Non-perturbative constraints for non-equilibrium matter

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arXiv:2512.07220, 2602.04961,
2607.XXXX

Far-from-equilibrium physics = anarchy?

…the preceding sections described approaches to the dynamic behaviour of systems close to an equilibrium critical point… However, in describing critical behaviour of systems which are far from equilibrium, no such guiding principles are available. Indeed, it would appear that the whole subject should crumble into anarchy. However, once again, the renormalization group and universality provide tools for attempting a classification.

(Chapter 10 of Cardy's book on RG)

Organizing principles?

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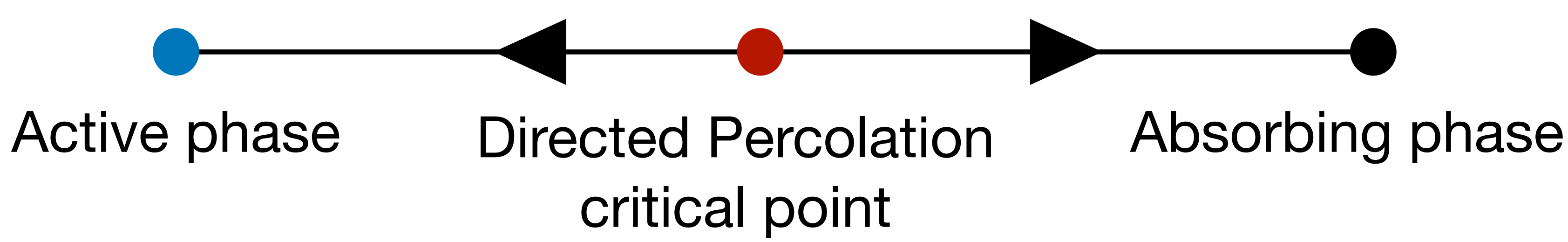
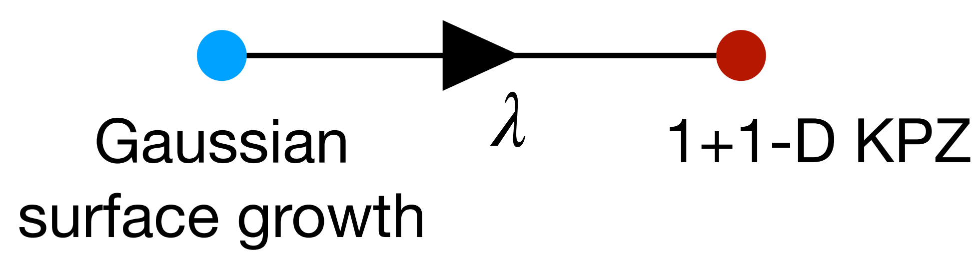
Two famous examples in 1+1-D:

Typical population dynamics problems:



Typical surface growth problems:



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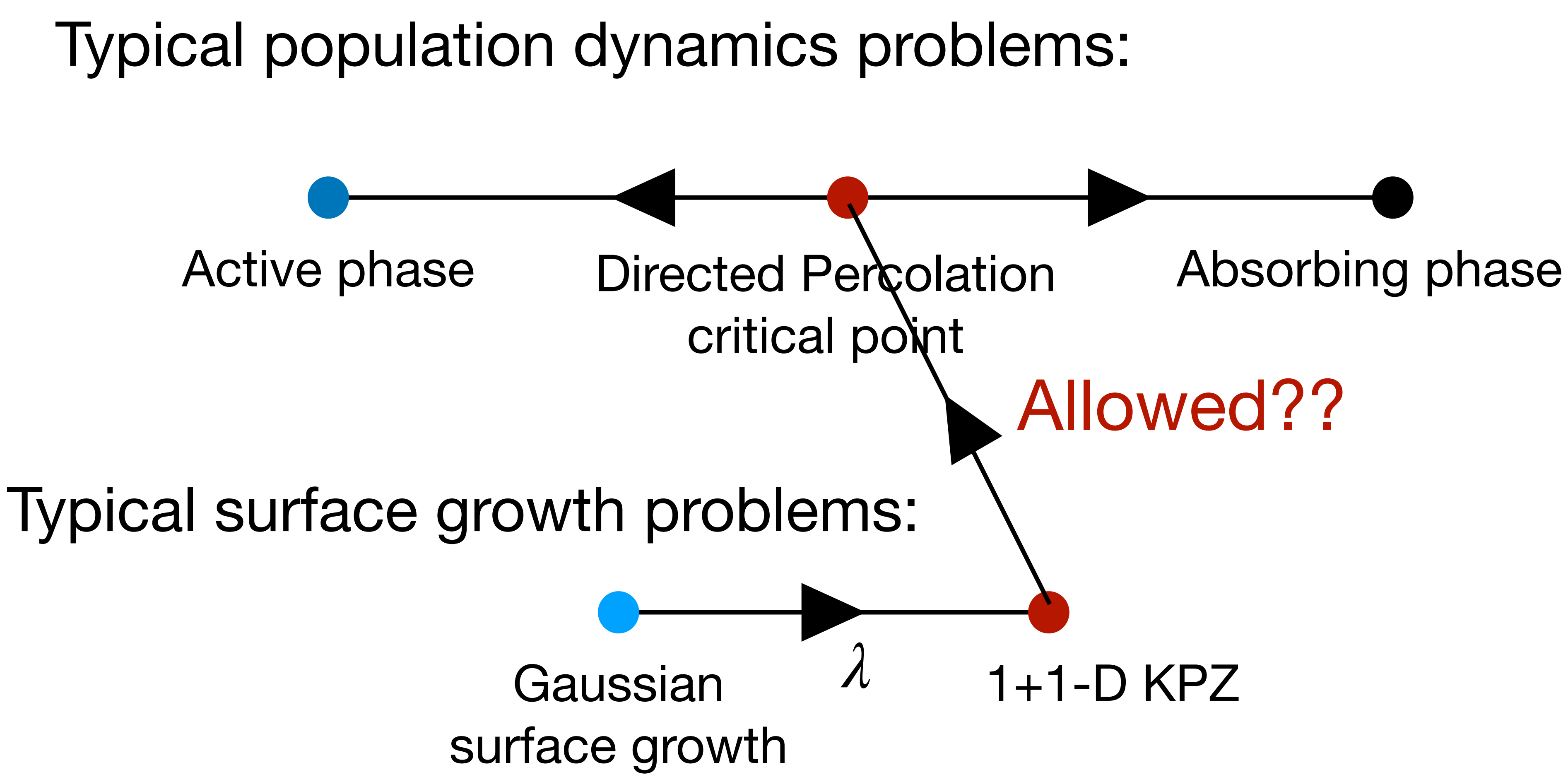

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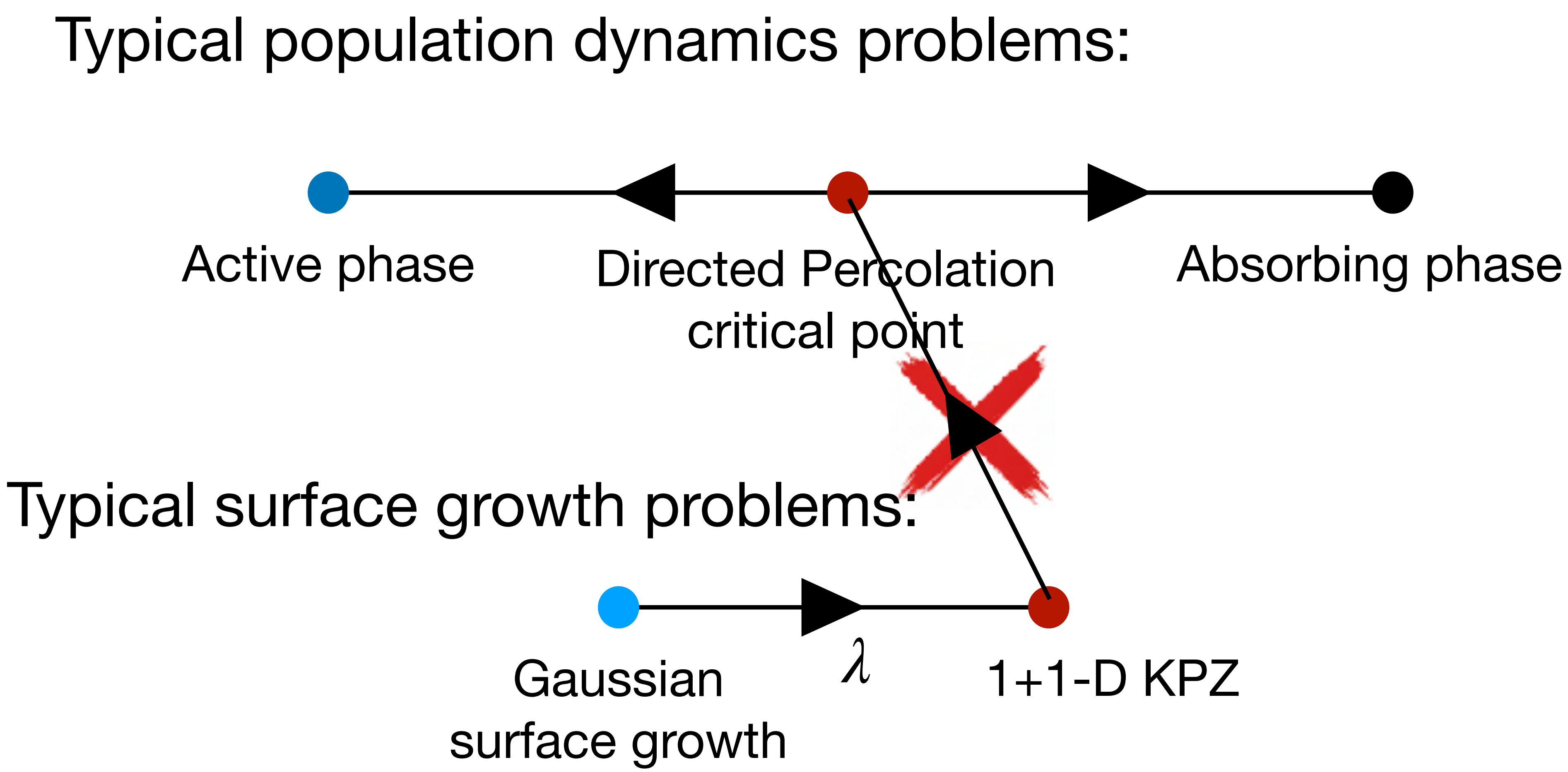

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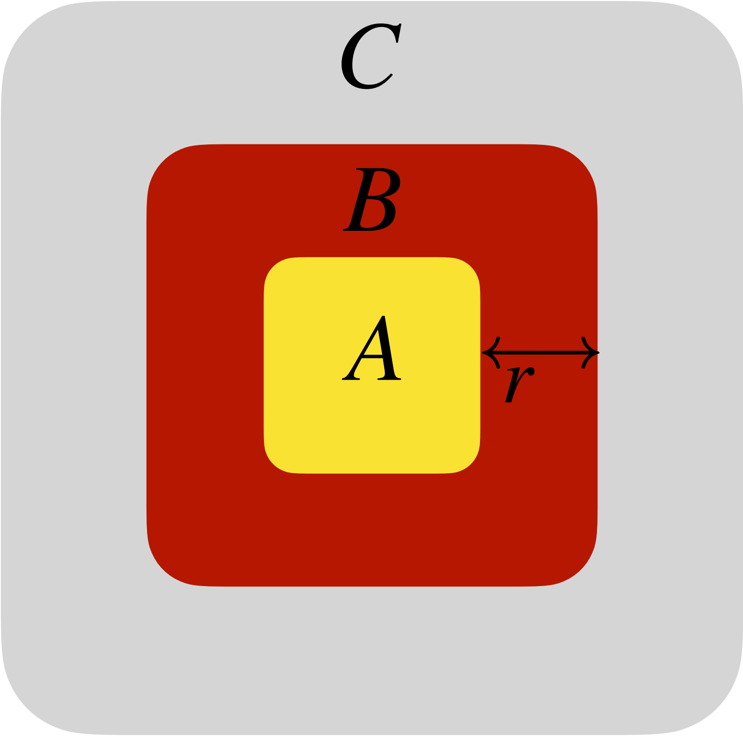

$$I(A:C\,|\,B) \sim e^{-r/\xi}$$

Markov length
$\sim$ 1/gap

[Coser, Perez-Garcia, 1810.05092; Sang, Hsieh 2404.07251; Sang, Lessa et al 2507.02292]

