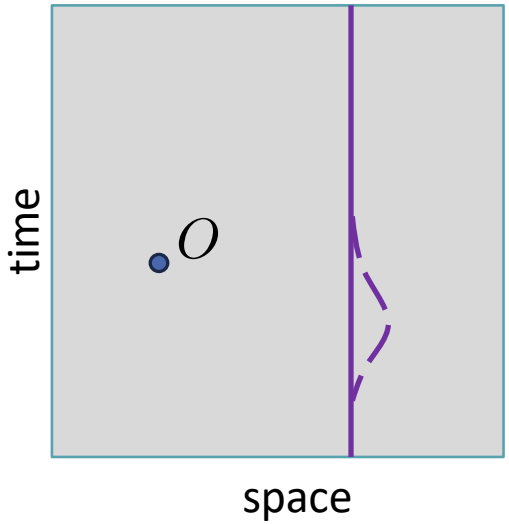
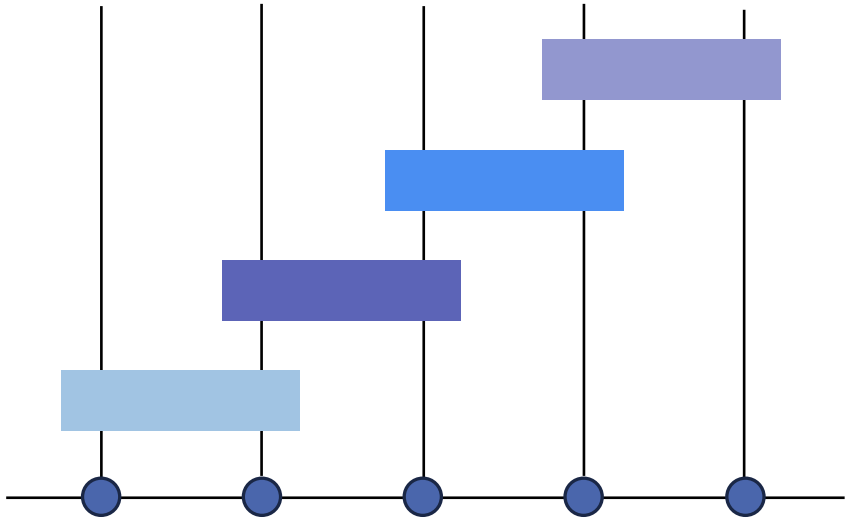




Sequential Quantum Circuit as Generalized Symmetry

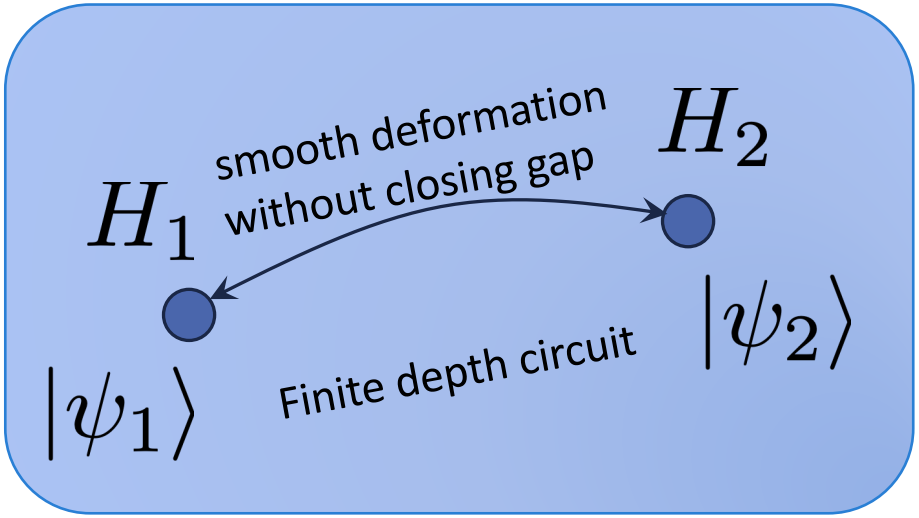
XIE CHEN, CALTECH

NORDITA, JULY 2026

IQIM INSTITUTE FOR QUANTUM INFORMATION AND MATTER
SIMONS FOUNDATION
BURKE INSTITUTE
Simons Collaboration on Ultra-Quantum Matter

