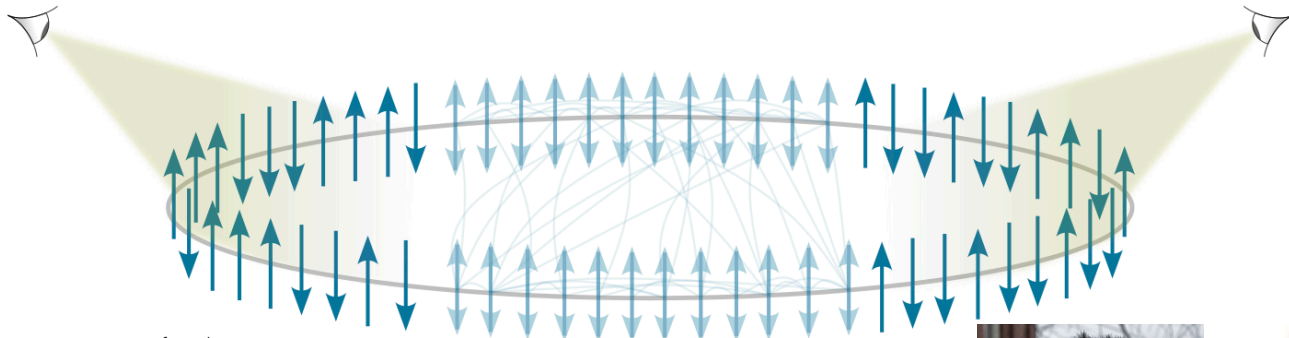


Monitored Conformal Field Theories

Romain Vasseur

(University of Geneva)

Nordita Workshop



K. Khanna & RV, Phys. Rev. Lett. 136, 160402 (2026)+ 2512.13809
+ work in progress with Sara Murciano



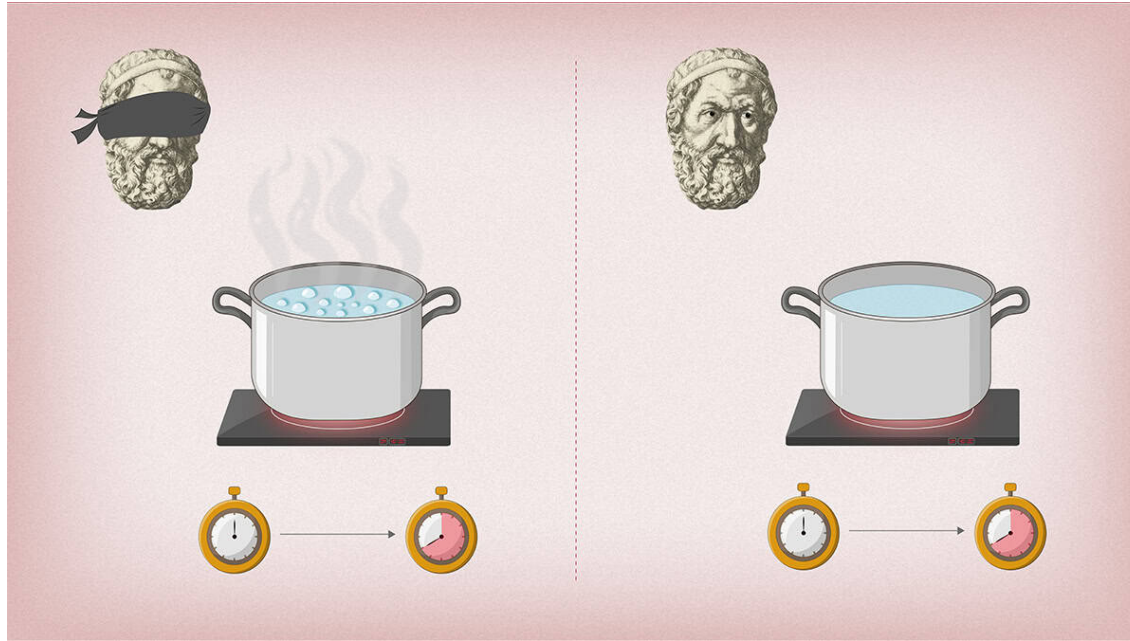
**UNIVERSITÉ
DE GENÈVE**



**Swiss National
Science Foundation**



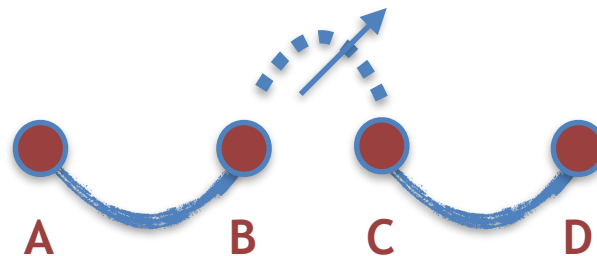
Measurements as a part of quantum dynamics



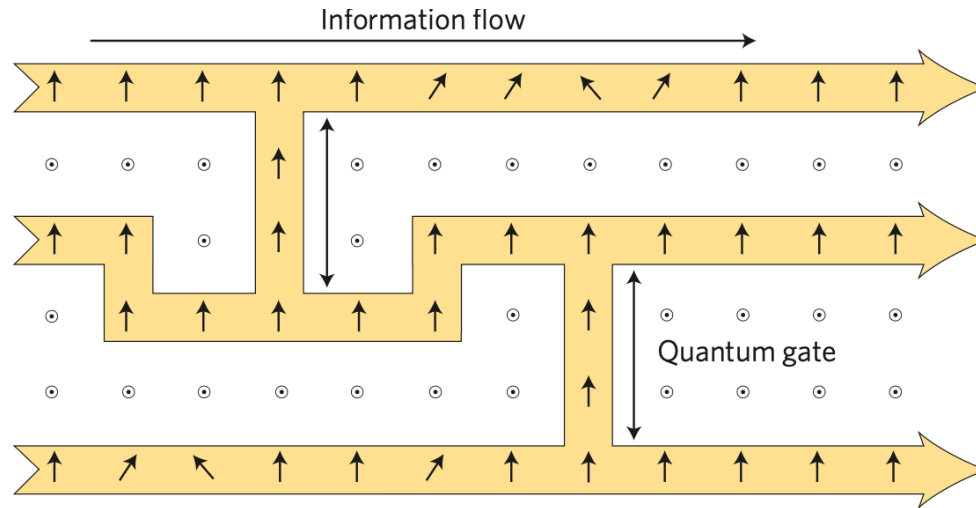
Quantum Zeno effect

Ref: phys.org

Quantum teleportation/swapping:



Measurements as a part of quantum dynamics



nature
physics

PROGRESS ARTICLE

PUBLISHED ONLINE: 2 JANUARY 2009 | DOI: 10.1038/NPHYS1157

Measurement-based quantum computation

H. J. Briegel^{1,2*}, D. E. Browne³, W. Dür^{1,2}, R. Raussendorf⁴ and M. Van den Nest^{2,5}

Measurement-based quantum computation

State preparation:

Article

Non-Abelian topological order and anyons on a trapped-ion processor

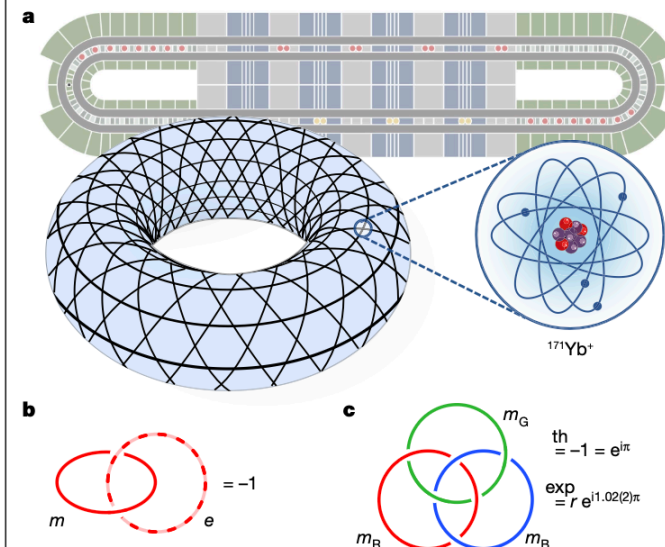
<https://doi.org/10.1038/s41586-023-06934-4>