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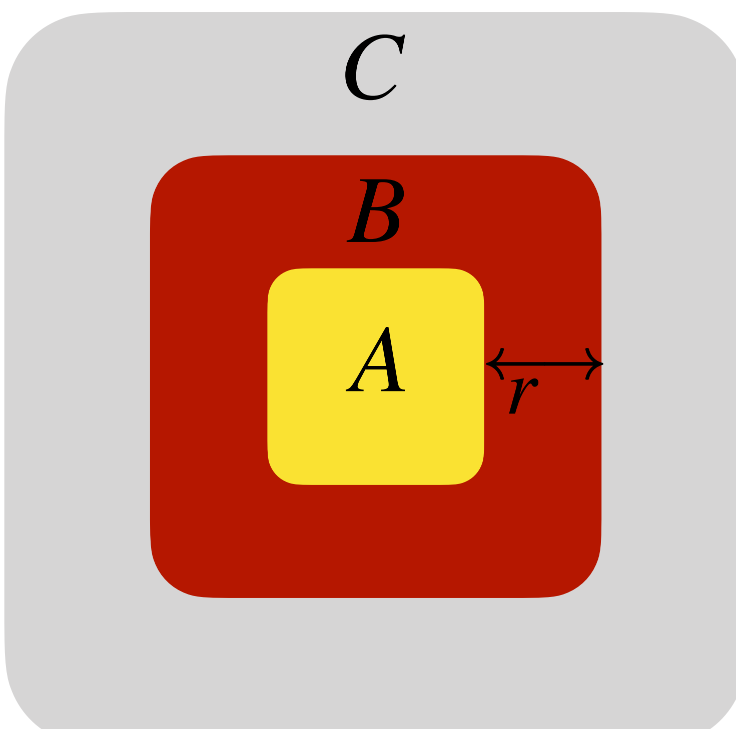

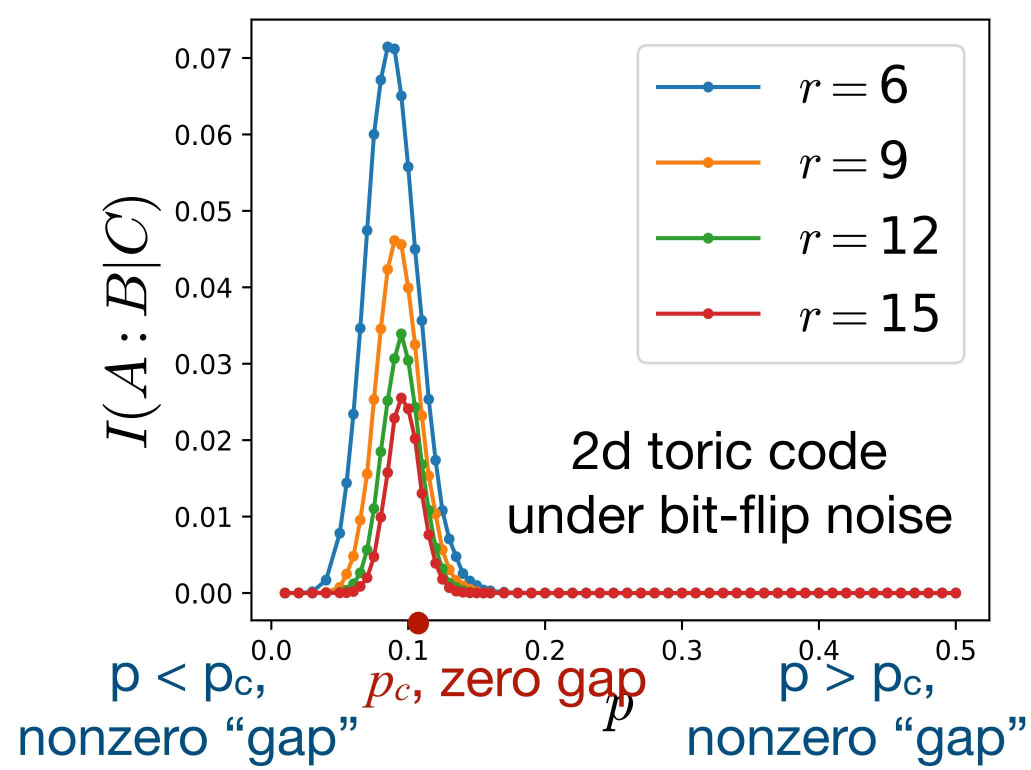

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   Does this framework work for classical non-equilibrium steady states? (directed percolation, KPZ, Toom's model, etc.). We tested it for 1+1-D directed percolation and it works; wider exploration would build confidence.

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Related broad theme: generalized symmetries (higher form, strong/weak etc.) in mixed-states and their anomalies.

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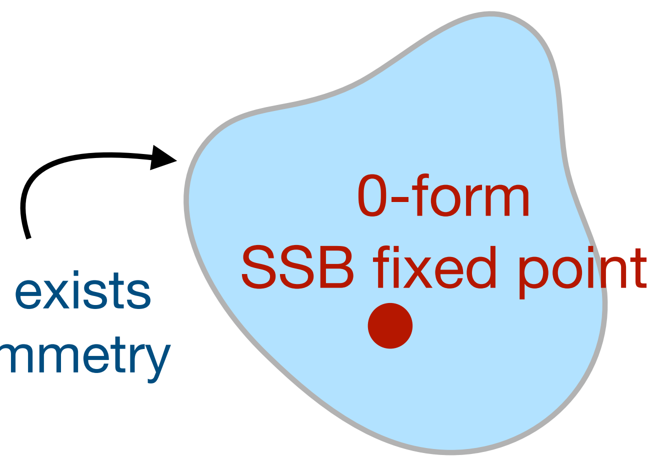
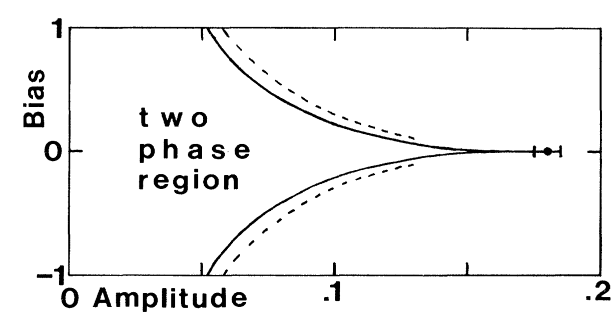
I will briefly discuss that even weak 0-form symmetries and their spontaneous symmetry breaking can be emergent: “Spontaneous symmetry breaking without symmetry”.

Open ball where SSB exists without any unitary symmetry

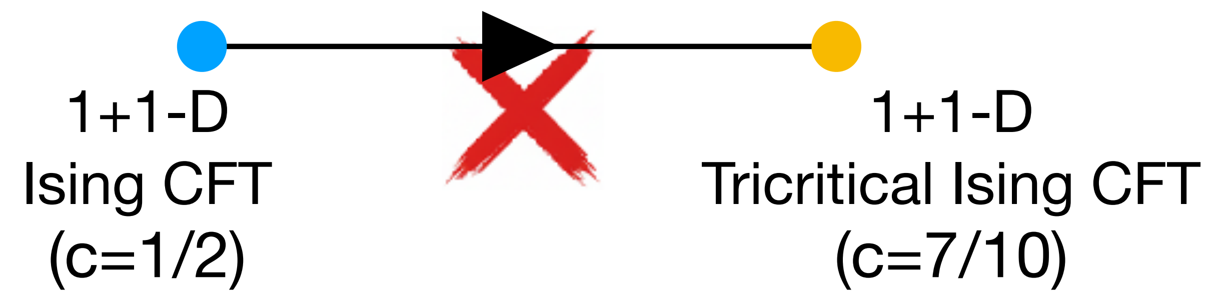




[Toom 1974, Bennett, Grinstein 1985]

Constraints on RG flows

For relativistic-invariant theories, there exist general constraints on RG flows.



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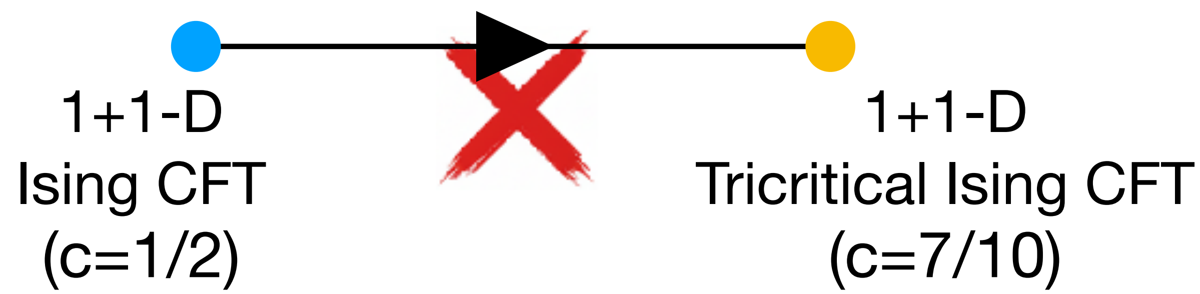


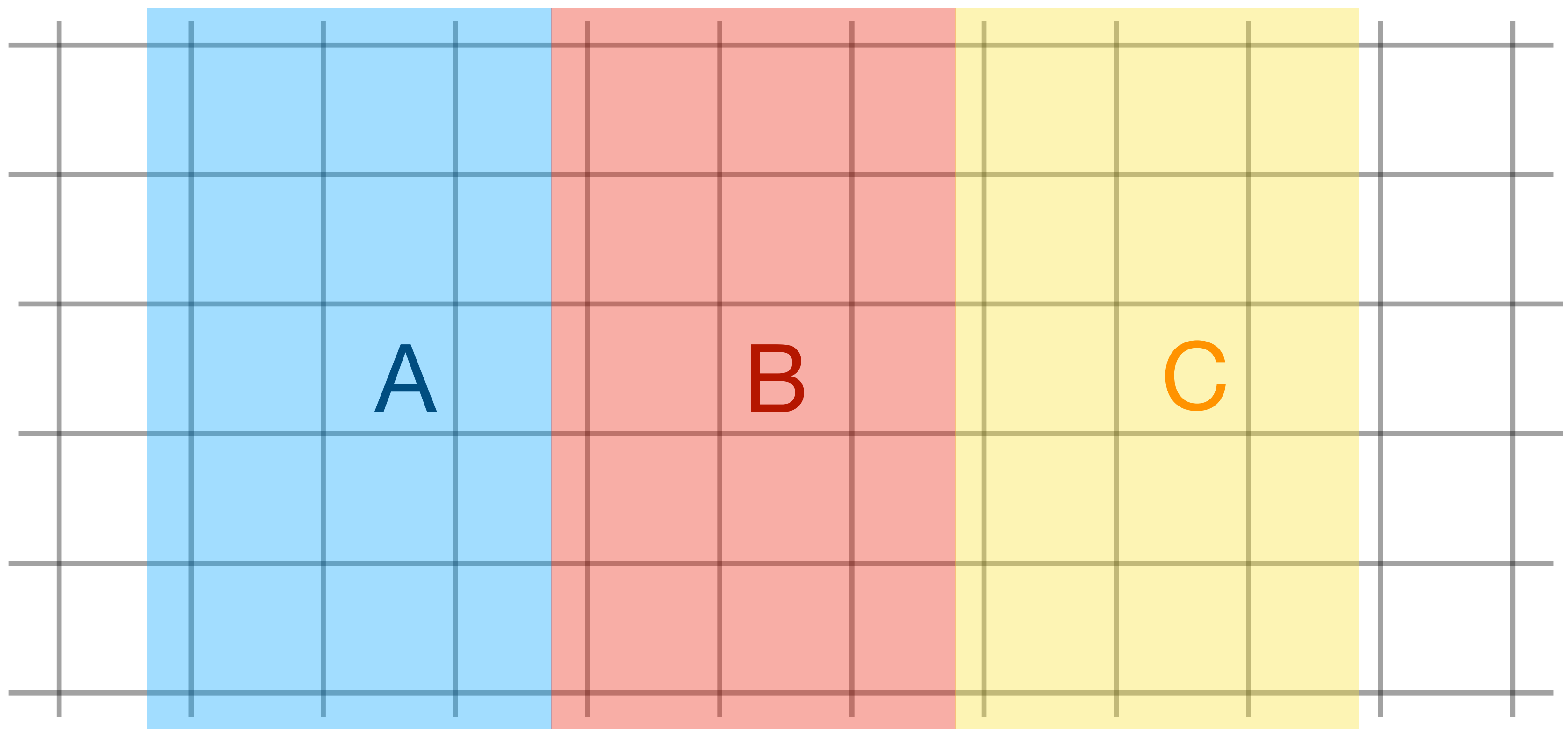
Such constraints ("c-theorems") exist in every dimension.

[1+1-d: Zamolodchikov 1986; 2+1-D: Casini, Huerta (2012); 3+1-D: Cardy (1988), Komargodski, Schwimmer (2011); recent generalizations to disordered systems by Patil, Ludwig 2507.07959.]

Remarkably, c-theorems can often be derived using strong subadditivity.

