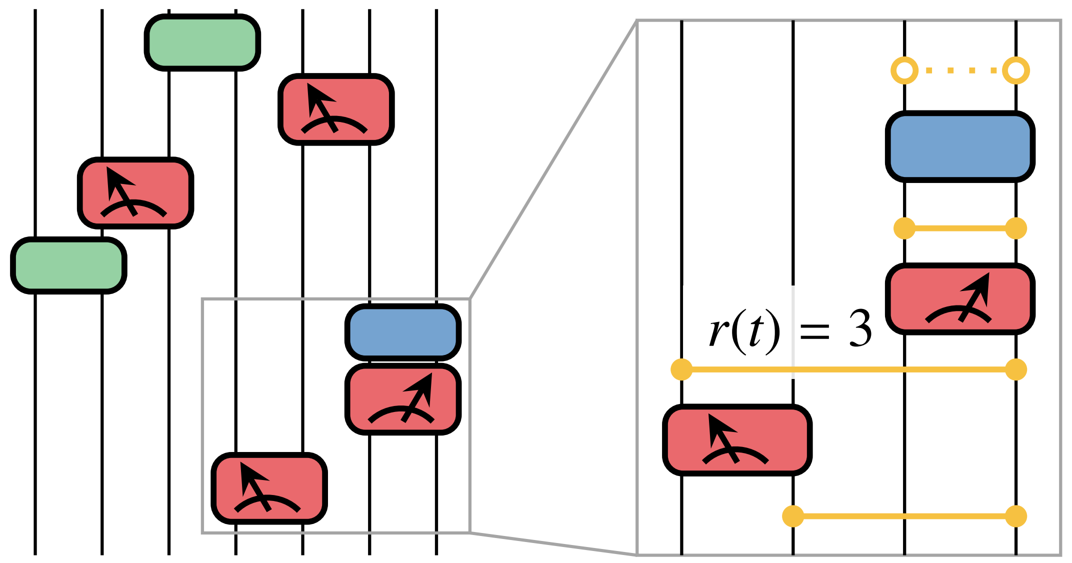
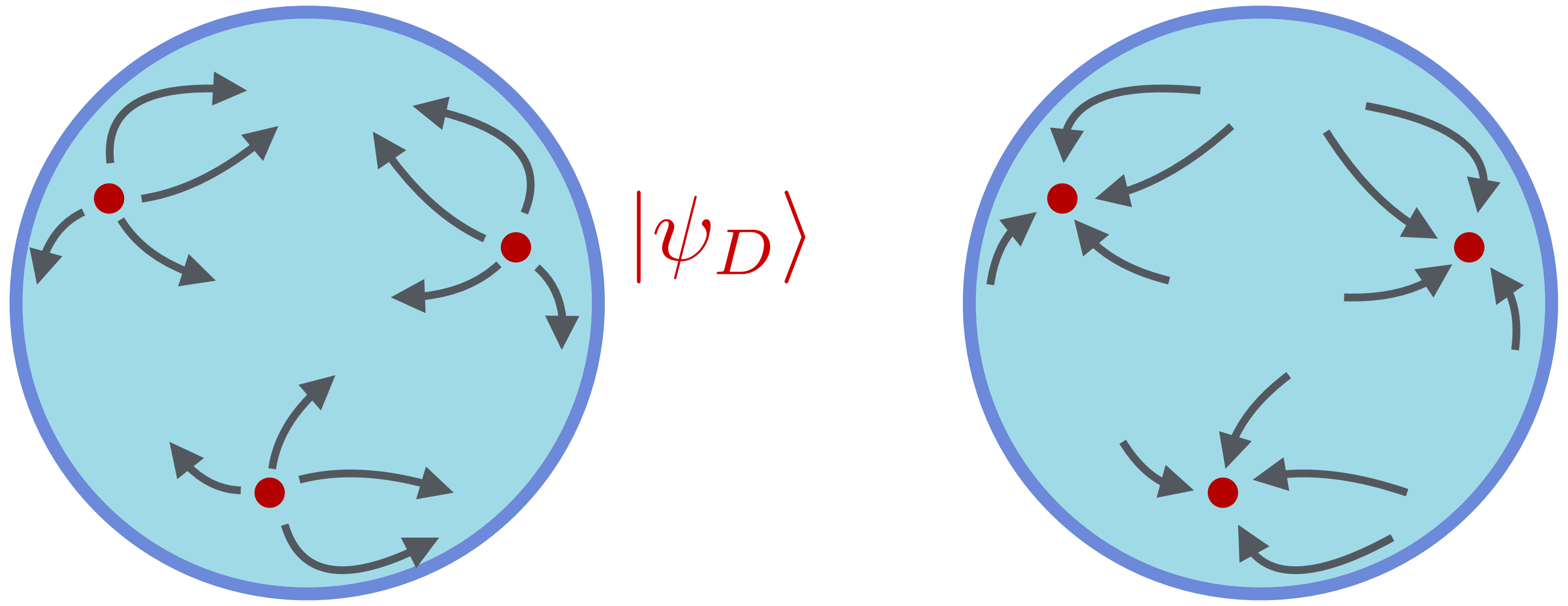
Entangled Attractors