Received: 23 April 2023

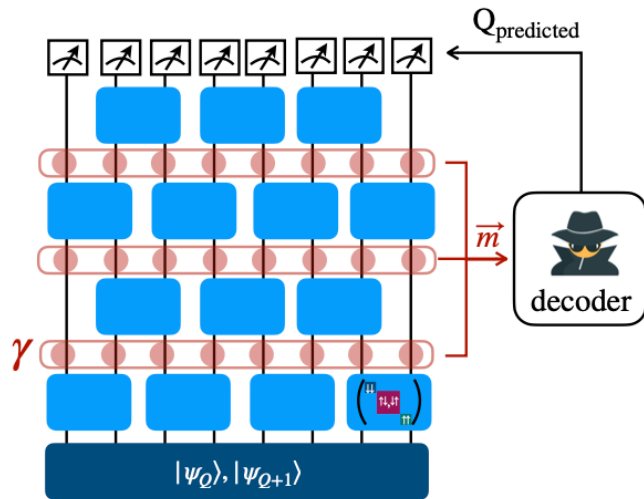
Accepted: 4 December 2023

Published online: 14 February 2024

Mohsin Iqbal^{1,5}, Nathanan Tantivasadakarn^{2,5}, Ruben Verresen^{3,5}, Sara L. Campbell⁴, Joan M. Dreiling⁴, Caroline Figgatt⁴, John P. Gaebler⁴, Jacob Johansen⁴, Michael Mills⁴, Steven A. Moses⁴, Juan M. Pino⁴, Anthony Ransford⁴, Mary Rowe⁴, Peter Siegfried⁴, Russell P. Stutz⁴, Michael Foss-Feig⁴, Ashvin Vishwanath³ & Henrik Dreyer^{1,5*}

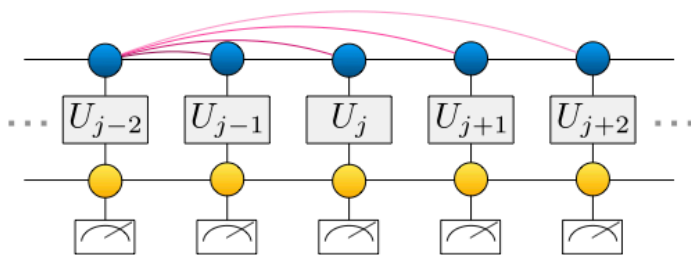


Measurement-induced phenomena



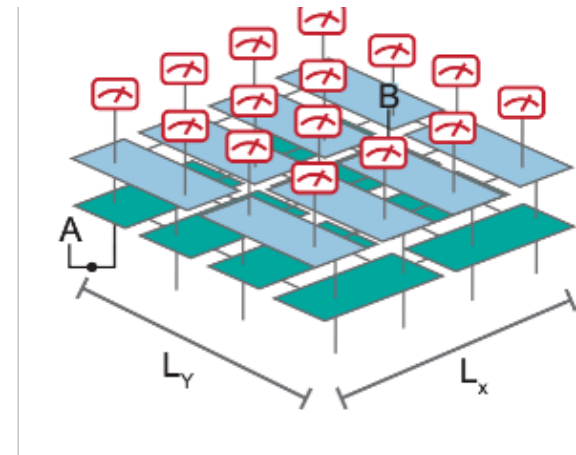
measurement-induced transitions

Skinner, Ruhman & Nahum '19
Li, Chen, Fisher '19



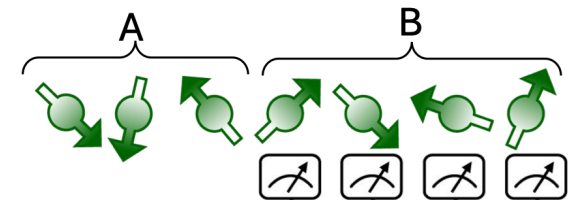
Measurement-altered criticality

Garratt, Weinstein & Altman '23, Murciano, Sala, Liu,
Mong, Alicea '23 + many others



Teleportation transitions

Bao, Block & Altman '24



$|\psi_{000}\rangle, |\psi_{001}\rangle, |\psi_{010}\rangle, \dots$

$$\mathcal{E}_{\text{proj}} = \{p_z, |\psi_z\rangle\}$$

Projected ensembles, deep thermalization

Ho, Choi '22, Ippoliti, Ho '23, ...

Measurement-altered criticality

Measurements Conspire Nonlocally to Restructure Critical Quantum States

Samuel J. Garratt¹, Zack Weinstein¹, and Ehud Altman^{1,2}

¹Department of Physics, University of California, Berkeley, California 94720, USA

²Materials Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA



(Received 6 September 2022; revised 7 February 2023; accepted 28 February 2023; published 18 May 2023; corrected 17 April 2024)

Measurement-Altered Ising Quantum Criticality

Sara Murciano^{1,2,*}, Pablo Sala^{1,2,*}, Yue Liu¹, Roger S. K. Mong³, and Jason Alicea^{1,2}

¹Department of Physics and Institute for Quantum Information and Matter, California Institute of Technology, Pasadena, California 91125, USA

²Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, California 91125, USA

³Department of Physics and Astronomy and Pittsburgh Quantum Institute, University of Pittsburgh, Pittsburgh, Pennsylvania 15260, USA

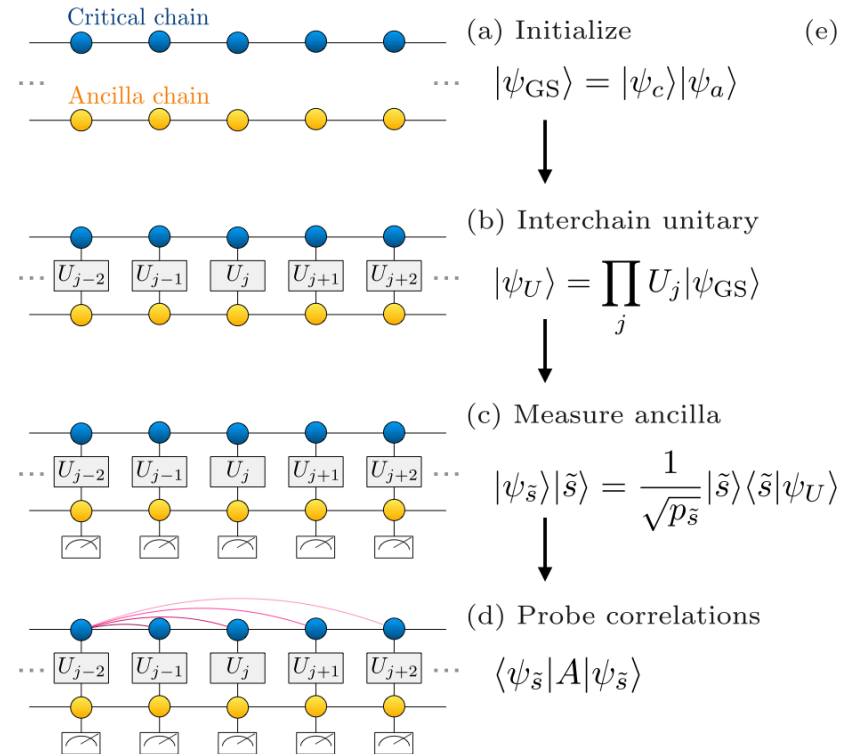


(Received 23 February 2023; accepted 20 October 2023; published 5 December 2023)

Entanglement in a one-dimensional critical state after measurements

Zhou Yang, Dan Mao, and Chao-Ming Jian

Show more



Measurement-induced/Localizable entanglement

$$\text{MIE}(A) = \sum_{\mathbf{m}} p_{\mathbf{m}} S_{\mathbf{m}}(A)$$

Lin, Ye, Zou, Sang and Hsieh '23.

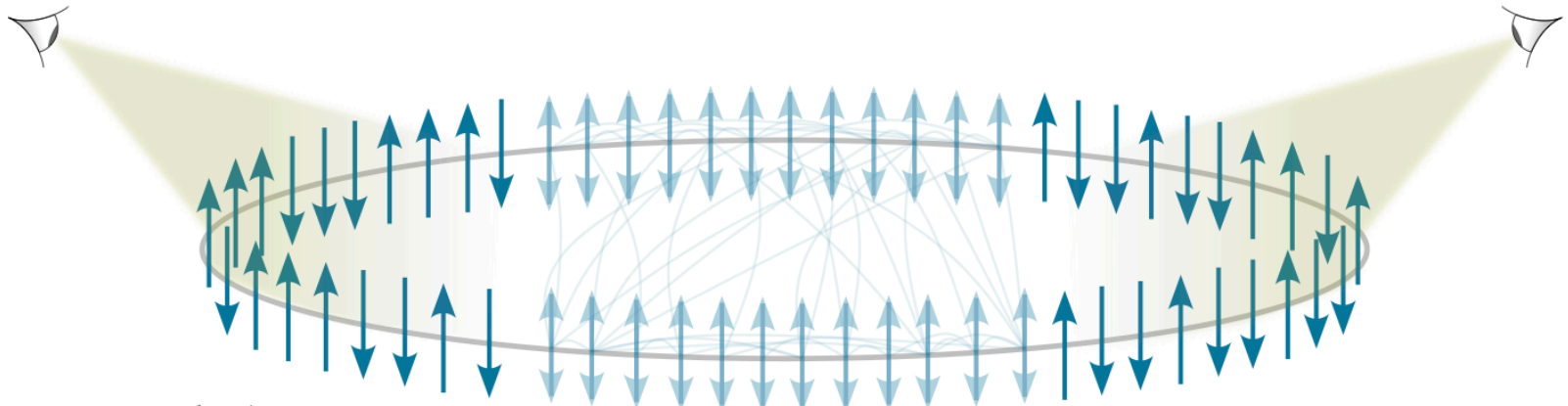
- Entanglement induced by measuring some regions. Also known as “localizable entanglement” if optimized over basis

Popp, Verstraete, Martin-Delgado, Cirac '04,
Verstraete, Popp, Cirac '04,

- Related to teleportation, MBQC, sign problems, strange correlators, projected ensembles ...

