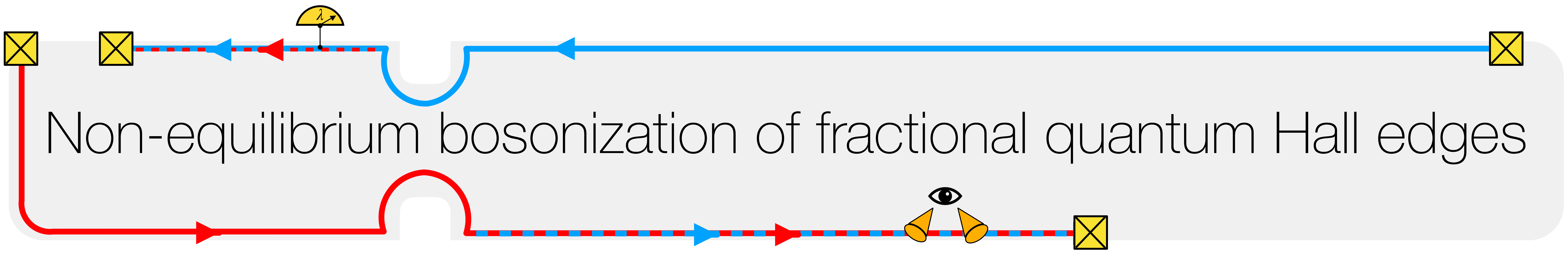
Non-equilibrium bosonization of fractional quantum Hall edges



Based on: C. Spånslätt, J. Park, and A. D. Mirlin, arXiv 2603.05088

Christian Spånslätt Rugarn

christian.spanslatt@kau.se
https://sites.google.com/view/christianspanslatt

Swedish
Research Council

Entanglement Dynamics: Open Quantum Systems, Monitored Circuits, and Topological Order

Nordita, June 24

KARLSTADS UNIVERSITET

Abelian FQH liquids and their boundary

D. C. Tsui et al.: Phys. Rev. Lett. 48, 1559 (1982)
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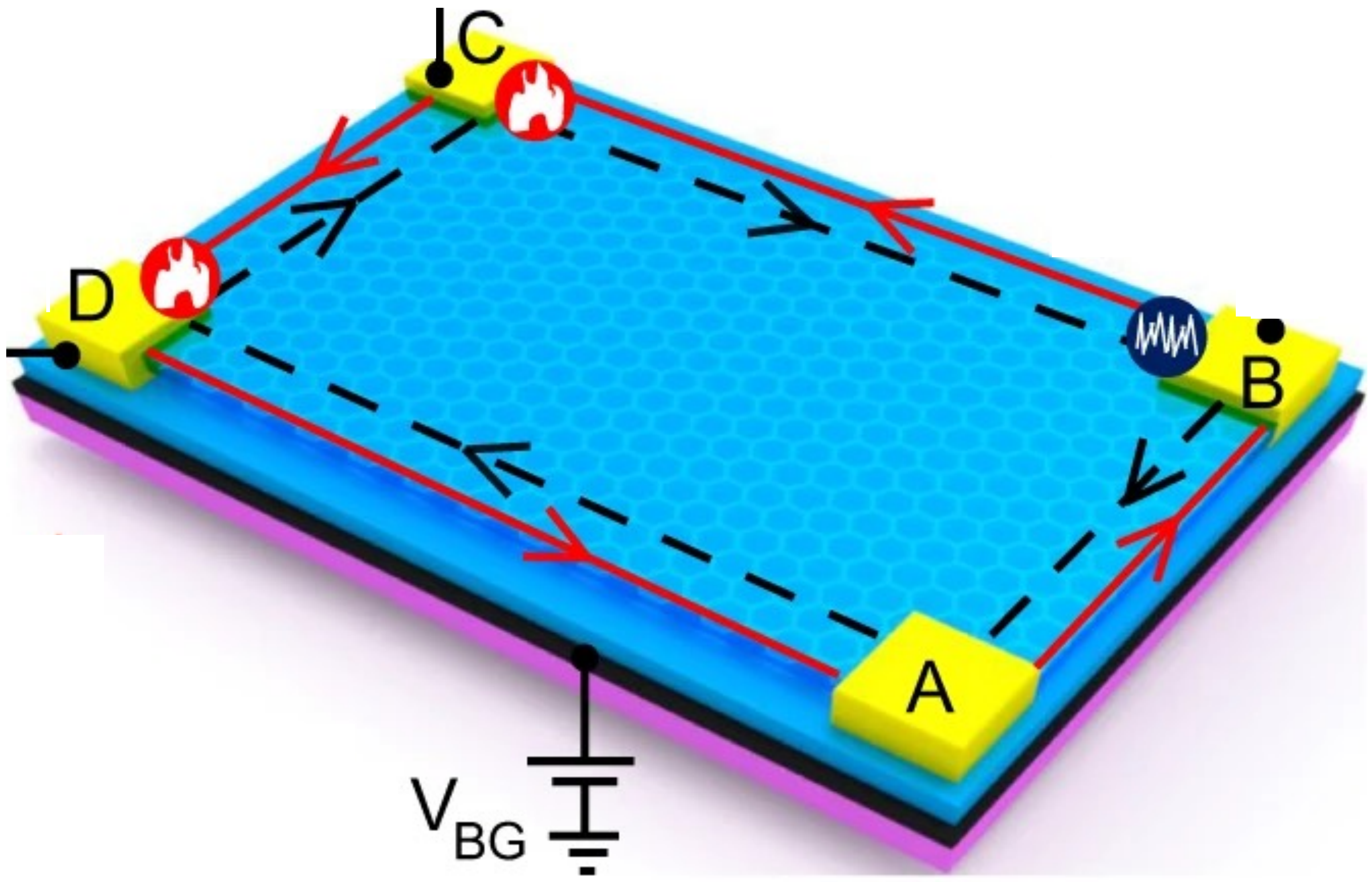

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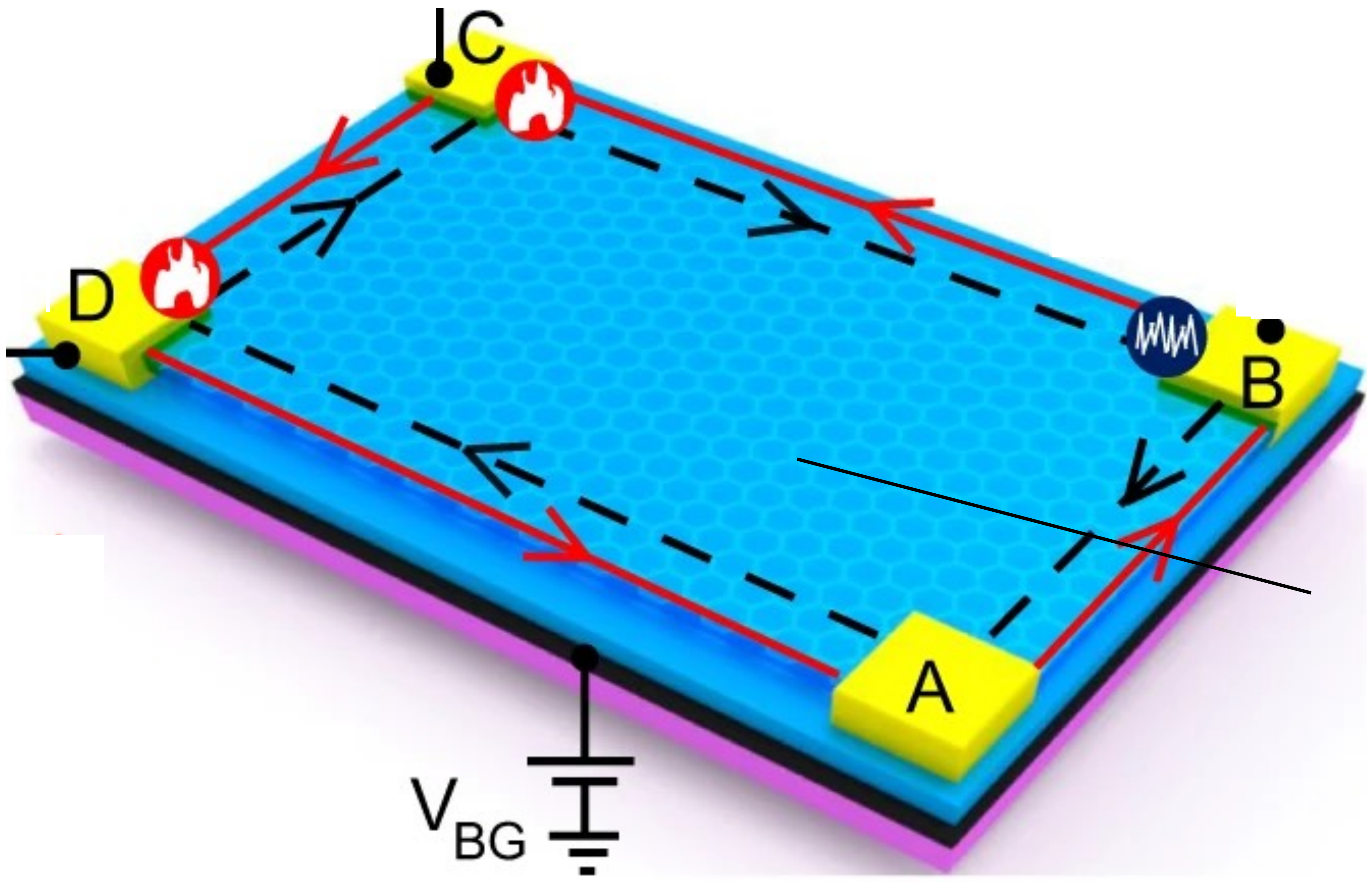

Charge insulating 2D bulk
Topological field theory, K-matrix
Quantum Statistics of excitations
Ground state degeneracy

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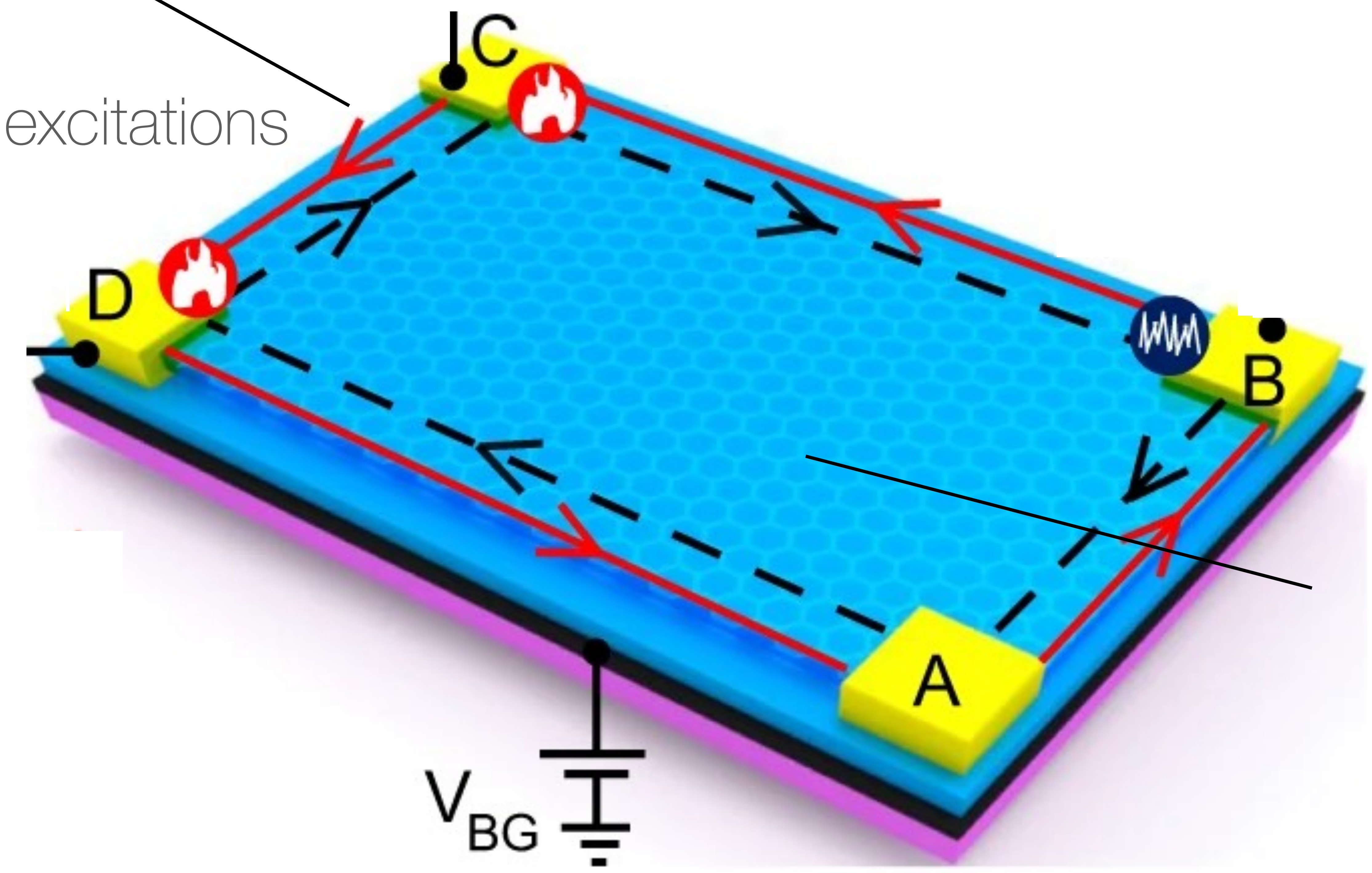

