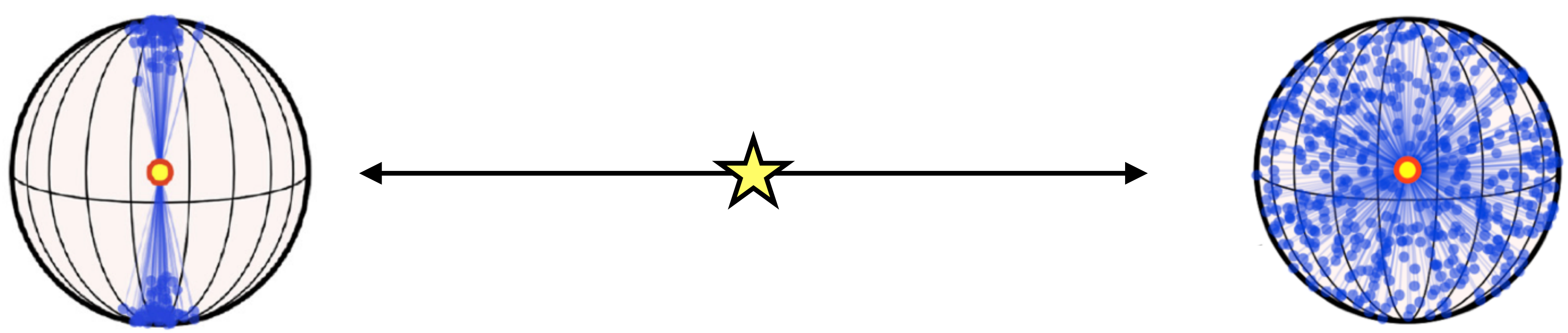
Entanglement Dynamics: Open Quantum Systems, Monitored Circuits, and Topological Order
Nordita, Stockholm, Sweden
30 June 2026

Quantum resource localizability transitions in deep thermalization

[Liu, Ippoliti, **WWH**, Phys. Rev. Lett. **136**, 100404 (2026)]
[Feng, Liu, Cheng, **WWH**, Ippoliti, arXiv:2606.08756]



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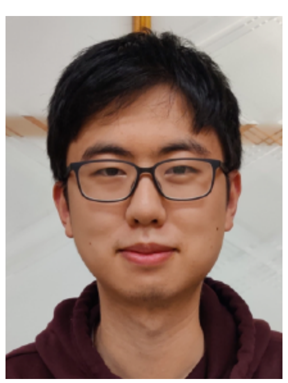

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Wen Wei Ho
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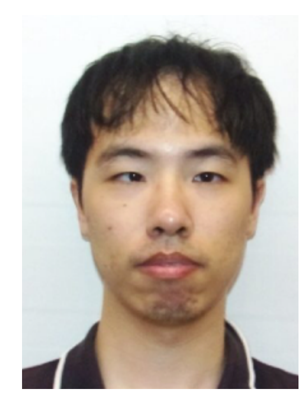

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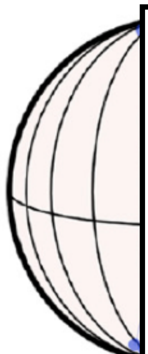
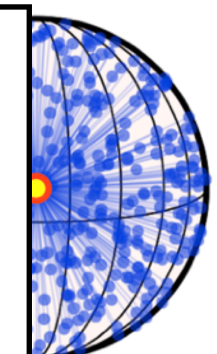


[**WWH**, S. Choi, PRL **128**, 060621 (2022)]
[Ippoliti and **WWH**, Quantum **6**, 886 (2022)]
[Ippoliti and **WWH**, PRX Q **4**, 030322 (2023)]
[Liu, Huang and **WWH**, PRL **113**, 260401 (2024)]
[Shrotriya and **WWH**, SciPost Physics **18** (2025)]
[Chang, Shrotriya, **WWH**, Ippoliti, PRX Q **6**, 020343 (2025)
[Yu, **WWH**, Kos, PRL **135** (26), 260402 (2025)]
[Mok, Haug, **WWH**, Preskill, arXiv:2601.00266]

Review on Deep Thermalization &
Hilbert space ergodicity appearing soon!
(With Daniel Mark, Manuel Endres, Matteo Ippoliti, Soonwon Choi)



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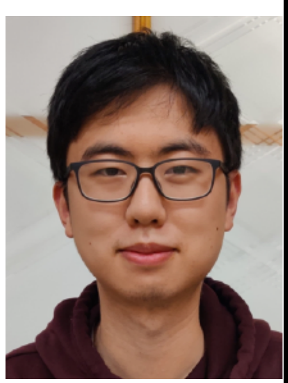

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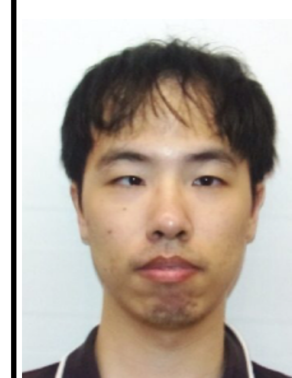

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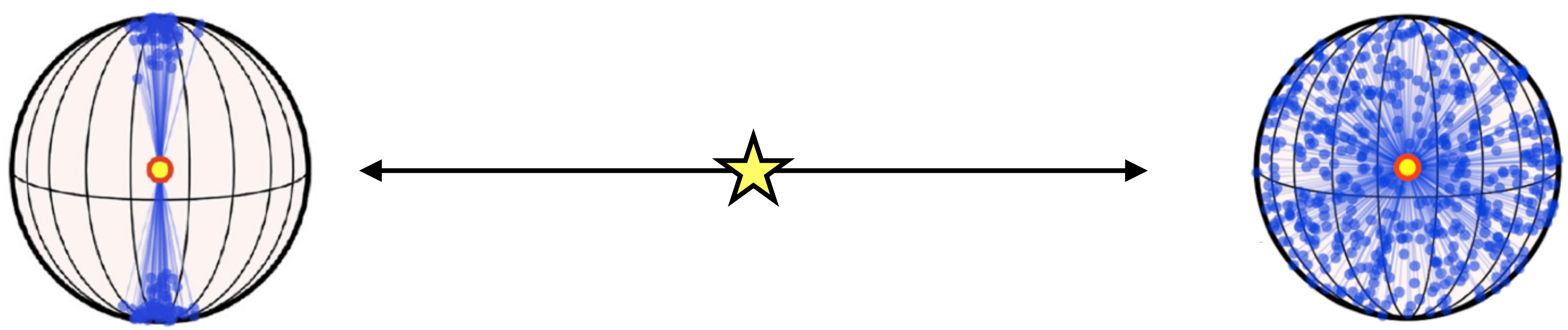

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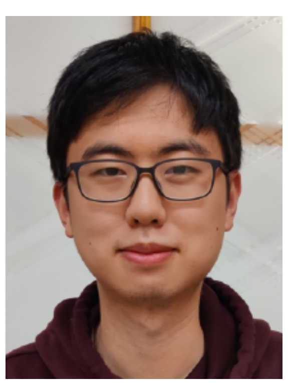

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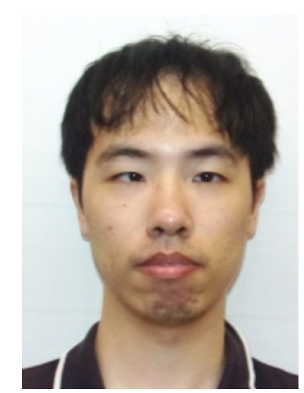

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Take-home messages

- Phase transitions in deeper form of quantum equilibration (deep thermalization), induced by measurements & quantum resources constraints in dynamics (coherence, magic, imaginarity, non-Gaussianity etc.)

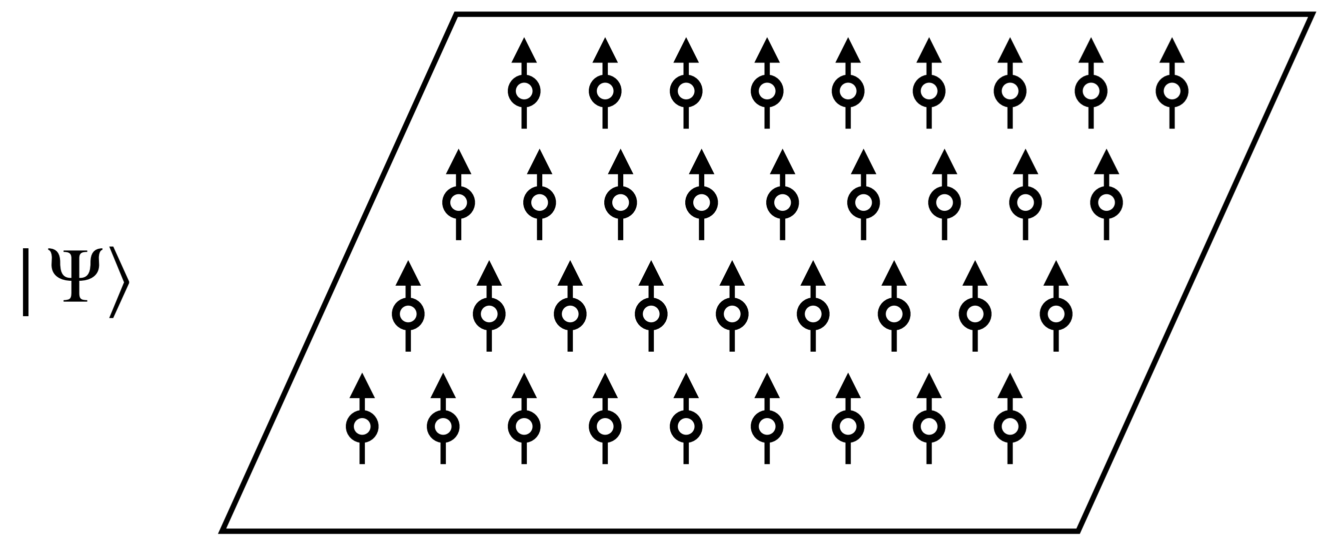
- Learnability/inference transition between a resource-fuzzy (thermalizing) phase and a resource-sharp (ergodicity-breaking) phase, mechanism: "block-sharpening"

- Classification of quantum resources into two classes: threshold localizable (existence of transition) vs smoothly localizable (no transition)

- Provides quantitative understanding of magic transitions in encoding-decoding dynamics

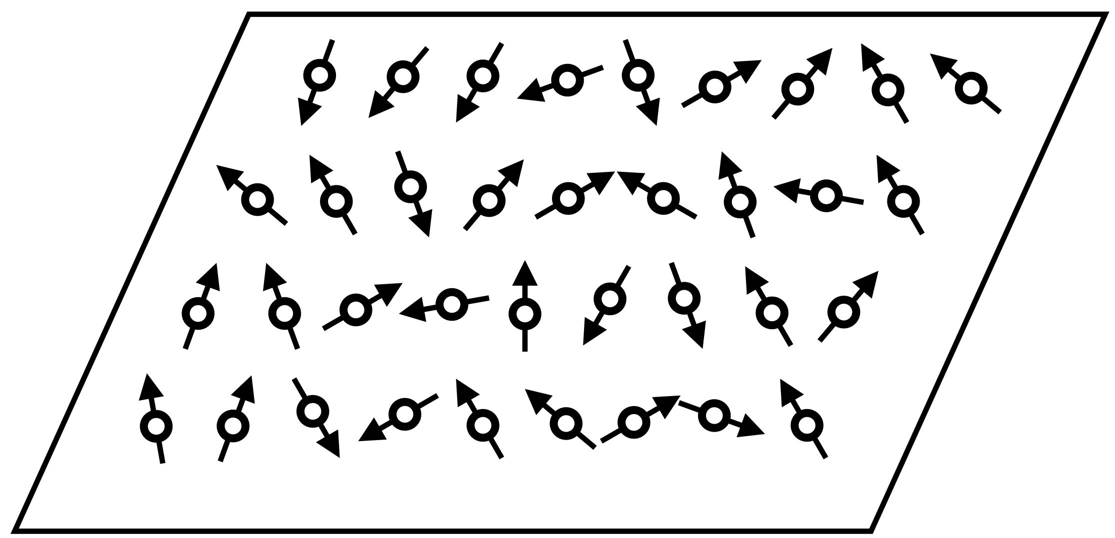
- Provides implications for quantum state certification protocols in quantum information science involving resource localization

Regular quantum thermalization



Regular quantum thermalization

$e^{-iHt}|\Psi\rangle$



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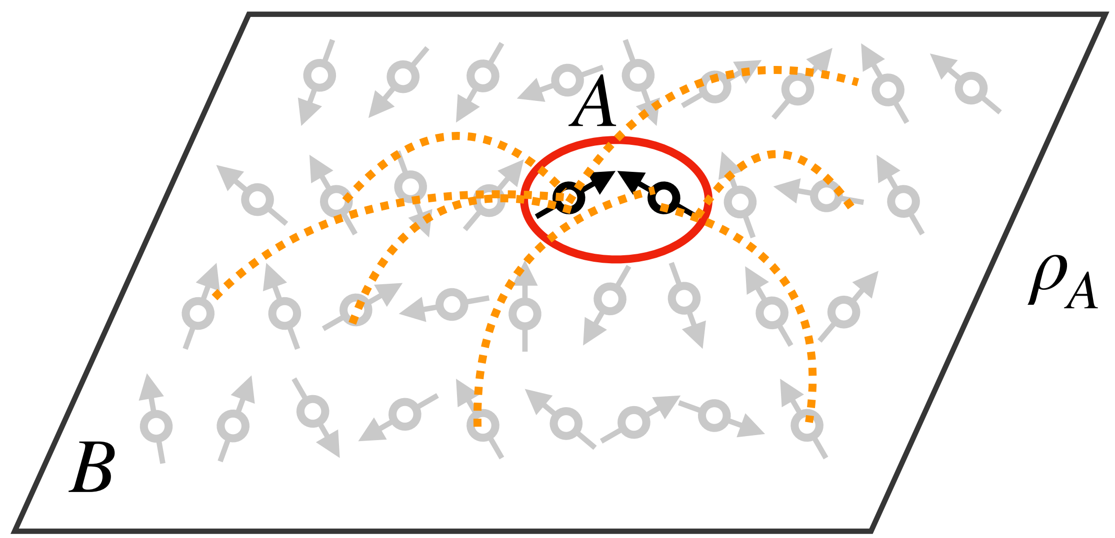
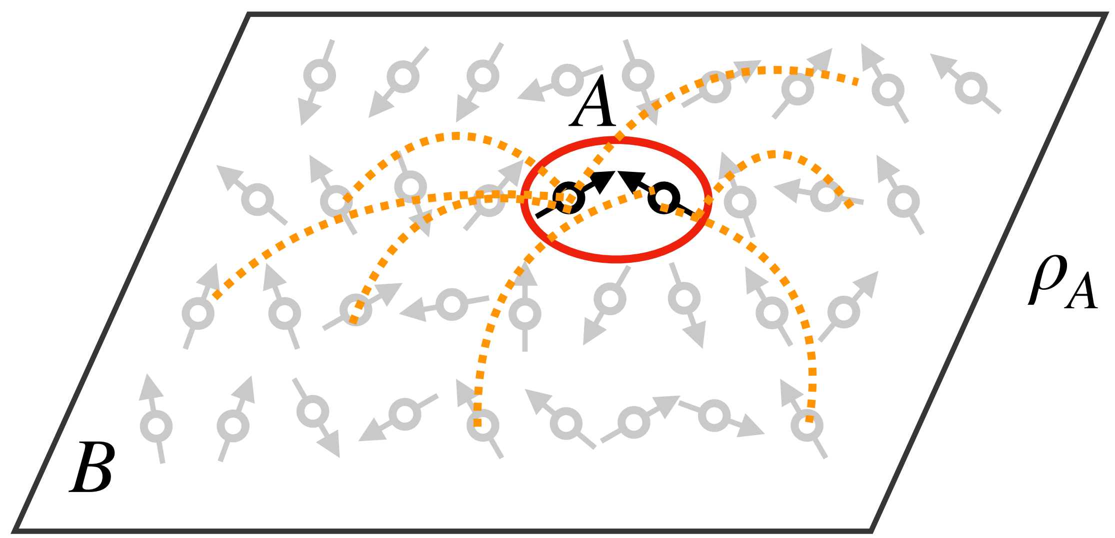


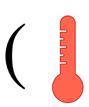
$\rho_A = \rho_{\text{Gibbs}}(\quad, \quad, \cdots)$

Universal!

Regular quantum thermalization

$$e^{-iHt}|\Psi\rangle$$



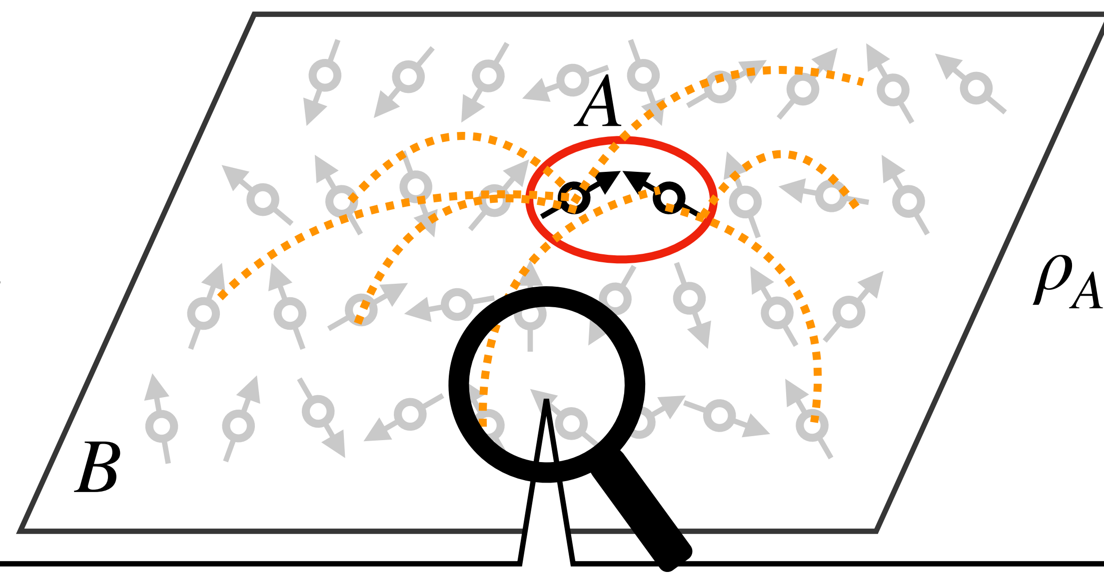
$\rho_A = \rho_{\mathrm{Gibbs}}($  $,$  $, \cdots)$

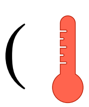
Universal!

But! This assumes we ignore the state of the bath B. What happens if we have access to more information about it?

Beyond regular thermalization

$e^{-iHt}|\Psi\rangle$



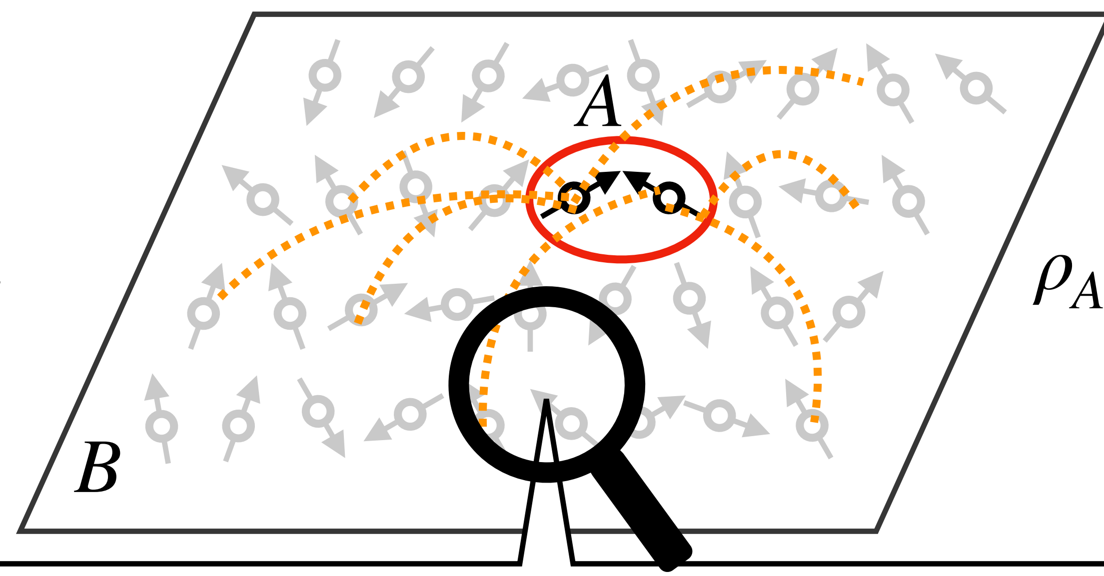
$\rho_A = \rho_{\text{Gibbs}}($  $,$  $, \cdots)$

Universal!

Bath state: $z_1 = \begin{matrix} 010101 \\ 101001 \\ 101011 \\ 010101 \end{matrix}$, $z_2 = \begin{matrix} 100010 \\ 001010 \\ 001010 \\ 101110 \end{matrix}$, $z_3 = \begin{matrix} 011110 \\ 110101 \\ 101010 \\ 111010 \end{matrix}$, $\cdots$

Beyond regular thermalization

$$e^{-iHt}|\Psi\rangle$$



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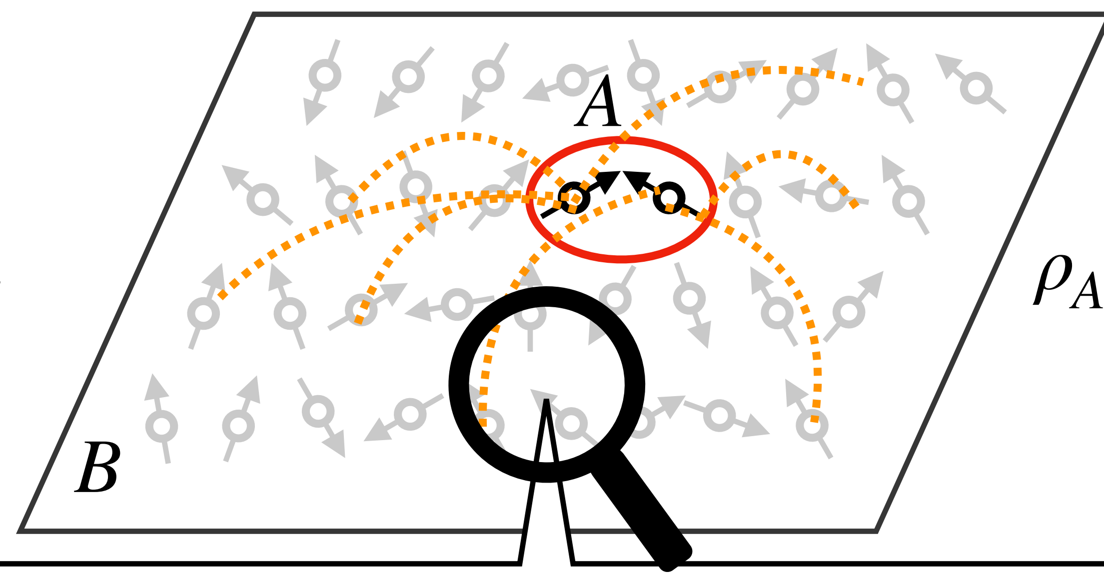
Local subsystem state: $|\psi_A(z_1)\rangle$, $|\psi_A(z_2)\rangle$, $|\psi_A(z_3)\rangle$, $\cdots$

(Conditional state)

Beyond regular thermalization

$$e^{-iHt}|\Psi\rangle$$



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(Conditional state)

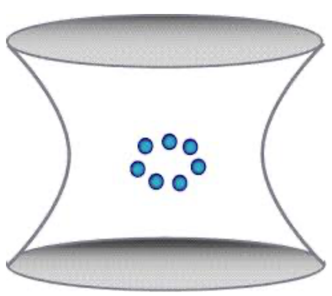
*Is there universality within the **ensemble** of quantum states (not just its mean)?*
If so, what are physical principles underlying emergence?

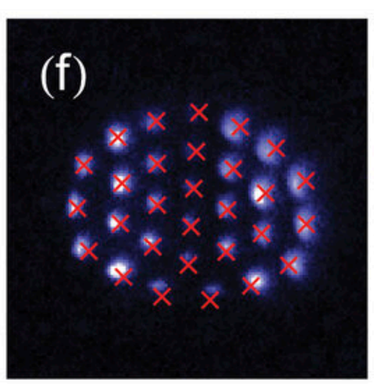
Why study this?

Measurement capabilities of modern quantum simulators yield fine-grained information $\Longrightarrow$ access to new physics!

Trapped ions

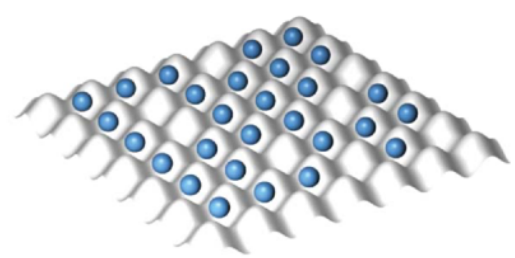
[Duke, Innsbruck, Honeywell…]

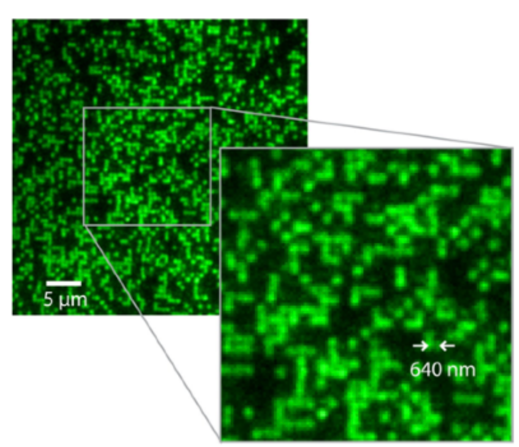




Cold atoms
(Quantum gas microscopes)

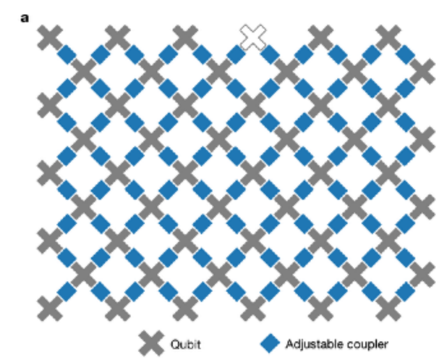
[Munich, Harvard, MIT…]

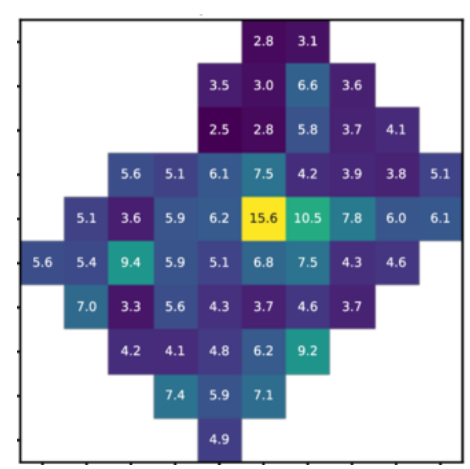




Superconducting qubits

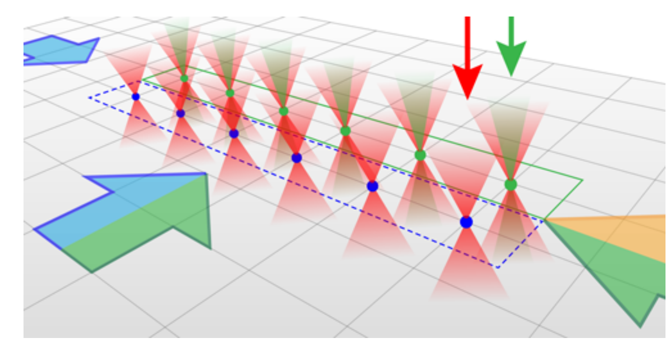
[Google, IBM,…]

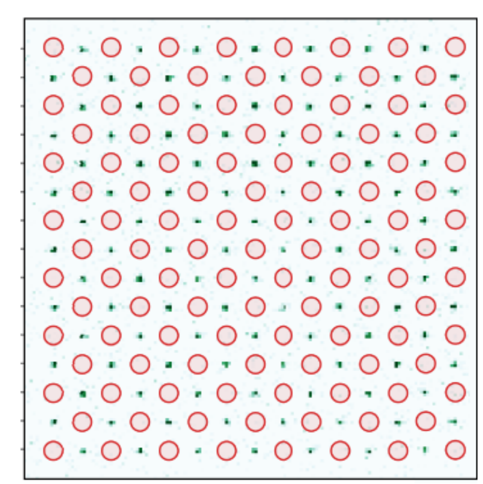




Rydberg atom arrays

[Harvard, Caltech, Paris,…]





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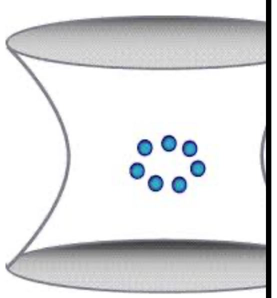
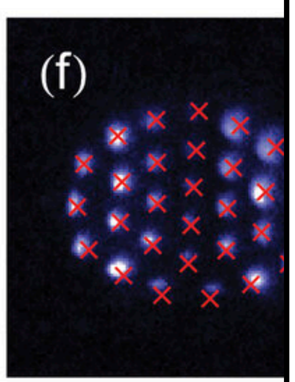
Measurement capabilities of modern quantum simulators yield fine-grained information $\implies$ access to new physics!

Trapped ior

[Duke, Innsbruck, Hor



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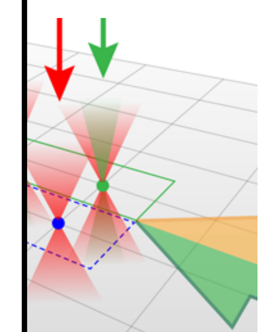

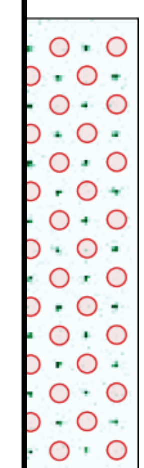

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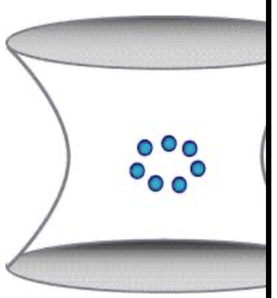

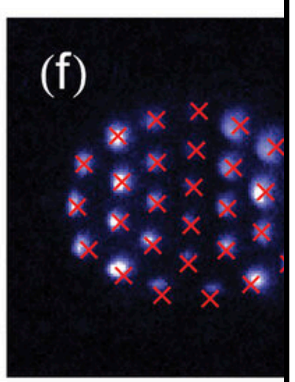

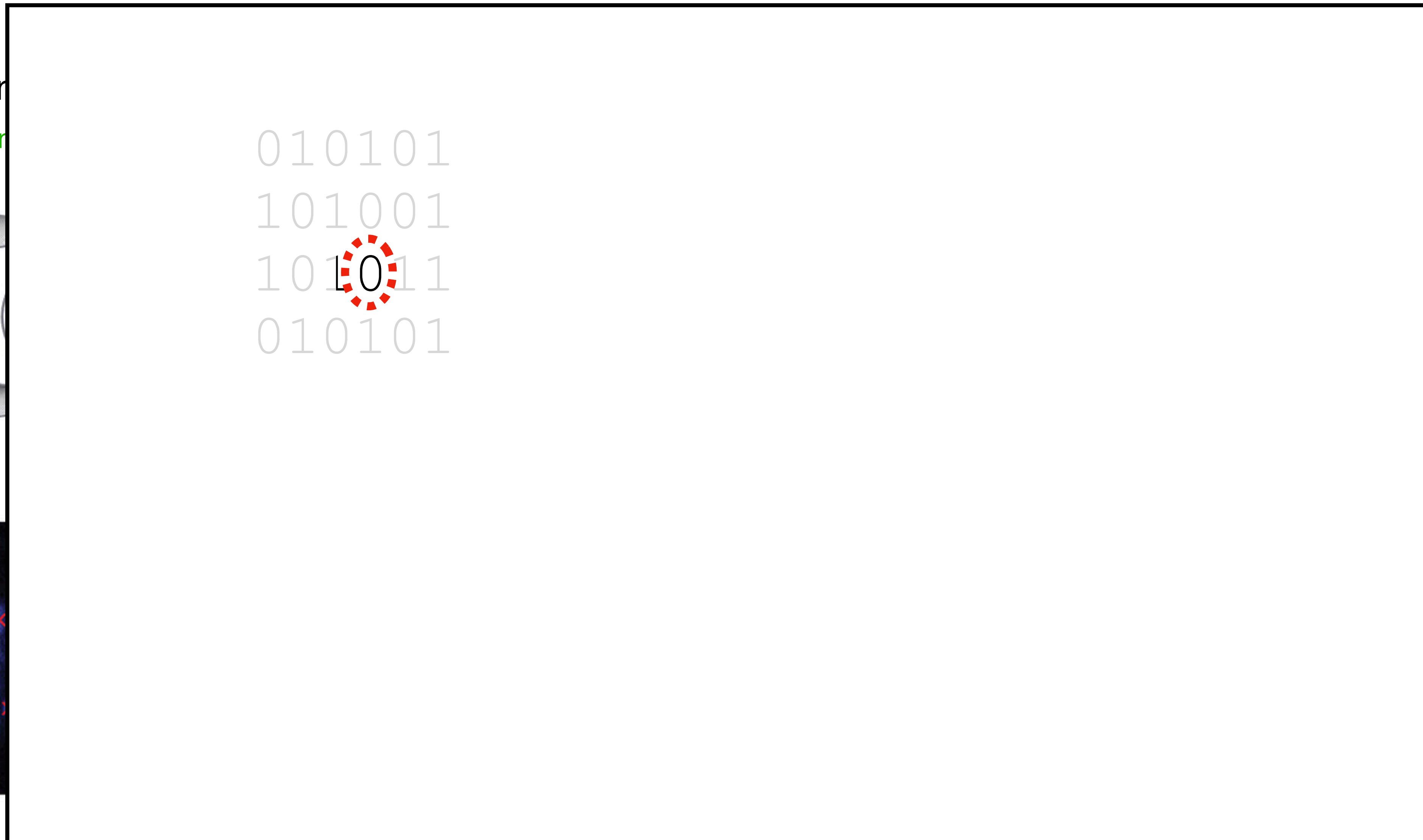

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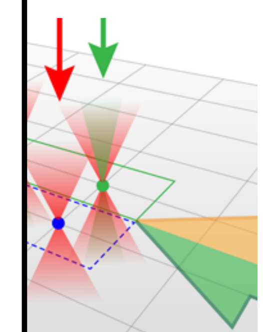

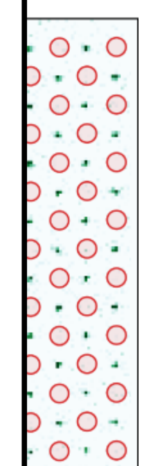

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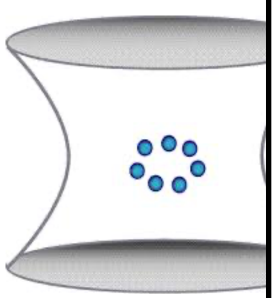

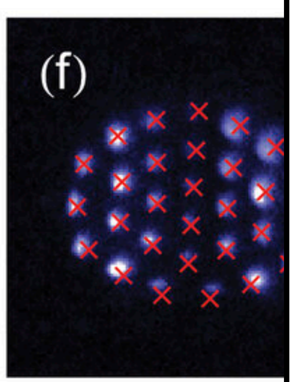

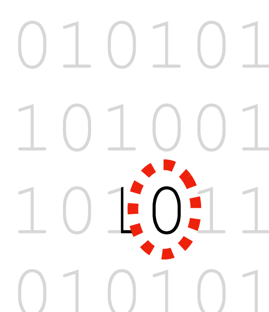

Local observable

$\approx \langle n_i \rangle$

"Marginal information"

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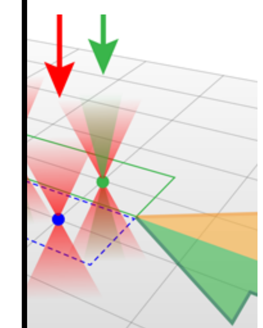

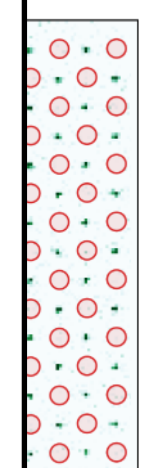

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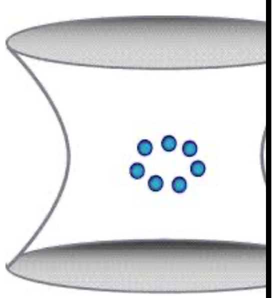

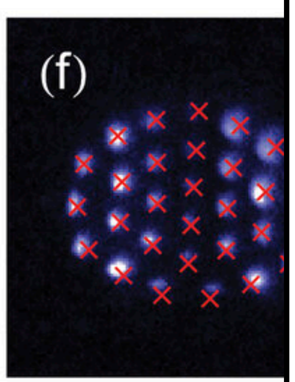

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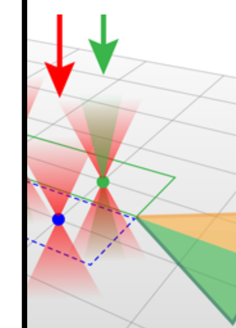

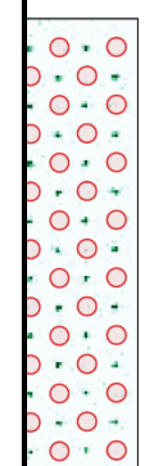

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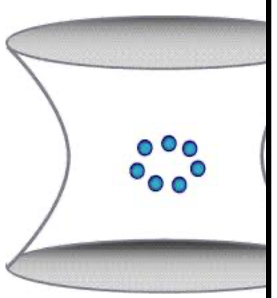
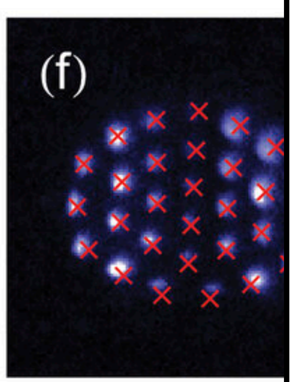
[Duke, Innsbruck, Hon





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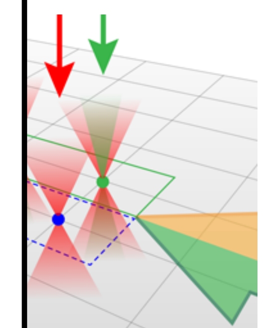
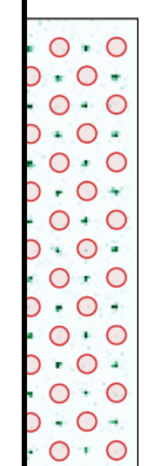




010101
101001
101011
010101

Local observable

$\approx \langle n_i \rangle$

"Marginal information"

010101
101001
101011
010101

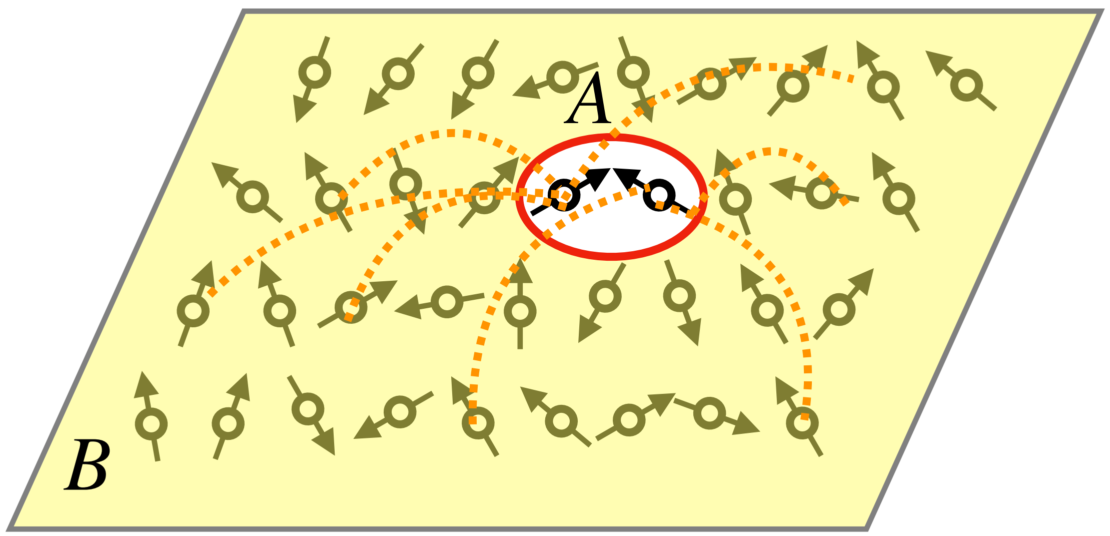
Environment correlated local observable

$\approx \langle n_i \rangle (z)$

"Conditional information"

[Cotler et al., PRX Quantum **4** (1), 010311 (2023)]
[Choi et al., Nature **613** 7944 (2023)]

Projected ensemble



Input: Global generator state $|\Psi\rangle$

Could be a state arising in dynamics $e^{-iHt}|\Psi_0\rangle$, an energy eigenstate $|n\rangle$ etc.