- Can also be thought of as CMI of system + measurement channel with register

Zhang & Gopalakrishnan '24



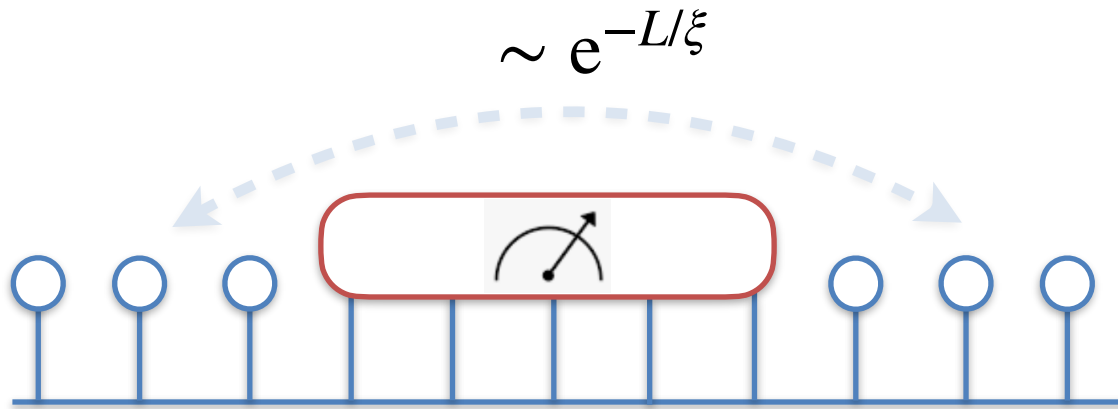
MIE in gapped phases

- For 1d positive states, decays exponentially

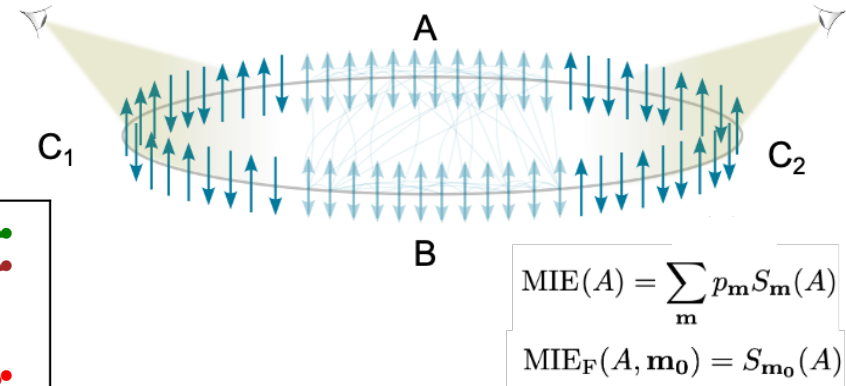
Hastings '15

- For SPT states (say cluster state): long-range MIE = $\log D$
In symmetric basis

Else, Schwarz, Bartlett & Doherty '12



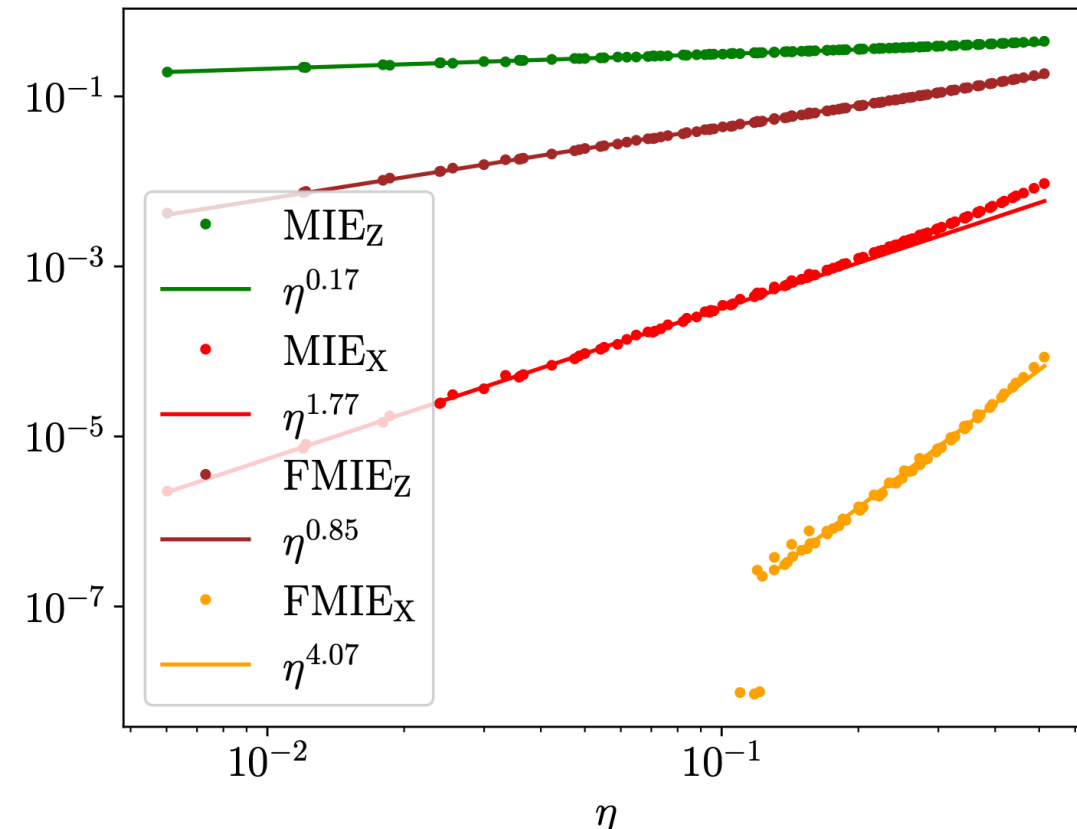
MIE in quantum critical states: numerics



**New exponents
for Ising model??**

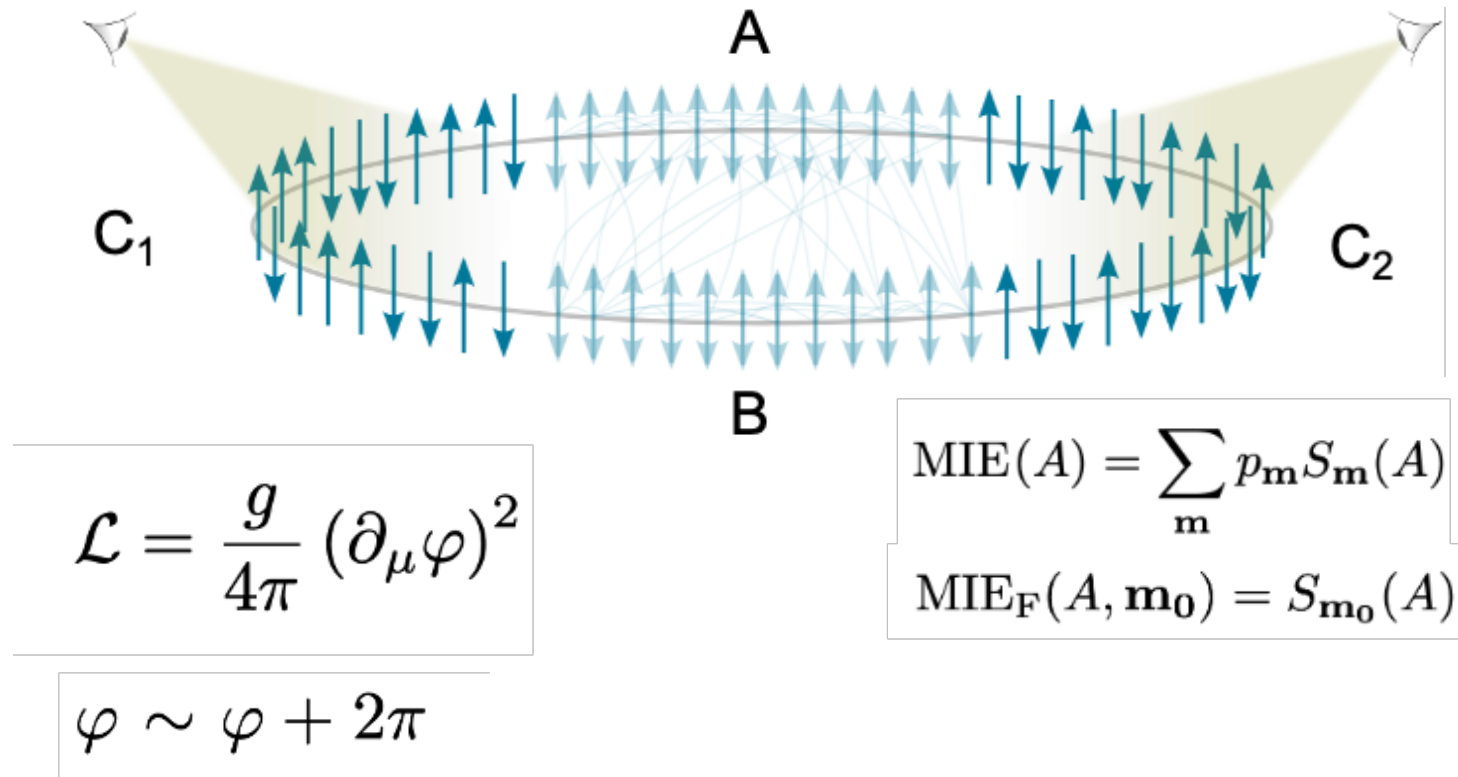
Universality?