[Casini, Huerta 2004; Casini, Huerta 2012]

Strong subadditivity of entropy



$$S(AB) + S(BC) - S(B) - S(ABC) \geq 0$$

[Lieb, Ruskai 1973]

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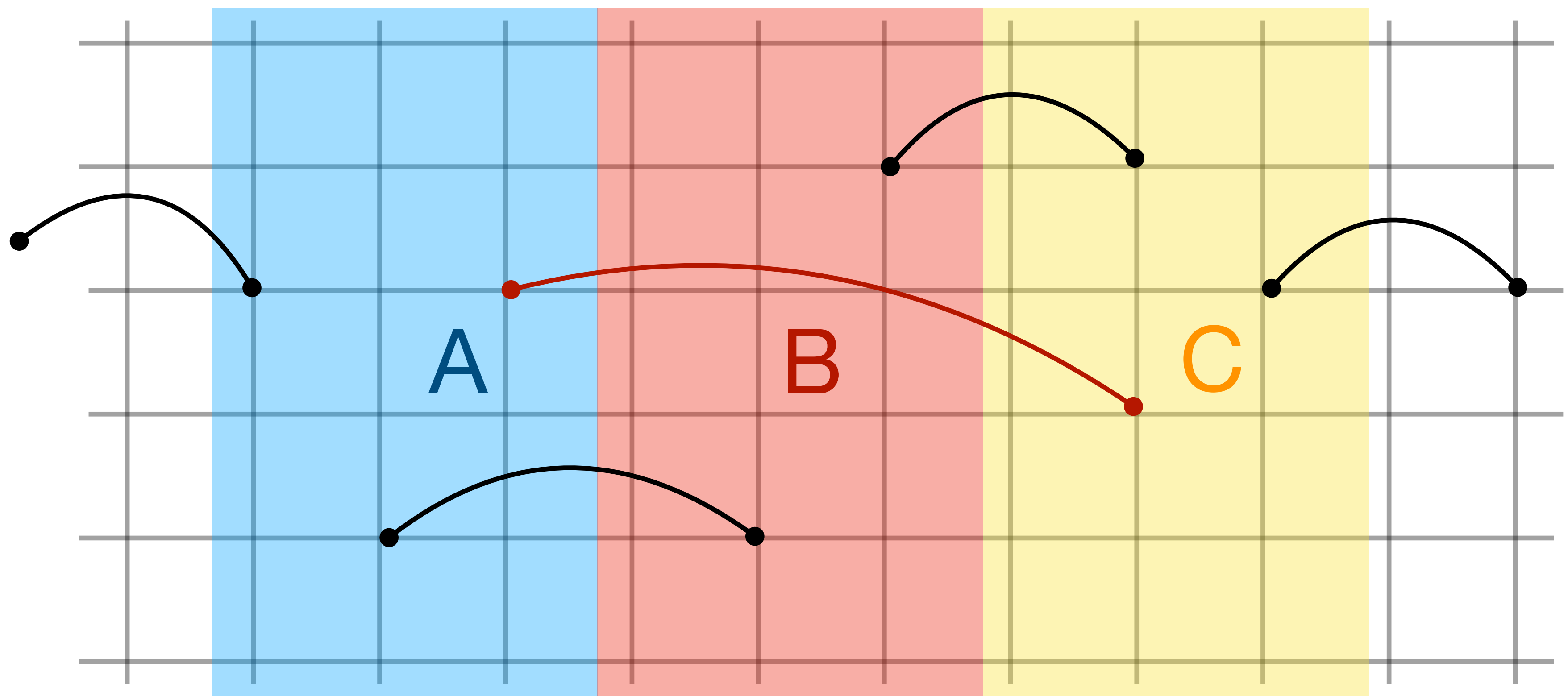


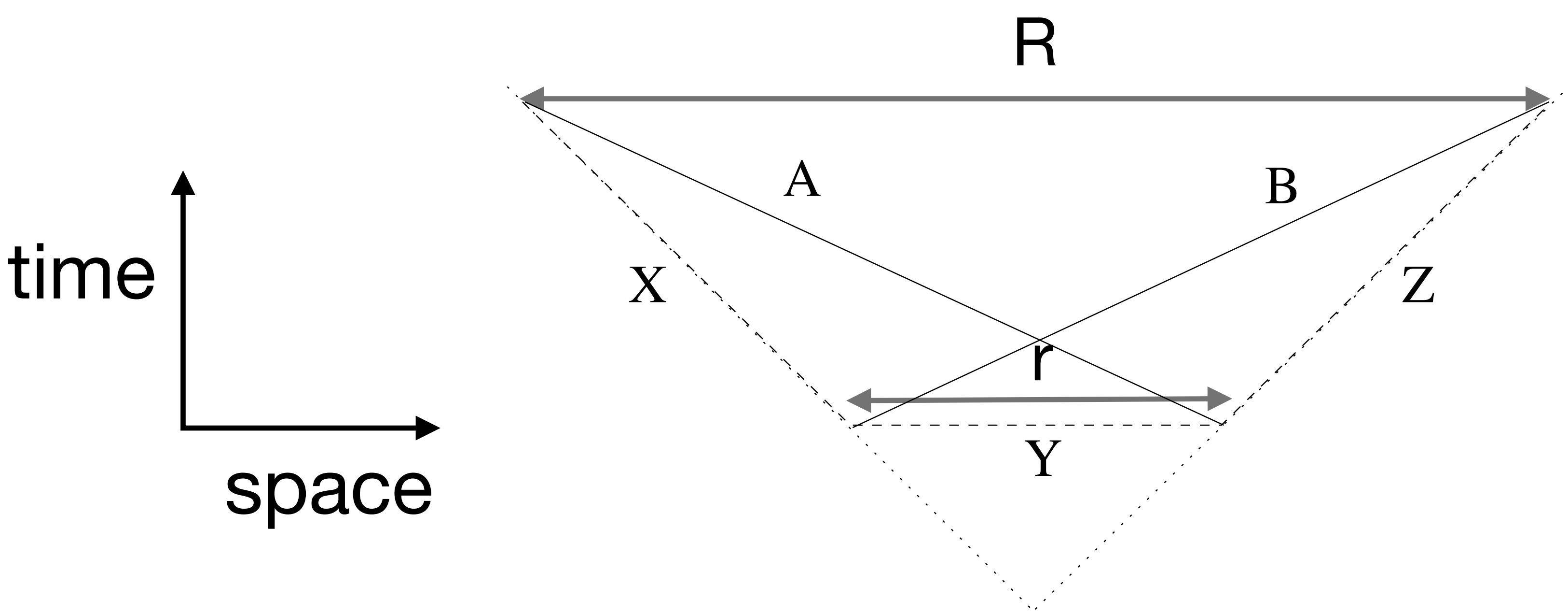
$$\underbrace{S(AB) + S(BC) - S(B) - S(ABC)} \geq 0$$

Correlations between A and C <u>not</u> mediated via B

= “Conditional mutual information” I(A:C|B)

c-theorem from Strong subadditivity

[Casini, Huerta, hep-th/0405111; cond-mat/0610375]



$$S(XY) + S(YZ) \geq S(Y) + S(XYZ)$$

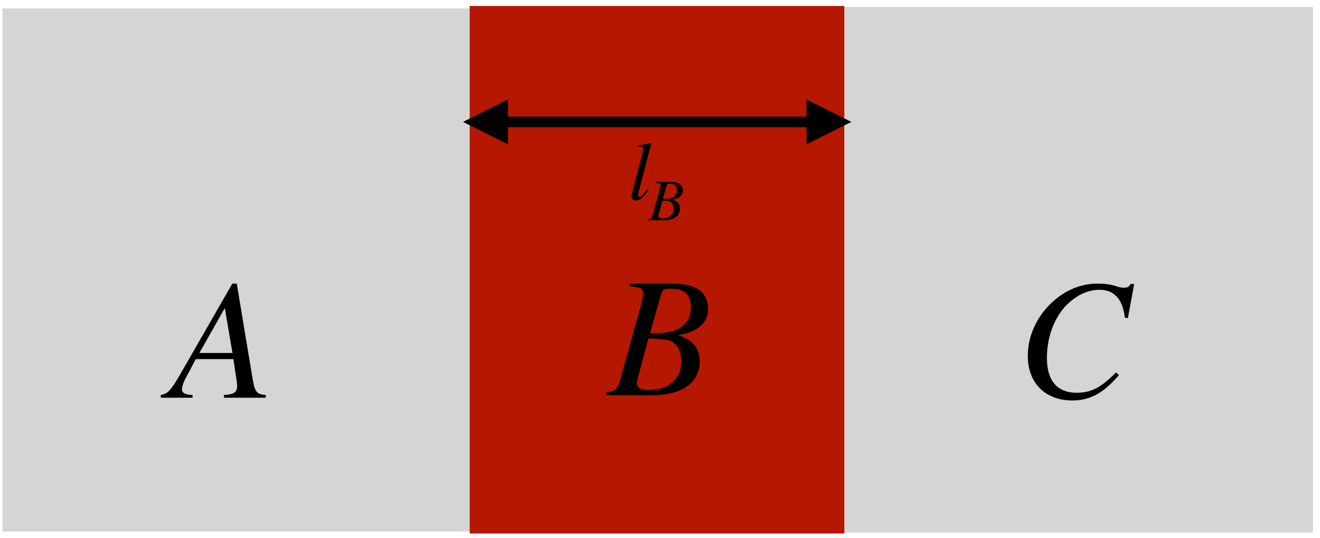
$$\Rightarrow \quad \frac{d^2 S(r)}{d\log(r)^2} \leq 0 \qquad \Rightarrow \quad \left.\frac{dS(r)}{d\log(r)}\right|_{UV} \geq \left.\frac{dS(r)}{d\log(r)}\right|_{IR}$$

$$\Rightarrow \quad \boxed{c_{UV} \geq c_{IR}}$$

No direct analog of c-theorems for non-relativistic systems.

Since strong subadditivity is always true, natural to seek its consequences nonetheless.

The scaling behavior of $I(A : C \,|\, B)$ will play a role in this pursuit.

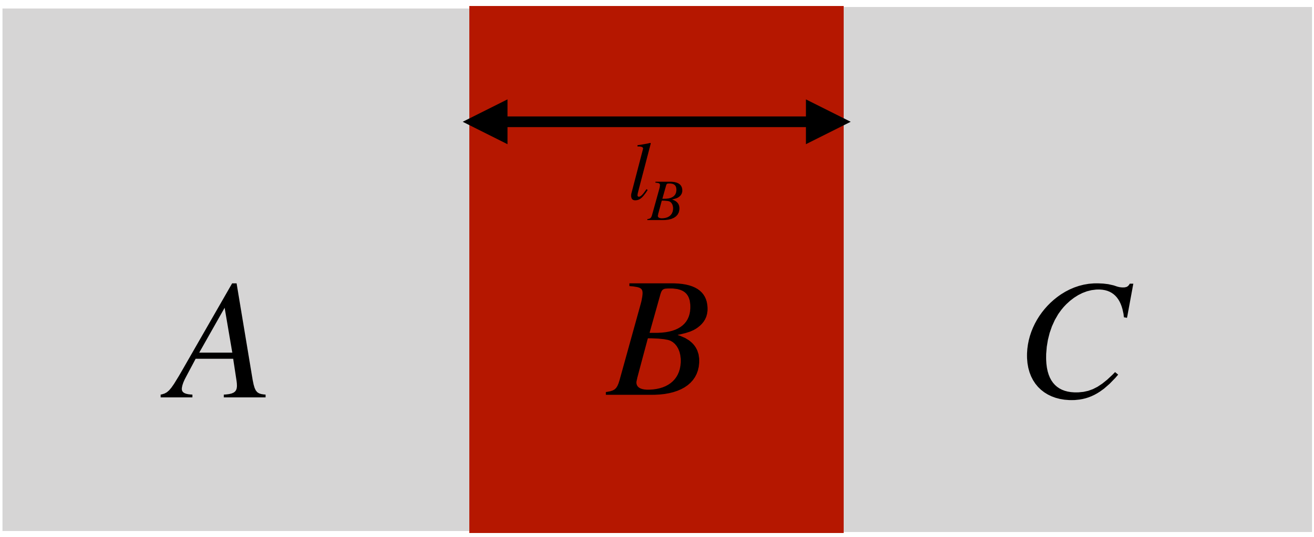


Typical behaviors of CMI at RG fixed points:

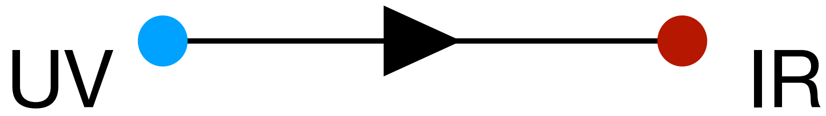
$I = 0$: This is expected for Gibbs states of local Hamiltonians. For classical Gibbs states, $I = 0$ exactly [Clifford-Hammersley 1971], even for critical states! For quantum Gibbs, when |A| = O(1), and arbitrary |B|, |C|, $I \sim e^{-l_B/\xi}$ with $\xi = O(1)$ [Chen, Rouze, 2504.02208].

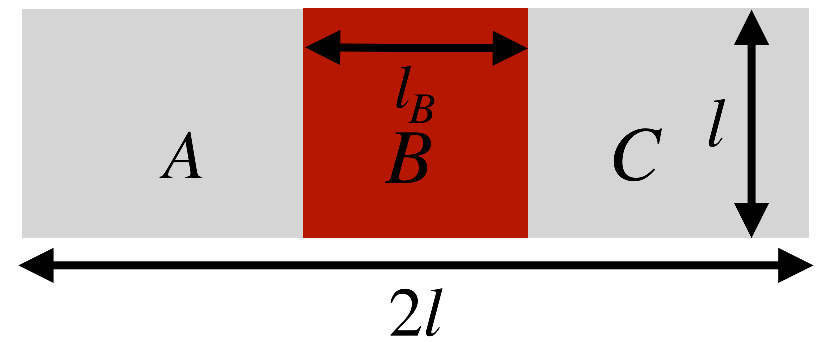
$I \sim$ O(1) constant: e.g. strong-to-weak symmetry breaking phases.

