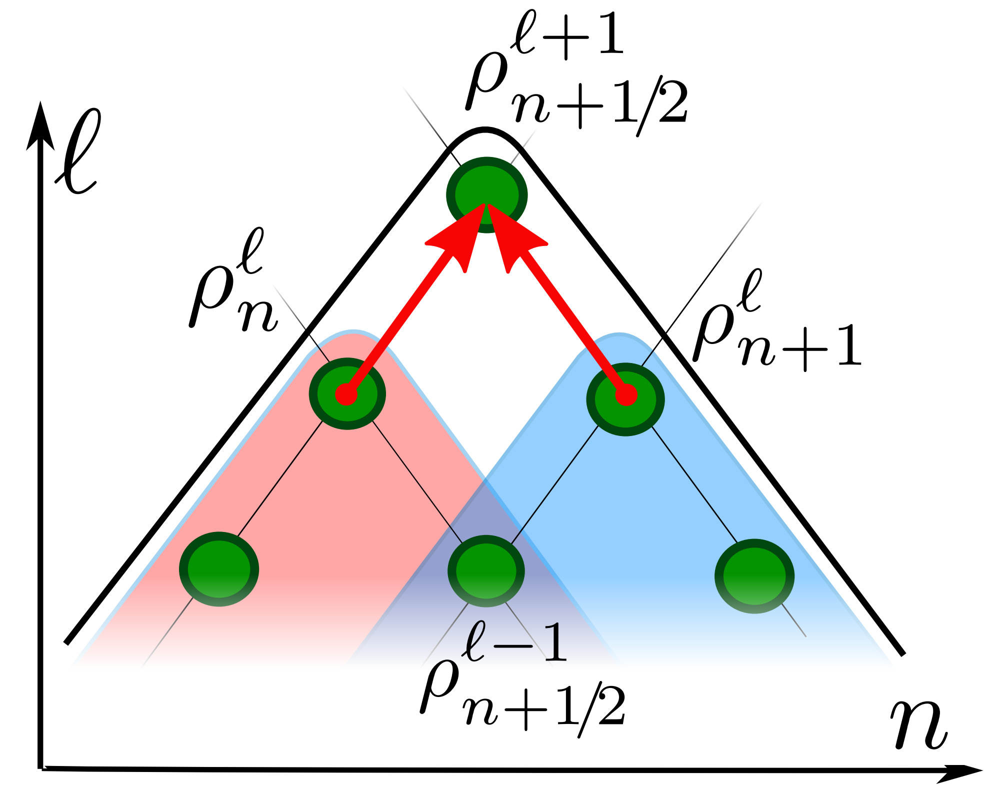
Large scale information $\Gamma = \sum_{\ell=\lfloor L/2 \rfloor}^{L-1} i_n^\ell$

$\boxed{\Gamma \notin \mathbb{N} \Rightarrow \text{long-range magic}}$

[Bilinskaya, Martínez, Ghosh, Klein Kvorning, Artiaco, JHB, arXiv:2510.26696]

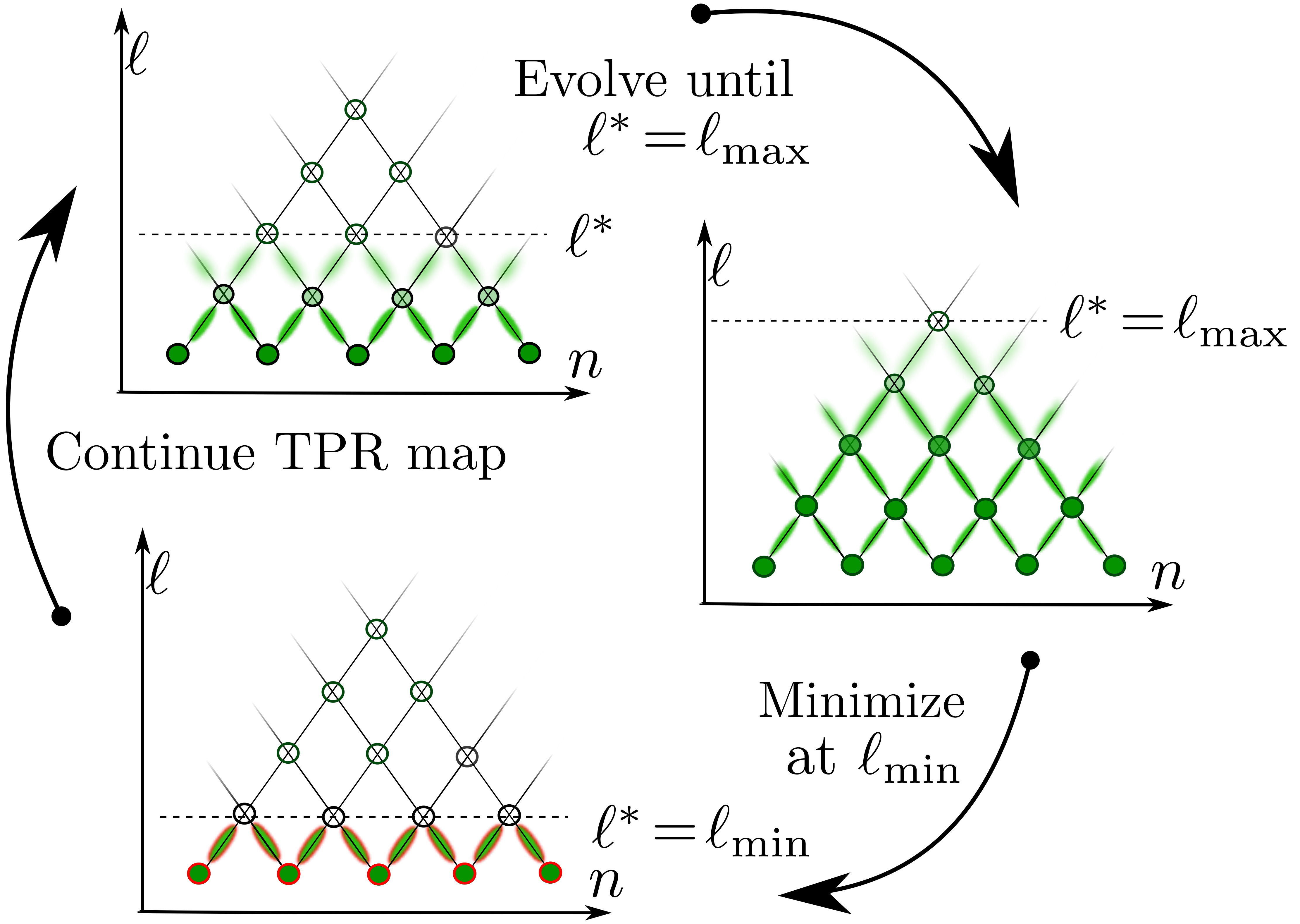
LITE – Local Information Time Evolution

If no information on scale $\ell+1$ can construct $\rho^{\ell+1}$



$$\rho^{\ell+1}_{n+1/2} = \exp\left[\ln(\rho^{\ell}_{n}) + \ln(\rho^{\ell}_{n+1}) + \ln(\rho^{l-1}_{n+1/2})\right]$$

(Petz) recovery map



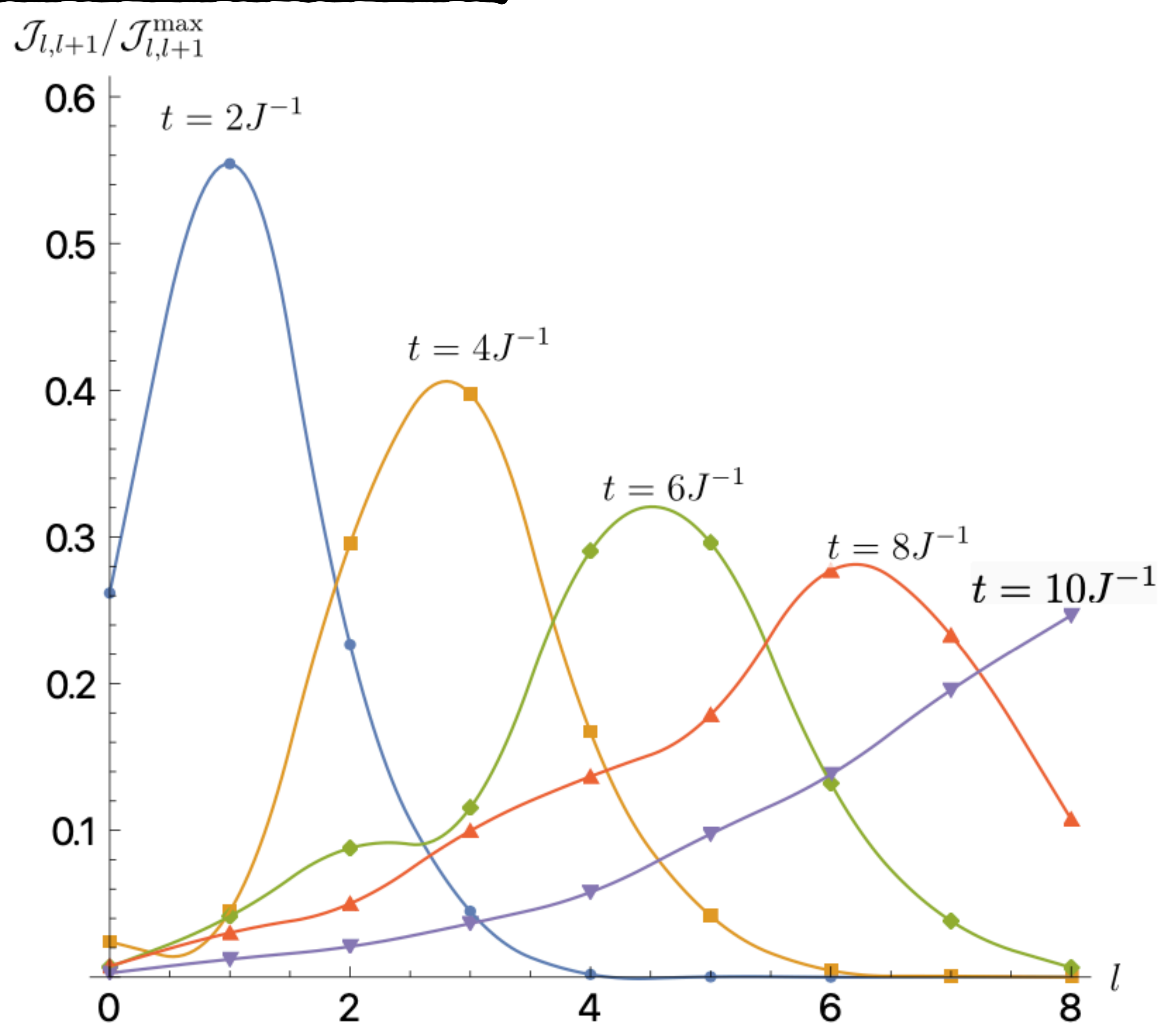
[Klein Kvorning, Herviou, JHB SciPost Physics (2022); Artiaco, Fleckenstein, Aceituno Chávez, Klein Kvorning, JHB PRX Quantum 2024]