Lin, Ye, Zou, Sang and Hsieh '23.



Found evidence of long-range, conformally invariant MIE in CFTs. New exponents?

MIE in Luttinger liquids: setup



- Measuring charge

$$\hat{n}(x) \simeq n_0 + \frac{1}{2\pi} \partial_x \varphi + A \cos(\varphi(x) + 2\pi n_0 x) + \dots$$

Overview

- Replica trick

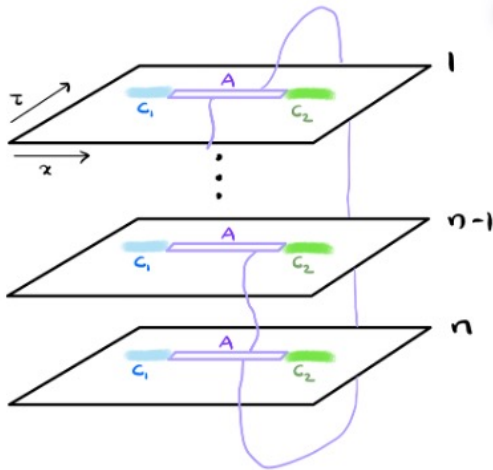
$$S_n = \frac{1}{n-1} \lim_{k \rightarrow 0} \frac{\partial}{\partial k} (F_A - F_0)$$

$$Z_0 = \sum_{\mathbf{m}} p_{\mathbf{m}}^{nk+1}$$

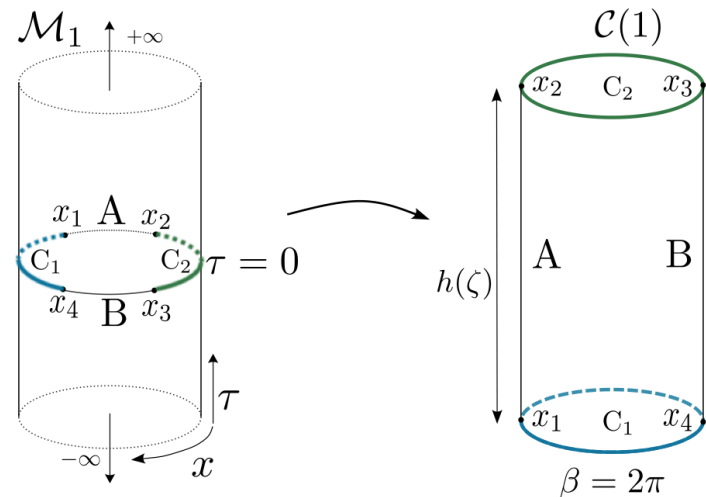
$$Z_A = \sum_{\mathbf{m}} p_{\mathbf{m}} (\text{tr} \rho_{\mathbf{m},A}^n)^k$$

- Path integral representation + conformal map

Cardy & Calabrese '04



Rajabpour et al '15, '16



- Quantum + classical decomposition

$$\varphi = \varphi_{\text{cl}} + \delta\varphi$$

$$\begin{aligned} \nabla^2 \varphi_{\text{cl}} &= 0 \\ \varphi_{\text{cl}}|_{\partial} &= m(x) \end{aligned}$$

- Sum over topological sectors/relative windings + measurement outcomes

Related to: Fradkin Moore '06 Oshikawa '10, Zaletel, Bardarson, Moore '11, Zhou, Chen, Faulkner, Fradkin '16 & related works by Stéphan, Misguish & Pasquier '11, '14

Replica trick

Randomness from measurement outcomes: $S_n = \frac{1}{1-n} \log \text{tr} \rho_{A,m}^n$

$$\log \text{tr} \rho_A^n = \lim_{k \rightarrow 0} \frac{\partial}{\partial k} (\text{tr} \rho_A^n)^k$$

Very similar to formalism used for random circuits/tensor networks, measurement-induced phase transitions:

$$S_n = \frac{1}{n-1} \lim_{k \rightarrow 0} \frac{\partial}{\partial k} (F_A - F_0)$$

RV, A.C. Potter, Y-Z. You and A.W.W. Ludwig '19
Zhou & Nahum '19

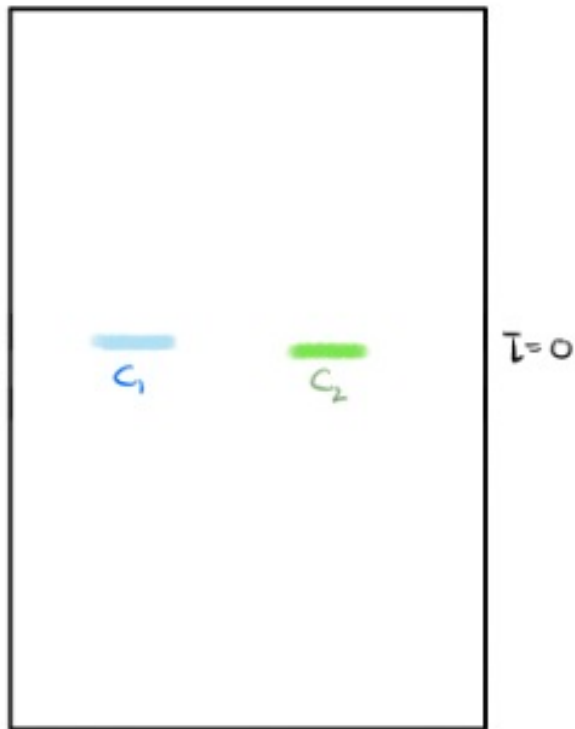
C-M. Jiang, RV, Y-Z. You and A.W.W. Ludwig '20
Bao, Choi & Altman '20

Need to compute (and analytically continue) the partition functions:

$$Z_0 = \sum_{\mathbf{m}} (\text{tr} \rho_{\mathbf{m}})^{nk+1} \quad Z_A = \sum_{\mathbf{m}} p_{\mathbf{m}} (\text{tr} \rho_{\mathbf{m},A}^n)^k$$

Path integral representation

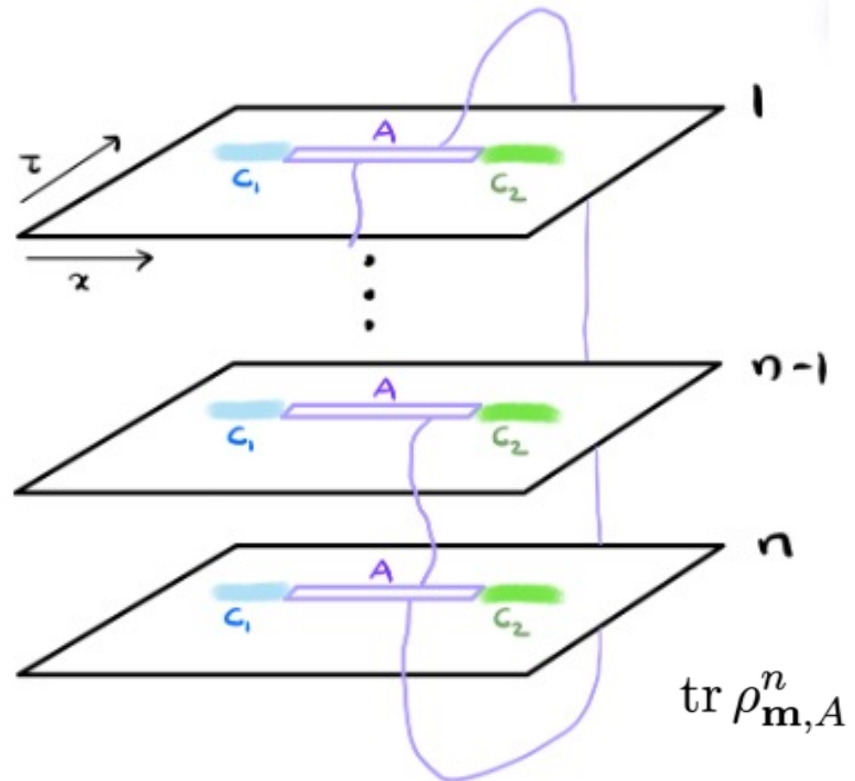
$$\text{tr } \rho_{\mathbf{m}} = Z_{\mathbf{m}} = \int \mathcal{D}[\varphi] e^{-S[\varphi]} \delta(\varphi(x, \tau = 0) - \mathbf{m}(x))$$