$I \sim 1/l_B^\alpha$: e.g. several non-equilibrium phase transitions.



Consider CMI along an RG flow triggered by a relevant perturbation





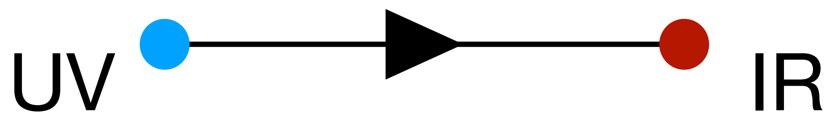
Four length scales:

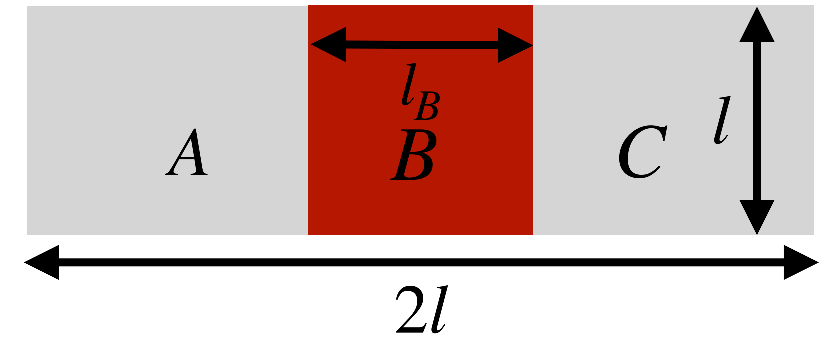
lattice spacing a, l, l_B, ξ = crossover scale between UV and IR

$$\Rightarrow \text{CMI} = f\left(\frac{l_B}{\xi}, \frac{l}{a}, \frac{l}{l_B}\right)$$

Meaning of RG flow from UV to IR:

Physics at distances much less (much greater) than ξ is characteristic of the UV (IR) fixed point.



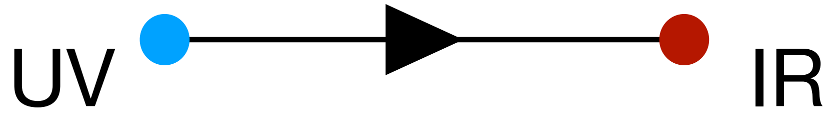


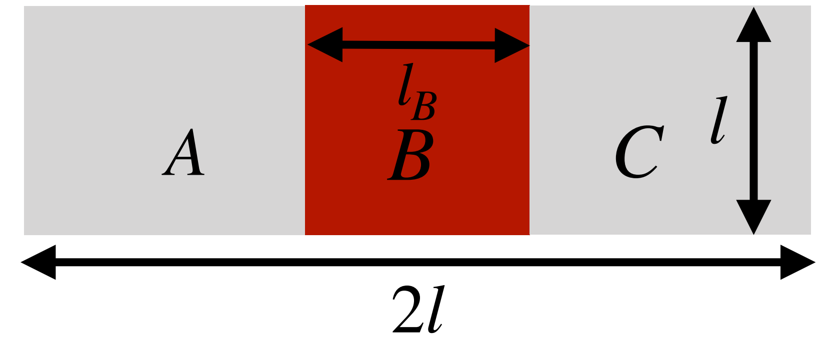
$$\text{CMI} = f\left(\frac{l_B}{\xi}, \frac{l}{a}, \frac{l}{l_B}\right)$$

<u>Assumptions:</u>

(i) CMI is "UV finite" i.e. non-divergent when $a \to 0$.

(ii) "Continuity": CMI at $0 < \dfrac{l_B}{\xi} \ll 1$ is representative of the UV fixed point, while CMI at $\dfrac{l_B}{\xi} \gg 1$ is representative of the IR fixed point.



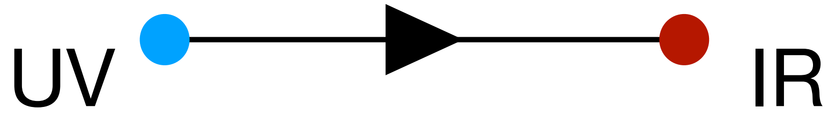


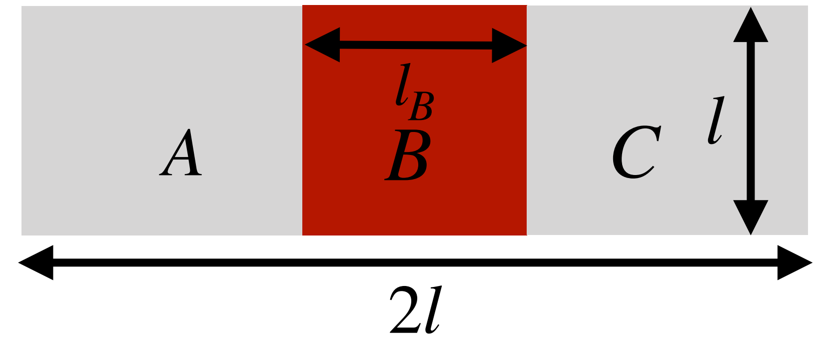
$$\text{CMI} = f\left(\frac{l_B}{\xi}, \frac{l}{a} = \infty, \frac{l}{l_B}\right)$$

At a scale-invariant fixed-point point, $l_B/\xi = 0$ or ∞ $\Rightarrow$

(i) If l held fixed, CMI expected to have a power-law dependence on l_B .

(ii) If l/l_B held fixed, CMI should be an O(1) shape-dependent universal number.





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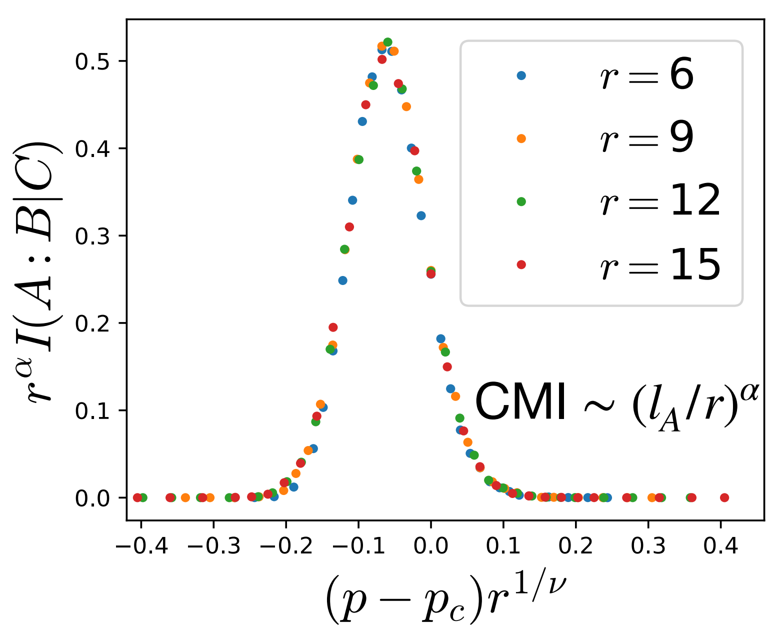
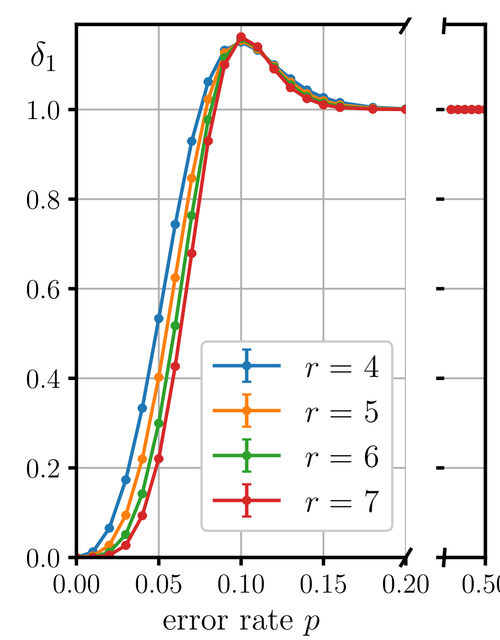
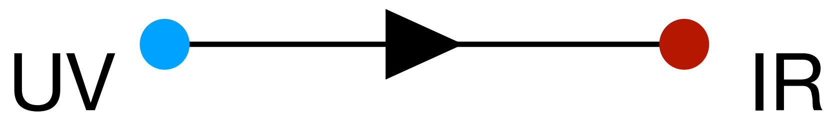


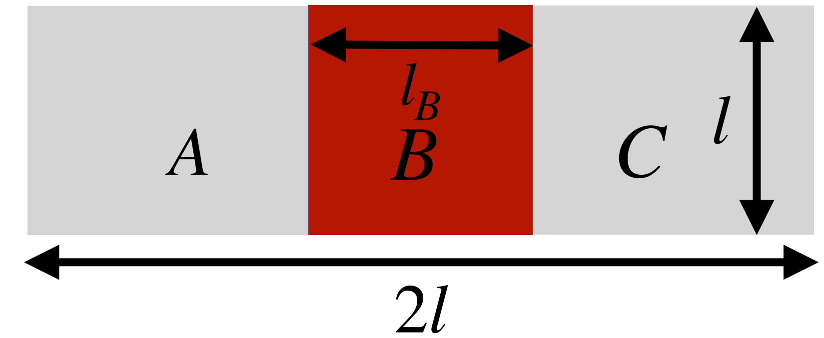
CMI for toric code's decodability transition.



Sang, Hsieh 2404.07251: l_A held fixed, $l_B \sim l_c = r$

Yang, Shi, Lee, 2506.04221: $l_A \sim l_B \sim l_c = r$





$$\text{CMI} = f\left(\frac{l_B}{\xi}, \frac{l}{a}, \frac{l}{l_B}\right)$$

Assumptions:

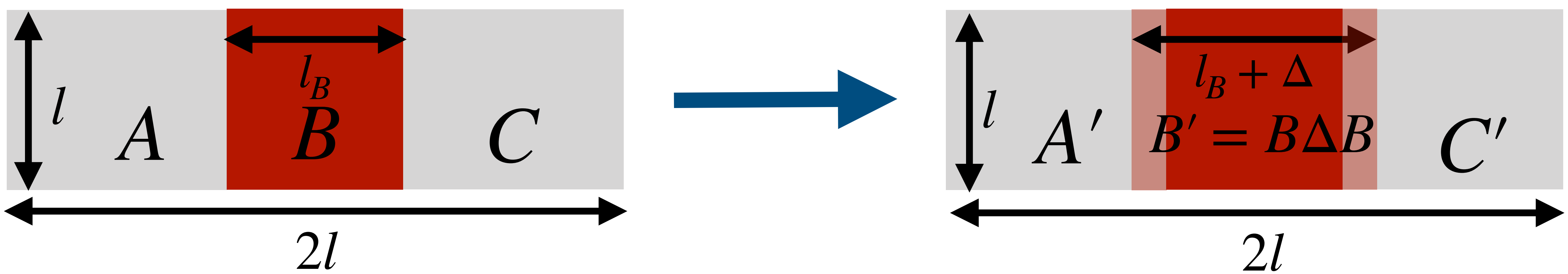
(i) CMI is "UV finite" i.e. non-divergent when $a \to 0$.

For now, we also take the limit $l/l_B \to \infty$ so that CMI $= f(l_B/\xi, \infty, \infty)$.

We will return to the more general case later.

(ii) "Continuity": CMI at $0 < \dfrac{l_B}{\xi} \ll 1$ is representative of the UV fixed point, while CMI at $\dfrac{l_B}{\xi} \gg 1$ is representative of the IR fixed point.

Constraint on RG flows from Strong Subadditivity

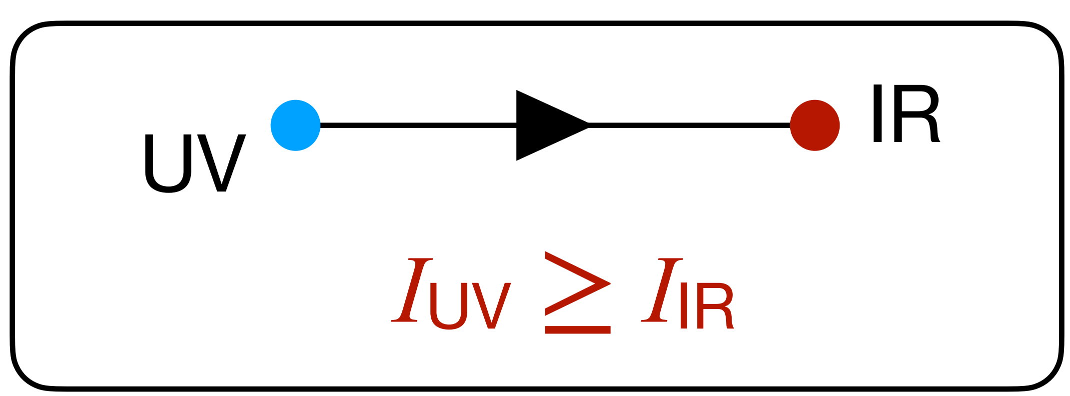


Notably, $I(A:C\,|\,B) - I(A:C'\,|\,B') = I(A':\Delta B\,|\,B) \geq 0 \;\Rightarrow\; \dfrac{\partial I}{\partial l_B} \leq 0$

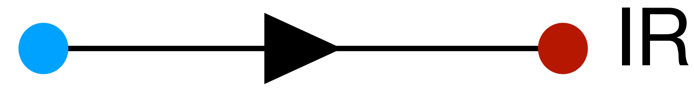
Due to the limit $l/l_B \to \infty, l/a \to \infty$,

CMI is only a function of $x = l_B/\xi$.

$$\Rightarrow \frac{dI}{d|x|} \leq 0 \qquad x = l_B/\xi$$



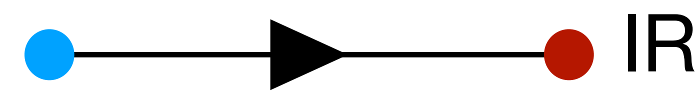
[Yu-Hsueh Chen, TG, 2602.04961]



$$\mathrm{CMI}_{\mathrm{UV}} \geq \mathrm{CMI}_{\mathrm{IR}}$$

Combination of scaling ansatz and Markovian property of local Gibbs states makes this a powerful constraint.