[Cotler et al., PRX Quantum **4** (1), 010311 (2023)]
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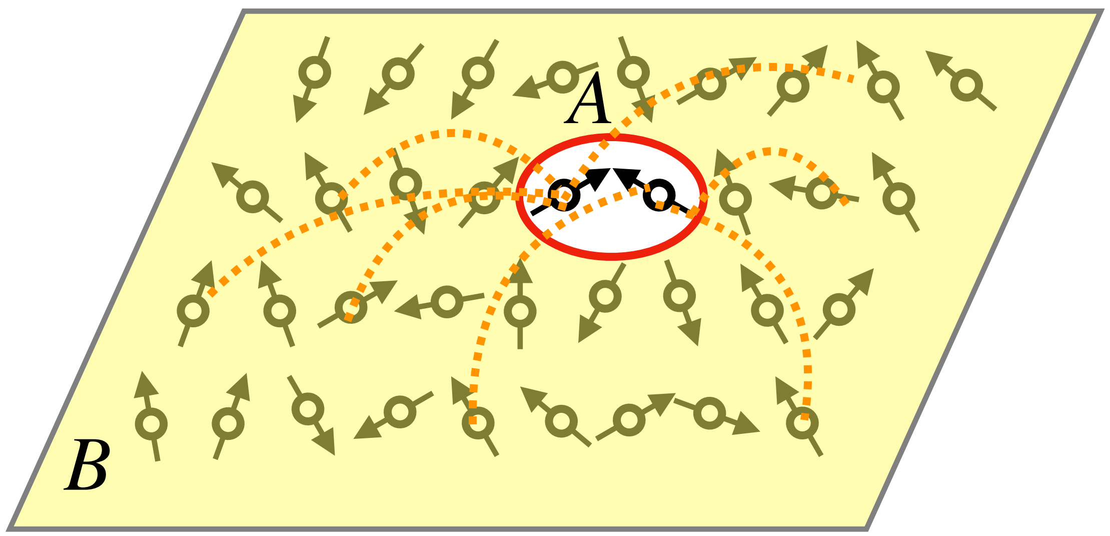

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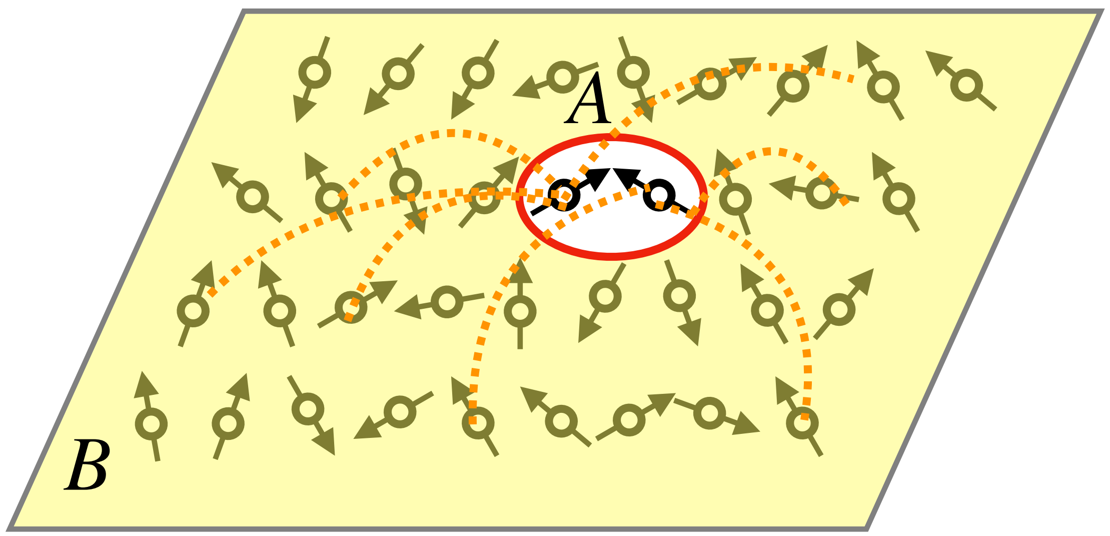

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Probability of observing z

Quantum state of local subsystem conditioned on bath

Classical state of bath (here in z-direction)

[Cotler et al., PRX Quantum **4** (1), 010311 (2023)]
[Choi et al., Nature **613** 7944 (2023)]

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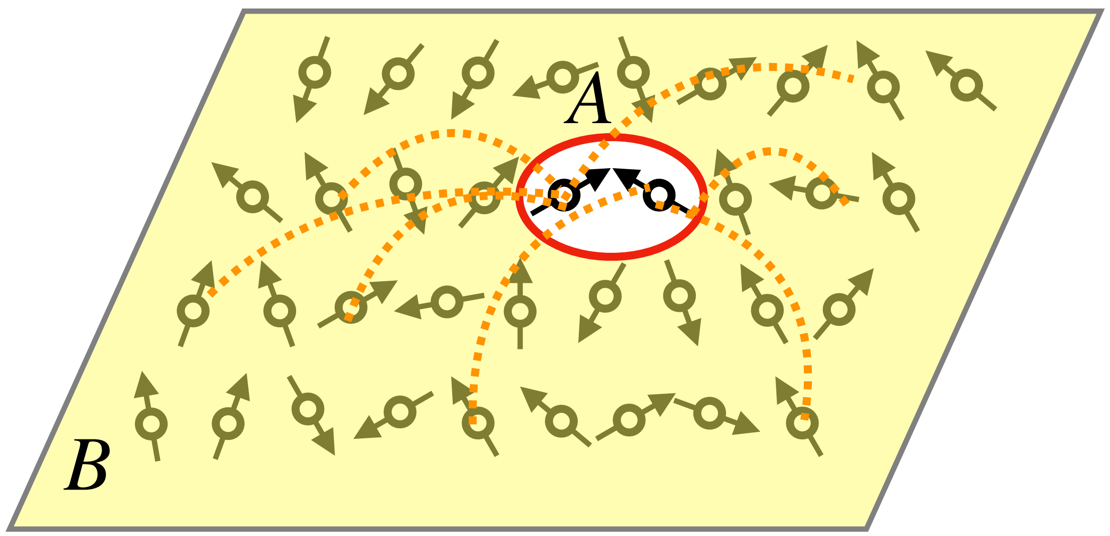

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Classical state of bath (here in z-direction)

Output: Projected ensemble

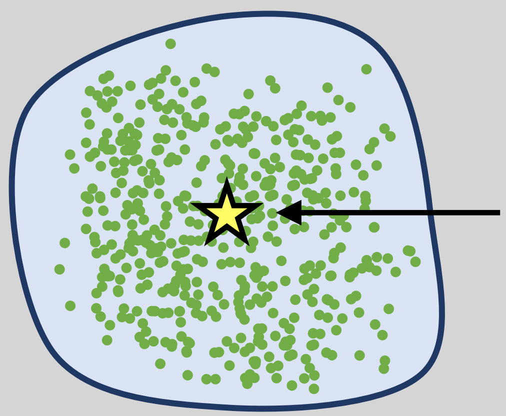
$$\boxed{\mathcal{E} := \{p(z), |\psi_A(z)\rangle\}}$$

Captures quantum fluctuations of local subsystem correlated with state of bath

[Cotler et al., PRX Quantum **4** (1), 010311 (2023)]
[Choi et al., Nature **613** 7944 (2023)]

Projected ensemble

Physically motivated unraveling of reduced density matrix



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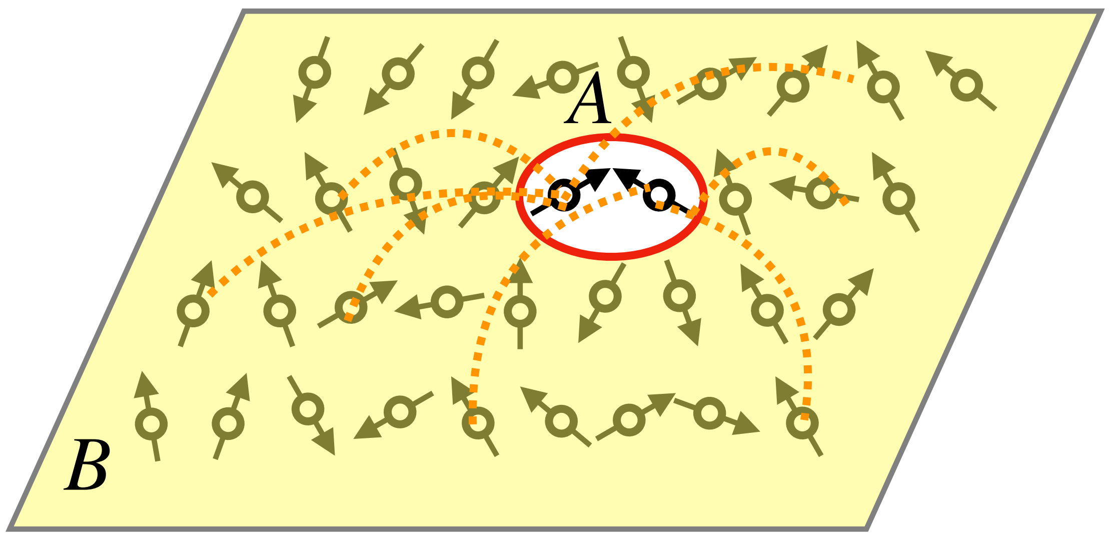

Input: Global generator state $|\Psi\rangle$

Could be a state arising in dynamics $e^{-iHt}|\Psi_0\rangle$, an energy eigenstate $|n\rangle$ etc.

Probability of observing z

$$|\Psi\rangle = \sum_z \sqrt{p_z}\,|\psi_A(z)\rangle\,|z\rangle_B$$

Quantum state of local subsystem conditioned on bath

Classical state of bath (here in z-direction)

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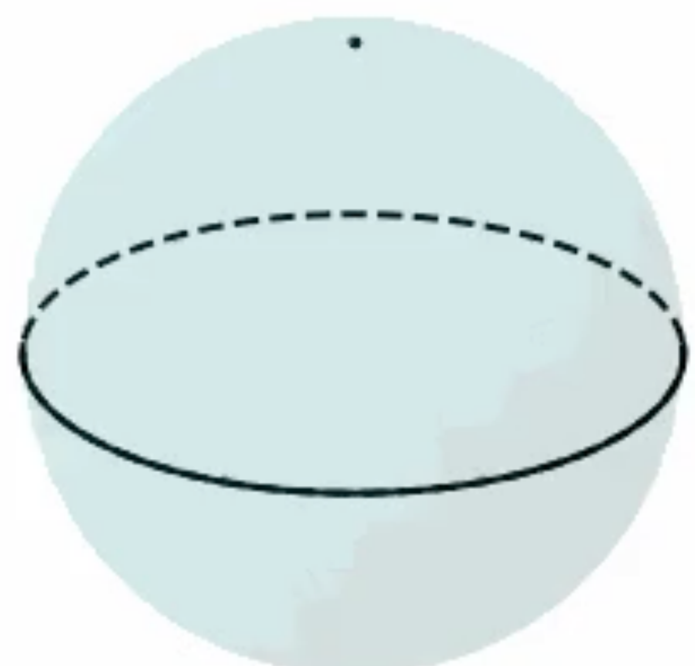
Visualization

Quench simulation of $e^{-iHt}|0000\cdots\rangle$ (nonintegrable H, at infinite temperature)

Projected ensemble of first spin over time:



Each dot is a conditional state $|\psi_A(z)\rangle$

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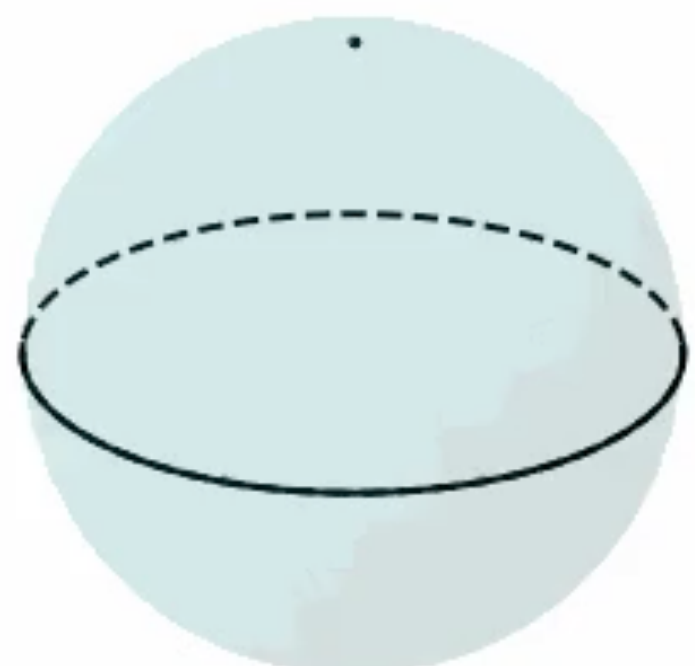

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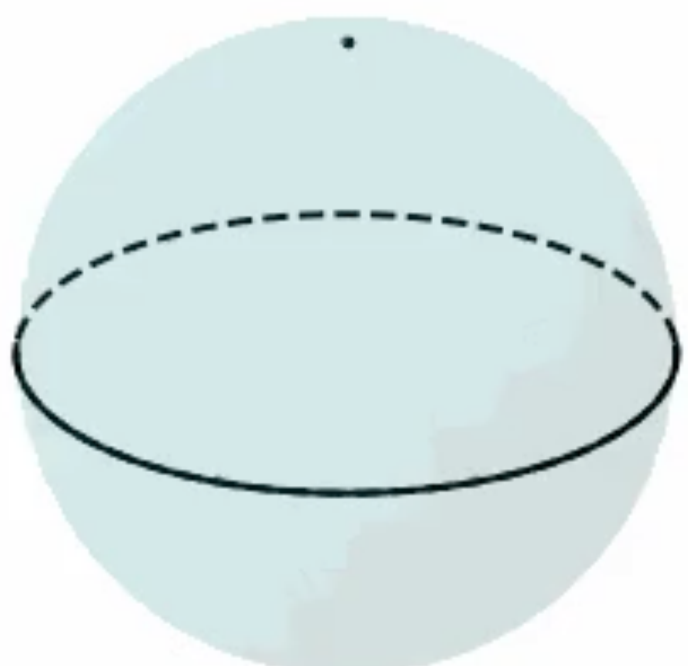

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[Liu, Huang and WWH, PRL **113**, 260401 (2024)]
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Information content of the ensemble of states

Communications viewpoint: encoding of classical information z into quantum states $|\psi_A(z)\rangle$

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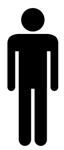

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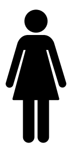

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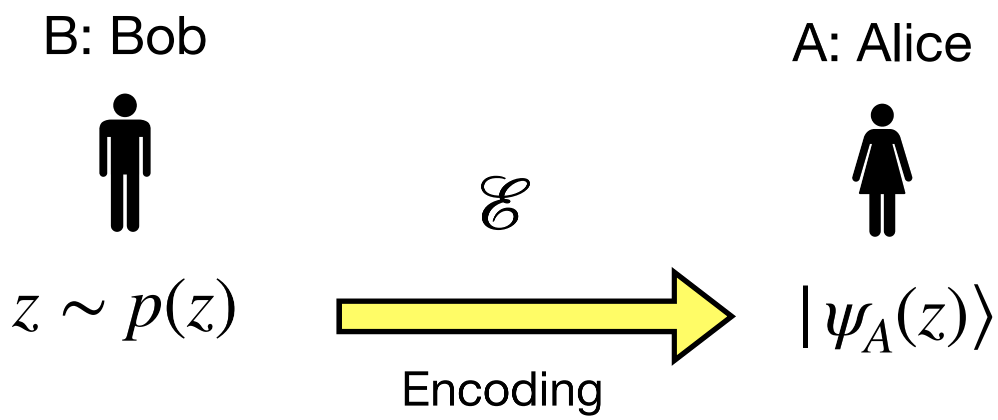

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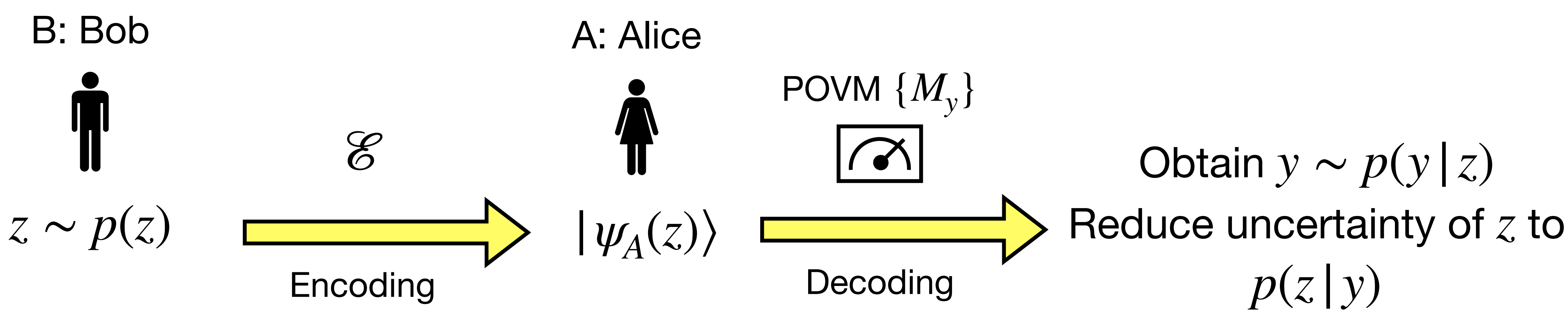

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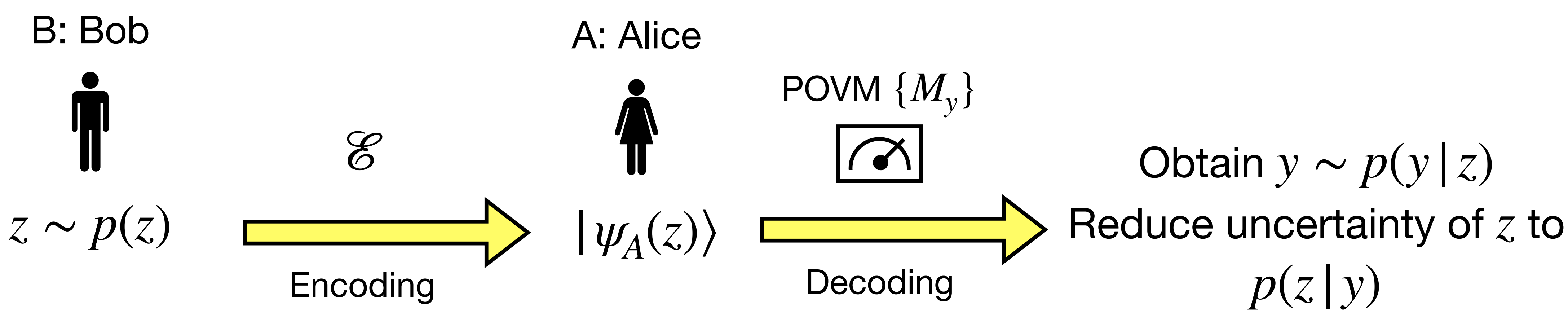

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$$I(\mathscr{E} : M) = H(Z) - H(Z|Y)$$

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$\mathscr{E}$

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POVM $\{M_y\}$

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Obtain $y \sim p(y|z)$
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Accessible information of ensemble definition

Meaning: maximum extractable classical information

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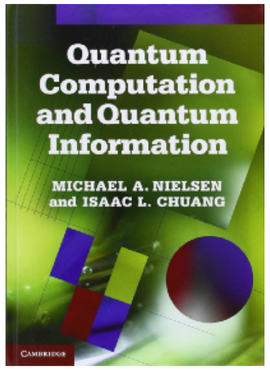

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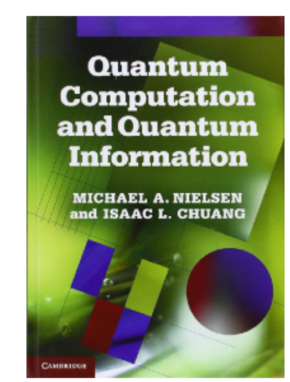

Why study this? Because distribution of ensemble governs its information content!

Bounds on accessible information

Consider a state ensemble $\mathscr{E}$ with mean $\rho = \mathbb{E}_{|\psi\rangle\sim\mathscr{E}}[\,|\psi\rangle\langle\psi|\,]$,

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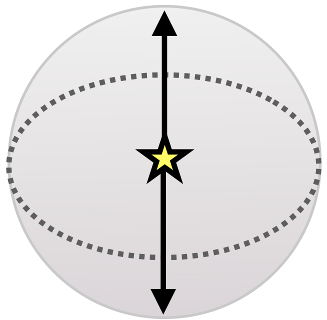

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"Minimally entropic"

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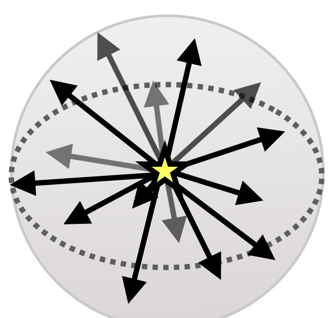

$\mathscr{E}$ = **overlapping** quantum states
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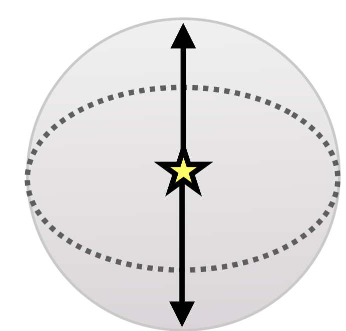

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(Saturation of $Q(\rho)$)

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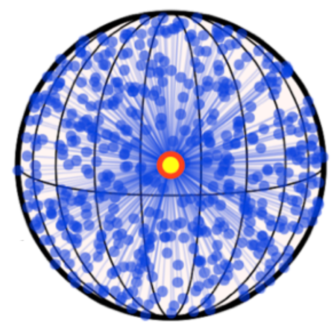

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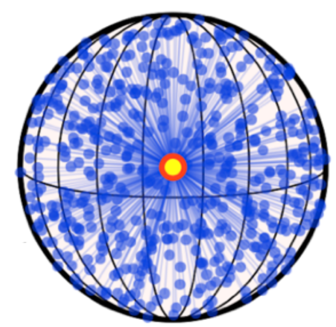

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For non-trivial ρ, "Scrooge" ensemble $\mathscr{E}_{Scrooge}[\rho]$.

[Jozsa, Robbs, Wootters PRA '94]

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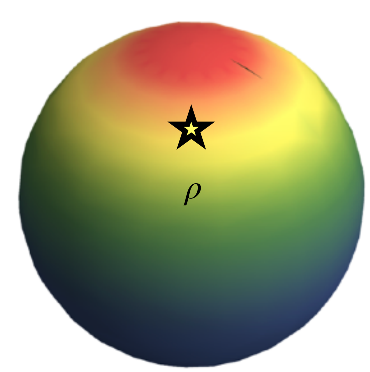

"Maximally entropic" up to constraint!

Named after Ebenezer Scrooge, stingy miser in "A Christmas Carol" (Charles Dickens)

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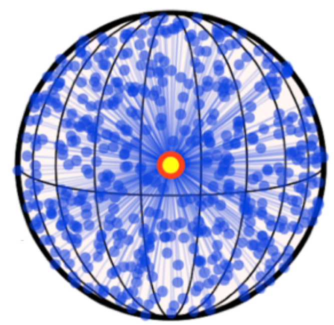

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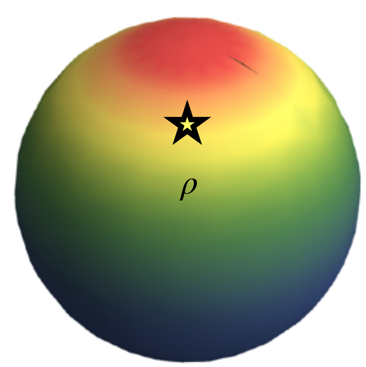

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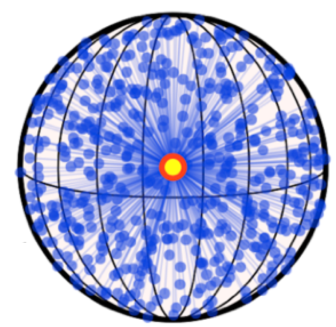

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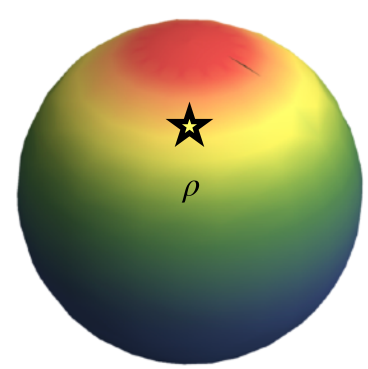

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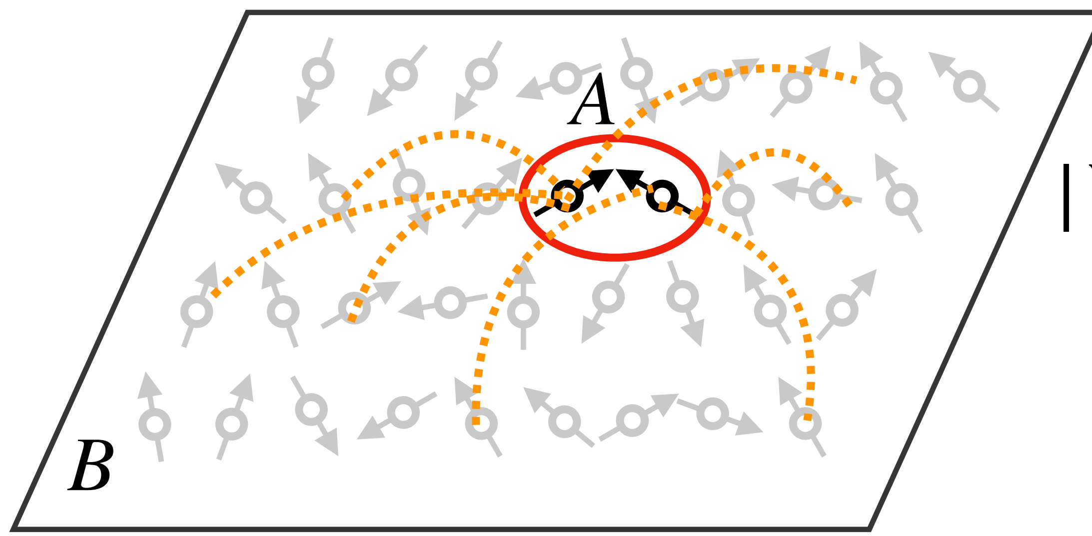
Also called the "Gaussian Adjusted Projected Ensemble", see e.g. [Goldstein, Lebowitz, Tumulka, J Stat Phys (2006)]

See also recent works [McGinley, Schuster, arXiv:2511.17172]

Deep thermalization

Let us return to the projected ensemble, the ensemble of local quantum states encoding classical information about the bath. **What should Nature typically do?**



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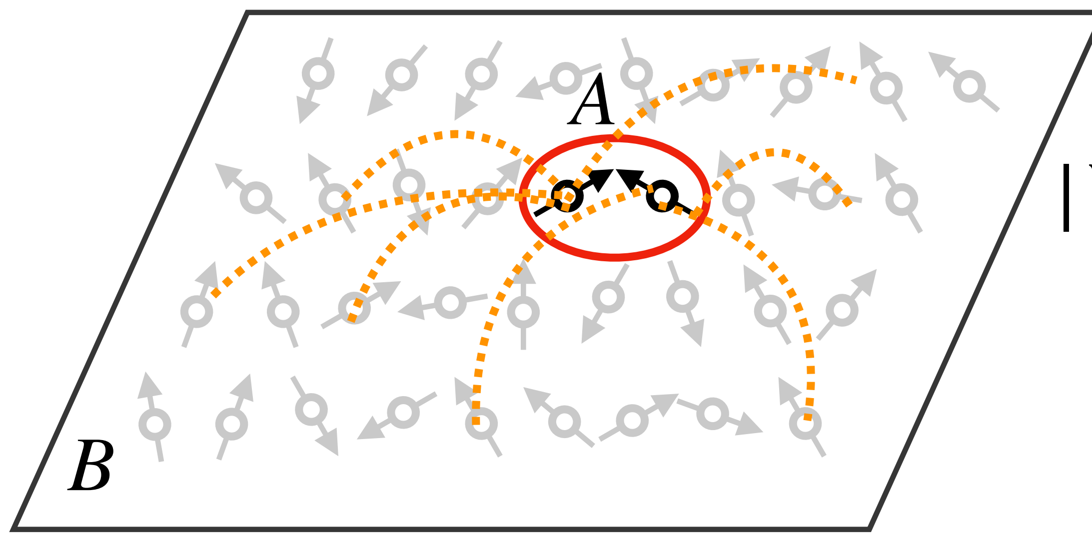

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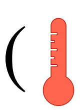
$\rho_A \to \rho_{\text{Gibbs}}($  $,$  $,\cdots)$ **(Regular thermalization)**

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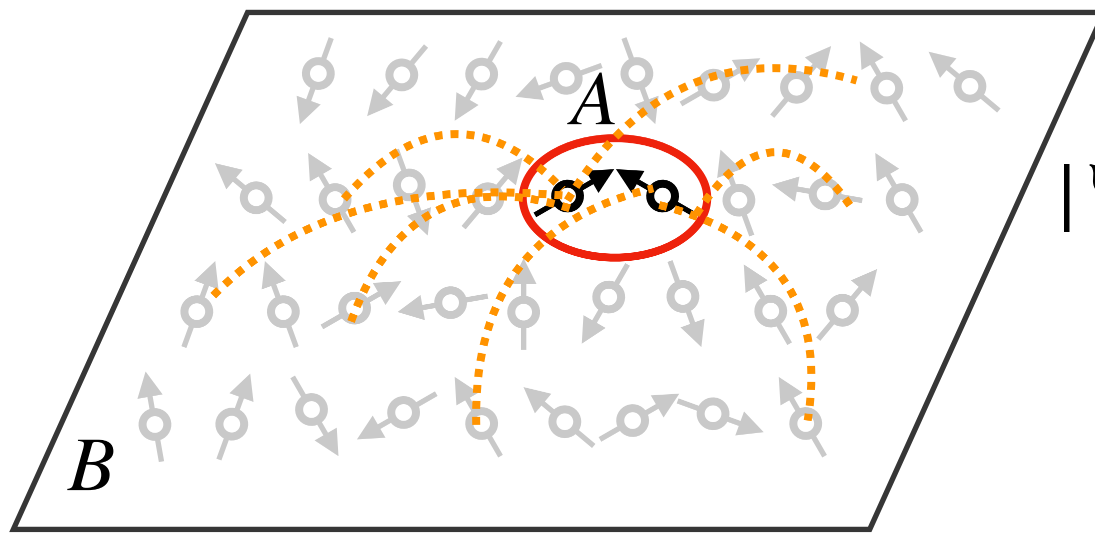

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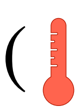
$\rho_A \rightarrow \rho_{\text{Gibbs}}($  ,  $, \cdots)$ **(Regular thermalization)**

But also, we might expect the encoding is information-hiding under complex dynamics (why should local subsystem and bath be highly correlated?):

[Conjecture/Principle]

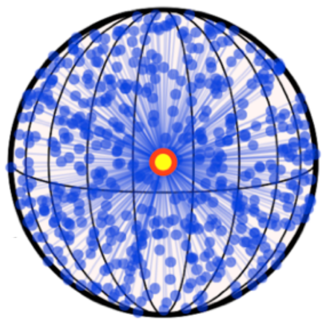
$\mathscr{E} \rightarrow \mathscr{E}_{\text{Haar}}, \mathscr{E}_{\text{Scrooge}}$, etc. **(Deep thermalization)**

Emergence of universal information-stingy distributions in nature

This can be viewed as a generalized maximum entropy principle

Evidence for deep thermalization to Haar

Dynamics with no conservation laws (Floquet, random circuits etc.) or at infinite eff. temperature



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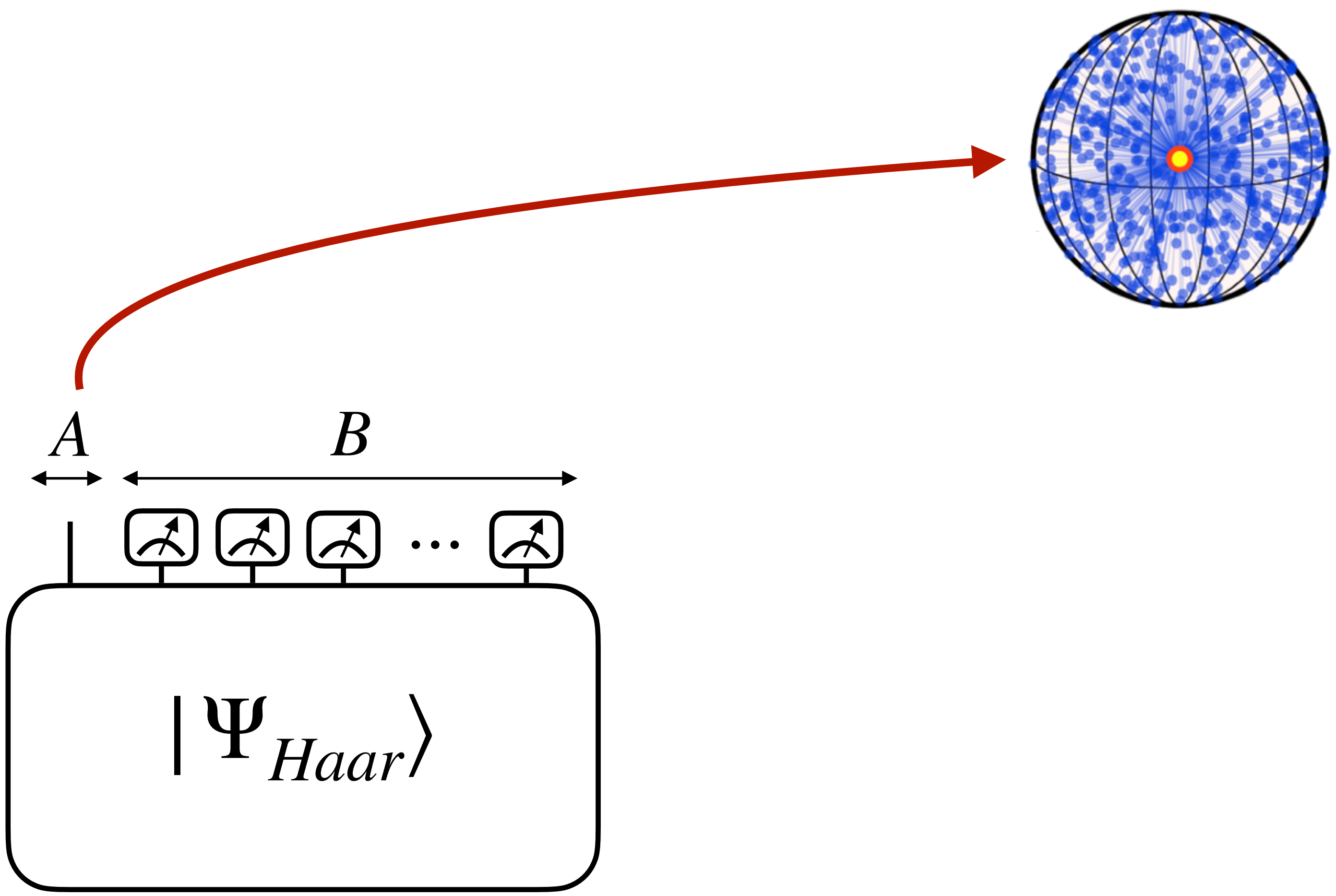

Theorem (RMT)

A single instance of a global Haar random state produces a PE close to Haar with high probability

[Cotler et al, PRXQ '23]

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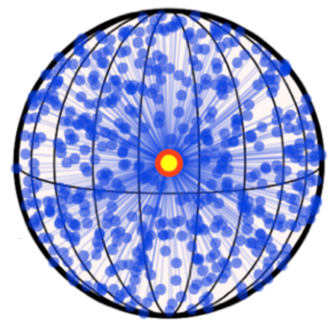