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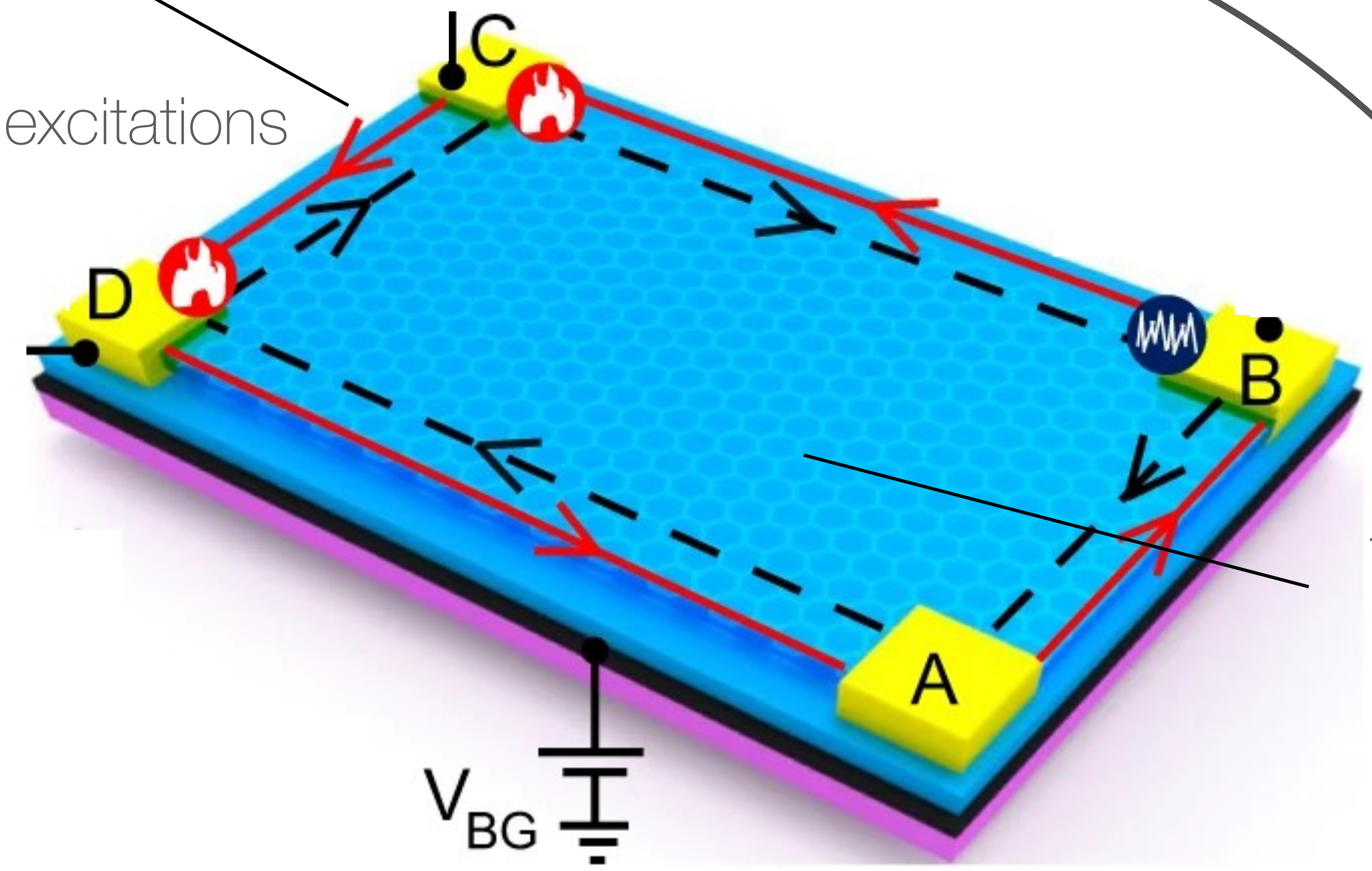

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Motivation

H. Bartolomei et al.: Science 368, 173 (2020); P. Glidic et al.: Phys. Rev. X 13, 011030 (2023)
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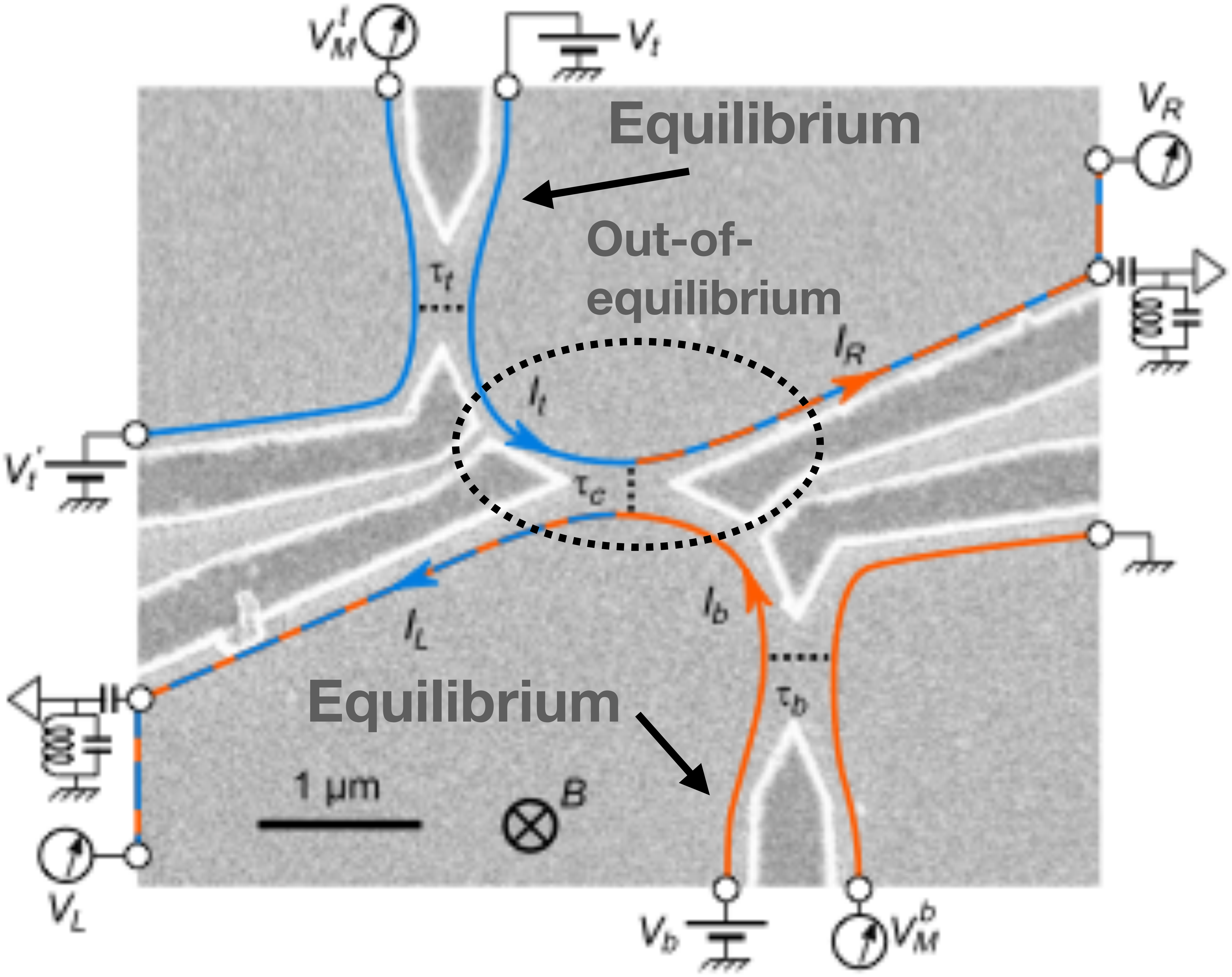

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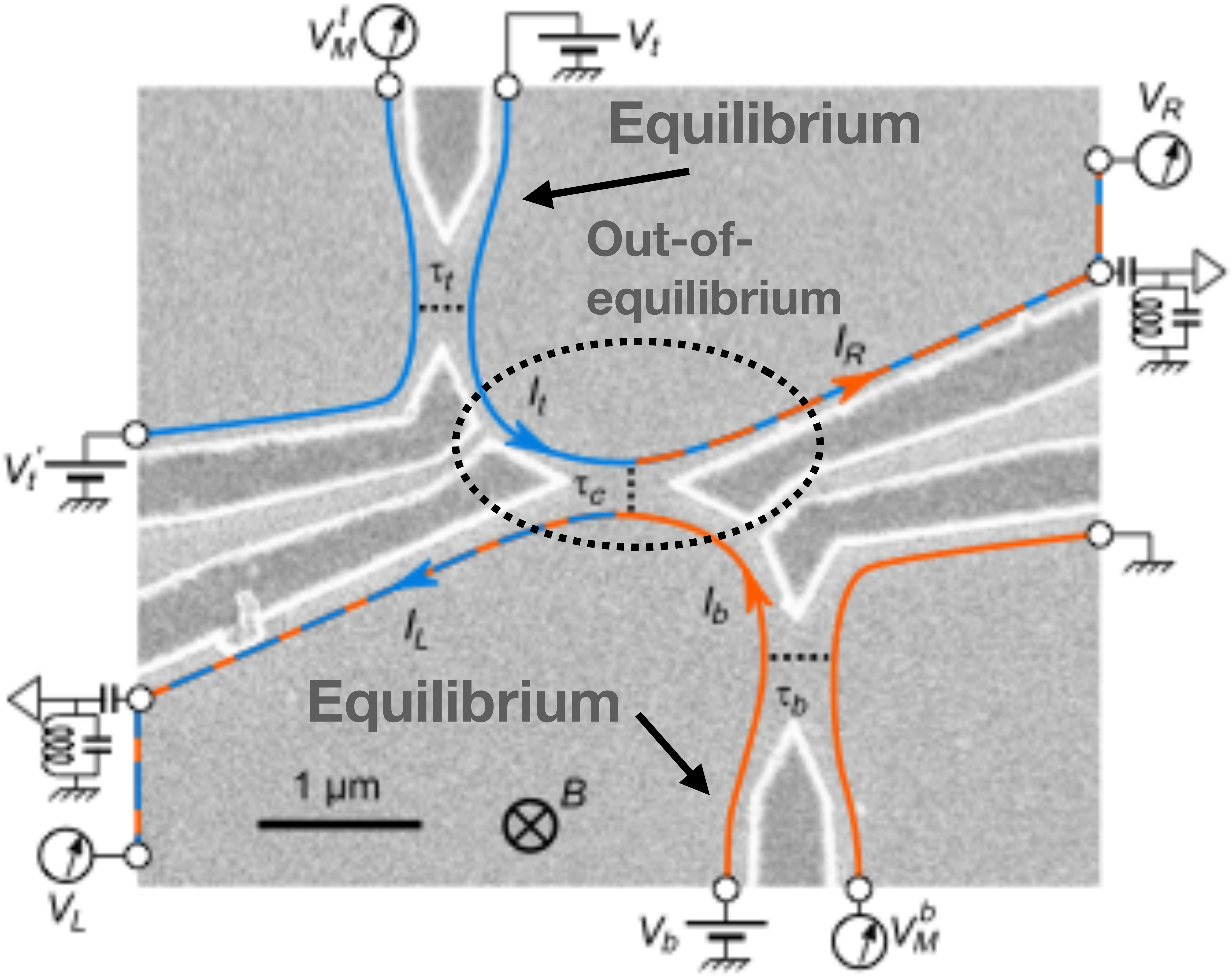

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This work:

We develop a unified framework for non-equilibrium FQH edge transport

Outline of results

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- Identifies experimental signatures of anyonic braiding
- Demonstrates fractionalizations of both charge and braiding angles for edges with multiple, interacting edge modes (see paper)