Immediate implication: If the effective Hamiltonian in the IR $H_{IR} \sim \log(\rho_{IR})$ is non-local, then so must be the effective Hamiltonian in the UV.



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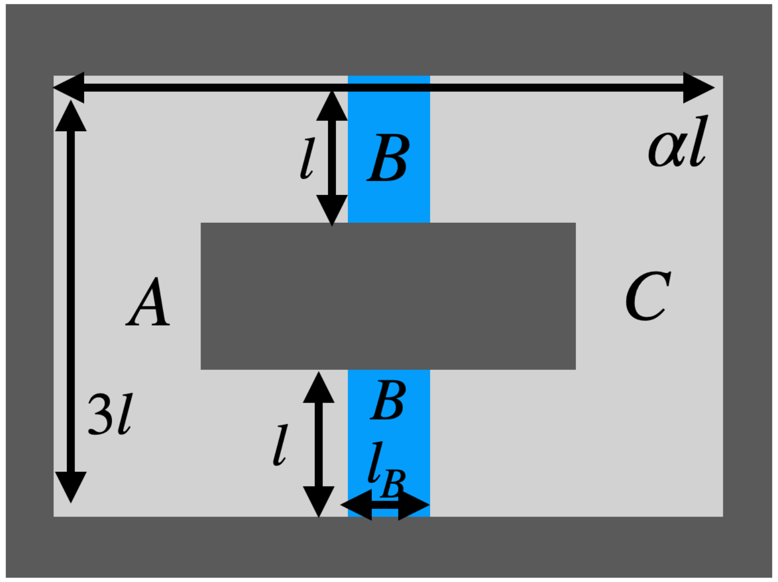
For example, the following is *not* allowed!

Critical 2d local Gibbs state e.g. BKT phase of XY model.

Flocking phase

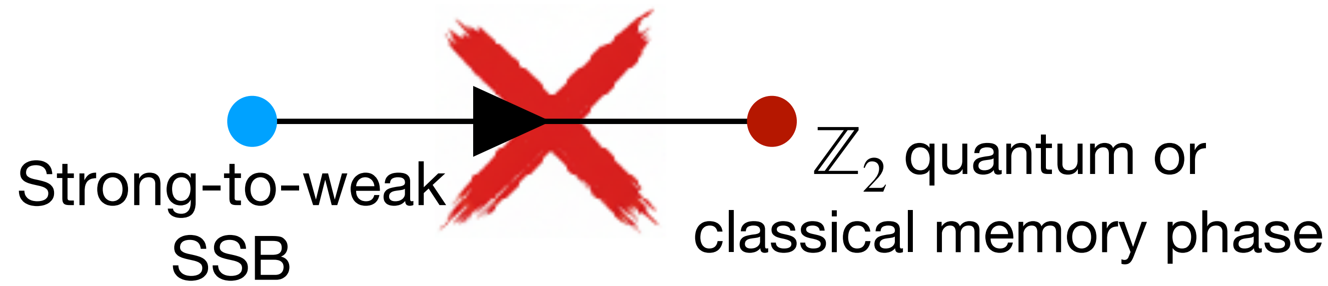
Consistent with the constraint, currently the only way to obtain a flocking phase from a Gibbs state is to add long-range dipolar couplings in the UV [Chen, Toner, Lee 2015].

Generalization of monotonicity to other geometries

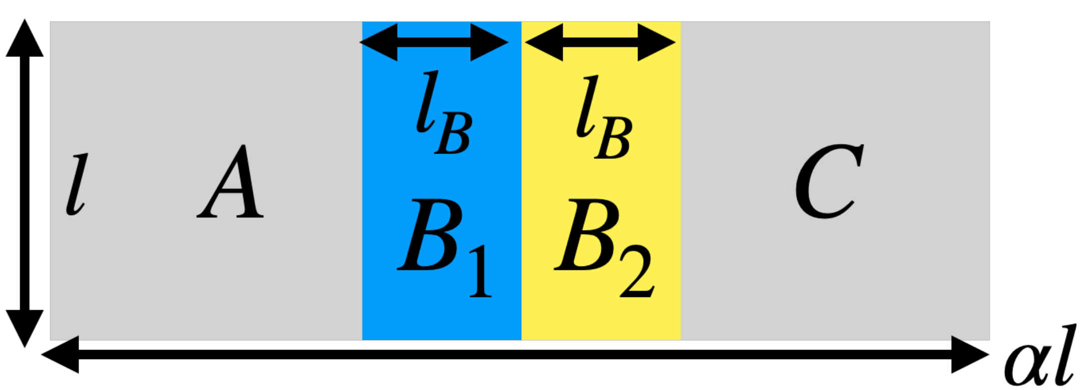


Levin-Wen geometry

$$\frac{dI(A:C|B)}{d|x|} \leq 0 \qquad x = l_B/\xi$$



$I_{\mathrm{UV}} = 0 \qquad I_{\mathrm{IR}} = 2\log(2)$ or $\log(2)$



yet another geometry

$$\frac{dI(A:C|B_1)}{d|x|} \leq 0 \qquad x = l_B/\xi$$

$I_{\mathrm{UV}} = 0 \qquad I_{\mathrm{IR}} = 2\log(2)$

A conjecture for *any* 1+1-D scale invariant theory

The assumption of UV finiteness of CMI is quite constraining.

Consider an infinite system with geometry $\longrightarrow$

$$\ell_A = \ell_C = \ell \ll \ell_B$$

CMI(A:C|B) fully determined by the entropy $S(x)$ of a block of size x:

$$\mathrm{CMI}(A:C\,|\,B) = -\,\ell^2 \frac{d^2 S}{dx^2}\bigg|_{x=\ell_B} + O((\ell/\ell_B)^3)$$

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Scale invariance leads to three distinct possibilities:

$(i)\, S(x) \sim x/\epsilon + (x/\epsilon)^\alpha,\ \alpha > 0$ $\Rightarrow$ CMI UV divergent.

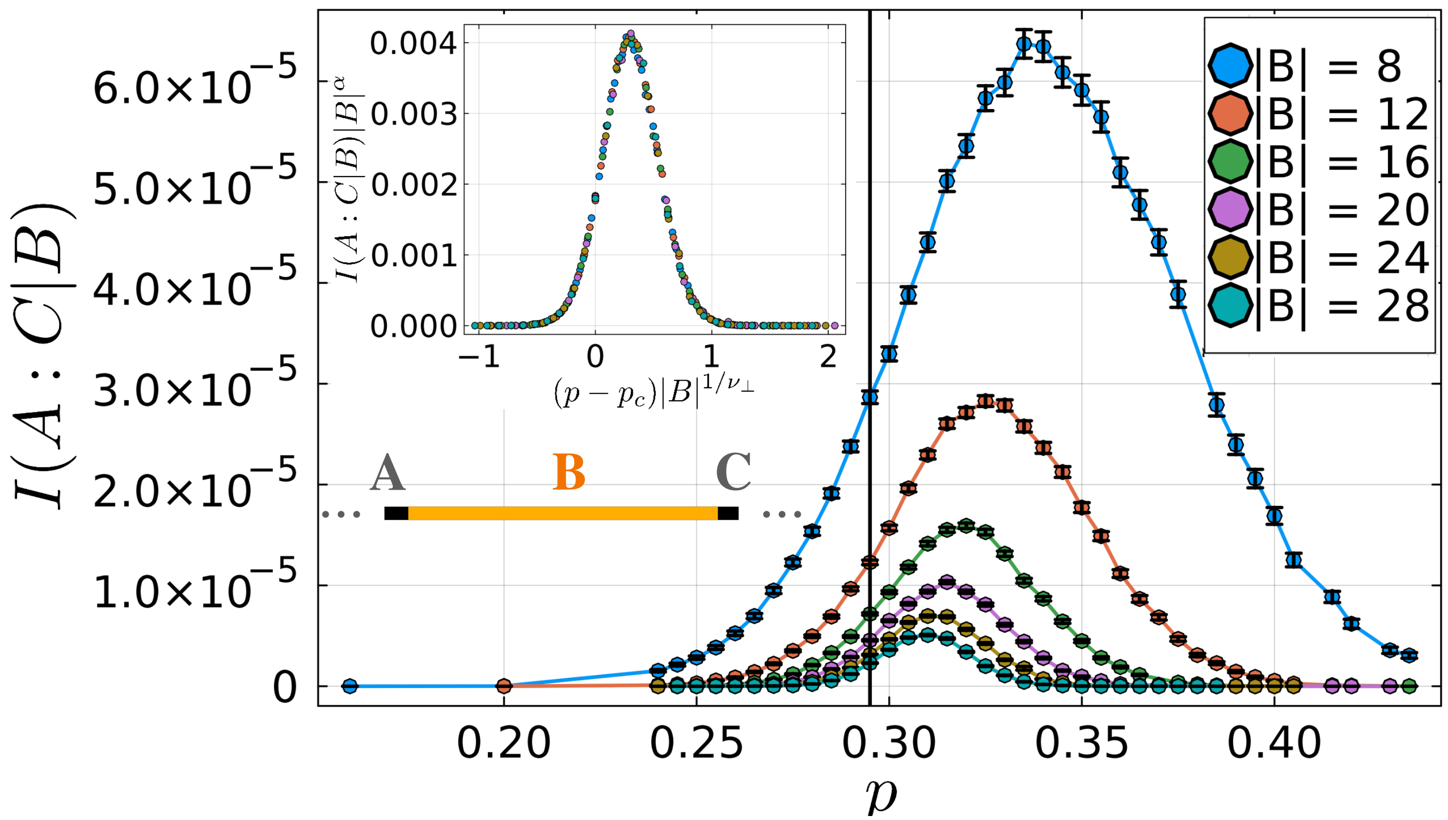
$(ii)\, S(x) \sim x/\epsilon + (x/\epsilon)^\alpha,\ \alpha < 0$ $\Rightarrow$ CMI vanishes.

$(iii)\, S(x) \sim x/\epsilon + \kappa \log(x/\epsilon)$ $\Rightarrow$ CMI $\sim \kappa(\ell/\ell_B)^2$

[Yu-Hsueh Chen, TG (upcoming)]

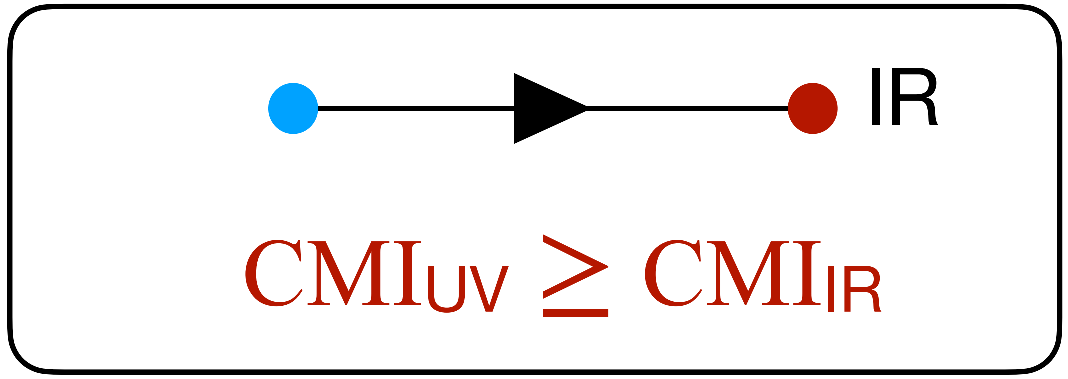
A conjecture for *any* 1+1-D scale invariant theory

This “explains” the CMI exponent for directed percolation in 1+1-d:



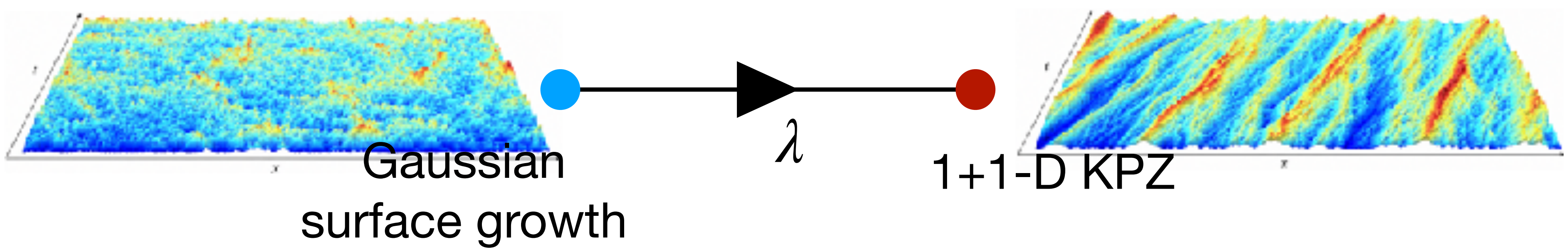
$$I(A:C|B) = \frac{1}{|B|^{\alpha}} f_2\Big((p-p_c)|B|^{1/\nu_\perp}\Big) \qquad \nu_\perp \approx 1.1 \quad \alpha \approx 2$$

[Yu-Hsueh Chen, TG, 2512.07220]



$\mathrm{CMI_{UV}}$ / $\mathrm{CMI_{IR}}$	**Zero**	**Non-zero constant**	**Power-law or logarithmic decay**
Zero	local Gibbs $\to$ local Gibbs. Application to KPZ and stability of critical Gibbs against developing CMI.	Z_n SW-SSB $\to$ trivial	$1+1$-D directed percolation critical point $\to$ active phase; Dephased toric code's RBIM critical point $\to$ Toric code,…
Non-zero constant	✗	$Z_p \to Z_q$ SW-SSB, $\Rightarrow p > q > 0$.	SW-SSB critical point $\to$ SW-SSB phase
Power-law or logarithmic decay	✗	✗	Anisotropic models of 'self-organized criticality'.