Theorem 1. *Let $|\Psi\rangle$ be chosen uniformly at random from the Hilbert space $\mathcal{H}$. The ensemble $\mathcal{E}_{A,\Psi}$ forms an ε-approximate k-design with probability at least $1-\delta$ if*

$$N_B = \Omega\left(k\, N_A + \log\left(\tfrac{1}{\varepsilon}\right) + \log\log\left(\tfrac{1}{\delta}\right)\right). \tag{6}$$

Theorem 2. *Let $|\Psi\rangle$ be a state sampled from an ensemble on $\mathcal{H}$ that forms an ε'-approximate k'-design. Then the projected ensemble $\mathcal{E}_{A,\Psi}$ forms an ε-approximate k-design with probability at least $1-\delta$ if*

$$N_B = \Omega\left(kN_A + \log\left(\tfrac{1}{\varepsilon\delta}\right)\right), \tag{7}$$

$$k' = \Omega\left(k\left(N_B + \log\left(\tfrac{1}{\varepsilon\delta}\right)\right)\right), \tag{8}$$

$$\log\left(\tfrac{1}{\varepsilon'}\right) = \Omega\left(kN_B\left(N_B + \log\left(\tfrac{1}{\varepsilon\delta}\right)\right)\right), \tag{9}$$

$$N_A = \Omega\left(\log(N_B) + \log(k) + \log\log\left(\tfrac{1}{\varepsilon\delta}\right)\right). \tag{10}$$

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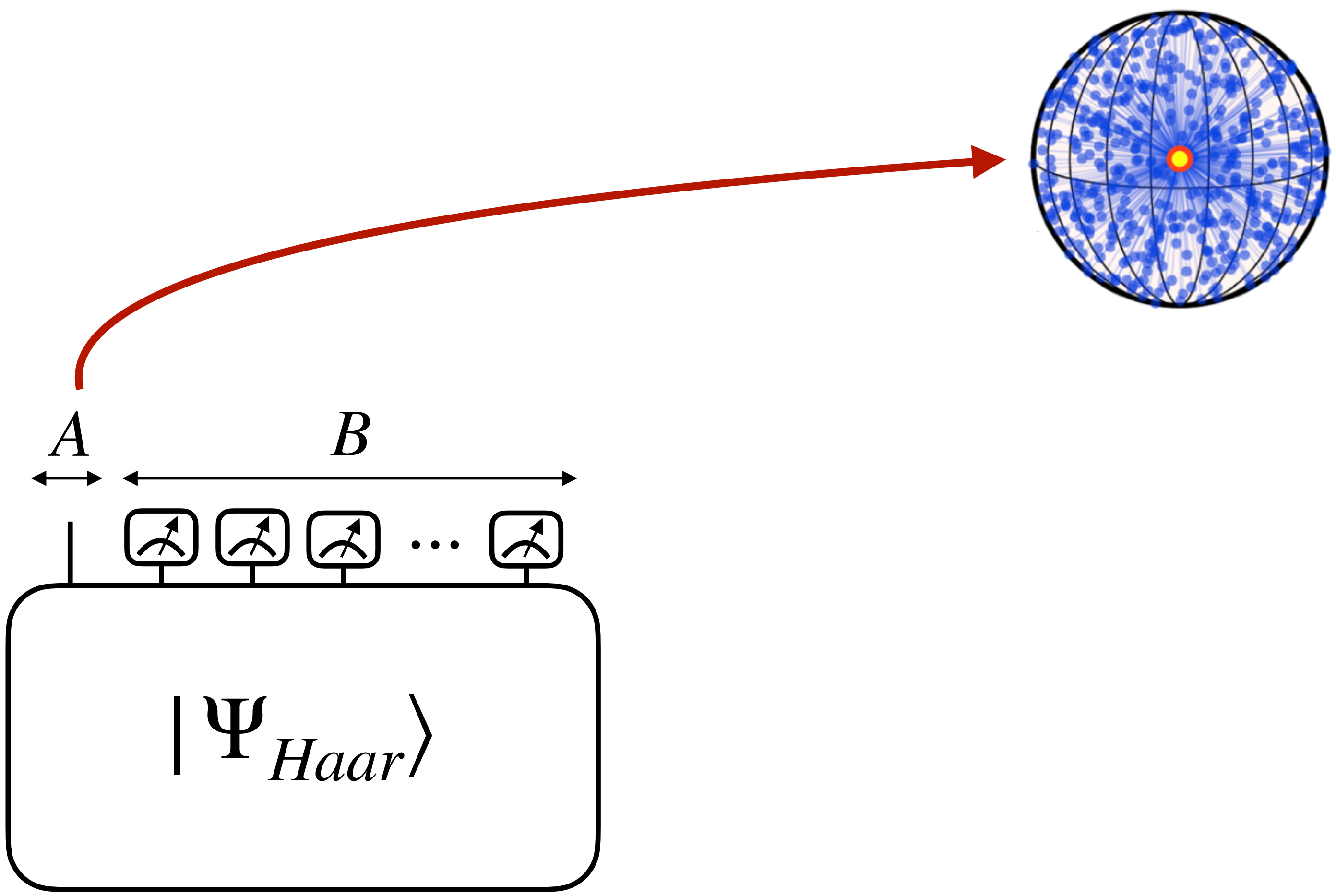

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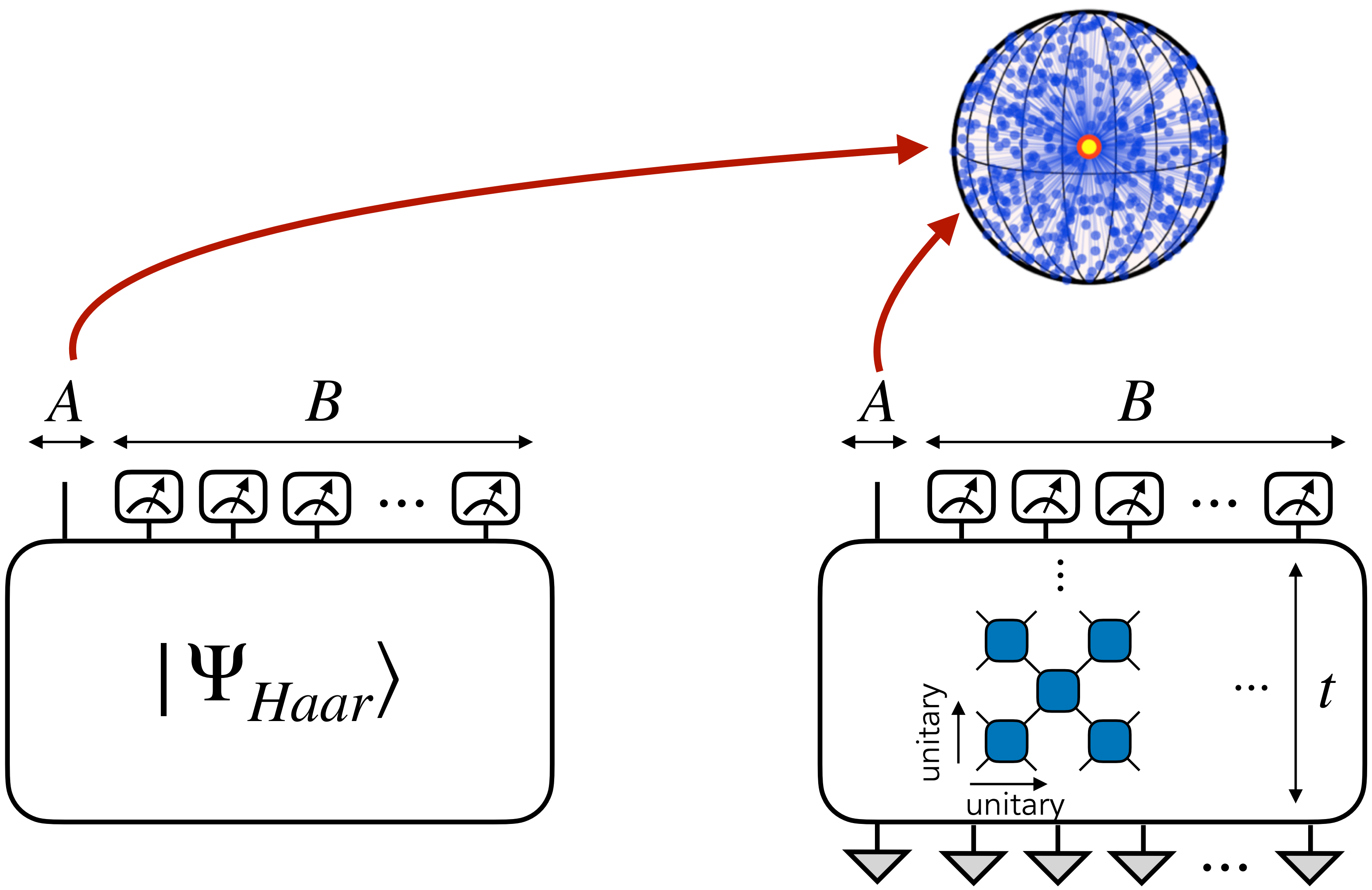

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Theorem (Dual-unitary circuits)

Almost all DU circuits have PEs which is Haar distributed exactly in the limit $N_B \to \infty$, when $t \geq N_A$

[WWH et al, PRL '22]
[Ippoliti,& WWH, PRXQ '22]
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Evidence for deep thermalization to Haar

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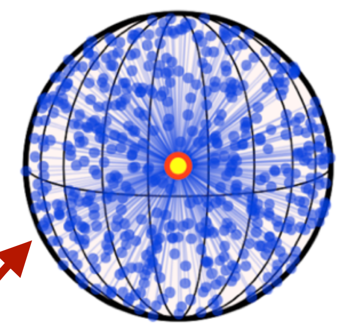

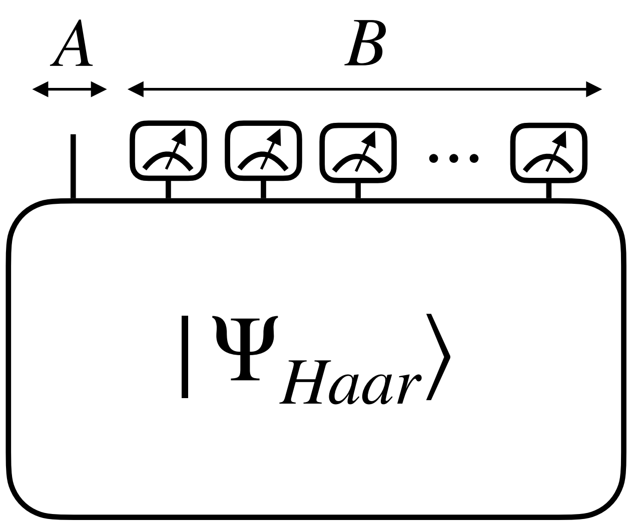

Theorem 1. *For $t \geq N_A$ and $g \notin \mathbb{Z}\pi/8$, the projected ensemble $\mathcal{E}$ forms an exact quantum state-design in the thermodynamic limit: for any k, $\lim_{N_B\to\infty} \rho_{\mathcal{E}}^{(k)} = \rho_{Haar}^{(k)}$.*

Definition. *A "circuit instance" (or just "instance") is a brickwork quantum circuit in 1+1 dimensions constructed by sampling two-qubit gates independently and identically from a probability distribution P on $\mathfrak{DU}$. The quantum circuit has finite depth t and is semi-infinite in space.*

Theorem. *The limit state $\rho_\infty^{(k)}$ is almost always the Haar moment $\rho_H^{(k)}$.*

Theorem (RMT)

A single instance of a global Haar random state produces a PE close to Haar with high probability

[Cotler et al, PRXQ '23]

Theorem (Dual-unitary circuits)

Almost all DU circuits have PEs which is Haar distributed exactly in the limit $N_B \to \infty$, when $t \geq N_A$

[WWH et al, PRL '22]
[Ippoliti,& WWH, PRXQ '22]
[Ippoliti & WWH, Quantum '22]

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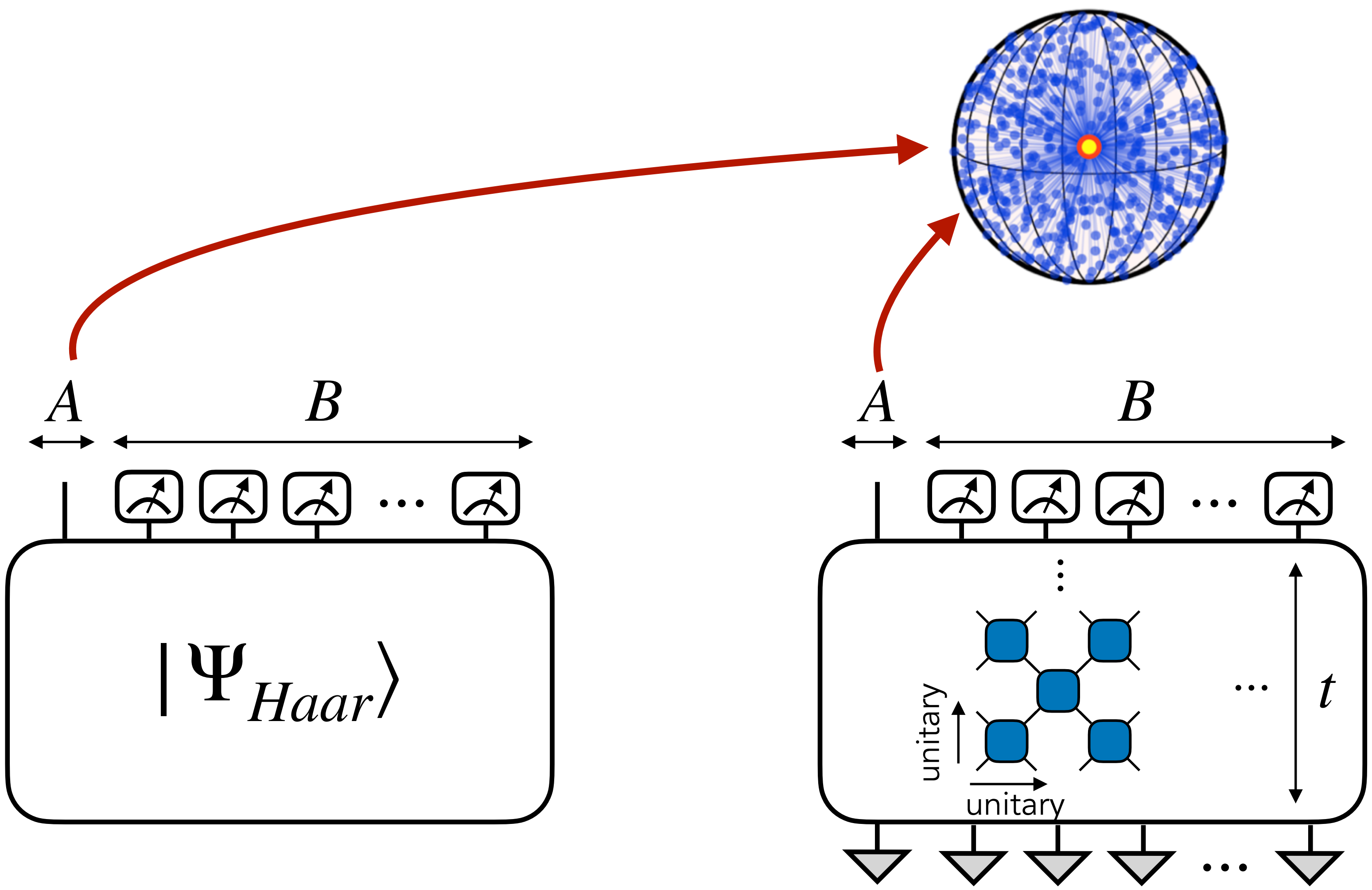

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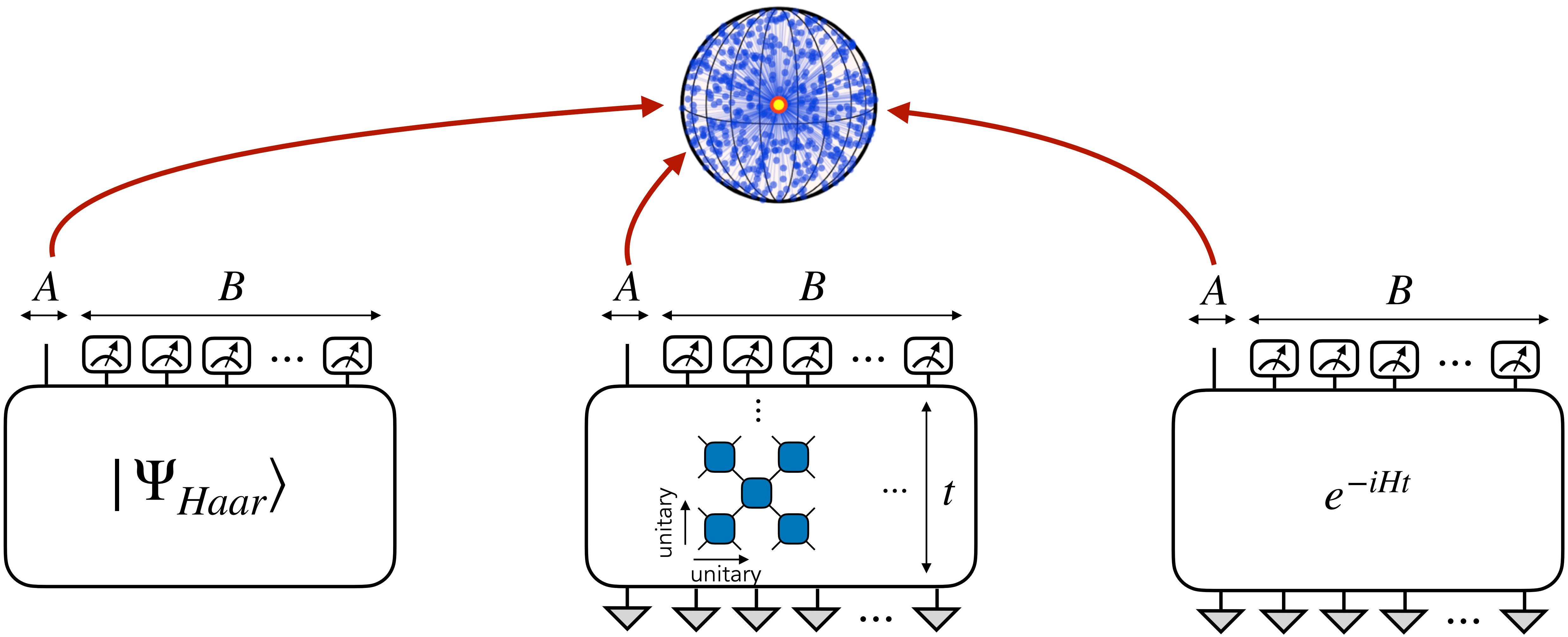

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Numerics + Experiments

Extensive numerics on mixed-field Ising model; quench expts on Rydberg atom array simulator

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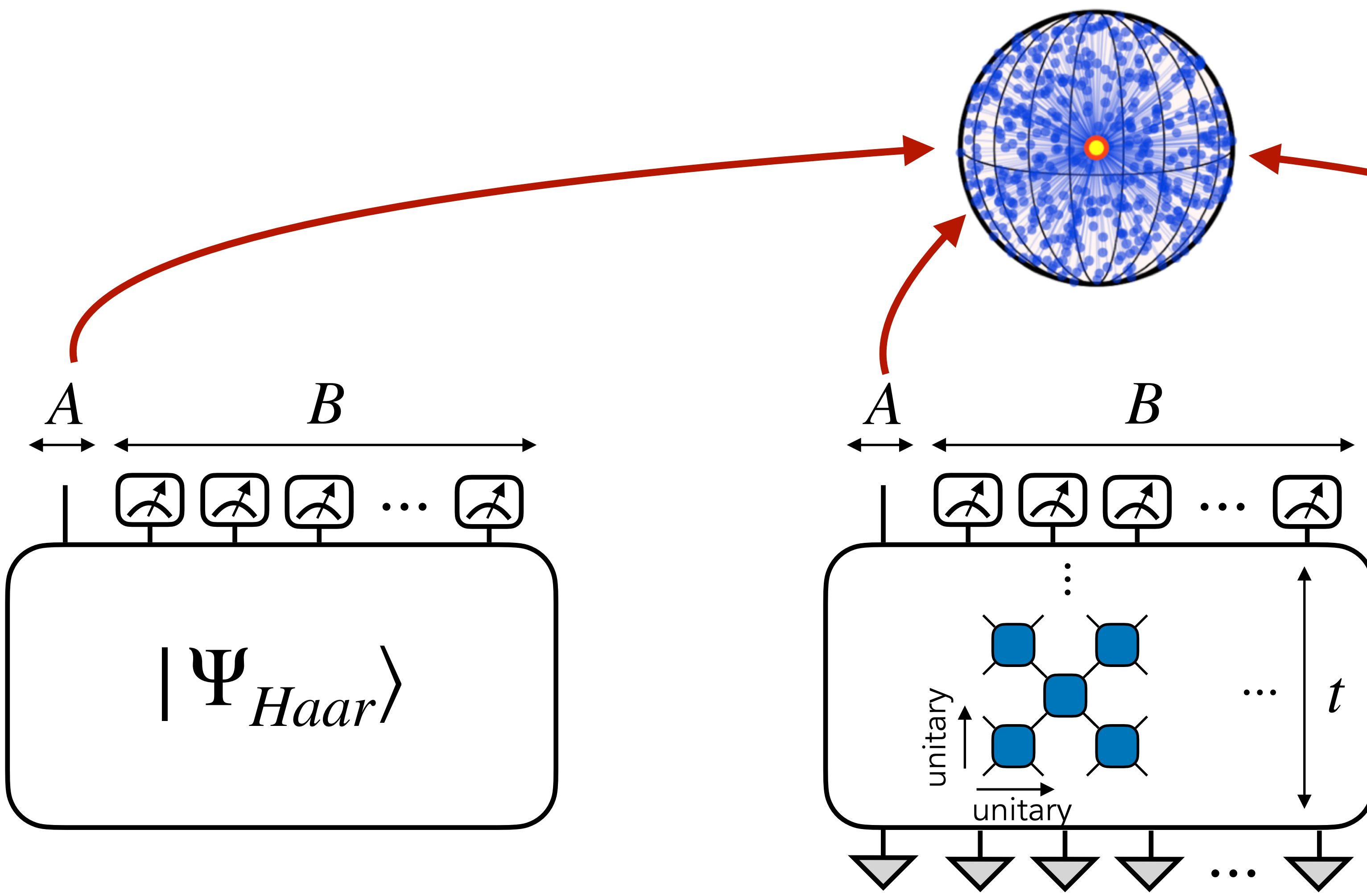

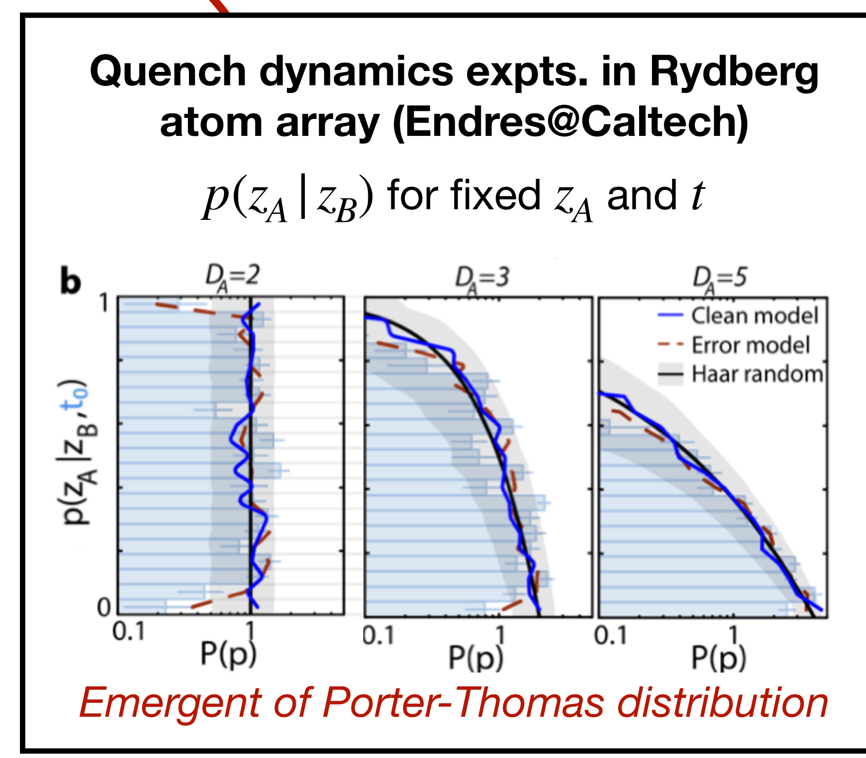

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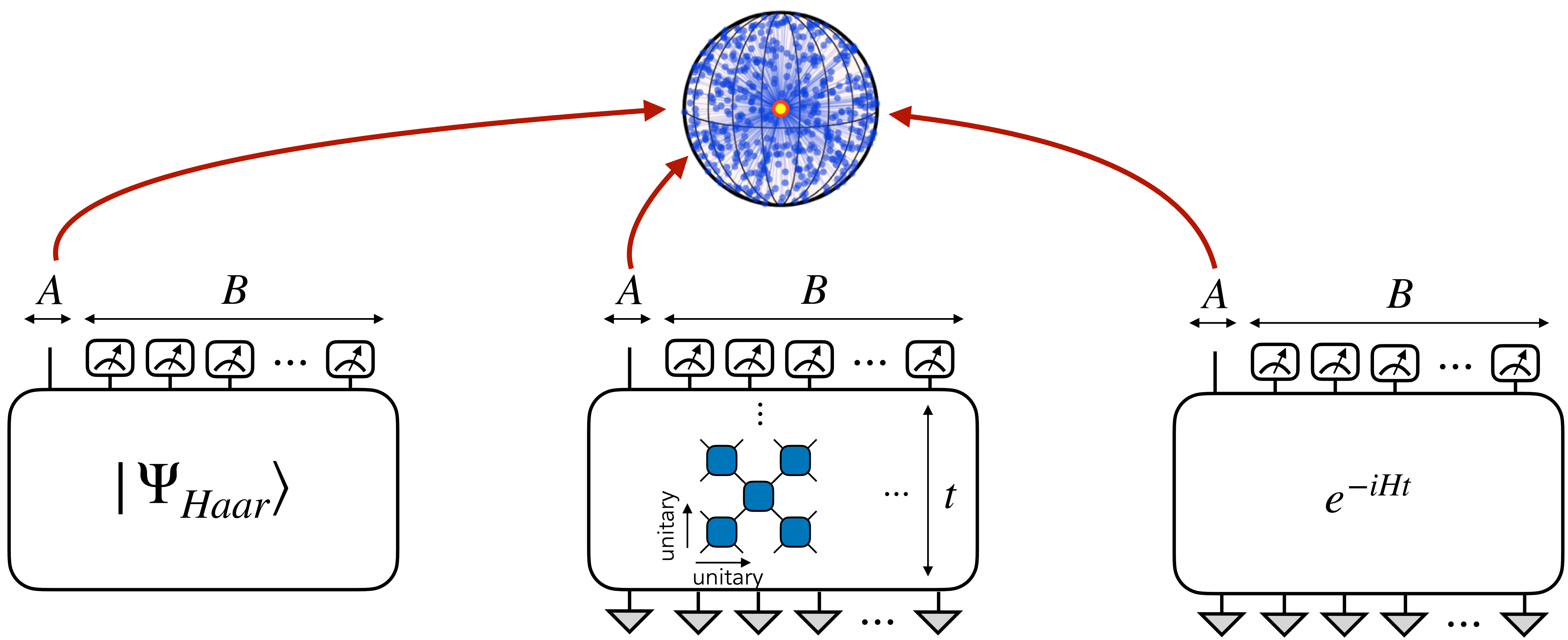

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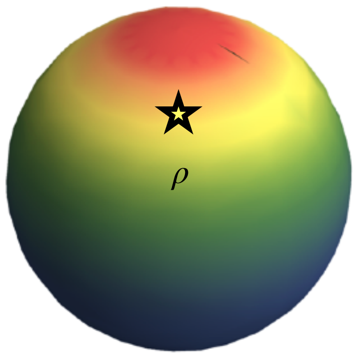

Deformed Haar

We should impose **constraints** on dynamics!

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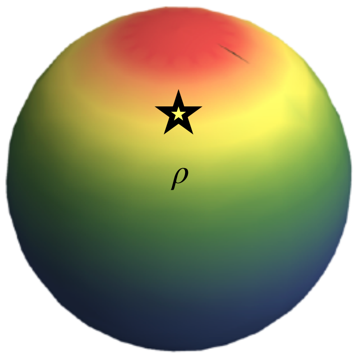

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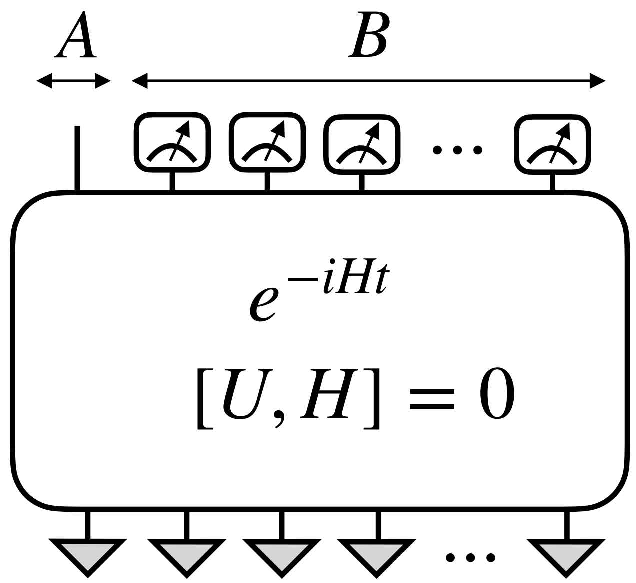

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[Mark et al, PRX’25]

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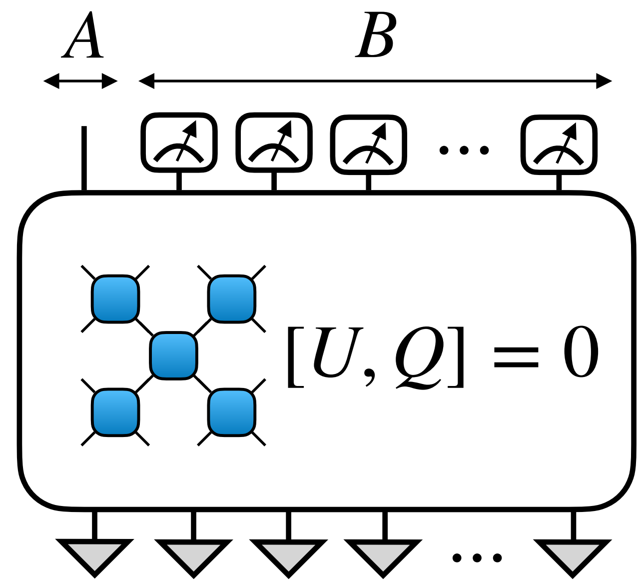

[Chang, Shrotriya, **WWH**, Ippoliti, PRX Q’25)]

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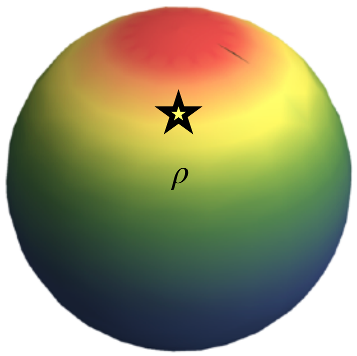

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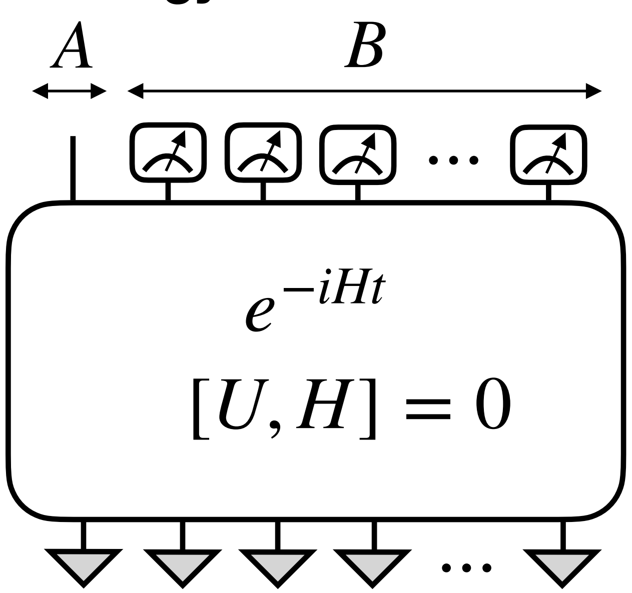

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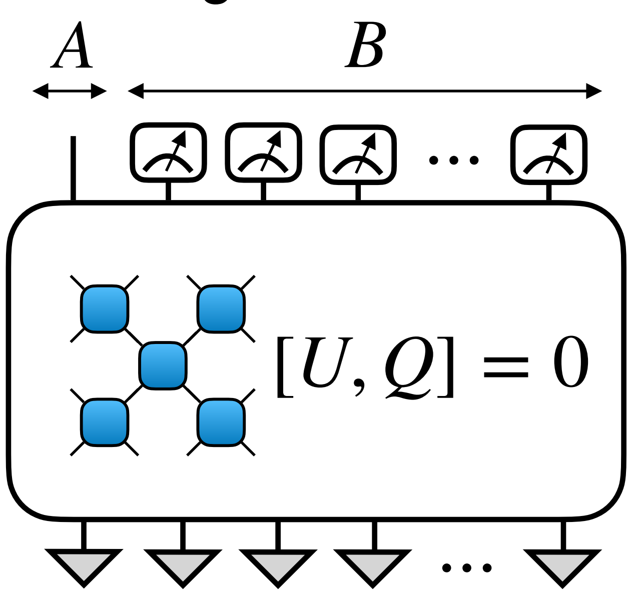

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Numerics*
Extensive numerics on mixed-field Ising model

**Importantly, measurements in energy non-revealing basis*

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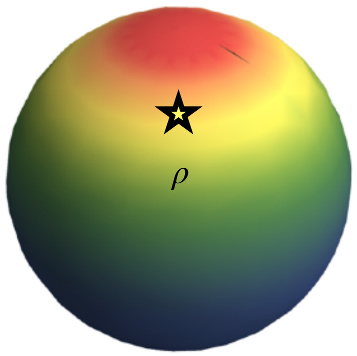

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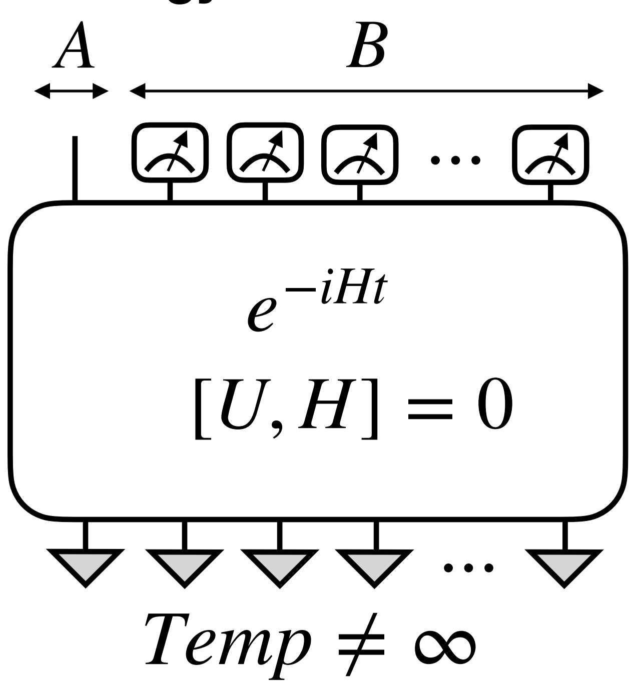

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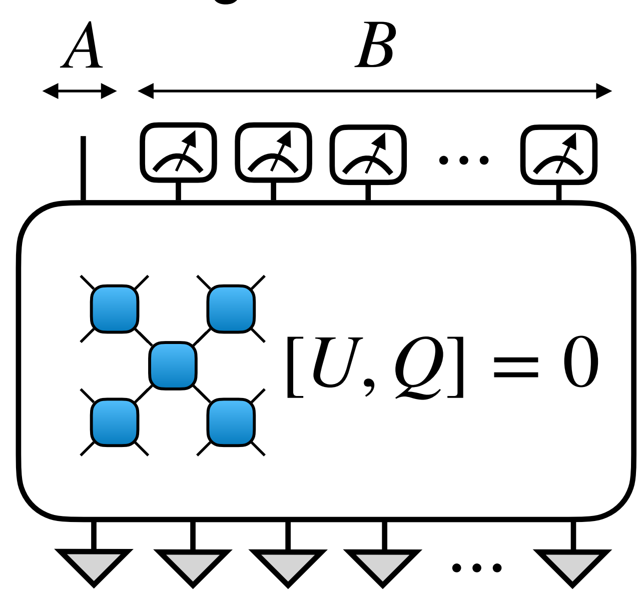

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Extensive numerics on mixed-field Ising model

**Importantly, measurements in energy non-revealing basis*

Rigorous theorems*

In "quantum chaotic" dynamics & other assumptions…

PHYSICAL REVIEW X **14**, 041051 (2024)

Maximum Entropy Principle in Deep Thermalization and in Hilbert-Space Ergodicity

Daniel K. Mark,[1] Federica Surace,[2] Andreas Elben,[2] Adam L. Shaw,[2] Joonhee Choi,[3]
Gil Refael,[2] Manuel Endres,[2] and Soonwon Choi[1,4]
[1]*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[2]*California Institute of Technology, Pasadena, California 91125, USA*
[3]*Department of Electrical Engineering, Stanford University, Stanford, California, USA*

[Mok, Haug, **WWH**, Preskill, arXiv:2601.00266]

Nature is stingy: Universality of Scrooge ensembles in quantum many-body systems

Wai-Keong Mok,[1] Tobias Haug,[2] Wen Wei Ho,[3,4] and John Preskill[1,5]
[1]*Institute for Quantum Information and Matter, California Institute of Technology, Pasadena, CA 91125, USA*
[2]*Quantum Research Centre, Technology Innovation Institute, Abu Dhabi, UAE*
[3]*Department of Physics, National University of Singapore, Singapore 117551*
[4]*Centre for Quantum Technologies, National University of Singapore, Singapore 117543*
[5]*AWS Center for Quantum Computing, Pasadena CA 91125*

**Precisely, prediction of "generalized Scrooge ensembles"*

[Chang, Shrotriya, **WWH**, Ippoliti, PRX Q **6**, 020343 (2025)]

Deep thermalization to Scrooge

How to account for different choices of measurements? Different initial charge fluctuations?