Finite depth circuit

Gapped phase

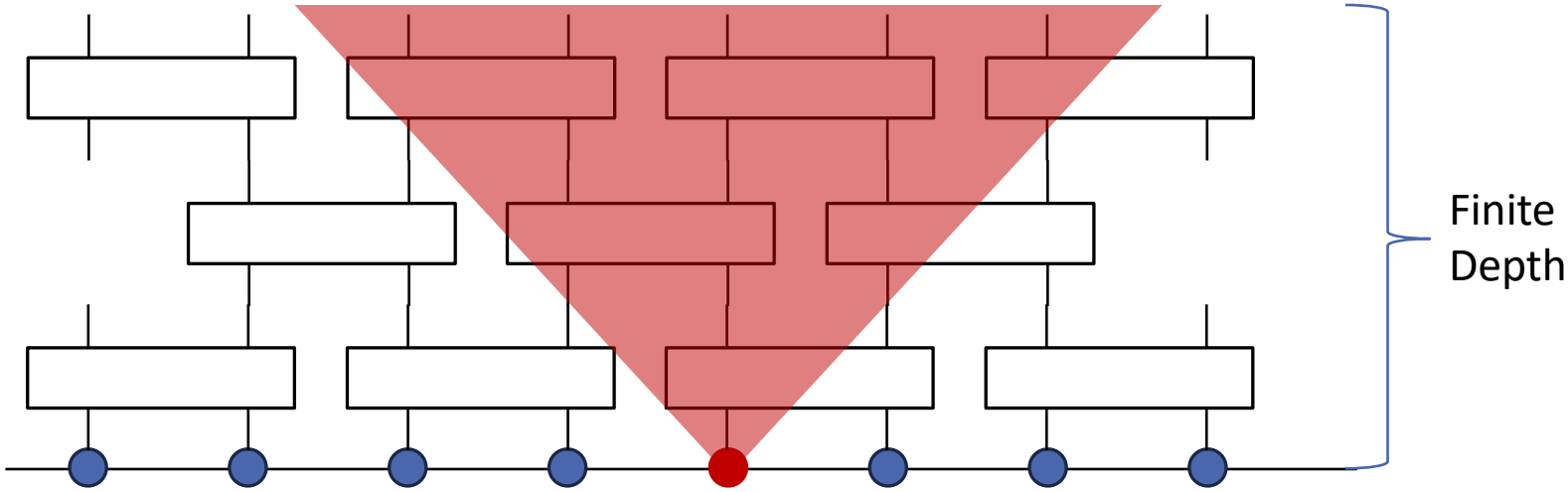


Necessary and sufficient

Chen, Gu, Wen, 2010

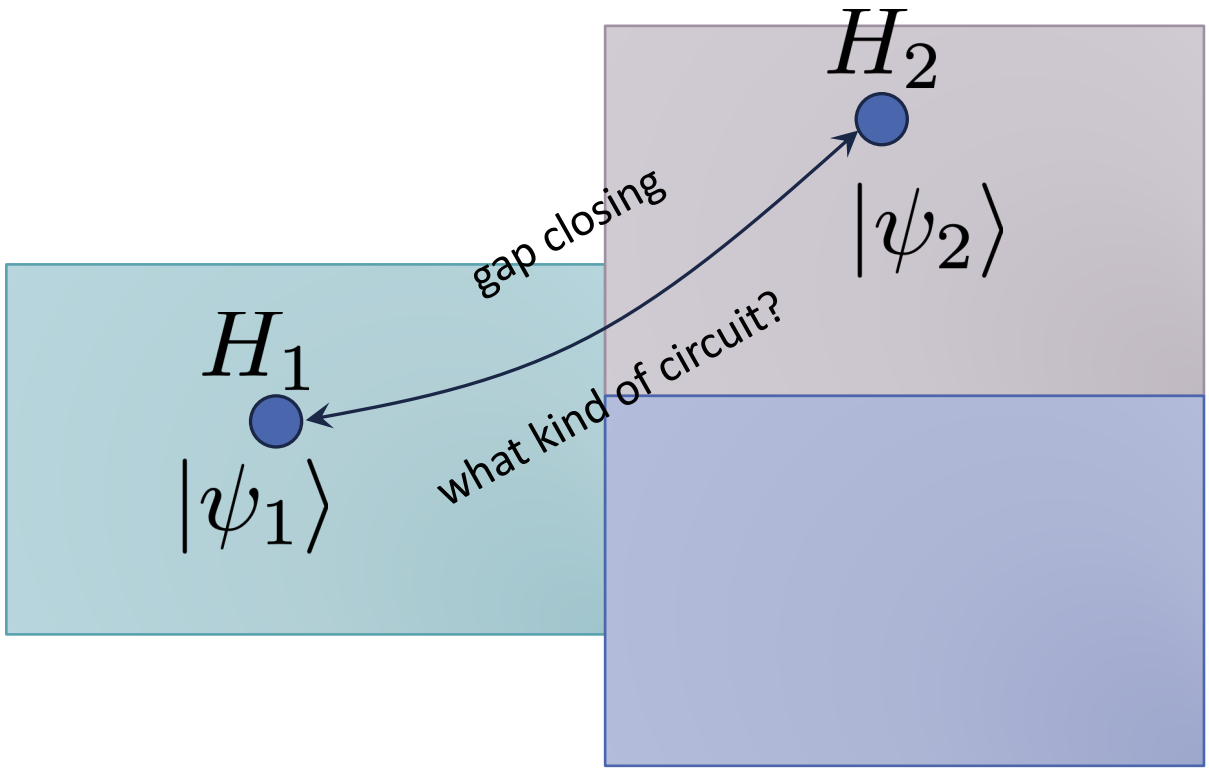
Finite depth circuit

- Everything stays local
- local operator → local operator
- short range correlation → short range correlation
- short range entanglement → short range entanglement



Chen, Gu, Wen, 2010

How to jump between gapped phases?



How to jump between gapped phases?

- Linear depth is lower bound, Bravyi, Hastings, Verstraete, 2006
- But generic linear depth circuit is too powerful
- entanglement volume law, high energy states, scrambling and chaos
- How to jump between gapped phases without leaving the gapped region?
- Sequential quantum circuit

Symmetric to Symmetry breaking

$$H = -\sum X_i \rightarrow H = -\sum Z_i Z_{i+1}$$

$$|\psi\rangle = |++...+\rangle \rightarrow |\psi\rangle = |00...0\rangle + |11...1\rangle$$

Global symmetry $U = \prod X_i$

$$< Z_i Z_{i+l} > - < Z_i >< Z_{i+l} >$$

short range correlation → long range correlation

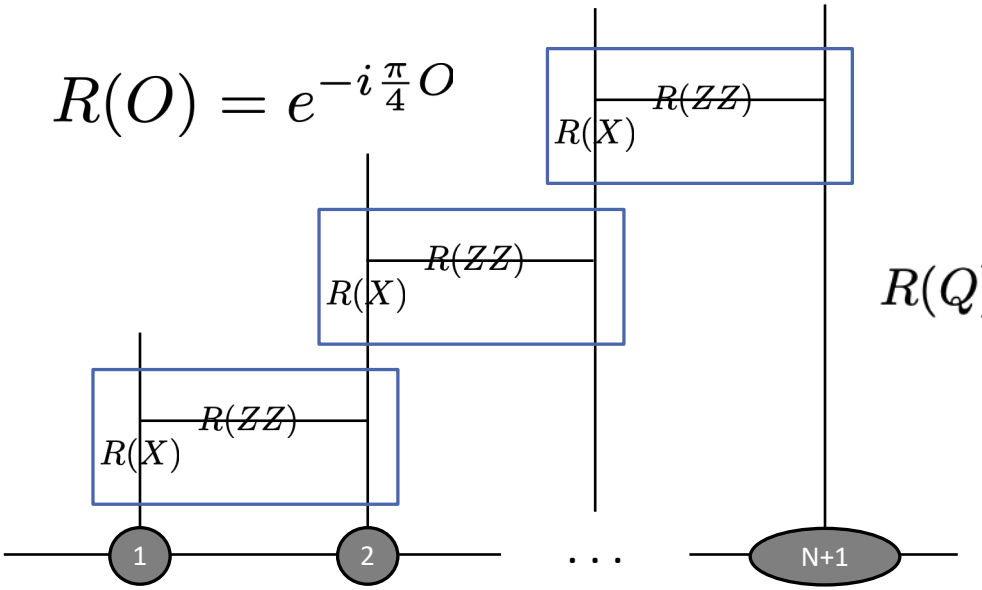
Schön, Solano, Verstraete, Cirac, Wolf, 2005; Ho, Hsieh, 2019

Symmetric to Symmetry breaking

$$H = -\sum X_i \rightarrow H = -\sum Z_i Z_{i+1} \qquad U = \prod X_i$$

$$|\psi\rangle = |++...+\rangle \rightarrow |\psi\rangle = |00...0\rangle + |11...1\rangle$$

$$R(O) = e^{-i\frac{\pi}{4}O}$$



$$R(Q)PR(Q)^\dagger = \begin{cases} P; & [P,Q] = 0 \\ iPQ; & \{P,Q\} = 0 \end{cases}$$