In microscopic models information flows to large scales leaving behind a tail of information at short scales

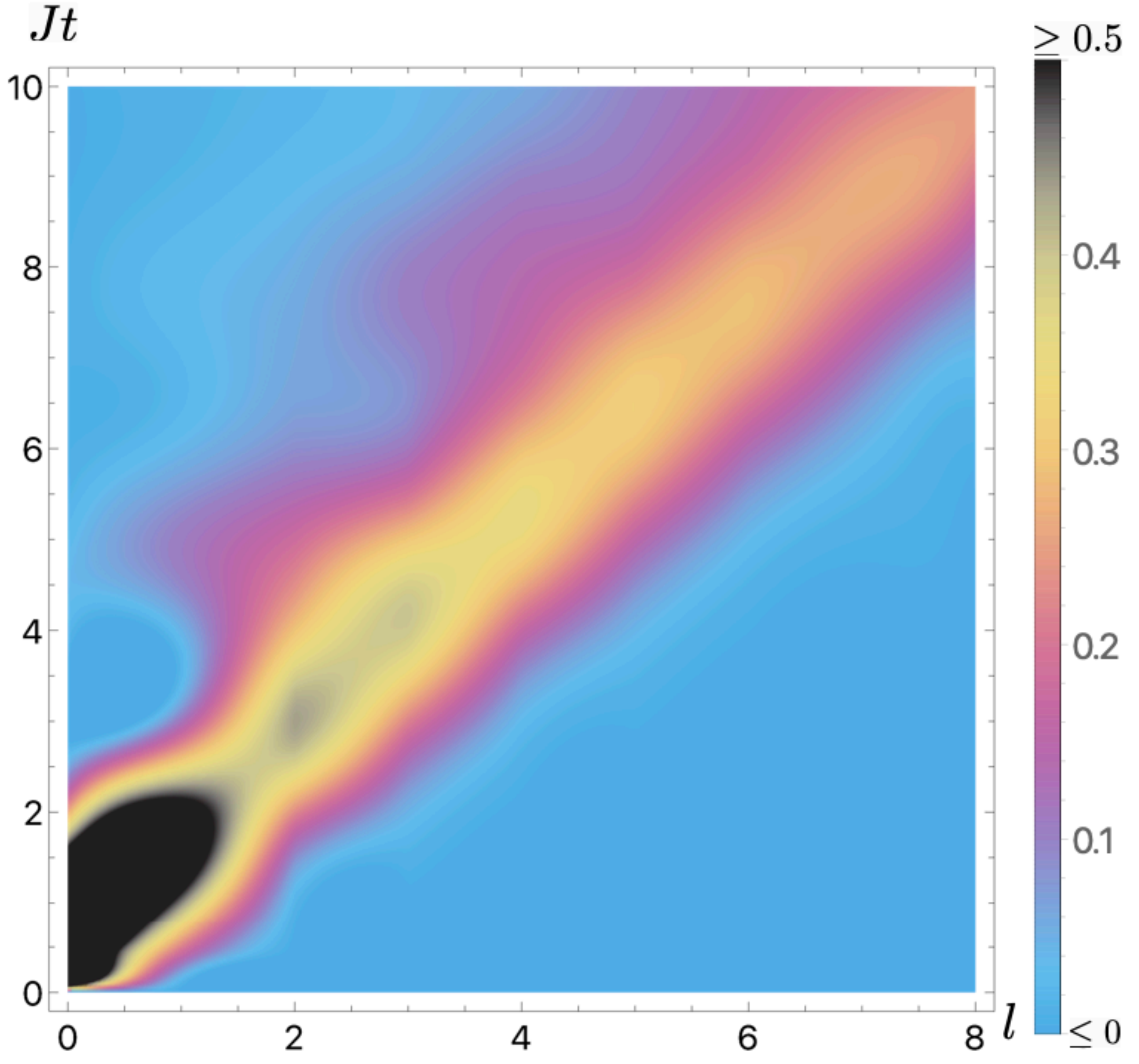
Mixed-field Ising model $H = \sum_i J\sigma_i^z\sigma_{i+1}^z + h_T\sigma_i^x + h_L\sigma_i^z$

$$\rho(t=0) = \cdots \otimes I_2 \otimes I_2 \otimes |\uparrow_x\rangle\langle\uparrow_x| \otimes I_2 \otimes I_2 \otimes \cdots$$

Total information on scale ℓ



Scale ℓ



[Klein Kvorning, Herviou, JHB SciPost Physics (2022)]

Example: energy diffusion in the mixed field Ising model

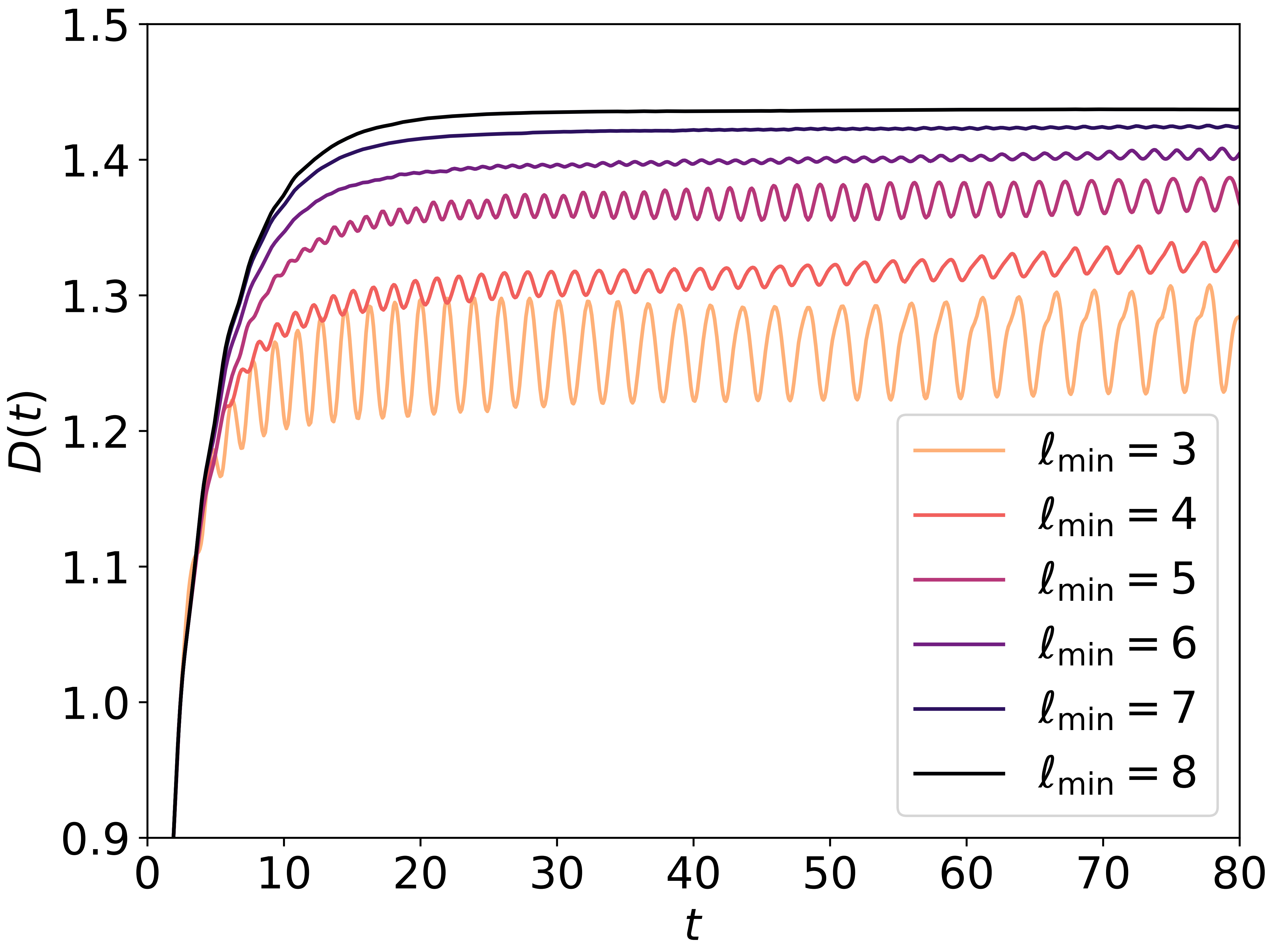
$$H = \sum_i J\sigma_i^z\sigma_{i+1}^z + h_T\sigma_i^x + h_L\sigma_i^z$$

$$J = 1,\ h_T = 1.4,\ h_L = 0.9045$$

$$\rho_{\text{init}} = \left(\bigotimes_{m<n-\ell/2} \rho_{m,\infty}\right) \otimes \rho^{\ell}_{n,\text{init}} \otimes \left(\bigotimes_{m>n+\ell/2} \rho_{m,\infty}\right)$$