$nk+1$ copies of this

$\text{tr } \rho_{\mathbf{m}}$

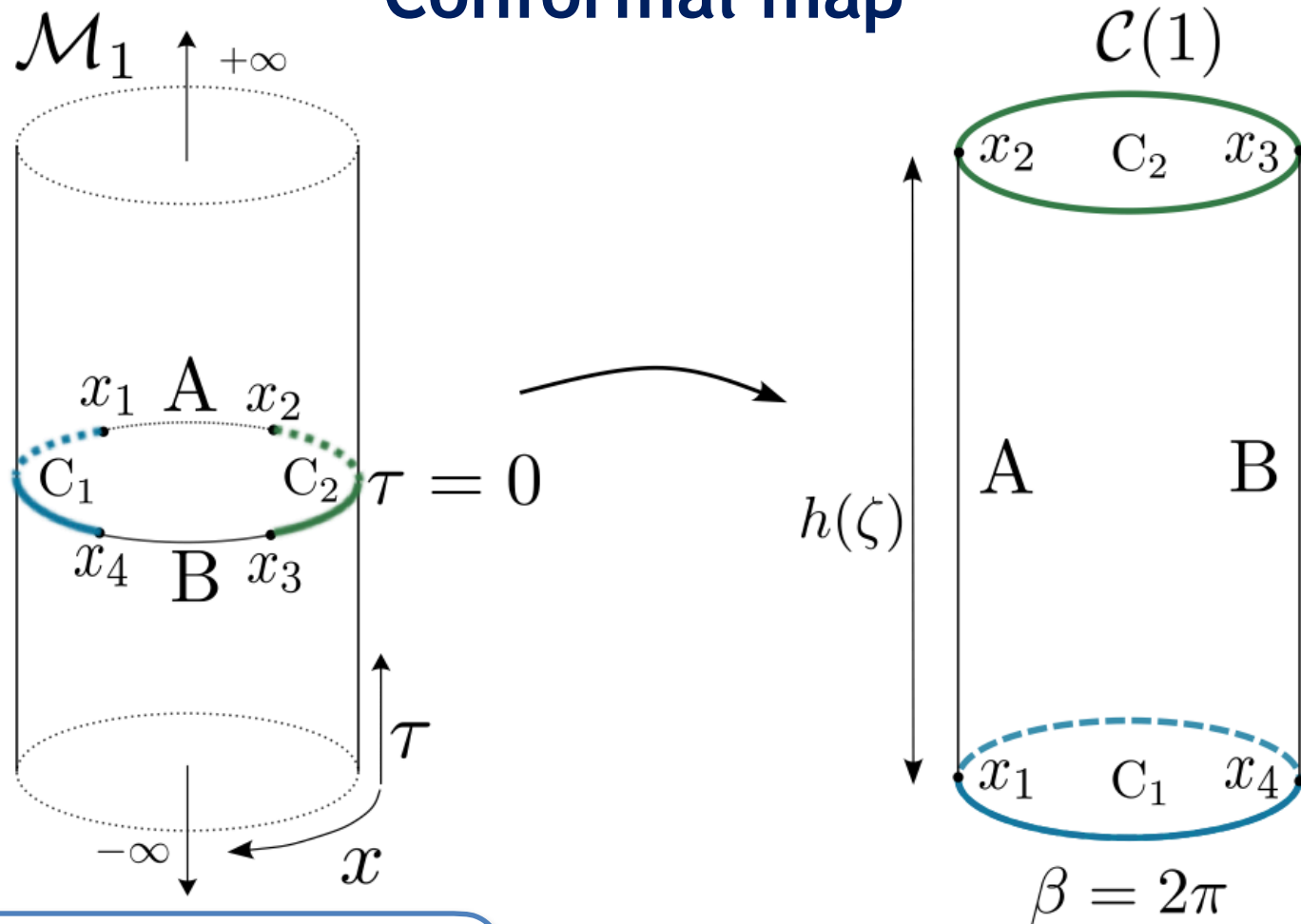
$$Z_0 = \sum_{\mathbf{m}} (\text{tr } \rho_{\mathbf{m}})^{nk+1}$$



k copies; all with *same* measurement outcome $\mathbf{m}$

$$Z_A = \sum_{\mathbf{m}} p_{\mathbf{m}} (\text{tr } \rho_{\mathbf{m},A}^n)^k$$

Conformal map



$$q_n = e^{-\pi\beta/(h/n)} = e^{-2\pi^2 n/h(\zeta)}$$

Cross ratio:

$$\zeta = \frac{\sin(\pi x_{12}/L) \sin(\pi x_{34}/L)}{\sin(\pi x_{13}/L) \sin(\pi x_{24}/L)}$$

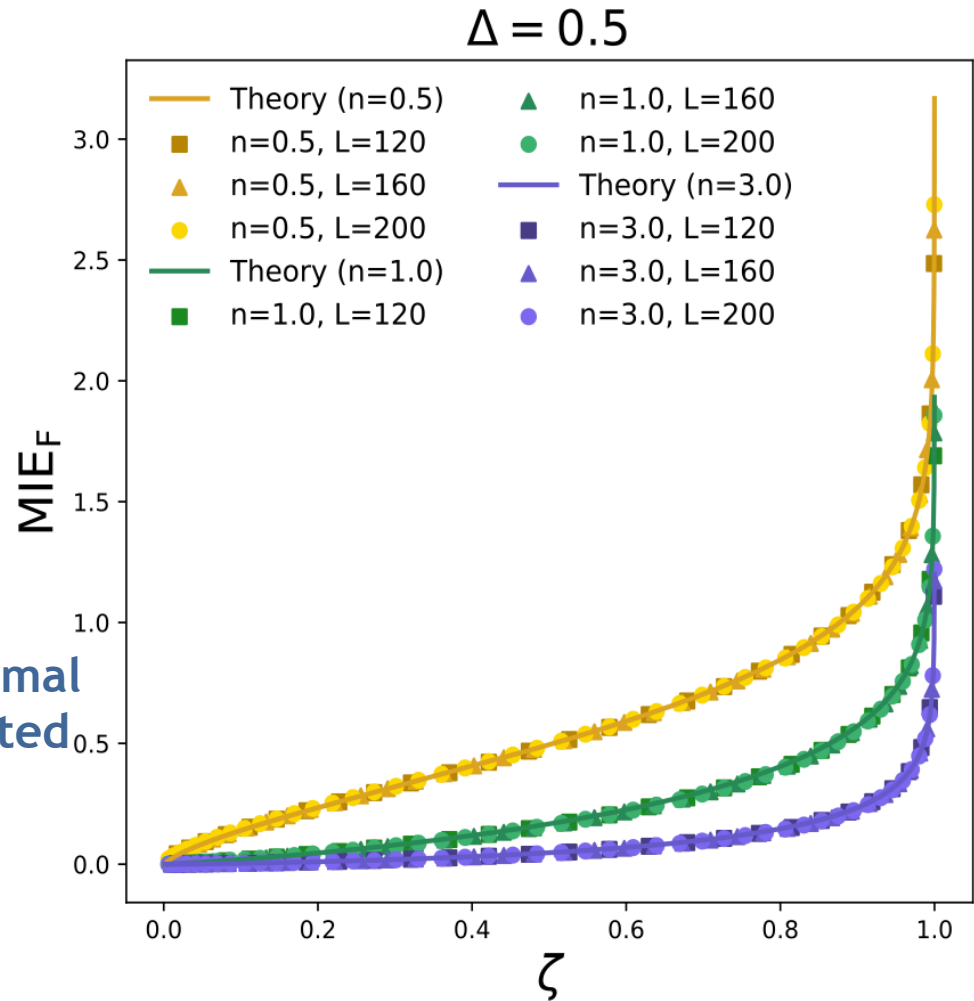
Note: Expect conformal invariance of final answer. Some specific measurement outcomes are known to be “problematic”, as they break conformal invariance...