Schön, Solano, Verstraete, Cirac, Wolf, 2005; Ho, Hsieh, 2019

Symmetric to Symmetry breaking

$$H = -\sum X_i \rightarrow H = -\sum Z_i Z_{i+1} \qquad U = \prod X_i$$

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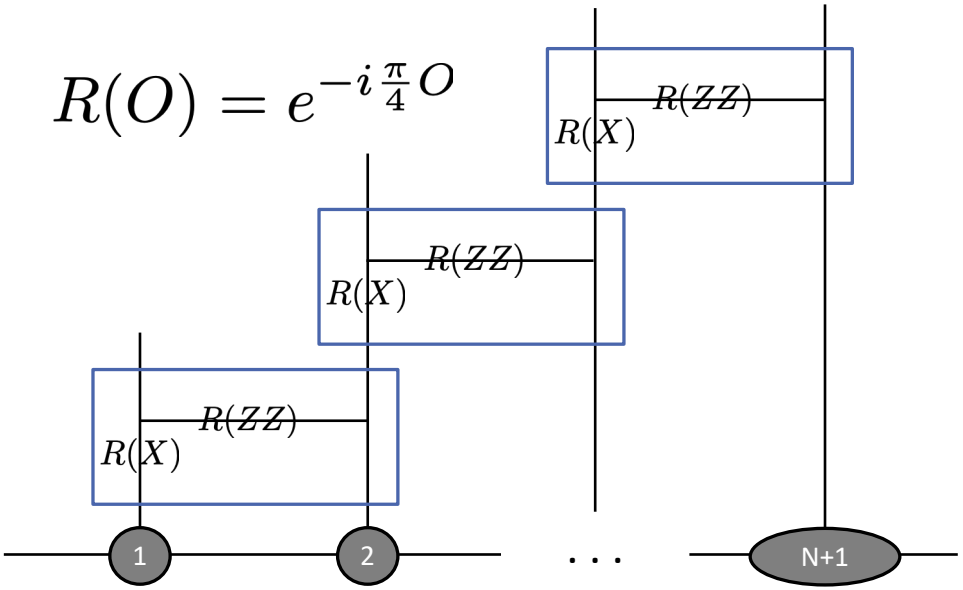


$$X_i \rightarrow Z_{i-1} Z_i$$

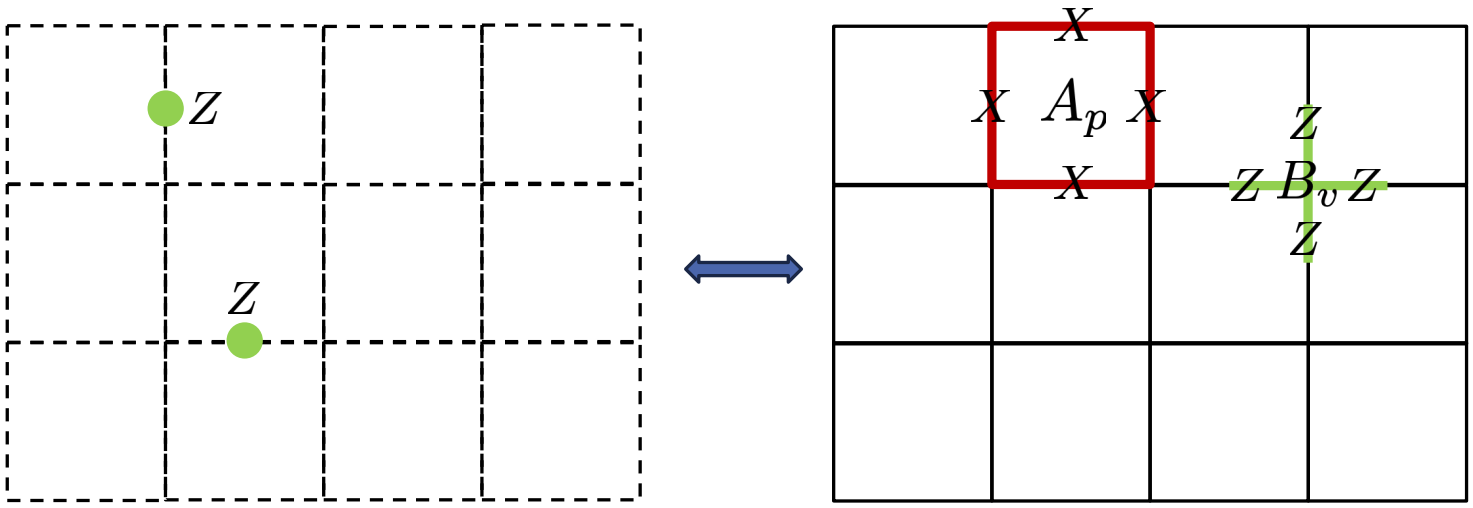
$$Z_i Z_{i+1} \rightarrow X_i$$

Depth $\sim L$

Sequential

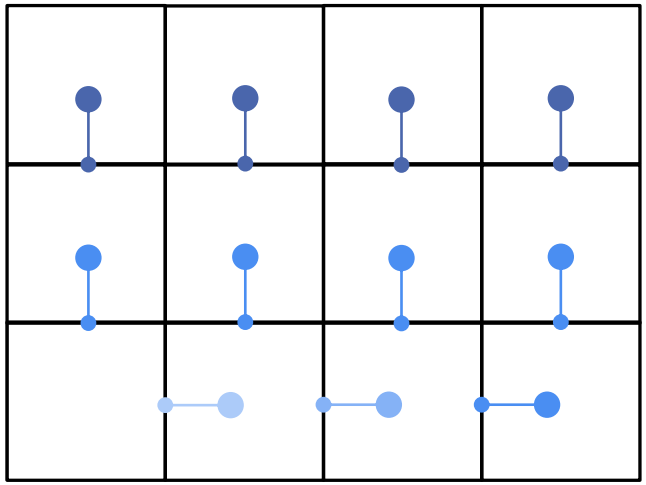
Schön, Solano, Verstraete, Cirac, Wolf, 2005; Ho, Hsieh, 2019