$$\sigma_E^2 = \sum_n (n-\bar{n})^2 \frac{E_n^r}{\langle H\rangle} - \left(\sum_n (n-\bar{n})\frac{E_n^r}{\langle H\rangle}\right)^2$$

$$D = \frac{1}{2}\partial_t \sigma_E^2$$

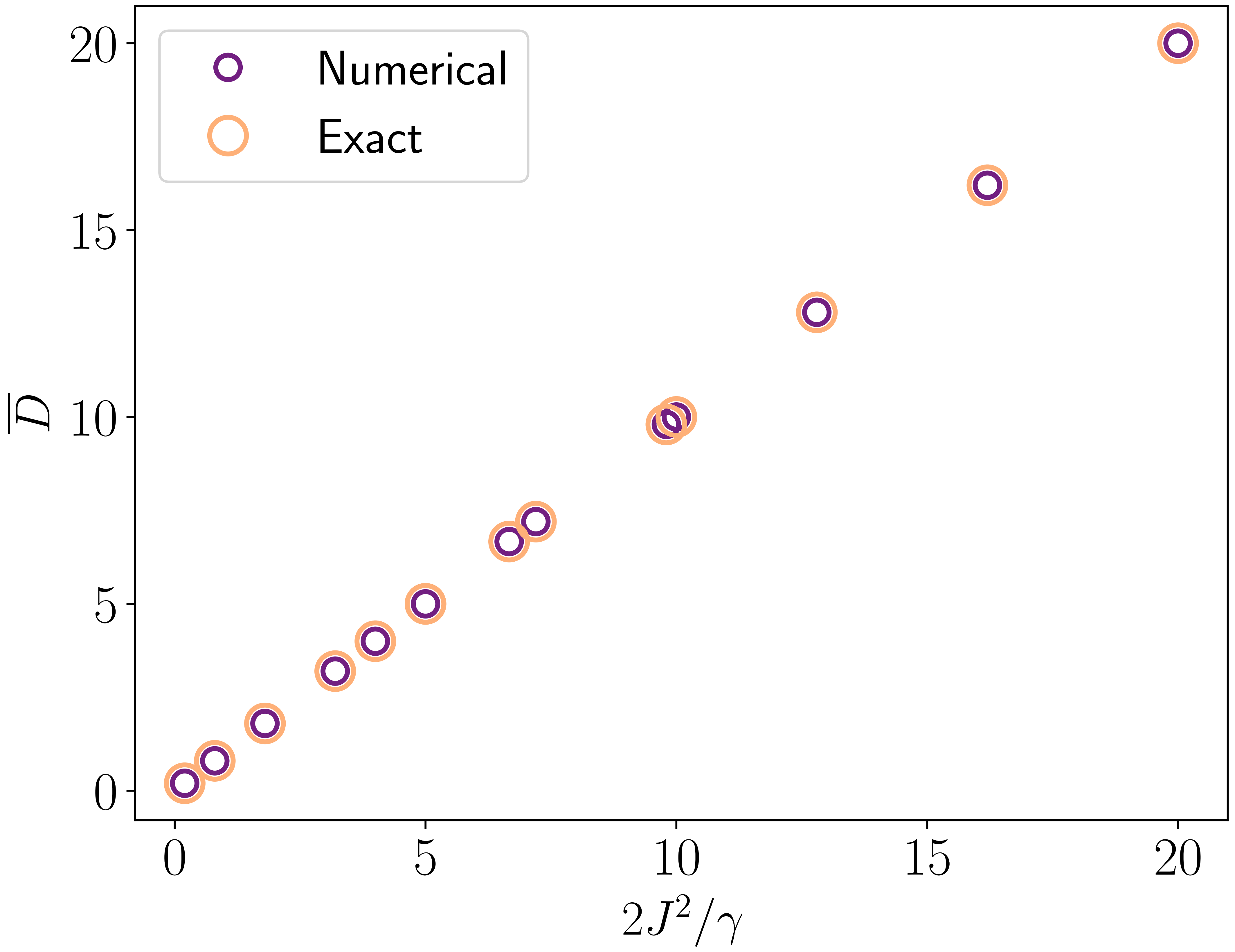


[See alos Rakovszky, von Keyserlingk, and Pollmann PRB 2022 for a different approach]

[Artiaco, Fleckenstein, Aceituno Chávez, Klein Kvorning, JHB PRX Quantum 2024]

Lindblad dynamics with local dissipators straightforwardly implemented on the information lattice

Göran Lindblad
KTH Royal Institute of Technology
9 July 1940–30 November 2022



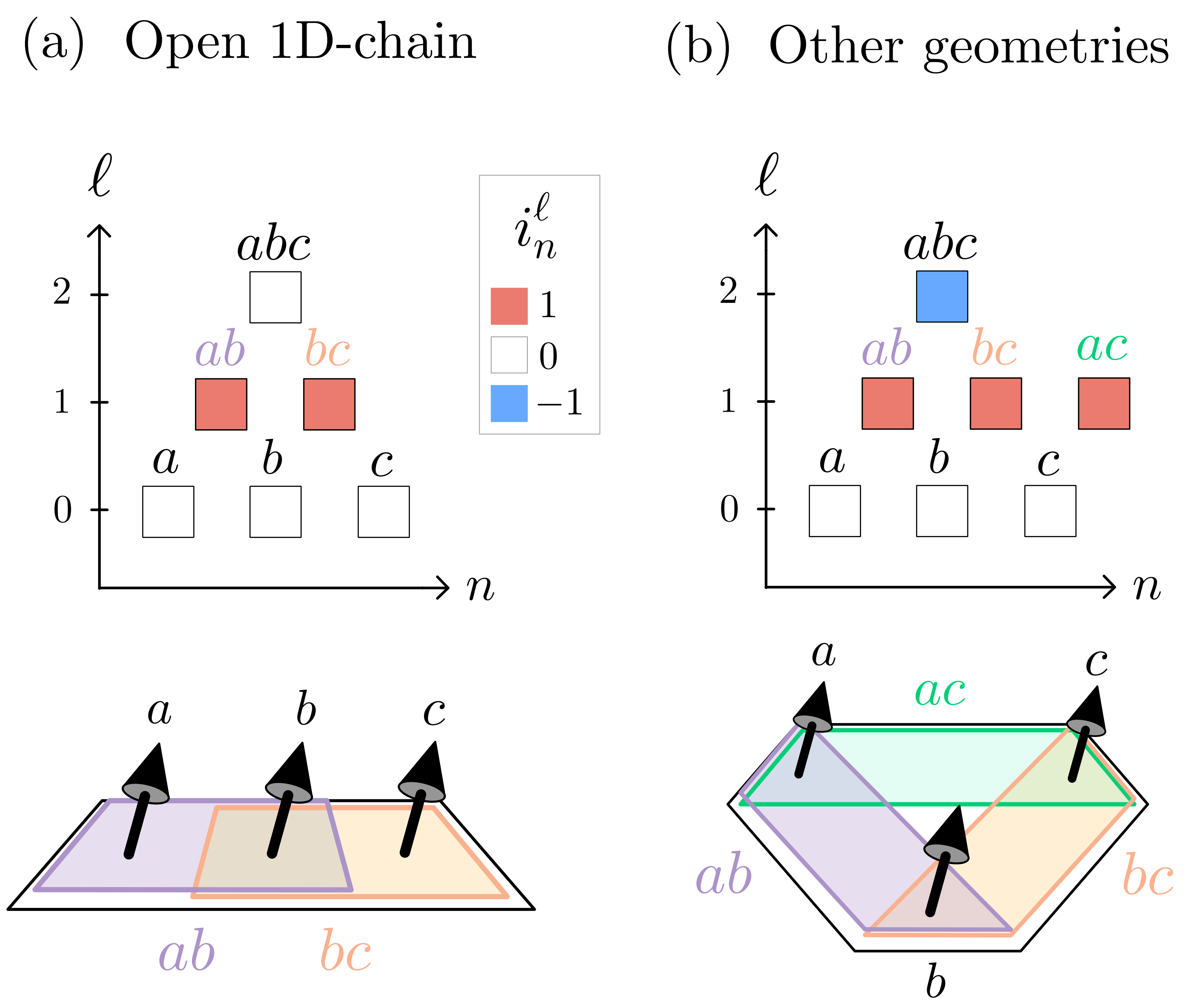
$$\partial_t \rho = -i\,[H,\rho] + \sum_j \gamma_j \left(L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\} \right)$$

$$H = \sum_j J(\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y) \qquad L_j = \sigma_j^z; \gamma_j = \gamma \,\forall j$$

[Compare with Jin, Ferreira, Filipone, Giamarchi, PRR 2022]

[Artiaco, Fleckenstein, Aceituno Chávez, Klein Kvorning, JHB PRX Quantum 2024]