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Key idea — ***measurements learn while nature hides information maximally!***

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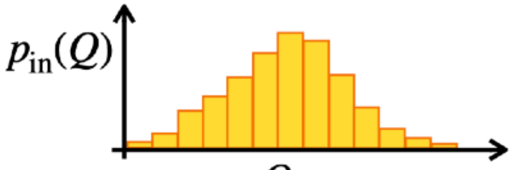

Initial state charge distribution

[Chang, Shrotriya, **WWH**, Ippoliti, PRX Q **6**, 020343 (2025)]

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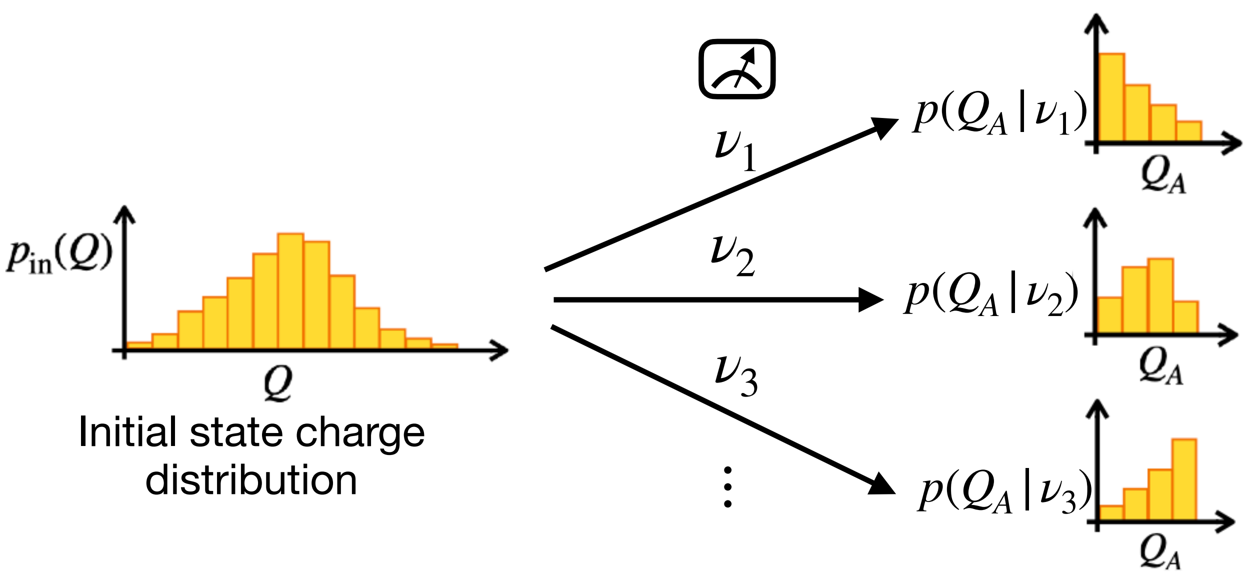

Measurements update our knowledge of charge on A (Bayesian inference)

[Chang, Shrotriya, **WWH**, Ippoliti, PRX Q **6**, 020343 (2025)]

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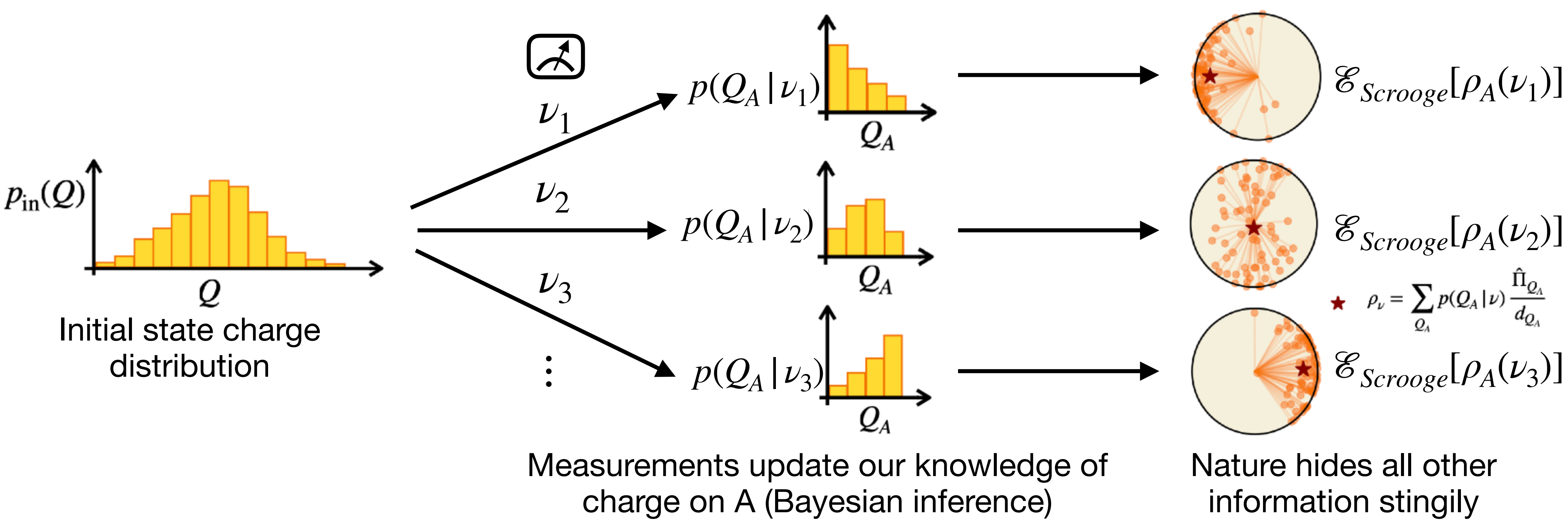

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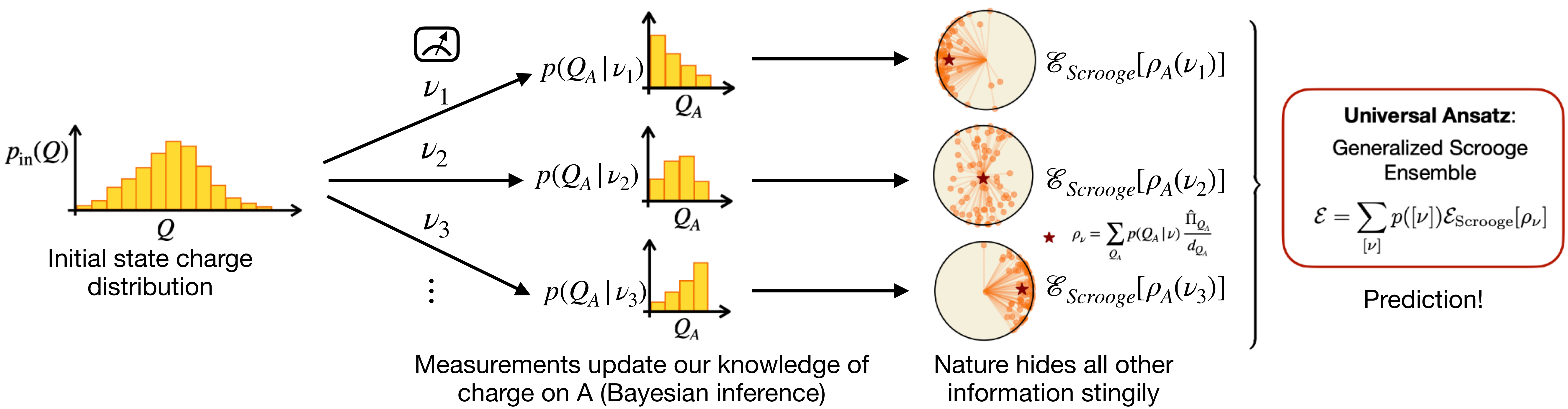

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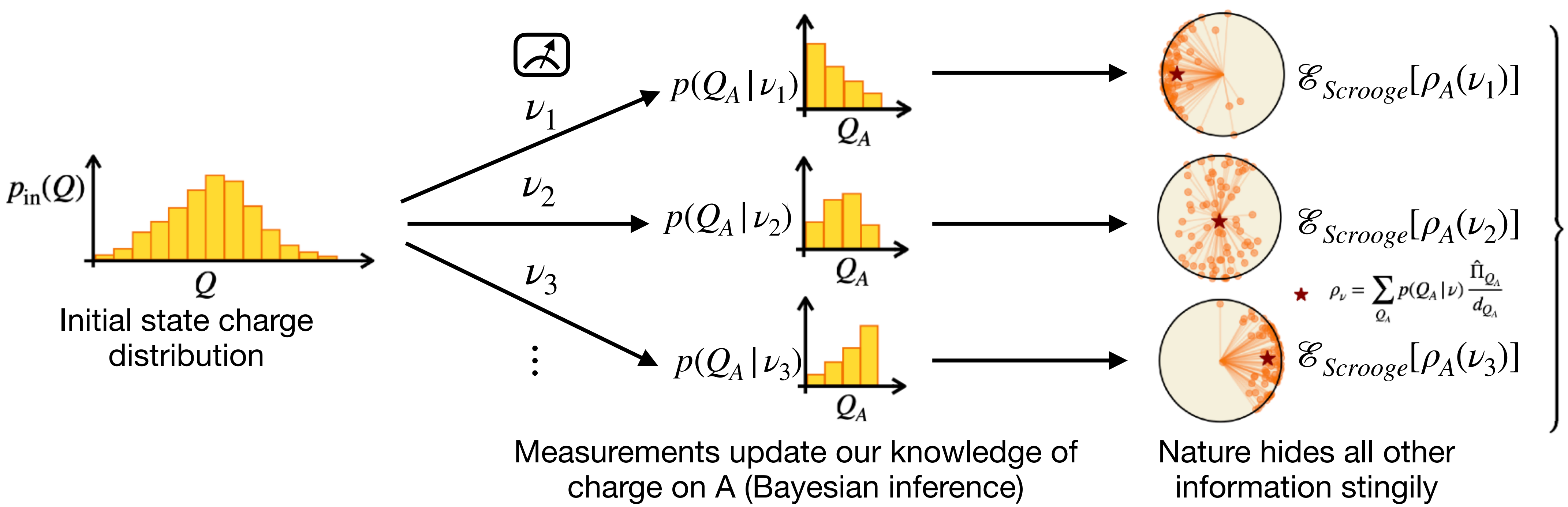


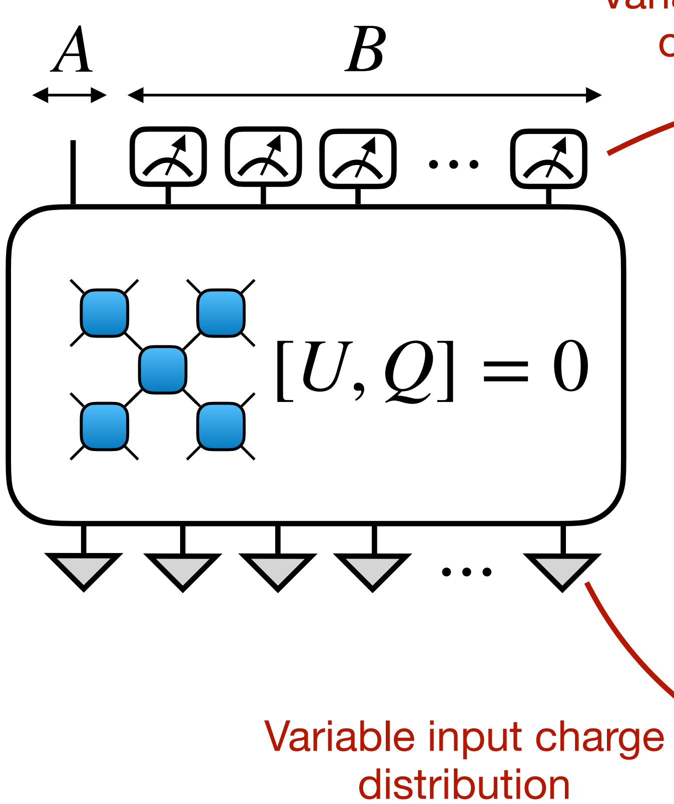
Universal Ansatz:
Generalized Scrooge Ensemble
$$\mathcal{E} = \sum_{[\nu]} p([\nu])\mathcal{E}_{\rm Scrooge}[\rho_\nu]$$

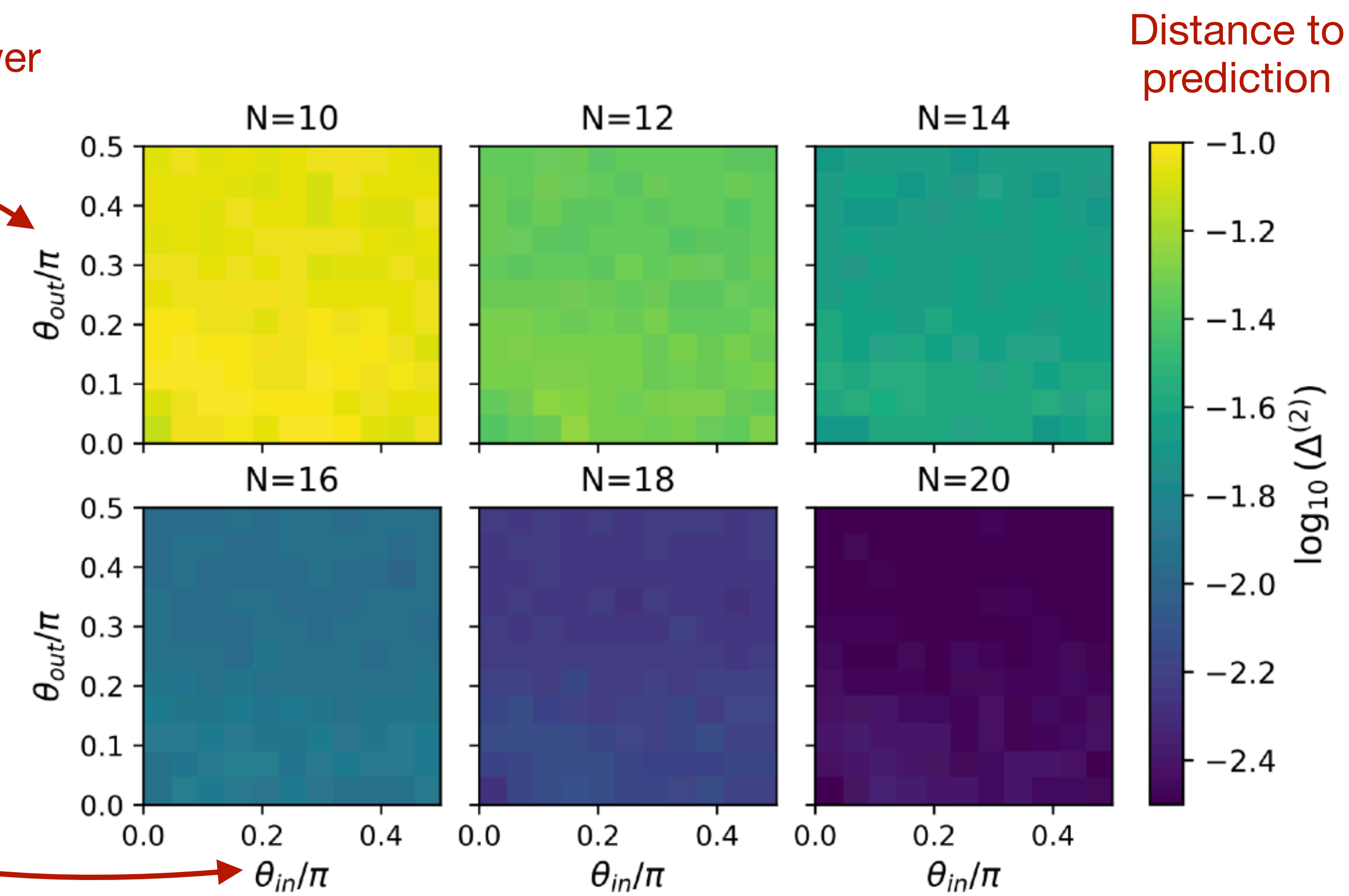
Prediction!

Family of maximally ergodic wavefunction distributions
(tuned by initial charge distribution & learning power of measurements)

[Chang, Shrotriya, **WWH**, Ippoliti, PRX Q **6**, 020343 (2025)]

Numerical evidence in charge-conserving dynamics





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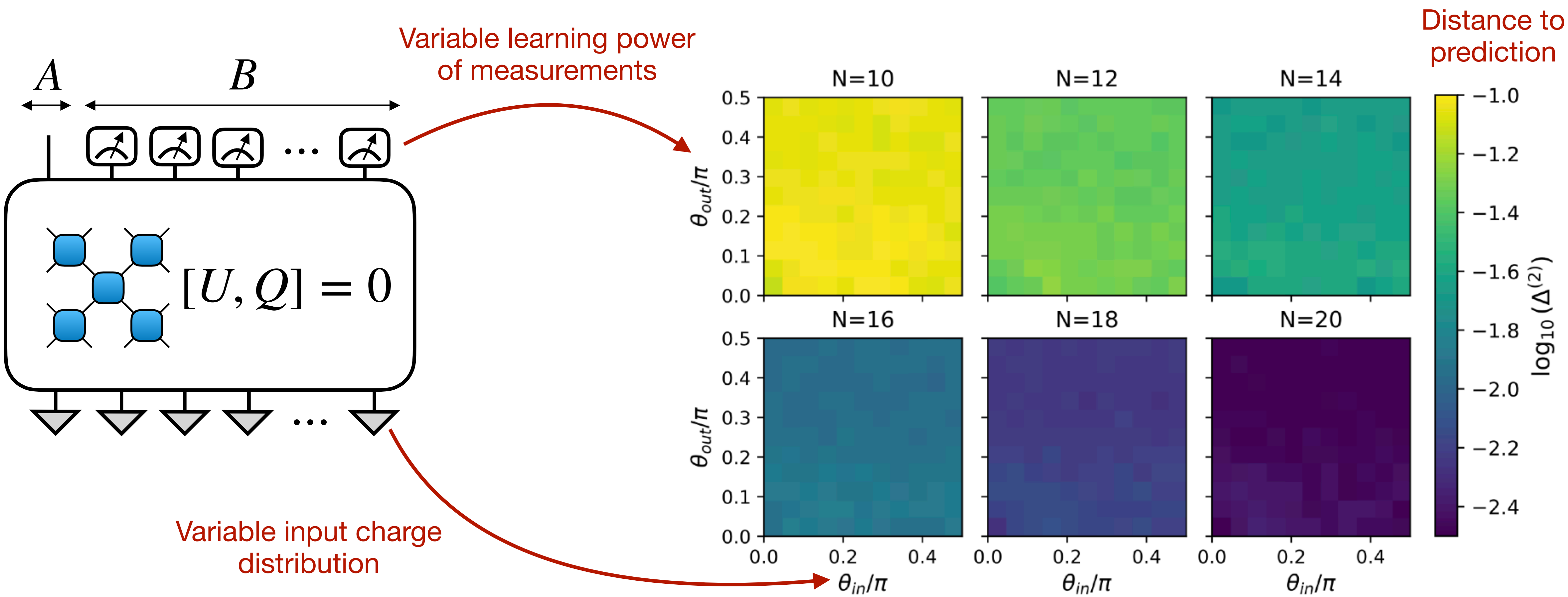
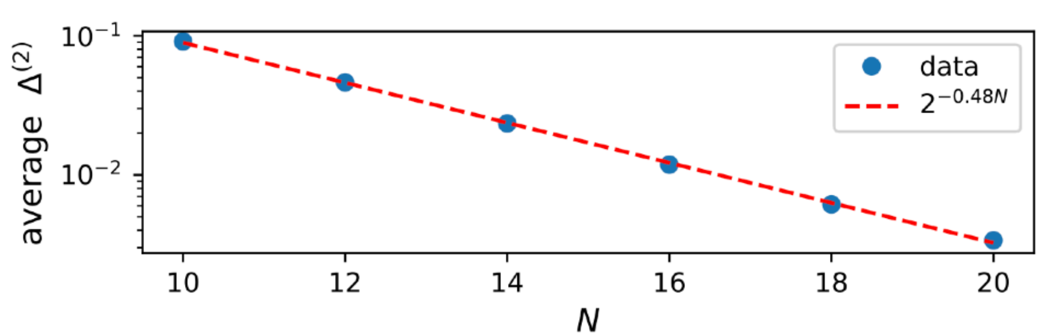


Convergence is exponential fast in system size!



Deep thermalization (emergence of Haar, Scrooge etc.) seems a generic behavior…

But does this maximum entropy paradigm always hold..?
Can there be ergodicity-breaking exceptions?

[Liu, Ippoliti, **WWH**, Phys. Rev. Lett. **136**, 100404 (2026)]
[Feng, Liu, Cheng, **WWH**, Ippoliti, arXiv:2606.08756]

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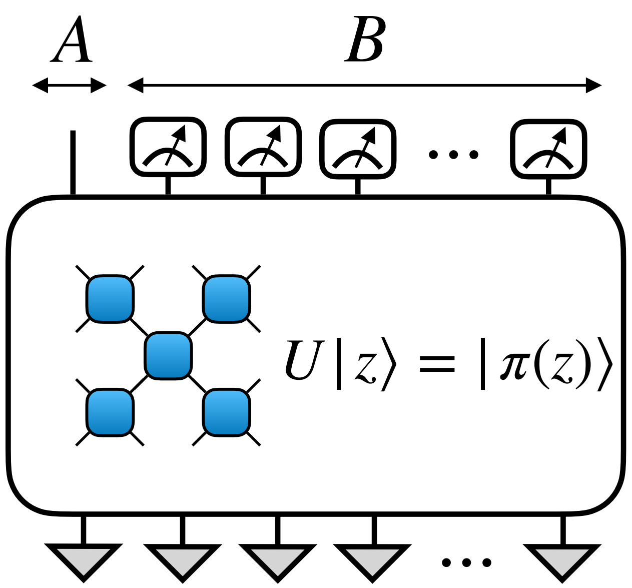
Another class of constrained dynamics… “Random Permutation Dynamics (RPD)”



Consider dynamics which map bit-strings to bit-strings

- Permutation π on computational basis states (“reversible cellular automaton”)
- Is classical: **conserves “coherence”**: $\sum_z c_z |z\rangle \mapsto \sum_z c_z |\pi(z)\rangle$
- But is also quantum: generates entanglement!

[Gopalakrishnan, Zakirov 2018; Iaconis et al 2019-20; Feng, MI, JHEP 2025; Bertini, Klobas, Kos, Malz PRX 2025; ...]

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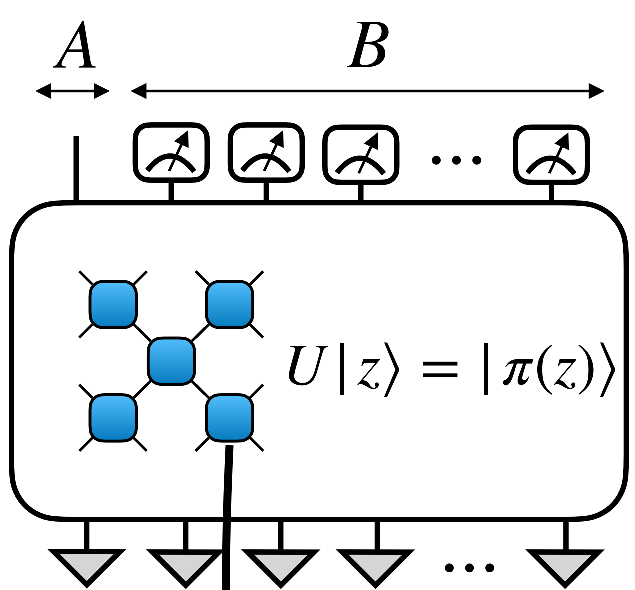

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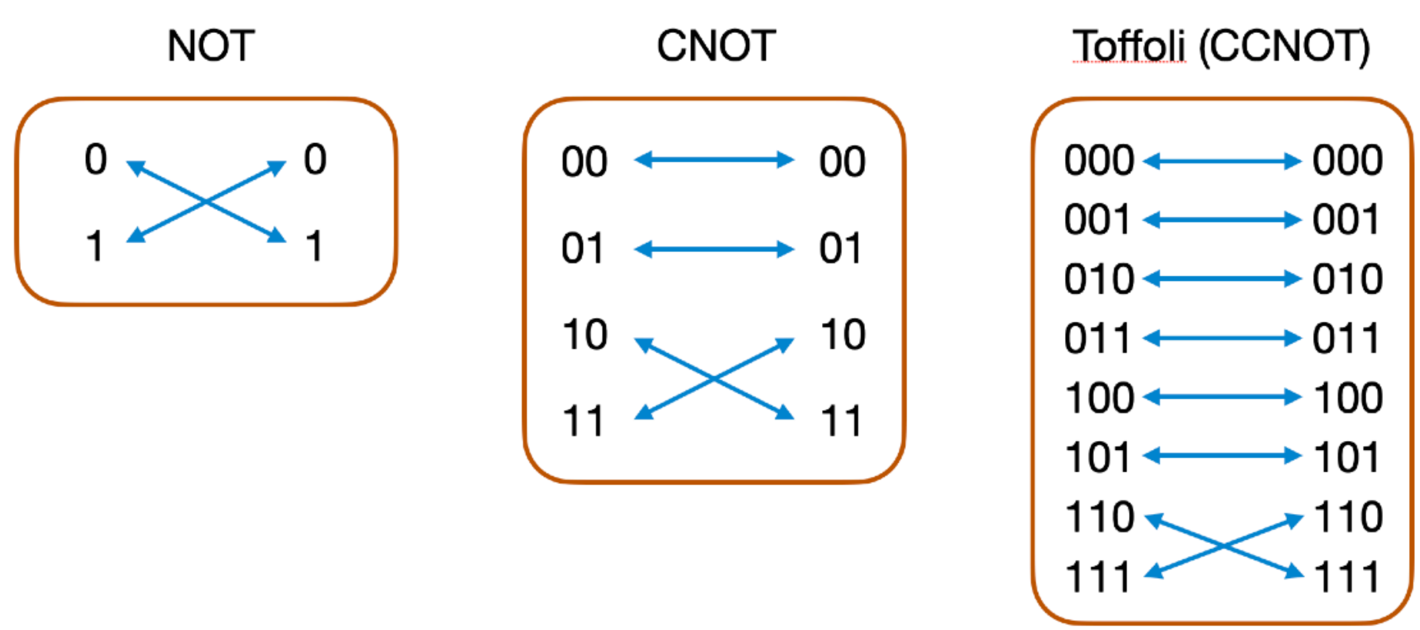

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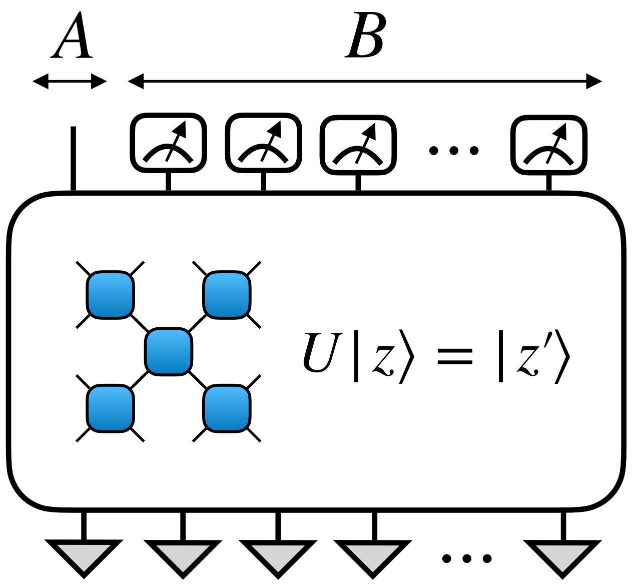
Deep thermalization under RPD

Local qubit measurement in z-x direction
(specified by Bloch angle θ_m)



$$|\,\Psi_0\rangle = (\cos(\theta_0/2)\,|\,0\rangle + i\sin(\theta_0/2)\,|\,1\rangle)^{\otimes N}$$

Uniform initial product state

$\theta_0 = \pi/2$

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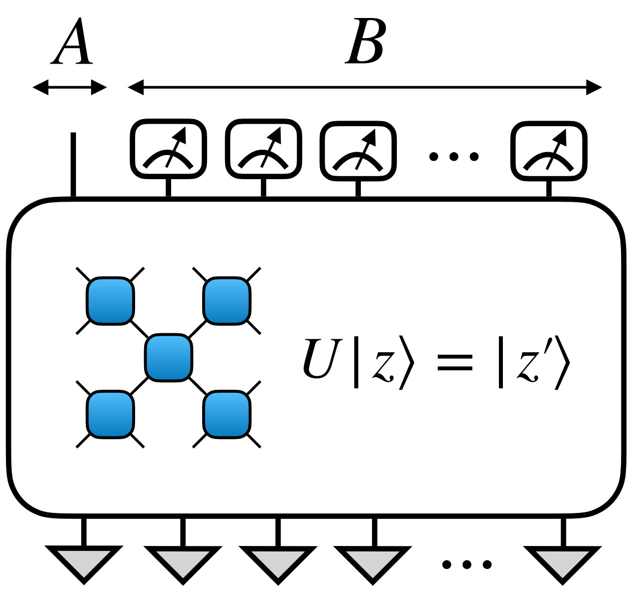

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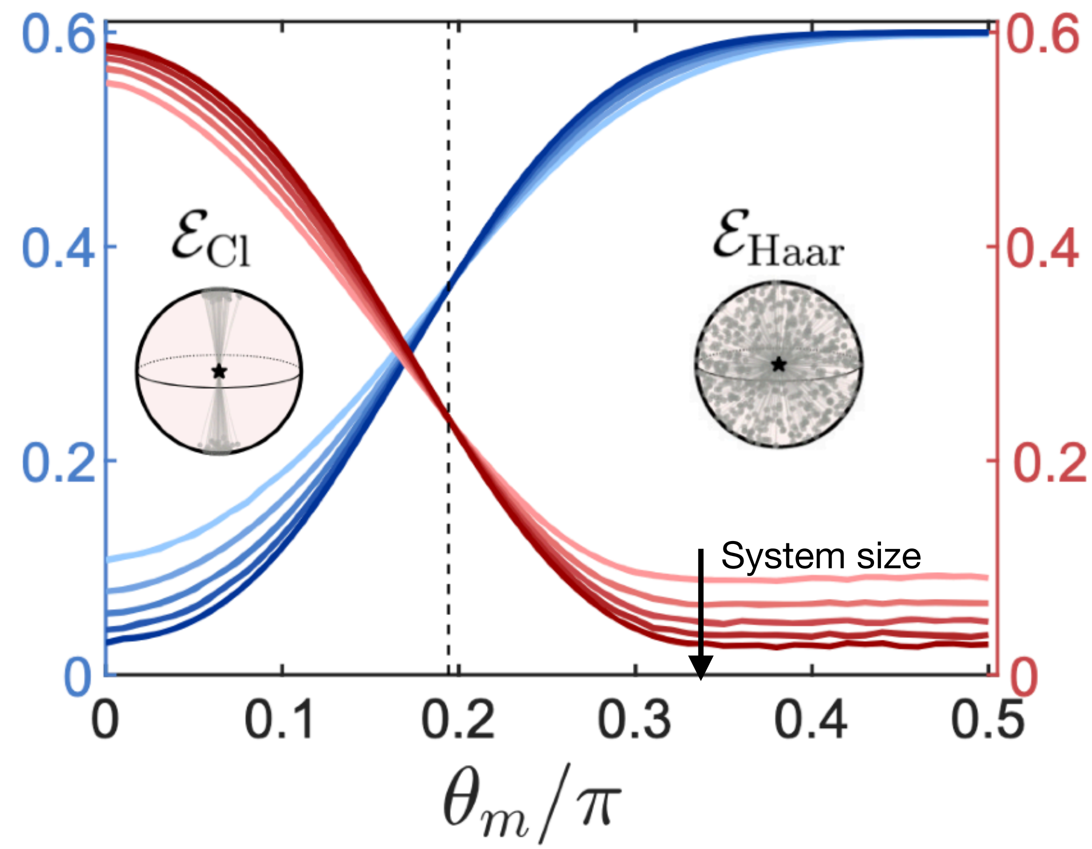
distance from ensemble of classical basis states

$\mathscr{E}_{cl} := \{\,|z_A\rangle\}$

distance from Haar distribution

$\mathscr{E}_{Haar} = \{d\psi_A, |\psi_A\rangle\}$



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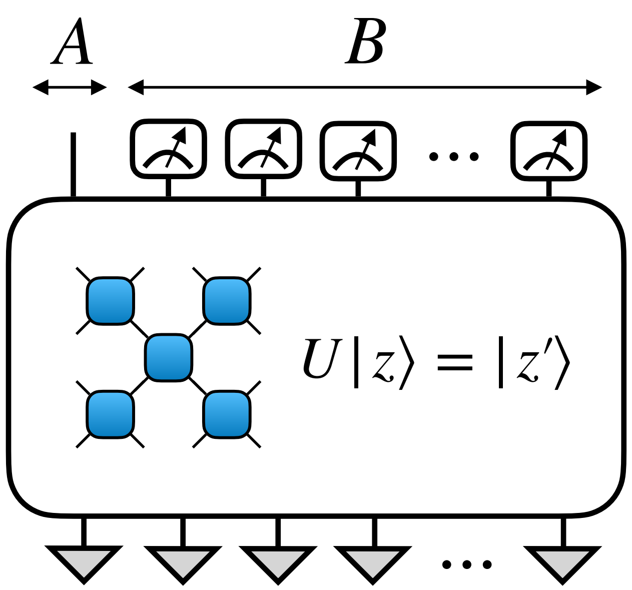

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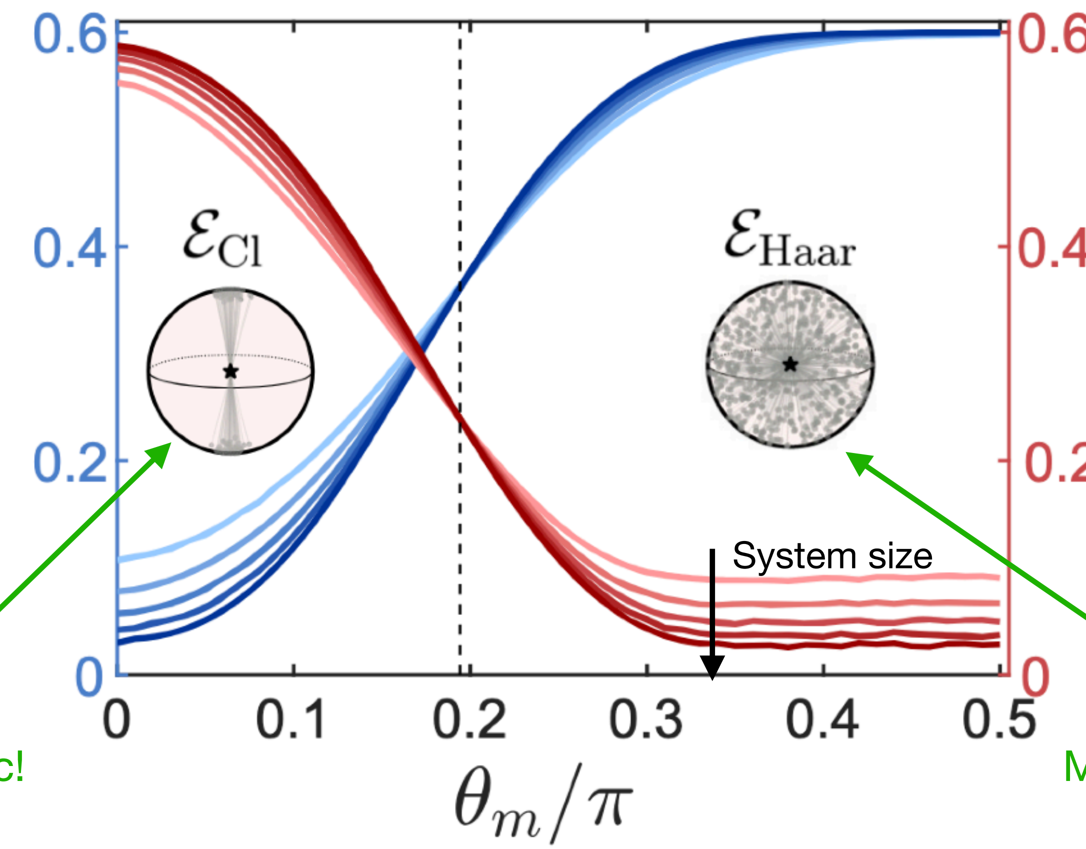

Minimally entropic!

Maximally entropic!