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For filling factor

$$\boxed{\nu = \frac{1}{m}, \quad m \in 2\mathbb{Z} + 1,}$$

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$$S = S_0 + S_V$$

$$S_0 = \frac{1}{4\pi\nu} \int_c dt \int dx\, \partial_x \phi_\mu(x,t) \left[-\eta \partial_t - v \partial_x\right] \phi_\mu(x,t)$$

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Chirality: right or left mover $\eta = \pm 1$

Non-universal edge velocity v

Commutation relations

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These relations encode the topological order of the bulk via the edge theory

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Braiding angle

$$2\theta_{12} = 2\pi n_1 n_2 \nu$$

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We will see how Green's functions can depend on such braiding angles

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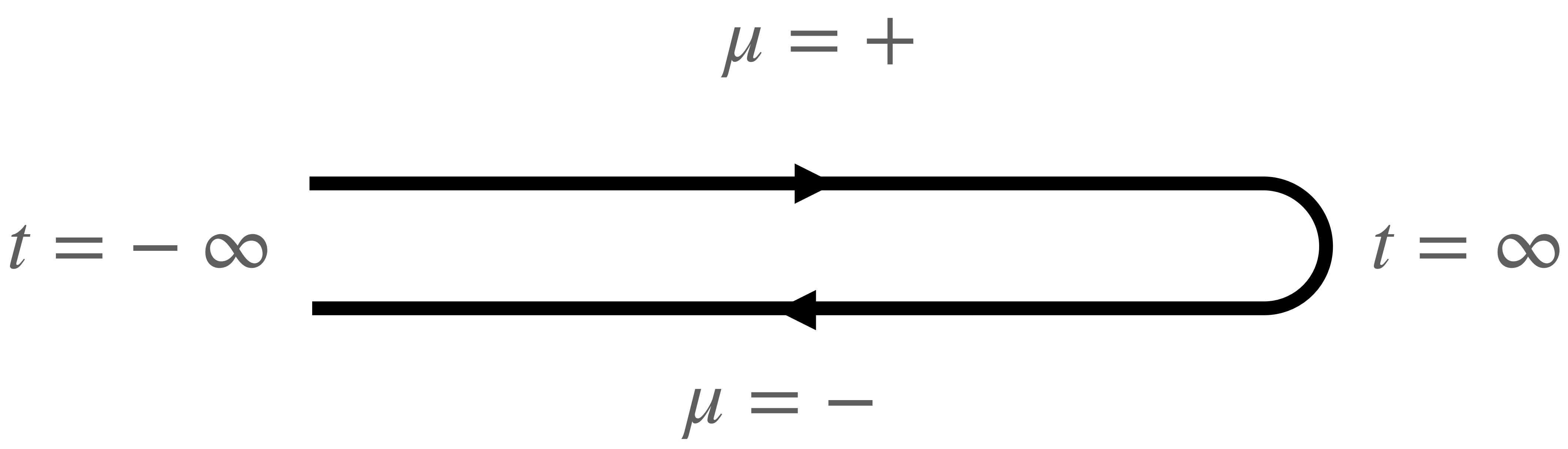
Out-of-equilibrium conditions distinguish future from past: need of time contour



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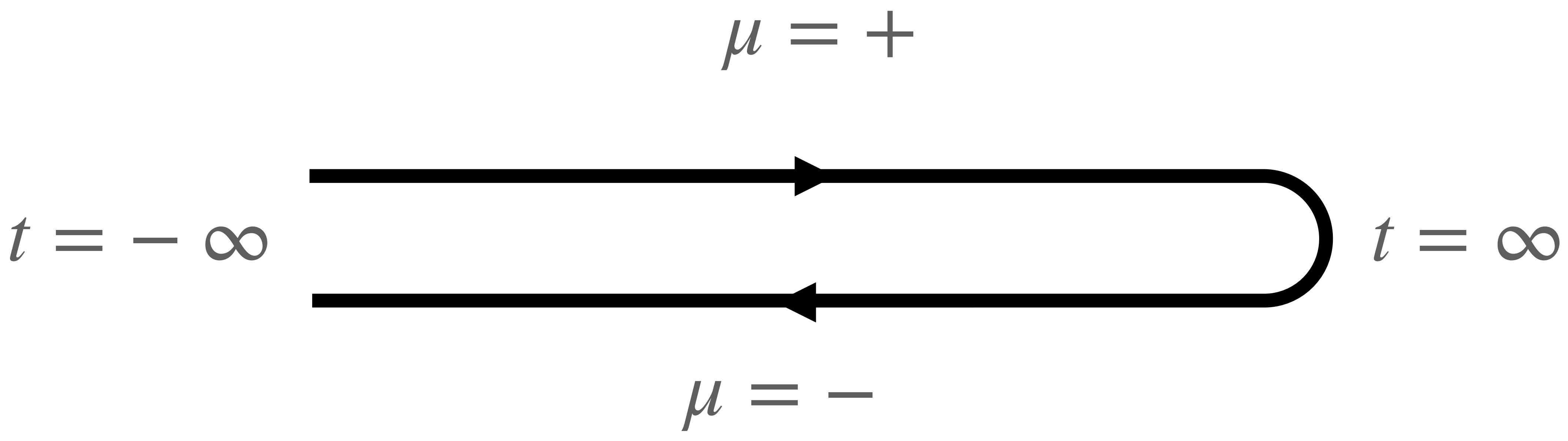

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The price to pay in comparison to equilibrium is keeping track of Keldysh indices

Basis change, Keldysh rotation:

$$\begin{aligned} \phi, \overline{\phi} &= (\phi_+ \pm \phi_-)/\sqrt{2} \\ \rho, \overline{\rho} &= (\rho_+ \pm \rho_-)/\sqrt{2} \\ V, \overline{V} &= (V_+ \pm V_-)/\sqrt{2}\,. \end{aligned}$$

Key object: the generating functional

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Depends only on $\overline{V}$!

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Central finding: The entire non-equilibrium part is determined by the density fluctuations (FCS)!

Fluctuations are represented as a determinant

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$$\delta_{\overline{V}}(t) = \nu\sqrt{2}\int_{-\infty}^{\infty} dt'\, \overline{V}[\eta v(t'+t), t']$$

Scattering phase in potential

$$f(\epsilon)$$

Generic distribution function of *fermions*

Ansatz: anyons "follow" the electrons

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The effective bosonic action of density fields is generally not free (Gaussian):

$$e^{iS[\rho,\overline{\rho}]} = \int \mathscr{D}V\mathscr{D}\overline{V}\, Z[V,\overline{V}]e^{-iV\overline{\rho}-i\overline{V}\rho} = e^{-\frac{i}{\nu}\rho\,\Pi_a^{-1}\overline{\rho}}F[\nu^{-1}\Pi_a^{-1}\overline{\rho}]$$

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In equilibrium, only $n = 2$ contributes: quadratic (Gaussian) theory

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All correlation functions are known: use to benchmark our theory!

Full counting statistics of charge

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The FCS generating function

$$\kappa(\lambda, x_0, \tau) \equiv \langle e^{i\lambda Q(x_0,\tau)/e} e^{-i\lambda Q(x_0,0)/e} \rangle$$

$$Q(x_0, \tau) \equiv e \int_{-\infty}^{x_0} dx \rho(x, \tau)$$

$$\langle\langle(\delta Q)^k\rangle\rangle = (ie\partial_\lambda)^k \ln \kappa(\lambda, x_0, \tau) \,|_{\lambda\to 0}$$

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$$\langle\langle (\delta Q)^k \rangle\rangle = (ie\partial_\lambda)^k \ln \kappa(\lambda, x_0, \tau) \mid_{\lambda \to 0}$$

We derive the determinant representation

$$\kappa(\lambda, x_0, \tau) = \left\{ \mathrm{Det}\left[1 + (e^{-i\delta_\tau(t)} - 1) f(\epsilon)\right] \right\}^{\frac{1}{\nu}}.$$

Full counting statistics of charge

L. S. Levitov et al.: JETP Letters 58, 230 (1993); L. S. Levitov et al.: J. Math. Phys. 37, 4845 (1996)
D. A. Ivanov et al.: Phys. Rev. B 56, 6839 (1997);

The FCS generating function

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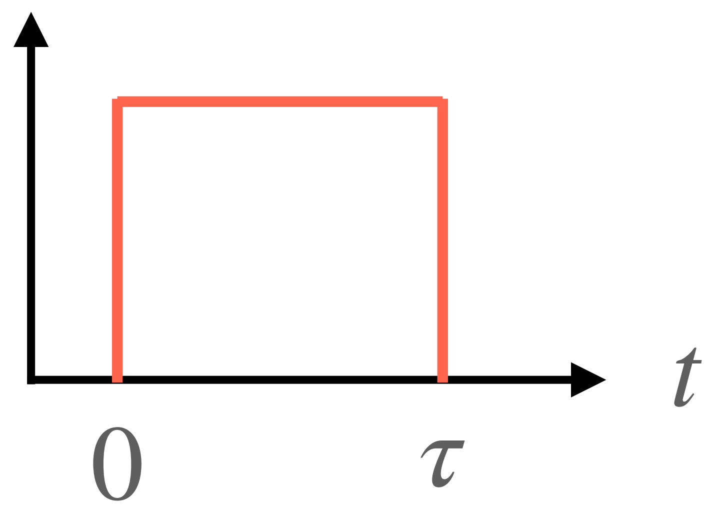
The scattering phase is a "Window" function

$$\delta_\tau(t) = \lambda \nu w_\tau(t,0)$$



Evaluating a Toeplitz-Fredholm determinant

G. Szegö: Math. Ann. 76, 490 (1915); P. Deift et al.: Ann. Of Math. 174, 1243 (2011)

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Long-time asymptotic obtained by the Szegö limit theorem (OK for $\lambda < \pi/2$)

$$\Delta[\delta_\tau(t)] \sim \exp\left[\frac{\tau}{2\pi}\int_{-\Lambda}^{\Lambda} d\epsilon \left\{ \ln\left[1 + (e^{-i\lambda\nu\tau} - 1)f(\epsilon)\right] \right\}\right] +$$

Proper regularization and normalization

Evaluating a Toeplitz-Fredholm determinant

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Equilibrium

$$f(\epsilon) = \frac{1}{e^{\epsilon/T} + 1}$$

$$\kappa(\lambda, x_0, \tau) \simeq e^{-\frac{\nu T \tau \lambda^2}{4\pi}}$$

$$\langle\langle(\delta Q)^k\rangle\rangle = \begin{cases} \frac{\nu e^2 T\tau}{2\pi}, & \text{for k=2,} \\ 0, & \text{otherwise} \end{cases}$$

Gaussian statistics

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Gaussian statistics

OOE: double step

$$f(\epsilon) = \mathcal{T}\Theta(-\epsilon + \nu eV) + (1 - \mathcal{T})\Theta(-\epsilon)$$

$$\kappa(\lambda, x_0, \tau) \simeq e^{-\frac{eV\mathcal{T}\tau}{2\pi}(1 - e^{-i\nu\lambda})}, \quad \mathcal{T} \ll 1$$

$$\langle\langle(\delta Q)^k\rangle\rangle = (e\nu)^{k-1}\langle Q\rangle$$

$$\langle Q\rangle = \frac{\nu e^2 V\mathcal{T}\tau}{2\pi}$$

Poissonian statistics

Green’s functions

A. Kamenev: Field Theory of Non-Equilibrium Systems (2011)

Green's functions

A. Kamenev: Field Theory of Non-Equilibrium Systems (2011)

Lesser and greater GFs

$$\mathcal{G}^{>}(\tau) = -\, i\langle \psi_n(\tau)\psi_n^\dagger(0)\rangle = \frac{-i}{2\pi a}\langle T_c e^{in\phi_-(0,\tau)} e^{-in\phi_+(0,0)}\rangle$$
$$\mathcal{G}^{<}(\tau) = +\, i\langle \psi_n^\dagger(\tau)\psi_n(0)\rangle = \frac{+i}{2\pi a}\langle T_c e^{-in\phi_-(0,0)} e^{in\phi_+(0,\tau)}\rangle\,.$$

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Light-cone variable

$$\tau = t_1 - t_2 - \eta(x_1 - x_2)/v$$

Green's functions

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Path integral representation of, e.g., greater GF for $n = 1$ (Laughlin anyon)

$$\mathcal{G}^>(\tau) = \frac{-i}{2\pi a}\int \mathcal{D}\rho\mathcal{D}\bar{\rho}\, e^{iS[\rho,\bar{\rho}]}e^{\frac{i}{\sqrt{2}}[\phi(0,\tau) - \phi(0,0) - \bar{\phi}(0,\tau) - \bar{\phi}(0,0)]}$$

Green's functions

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With the action

$$e^{iS[\rho,\bar{\rho}]} = e^{-\frac{i}{\nu}\rho\,\Pi_a^{-1}\bar{\rho}}F[\nu^{-1}\Pi_a^{-1}\bar{\rho}] = e^{-\frac{i}{\nu}\rho\,\Pi_a^{-1}\bar{\rho}}\left\{\Delta[\nu^{-1}\Pi_a^{-1}\bar{\rho}]\right\}^{\frac{1}{\nu}}$$