From Frustration-Free Control to Absorbing Scar States





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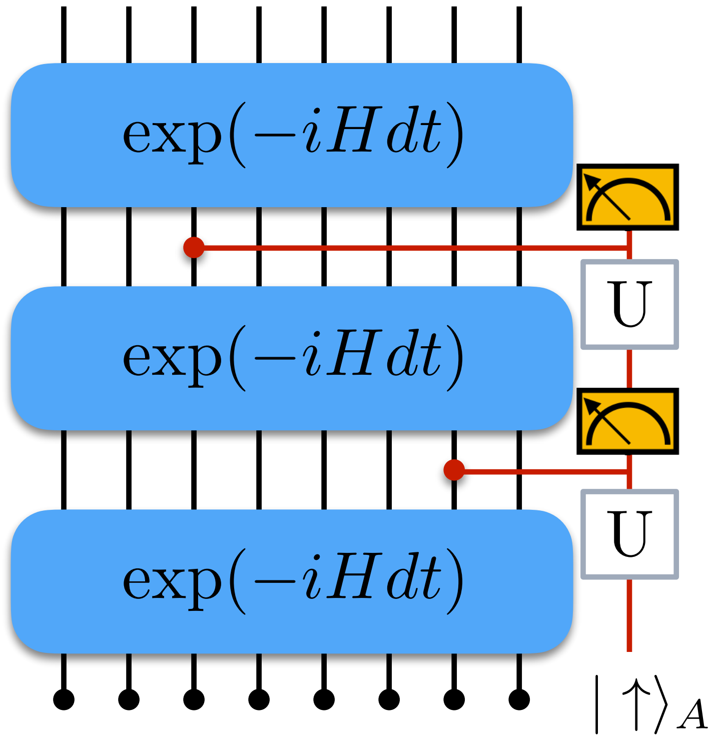
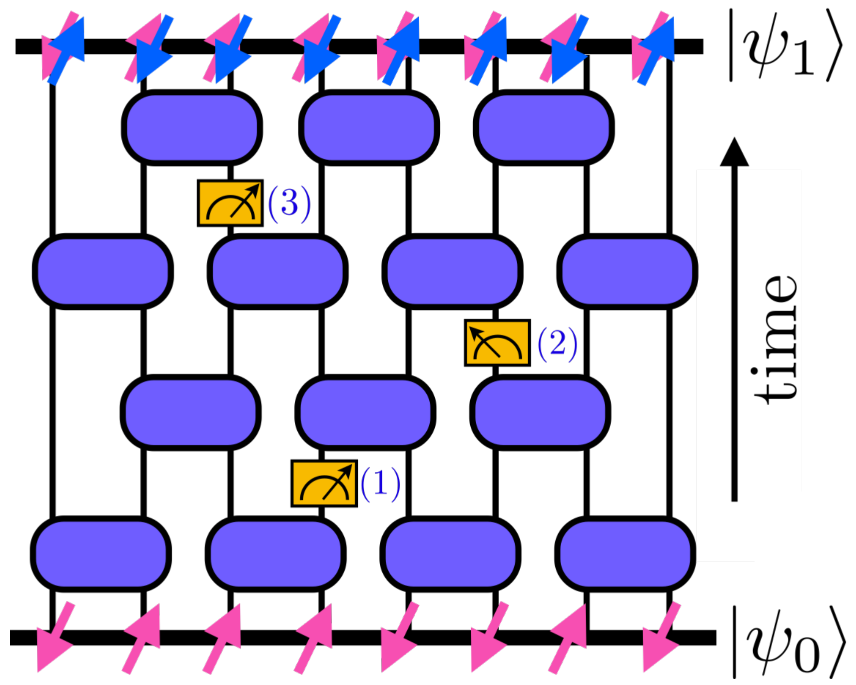
CRC183 ENTANGLED STATES OF MATTER
ML4Q
Heisenberg-Programm
DFG Deutsche Forschungsgemeinschaft

Motivation

midcircuit measurement



Hamiltonian & measurement



digital monitored circuit

measure ‚on-the-fly' — experiment continues

neutral atoms — photonics — trapped ions — SC circuits

monitored quantum many-body physics

- unitary evolution
- non-unitary evolution

monitored matter

- robustness — phases & transitions
- universality
- classification

Measurement as a dynamical resource

gain information

local projective / weak measurement

$$|\psi\rangle \rightarrow \lambda$$

collapse wave function

project onto eigenspace

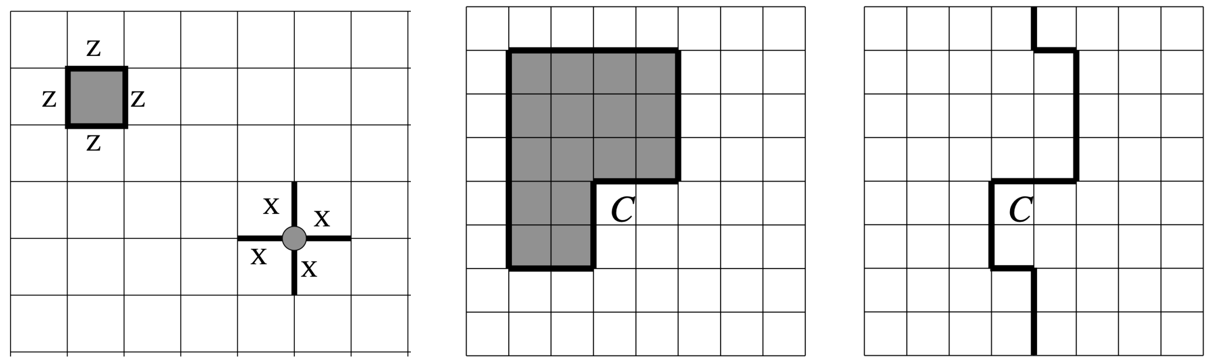
$$|\psi\rangle \rightarrow \lambda \rightarrow \hat{P}_\lambda|\psi\rangle$$

adaptive feedback

unitary conditioned on outcome

$$|\psi\rangle \rightarrow \lambda \rightarrow \hat{U}_\lambda \hat{P}_\lambda|\psi\rangle$$

quantum error correction

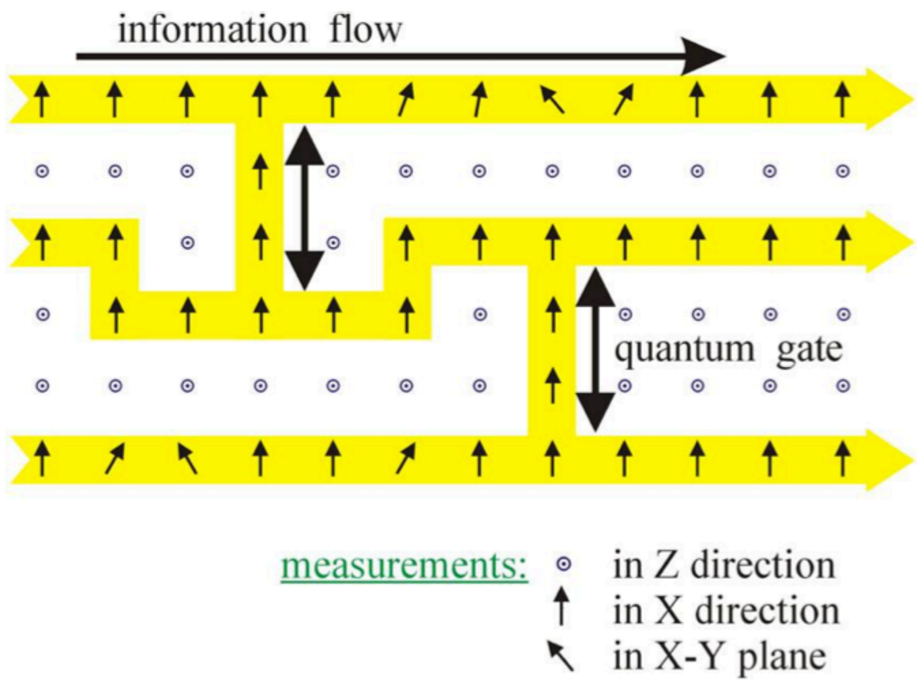


surface code:

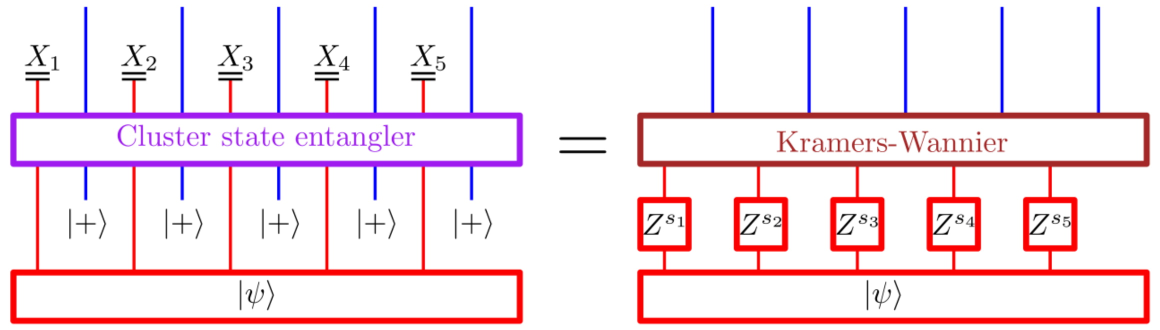
Dennis, Kitaev, Landahl, Preskill, J. Math. Phys. 2002

measurement-based QC

Raussendorf, Briegel PRL 2001
Briegel et al. Nature Physics 2009



state preparation



Piroli, Styliaris, Cirac, PRL 2021
Tantivasdakarn et al. PRX 2024
Iqbal et al. (Quantinuum) Nature 2024
Chen et al (IBM) Nature Physics 2024

State preparation as an absorbing state problem

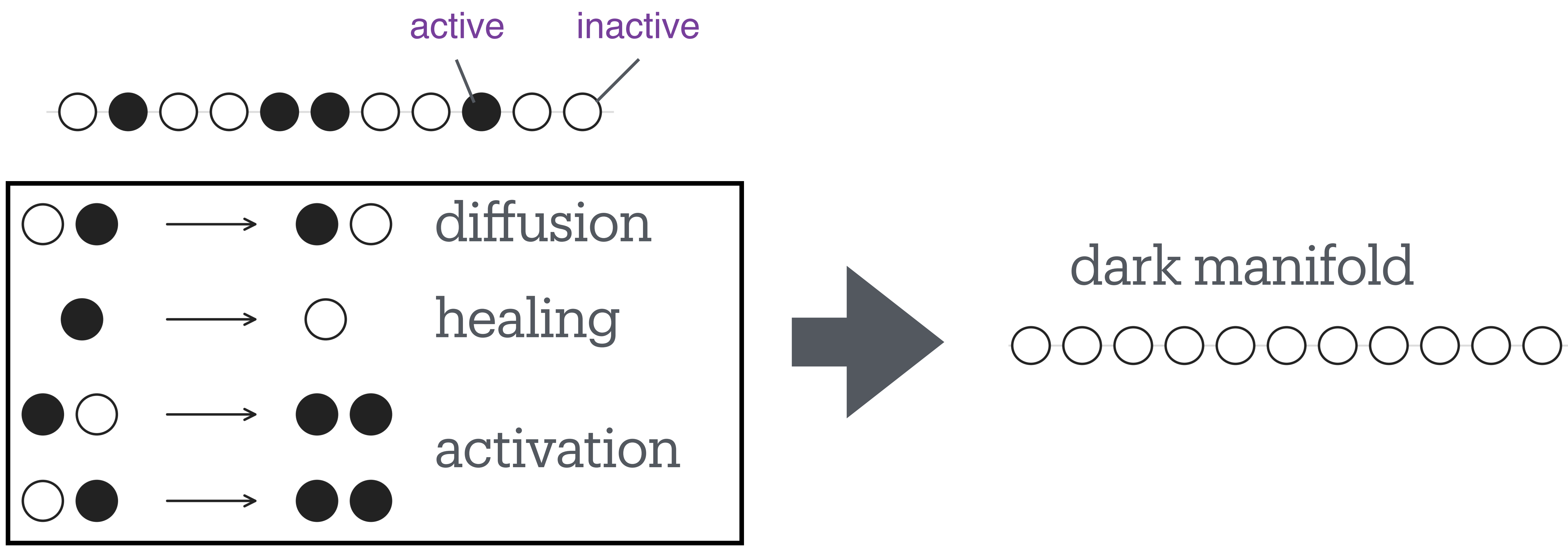
Markov dynamics & absorbing state

once entered cannot be left

paradigmatic example:

contact process / directed percolation

Janssen, Z. Phys. B (1981), Grassberger, Z. Phys. B (1982)
Hinrichsen, Advances in Physics (2000)



translate to state preparation

projectors $\hat{P}_\ell^2 = \hat{P}_\ell$

dark manifold $\hat{P}_\ell |\psi_D\rangle = 0$

(how) is the dark manifold reached?

example: GS of frustration free Hamiltonian

$$\hat{H} = \sum_\ell \hat{P}_\ell$$

e.g., Ferromagnetic Heisenberg, AKLT, Toric Code, Cluster State

healing as measurement-feedback dynamics

$$\hat{P}_\ell = \begin{cases} 1 & \text{active} \\ 0 & \text{inactive} \end{cases}$$

active → local rotation to target $\hat{V}_\ell$

inactive → leave unchanged

feedback condition $[\hat{V}_\ell, \hat{P}_\ell] \neq 0$

Dörstel, Iadecola, Wilson, **MB**, PRL (2026)
Feldmeier, Liu, Lukin, Choi, arXiv (2026)

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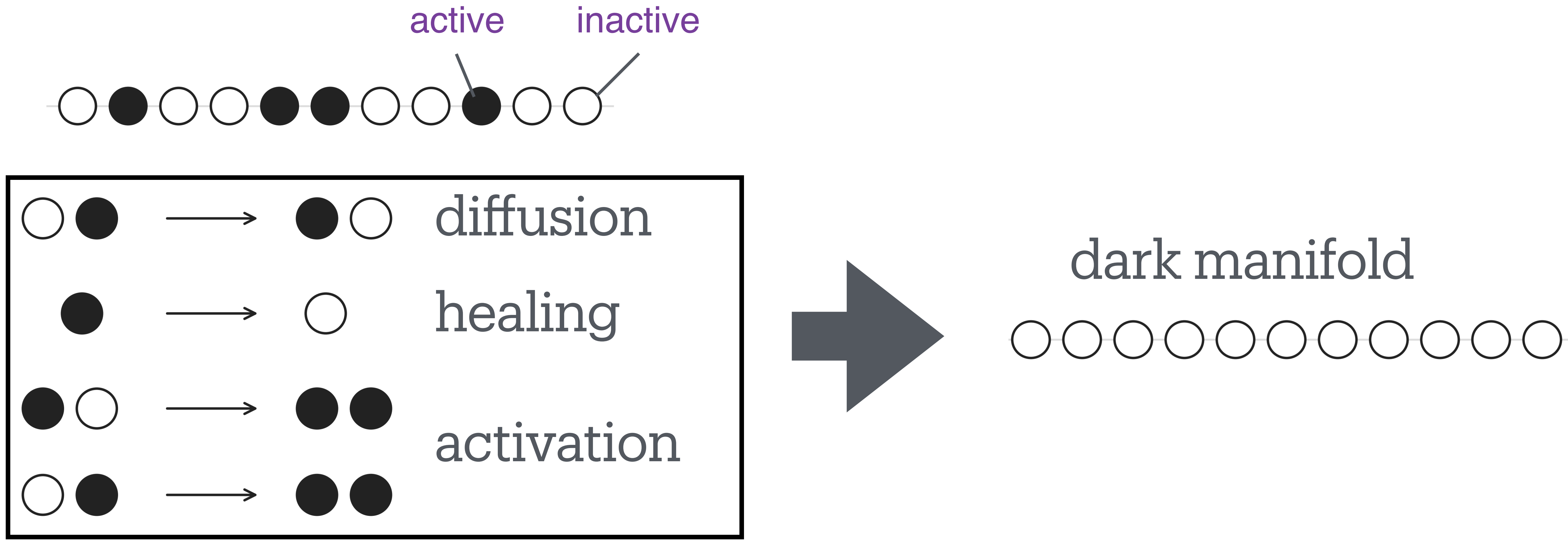