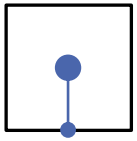
Trivial to topological



$\gamma = 0$ $\gamma = 1$

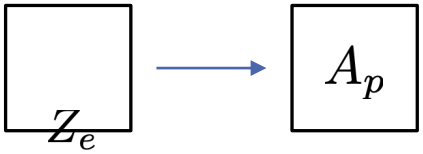
Trivial to topological





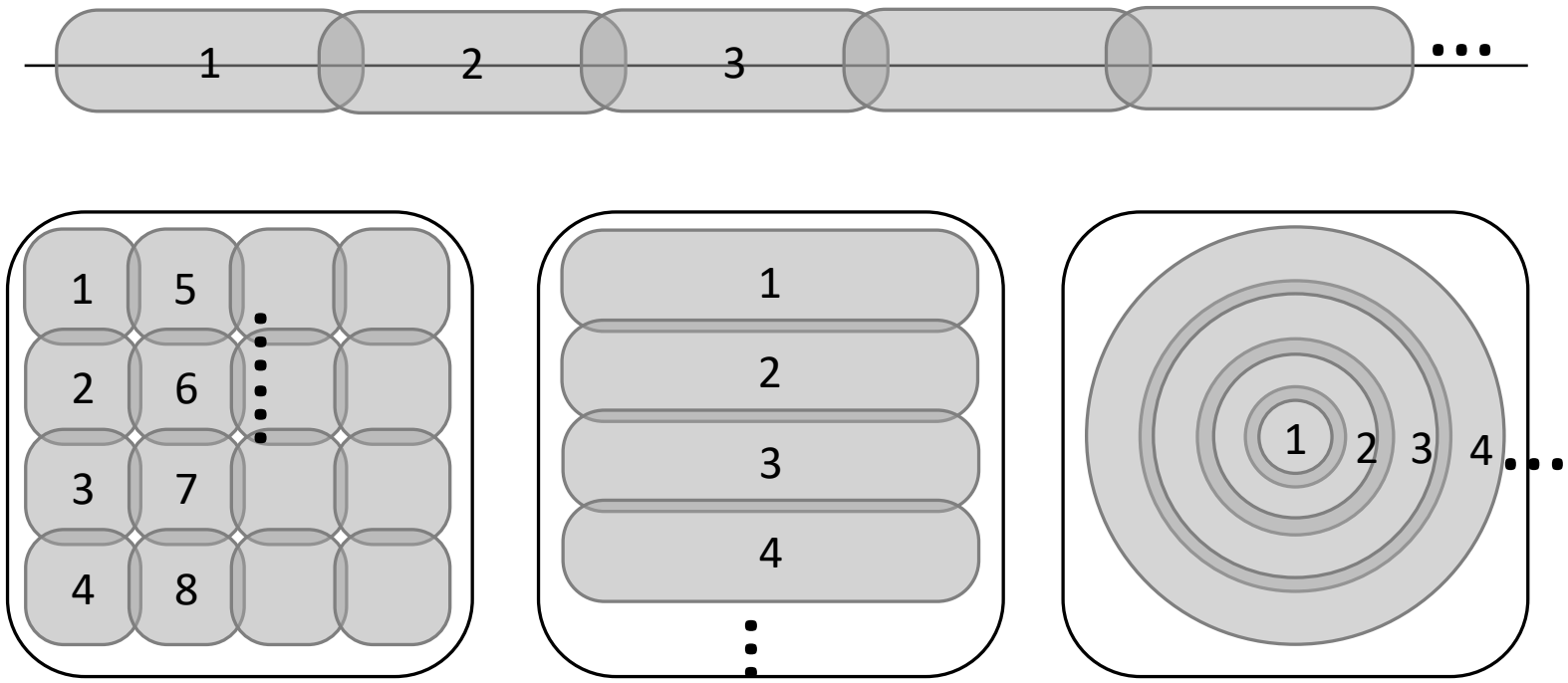
$= R(Z_e)R(A_p)$

$$R(O) = e^{-i\frac{\pi}{4}O}$$



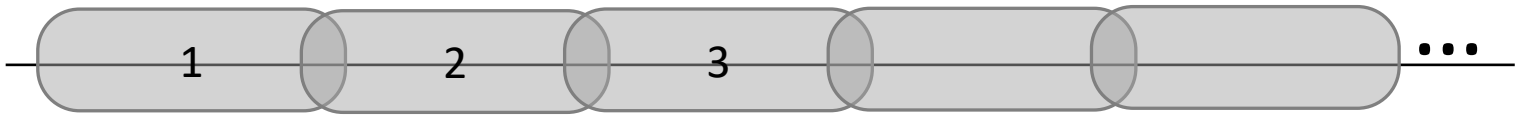
Depth $\sim L_x + L_y$

Sequential circuit



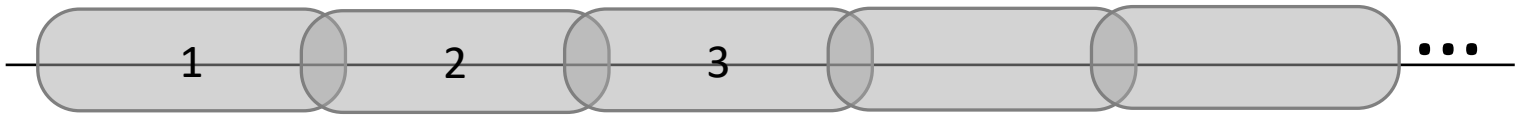
Schön, Solano, Verstraete, Cirac, 2005; Schön, Hammerer, Wolf, Cirac, 2007; Bañuls, Pérez-Garcia, Wolf, Verstraete, Cirac, 2008; Wei, Malz, Cirac, 2022 ….

Sequential circuit



- Sequential application of local unitaries / finite depth circuits
- Generically linear depth of higher
- Each local DOF acted on by a finite number of gates
- total # of gates ~ volume of system

Sequential circuit



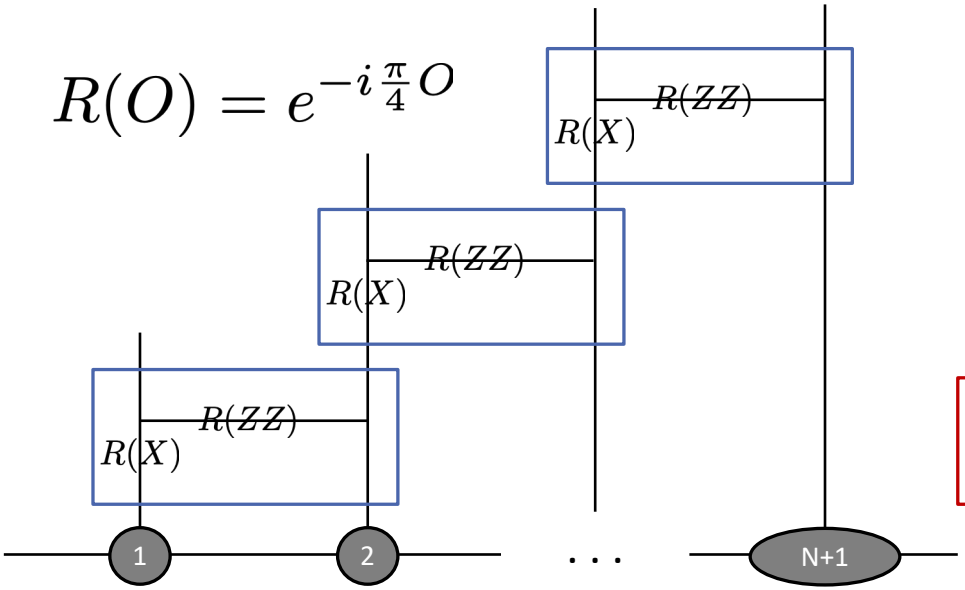
- Does not preserve locality
- Does not preserve ground state degeneracy
- Does not preserve short range correlation
- Does not preserve short range entanglement
- Preserves entanglement area law
- Preserves gap