The integral over the classical field is Gaussian

The integral over the classical field is Gaussian

Path integral representation

$$\mathcal{G}^{>}(\tau) = \frac{-i}{2\pi a}\int \mathcal{D}\rho\mathcal{D}\overline{\rho}e^{-\frac{i}{\nu}\rho\,\Pi_a^{-1}\overline{\rho}}\Delta[\nu^{-1}\Pi_a^{-1}\overline{\rho}]e^{\frac{i}{\sqrt{2}}\left[\phi(0,\tau)-\phi(0,0)-\overline{\phi}(0,\tau)-\overline{\phi}(0,0)\right]}$$
$$= \frac{-i}{2\pi a}\int \mathcal{D}\overline{\rho}\,\left\{\Delta[\delta_{\overline{V}}(t)]\right\}^{\frac{1}{\nu}}\Big|_{\overline{V}=\nu^{-1}\Pi_a^{-1}\overline{\rho}} e^{-\frac{i}{\sqrt{2}}\left(\overline{\phi}(0,\tau)+\overline{\phi}(0,0)\right)}\delta\left(\overline{\rho}-\overline{\rho}_{\mathrm{sol}}\right),$$

$$\rho \propto \partial_x \phi$$

The integral over the classical field is Gaussian

Path integral representation

$$\begin{aligned}\mathcal{G}^{>}(\tau) &= \frac{-i}{2\pi a}\int \mathcal{D}\rho\mathcal{D}\overline{\rho}e^{-\frac{i}{\nu}\rho\,\Pi_a^{-1}\overline{\rho}}\Delta[\nu^{-1}\Pi_a^{-1}\overline{\rho}]e^{\frac{i}{\sqrt{2}}\left[\phi(0,\tau)-\phi(0,0)-\overline{\phi}(0,\tau)-\overline{\phi}(0,0)\right]}\\ &= \frac{-i}{2\pi a}\int \mathcal{D}\overline{\rho}\,\left\{\Delta[\delta_{\overline{V}}(t)]\right\}^{\frac{1}{\nu}}\Big|_{\overline{V}=\nu^{-1}\Pi_a^{-1}\overline{\rho}}\; e^{-\frac{i}{\sqrt{2}}\left(\overline{\phi}(0,\tau)+\overline{\phi}(0,0)\right)}\delta\left(\overline{\rho}-\overline{\rho}_{\rm sol}\right),\end{aligned}$$

$$\rho \propto \partial_x\phi$$

where

$$\eta\partial_t\overline{\rho}_{\rm sol}((x,t)+\partial_x v\overline{\rho}_{\rm sol}(x,t) = -\,\eta\nu\frac{1}{\sqrt{2}}\delta(x)\big(\delta(t-\tau)-\delta(t)\big)$$

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Path integral representation

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Putting back again gives after proper regularization

$$\mathcal{G}^{\gtrless}(\tau) = \frac{\mp i}{2\pi a}\left(\frac{a}{a\pm i v\tau}\right)^{\nu}\left\{\overline{\Delta}[\delta_\tau(t)]\right\}^{\frac{1}{\nu}}$$

Determinant normalized to Zero temperature distribution

The integral over the classical field is Gaussian

Path integral representation

$$\mathcal{G}^{>}(\tau) = \frac{-i}{2\pi a}\int \mathcal{D}\rho\mathcal{D}\overline{\rho}e^{-\frac{i}{\nu}\rho\,\Pi_a^{-1}\overline{\rho}}\Delta[\nu^{-1}\Pi_a^{-1}\overline{\rho}]e^{\frac{i}{\sqrt{2}}[\phi(0,\tau)-\phi(0,0)-\overline{\phi}(0,\tau)-\overline{\phi}(0,0)]}$$
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Determinant normalized to
Zero temperature distribution

with the scattering phase

$$\delta_\tau(t) = 2\pi\nu w_\tau(t,0)$$

Generic expression of OOE GFs

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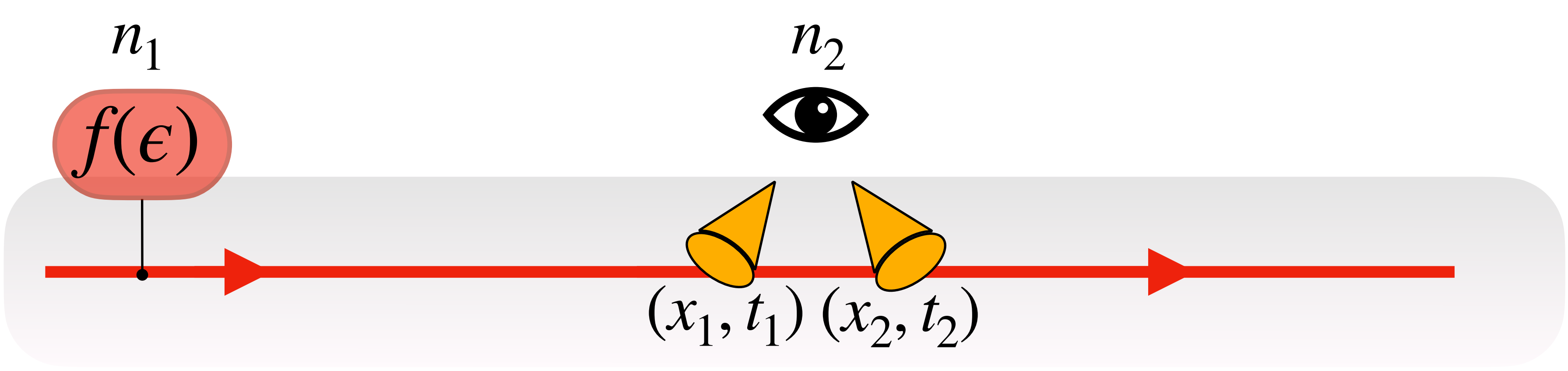
Generally, for injecting particles of type n_1 and probing correlations of type n_2



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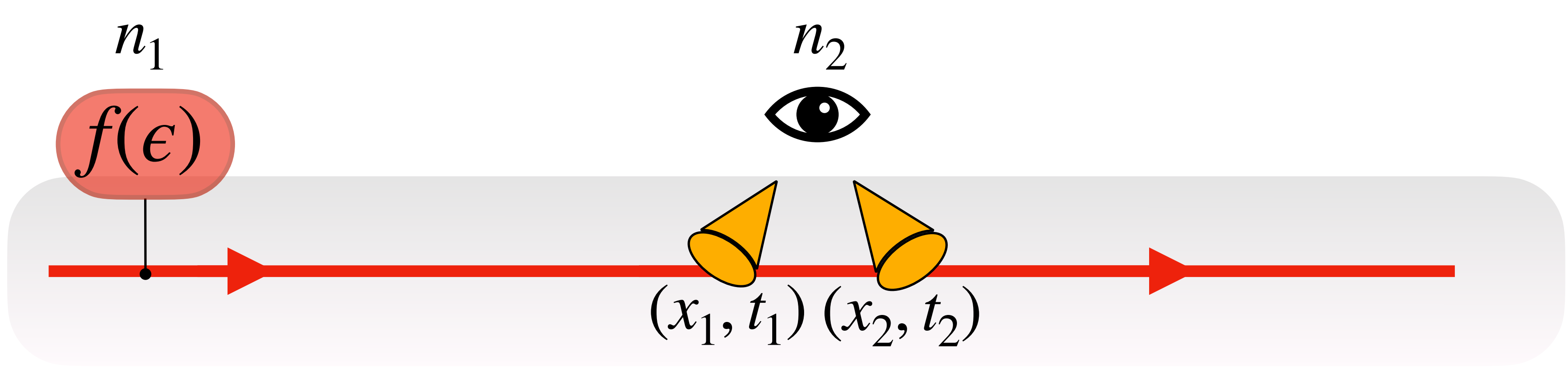

$$\mathcal{G}^{\gtrless}(\tau) = \frac{\mp i}{2\pi a}\left(\frac{a}{a \pm i v \tau}\right)^{n_2^2 \nu} \overline{\Delta}[\delta_\tau(t)]^{\frac{1}{n_1^2\nu}}.$$

$$\delta_\tau(t) = 2\pi n_1 n_2 \nu w_\tau(t,0) = 2\theta_{12} w_\tau(t,0)$$

GFs probe braiding between injected particles and probed particles

Evaluating a Toeplitz-Fredholm determinant again

D. B. Gutman et al.: J. Phys. A: Math. Theor. 44, 165003 (2011); P. Deift et al.: Ann. Of Math. 174, 1243 (2011)

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Now $0 < |2\theta_{12}| \leq 2\pi$, depending on the involved quasiparticles

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1. Take minimal anyons: $2\theta_{12} = 2\pi\nu$. Szegö works well

$$\mathcal{G}^{\gtrless}(\tau) \simeq \frac{\mp i}{2\pi a}\left(\frac{a}{a \pm iv\tau}\right)^{\nu} e^{-\gamma_\phi|\tau|}e^{-i\omega_V\tau}$$

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$$\omega_V = \frac{eV\mathcal{T}\sin(2\theta_{12})}{2\pi}$$

Shift of electro-chemical potential

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$$\gamma_\phi = \frac{|eV|\mathcal{T}\left(1 - \cos(2\theta_{12})\right)}{2\pi}$$

Non-equilibrium dephasing rate

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2. Take $2\theta_{12} = 2\pi\nu$

- Free fermions
- Injecting anyons and correlations of electrons
- Injecting electrons and correlations of anyons

Evaluating a Toeplitz-Fredholm determinant again

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???

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$$\gamma_\phi = \omega_V = 0 \Rightarrow \mathcal{G}^{\gtrless}(\tau) \simeq \frac{\mp i}{2\pi a}\left(\frac{a}{a \pm i v\tau}\right)^{\nu} = \mathcal{G}^{\gtrless}_{\rm eq}(\tau)$$

???

Szegö misses additional logarithmic branches in the determinant
(Generalized) Fisher-Hartwig conjecture solves this problem by summing over branches

Hartwig-Fisher result for the Green's function

D. B. Gutman et al.: J. Phys. A: Math. Theor. 44, 165003 (2011); P. Deift et al.: Ann. Of Math. 174, 1243 (2011)

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Summing over two logarithmic branches gives, with $\delta_0 = 2\theta_{12}$, the GFs

$$\mathcal{G}^{\gtrless}(\tau) \simeq \frac{\mp i}{2\pi a}\frac{a^{n_2^2\nu}}{(a \pm iv\tau)^{n_2^2\nu}}\exp[-\frac{\mathcal{T}\,|e_1^* V\tau|}{2\pi n_1^2}(1-e^{-i\delta_0 \mathrm{sgn}(e_1^* V\tau)})]\left(\frac{1}{|e_1^* V\tau|}\right)^{-\frac{2n_2}{\pi n_1}\mathcal{T}\sin\left(\frac{\delta_0}{2}\right)\exp[i\frac{\delta_0}{2}\mathrm{sgn}(e_1^* V\tau)]}$$

$$\times\left[1+\frac{\mathcal{T}}{n_1^2\nu}e^{-i\delta_0\mathrm{sgn}(e_1^* V\tau)/2}\left\{\frac{\delta_0}{2\pi^2}\sin\left(\frac{\delta_0}{2}\right)\left(2-\psi(1-\frac{\delta_0}{2\pi})-\psi(1+\frac{\delta_0}{2\pi})\right)-e^{-ie_1^* V\tau}\frac{1}{(\Gamma(\frac{\delta_0}{2\pi}))^2}\left(\frac{1}{|e_1^* V\tau|}\right)^{2(1-\frac{\delta_0}{2\pi})}\right\}\right]$$

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$$\times\left[1 + \frac{\mathcal{T}}{n_1^2\nu}e^{-i\delta_0 \mathrm{sgn}(e_1^* V\tau)/2}\left\{\frac{\delta_0}{2\pi^2}\sin\left(\frac{\delta_0}{2}\right)\left(2 - \psi(1 - \frac{\delta_0}{2\pi}) - \psi(1 + \frac{\delta_0}{2\pi})\right) - e^{-ie_1^* V\tau}\frac{1}{(\Gamma(\frac{\delta_0}{2\pi}))^2}\left(\frac{1}{|e_1^* V\tau|}\right)^{2(1-\frac{\delta_0}{2\pi})}\right\}\right]$$

The limit $2\theta_{12} \to 2\pi$ correctly reproduces the exact free-fermion result for double step

$$\mathcal{G}^{\gtrless}(\tau) = \frac{\mp i}{2\pi a}\frac{a}{(a \pm i v\tau)}(1 - \mathcal{T} + \mathcal{T}e^{-ieV\tau})$$