Generalising to higher dimensions – overlap redundancies



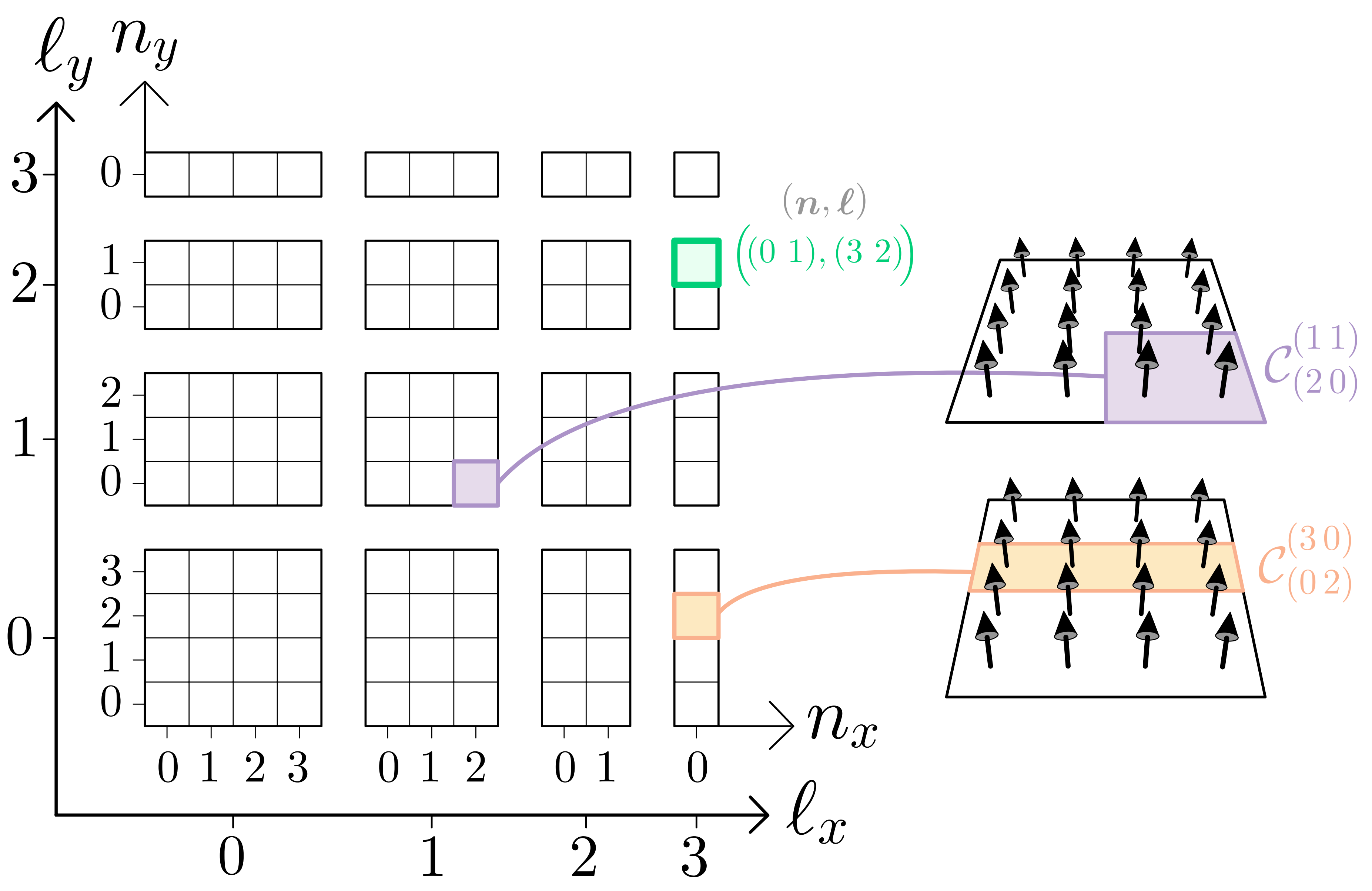
Example state $\rho = \frac{1}{2}(|000\rangle\langle 000| + |111\rangle\langle 111|)$

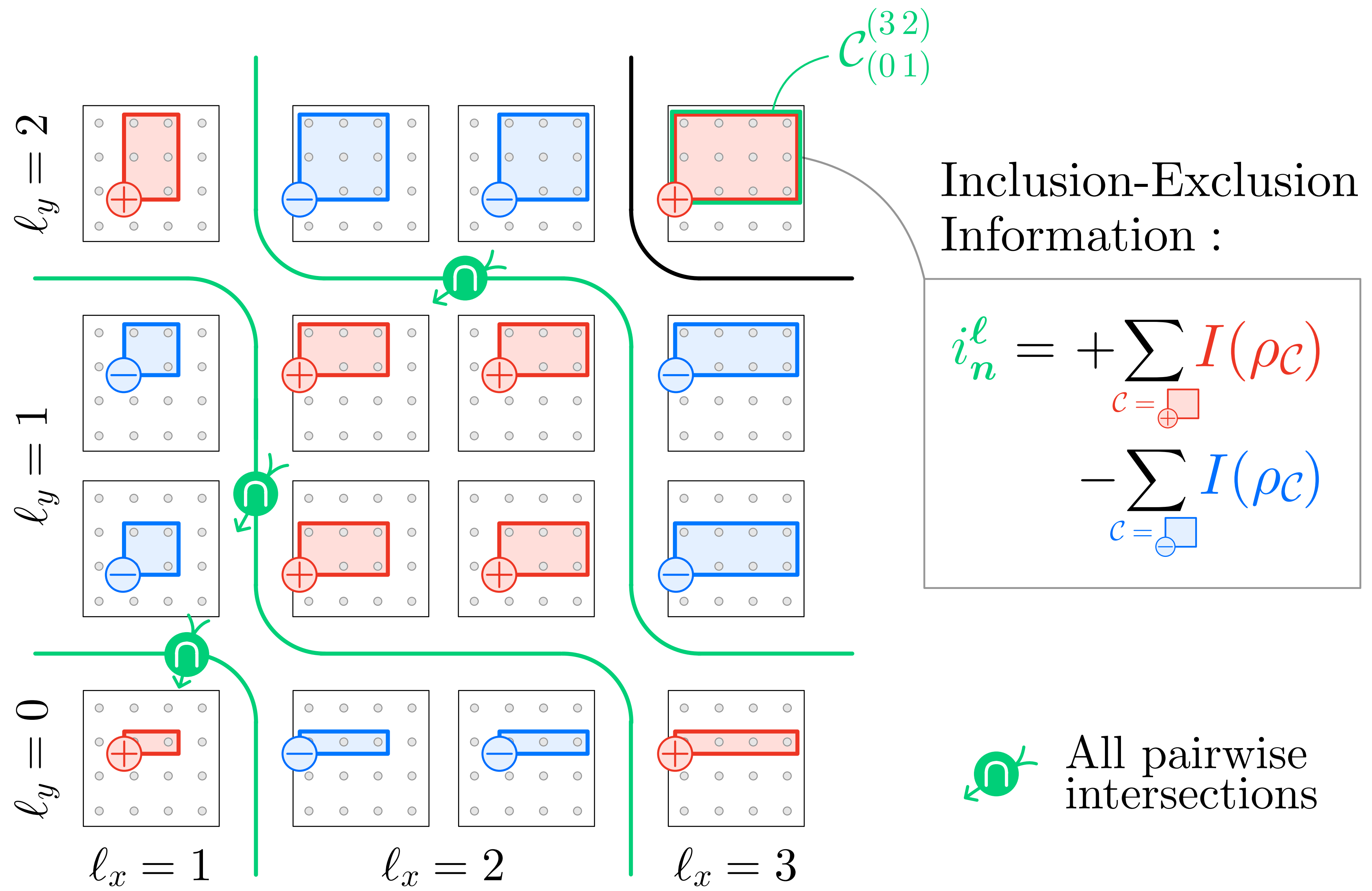
[Flór, Artiaco, Klein Kvorning, JHB, arXiv:2512:20793]

Generalising to higher dimensions – Inclusion-Exclusion information

In higher dimension there are many different choices of set of subsystems

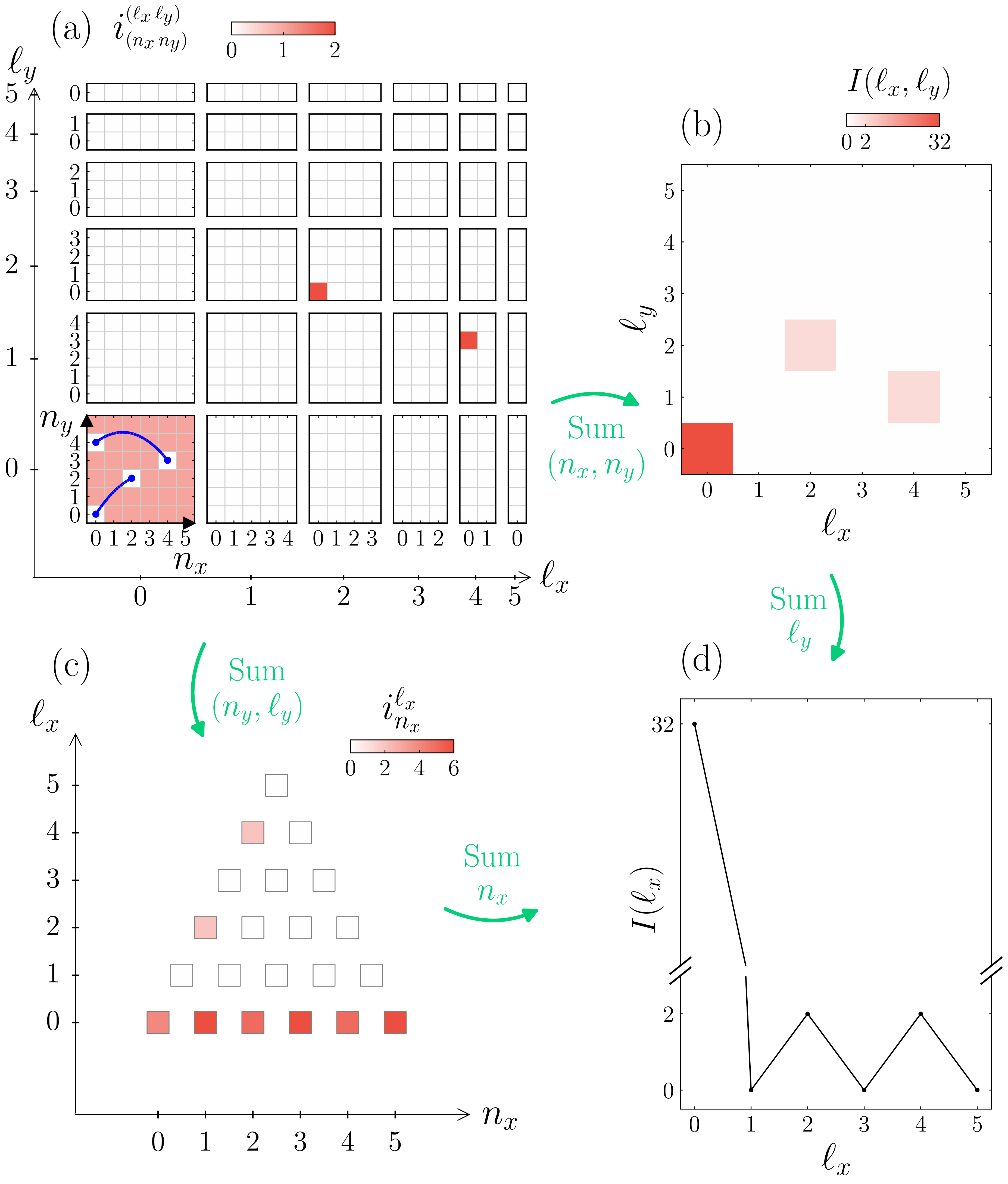
Any set that is closed under intersections allows for defining inclusion-exclusion information