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(how) is the dark manifold reached?

quantum channel

$$\hat{\rho} \mapsto \hat{K}_{\ell,0}\hat{\rho}\hat{K}_{\ell,0}^\dagger + \hat{K}_{\ell,1}\hat{\rho}\hat{K}_{\ell,1}^\dagger$$

$$\hat{K}_{\ell,0} = \mathbb{I} - \hat{P}_\ell$$

$$\hat{K}_{\ell,1} = \hat{V}_\ell \hat{P}_\ell$$

non-unital

$$\hat{\rho}_D = |\psi_D\rangle\langle\psi_D| \mapsto \hat{\rho}_D$$

Defect detection & annihilation as nonlocal dynamics

so far: target = absorbing

entangled states $\neq$ product states

defects are nonlocal $[\hat{P}_{\ell'}, \hat{P}_{\ell}] \neq 0$ singlets

feedback acts nonlocal $[\hat{P}_{\ell'}, \hat{V}_{\ell}] \neq 0$ singlets / cluster state

example: Dicke states from singlet constraints

$\vec{J}^2|\psi_D^Z\rangle = \frac{L(L+2)}{4}|\psi_D^Z\rangle$ maximum total angular momentum

$\hat{Z}_{\text{tot}}|\psi_D^Z\rangle = Z|\psi_D^Z\rangle$ total-Z magnetization

$S_{L/2}(Z=0) = \frac{1}{2}\log(L) + \mathcal{O}(1)$ half-chain entanglement

$\hat{P}_{n,m} = \frac{1}{4}\left[\mathbb{I} - \vec{\sigma}_n \cdot \vec{\sigma}_m\right]$ singlet

$\hat{P}_{n,m}|\psi_D\rangle = 0$ fully symmetric

$\hat{H}_{\text{FMH}} = J\sum_{\ell}\hat{P}_{\ell,\ell+1}$ ferromagnetic Heisenberg

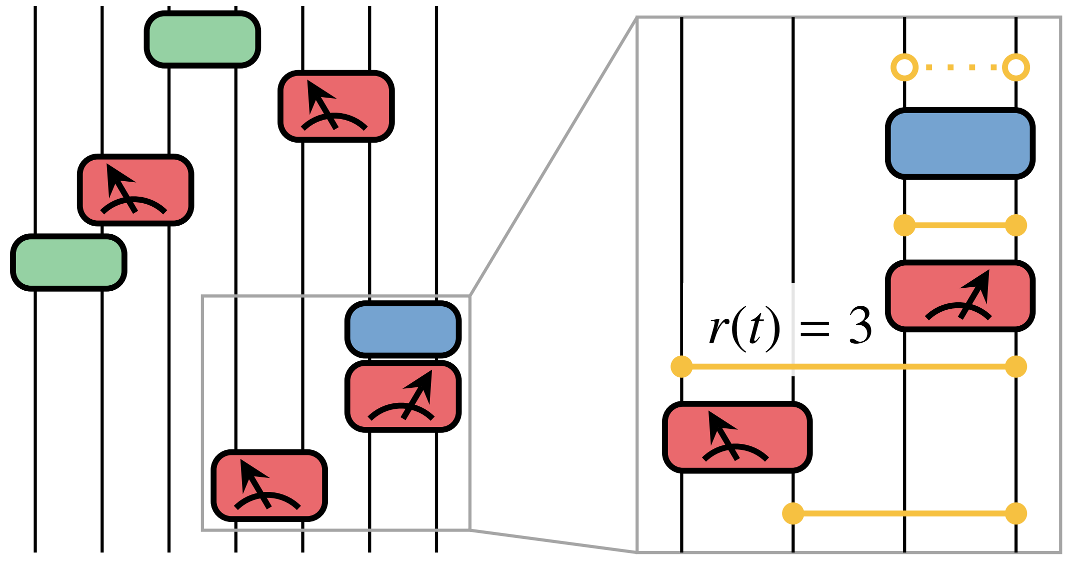
preparation as measurement-feedback dynamics

$$\hat{P}_{\ell,\ell+1} = \begin{cases} 1 & \text{singlet} \\ 0 & \text{triplet} \end{cases}$$

rotate $\hat{V}_{\ell,\ell+1} = \hat{Z}_{\ell}$

$$\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}} \to \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}$$

Hauser, Li, Vijay, Fisher, PRB (2024)



Defect detection & annihilation as nonlocal dynamics

measurement-only — w/o feedback $\partial_t \hat{\rho} = \mathcal{L}\hat{\rho} = -\sum_{\ell} [\hat{P}_{\ell,\ell+1}, [\hat{P}_{\ell,\ell+1}, \hat{\rho}]]$

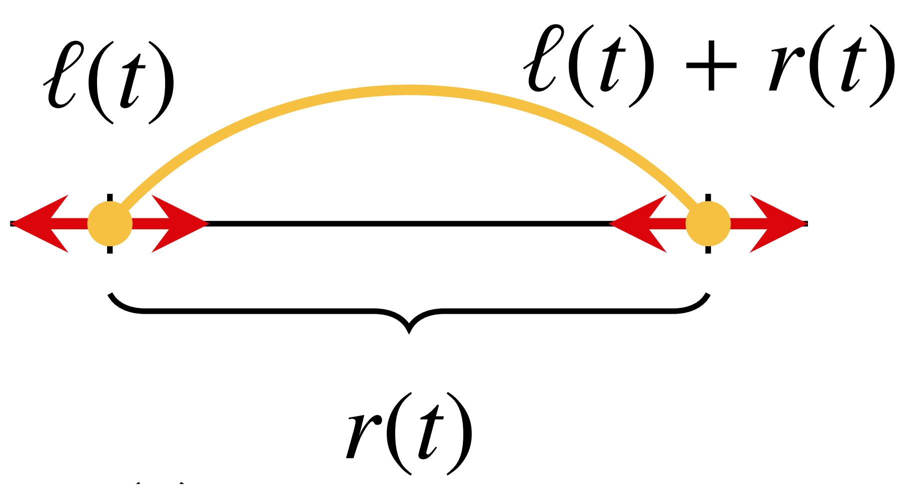
Dörstel, Iadecola, Wilson, **MB**, PRL (2026)
Li, Pollmann, Read, Sala, PRX (2025)