Forced case

$$\text{MIE}_F^{(n)}(A) = \frac{1}{1-n} \log \frac{Z_{D, \mathcal{M}_n}}{Z_{D, \mathcal{M}_1}^n}$$

- Force outcomes onto Dirichlet BC
- Cylinder partition functions known exactly for compact free boson
- Previous works missed (?) the conformal invariance of this quantity. Only reported asymptotic limits

Rajabpour et al '15, '16, Antonini, Bentsen, Cao, Harper, Jian Swingle '22, ...



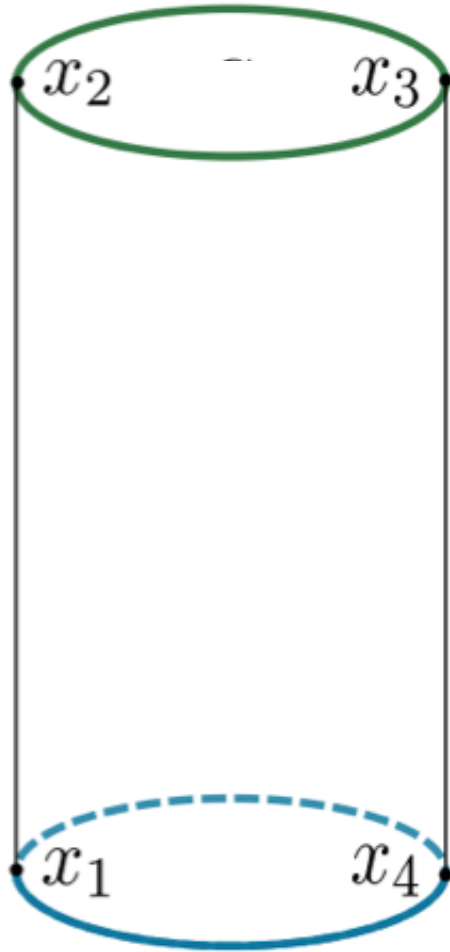
XXZ model:
$$H = \sum_j \sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \Delta \sigma_j^z \sigma_{j+1}^z$$

$$\Delta = -\cos \pi g$$

$$\Delta \in (-1, 1)$$

MIE: classical + quantum fluctuations

$$\varphi|_{\partial} = m(x)$$



- Need to deal with (random) measurement outcome
- Absorb measurements in classical solution:

$$\varphi = \varphi_{\text{cl}} + \delta\varphi$$

$$\begin{aligned}\nabla^2 \varphi_{\text{cl}} &= 0 \\ \varphi_{\text{cl}}|_{\partial} &= m(x)\end{aligned}$$

- Using the quadratic nature of the action, we loosely have:

$$Z_m = e^{-S[\varphi_{\text{cl}}]} Z_D$$

- BUT: we have to sum over topological sectors (compact field)

Winding contribution

Need to evaluate a sum over winding sectors: $\mathcal{W}_{n,k} := \sum_{\vec{w} \in \mathbb{Z}^k} \exp \left[- \sum_{i=1}^k S_{\mathcal{C}(n)} [\bar{\varphi}_{\text{cl}}^{(i)}] \right]$

Action of classical solution satisfying $\{m\}$ at boundaries

Borrow “trick” from Fradkin & Moore (in context of 2+1d QC points):

Entanglement entropy of 2D conformal quantum critical points:
hearing the shape of a quantum drum

Eduardo Fradkin¹ and Joel E. Moore^{2,3}

Subtlety related to compactification: Oshikawa '10, Zaletel, Bardarson, Moore '11, Zhou, Chen, Faulkner, Fradkin '16 & related works by Stéphan, Misguish & Pasquier '11, '14

Use relative coordinates that do not see measurements $(\phi_{i+1} - \phi_i)/\sqrt{2}$

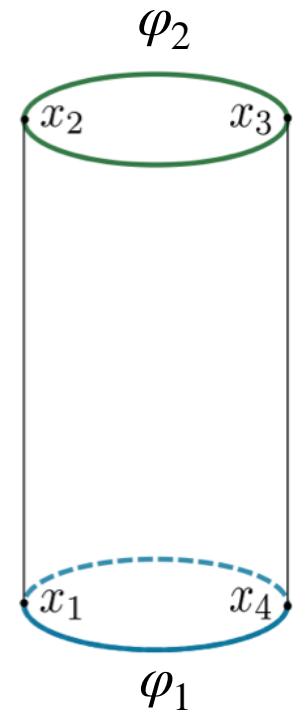
... and “center of mass”, which can be simplified using $\sum_m Z_m = Z$

Find:

$$\mathcal{W}_{n,k}(\zeta, g) = \sqrt{\frac{Qg}{2\pi h}} \int_{-\infty}^{\infty} d\delta_{\varphi} e^{-\frac{g\delta_{\varphi}^2}{2h}} \left[\sum_{w \in \mathbb{Z}} q_n^g \left(w + \frac{\delta_{\varphi}}{2\pi} \right)^2 \right]^k$$

Final result and asymptotics

- Closed form analytic answer!
- Natural BCFT interpretation in terms of Born averaging over Dirichlet boundary conditions
- Small cross-ratio dominated by winding contribution:

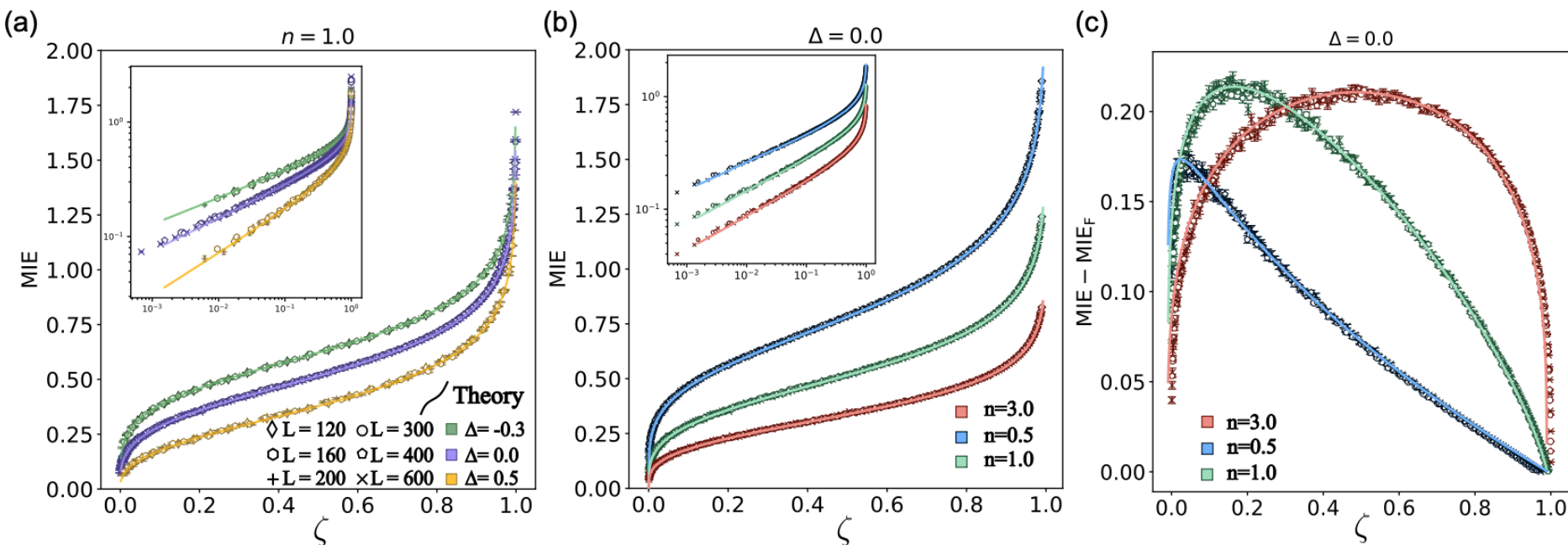


$$\text{MIE}^{(n)}(A) \underset{\zeta \rightarrow 0}{\sim} \begin{cases} \frac{\zeta^{g/2}}{\sqrt{\log(1/\zeta)}} & n > \frac{1}{2}, \\ \zeta^{g/2} & n = \frac{1}{2}, \\ \zeta^{2gn(1-n)} & 0 < n < \frac{1}{2}, \end{cases}$$

$$\text{MIE}_F^{(n)} \underset{\zeta \rightarrow 0}{\sim} \begin{cases} \zeta^{2g} & n > 1, \\ \zeta^{2g} \log \zeta & n = 1, \\ \zeta^{2ng} & n < 1, \end{cases}$$

- Logarithms everywhere, makes extracting exponents numerically tricky...
- Different universality for the “real MIE”. Qualitatively very different, **larger** that pre-existing mutual information only for “real” measurements!