[Flór, Artiaco, Klein Kvorning, JHB, arXiv:2512:20793]

Simple example of a product state with two singlets

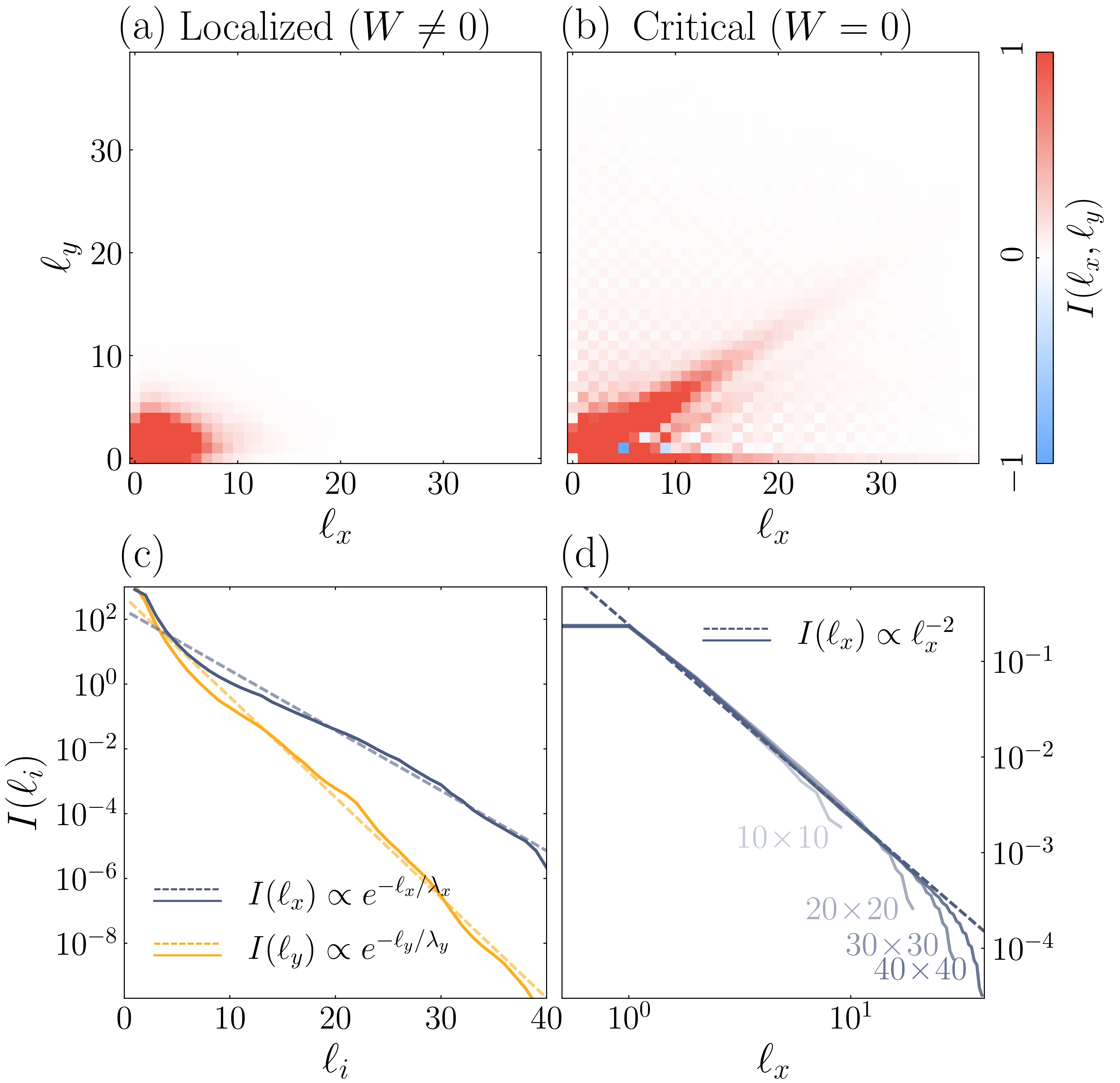


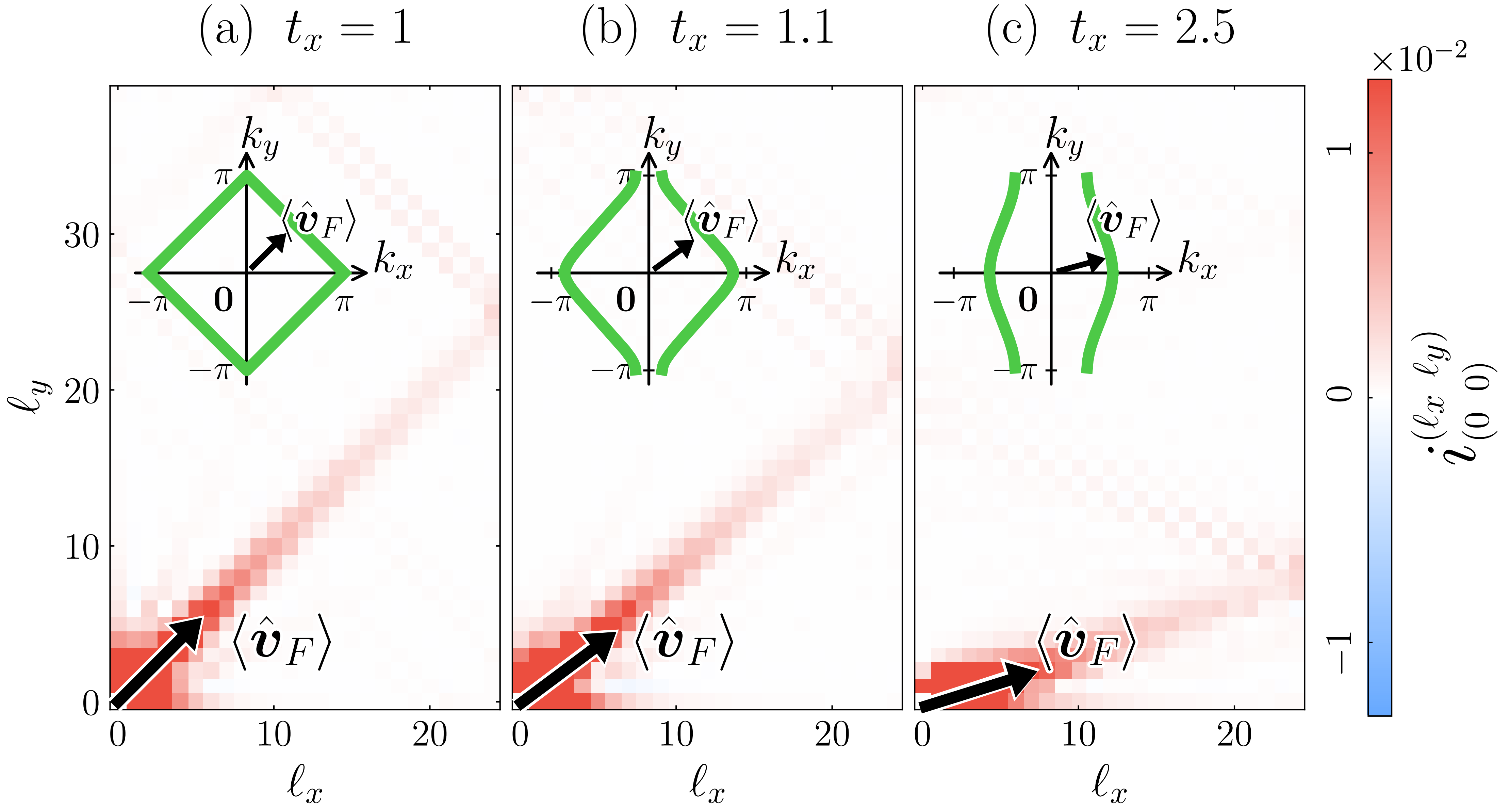
Partial summing gives quasi-1d information lattices with positive local information

Summing over one scale direction gives direction dependent information per scale

[Flór, Artiaco, Klein Kvorning, JHB, arXiv:2512:20793]