$[\vec{J}^2, \hat{P}_{n,m}] = 0$ total angular momentum fixes singlet sector

$$\partial_t \langle \hat{P}_{n,m} \rangle = \mathrm{tr}(\hat{P}_{n,m}\mathcal{L}\hat{\rho}) = \frac{1}{2}\sum_{\ell} D_{n,\ell}\langle \hat{P}_{\ell,m}\rangle + \langle \hat{P}_{n,\ell}\rangle D_{\ell,m}$$

random walk of singlet endpoints

lattice Laplace



random walk of singlet center & length

$\hat{\rho} \mapsto \mathbb{I}_J$ uniform distribution of singlet length & center

Feedback adds an absorbing boundary

measurement-feedback

$$\partial_t \hat{\rho} = \mathcal{L}\hat{\rho} = -\sum_{\ell} \left\{ \underbrace{[\hat{P}_{\ell,\ell+1}, [\hat{P}_{\ell,\ell+1}, \hat{\rho}]]}_{\text{dephasing}} + \underbrace{[\hat{P}_{\ell,\ell+1}\hat{\rho}\hat{P}_{\ell,\ell+1}, \hat{V}_\ell]\hat{V}_\ell^\dagger}_{\text{feedback}} \right\}$$

$$\partial_t \langle \hat{P}_{n,m} \rangle = \mathrm{tr}(\hat{P}_{n,m}\mathcal{L}\hat{\rho}) = \frac{1}{2}\sum_{\ell} D_{n,\ell}\langle \hat{P}_{\ell,m}\rangle + \langle \hat{P}_{n,\ell}\rangle D_{\ell,m} \qquad -\lambda \langle \hat{P}_{n,m}\rangle\, \delta_{n,m\pm 1}$$

lattice Laplace

annihilation of singlets
with length one

linear absorbing hydrodynamics $\partial_t \mathcal{P} = M\mathcal{P}$

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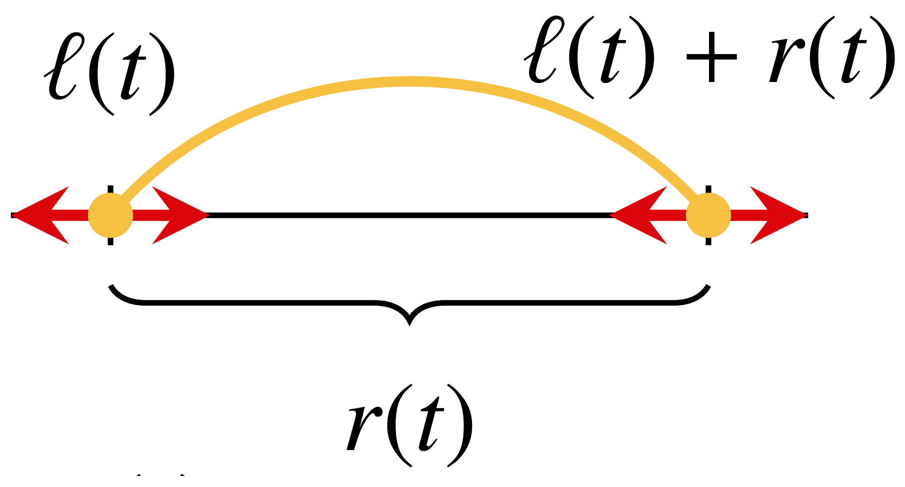
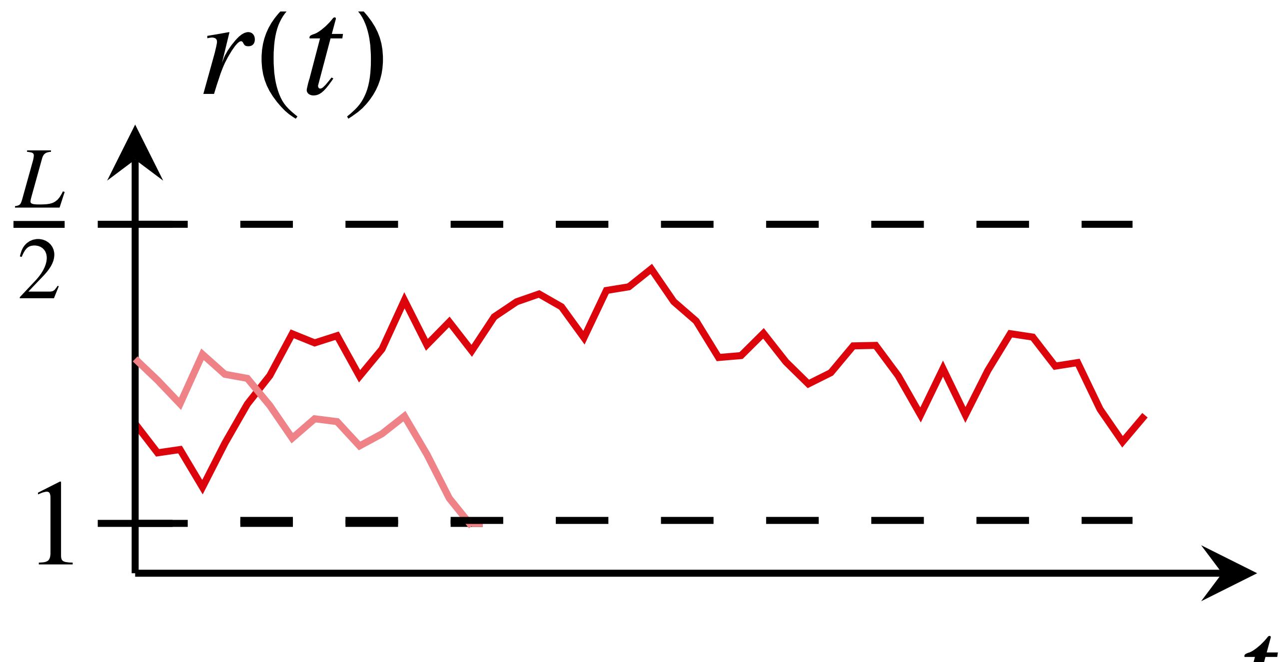