Quantum point contact geometry

H. Bartolomei et al.: Science 368, 173 (2020); P. Glidic et al.: Phys. Rev. X 13, 011030 (2023
J.-Y. M. Lee et al.: Nature 617, 277 (2023); M. Ruelle et al.: Phys. Rev. X 13, 011031 (2023))

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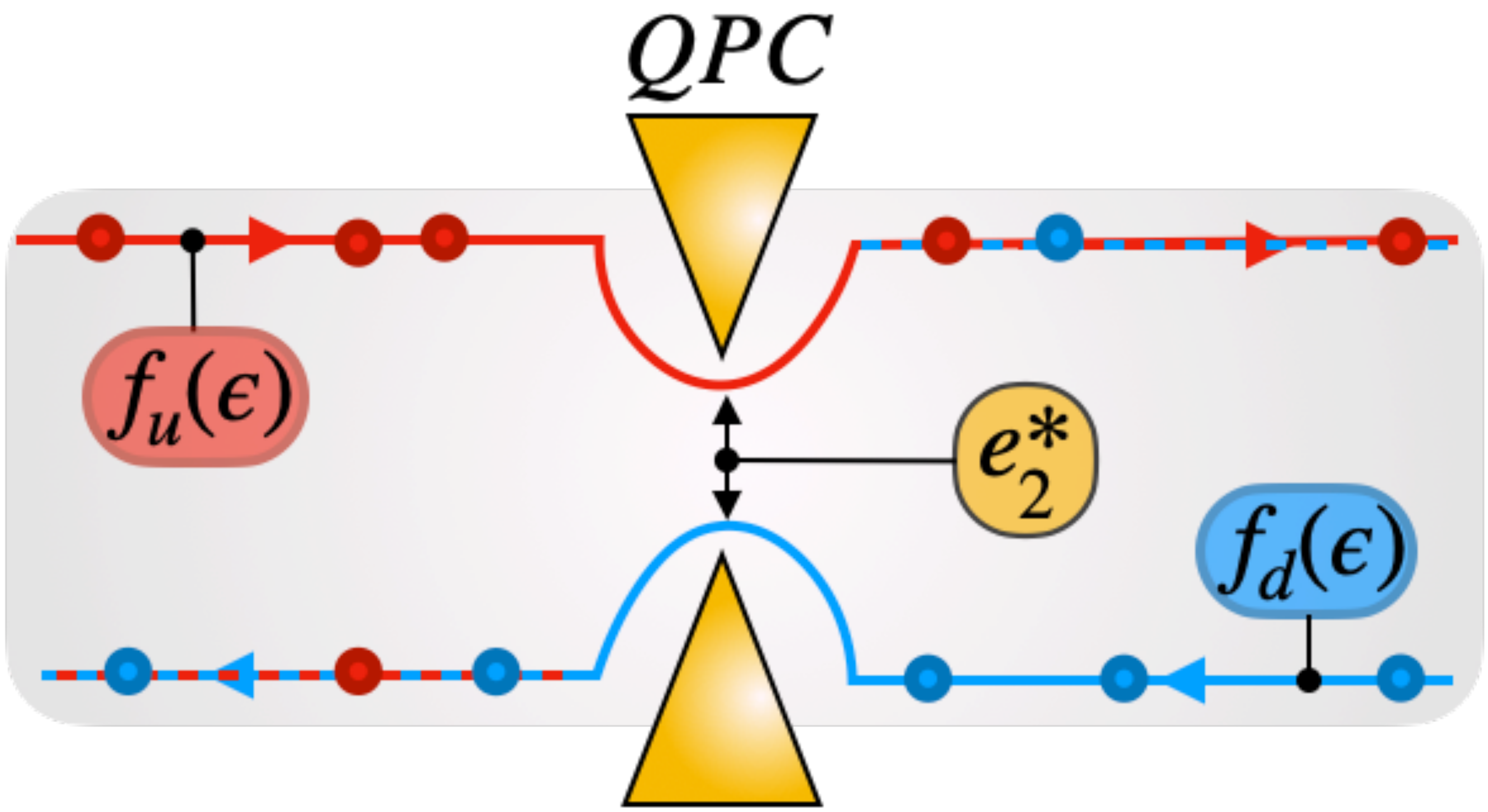

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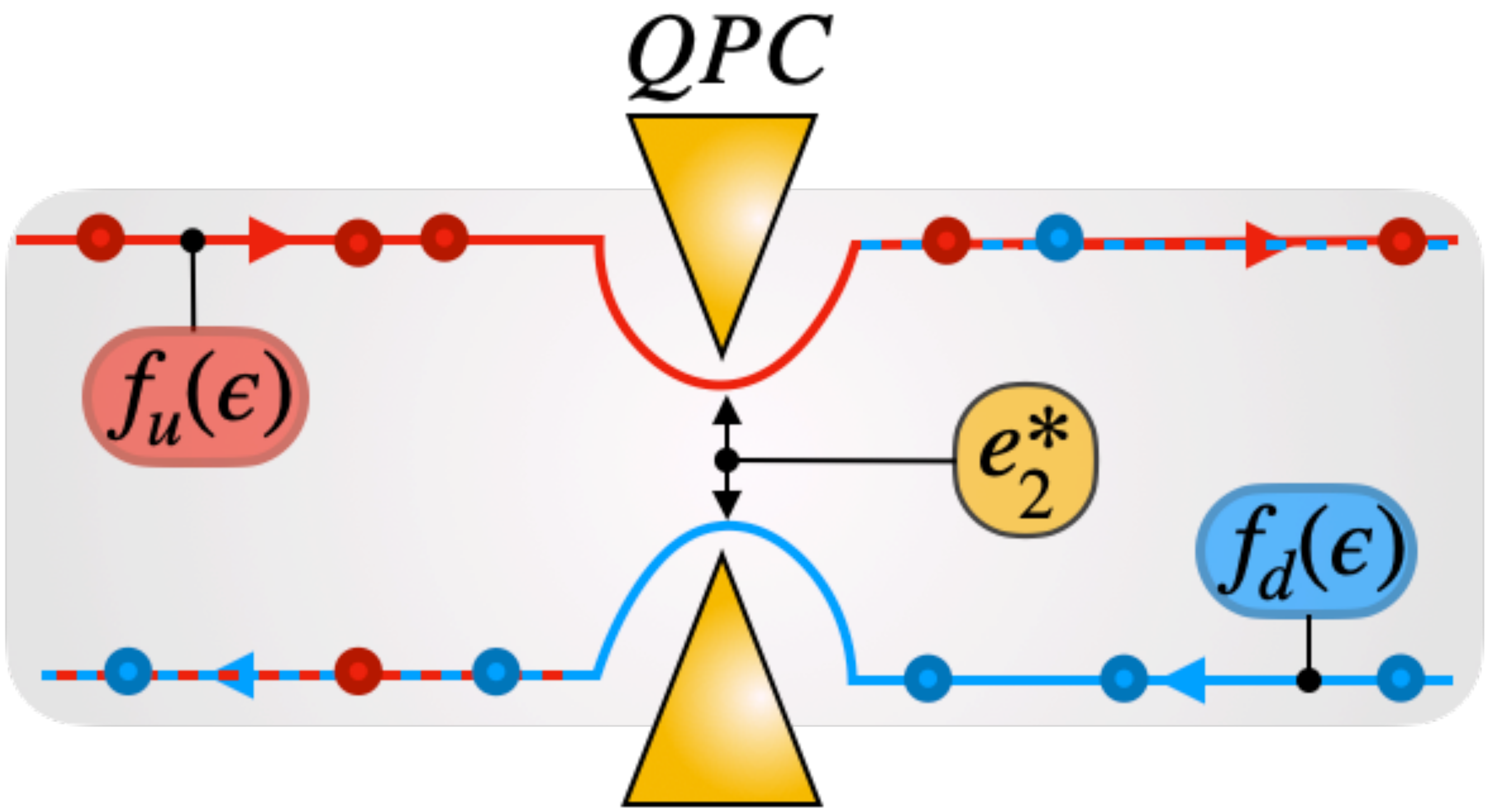

Two Laughlin edges u and d connected by a weak link transferring charges e_2^*

Tunneling current and noise

Tunneling current and noise

Tunneling Hamiltonian

$$H_T = A^\dagger + A, \quad A = \xi\, \psi^\dagger_{n_2,u} \psi_{n_2,d}$$

Tunneling current and noise

Tunneling Hamiltonian

$$H_T = A^\dagger + A, \quad A = \xi \psi^\dagger_{n_2,u}\psi_{n_2,d}$$

Tunneling current operator

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Note: All expectation values are taken w.r.t. to the generic input states!

Benchmark: standard voltage bias

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We recover usual results: shot noise probes the tunneling charge

Scaling dimension

$$\zeta = \nu n_2^2$$

$$\langle I_T \rangle \propto e_2^* \, | e_2^* (V_u - V_d) |^{2\zeta - 1}$$

$$S_T \propto 2 (e_2^*)^2 \, | e_2^* (V_u - V_d) |^{2\zeta - 1}$$

$$F \equiv \frac{S_T}{2e\langle I_T \rangle} = \frac{e_2^*}{e} = \nu n_2$$

Non-equilibrium injection

Non-equilibrium injection

Inject double step distributions (e.g., generated by separate QPCs) with charges $e_1^* = \nu n_1 e$

$$f_{u/d}(\epsilon) = \mathcal{T}_{u/d}\Theta(-\epsilon + e_1^* V_{u/d}) + (1 - \mathcal{T}_{u/d})\Theta(-\epsilon)$$

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$$\gamma_{u/d} = \frac{|eV_{u/d}|\mathcal{T}_{u/d}\left(1 - \cos(2\theta_{12})\right)}{2\pi n_1}$$

$$\omega_{u/d} = \frac{eV_{u/d}\mathcal{T}_{u/d}\sin(2\theta_{12})}{2\pi n_1}$$

Non-equilibrium injection

Non-equilibrium injection

Tunneling current and noise

$$\langle I_T \rangle \propto e_2^* \cos(\pi\zeta) \frac{\sin\left[(1-2\zeta)\tan^{-1}(p)\right]}{(|\omega_u - \omega_d|^2 + (\gamma_u + \gamma_d)^2)^{1/2-\zeta}}$$

$$S_T \propto (e_2^*)^2 \sin(\pi\zeta) \frac{\cos\left[(1-2\zeta)\tan^{-1}(p)\right]}{(|\omega_u - \omega_d|^2 + (\gamma_u + \gamma_d)^2)^{1/2-\zeta}}\,.$$

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The Fano factor depends on scaling dimension $\zeta = \nu n_2^2$ and braiding angle $2\theta_{12} = 2\pi\nu n_1 n_2$