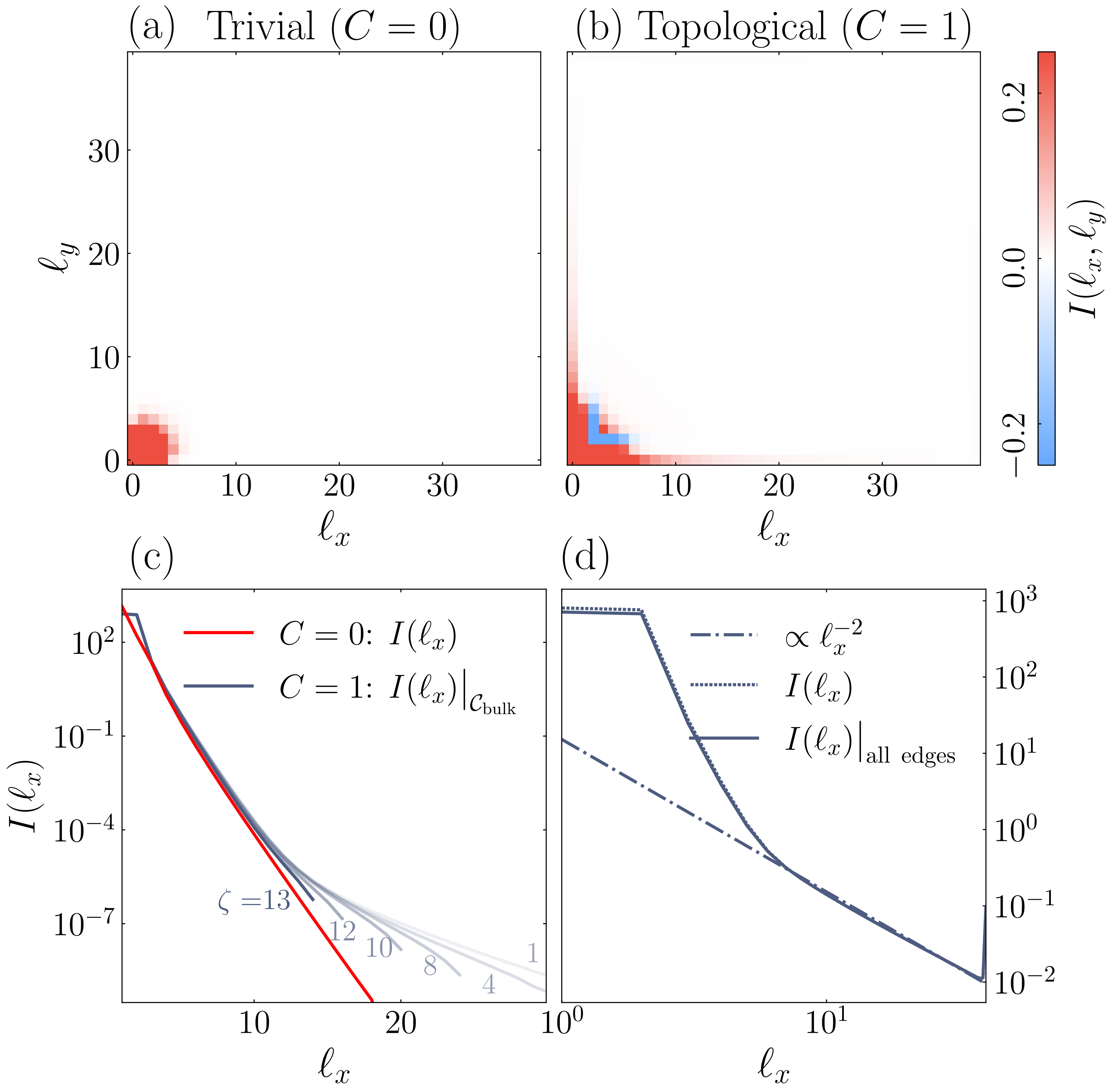
2D Anderson model with anisotropic hopping

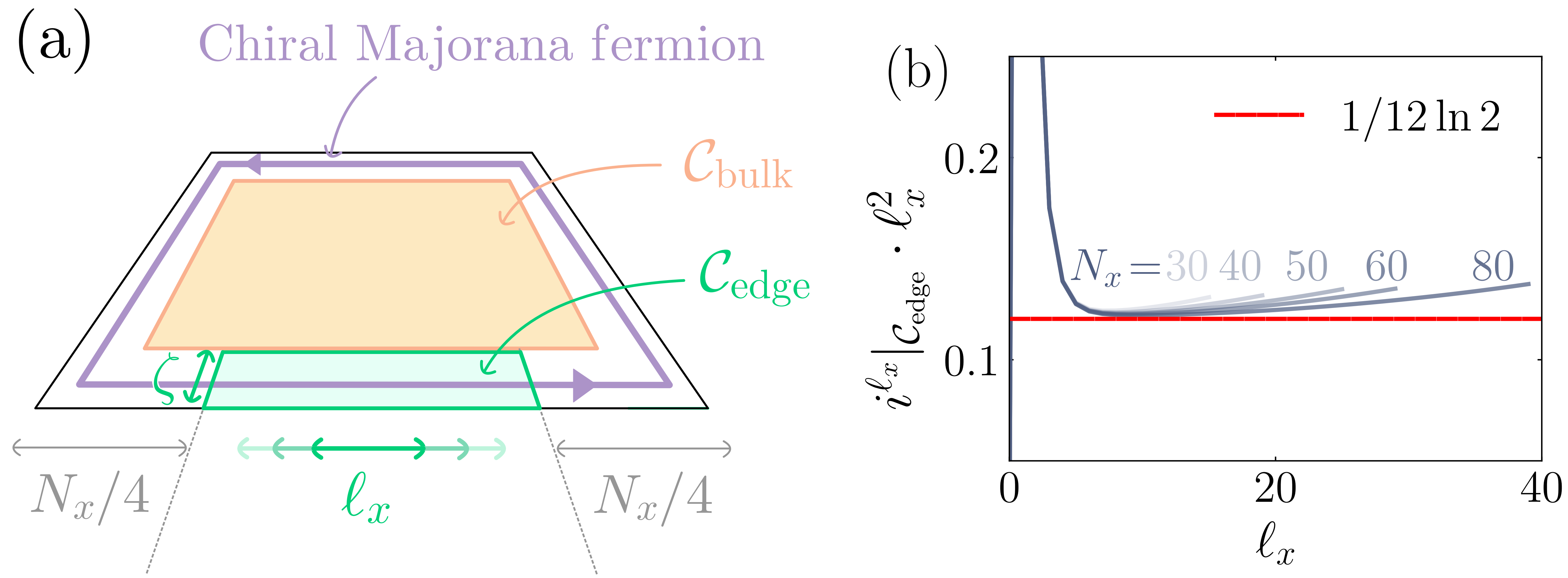




[Flór, Artiaco, Klein Kvorning, JHB, arXiv:2512:20793]

$p + ip$ superconductor



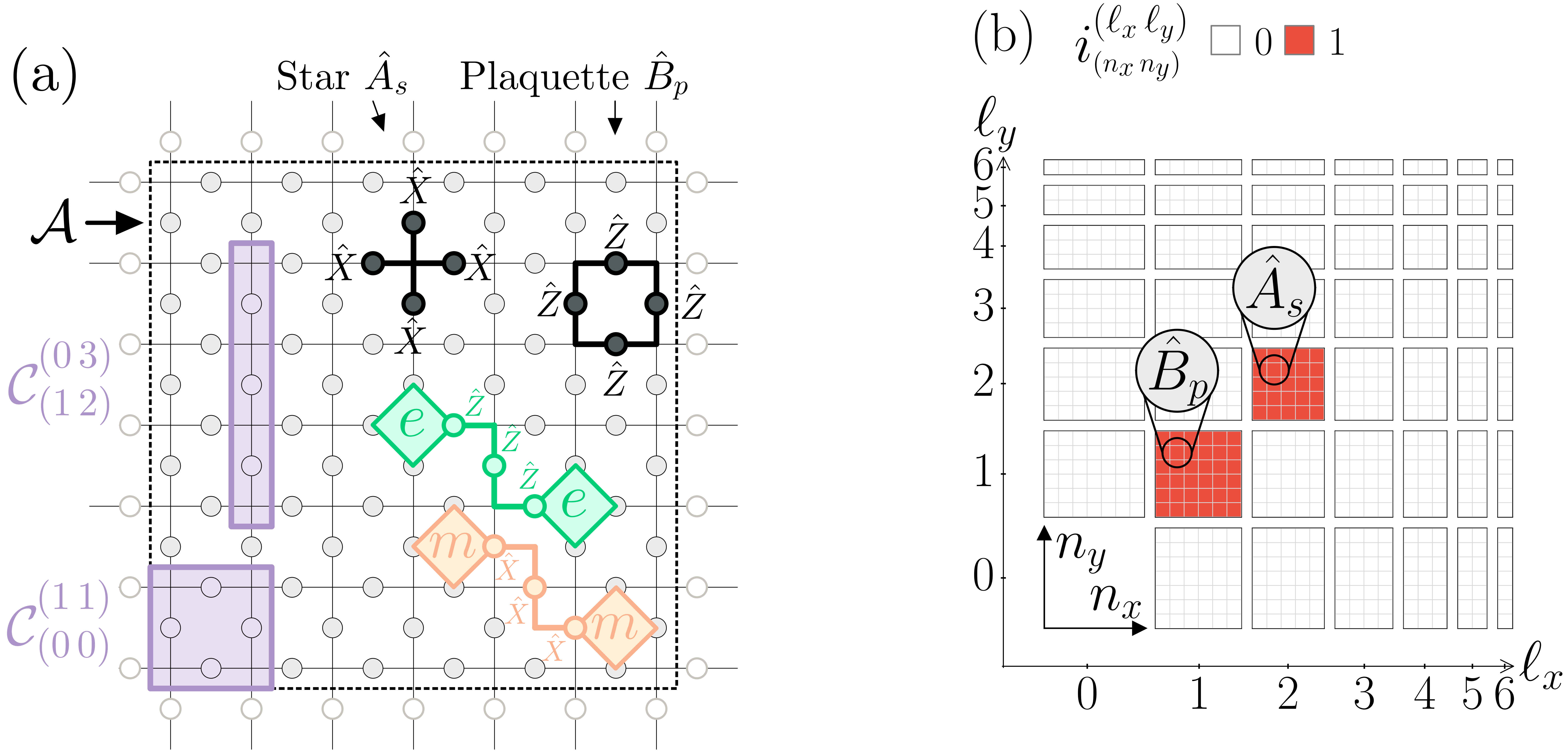


Can separate information in bulk and at boundary

Bulk exponentially decaying, edge follows a power law with appropriate central charge