Symmetric to Symmetry breaking

$$H = -\sum X_i \rightarrow H = -\sum Z_i Z_{i+1} \qquad U = \prod X_i$$

$$|\psi\rangle = |++...+\rangle \rightarrow |\psi\rangle = |00...0\rangle + |11...1\rangle$$

$$R(O) = e^{-i\frac{\pi}{4}O}$$



$$X_i \rightarrow Z_{i-1} Z_i$$

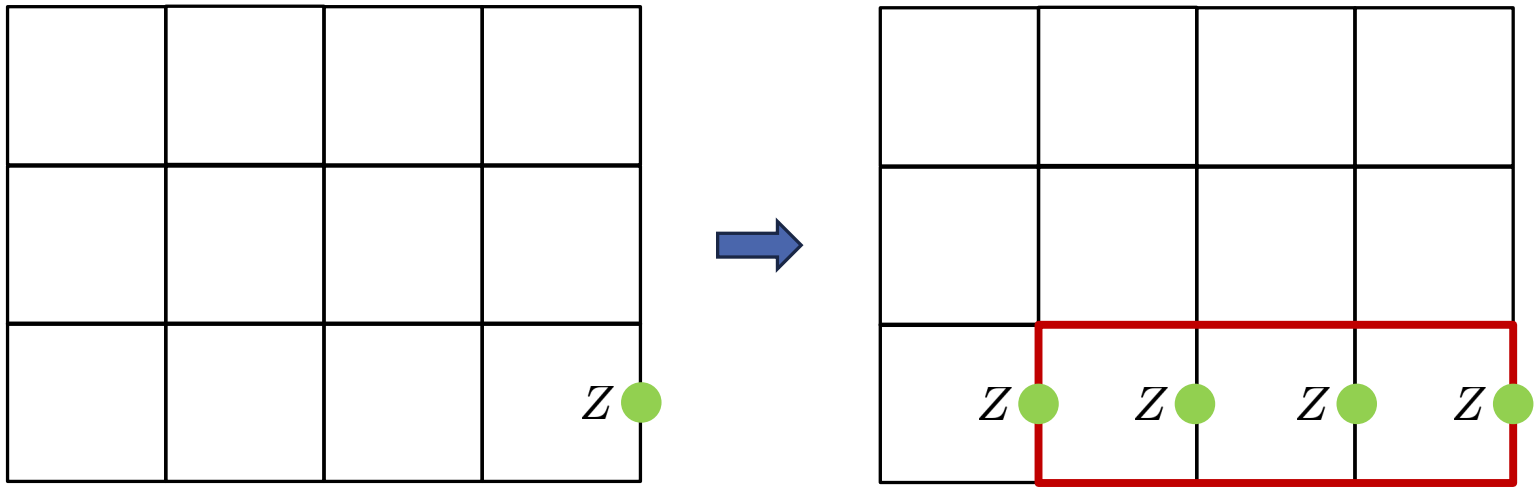
$$Z_i Z_{i+1} \rightarrow X_i$$

Depth $\sim L$

$$X_1 \rightarrow X_1 X_2 ... X_N \cdot Z_1 Z_N$$

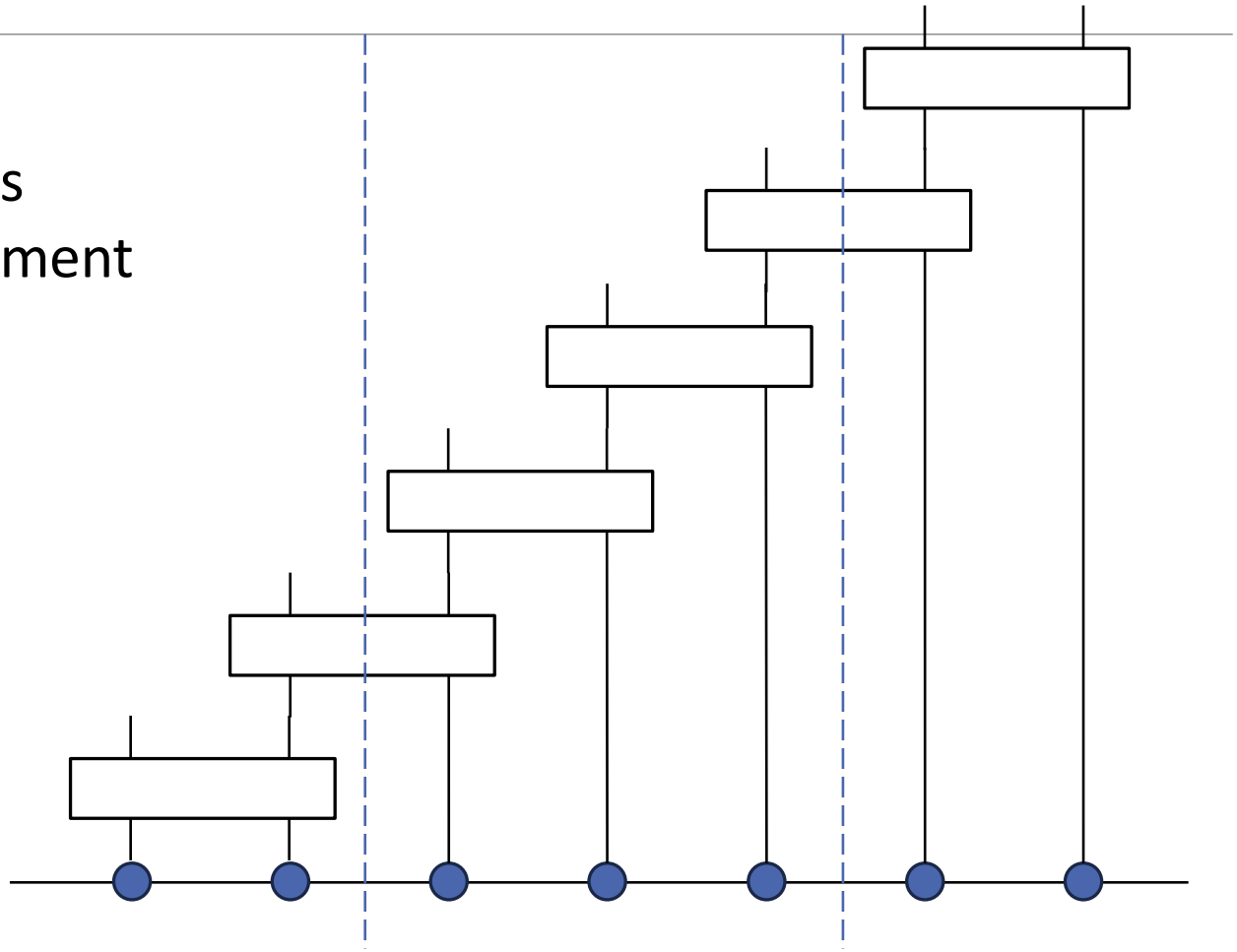
Schön, Solano, Verstraete, Cirac, Wolf, 2005; Ho, Hsieh, 2019