$$F = \frac{e_2^*}{|e|} \cot(\pi\zeta) \cot\left[(1-2\zeta)\tan^{-1}(p)\right]$$

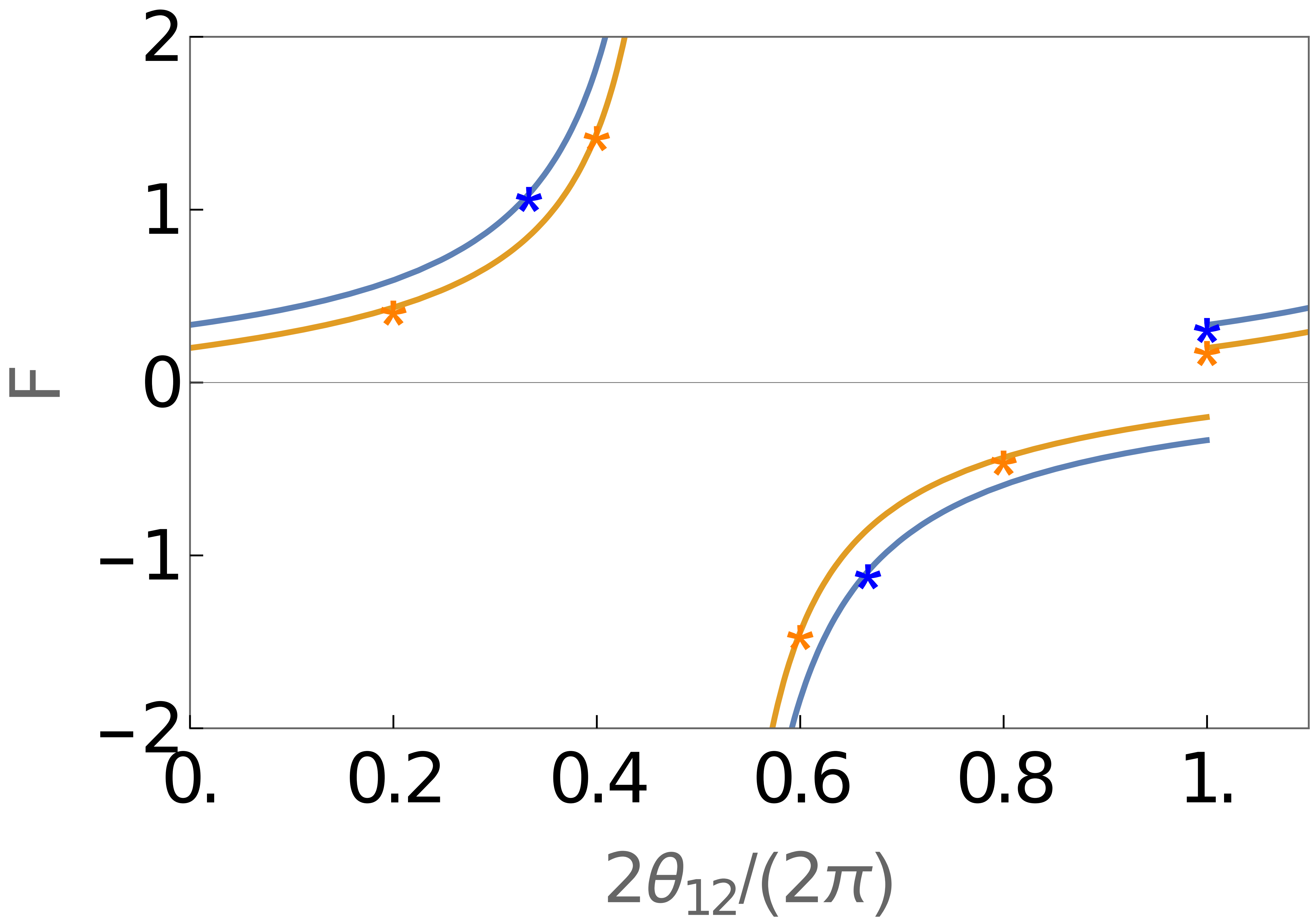
$$p = \frac{eV_u\mathcal{T}_u - eV_d\mathcal{T}_d}{|eV_u\mathcal{T}_u| + |eV_d\mathcal{T}_d|} \cot(\theta_{12})$$

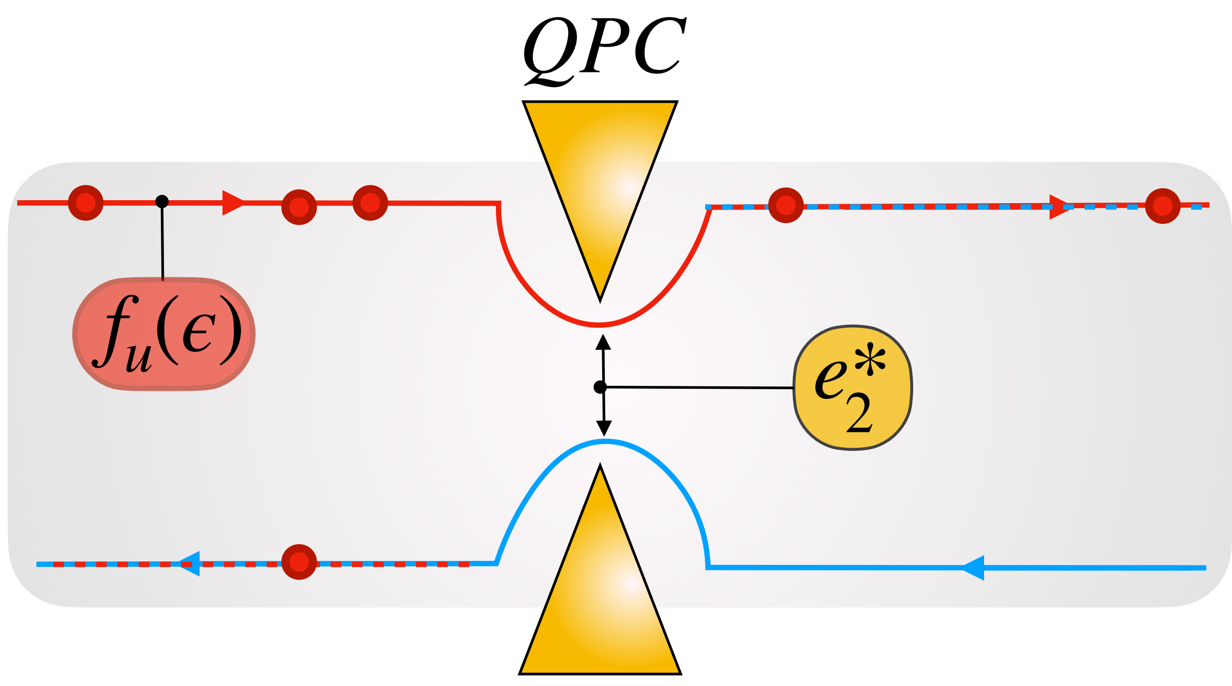
Fano factor vs braiding angle

Fano factor vs braiding angle

$\zeta = \nu$

Tunneling of minimal anyon





$\nu = 1/3$ — $*$ $2\theta_{12}/(2\pi) = 1/3, 2/3, 1$

$\nu = 1/5$ — $*$ $2\theta_{12}/(2\pi) = 1/5, 2/5, 3/5, 4/5, 1$

Fano factor vs braiding angle

$\boxed{\zeta = \nu}$

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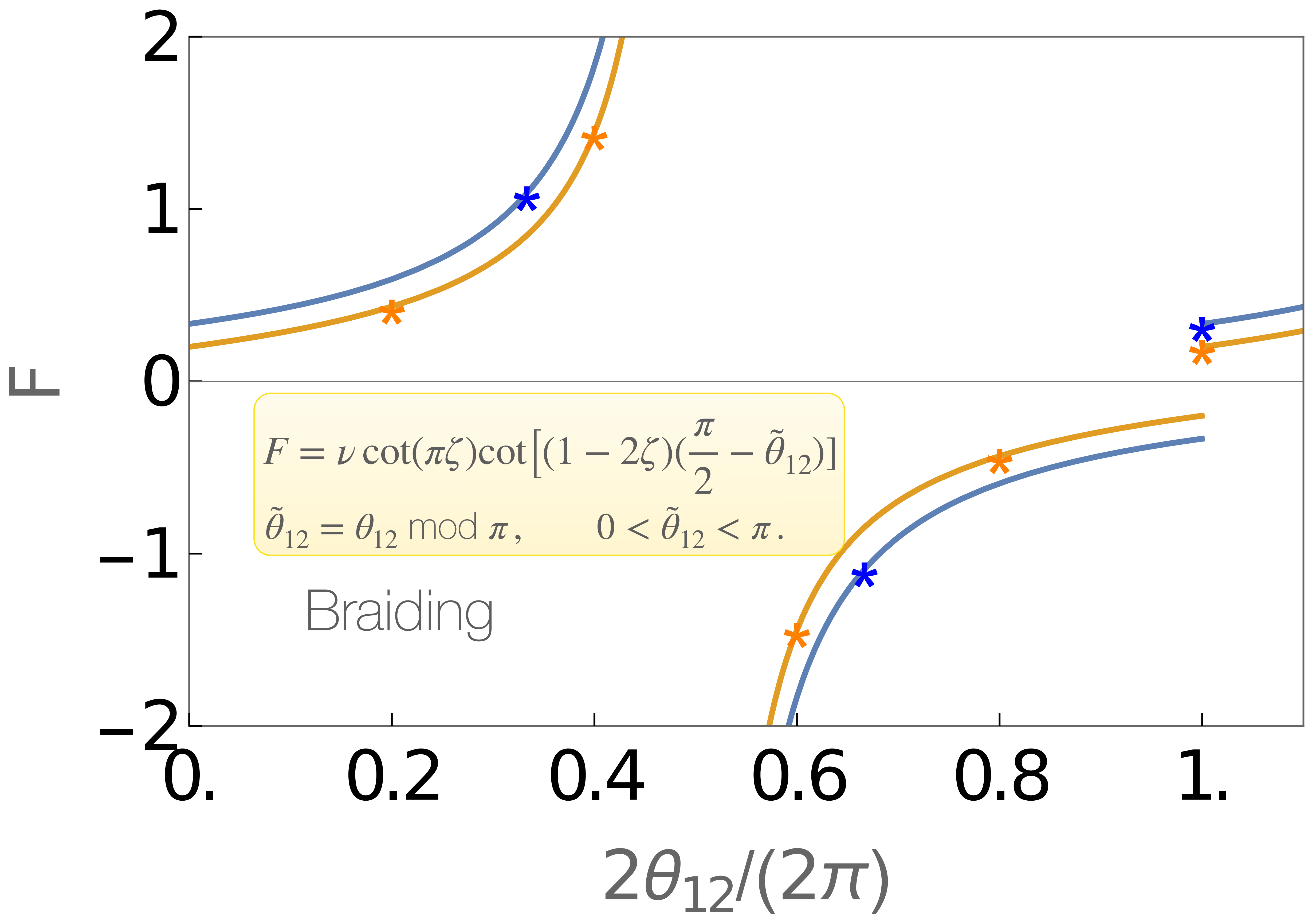

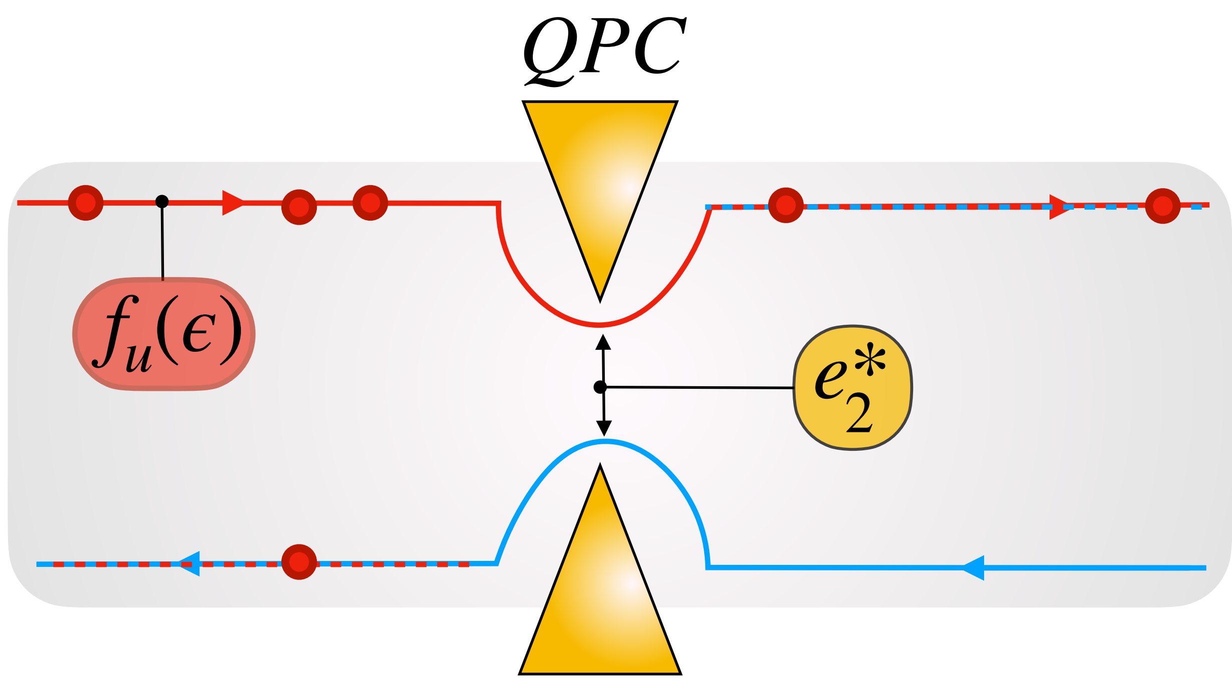

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Fano factor vs braiding angle