1+1-D KPZ: $$\partial_t h(x,t) = \nu \nabla^2 h(x,t) + \lambda(\partial_x h)^2 + \zeta(x,t)$$



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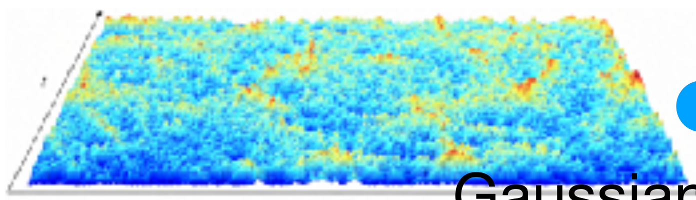
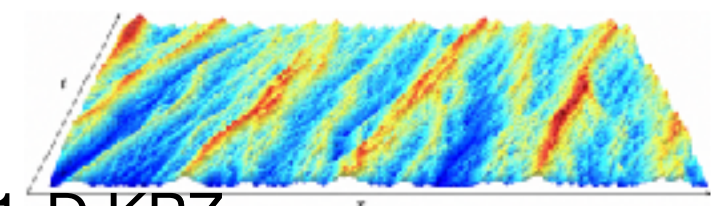


λ



Gaussian surface growth

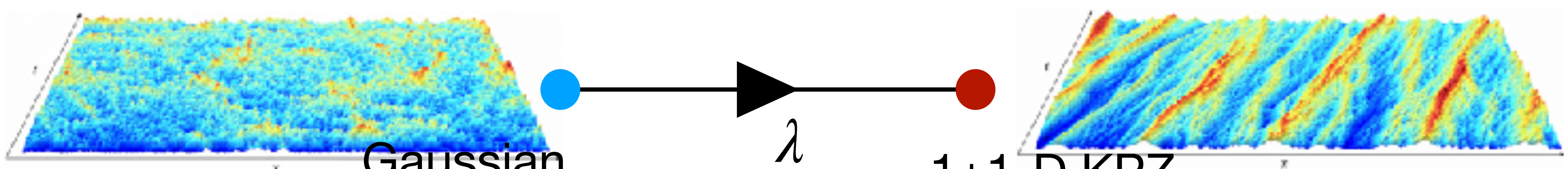
CMI = 0

1+1-D KPZ

CMI = 0

Steady-state must be local Gibbs (which it indeed is).

1+1-D KPZ: $\partial_t h(x,t) = \nu \nabla^2 h(x,t) + \lambda(\partial_x h)^2 + \zeta(x,t)$



Gaussian surface growth

1+1-D KPZ

CMI = 0

CMI = 0

Steady-state must be local Gibbs (which it indeed is).

Stability of local Gibbs against developing CMI:

UV

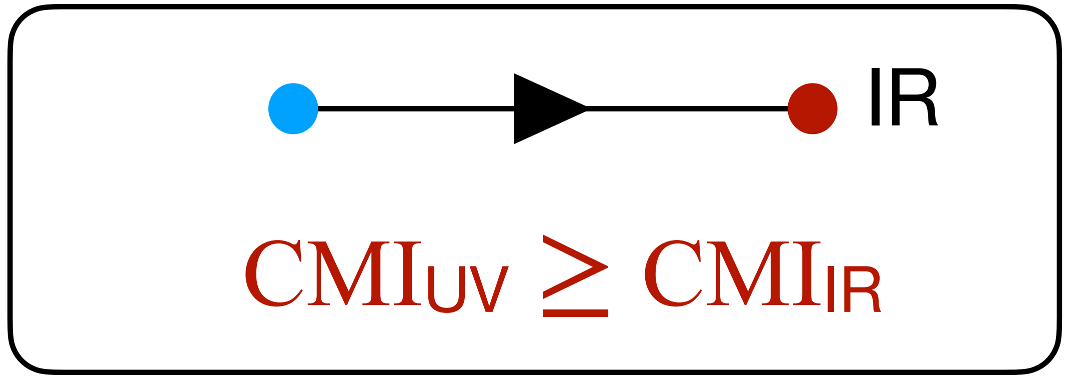
IR

$\mathrm{CMI}_{\mathrm{UV}} = 0$

$\mathrm{CMI}_{\mathrm{IR}} \neq 0$

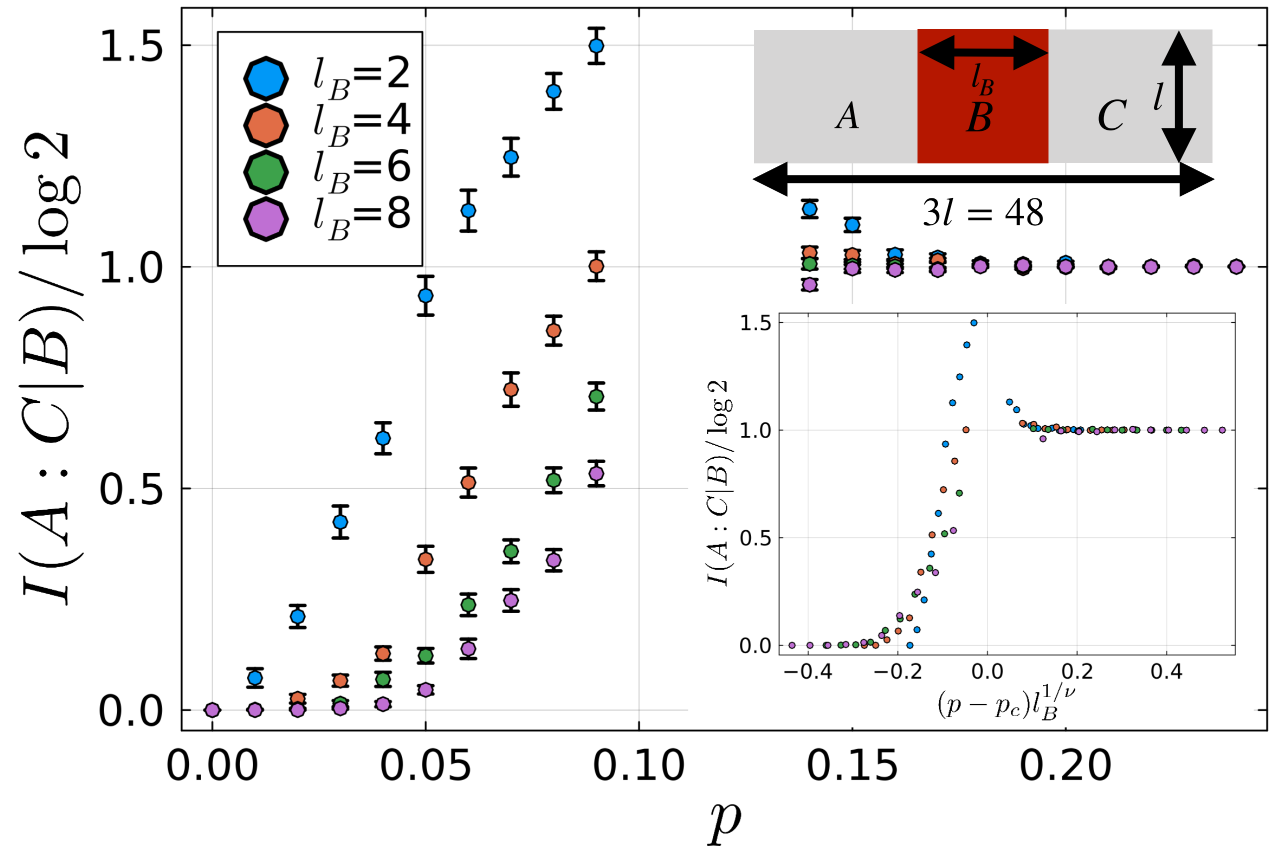
e.g. a local Gibbs state including critical, local Gibbs

Consistent with perturbative calculation in [Zhang and S. Gopalakrishnan 2511.01976].

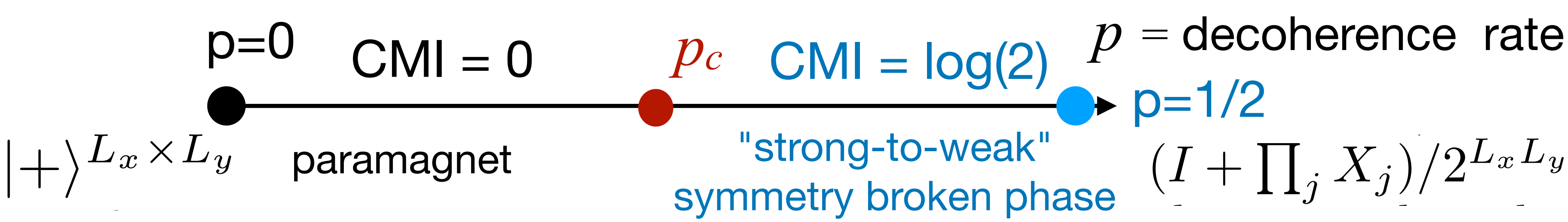


$\mathrm{CMI}_{\mathrm{IR}}$ \ $\mathrm{CMI}_{\mathrm{UV}}$	**Zero**	**Non-zero constant**	**Power-law or logarithmic decay**
Zero	local Gibbs $\to$ local Gibbs. Application to KPZ and stability of critical Gibbs against developing CMI.	Z_n SW-SSB $\to$ trivial	$1+1$-D directed percolation critical point $\to$ active phase; Dephased toric code's RBIM critical point $\to$ Toric code,…
Non-zero constant	✗	$Z_p \to Z_q$ SW-SSB, $\Rightarrow p > q > 0$.	SW-SSB critical point $\to$ SW-SSB phase
Power-law or logarithmic decay	✗	✗	Anisotropic models of 'self-organized criticality'.

CMI for SW-SSB transition

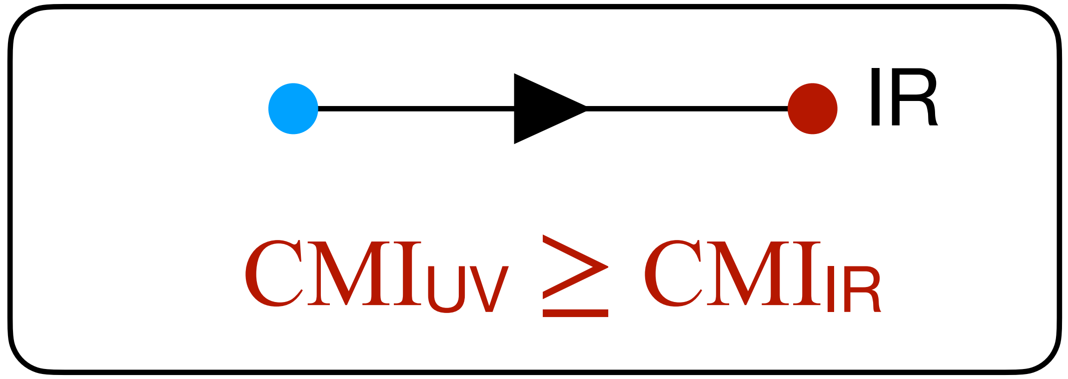


$$\mathcal{E}_{\langle i,j\rangle}[\cdot] = (1-p)(\cdot) + p\, Z_i Z_j (\cdot) Z_i Z_j$$



Due to monotonicity, at the critical point, CMI must exceed log(2).