Numerics



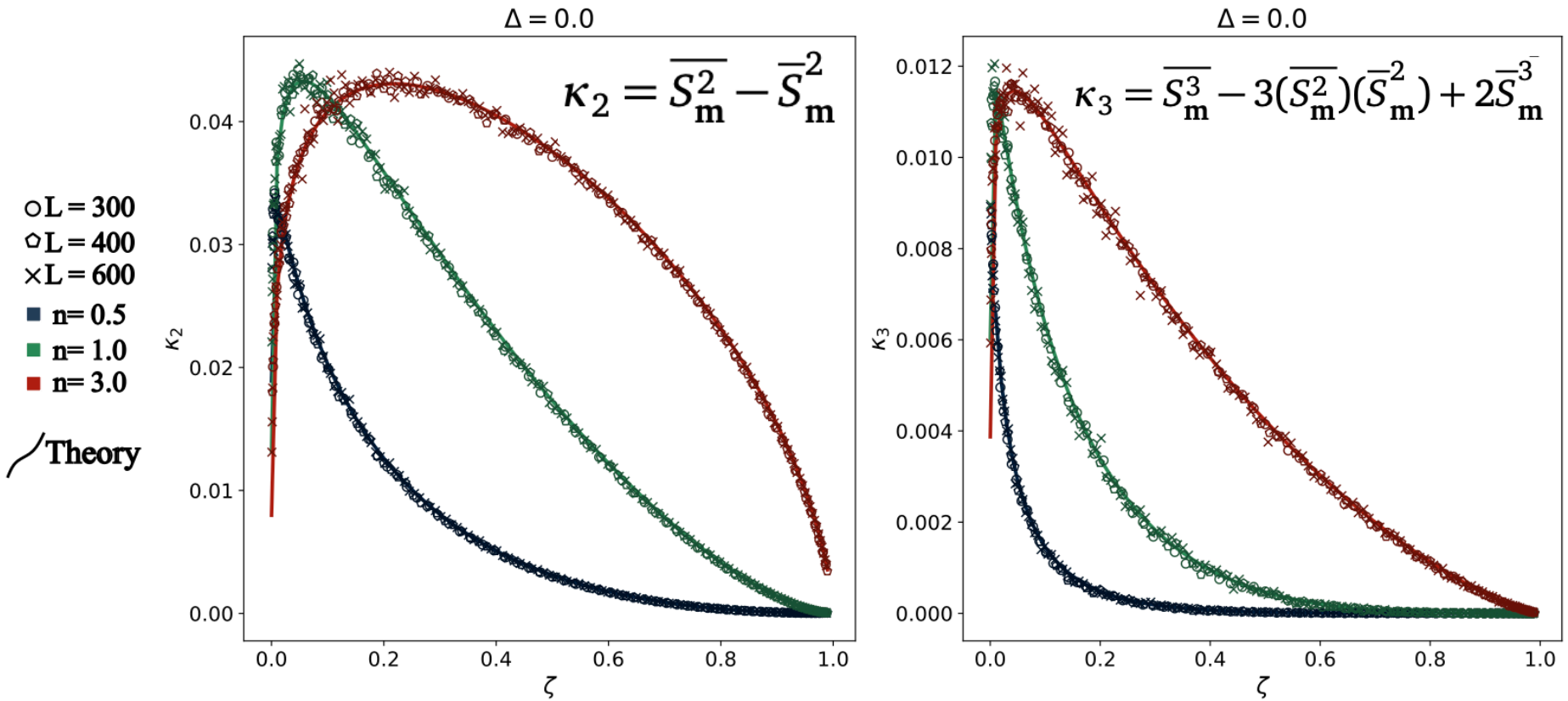
XXZ model:
$$H = \sum_j \sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \Delta \sigma_j^z \sigma_{j+1}^z$$

$$\Delta = -\cos \pi g$$

$$\Delta \in (-1, 1)$$

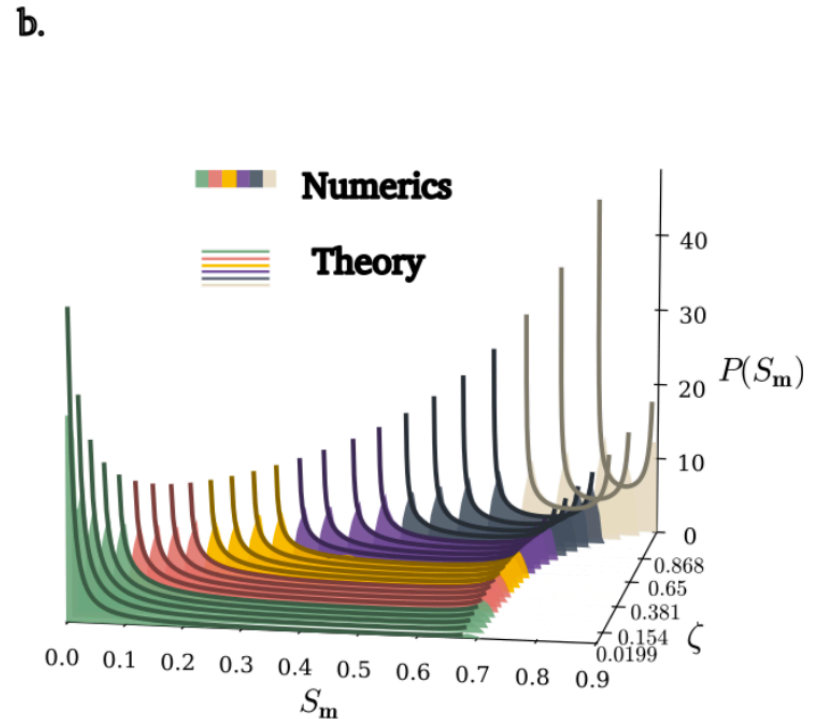
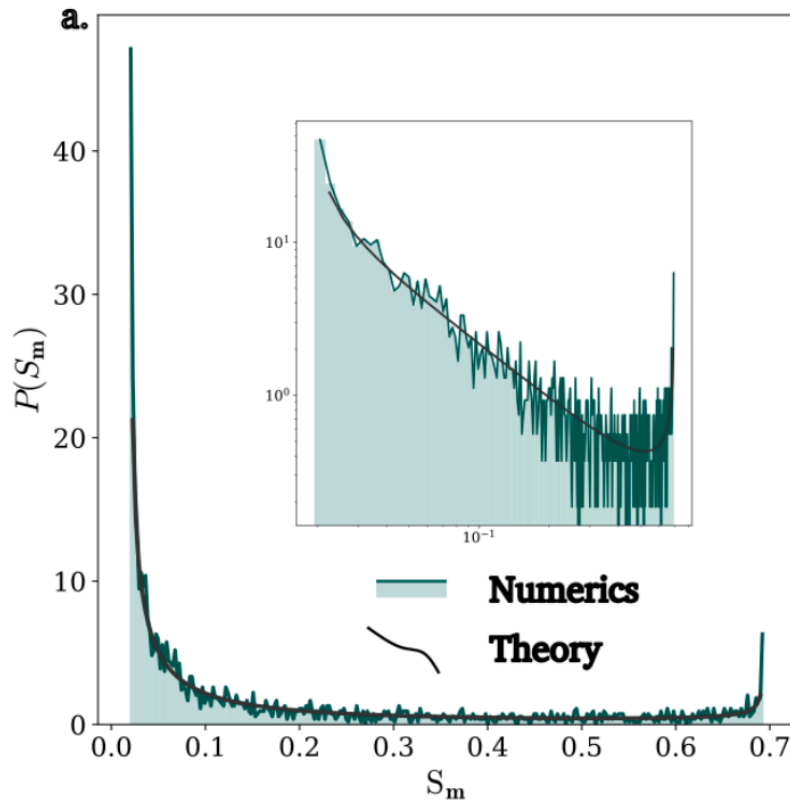
Note: even the case of entanglement of two disjoint intervals in the free boson CFT doesn't have a closed form solution for any cross ratio! Calabrese, Cardy & Tonni '09

Full distribution of MIE



- Can access higher cumulants by computing: $\mathcal{Z}_A(k_1, k_2) = (\text{tr} \rho_{m,A}^n)^{k_1} \overline{\text{tr} \rho_m^{k_2}}$
- Broad distribution distribution at small cross ratio.
Peak at $\log 2$ (maximum “quantum wire” behavior)

Full distribution of MIE

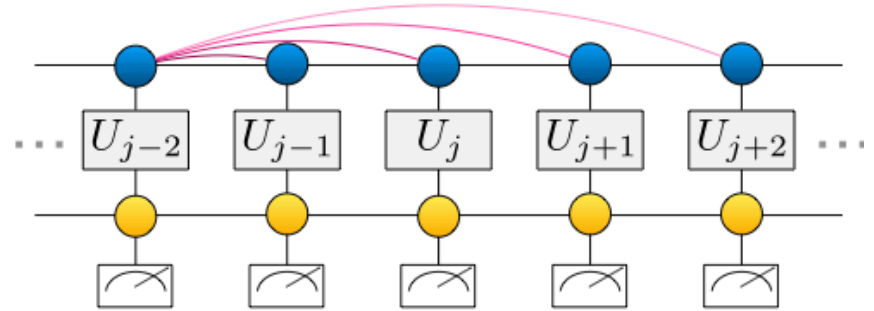


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- Broad distribution at small cross ratio.
Peak at $\log 2$ (maximum “quantum wire” behavior)

Measurement altered criticality

Measurement-altered criticality

Garratt, Weinstein & Altman '23, Murciano, Sala, Liu,
Mong, Alicea '23 + many others



- New interesting critical phenomena from weakly monitoring ground states
- Very scarce analytical results, with notable exception of Patil, Ludwig '24 (found monitored fixed points perturbatively close to clean limit)
- Exact analytical results for monitored Luttinger liquids!