Trivial to topological



Sequential circuit

Preserves entanglement area law



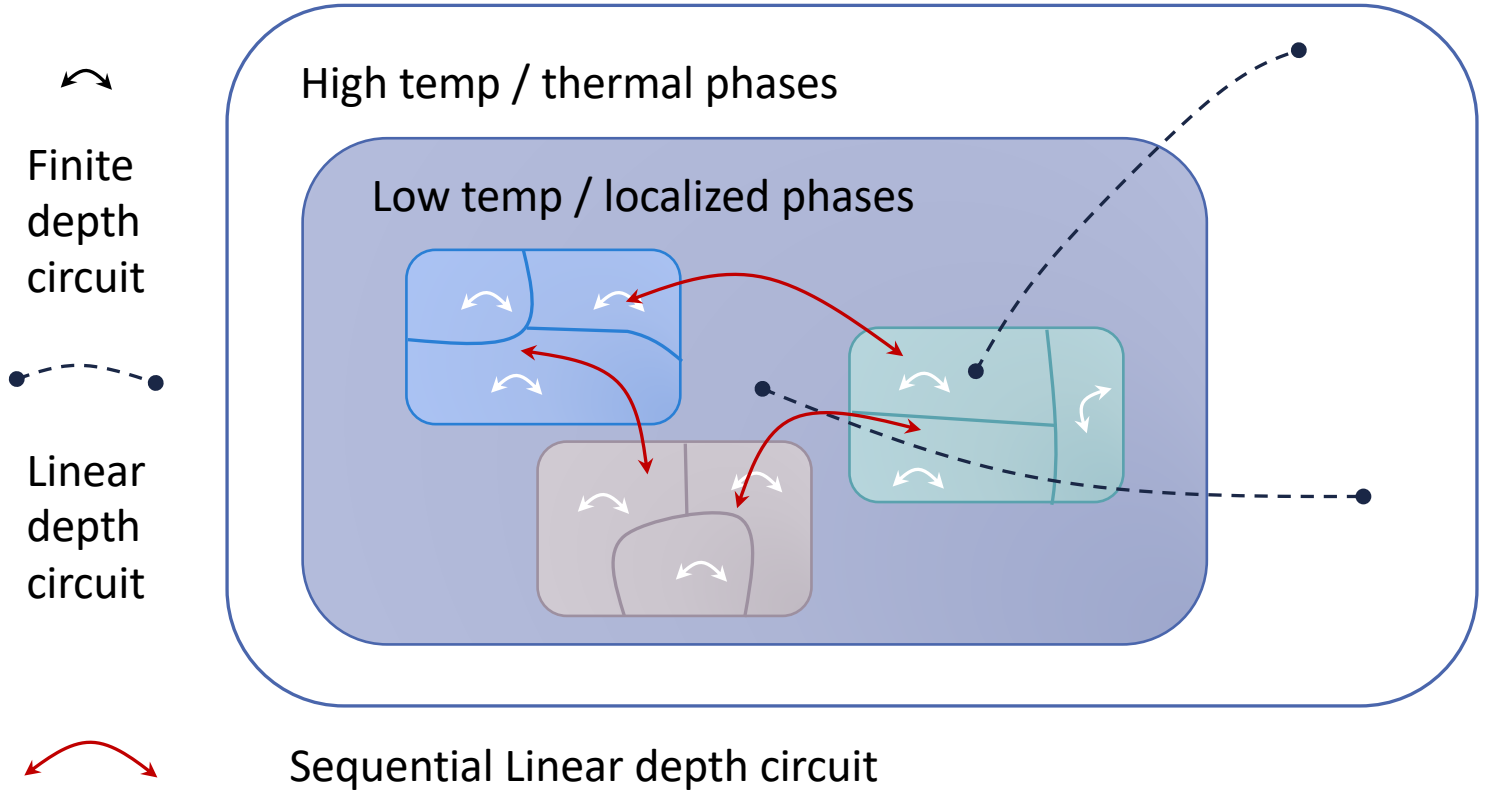
Sequential circuit

Maps between

- Trivial state
- Symmetry breaking GHZ state
- Symmetry protected topological state
- Fractional topological state
- Fracton state

XC, Dua, Hermele, Stephen, Tantivasadakarn, Vanhove, and Zhao, *Phys. Rev. B* 109, 075116 (2024)

Sequential circuit



Comparison

preserves	Finite depth circuit	QCA	Sequential circuit	Linear depth circuit
Area law	Yes	Yes	Yes	No
Locality	Yes	Yes	No	No
Short range correlation	Yes	Yes	No	No
Short range entanglement	Yes	Yes (in the bulk)	No	No