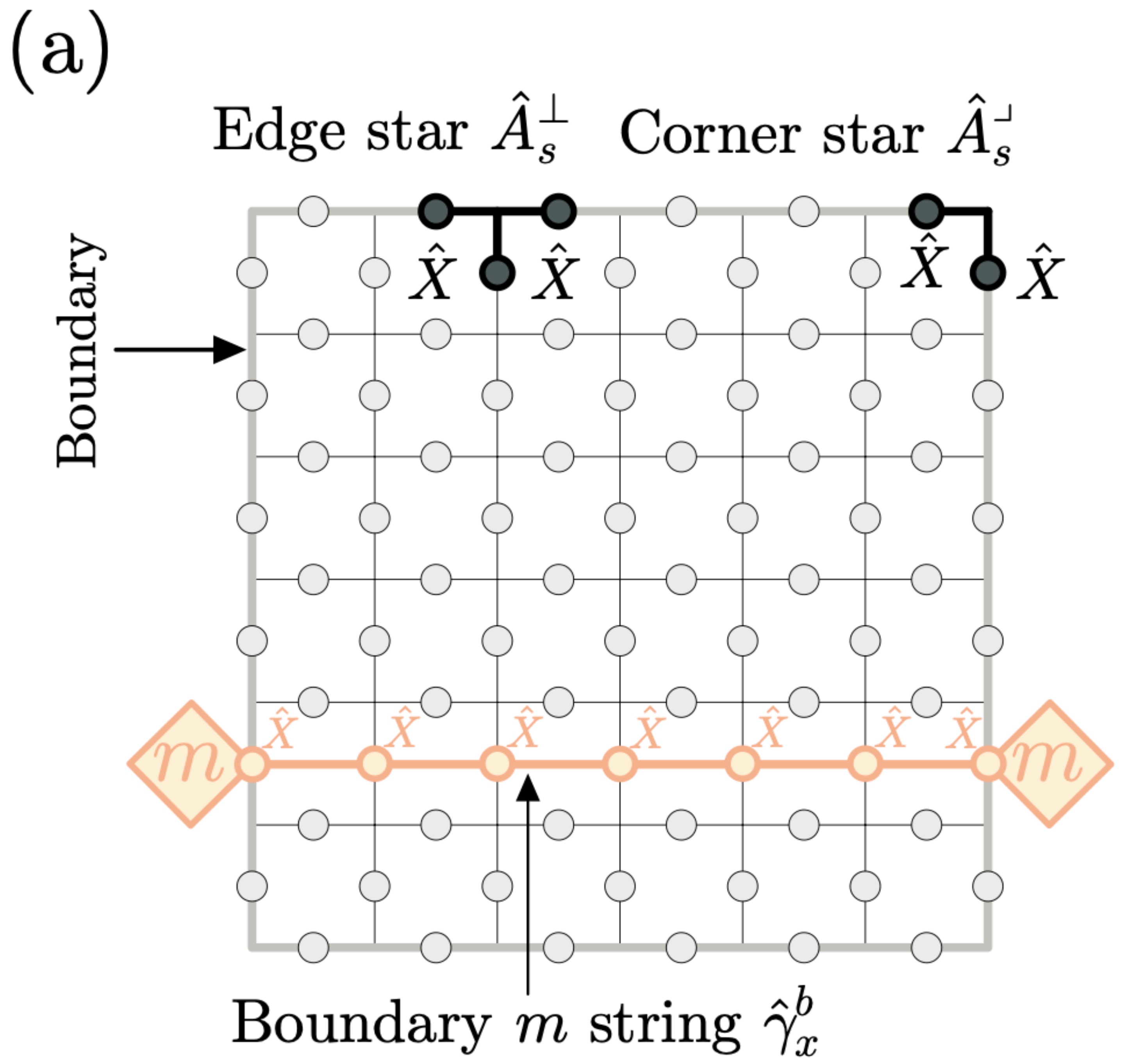
[Flór, Artiaco, Klein Kvorning, JHB, arXiv:2512:20793]

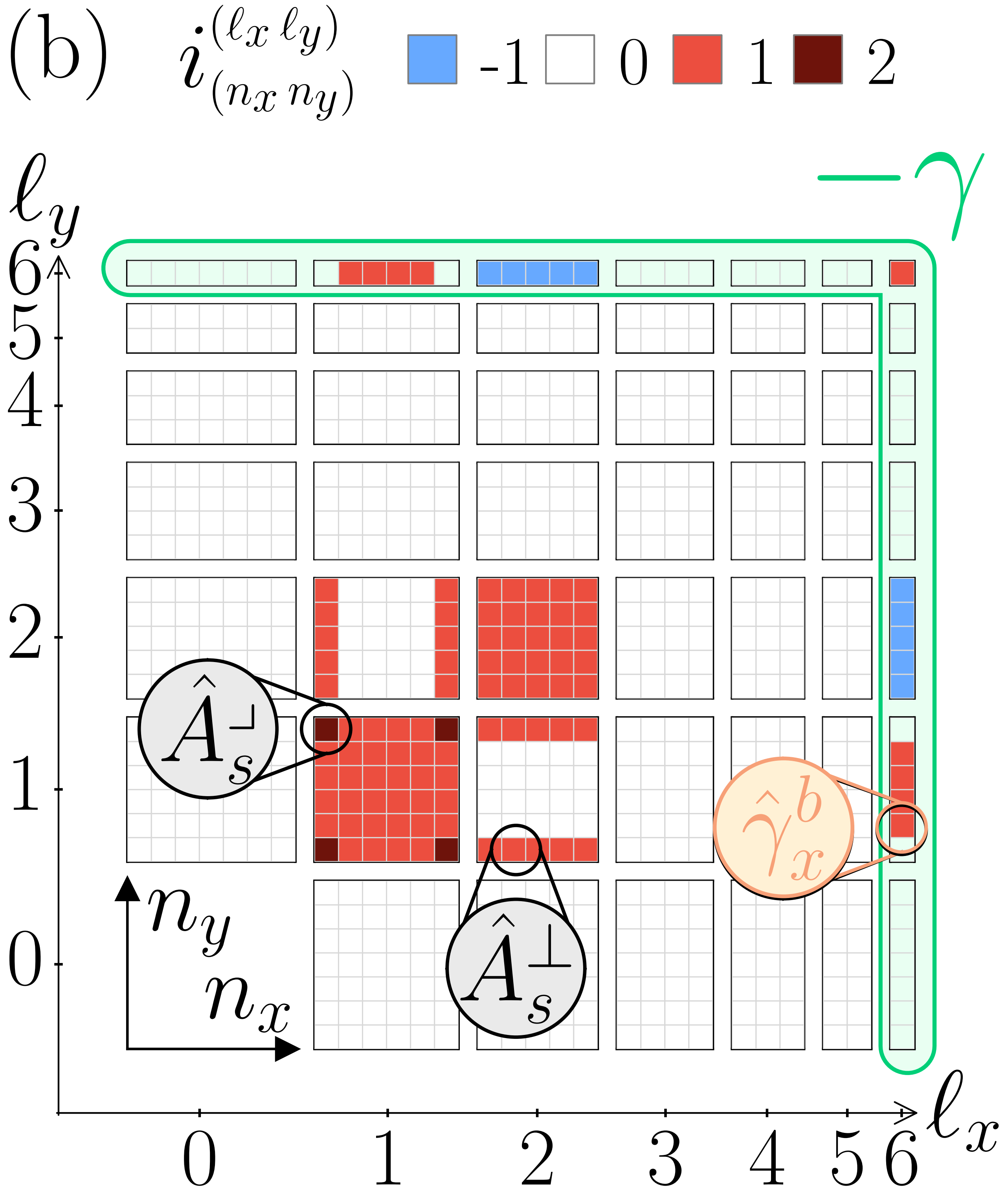
Toric code – bulk information is local



[Flór, Artiaco, Klein Kvorning, JHB, arXiv:2512:20793]