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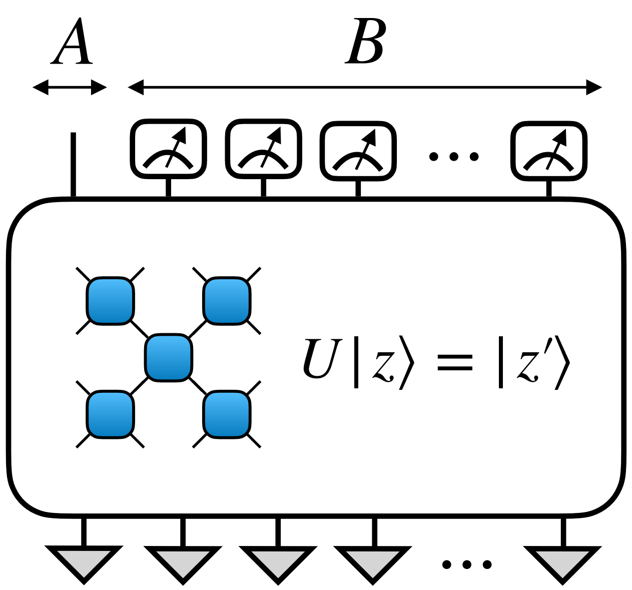

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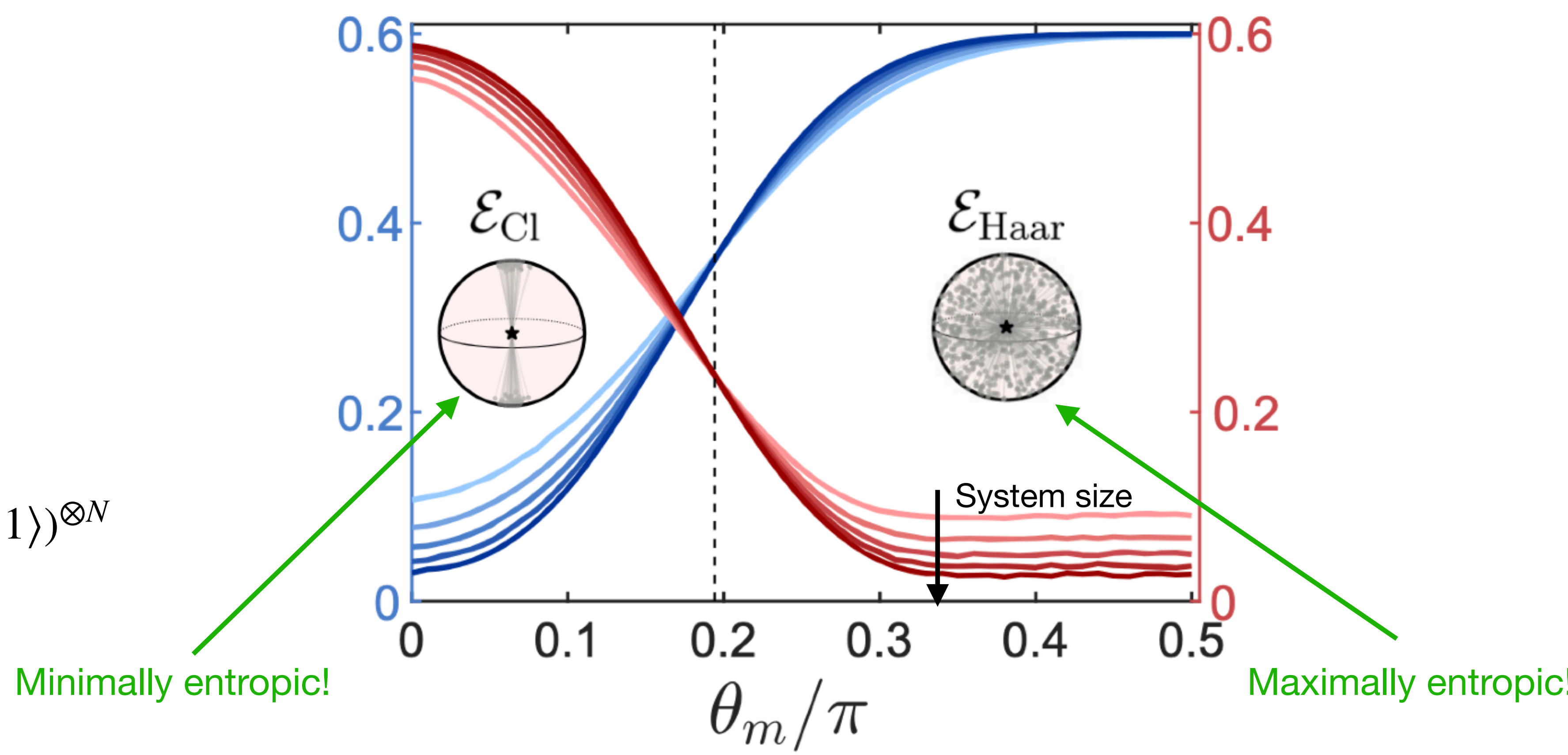
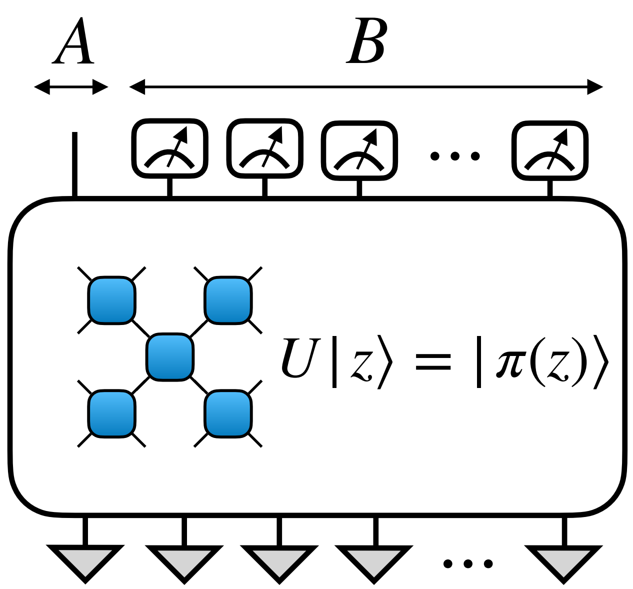


Phase transition in deep thermalization to a deep ergodicity breaking phase!
(Note: blind to local observables)

Puzzle: role of constraints??

Coherence conservation



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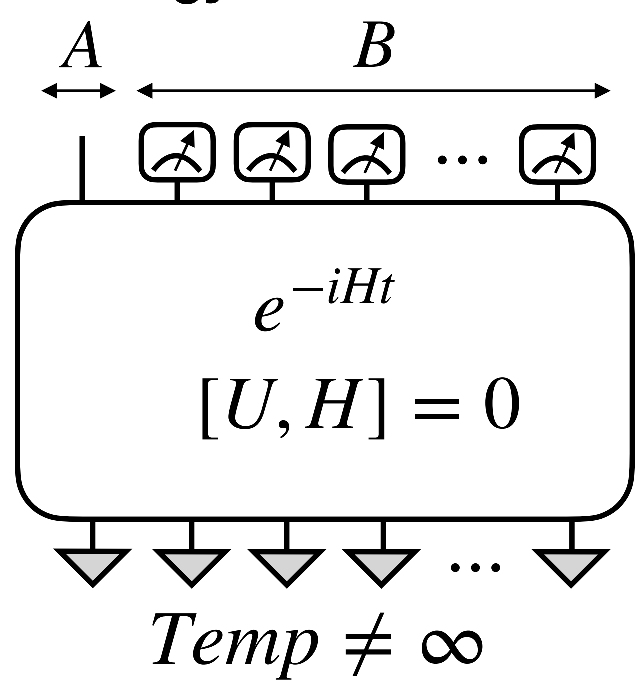

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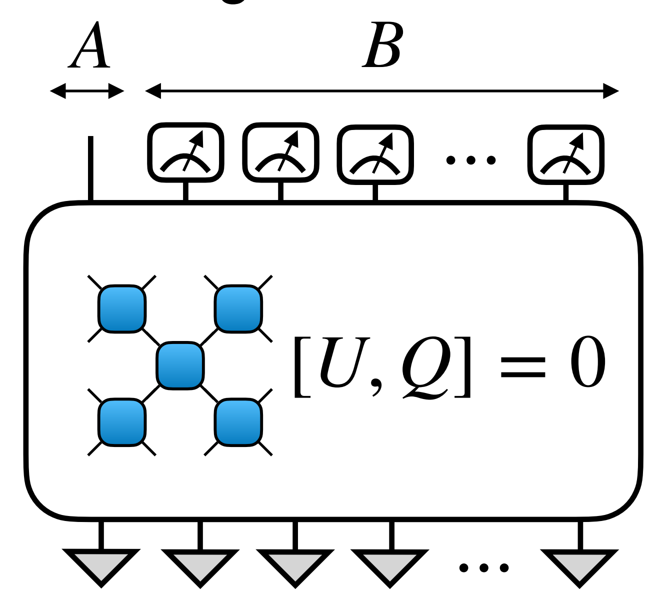

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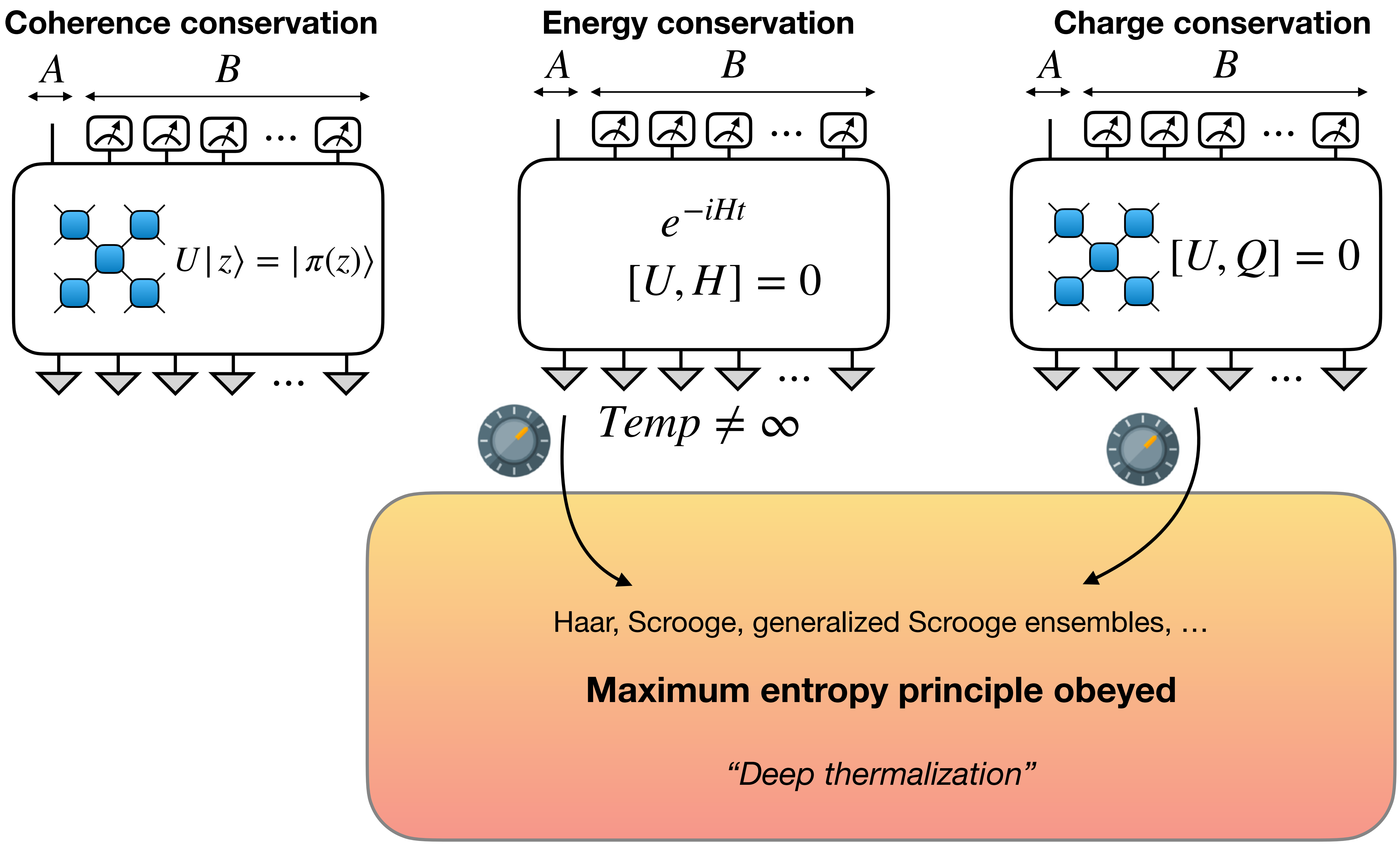

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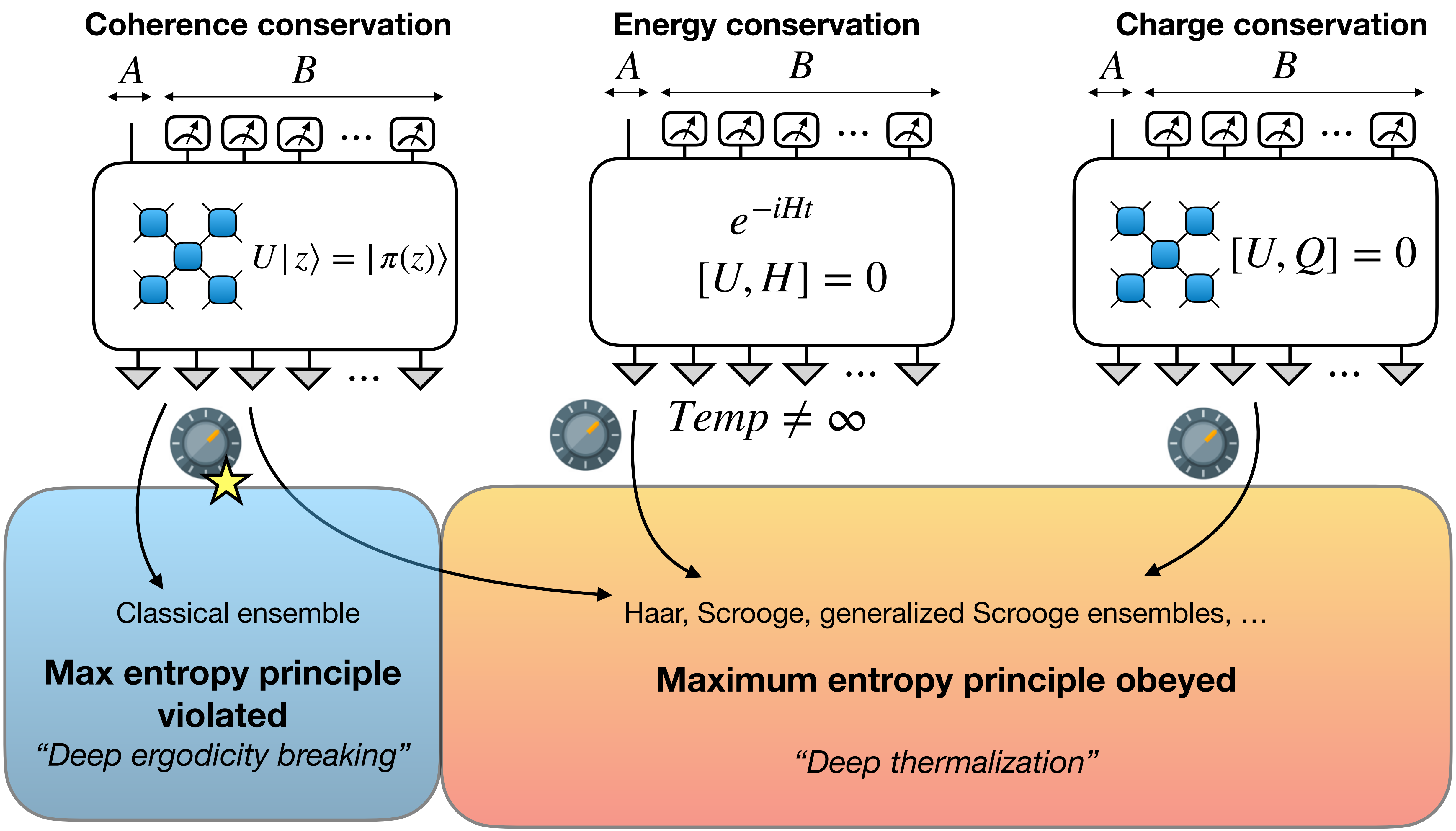
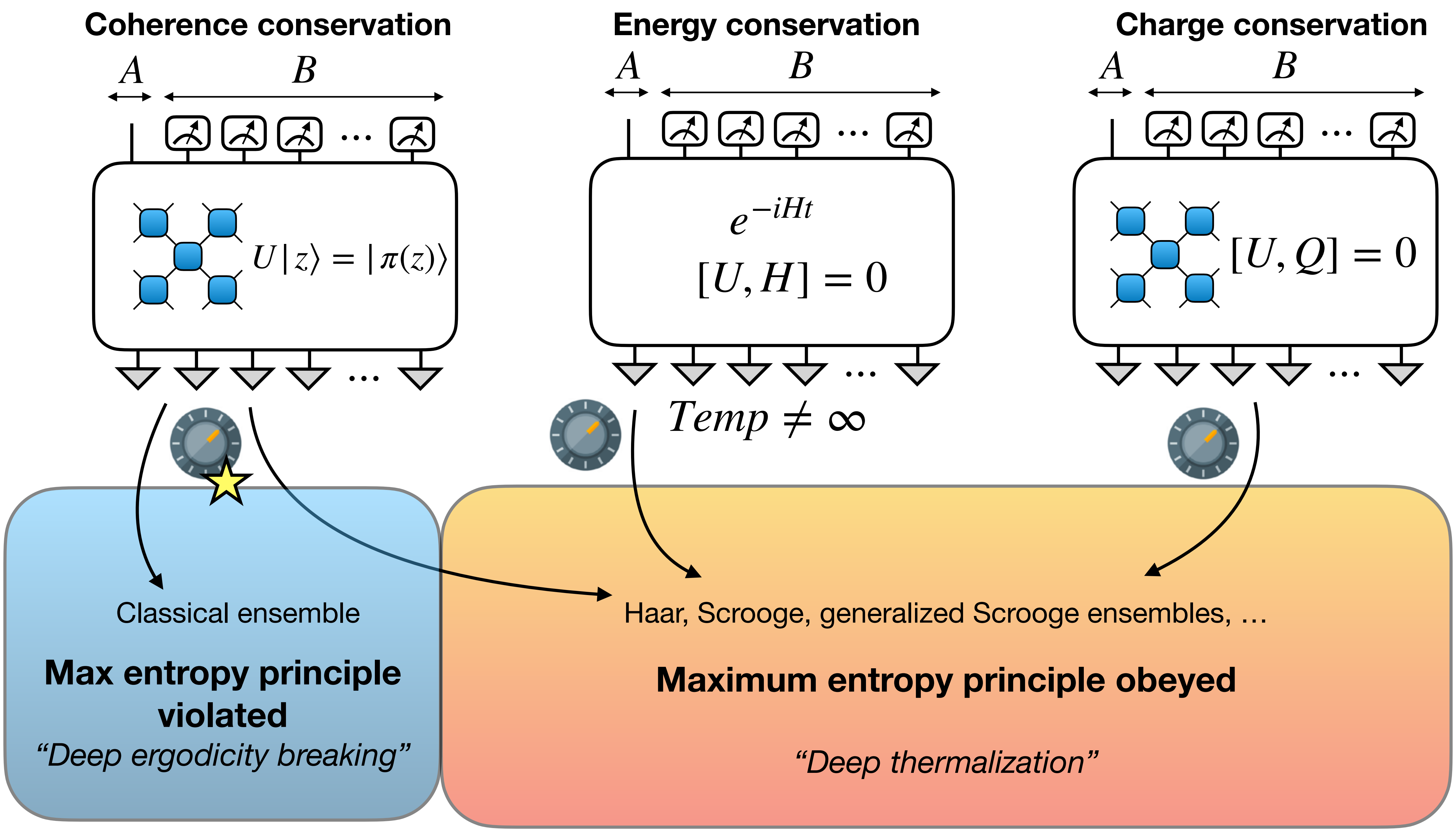
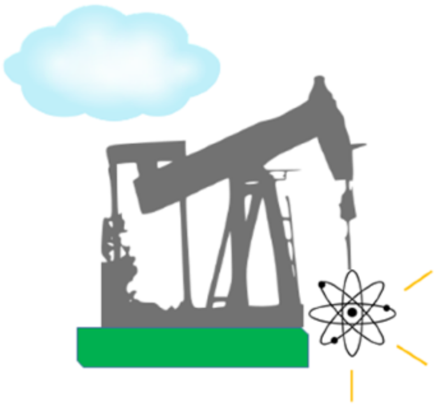


Puzzle: role of constraints??



Why do some constraints in dynamics lead to threshold behavior while others smooth behavior???

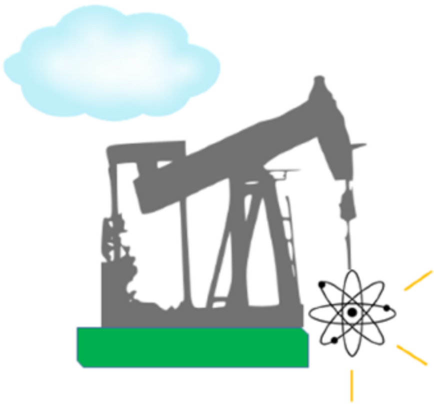
[Feng, Liu, Cheng, **WWH**, Ippoliti, arXiv:2606.08756]



Constraints through the lens of quantum resource theory

Key insight: constraints limit quantum information processing power of dynamics.
View in the unifying language of quantum resource theories

[Feng, Liu, Cheng, **WWH**, Ippoliti, arXiv:2606.08756]



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Crash course, QRT101: [Chitambar, Gour, RMP 91 025001 (2019)]

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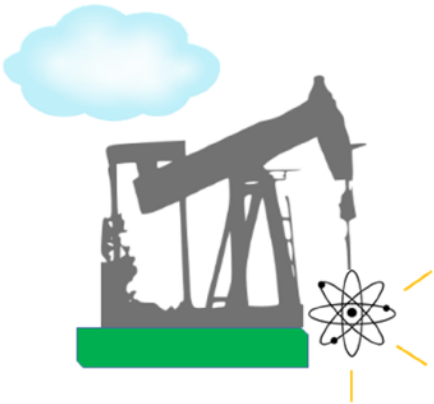

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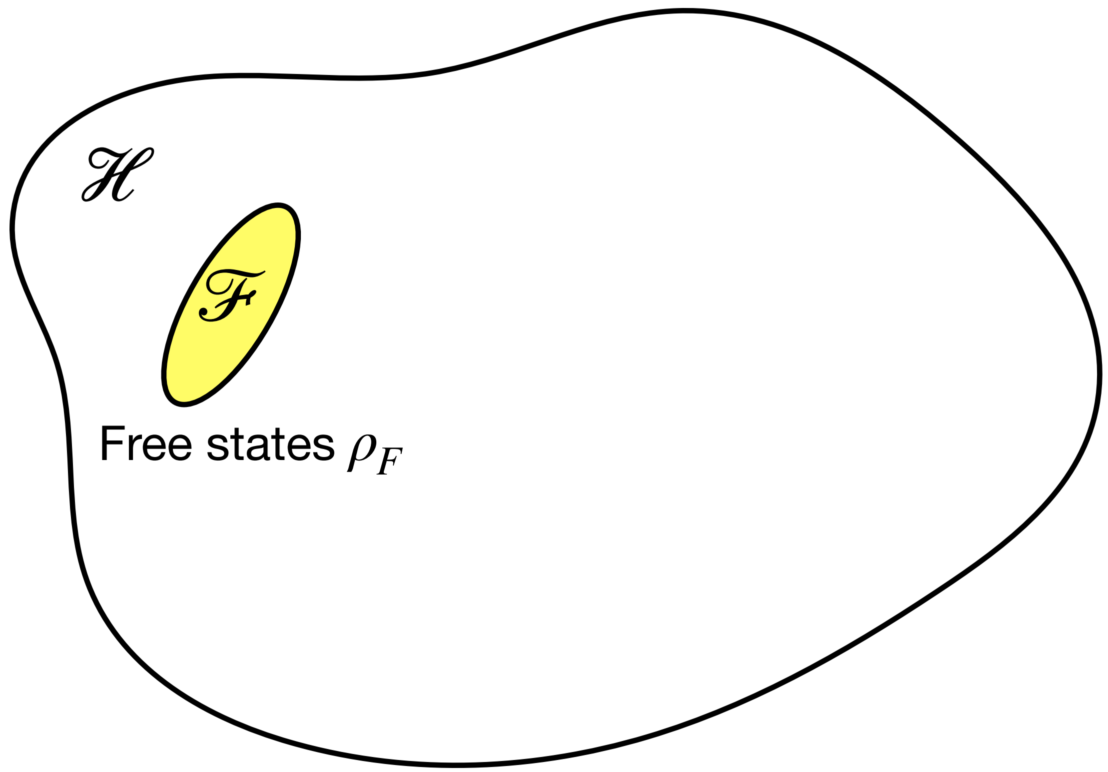

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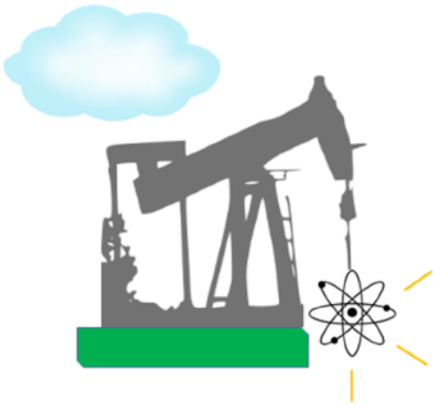

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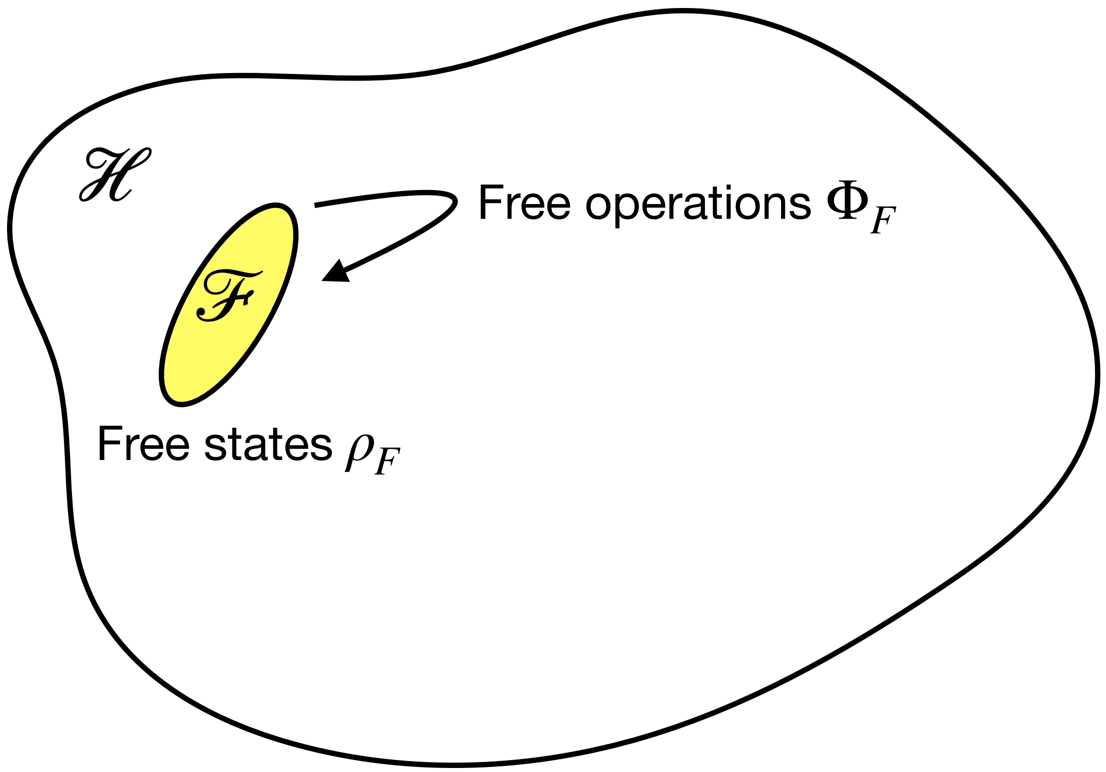

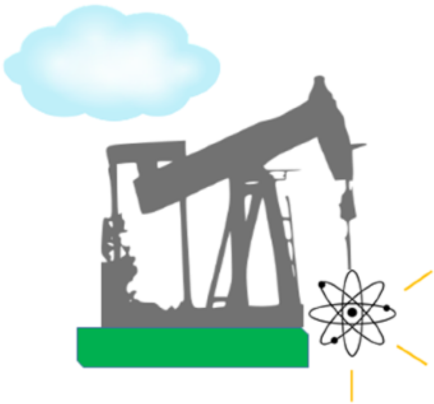

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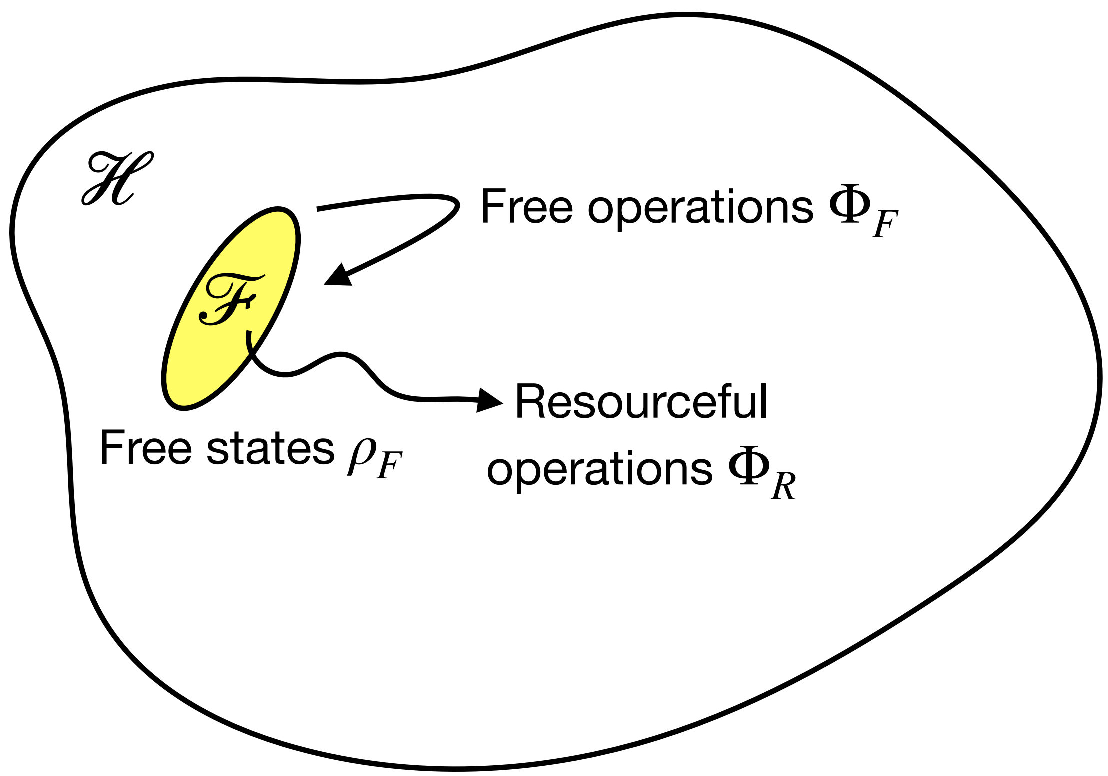

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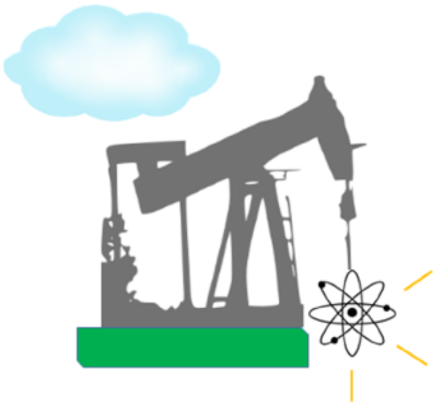

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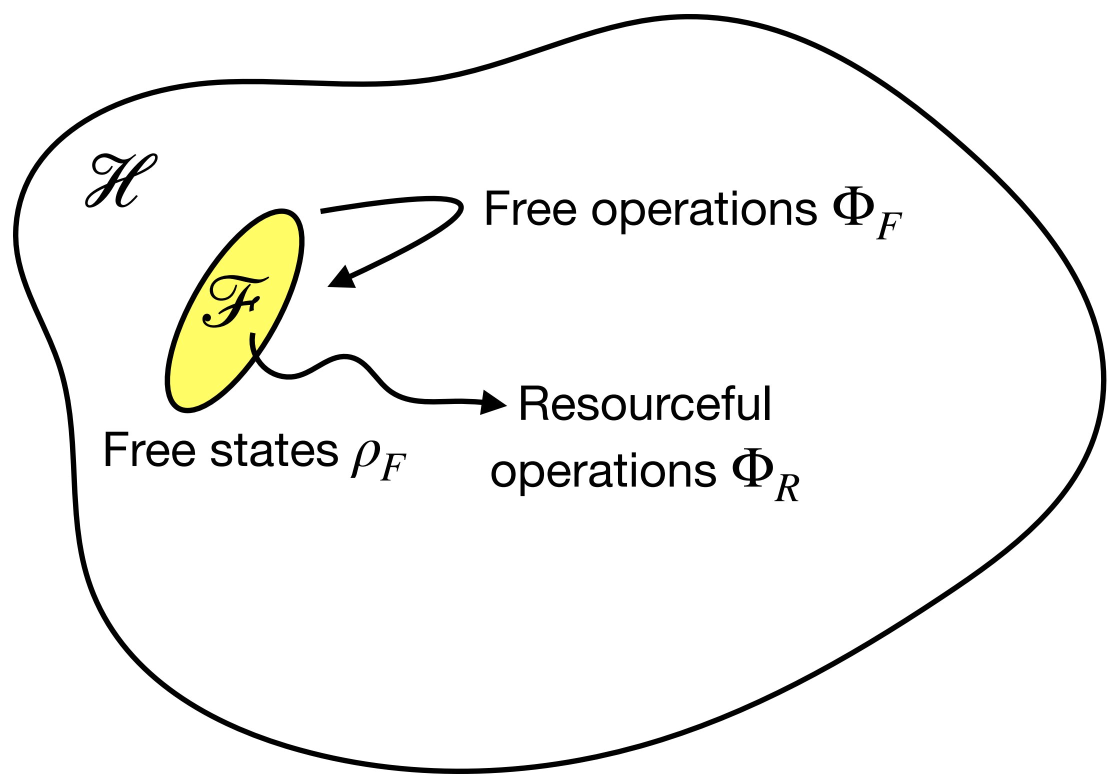
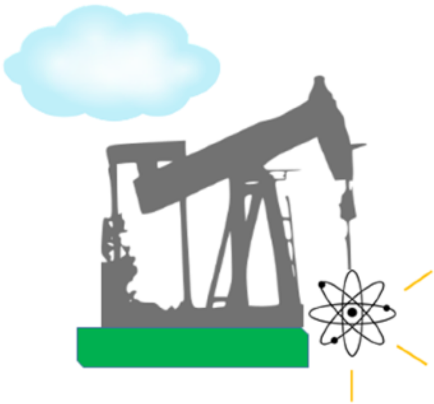


Resource monotone
R

[Feng, Liu, Cheng, **WWH**, Ippoliti, arXiv:2606.08756]



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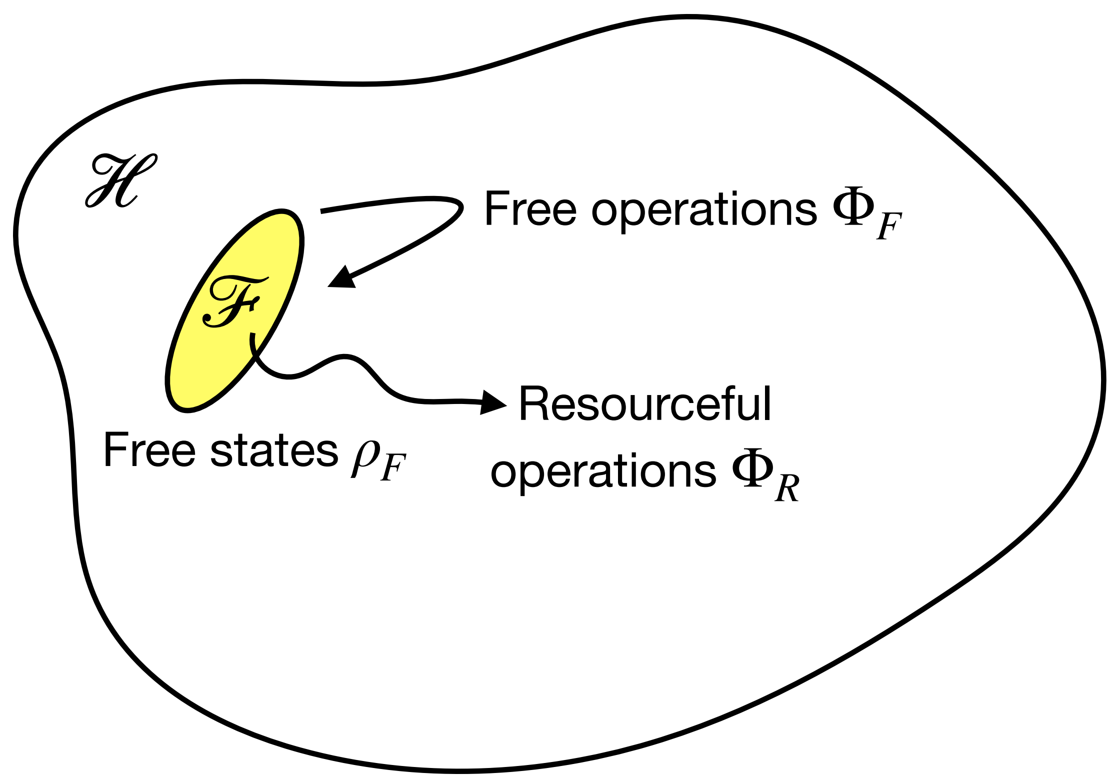
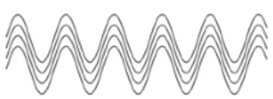
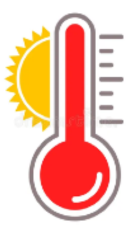


Resource monotone
R

By construction, operations limited to free states and free operations cannot perform **general** quantum information processing. **Injection** of resources necessary to go beyond.