[Yu-Hsueh Chen, TG, 2602.04961]



$\mathrm{CMI}_{\mathrm{UV}}$ \ $\mathrm{CMI}_{\mathrm{IR}}$	**Zero**	**Non-zero constant**	**Power-law or logarithmic decay**
Zero	local Gibbs $\to$ local Gibbs. Application to KPZ and stability of critical Gibbs against developing CMI.	Z_n SW-SSB $\to$ trivial	$1+1$-D directed percolation critical point $\to$ active phase; Dephased toric code's RBIM critical point $\to$ Toric code,...
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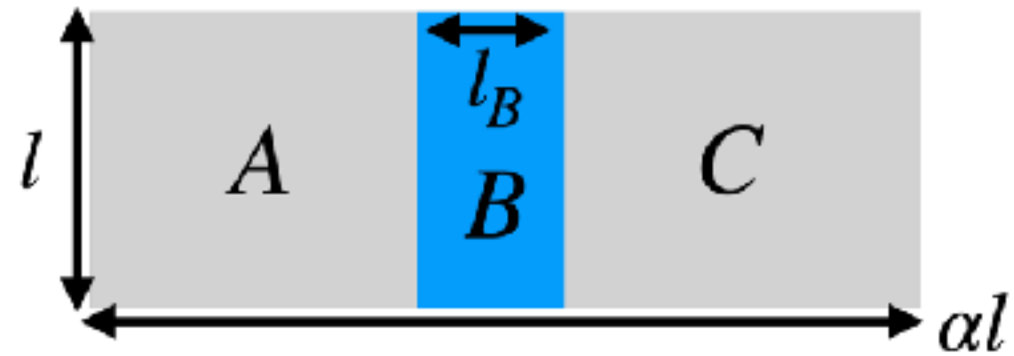
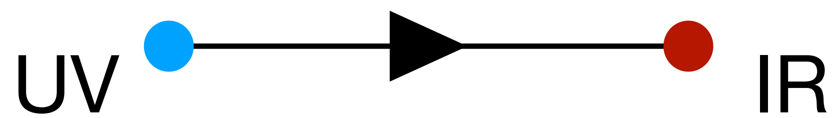
Power-law or logarithmic decay	✗	✗	Anisotropic models of 'self-organized criticality'.

Strong subadditivity constraint on critical exponents





$$\mathrm{CMI}_{\mathrm{UV}} \sim \left(\frac{l}{l_{B,UV}}\right)^{\alpha_{UV}} \qquad \mathrm{CMI}_{\mathrm{IR}} \sim \left(\frac{l}{l_{B,IR}}\right)^{\alpha_{IR}} \qquad l_{B,UV} \ll l_{B,IR}$$

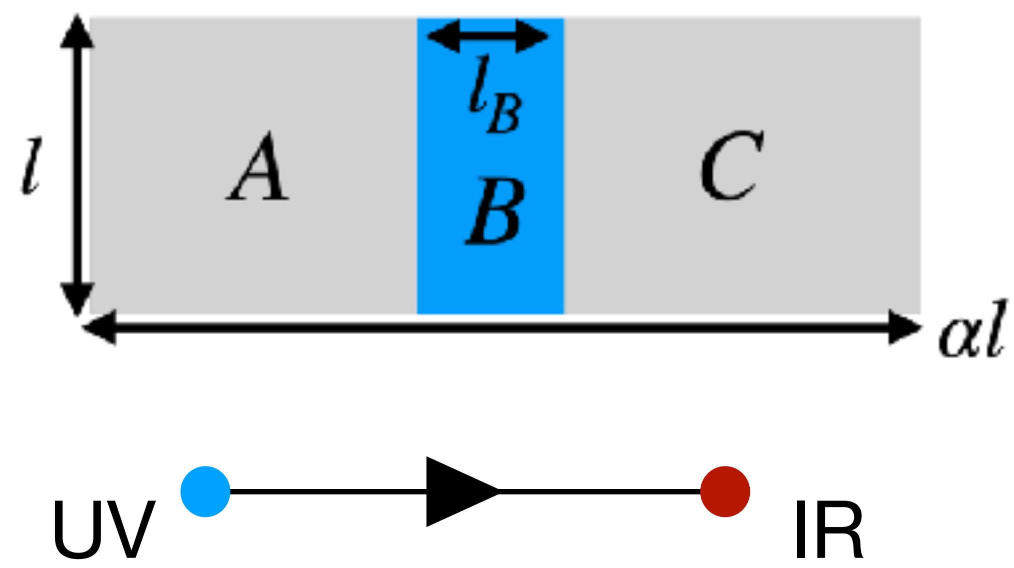
SSA implies $\mathrm{CMI}_{\mathrm{UV}} \geq \mathrm{CMI}_{\mathrm{IR}}$

Assumption: Above scaling holds for arbitrarily large l at fixed $l_{B,UV}, l_{B,IR}$.

Taking $l \to \infty \Rightarrow \boxed{\alpha_{UV} \geq \alpha_{IR}}$

CMI decays *slower* in the IR!

Strong subadditivity constraint on critical exponents



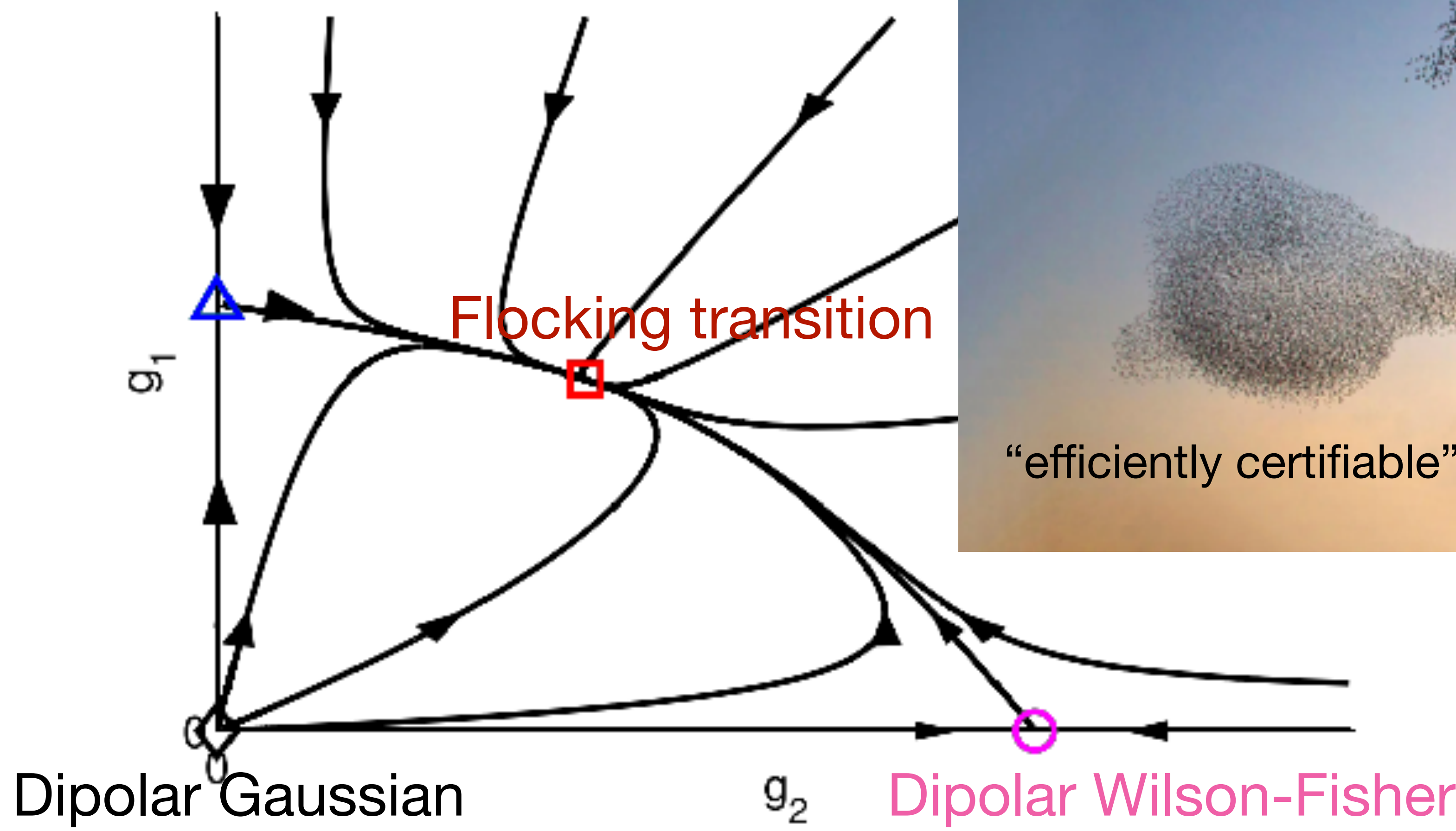
If $\alpha_{UV} = \alpha_{IR}$, then natural to consider the scaling ansatz,

$$\text{CMI} = \left(\frac{\ell}{\ell_B}\right)^{\alpha} f\left(\frac{\ell_B}{\xi}\right) = \left(\frac{\ell}{\xi}\right)^{\alpha} \frac{f(x)}{x^{\alpha}} \quad \text{where } x = \frac{\ell_B}{\xi}$$

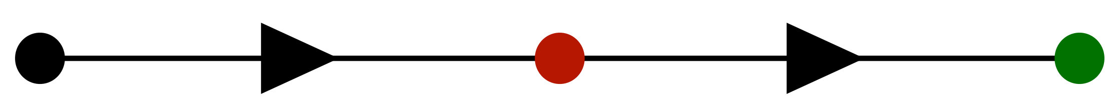
Strong subadditivity then implies that $\dfrac{dg(x)}{dx} \leq 0$ where $g(x) = \dfrac{f(x)}{x^{\alpha}}$.

Application to flocking transitions

[Chen, Toner, Lee 2015]



$$\mathrm{CMI} \sim \left(\frac{l}{l_B}\right)^{\alpha}$$



Dipolar Gaussian

Flocking transition

Flocking phase. Must have long-range effective Hamiltonian!

$$\alpha_{\text{Dipolar}} \geq \alpha_{\text{Flocking transition}} \geq \alpha_{\text{Flocking phase}}$$

Testing monotonicity in "self-organized criticality"

$$\partial_t \phi = \mu \nabla^2 \phi + \nabla^4 \phi + \zeta$$

[Hwa, Kardar 1989; Grinstein, Lee, Sachdev 1990]

$$\langle \zeta(\mathsf{x}, t) \zeta(\mathsf{x}', t') \rangle \sim \left(a \nabla_\perp^2 + \nabla_\parallel^2 \right) \delta(\mathsf{x} - \mathsf{x}') \delta(t - t') \qquad a \neq 1$$

Steady state distribution $\rho \sim e^{-H}$ where

$$H[\phi] = \frac{1}{2} \int \frac{\mathrm{d}^2 k}{(2\pi)^2} K(\boldsymbol{k}) \, \phi_{\boldsymbol{k}} \phi_{-\boldsymbol{k}},$$

$$K(\boldsymbol{k}) = \frac{(k_x^2 + k_y^2)(k_x^2 + k_y^2 + \mu)}{a k_x^2 + k_y^2}$$

Yu-Hsueh Chen, TG
(upcoming)

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Dynamics of a locally conserved order-parameter (“Model B”) with spatially anisotropic noise.

Steady state distribution $\rho \sim e^{-H}$ where

$$H[\phi] = \frac{1}{2}\int \frac{\mathrm{d}^2 k}{(2\pi)^2}\, K(\boldsymbol{k})\, \phi_{\boldsymbol{k}}\phi_{-\boldsymbol{k}},$$

$$K(\boldsymbol{k}) = \frac{(k_x^2 + k_y^2)(k_x^2 + k_y^2 + \mu)}{a k_x^2 + k_y^2}$$

Anisotropy $a \neq 1$ leads to long-range interactions in the steady-state

$\Rightarrow$ expect long-range CMI.

Yu-Hsueh Chen, TG
(upcoming)

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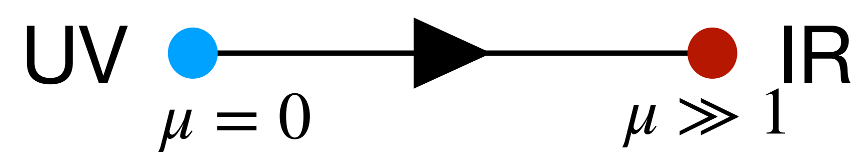
μ is a relevant perturbation:



How does CMI monotonicity work here?

Yu-Hsueh Chen, TG
(upcoming)

Testing monotonicity in "self-organized criticality"

$$\partial_t \phi = \mu \nabla^2 \phi + \nabla^4 \phi + \zeta$$

[Hwa, Kardar 1989; Grinstein, Lee, Sachdev 1990]

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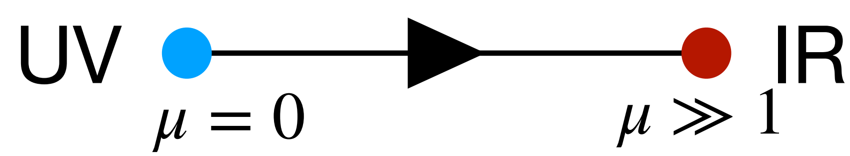

$$\mathrm{CMI}_{UV}(\ell \gg \xi \gg \ell_B) \sim (a-1)^4 \frac{\ell}{\ell_B}$$

$$\mathrm{CMI}_{IR}(\ell \gg \ell_B \gg \xi) \sim (a-1)^2 \frac{\ell}{\ell_B}$$

By making anisotropy $(a-1)$ sufficiently small, one can apparently make $\mathrm{CMI}_{\mathrm{UV}} < \mathrm{CMI}_{\mathrm{IR}}$!