annihilation random walk of singlet lengths



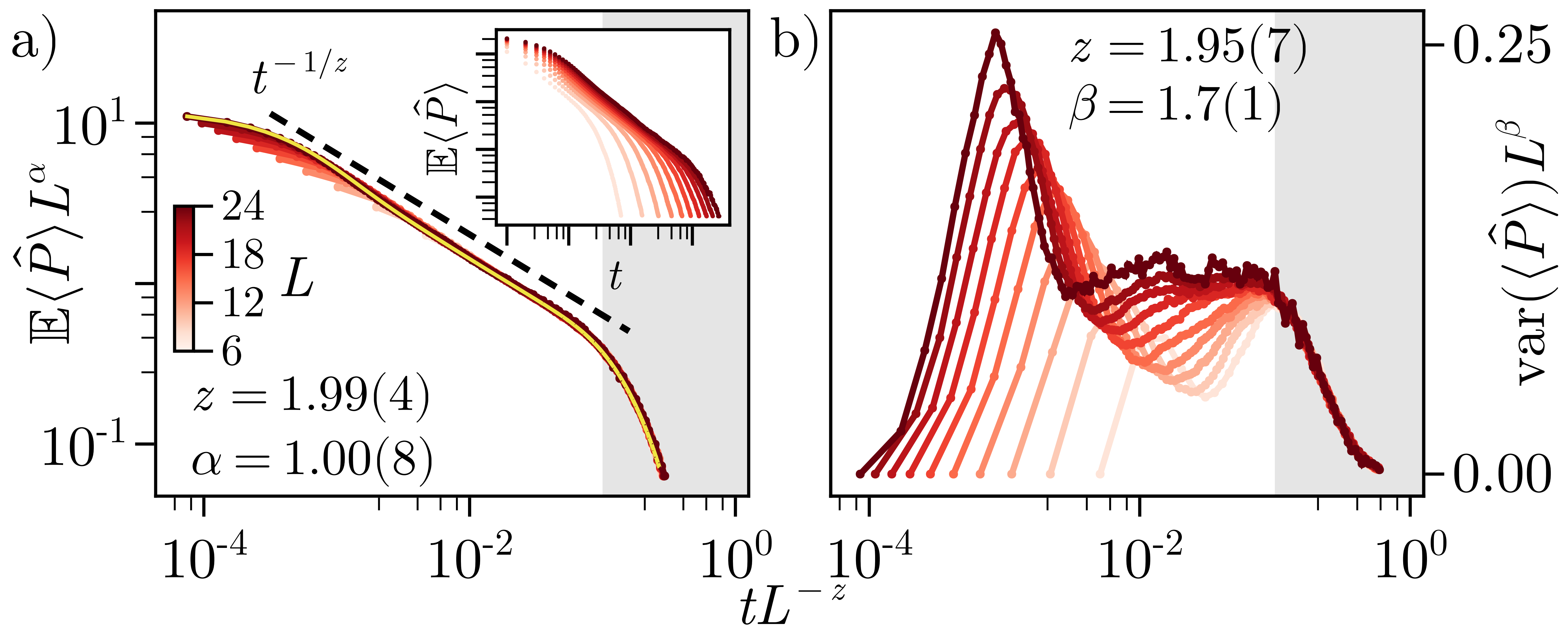
absorbing boundary at length one

Convergence is an annihilation random walk

time governed by diffusive transport

$$\text{d=1:} \quad \mathbb{E}(\langle \hat{P} \rangle) \sim \begin{cases} t^{-1/2} & \text{for } t < L^2 \\ \exp(-tL^{-2}) & \text{for } t > L^2 \end{cases}$$

number of singlets



preparation time = number of measurements / qubit

$$t_{\text{prep}}(L) \sim \begin{cases} L^{2d} & \text{for } d = 1 \\ L^{d}\log(L) & \text{for } d = 2 \\ L^{d} & \text{for } d \geq 3 \end{cases}$$

diffusion

mean-field

linear dimension

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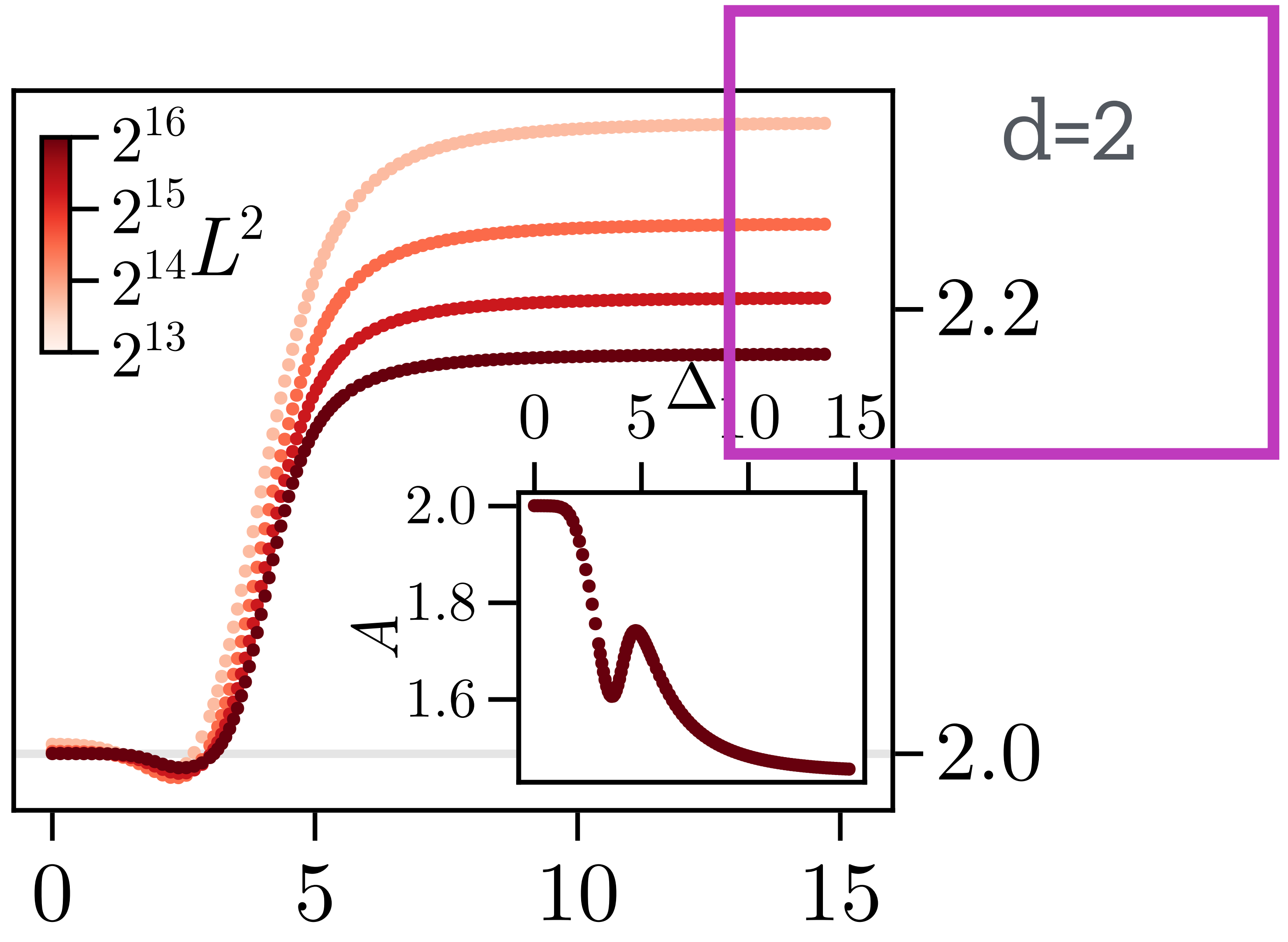
Feldmeier, Liu, Lukin, Choi, arXiv (2026)
Hauser, Li, Vijay, Fisher, PRB (2024)