Sequential Circuit = Generalized Symmetry on Lattice

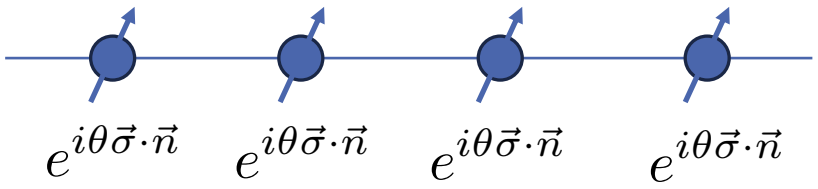
Generalized Symmetry

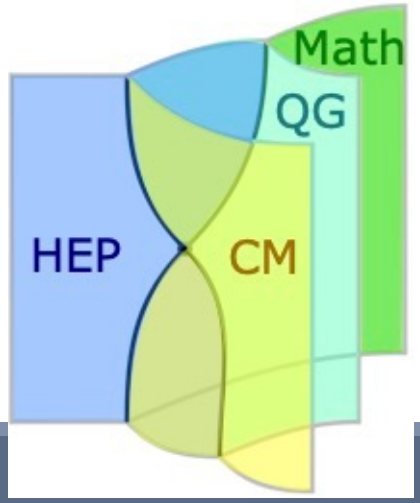
Conventional

- Global action
- Point-like charges
- Unitary / anti-unitary
- Forms a group

Generalized

- subsystem
- higher-form
- non-invertible
- fusion category

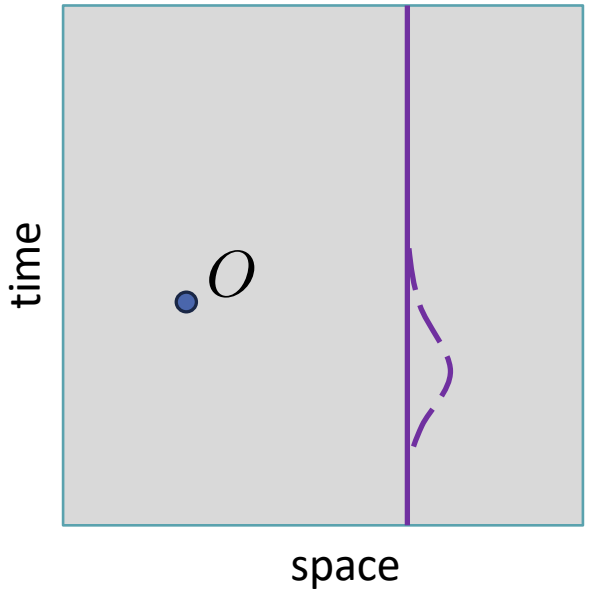




Gaiotto, Kapustin, Seiberg, Willett, 2014

Generalized Symmetry

Topological defect



$$\mathrm{Tr}\left(Oe^{-\beta H}\right)$$

Ising CFT

Z2 symmetry η

Kramers-Wannier σ

$$\sigma \times \sigma = 1 + \eta$$

non-invertible

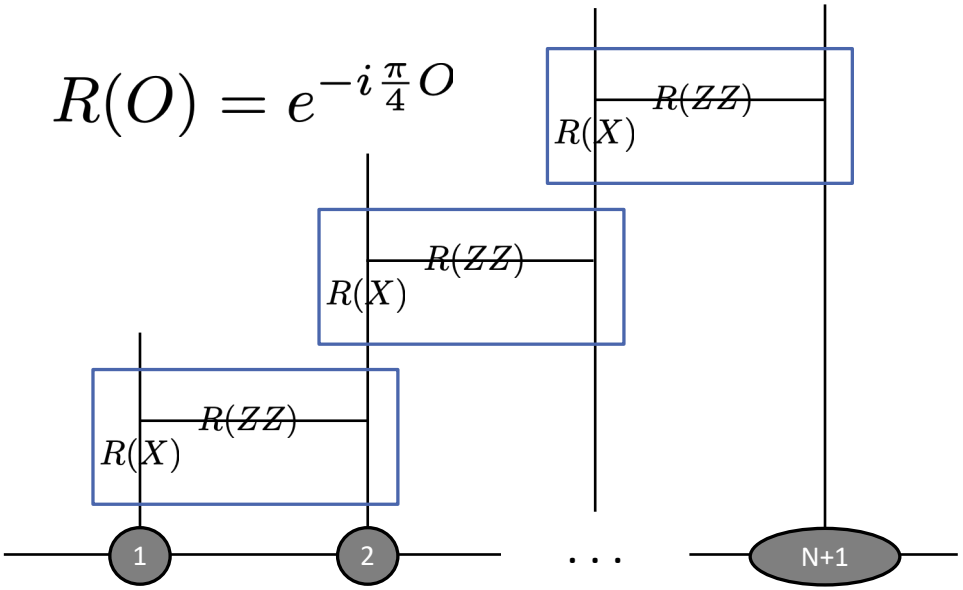
Aasen, Fendley, Mong, 2020

Symmetric to Symmetry breaking

$$H = -\sum X_i \rightarrow H = -\sum Z_i Z_{i+1} \qquad U = \prod X_i$$

$$|\psi\rangle = |++...+\rangle \rightarrow |\psi\rangle = |00...0\rangle + |11...1\rangle$$