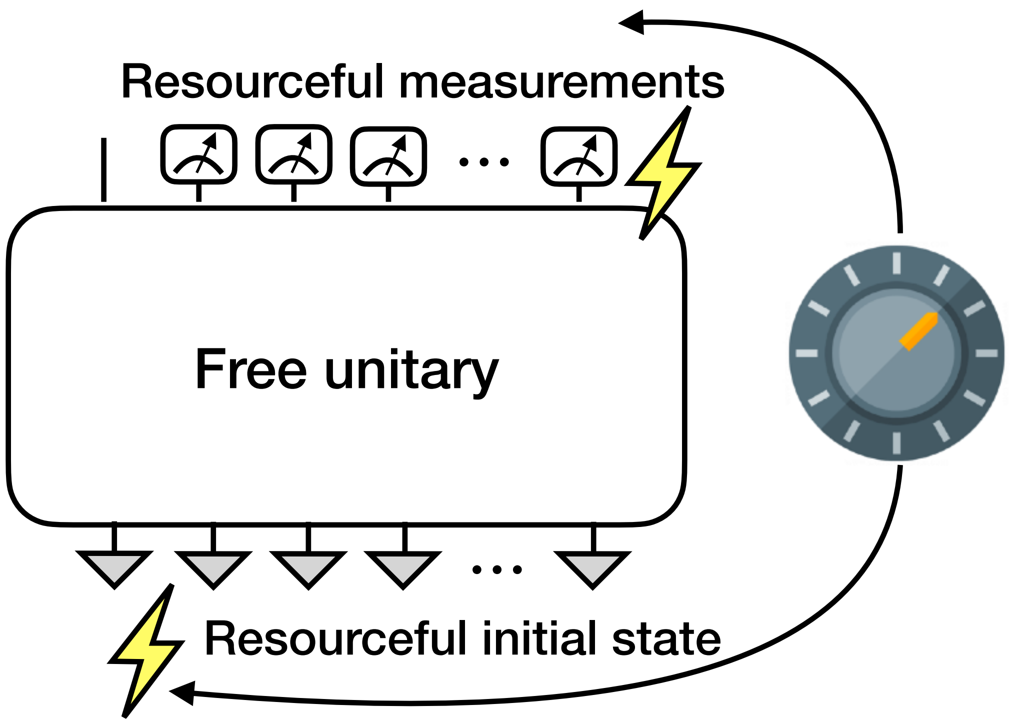
[Chitambar, Gour, RMP 91 025001 (2019)]

Examples of QRTs…

QRT	Free states	Free operations	Resource monotone
 Coherence	Computational basis states $\|z\rangle$ and mixtures	Permutation dynamics, Diagonal phase gates, Dephasing etc.	Relative entropy of coherence $C(\rho) = S(\rho_{Dephased}) - S(\rho)$
 Magic	Stabilizer states and mixtures	Clifford unitaries etc.	Stabilizer Renyi Entropy $M(\|\psi\rangle) = -\log_2(\sum_P \langle\psi\|P\|\psi\rangle^4/2^N)$
 Athermality “Non-equilibrium”	Gibbs states	Energy preserving operations, e.g. Hamiltonian dynamics	Nonequilibrium Helmholtz free energy $F(\rho) = \mathrm{Tr}(\rho H) - k_B T S(\rho)$
 Asymmetry “Breaking of charge conservation”	Block diagonal mixtures of charge sectors	Charge conserving dynamics etc.	Relative entropy of athermality $\Delta S(\rho) = S(\mathcal{G}(\rho)) - S(\rho)$ $\mathcal{G}$: Twirl of symmetry group

[Feng, Liu, Cheng, **WWH**, Ippoliti, arXiv:2606.08756]

Deep thermalization with quantum resource constraints



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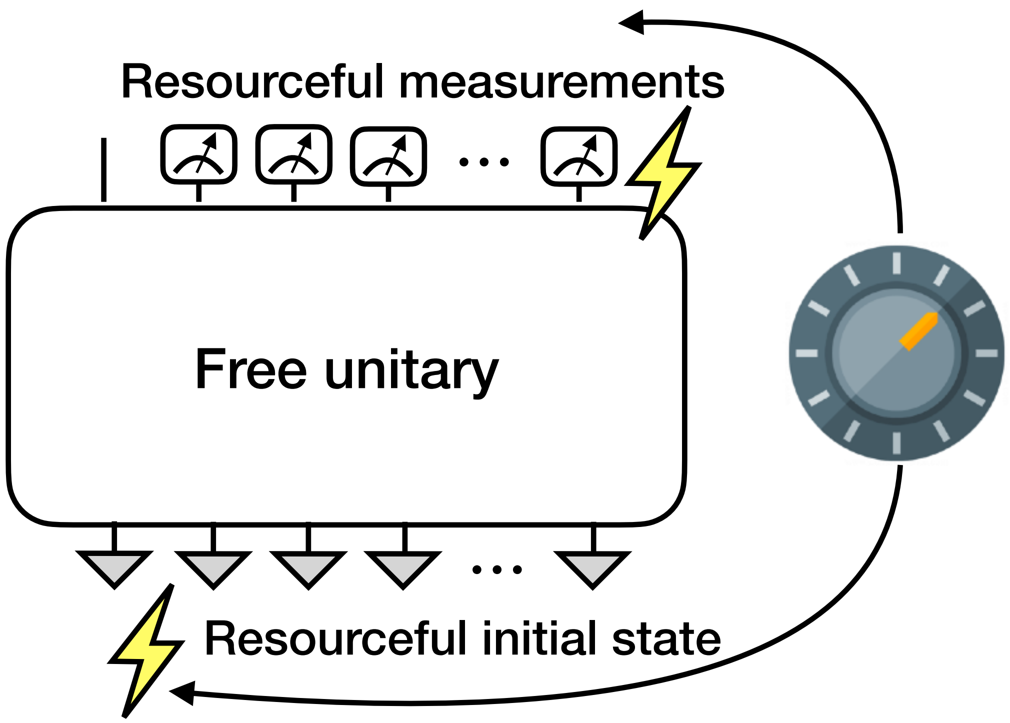

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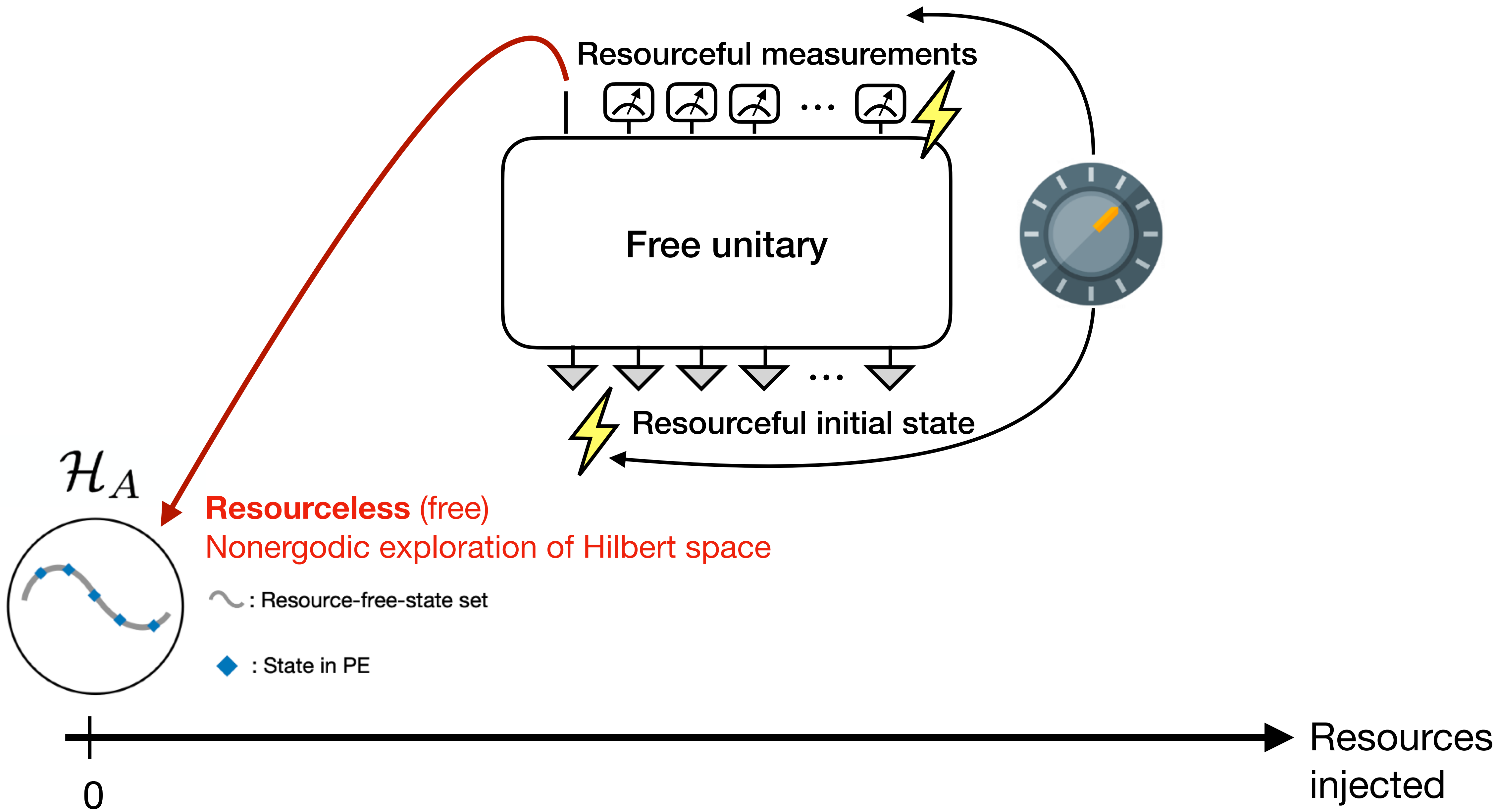

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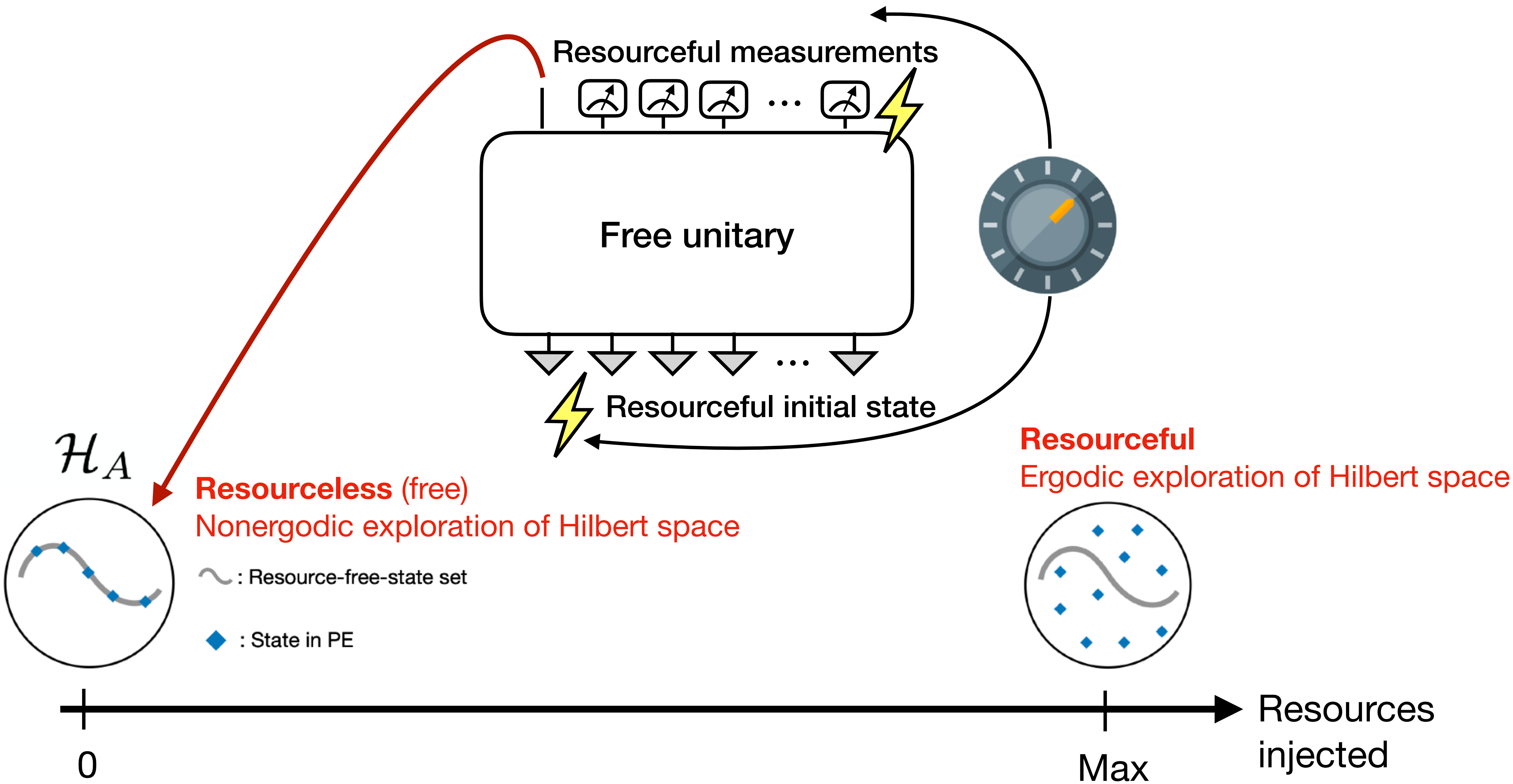

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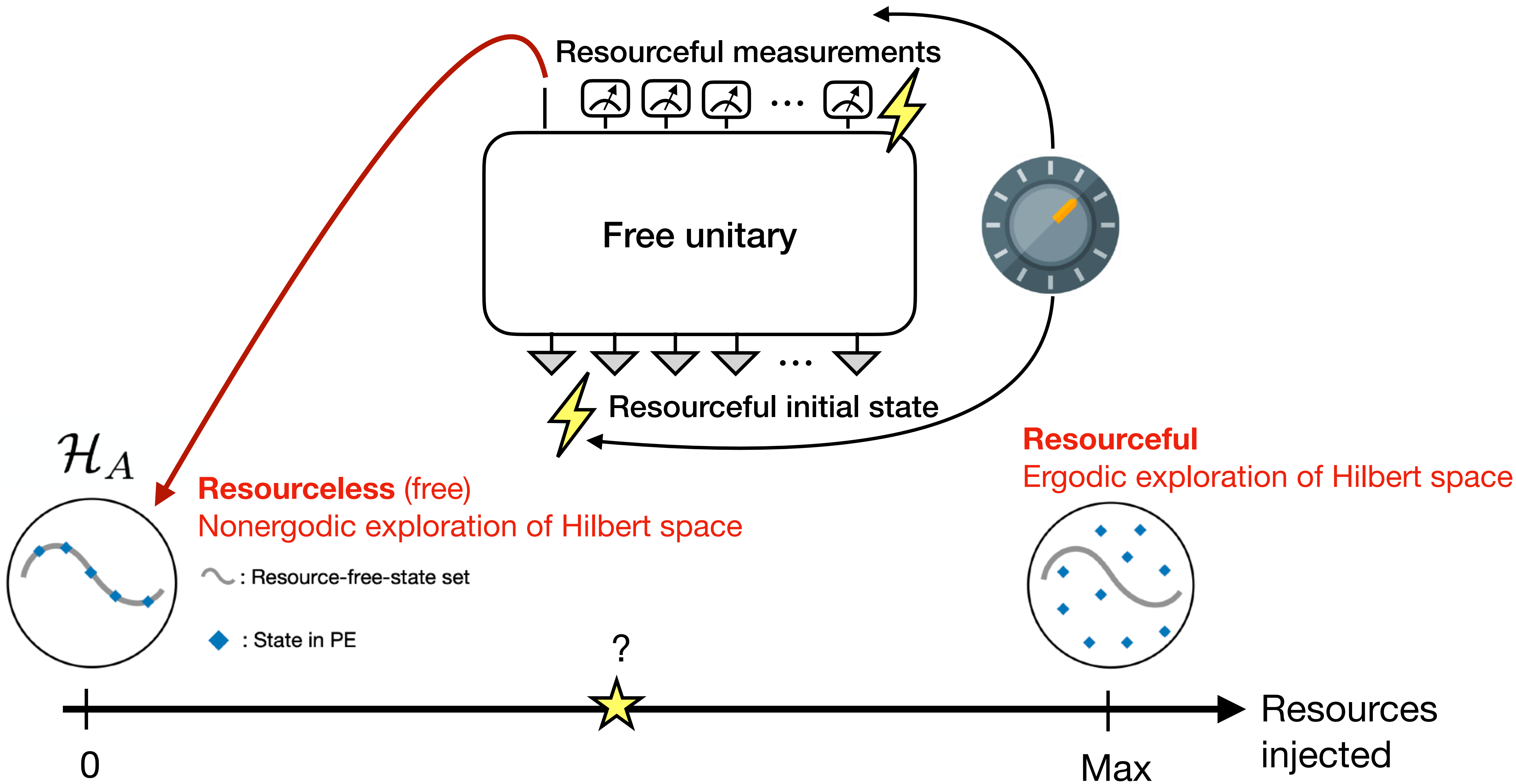
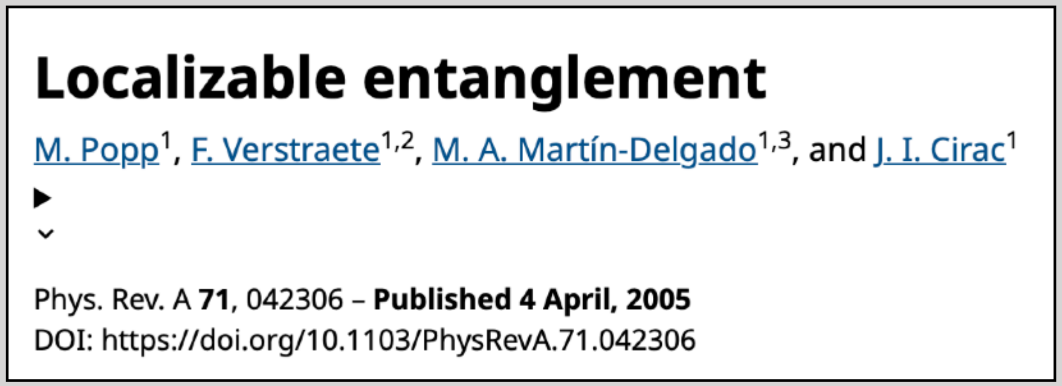


Deep thermalization with quantum

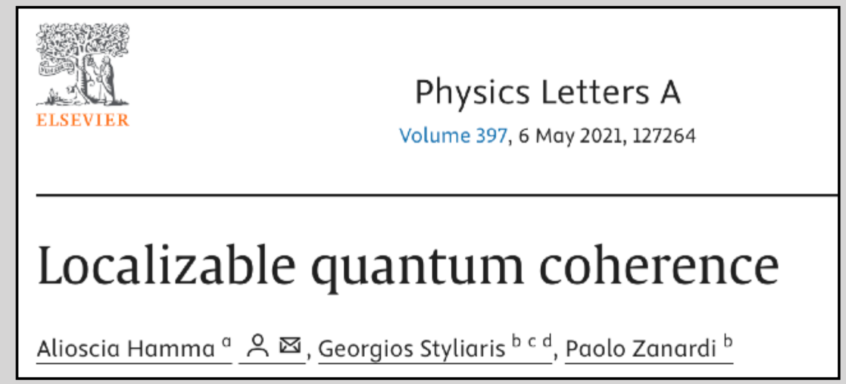
Key concept: concentration of resource in local subsystem: "**quantum resource localization**"



Localizable entanglement

M. Popp[1], F. Verstraete[1,2], M. A. Martín-Delgado[1,3], and J. I. Cirac[1]

Phys. Rev. A **71**, 042306 – **Published 4 April, 2005**
DOI: https://doi.org/10.1103/PhysRevA.71.042306



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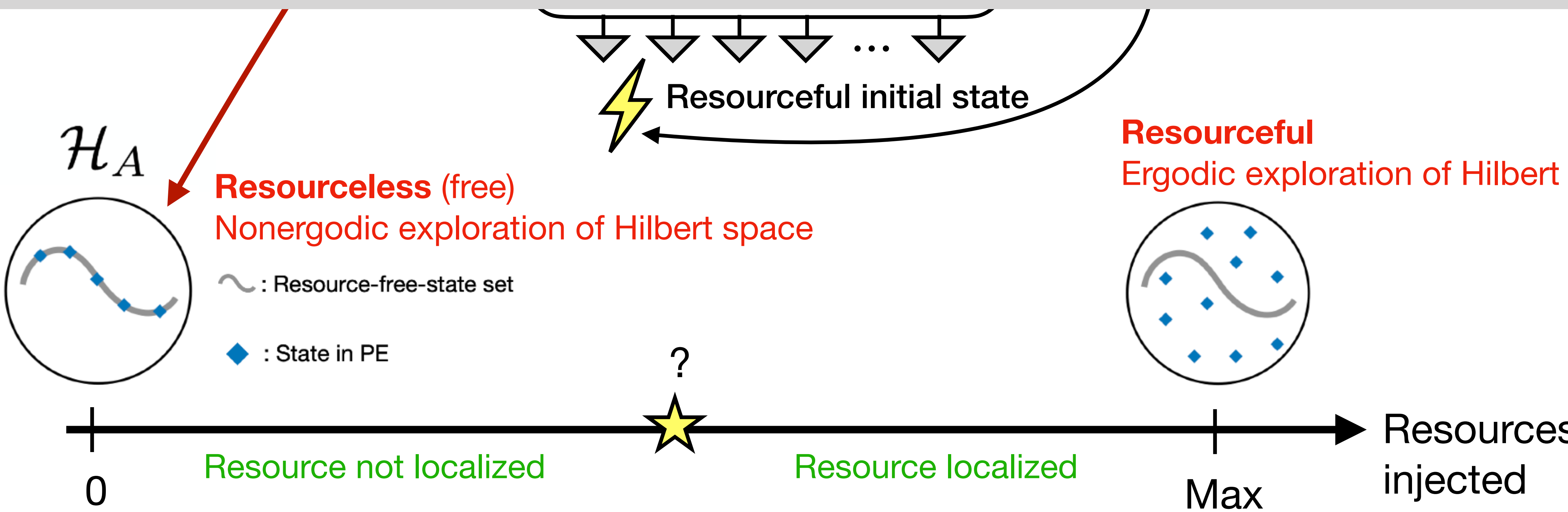
Physics Letters A
Volume 397, 6 May 2021, 127264

Localizable quantum coherence

Alioscia Hamma[a], Georgios Styliaris[b c d], Paolo Zanardi[b]



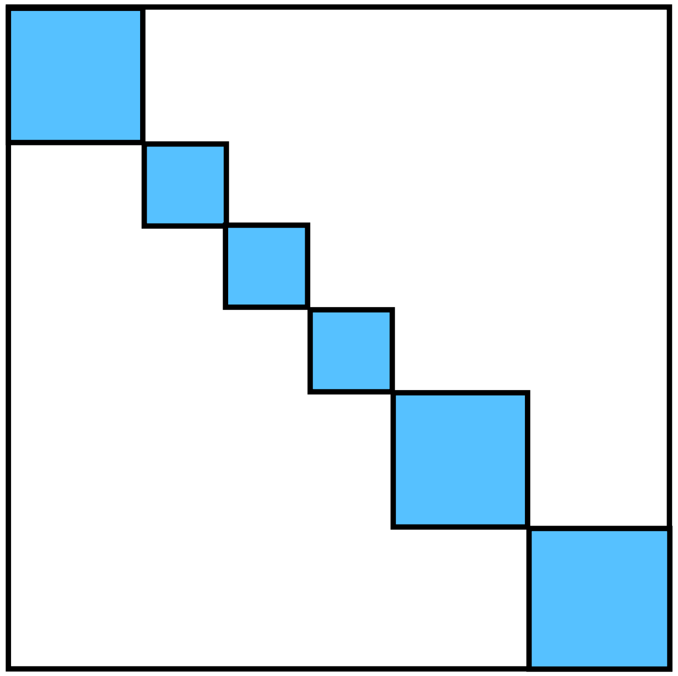
Is there a finite localization threshold?

Theory of quantum resource localization

Focus on "subspace coherence" QRTs: subspaces (blocks) in Hilbert space of bit-strings z

$$\mathscr{H} = \oplus_i \mathscr{B}_i$$

$\dim(\mathscr{B}_i) = d_i$



Note: generalization of charge-sharpening transitions in MIPT [Agrawal et al PRX'22, Barratt et al PRL'22, Feng et al Quantum '25]

[Feng, Liu, Cheng, **WWH**, Ippoliti, arXiv:2606.08756]

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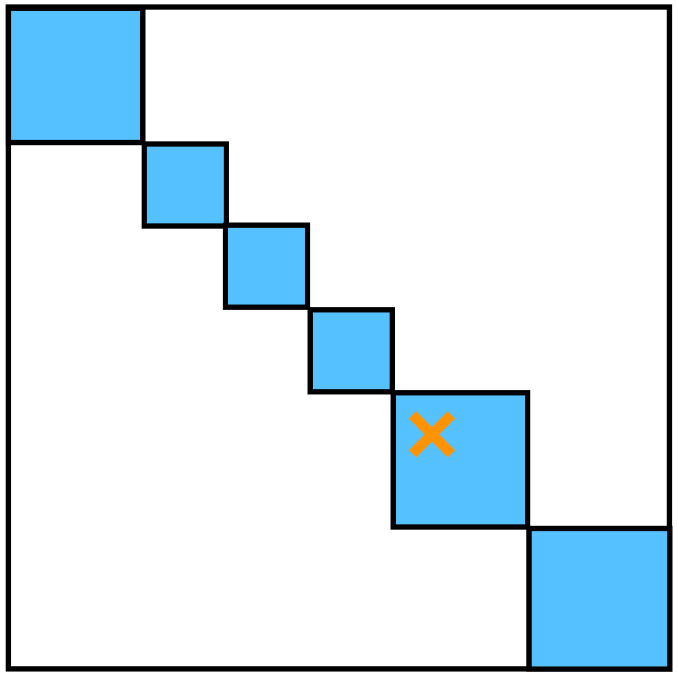

Free (pure) state:
$|\Psi\rangle = |\Phi_i\rangle$

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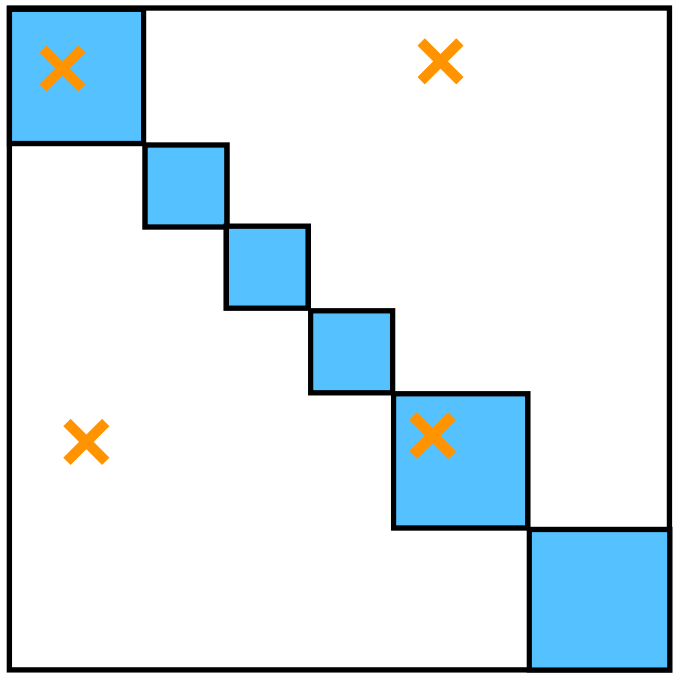

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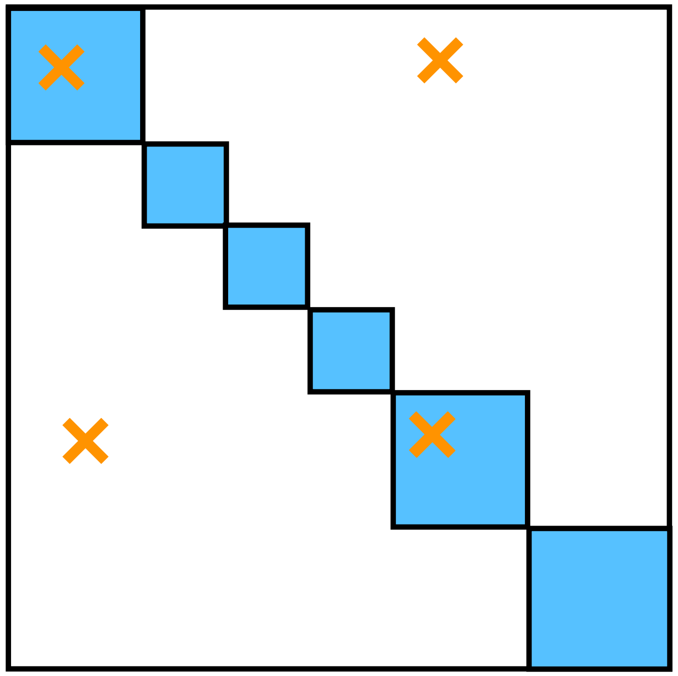

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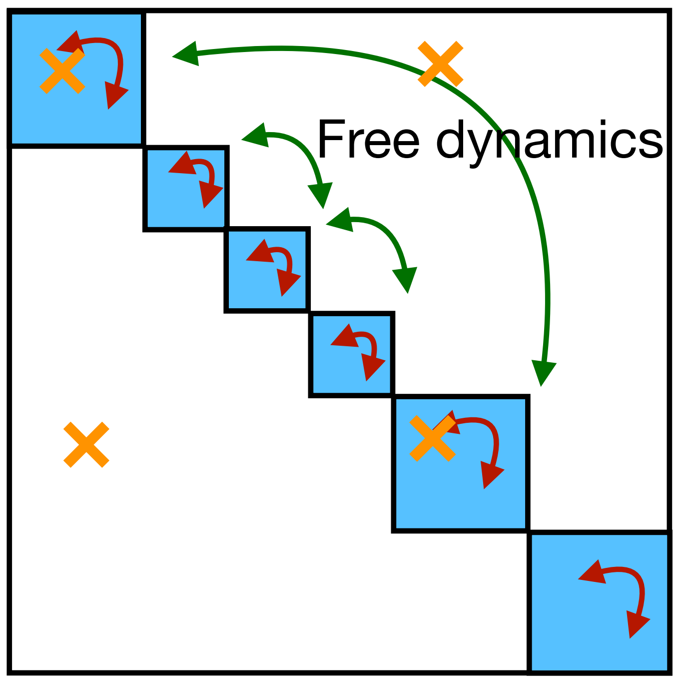

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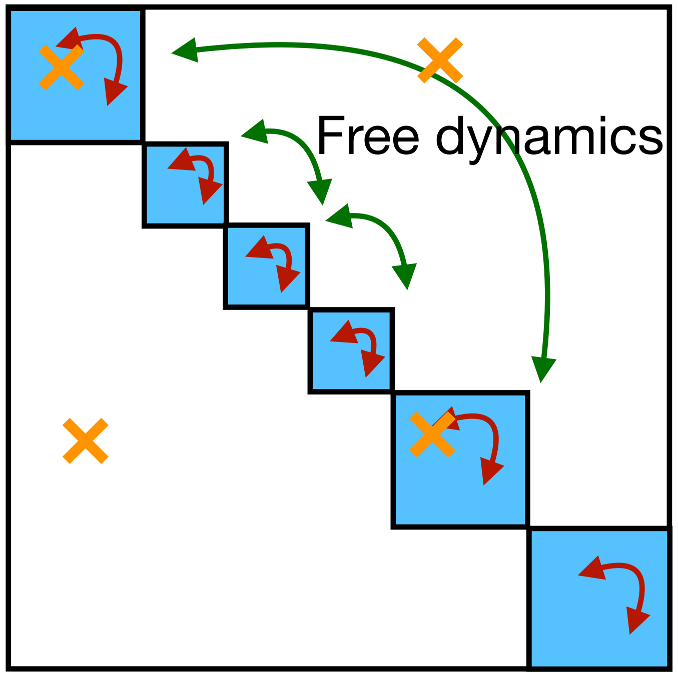

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Key question:

Given initial state $|\Psi_0\rangle$ which has some fuzziness in resource and measurements in a basis which is also fuzzy, can the observer, upon receiving measurement outcome $|\Psi_m\rangle$, infer which the block $\mathscr{B}_i$ the system has collapsed to?

If yes: block-sharp, with high probability post-measurement state is resourceless (single bit-string z)
If not: block-fuzzy, with high probability post-measurement state is resourceful (random looking)

"Block sharpening"

Note: generalization of charge-sharpening transitions in MIPT [Agrawal et al PRX'22, Barratt et al PRL'22, Feng et al Quantum '25]

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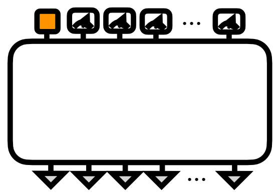
Microscopically, resource content of projected state governed by matrix element

$\langle z_A | \tilde{\psi}_A(\nu) \rangle =$  $= \langle \Psi_m | U_{free} | \Psi_0 \rangle$ (Coefficient of post-measurement state)

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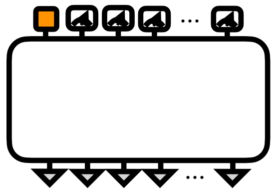
$\langle z_A | \tilde{\psi}_A(\nu) \rangle =$  $= \langle \Psi_m | U_{free} | \Psi_0 \rangle$ (Coefficient of post-measurement state)

Injection of initial (I) resource
Superposition of $2^{\alpha_0 N}$ bit-strings

[Feng, Liu, Cheng, **WWH**, Ippoliti, arXiv:2606.08756]

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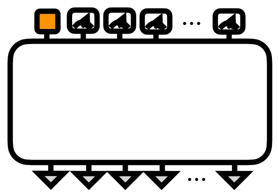
$\langle z_A | \tilde{\psi}_A(\nu) \rangle =$  $= \langle \Psi_m | U_{free} | \Psi_0 \rangle$ (Coefficient of post-measurement state)

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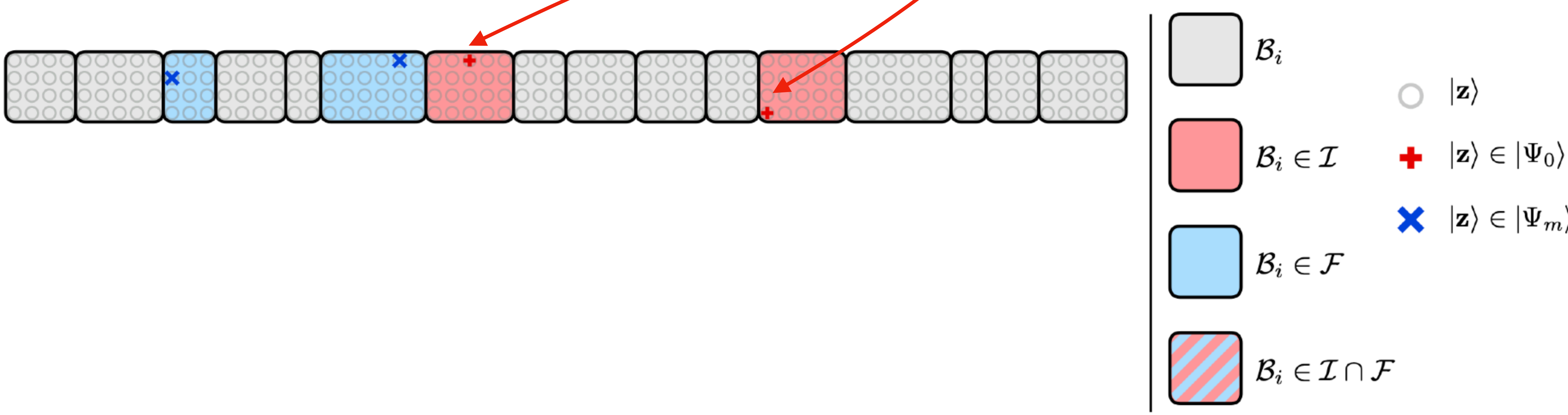

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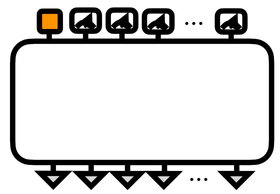
$\langle z_A | \tilde{\psi}_A(\nu) \rangle =$  $= \langle \Psi_m | U_{free} | \Psi_0 \rangle$ (Coefficient of post-measurement state)

Scrambler

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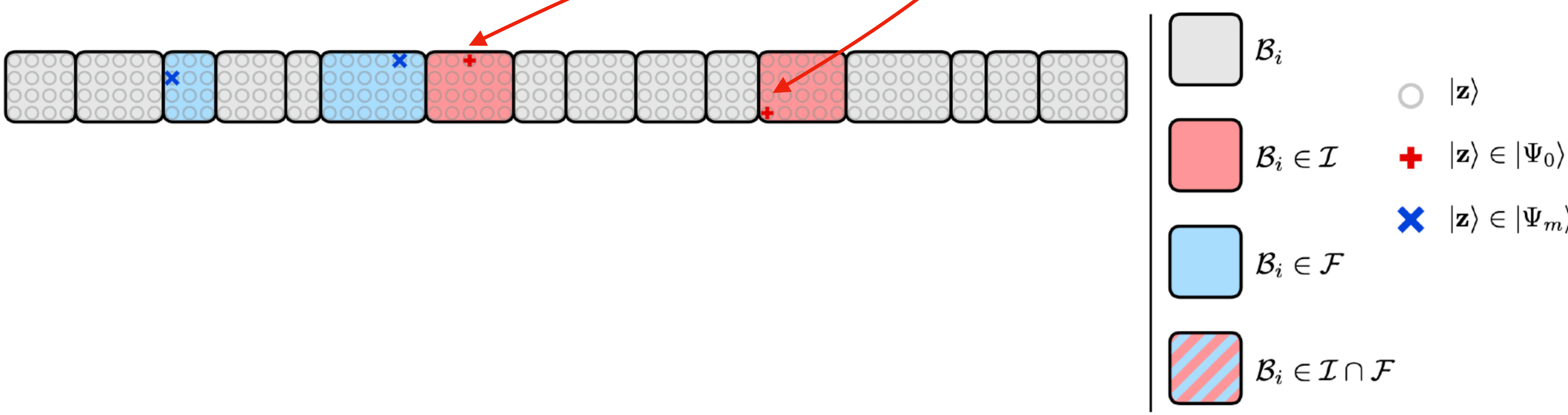

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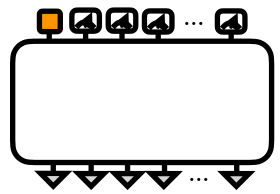
$\langle z_A | \tilde{\psi}_A(\nu) \rangle =$  $= \langle \Psi_m | U_{free} | \Psi_0 \rangle$ (Coefficient of post-measurement state)

Injection of final (F) resource
Superposition of $2^{\alpha_m N}$ bit-strings

Scrambler

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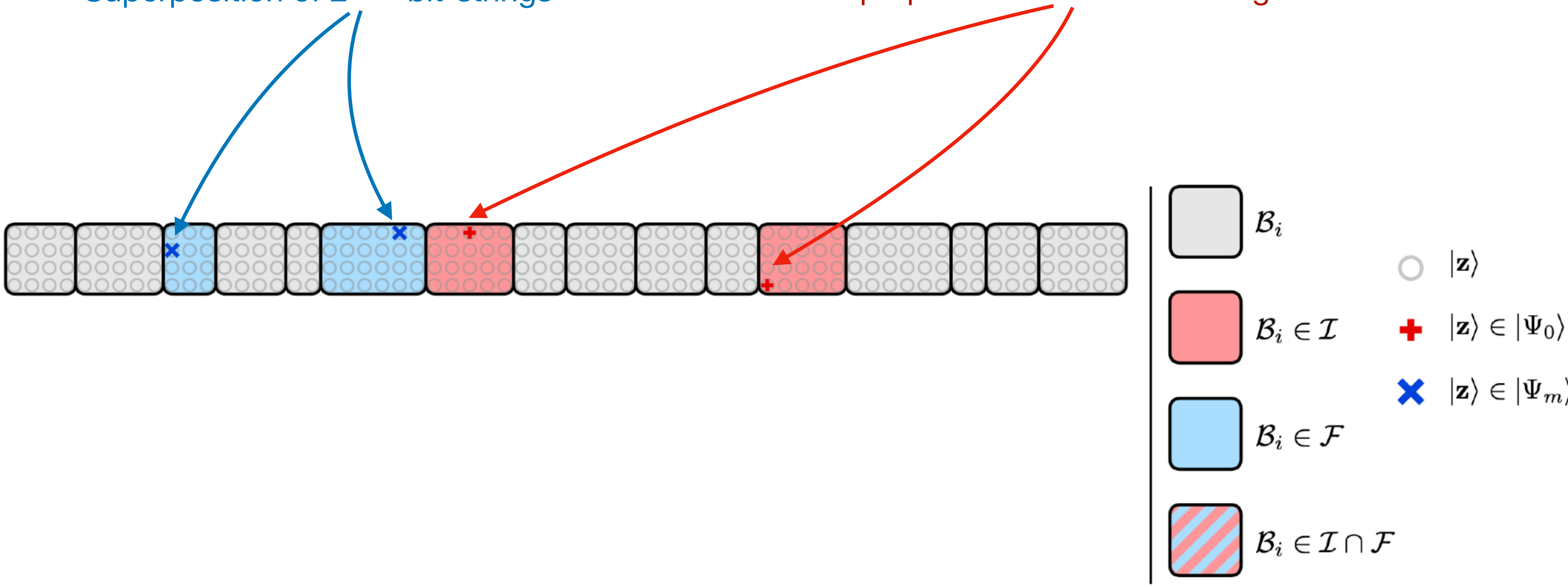
Superposition of $2^{\alpha_0 N}$ bit-strings