Yu-Hsueh Chen, TG (upcoming)

Testing monotonicity in "self-organized criticality"

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[Hwa, Kardar 1989; Grinstein, Lee, Sachdev 1990]

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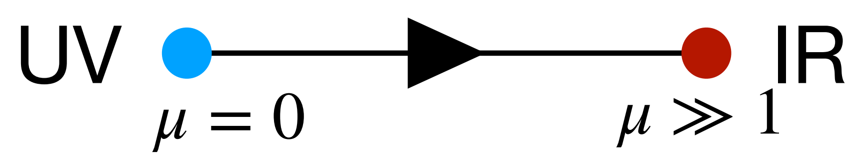


$$\mathrm{CMI}_{UV}(\ell \gg \xi \gg \ell_B) \sim (a-1)^4 \frac{\ell}{\ell_B} + (a-1)^2 \ell/\xi$$

$$\mathrm{CMI}_{IR}(\ell \gg \ell_B \gg \xi) \sim (a-1)^2 \frac{\ell}{\ell_B}$$

However, there is a "subleading" correction, which dominates $\mathrm{CMI}_{\mathrm{UV}}$ when $1 - a \to 0$. This correction rescues CMI monotonicity and can be thought of as a consequence of strong subadditivity!

Yu-Hsueh Chen, TG (upcoming)

Organizing principles?

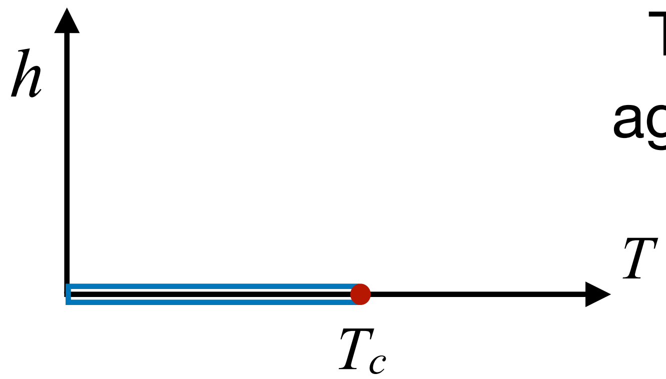
1. Steady states of long time evolution as fixed-points of RG.

   Non-perturbative constraints on RG flows? (e.g. are there any c-theorem type statements or anything goes?). Scaling arguments combined with information-theoretic inequalities are surprisingly powerful.

2. Mixed-state phases as equivalence classes of states connected by short-time evolution along which "gap" remains non-zero.

   Spontaneous symmetry breaking without symmetry.

Spontaneous symmetry breaking without symmetry



The SSB phase in the Gibbs state of Ising model is unstable against explicit symmetry breaking, i.e., under the replacement

$$e^{-\beta \sum_{\langle i,j\rangle} s_i s_j} \to e^{-\beta\left(\sum_{\langle i,j\rangle} s_i s_j + h \sum_i s_i\right)}$$

However, this replacement is not achievable via a short-depth local channel.

Is the SSB phase *perturbatively* stable against action of arbitrary, explicit symmetry breaking channels?

Given $\rho_0 = (|\uparrow\rangle\langle\uparrow|)^l + (|\downarrow\rangle\langle\downarrow|)^l$

Consider $\rho = \mathcal{N}[\rho_0] = \mathcal{N}[(|\uparrow\rangle\langle\uparrow|)^l] + \mathcal{N}[(|\downarrow\rangle\langle\downarrow|)^l]$

where $\mathcal{N}$ is any p-bounded quantum channel.

Is ρ in the same phase of matter as ρ_0?

Using SSA, one can show that given a state of the form $\rho = \sum_\alpha p_\alpha \rho_\alpha$

$$\boxed{I_\rho(A:C|B) \leq \sum_\alpha p_\alpha I_{\rho_\alpha}(A:C|B) + S_\sigma(D|B)}$$

where $\sigma = \sum_\alpha p_\alpha |\alpha\rangle\langle\alpha|_D \otimes \rho_\alpha$

Consider $\rho = \mathcal{N}[\rho_0] = \mathcal{N}[(|\uparrow\rangle\langle\uparrow|)^l] + \mathcal{N}[(|\downarrow\rangle\langle\downarrow|)^l]$

where $\mathcal{N}$ is any p-bounded local channel.

Decodability of repetition code $\Rightarrow S_\sigma(D|B) \sim e^{-|B|/\xi'}$.

Assuming that the trivial state is a stable phase of matter,
both terms in the inequality decay exponentially.

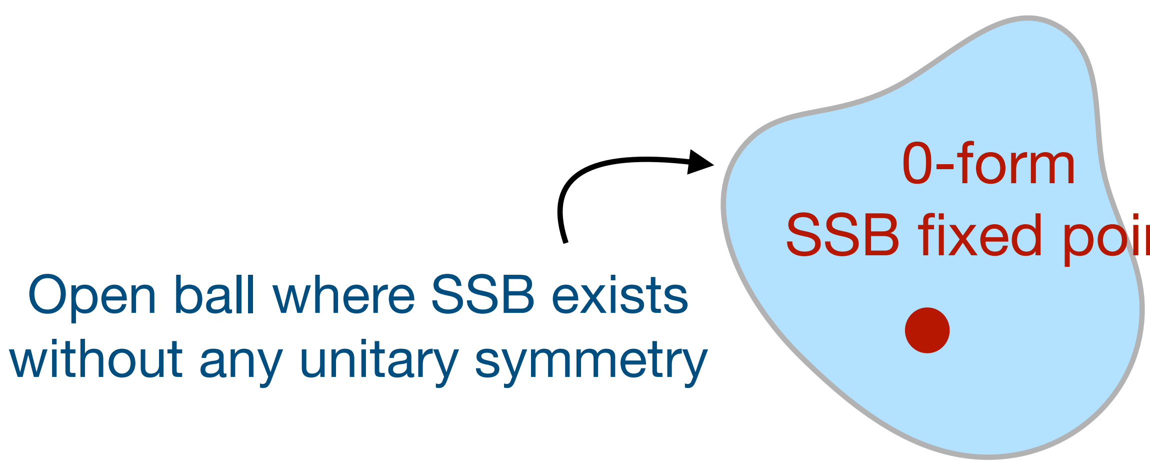
$\Rightarrow I_\rho(A:C|B) \sim e^{-l_B/\xi} \Rightarrow \rho$ is in the same phase of matter as ρ_0.

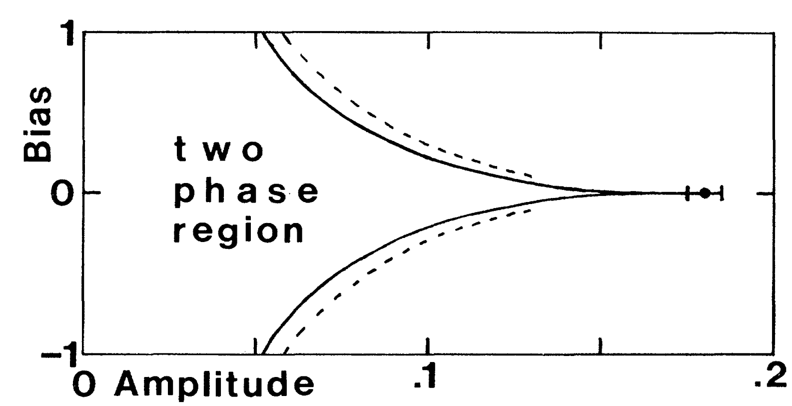
$\exists$ "inverse" channel $\mathcal{N}^{-1}$, such that $\rho_0 = \mathcal{N}^{-1}[\rho]$.

Spontaneous symmetry breaking without symmetry

Consider a local channel T (e.g. Glauber dynamics) whose long-time steady state is the Ising SSB state $\rho_0 = (|\uparrow\rangle\langle\uparrow|)^l + (|\downarrow\rangle\langle\downarrow|)^l$. Then the corrupted, non-symmetric state ρ is the long-time steady state of $\mathcal{N}_{\rho_0\to\rho} T \mathcal{N}^{-1}_{\rho\to\rho_0}$, a local channel *without* Ising symmetry.

Therefore, in the steady state, one obtains "symmetry breaking" (long-range order) without any unitary symmetry. This is a generic phenomena, and also reminiscent of Toom's automata.



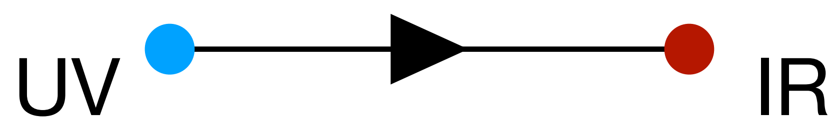


[Toom 1974, Bennett, Grinstein 1985]

Summary

Strong subadditivity constrains non-equilibrium phase diagrams.

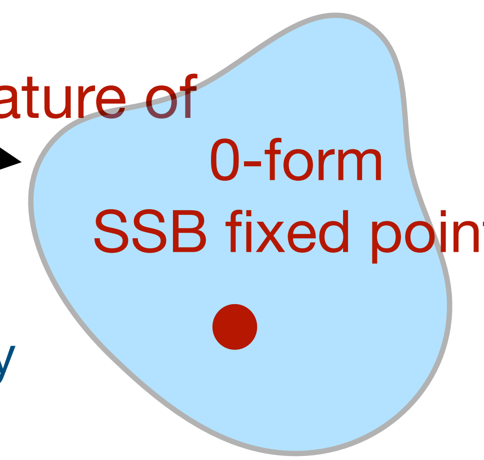
CMI not only defines phases of matter, but also informs their stability.



$$\mathrm{CMI}_{\mathrm{UV}} \geq \mathrm{CMI}_{\mathrm{IR}}$$

Spontaneous symmetry breaking without symmetry is a generic feature of non-equilibrium systems.

Open ball where SSB exists without any unitary symmetry



Future directions…

Field-theory substantiation of the UV finiteness of CMI?
Analytic calculation of universal CMI scaling function?

More exploration of non-equilibrium phase diagrams and interplay with CMI (2+1-D DP, KPZ,…)

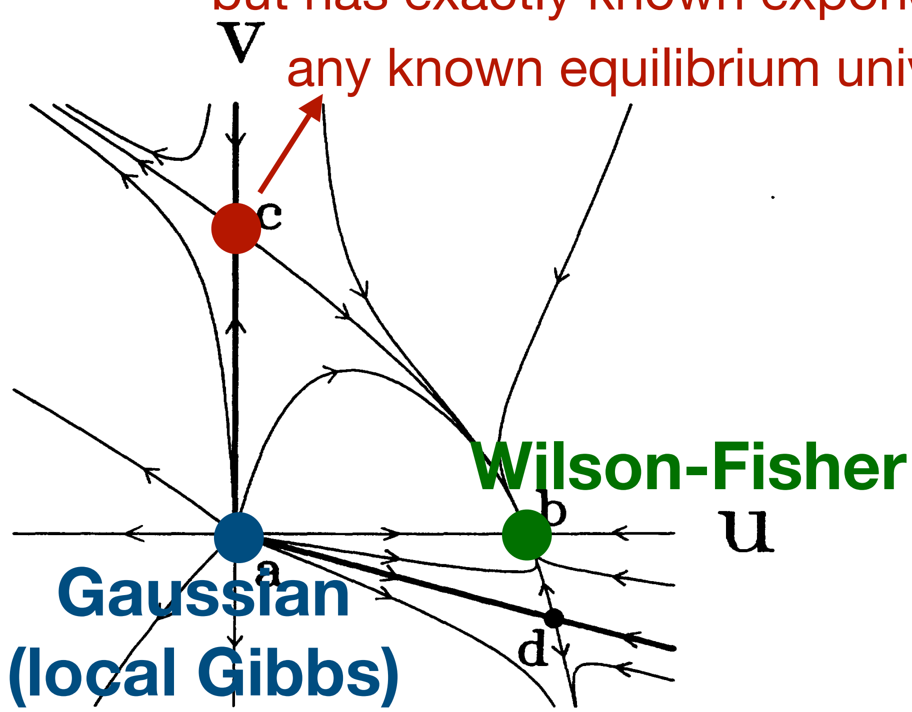
Better numerical algorithms to calculate CMI in d > 1?
(let’s ask Google to fund “alpha-CMI”! 😉)

A couple puzzles

$$\partial_t \phi = \nabla^2 \phi - u\phi^3 + \underbrace{v\partial_x \phi^2}_{\substack{\text{breaks} \\ \text{detailed balance}}} + \zeta$$

[Bassler, Schmittmann 1994]

Based on monotonicity, this fixed point expected to be local Gibbs but has exactly known exponents that do not correspond to any known equilibrium universality [Hwa, Kardar 1989]



2+1-D KPZ: $\partial_t h(x,t) = \nu \nabla^2 h(x,t) + \lambda (\nabla h)^2 + \zeta(x,t)$

UV $\xrightarrow{\lambda}$ IR

UV	IR
Gibbs state of local, Gaussian H	2+1-D KPZ fixed-point
CMI = 0	CMI $\stackrel{?}{=}$ 0

Is 2+1-D KPZ steady-state a local Gibbs state?