diffusion limited annihilation

Speedup through transport

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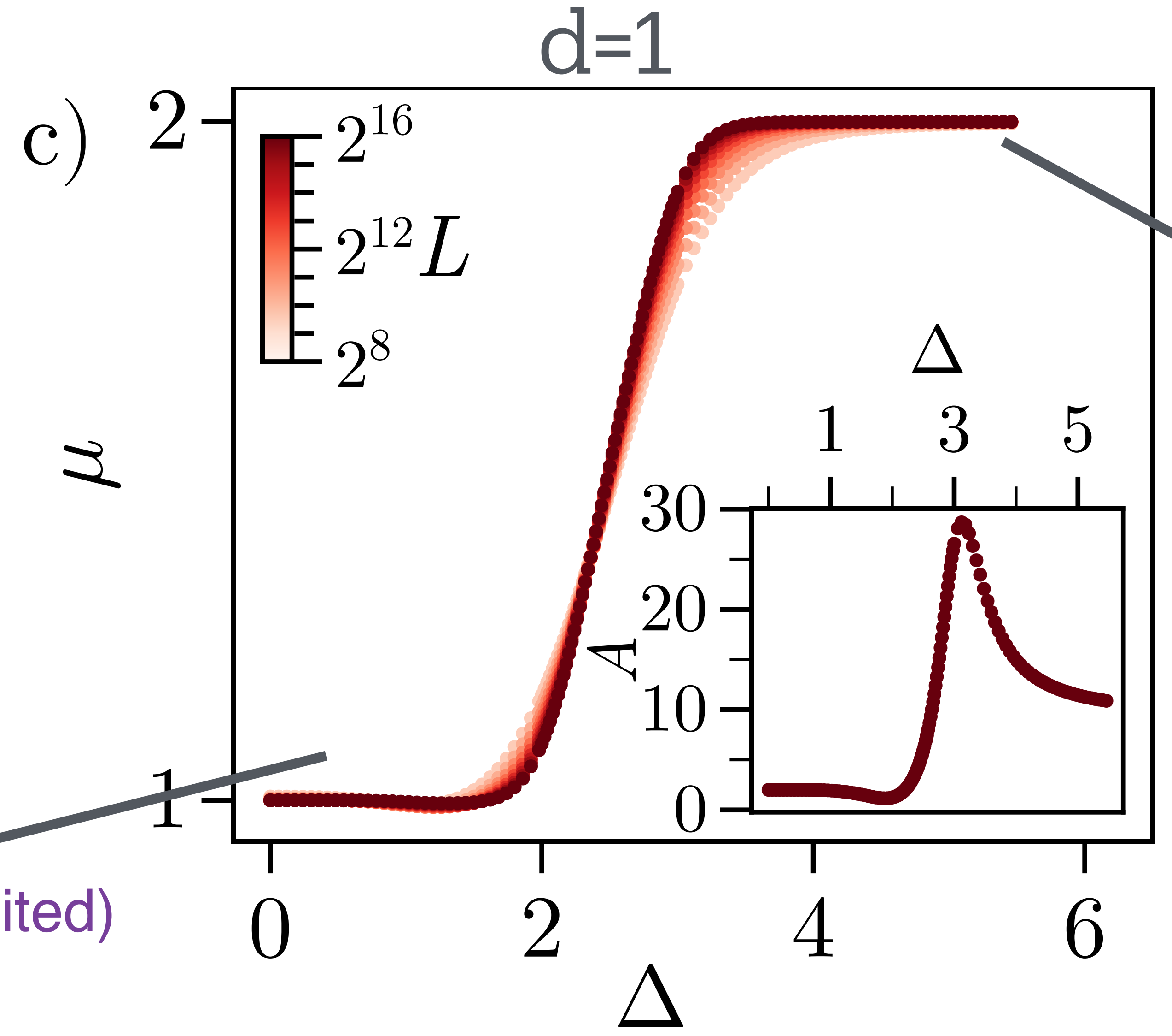
(a) modify geometry/range

measurement range $P_{\ell,\ell+m} \sim m^{-\Delta}$

$$t_{\text{prep}}(L) \sim AL^{\mu}$$



short-range (diffusion limited)

infinite-range (reaction limited)

(b) accelerate transport w/o measurement

add scrambling unitaries $\hat{U}_{\ell,\ell+1} = \exp(i\varphi\mathcal{S}_{\ell,\ell+1})$

SWAP

$[\hat{U}_{\ell,\ell+1}, \hat{J}_{\text{tot}}] = 0$

faster transport - no additional generation/annihilation

$$t_{\text{prep}}(L) \sim \frac{A}{1+R} L^{2-\alpha}$$

scrambling rate

Imperfections and generalization

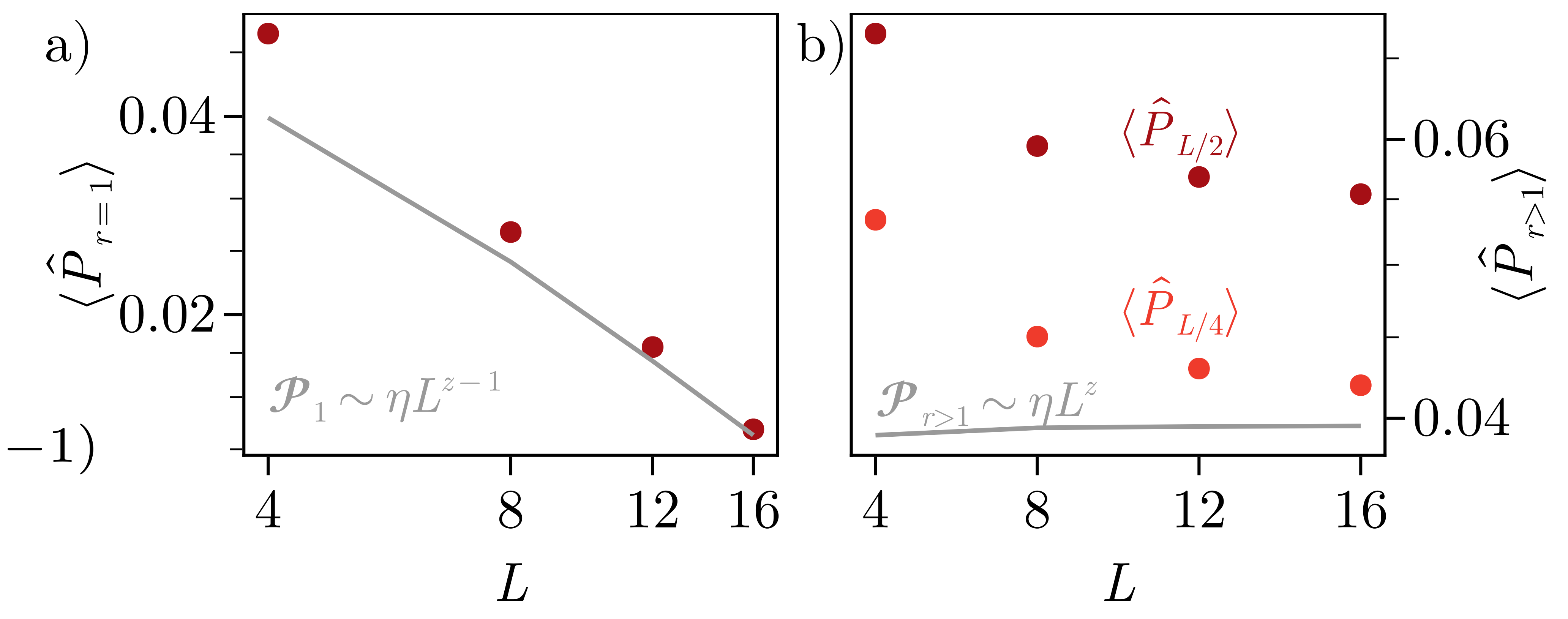
measurement-feedback **imperfections** with rate η