Same setup as:
Garratt, Weinstein & Altman '23,

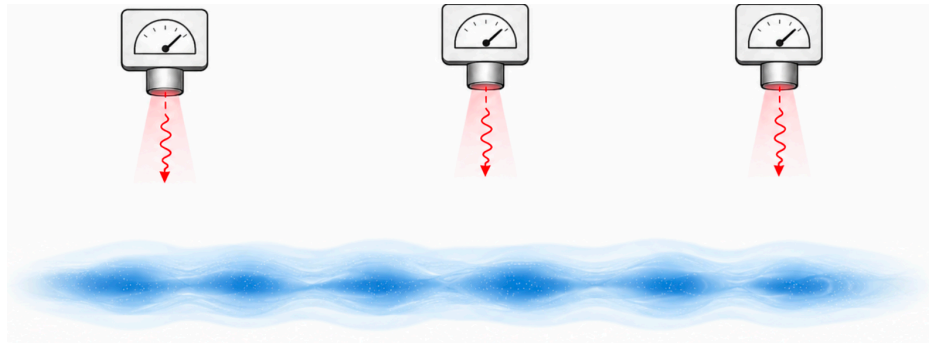


Sara Murciano
(CNRS)



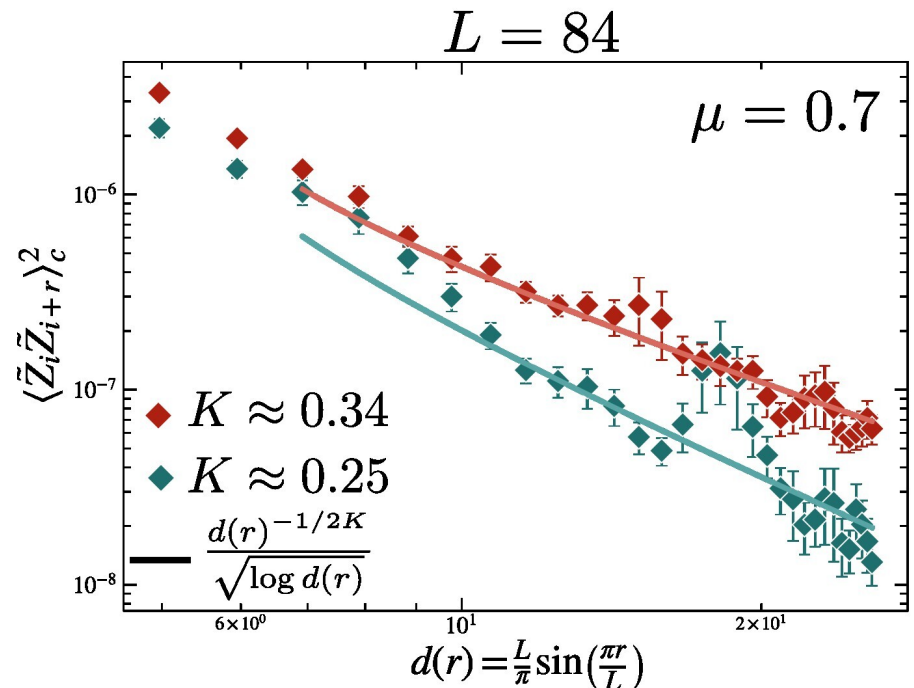
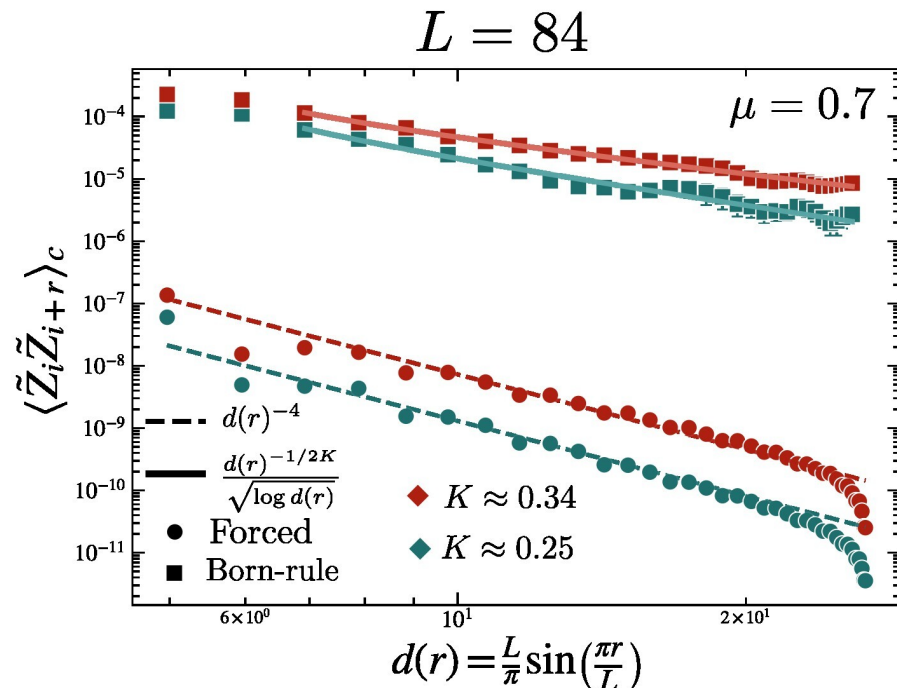
Kabir Khanna
(Geneva)

“Strong-measurement” altered criticality



$$\mathcal{N}_k(x) = \sum_m p_m [\langle \hat{n}(x) \hat{n}(0) \rangle_m - \langle \hat{n}(x) \rangle_m \langle \hat{n}(0) \rangle_m]^k$$

$$\Theta_k(x) = \sum_m p_m \langle e^{i[\theta(x) - \theta(0)]} \rangle_m^k,$$



work in progress with Sara Murciano & Kabir Khanna

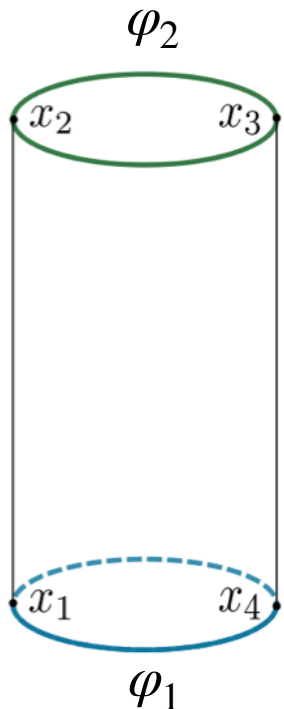
Born averaging over conformal boundary conditions

- Analytic solution but technique specific to compact free boson.



- ... BUT, the final answer has a very appealing BCFT-friendly form:

$$\text{MIE}^{(n)} = \int_0^{2\pi} P(\delta_\varphi) \text{MIE}_F^{(n)}(\delta_\varphi)$$



Forced MIE with uniform outcomes φ_1 and φ_2

$$\delta_\varphi = \varphi_2 - \varphi_1$$

- Suggests that (almost) all measurement outcomes flow to “nice” uniform Dirichlet boundary conditions

Conclusion

- **Exact** replica trick calculation of **MIE for Luttinger liquids (aka compact boson CFT)**
- Promising angle to deal with **measurements in CFT** and field theory?
- Ising CFT? Other observables? Measurements = average over boundary conditions? BCFT interpretation? *ongoing work(s) with A. Kumar, S. Murciano, A. Ludwig, R. Patil*
- Measurement altered criticality *ongoing work with S. Murciano*