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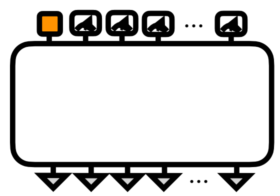
$\langle z_A | \tilde{\psi}_A(\nu) \rangle =$  $= \langle \Psi_m | U_{free} | \Psi_0 \rangle$ (Coefficient of post-measurement state)

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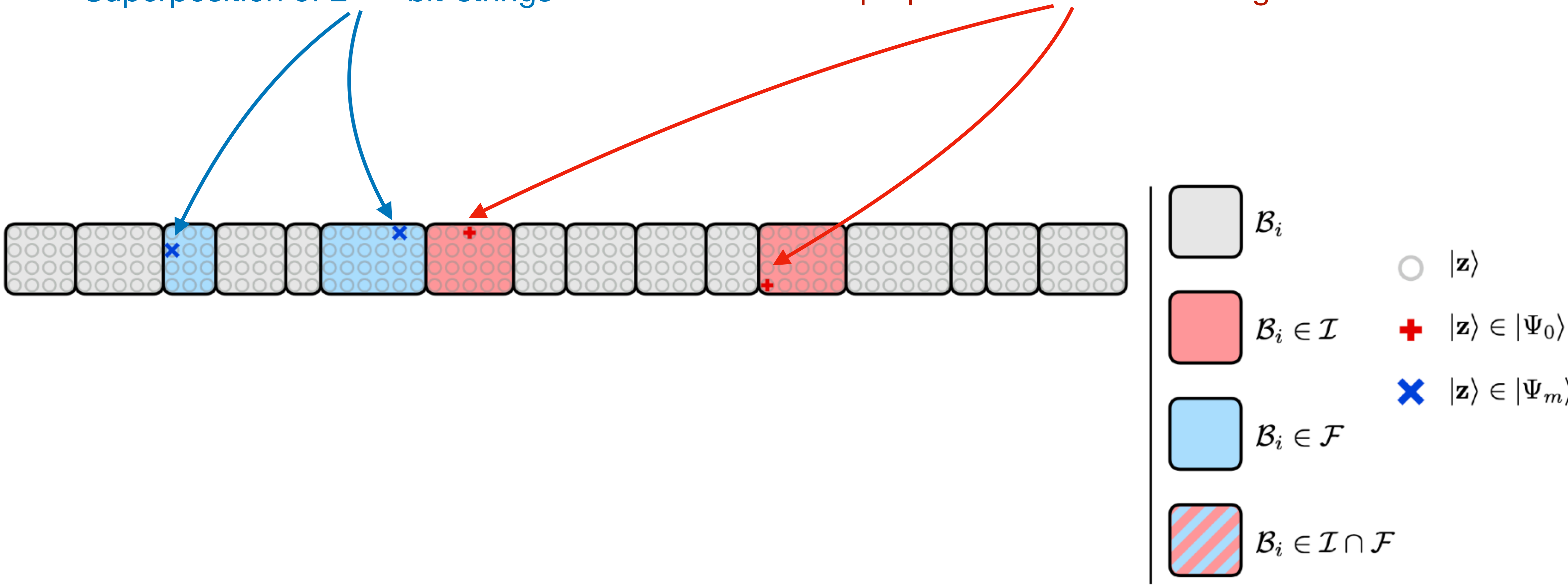
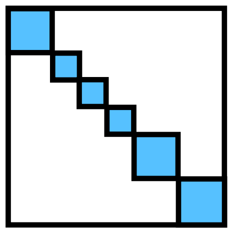


Number of intersections of blocks $\mathscr{B}_i$: **random subset intersection problem or generalized birthday problem!**

Exhibits a threshold behavior depending on **block-size distribution** of QRT and **density** α_0, α_m of injected resource

[Feng, Liu, Cheng, **WWH**, Ippoliti, arXiv:2606.08756]



$$\mathscr{H} = \bigoplus_i \mathscr{B}_i$$

$$p_i = \mathrm{dim}(\mathscr{B}_i)/2^N$$

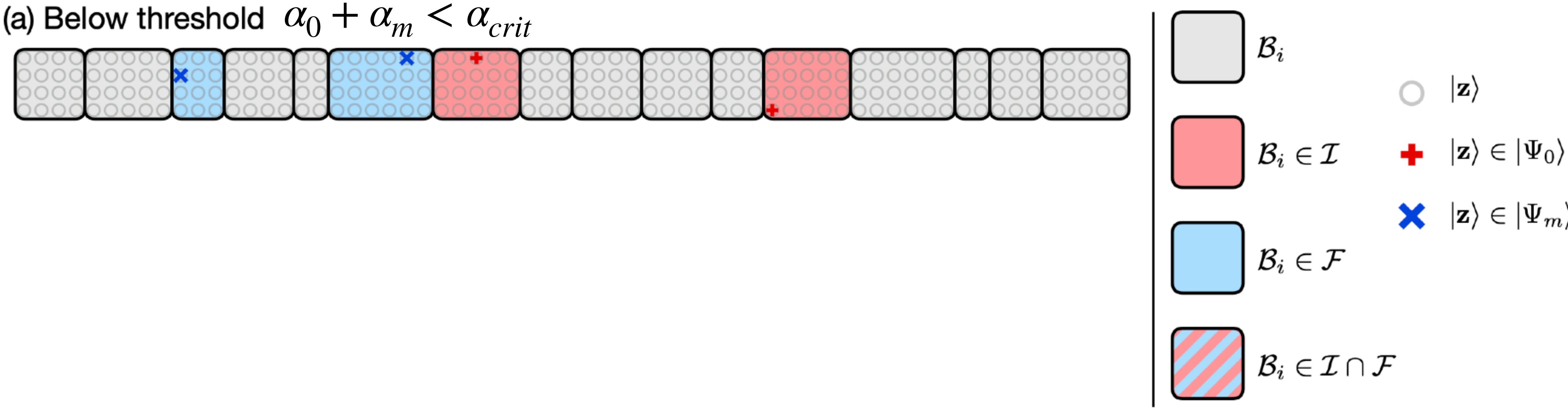
$$H_2(\{p_i\}) = -\log_2 \sum_i p_i^2$$

Collision entropy/2nd-Renyi entropy of block probabilities

Threshold condition

$$\boxed{\alpha_{crit} := \lim_{N\to\infty} \frac{1}{N} H_2(\{p_i\})}$$

(a) Below threshold $\alpha_0 + \alpha_m < \alpha_{crit}$



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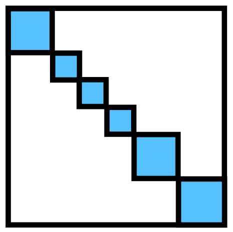

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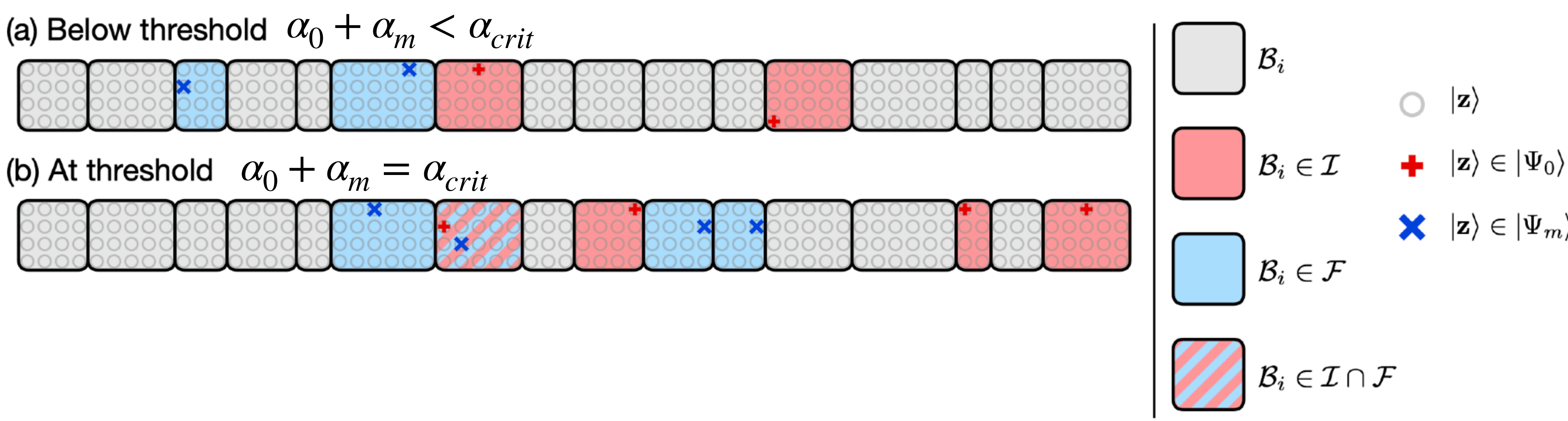

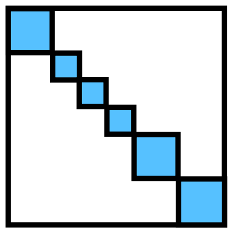

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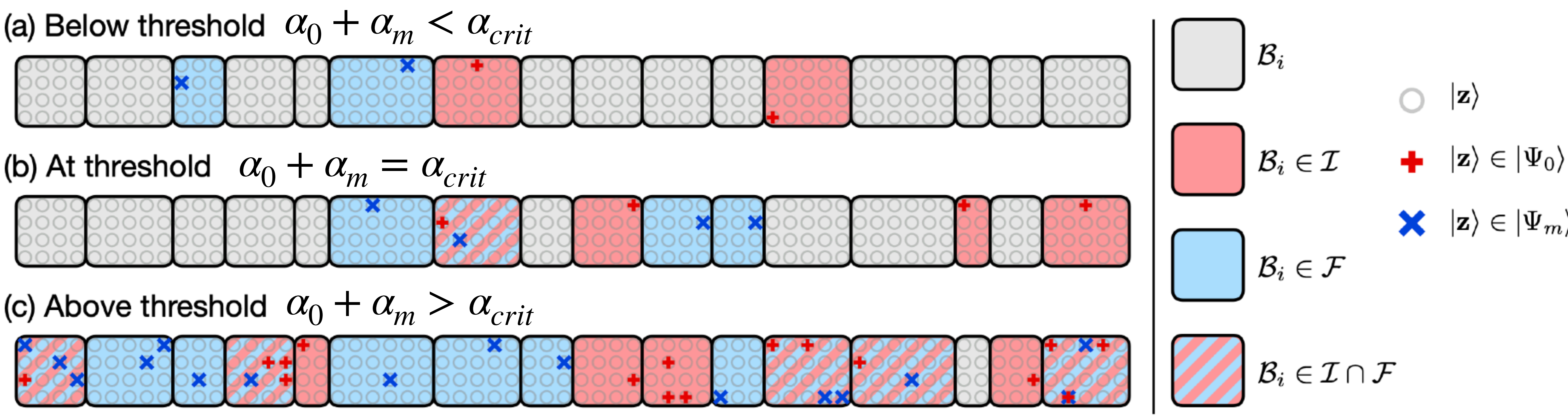
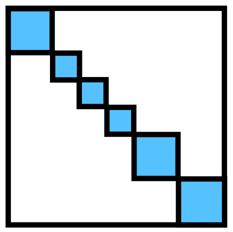
$$H_2(\{p_i\}) = -\log_2 \sum_i p_i^2$$

Collision entropy/2nd-Renyi entropy of block probabilities

Threshold condition

$$\boxed{\alpha_{crit} := \lim_{N\to\infty} \frac{1}{N} H_2(\{p_i\})}$$





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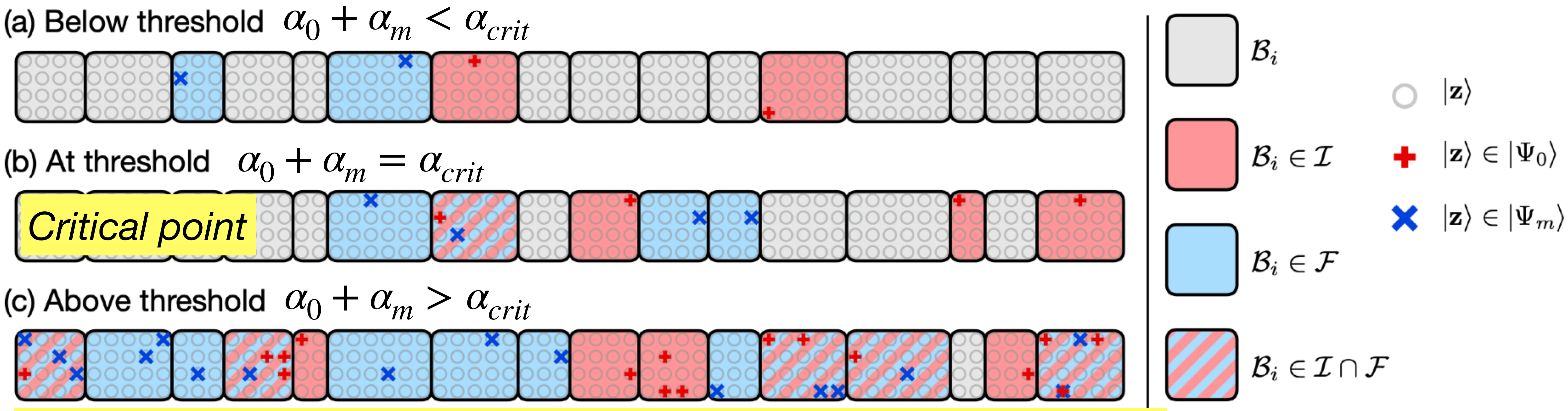
Vanishing intersection: Resource-sharp; deep ergodicity-breaking



Proliferation of intersections: Resource-fuzzy; Deep thermalization

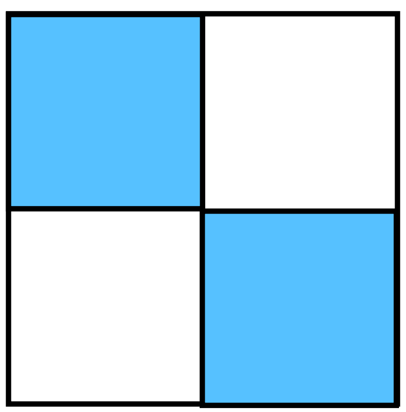
Classification of QRTs

Smoothly localizable (SL)

$\alpha_{crit} = 0$

Intensive resource

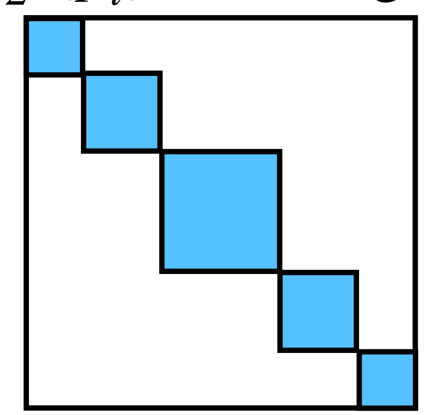
$H_2(\{p_i\}) = O(1)$



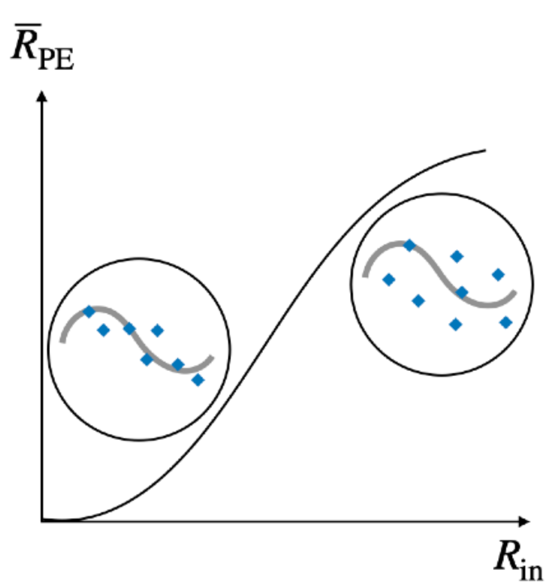
e.g. $\mathbb{Z}_2$ asymmetry
imaginarity

Marginal resource

$H_2(\{p_i\}) = O(\log N)$



e.g. $U(1)$ asymmetry,
$SU(2)$-asymmetry…

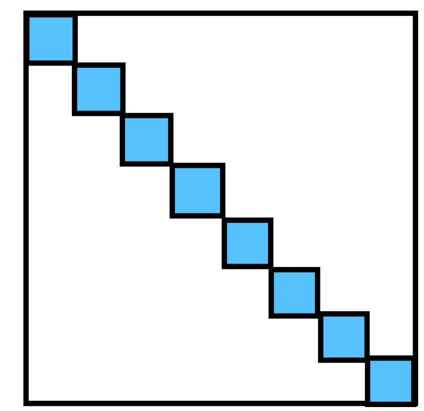


Threshold localizable (TL)

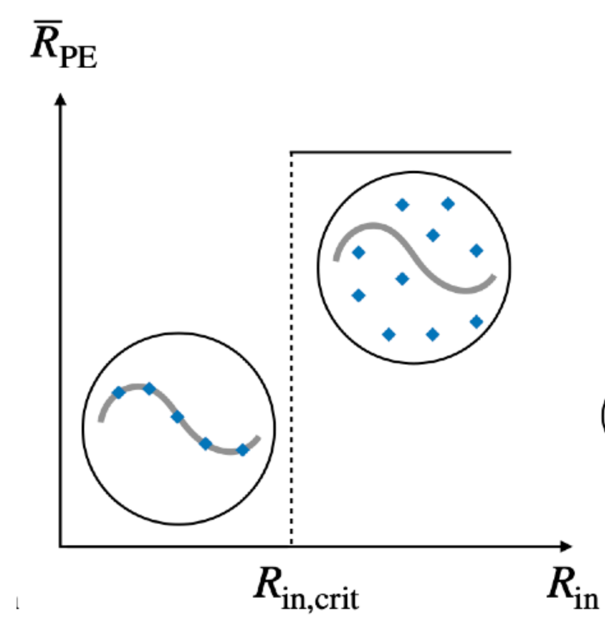
$\alpha_{crit} > 0$

Extensive resource

$H_2(\{p_i\}) = \sigma N$



e.g. coherence, coherence
between syndrome subspaces of
classical codes



[Feng, Liu, Cheng, **WWH**, Ippoliti, arXiv:2606.08756]

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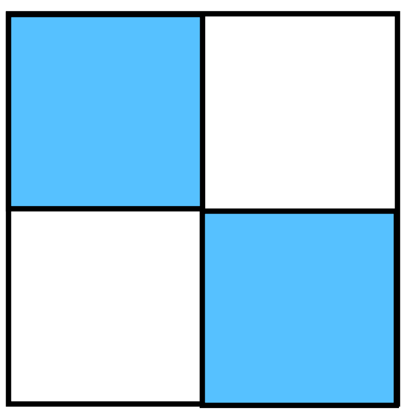

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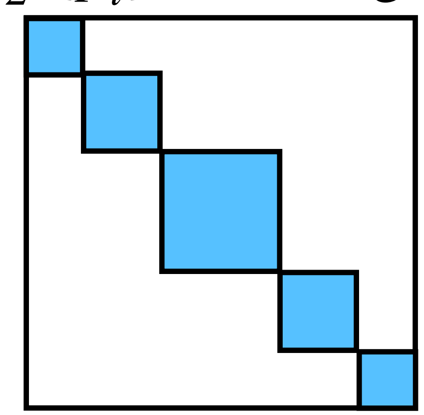

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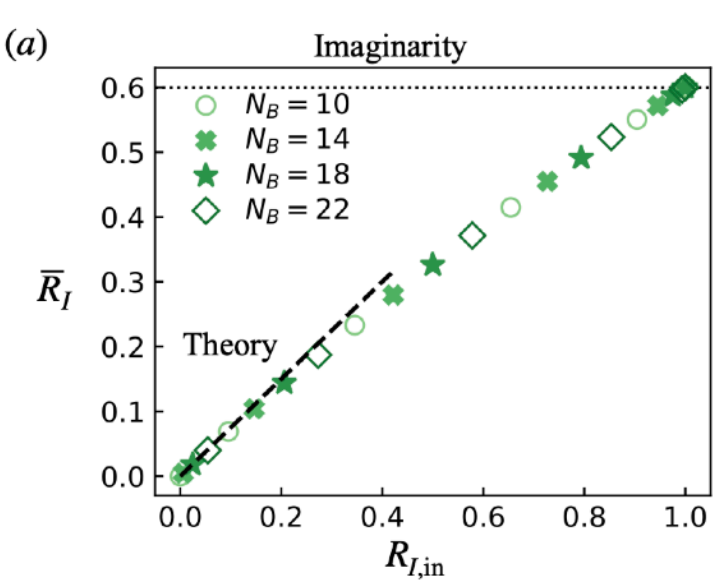

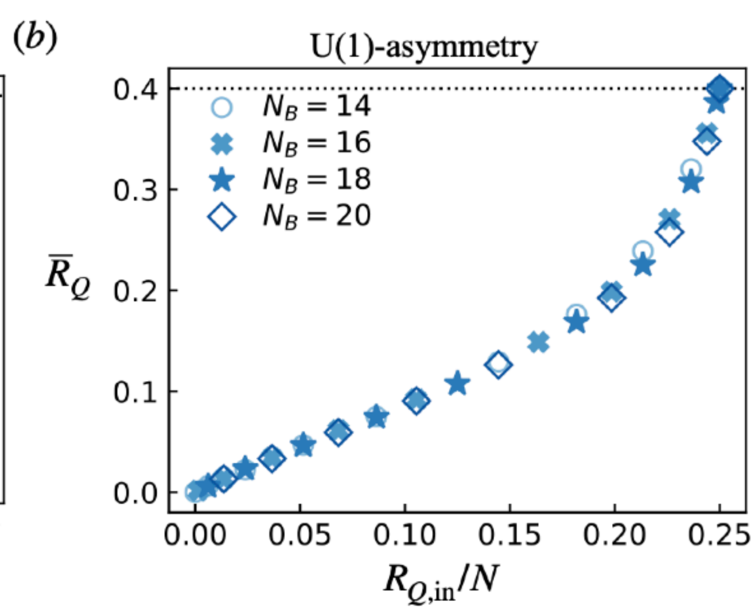

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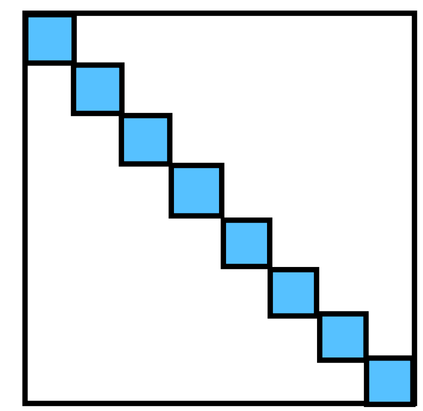

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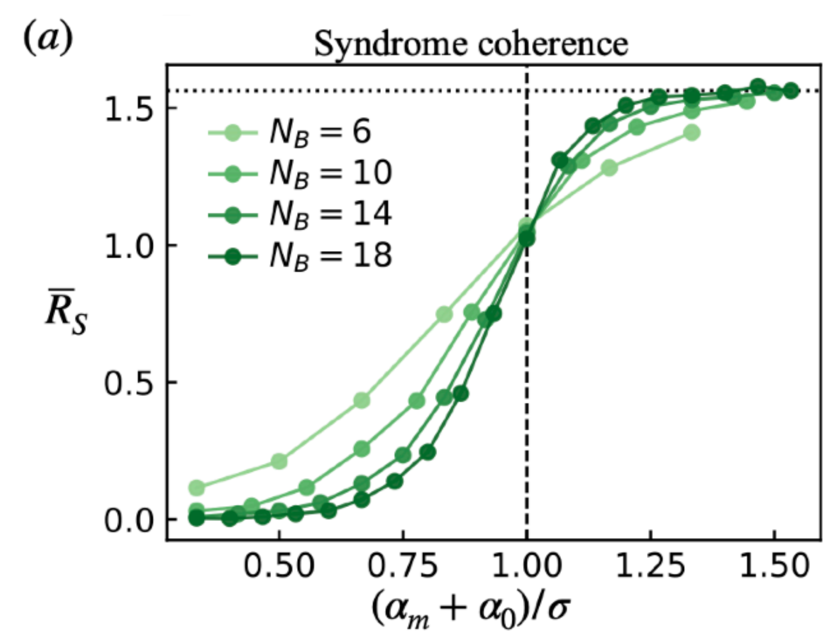

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Magic phase transition in encoding-decoding dynamics

Our results also provide insights into magic transitions in encoding-decoding dynamics

nature physics

Trapped ion experiment

Article https://doi.org/10.1038/s41567-024-02637-3

Phase transition in magic with random quantum circuits

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Pradeep Niroula [1,2], Christopher David White[1], Qingfeng Wang [2,3], Sonika Johri[4], Daiwei Zhu [4], Christopher Monroe [1,2,4,5], Crystal Noel [5] & Michael J. Gullans [1]

[Nature Physics 20, 1786 (2024)]

Theory of Magic Phase Transitions in Encoding-Decoding Circuits

Piotr Sierant [1,*] and Xhek Turkeshi [2,†]
[1] *Barcelona Supercomputing Center Plaça Eusebi Güell, 1-3 08034, Barcelona, Spain*
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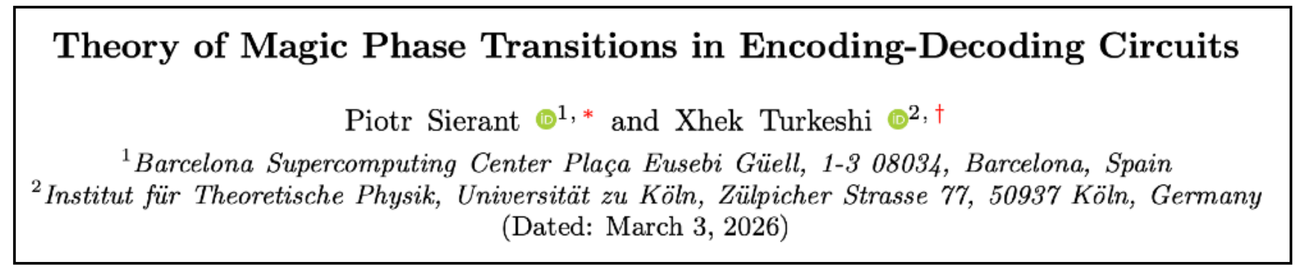
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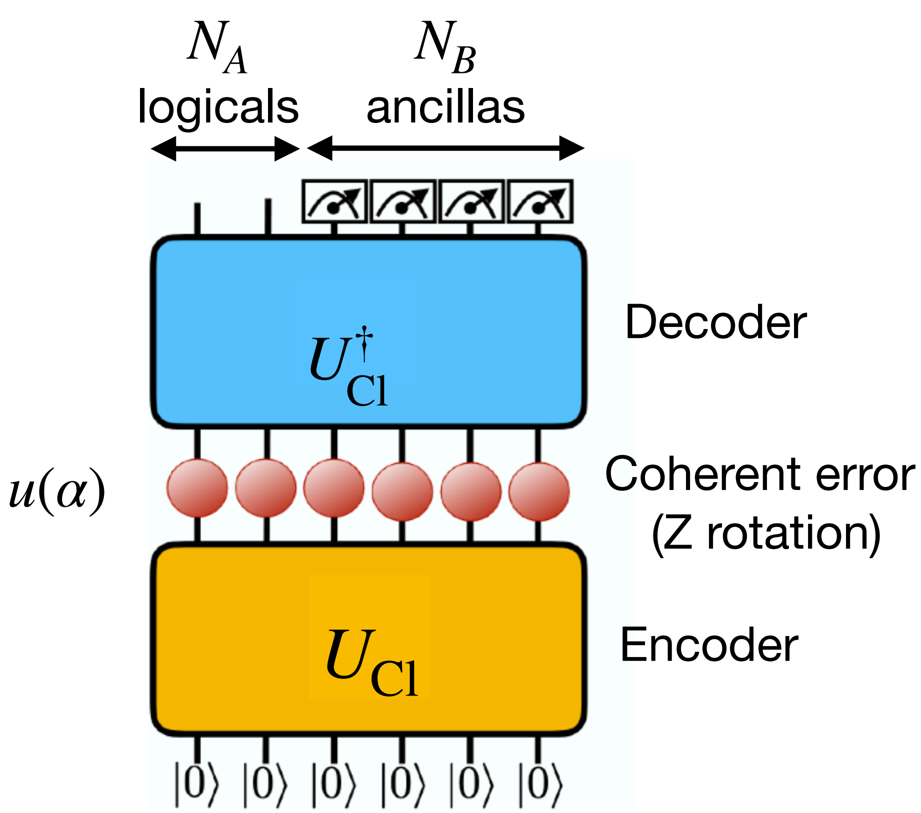
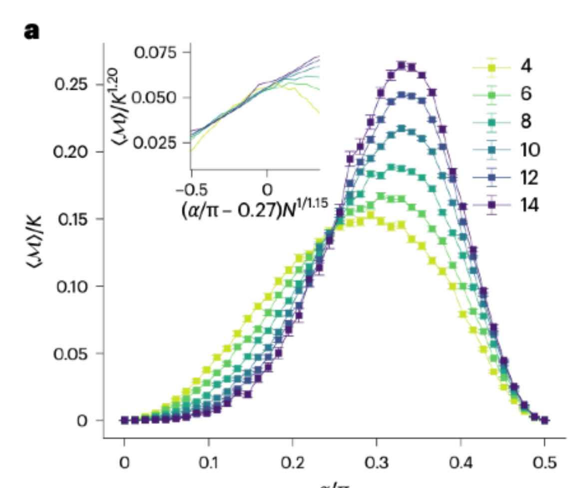


Idea: study the threshold of error correction as the proliferation of magic in the post-measurement states

A belief was that successful decoding required finite code rates $r := N_A/N > 0$



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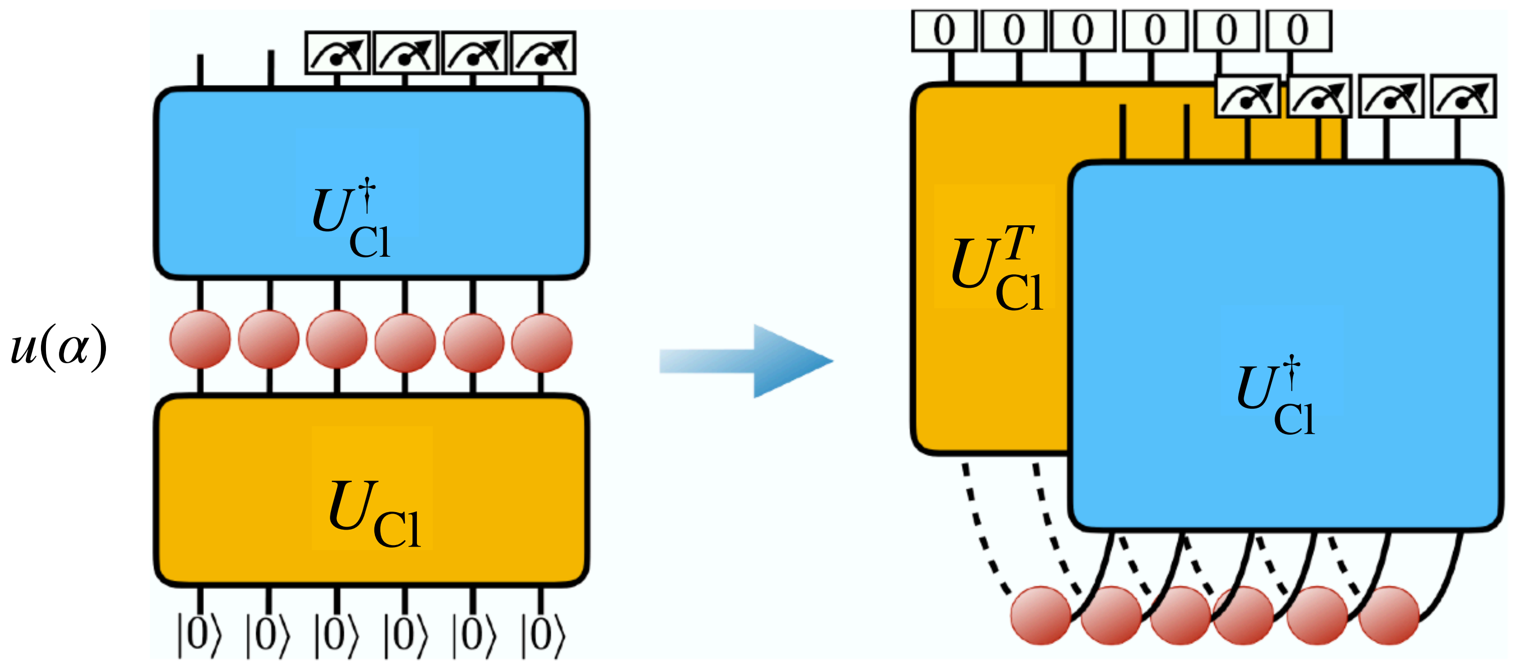
Magic phase transition in encoding-decoding dynamics

Consider projected ensemble in the Choi representation:



Forced + Born measurements

Dynamics

$|u(\alpha)\rangle\rangle^{\otimes N}$

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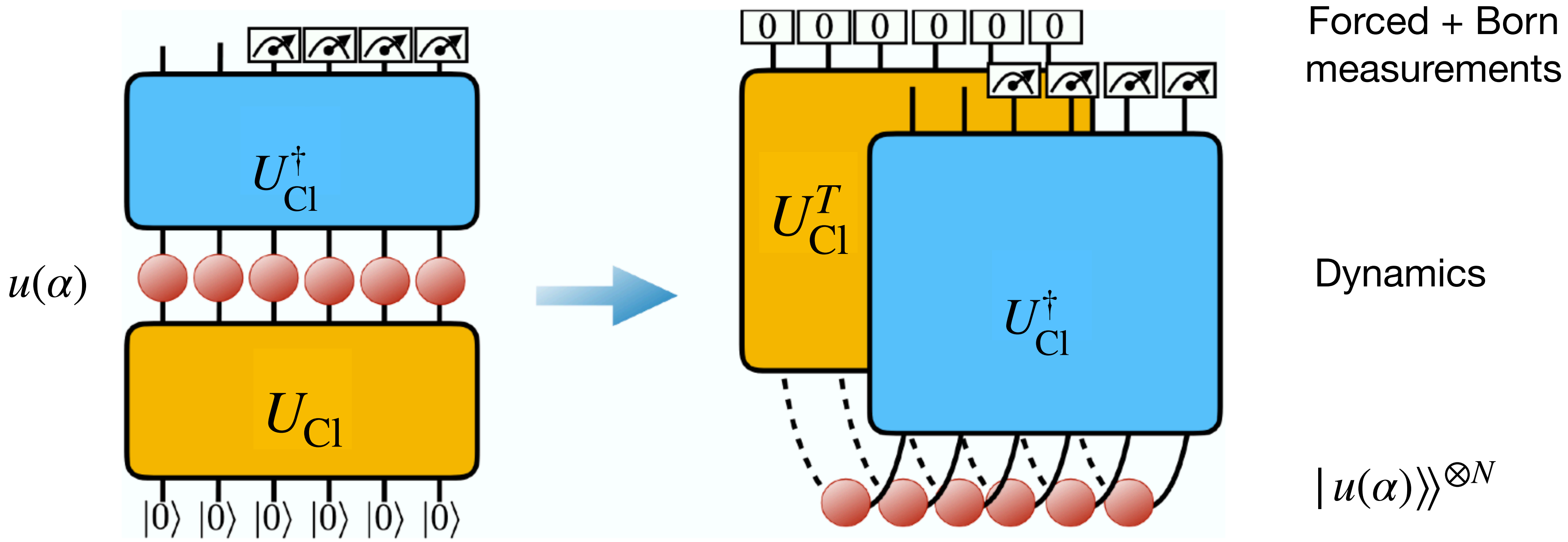

Key observation: Dynamics $U_{cl}^{T} \otimes U_{cl}^{\dagger}$ is not generic evolution, it is constrained!

Specifically, it preserves the “amount” of superposition over Pauli strings present in the initial state (“Pauli coherence”).