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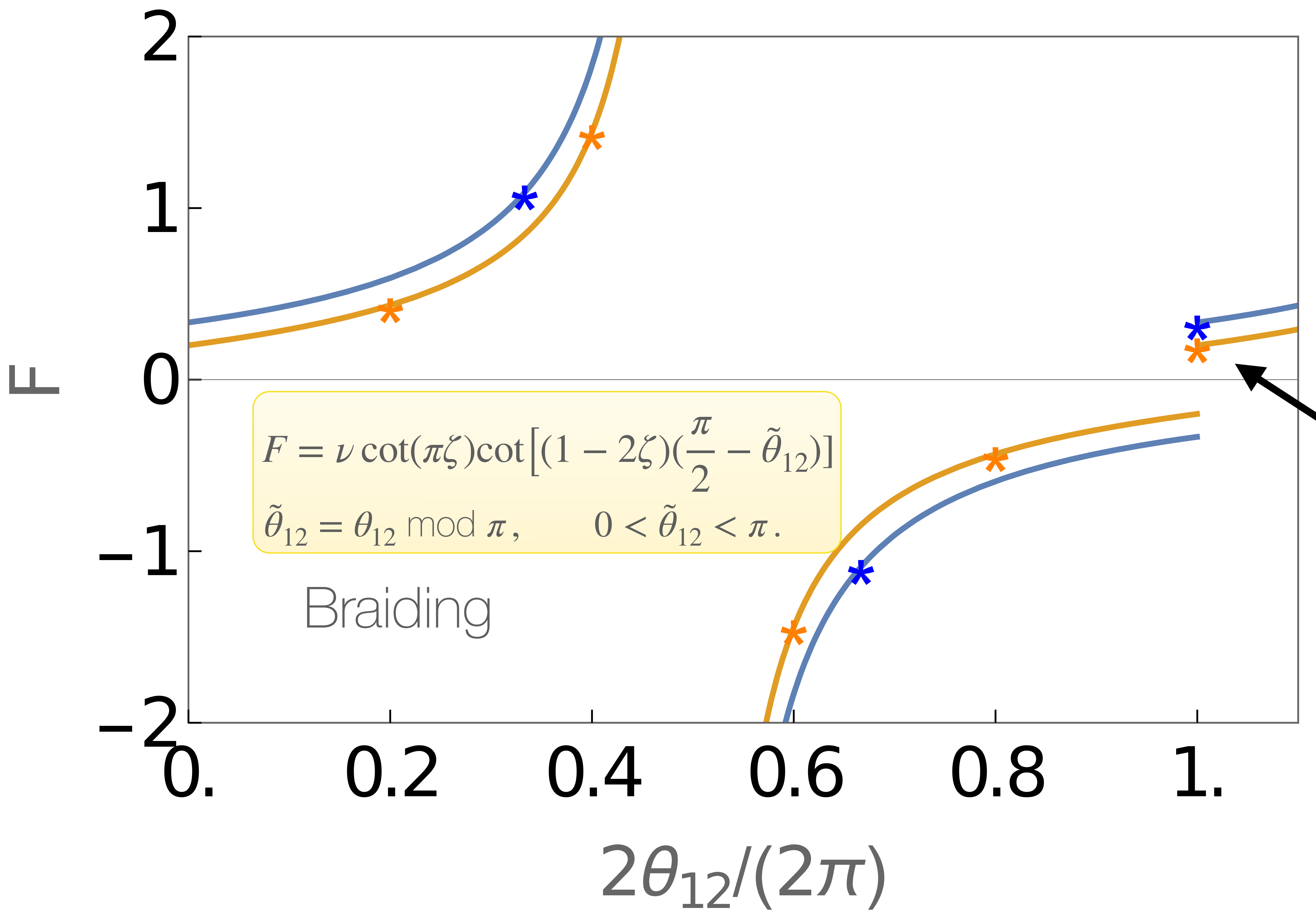

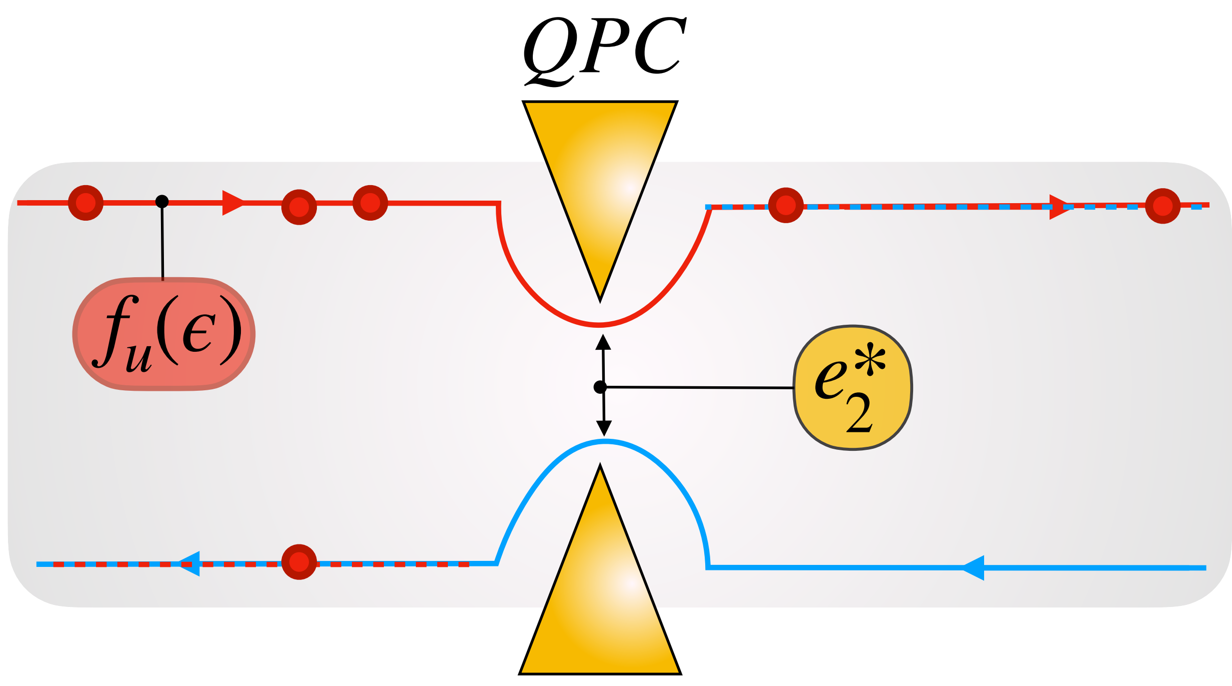

$F = \nu$

Trivial braiding

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Physical picture: time-domain braiding

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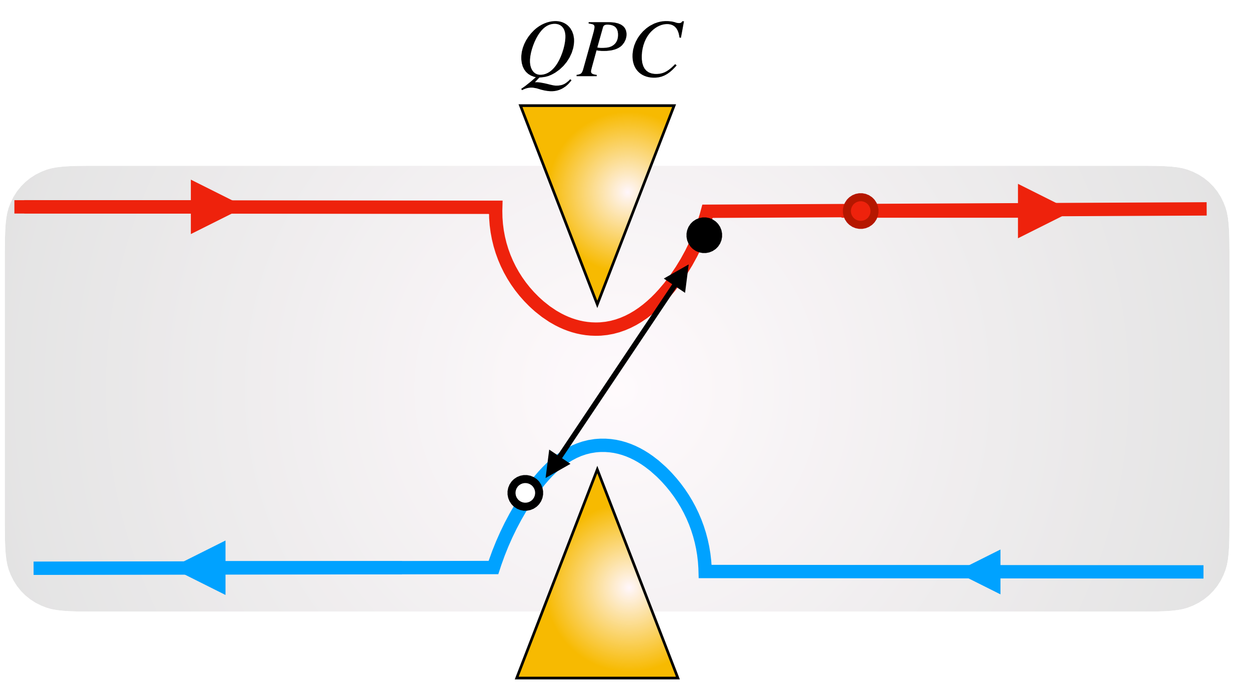

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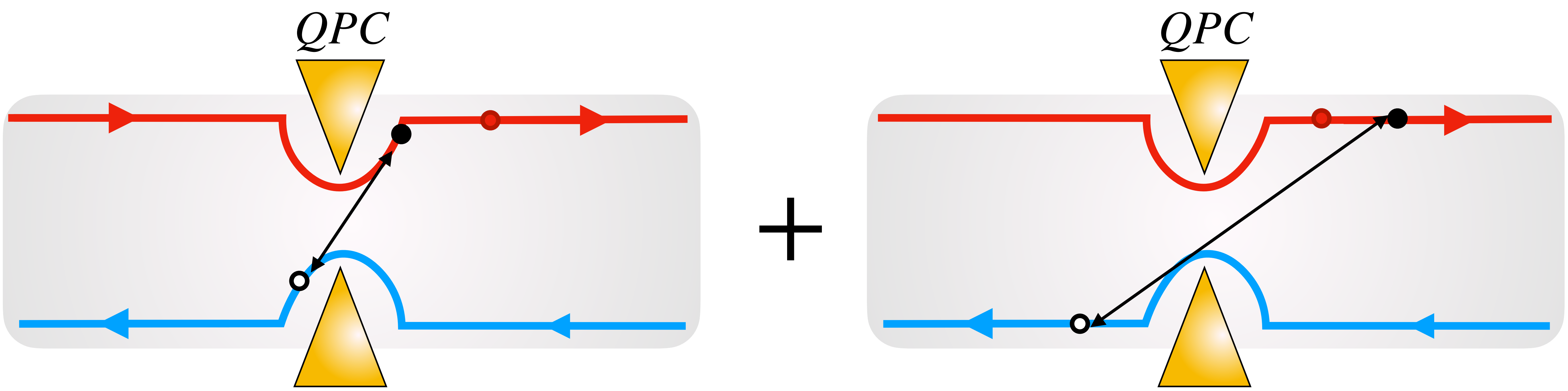

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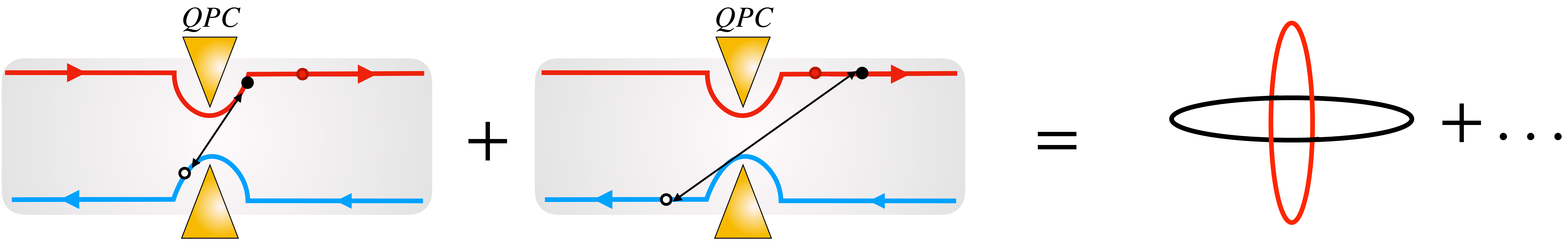

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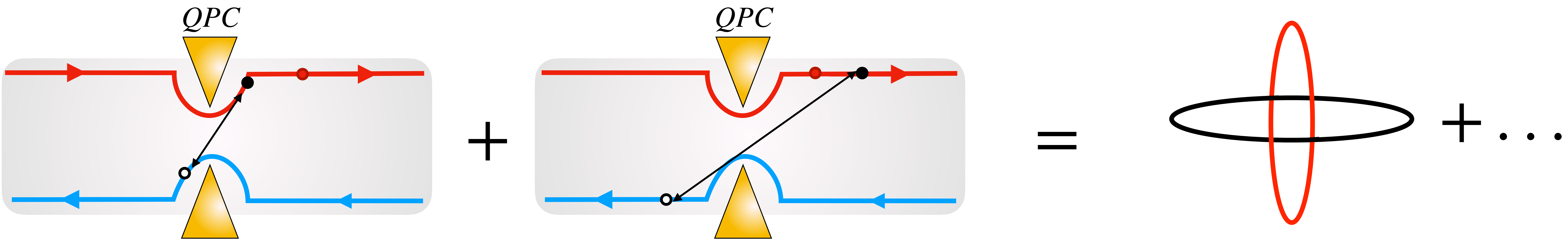