Toric code – topological entanglement is an overlap redundancy



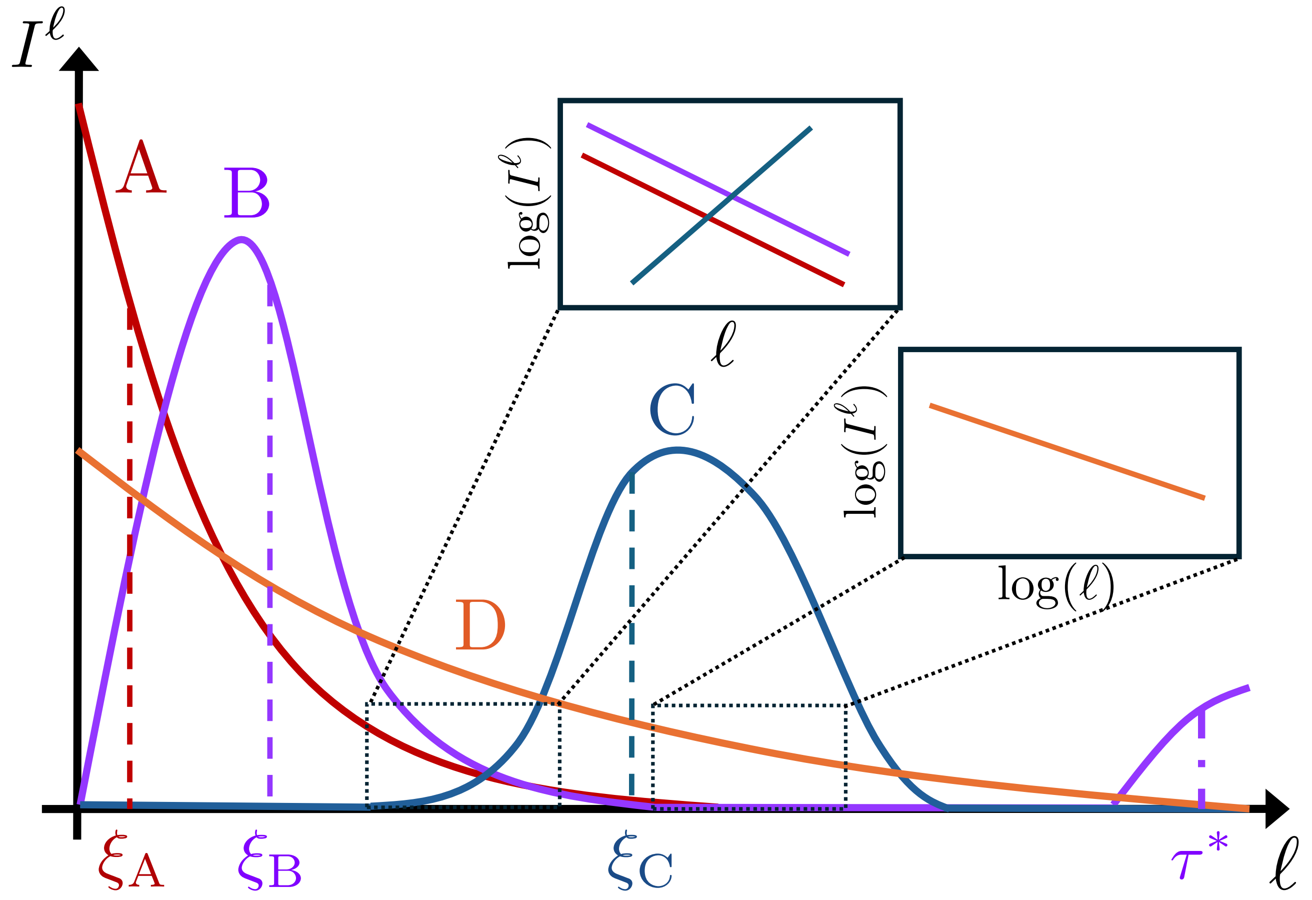


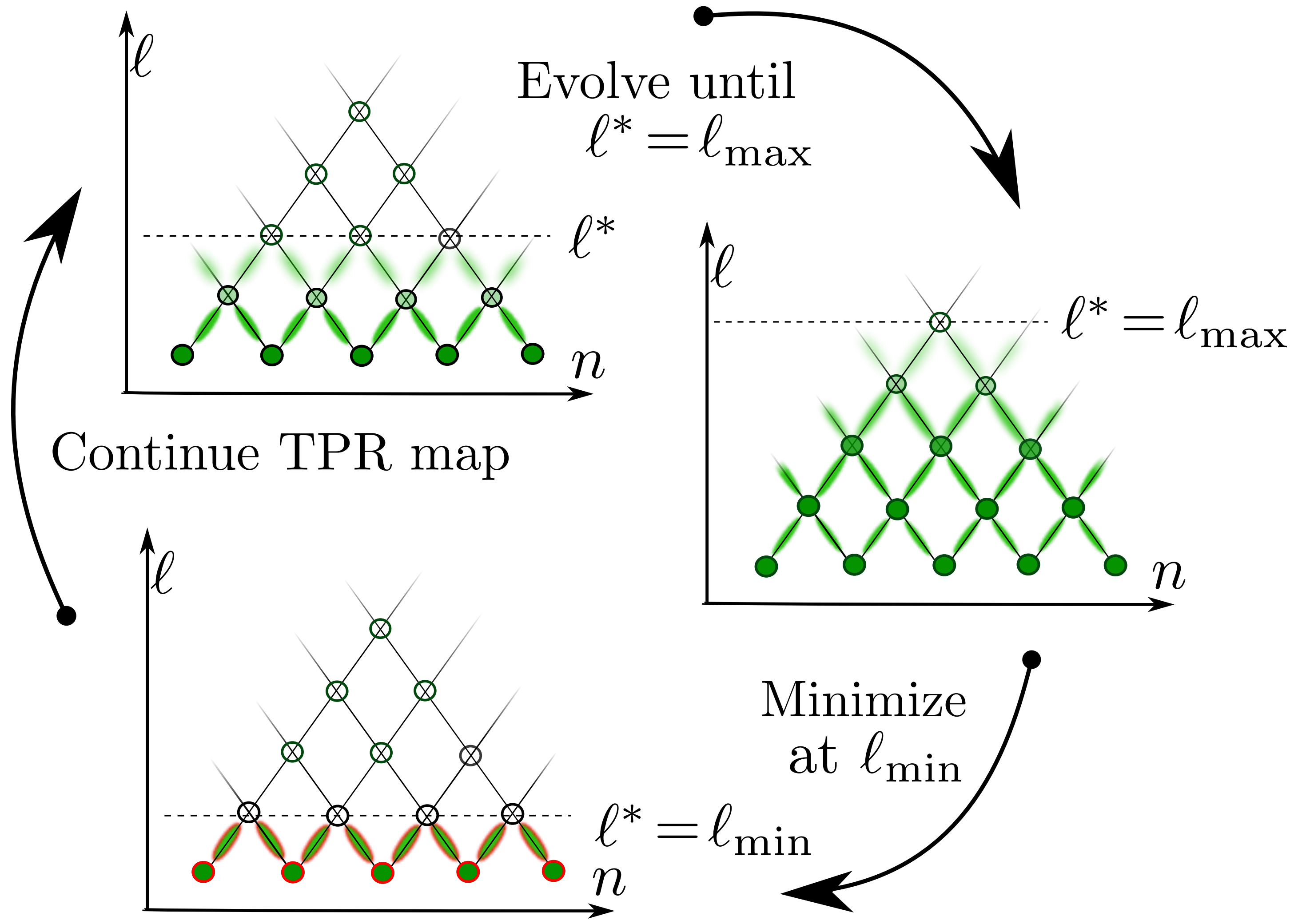
Topological entanglement entropy γ arises from redundancies of strings

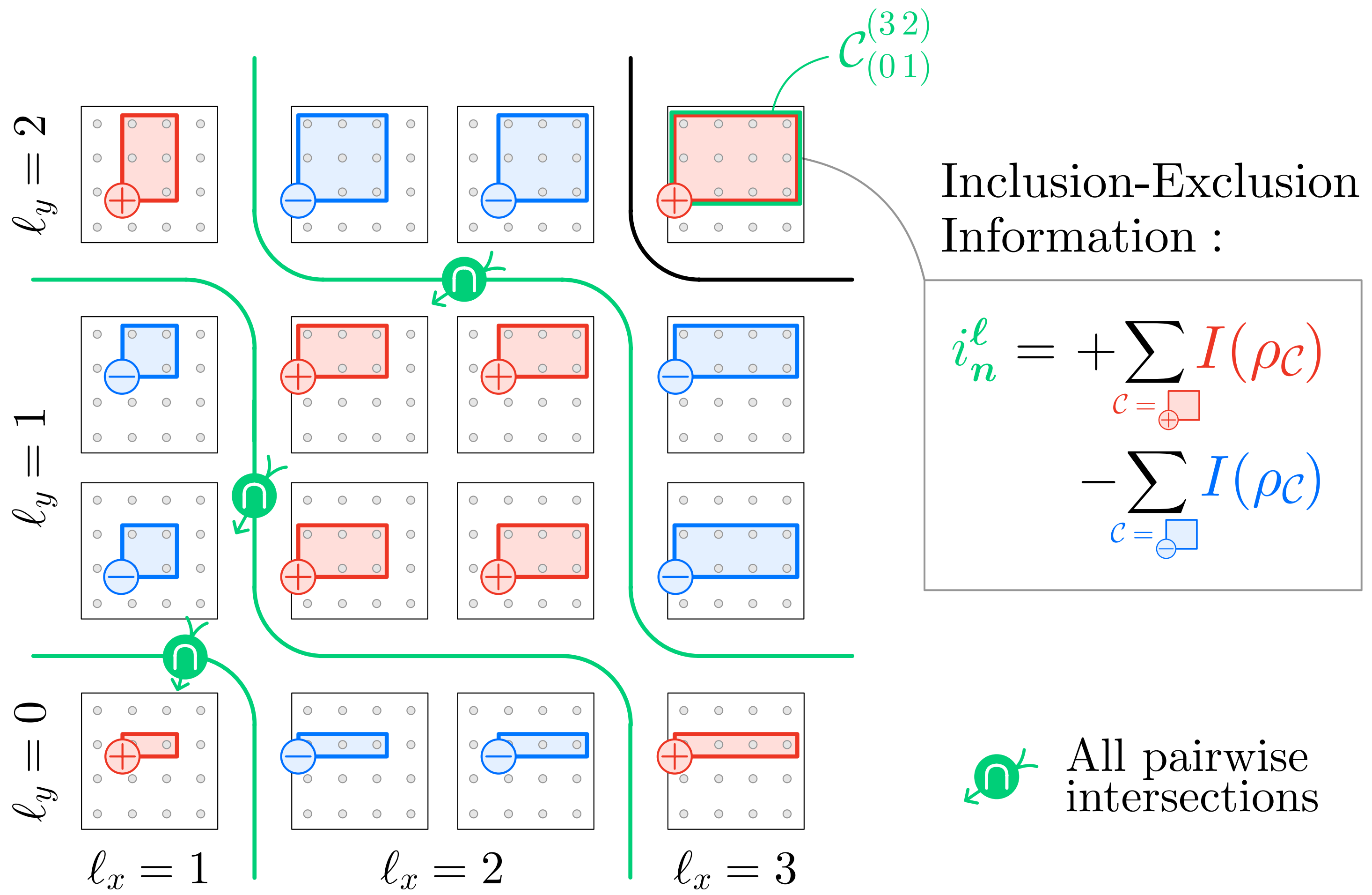
Can not be changed by local transformations

[Flór, Artiaco, Klein Kvorning, JHB, arXiv:2512:20793]

Summary







SciPost Physics **13**, 080 (2022); PRX Quantum **5**, 020352 (2024); PRL **134**, 190401 (2025); arXiv:2510.26696; arXiv:2512:20793