compatible with absorbing state:

irrelevant

incompatible

$\mathbb{E}(\langle\hat{P}\rangle) \sim \eta L^{\mu(-1)}$



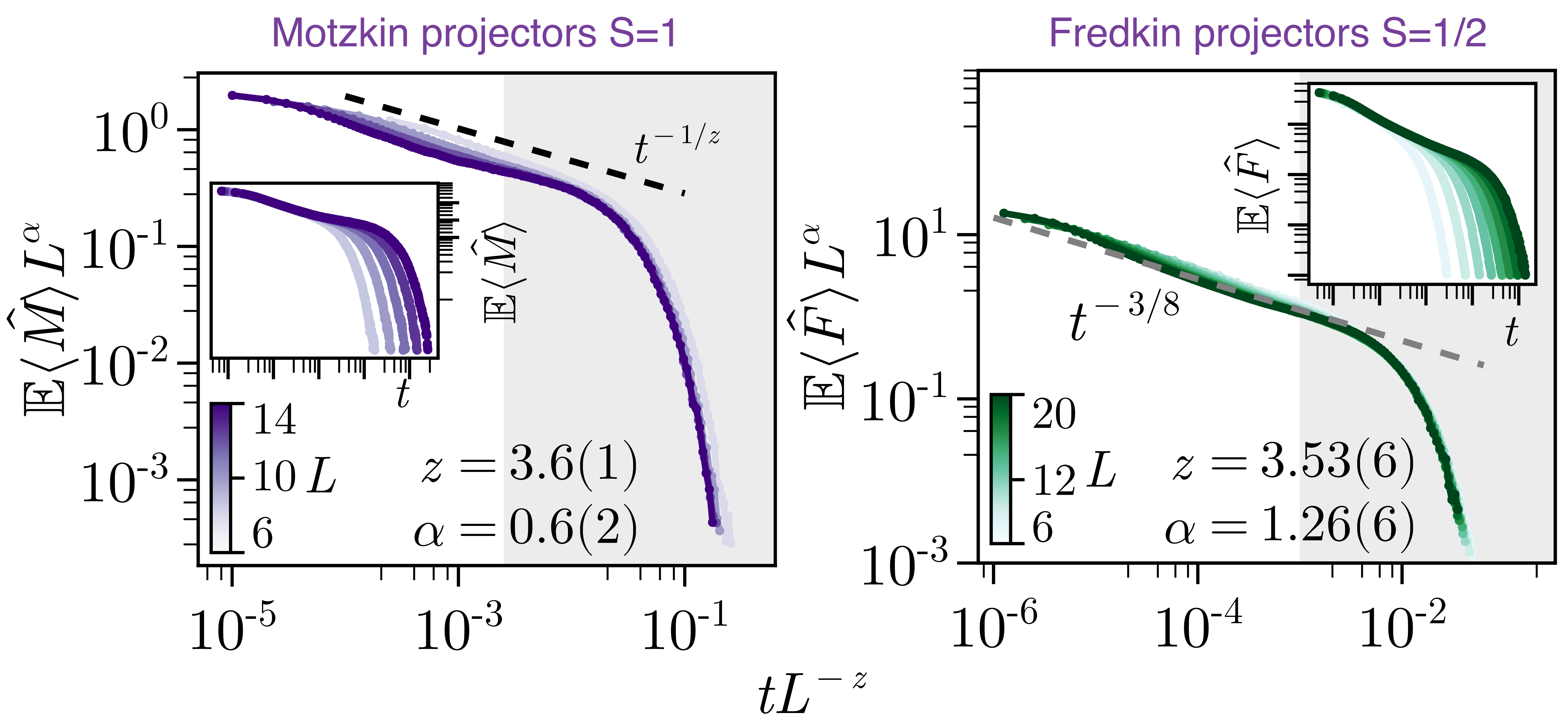
larger on-site Hilbert space $\mathrm{SU}(2) \to \mathrm{SU}(N)$

exactly solvable / same results

no closed defect picture

non-linear hydrodynamics: subdiffusion

Dörstel, Iadecola, Wilson, **MB**, PRL (2026)
Feldmeier, Liu, Lukin, Choi, arXiv (2026)

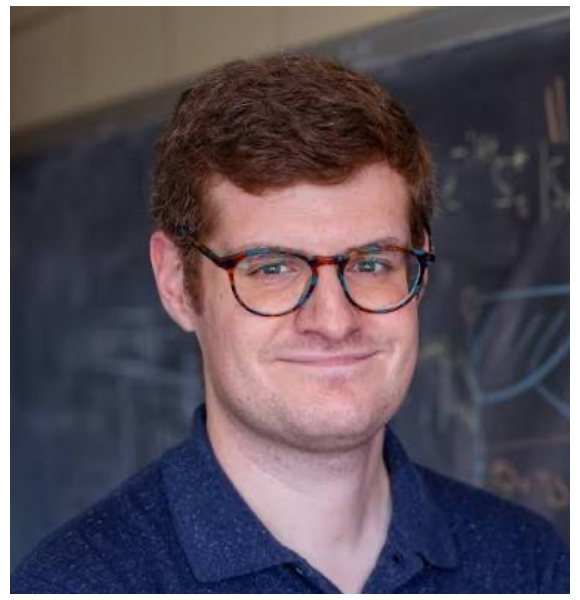


Takeaways

(Some) structured subspaces can be engineered as dynamical attractors

- measurements define defects
- feedback annihilates defects / makes subspace absorbing

J.H. Wilson



T. Iadecola

Thank you!