Injected quasiparticles, n_1, braid in the time-domain with the tunneling quasiparticles, n_2

$$2\theta_{12} = 2\pi\nu n_1 n_2$$

Central messages

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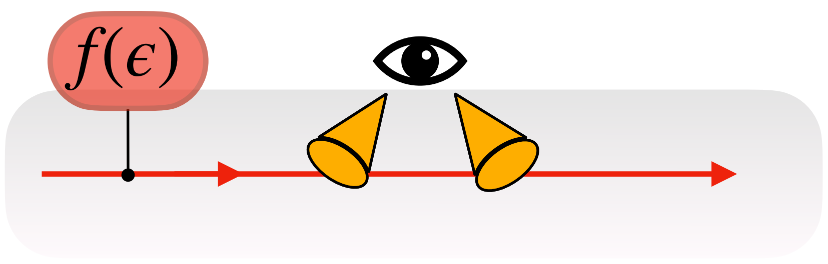
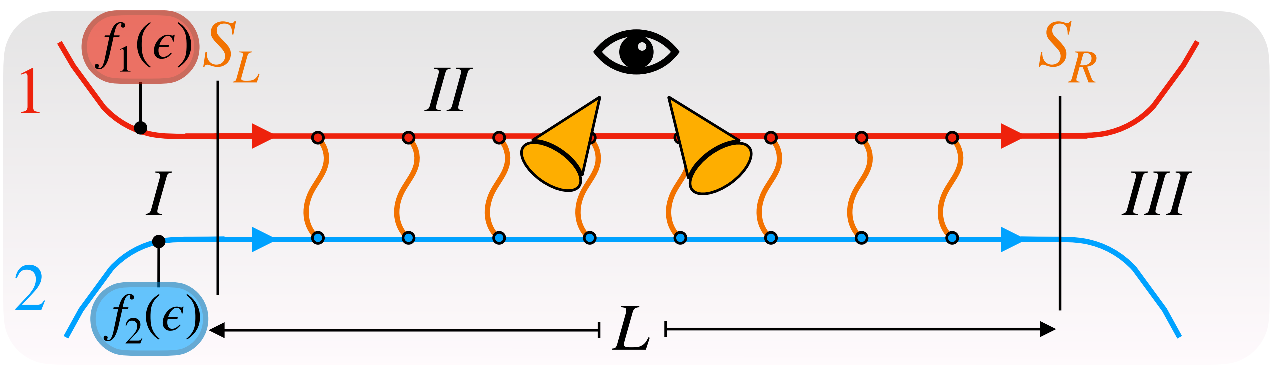
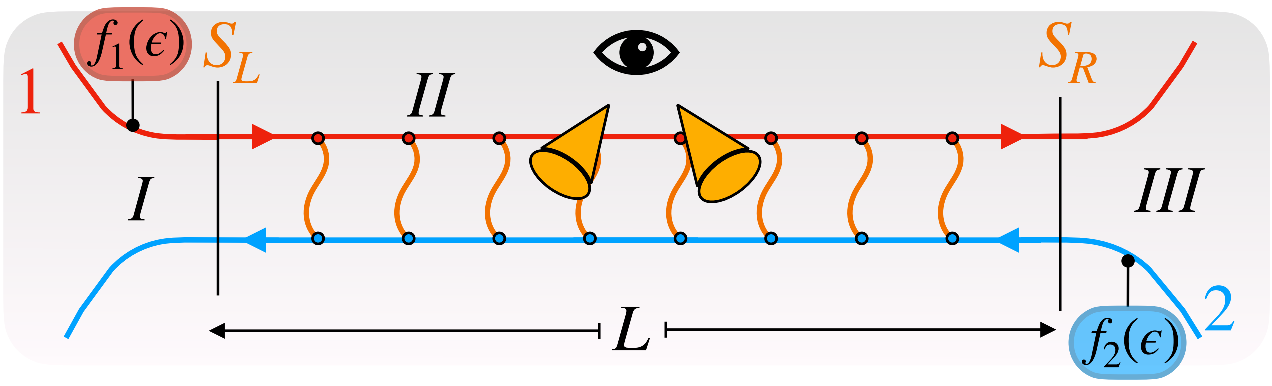
- Gives information about, e.g., *anyonic statistics,* $2\theta_{12}$
- Generalizes to *any Abelian FQH* edge structure (see paper): interactions modify scattering phases: *fractionalization of both charge and braiding angle*



Single mode (this talk)



Co-propagating modes



Counter-propagating modes

Outlook

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Can we generalize to non-Abelian edge structures?

P. Fendley, M. P. A. Fisher, C. Nayak: Phys. Rev. B 75, 045317 (2007)

Thank you!

Non-equilibrium bosonization of fractional quantum Hall edges

C. Spånslätt, J. Park, and A. D. Mirlin, arXiv 2603.05088

christian.spanslatt@kau.se

https://sites.google.com/view/christianspanslatt

Swedish
Research Council

Bonus Material

Evaluation of Fredholm-Toeplitz determinants

Toeplitz form

$$\Delta[\delta_\tau(t)] = \mathrm{Det}\left[1 + (e^{-i\delta_\tau(t)} - 1)f(\epsilon)\right]$$

$$\delta_\tau(t) = \delta_0\left[\Theta(-t) - \Theta(-t-\tau)\right]$$

$$[\epsilon, t] = i$$

Zero temperature

$$f(\epsilon) = \Theta(-\epsilon) \equiv f_0(\epsilon)$$

$$\Delta[\delta_\tau(t)]_{T=0,V=0} = \frac{e^{-i\delta_0\tau\Lambda/(2\pi)}}{(1+\tau^2\Lambda^2)^{\frac{1}{2}\left(\frac{\delta_0}{2\pi}\right)^2}}$$

$$f(\epsilon) = f_0(\epsilon - e_1^* V) \equiv \Theta(-\epsilon + e_1^* V)$$

$$\Delta[\delta_\tau(t)]_{T=0,V} = e^{-i\delta_0\tau e_1^* V/(2\pi)}\Delta[\delta_\tau(t)]_{T=0,V=0}$$

$$\left\{\overline{\Delta}[\delta_\tau(t)]_{T=0,V}\right\}^{\frac{1}{n_1^2\nu}} \equiv \left\{\frac{\Delta[\delta_\tau(t)]_{T=0,V}}{\Delta[\delta_\tau(t)]_{T=0,V=0}}\right\}^{\frac{1}{n_1^2\nu}} = \left[e^{-in_1\nu e(\delta_0/2\pi)V\tau}\right]^{\frac{1}{n_1^2\nu}} = e^{-ie_2^* V\tau}$$

Evaluation of Fredholm-Toeplitz determinants

Finite temperature

$$f(\epsilon) = f_T(\epsilon - e_1^* V) \equiv \frac{1}{e^{(\epsilon - e_1^* V)/T} + 1}$$

$$\Delta[\delta_\tau(t)]_{T,V} = \left(\frac{\pi T \tau}{\sinh(\pi T \tau)}\right)^{\left(\frac{\delta_0}{2\pi}\right)^2} \times \frac{e^{-i\delta_0 \tau \Lambda/(2\pi)}}{(1+\tau^2\Lambda^2)^{\frac{1}{2}\left(\frac{\delta_0}{2\pi}\right)^2}} e^{-i\delta_0 \tau e_1^* V/(2\pi)}$$

$$\left\{\overline{\Delta}[\delta_\tau(t)]_{T,V}\right\}^{\frac{1}{n_1^2 \nu}} \equiv \left\{\frac{\Delta[\delta_\tau(t)]_{T,V}}{\Delta[\delta_\tau(t)]_{T=0,V=0}}\right\}^{\frac{1}{n_1^2 \nu}} = \left(\frac{\pi T \tau}{\sinh(\pi T \tau)}\right)^{n_2^2 \nu} e^{-i e_2^* V \tau}$$

Evaluation of Fredholm-Toeplitz determinants

Szegö theorem

"Symbol"

$$g(z) = \exp\{U(z)\} \qquad z = e^{i\varphi} \quad \varphi \in [-\pi, \pi]$$

Fourier transform

$$U(z) = \sum_{k=-\infty}^{\infty} U_k e^{i\varphi k}, \quad \text{where } U_k = \int_{-\pi}^{\pi} \frac{d\varphi}{2\pi} U(z) e^{-i\varphi k}.$$

Large size N asymptotics

$$\det(g) \sim \det(g_N) \approx \exp(NU_0 + \sum_{k=1}^{N} kU_k U_{-k}),$$

Evaluation of Fredholm-Toeplitz determinants

Double step

$$f(\epsilon) = \mathcal{T} f_0(\epsilon - e_1^* V) + (1 - \mathcal{T}) f_0(\epsilon) .$$

Apply Szegö theorem

$$g(z = e^{i\pi\epsilon/\Lambda}) = \left[1 + (e^{-i\delta_0} - 1) f(\epsilon)\right] e^{-i\epsilon\delta_0/(2\Lambda)} .$$

$$\Delta[\delta_\tau(t)] \approx e^{-i\frac{\delta_0\tau\Lambda}{2\pi}} e^{ie_1^* V\beta_1\tau} (\Lambda|\tau|)^{-(\beta_0^2+\beta_1^2)} \left(\frac{\Lambda}{|e_1^* V|}\right)^{-2\beta_0\beta_1}$$

$$\overline{\Delta}[\delta_\tau(t)] \equiv \frac{\Delta[\delta_\tau(t)]}{\Delta[\delta_\tau(t)]_{T=0,V=0}} \approx e^{ie^* V\beta_1\tau} \left(\frac{1}{|e_1^* V\tau|}\right)^{-2\beta_0\beta_1} .$$

$$\beta_0 = -\beta_1 - \frac{\delta_0}{2\pi}, \quad \beta_1 = -\frac{i}{2\pi} \mathrm{sgn}(e_1^* V) \ln[1 + \mathcal{T}(e^{-i\delta_0 \mathrm{sgn}(e_1^* V)} - 1)] .$$

$$\overline{\Delta}[\delta_\tau(t)] \approx \exp\left[-\frac{|e_1^* V\tau|\mathcal{T}}{2\pi} (1 - e^{-i\delta_0 \mathrm{sgn}(eV\tau)})\right]$$

Evaluation of Fredholm-Toeplitz determinants

Generalized HF conjecture
(Symbol with singularities)

$$g(z) = e^{U(z)} z^{\sum_{j=0}^{m} \beta_j} \prod_{j=0}^{m} |z - z_j|^{2\alpha_j} h_{z_j,\beta_j}(z) z_j^{-\beta_j}$$

$$h_{z_j,\beta_j}(z) = \begin{cases} e^{i\pi\beta_j}, & -\pi < \arg z < \varphi_j \\ e^{-i\pi\beta_j}, & \varphi_j < \arg z < \pi, \end{cases}$$

Large size asymptotics with sum over logarithm branches

$$\Delta_N \simeq e^{NU_0} \sum_{k_0+\cdots+k_m=0} \prod_{j=0}^{m} z_j^{k_j N} \left[N^{-\sum_{j=0}^{m} \beta_j^2} \prod_{0\le j<l\le m} |z_j - z_l|^{2\beta_j\beta_l} \prod_{j=0}^{m} G(1+\beta_j) G(1-\beta_j) \right]_{\beta_j \to \beta_j + k_j}$$

Evaluation of Fredholm-Toeplitz determinants

Evaluation of Fredholm-Toeplitz determinants

Summing over two logarithmic branches gives, with $\delta_0 = 2\theta_{12}$, the GFs

$$\mathcal{G}^{\gtrless}(\tau) \simeq \frac{\mp i}{2\pi a}\frac{a^{n_2^2\nu}}{(a \pm iv\tau)^{n_2^2\nu}} \exp[-\frac{\mathcal{T}\,|e_1^* V\tau|}{2\pi n_1^2}(1 - e^{-i\delta_0 \mathrm{sgn}(e_1^* V\tau)})]\left(\frac{1}{|e_1^* V\tau|}\right)^{-\frac{2n_2}{\pi n_1}\mathcal{T}\sin\left(\frac{\delta_0}{2}\right)\exp[i\frac{\delta_0}{2}\mathrm{sgn}(e_1^* V\tau)]}$$

$$\times\left[1 + \frac{\mathcal{T}}{n_1^2\nu}e^{-i\delta_0 \mathrm{sgn}(e_1^* V\tau)/2}\left\{\frac{\delta_0}{2\pi^2}\sin\left(\frac{\delta_0}{2}\right)\left(2 - \psi(1 - \frac{\delta_0}{2\pi}) - \psi(1 + \frac{\delta_0}{2\pi})\right) - e^{-ie_1^* V\tau}\frac{1}{\left(\Gamma(\frac{\delta_0}{2\pi})\right)^2}\left(\frac{1}{|e_1^* V\tau|}\right)^{2(1-\frac{\delta_0}{2\pi})}\right\}\right]$$