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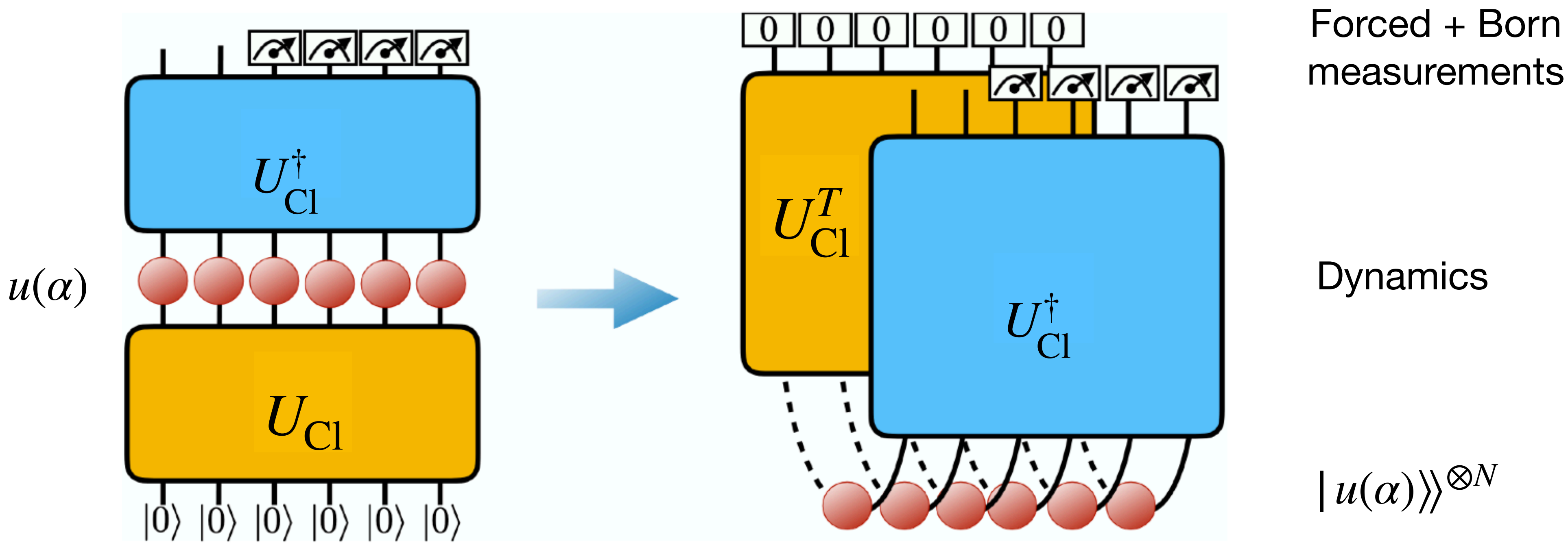

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E.g. if $|u\rangle\rangle = c_{\mathbb{1}}|\mathbb{1}\rangle\rangle + c_X|X\rangle\rangle + c_Y|Y\rangle\rangle + c_Z|Z\rangle\rangle$

$$U_{Cl}^{T} \otimes U_{Cl}^{\dagger}|u\rangle\rangle = c_{\mathbb{1}}|\mathbb{1}\rangle\rangle + c_X|Z\rangle\rangle + c_Y|X\rangle\rangle + c_Z|Y\rangle\rangle$$

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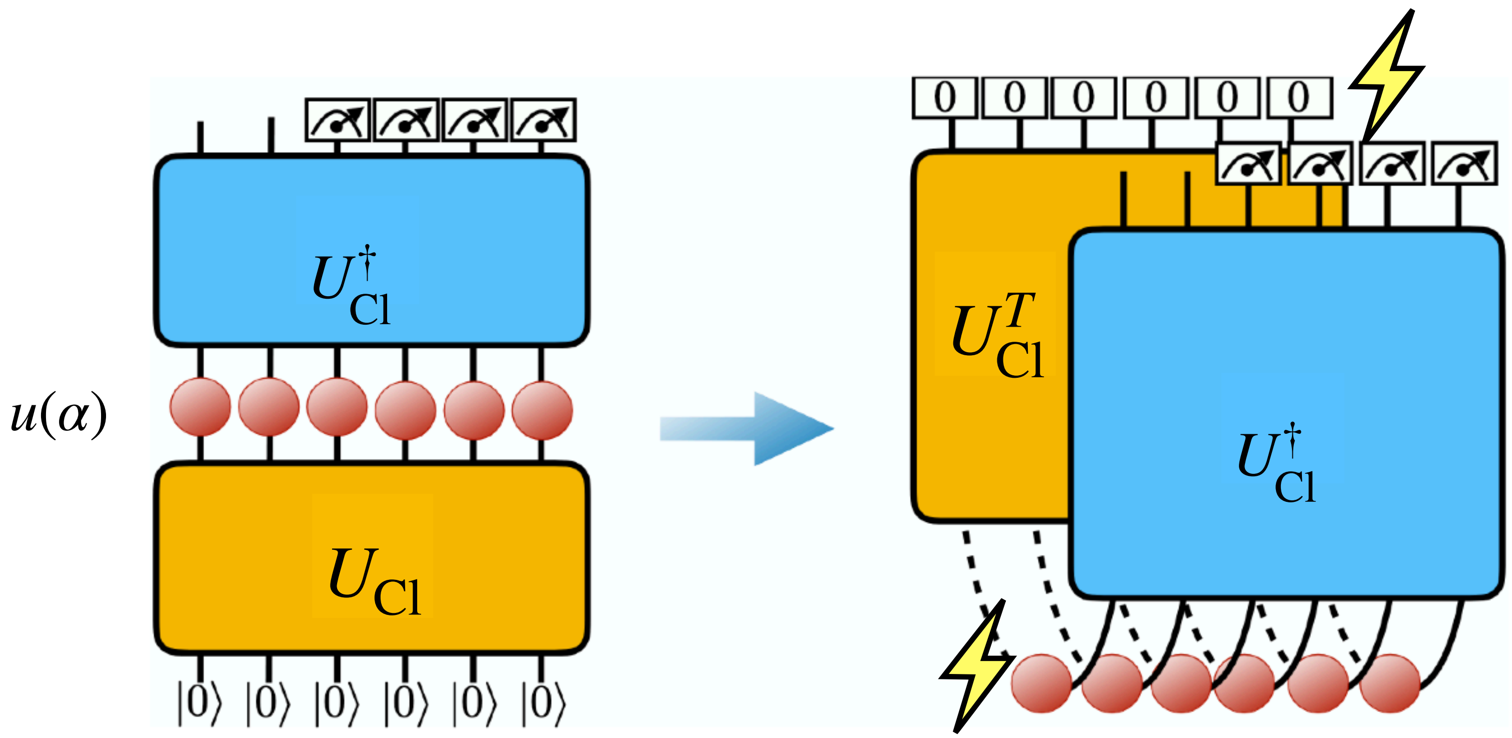

Resourceful (fixed)
Forced + Born measurements

Free
Dynamics

Resourceful (tunable)
$|u(\alpha)\rangle\rangle^{\otimes N}$

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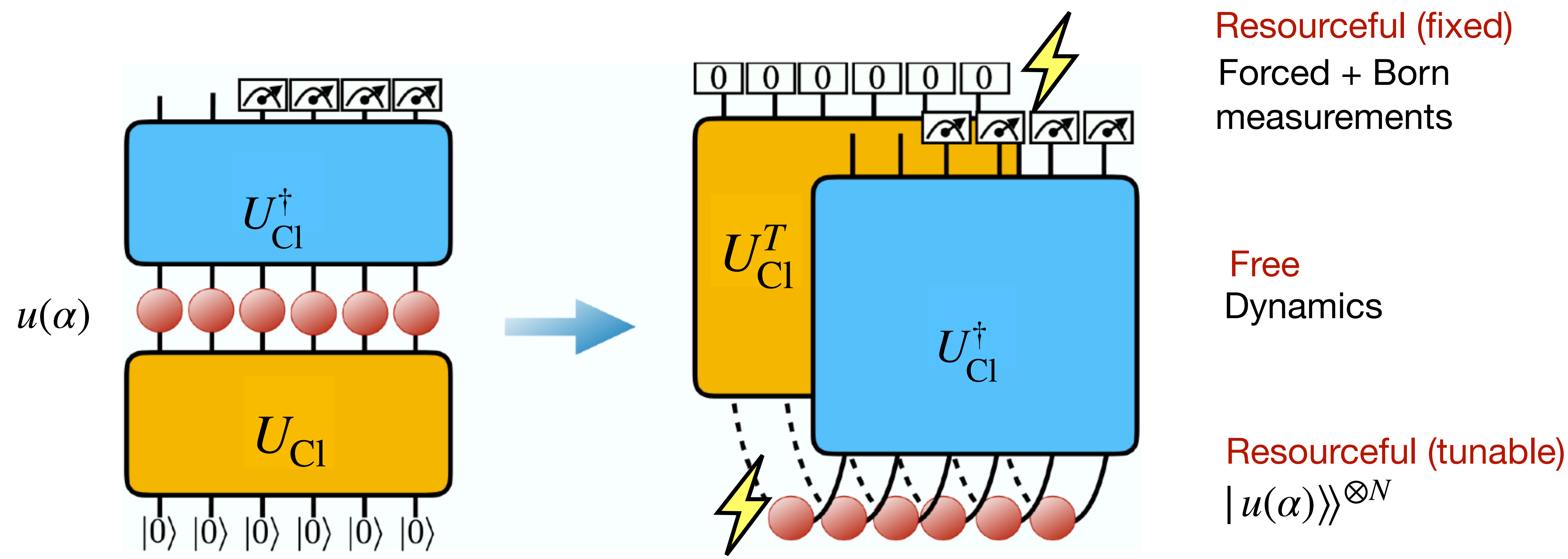

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Expect: Pauli Coherence threshold $\approx$ Magic threshold = error correctability

[Feng, Liu, Cheng, **WWH**, Ippoliti, arXiv:2606.08756]

Magic transition

Prediction: error correction threshold at critical injected Pauli coherence

$$R_{PC}[u(\alpha)] := -\sum_{P=\mathbb{I},X,Y,Z} |c_P|^2 \log_2 |c_P|^2 = \frac{1}{2}$$

$$u = \sum_{P=\mathbb{I},X,Y,Z} c_P P$$

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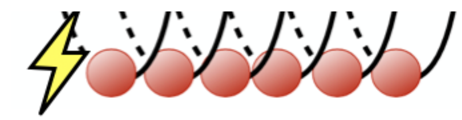

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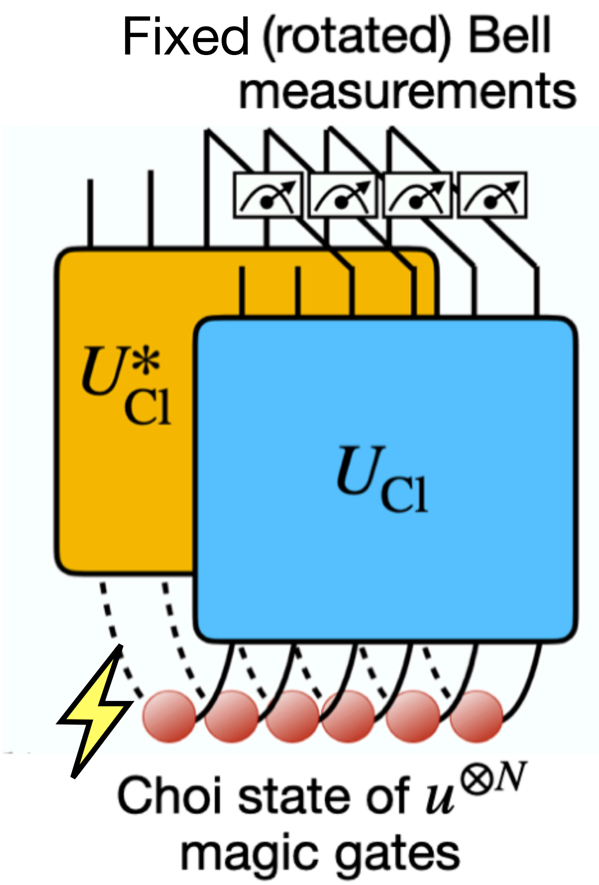


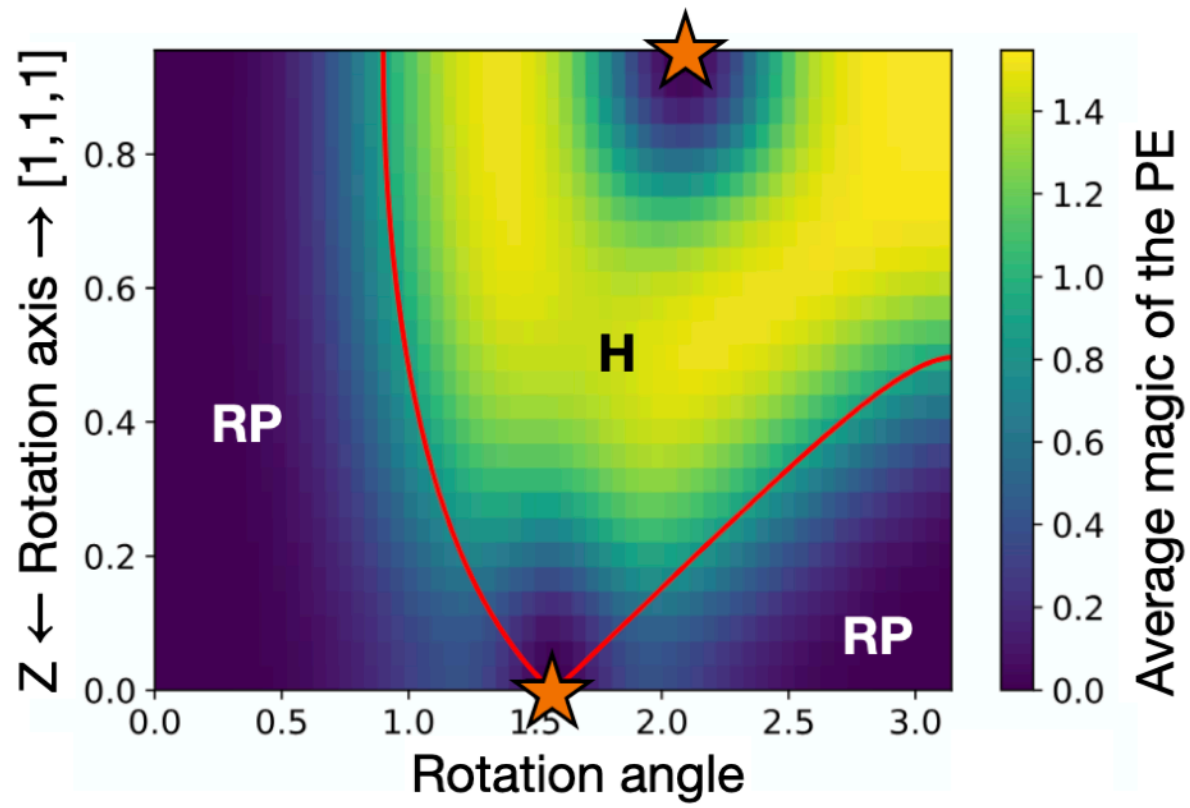
$$u(\theta, \mathbf{n}) = e^{-i\frac{\theta}{2}\mathbf{n}\cdot\boldsymbol{\sigma}}$$

RP: random Pauli phase

H: Haar phase

★: Clifford points

/ : predicted phase boundary (Pauli coherence density = 1/2)



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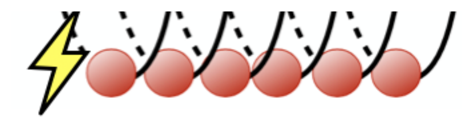

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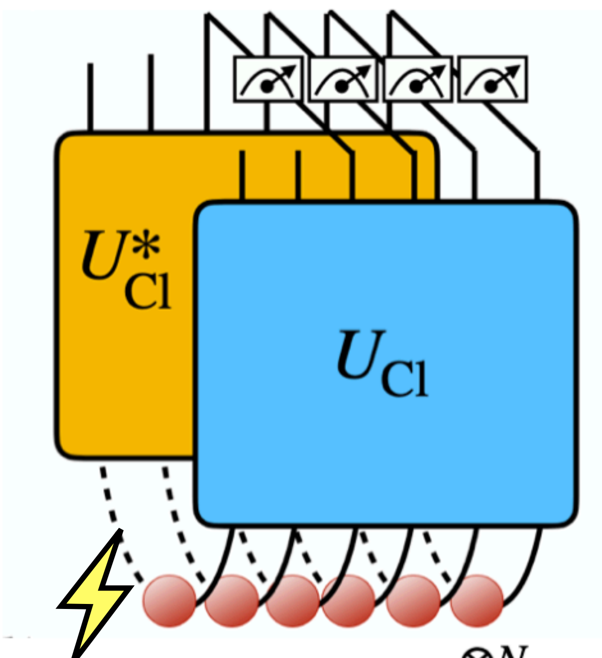
Fixed (rotated) Bell measurements



Choi state of $u^{\otimes N}$ magic gates

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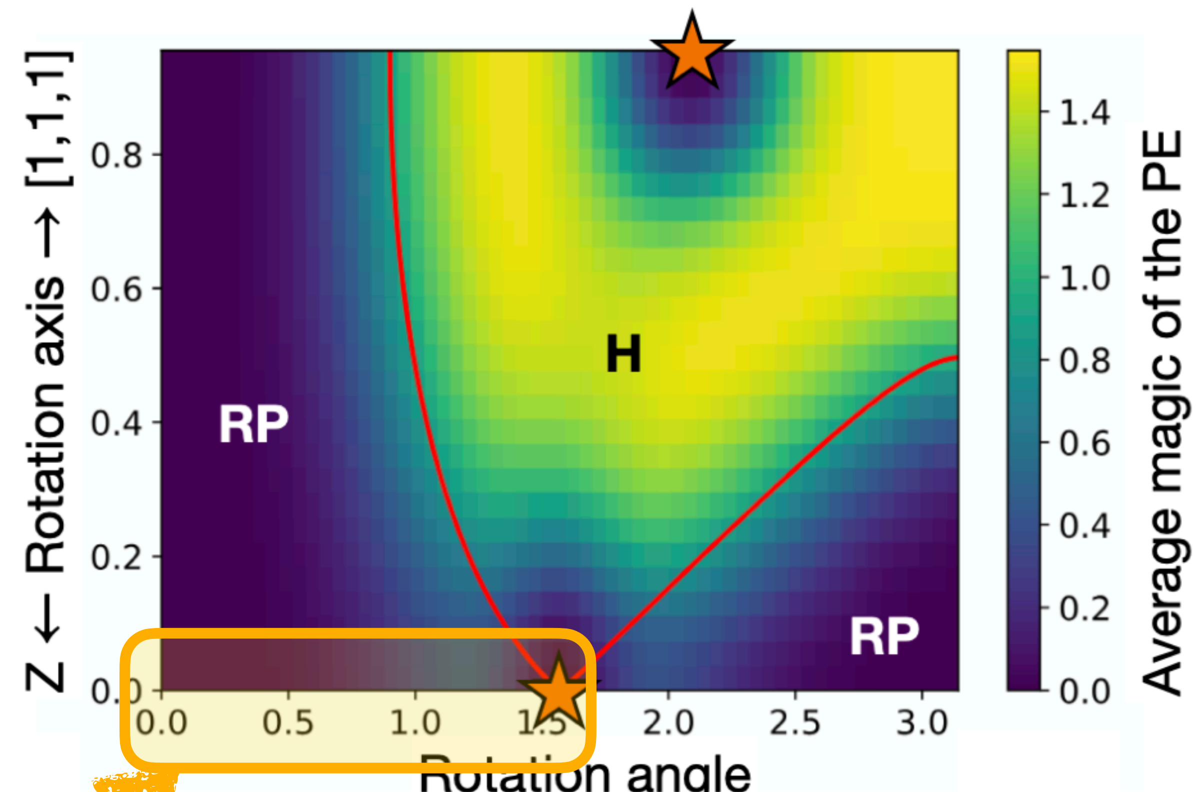

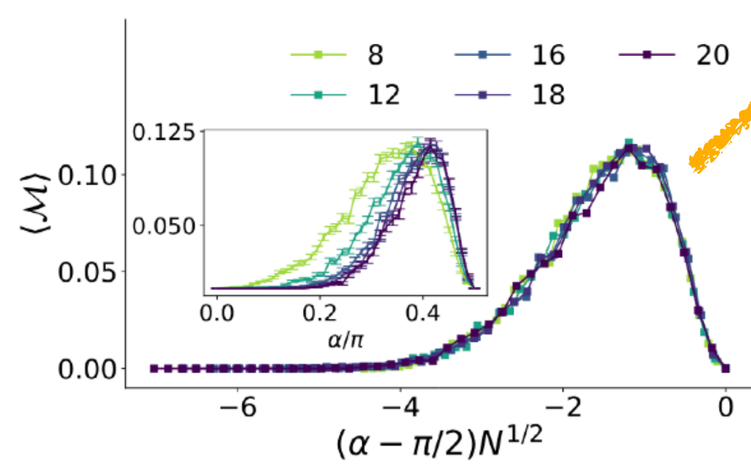

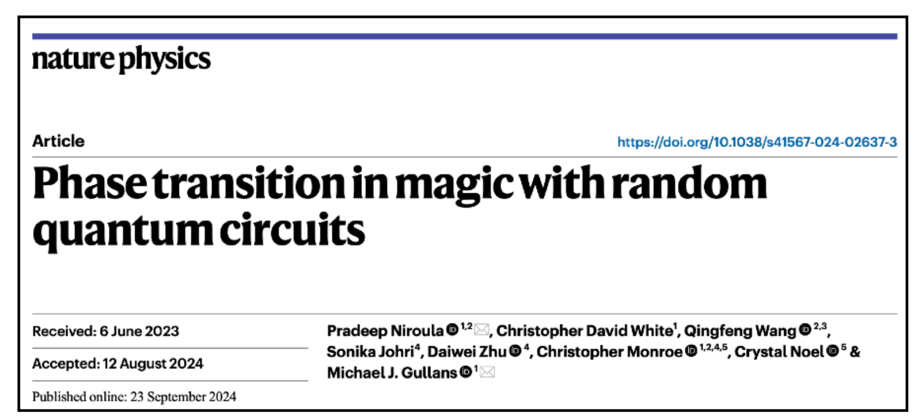
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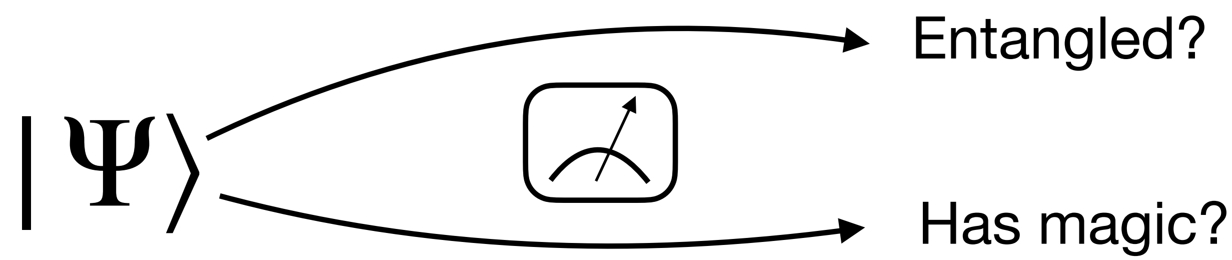
Theoretical explanation of experimentally observed magic transition!

Implications for quantum state certification

Task: Certifying properties (e.g. entanglement, magic, complexity) of a many-body state



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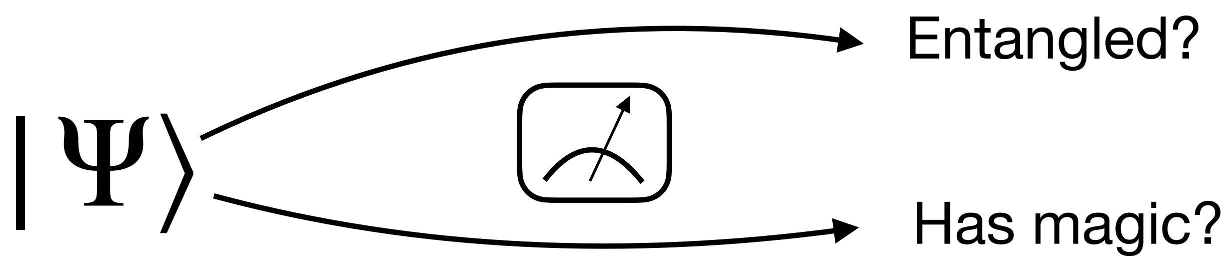

Challenge: estimating complicated **global** quantities entails humongous sample complexity 🙁

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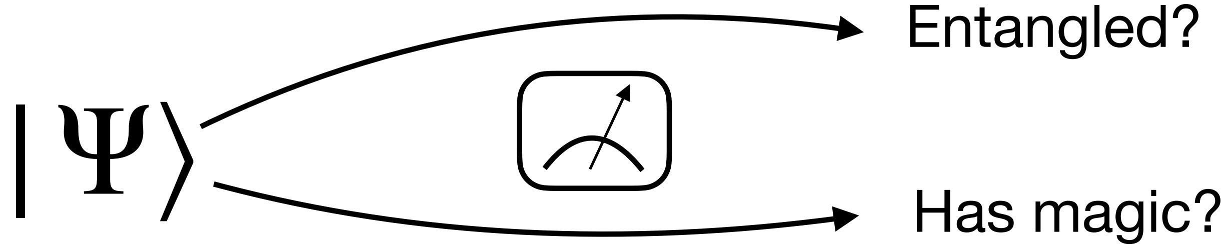

Challenge: estimating complicated **global** quantities entails humongous sample complexity

arXiv:2509.17580

Certifying localizable quantum properties with constant sample complexity

Zhenyu Du,[1] Jinchang Liu,[2] Elias X. Huber,[3] Zi-Wen Liu,[4,*] and Xiongfeng Ma[1,†]

[1] *Center for Quantum Information, Institute for Interdisciplinary Information Sciences, Tsinghua University, Beijing 100084, China*
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[3] *Yenching Academy, Peking University, Beijing 100871, China*
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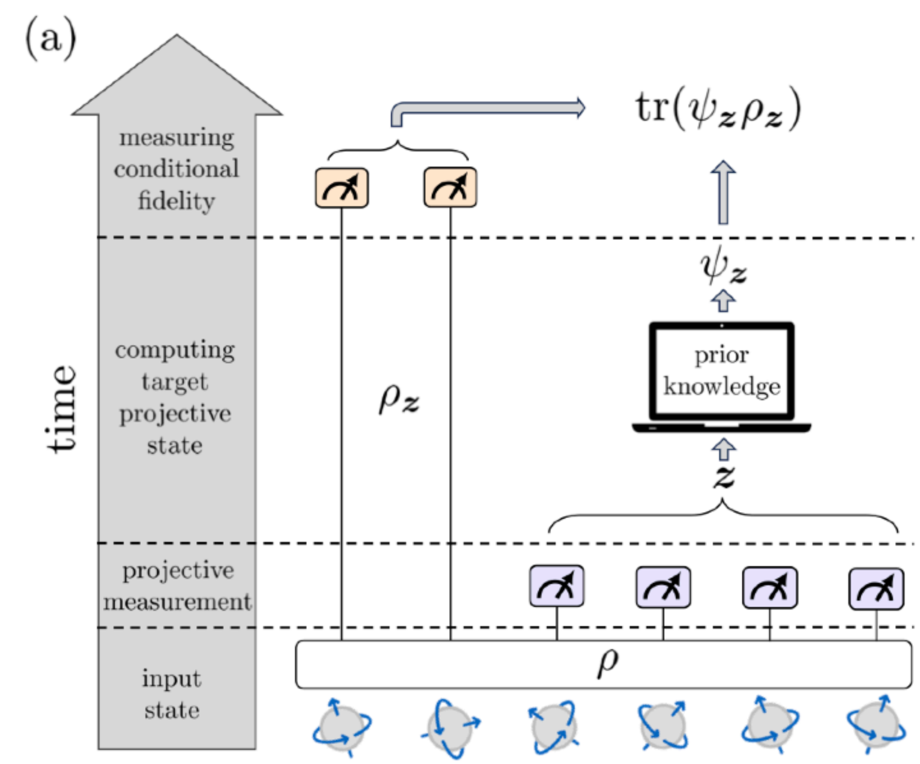
Proposal: localize global property on local subsystem — certifying local states is easy (no free lunch though: requires computational resources)



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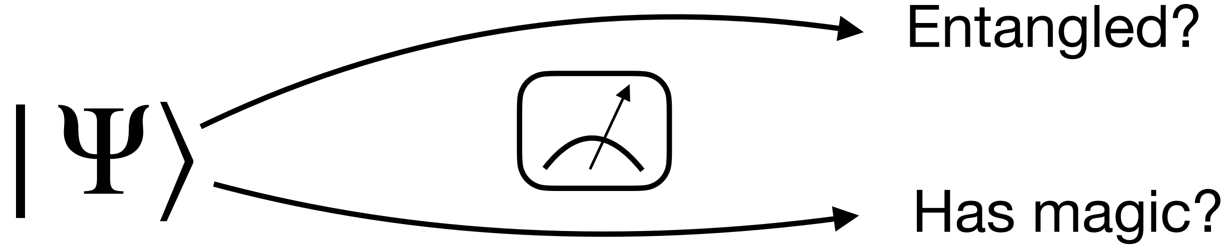

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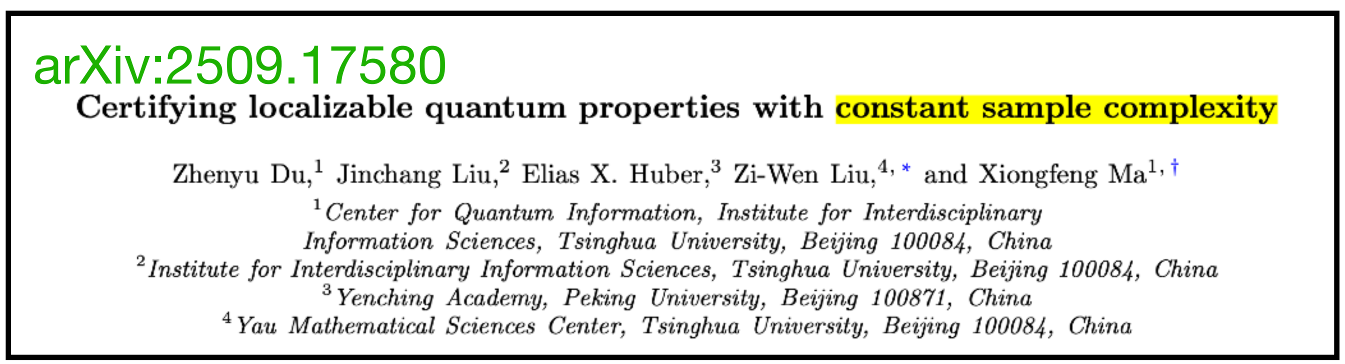
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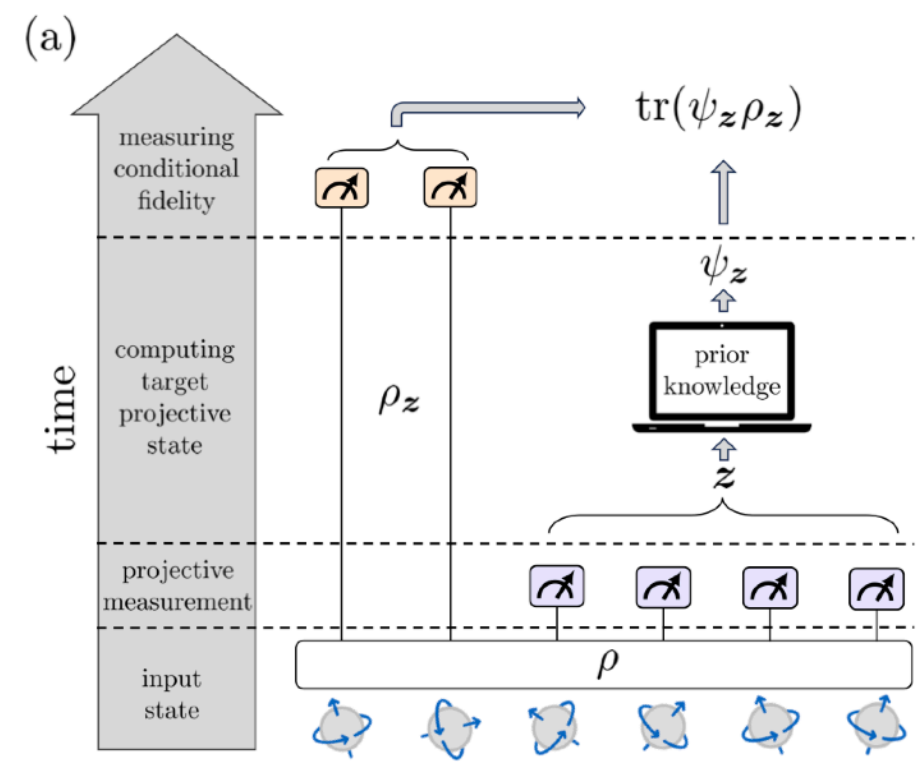

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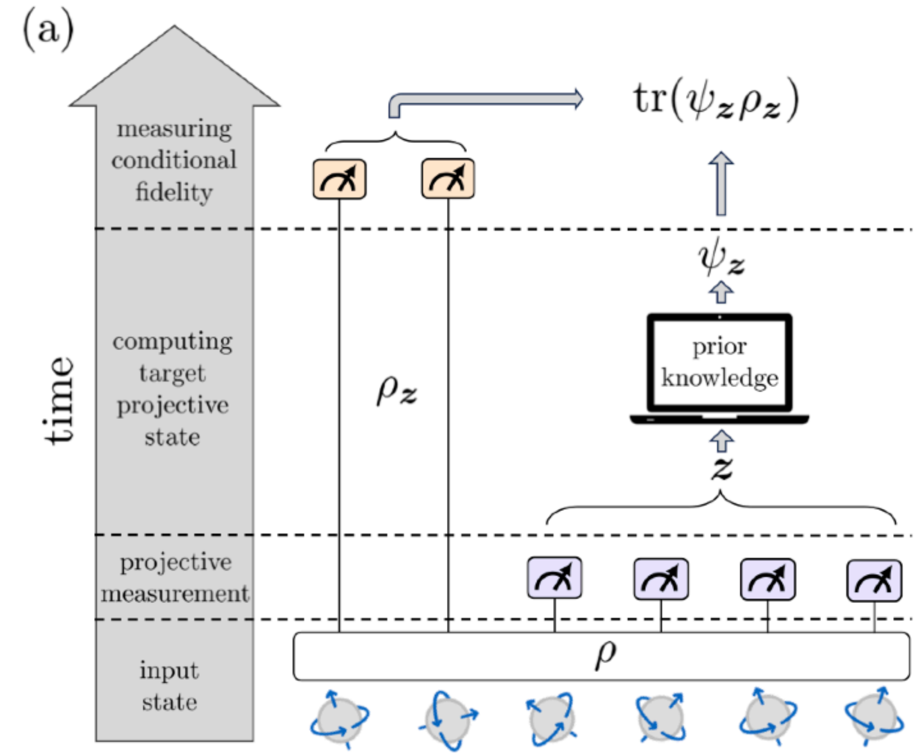

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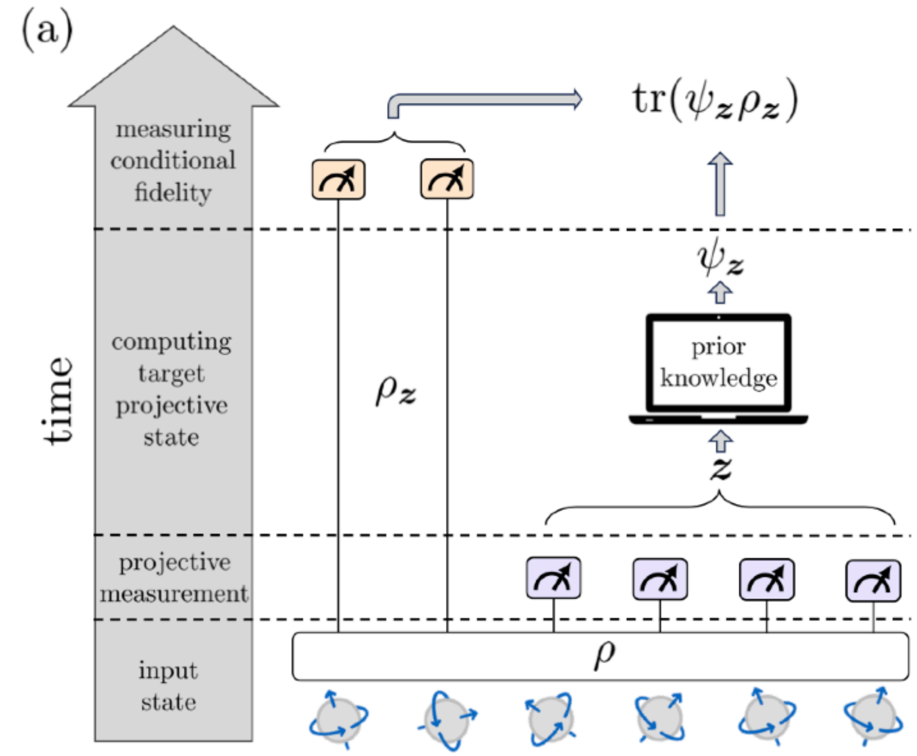

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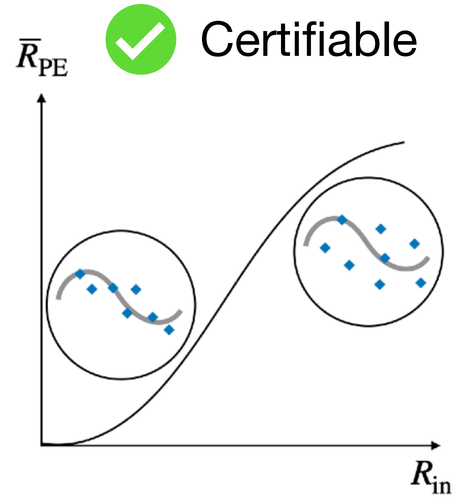
Our results inform us on limitations on any state certification protocols based on resource localization

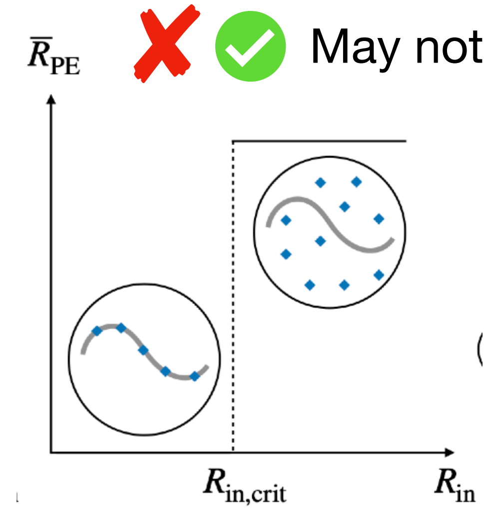
Smoothly localizable (SL)

Certifiable



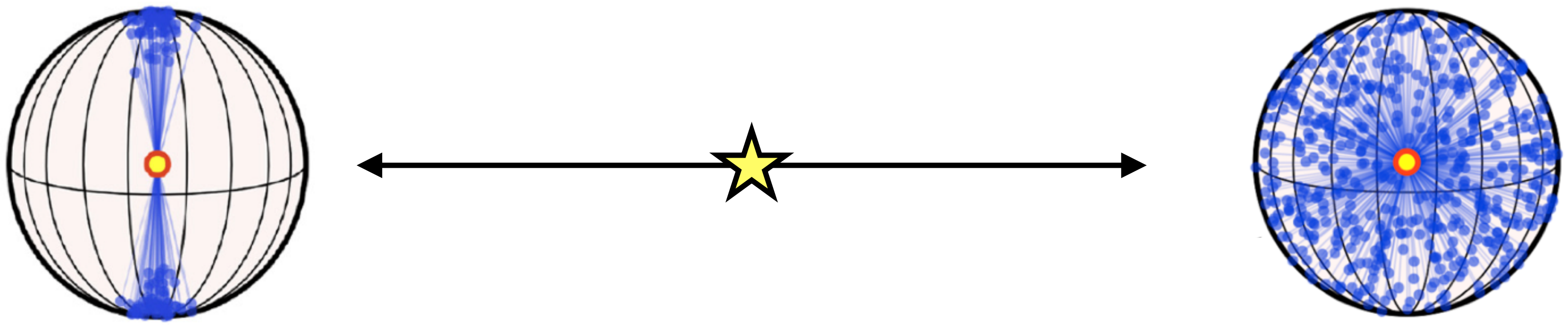
Threshold localizable (TL)

May not be certifiable



Conclusions

- Deep thermalization: emergence of universal local wavefunction distributions
- Underpinned by maximum entropy principles in QI
- Exceptions to maximally entropic paradigm: deep-ergodicity breaking from quantum resource constraints
- Phase transitions as resource localizability / block-sharpening
- Implications in quantum error correction and quantum information science



[Liu, Ippoliti, **WWH**, PRL **136**, 100404 (2026)]
[Zhou, Liu, Cheng, **WWH**, Ippoliti, arXiv:2606.08756]

Thank you for your attention!