$$R(O) = e^{-i\frac{\pi}{4}O}$$



$$X_i \rightarrow Z_{i-1} Z_i$$

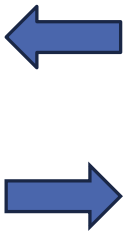
$$Z_i Z_{i+1} \rightarrow X_i$$

Depth $\sim L$

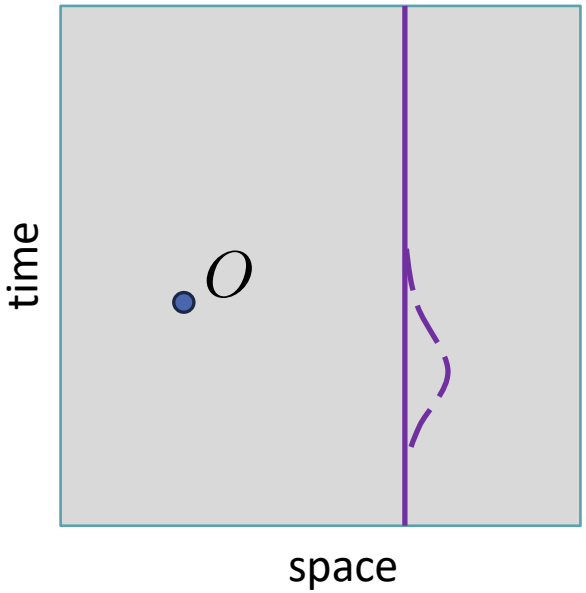
Kramers-Wannier transformation σ

Schön, Solano, Verstraete, Cirac, Wolf, 2005; Ho, Hsieh, 2019

Sequential Circuit = Generalized Symmetry on Lattice

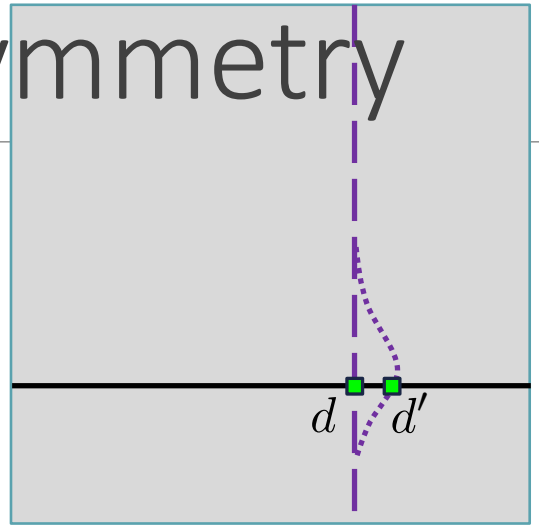


Generalized Symmetry

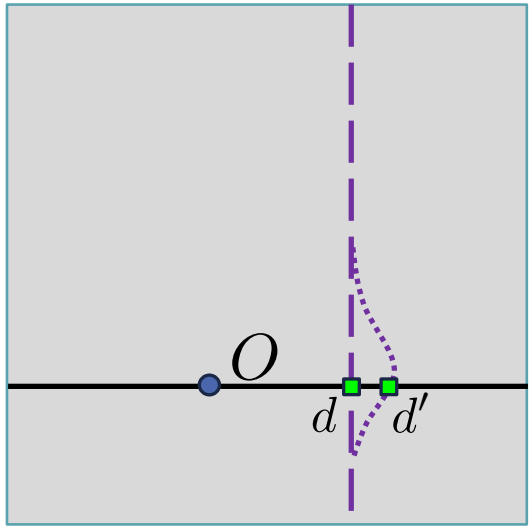


$$\mathrm{Tr}\left(Oe^{-\beta H}\right)$$

Symmetry twist



Unitary



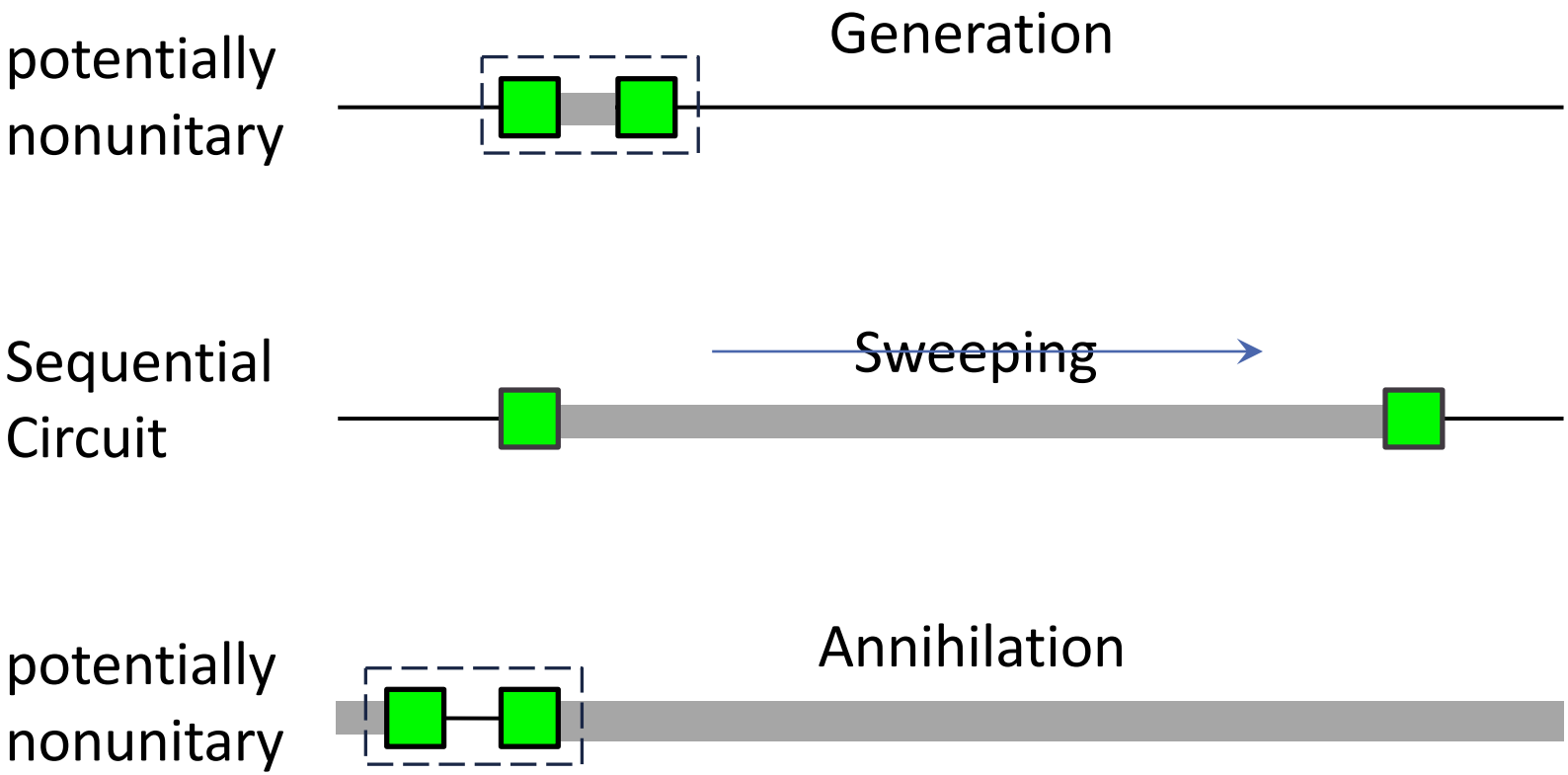
Local Unitary

Gaiotto, Kapustin, Seiberg, Willett, 2014

Sequential Circuit = Generalized Symmetry on Lattice

?

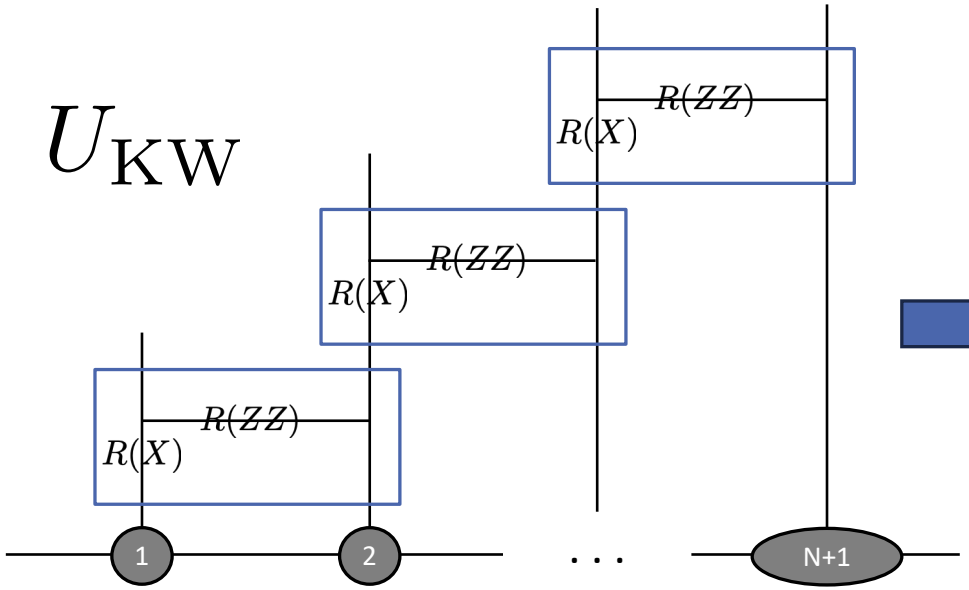
Generalized Symmetry



Kramers-Wannier

$$H = -\sum X_i \rightarrow H = -\sum Z_i Z_{i+1} \qquad U = \prod X_i$$

$$|\psi\rangle = |++...+\rangle \rightarrow |\psi\rangle = |00...0\rangle + |11...1\rangle$$



Kramers-Wannier σ

$$\sigma \times \sigma = 1 + \eta$$

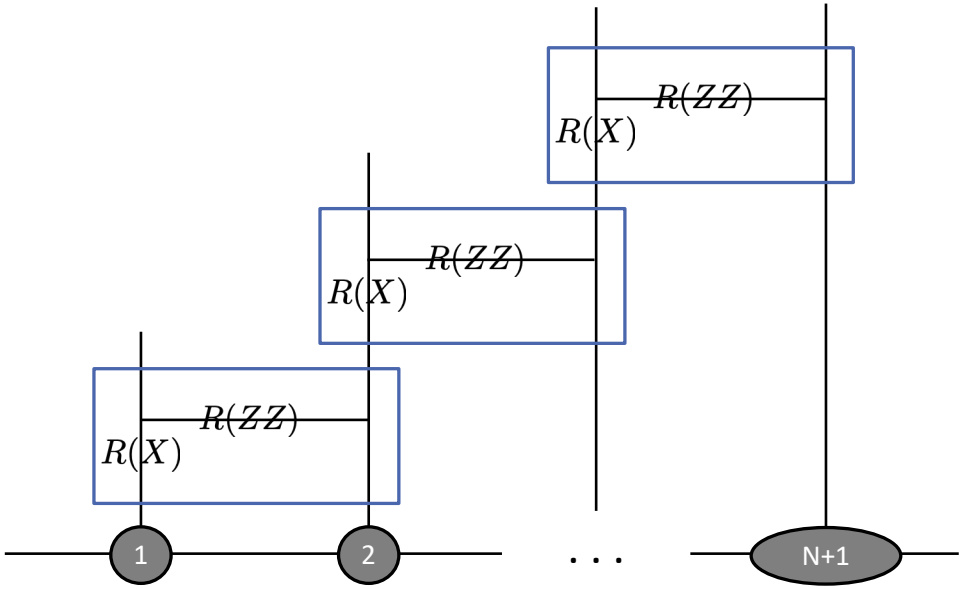
Kramers-Wannier

U_{KW}

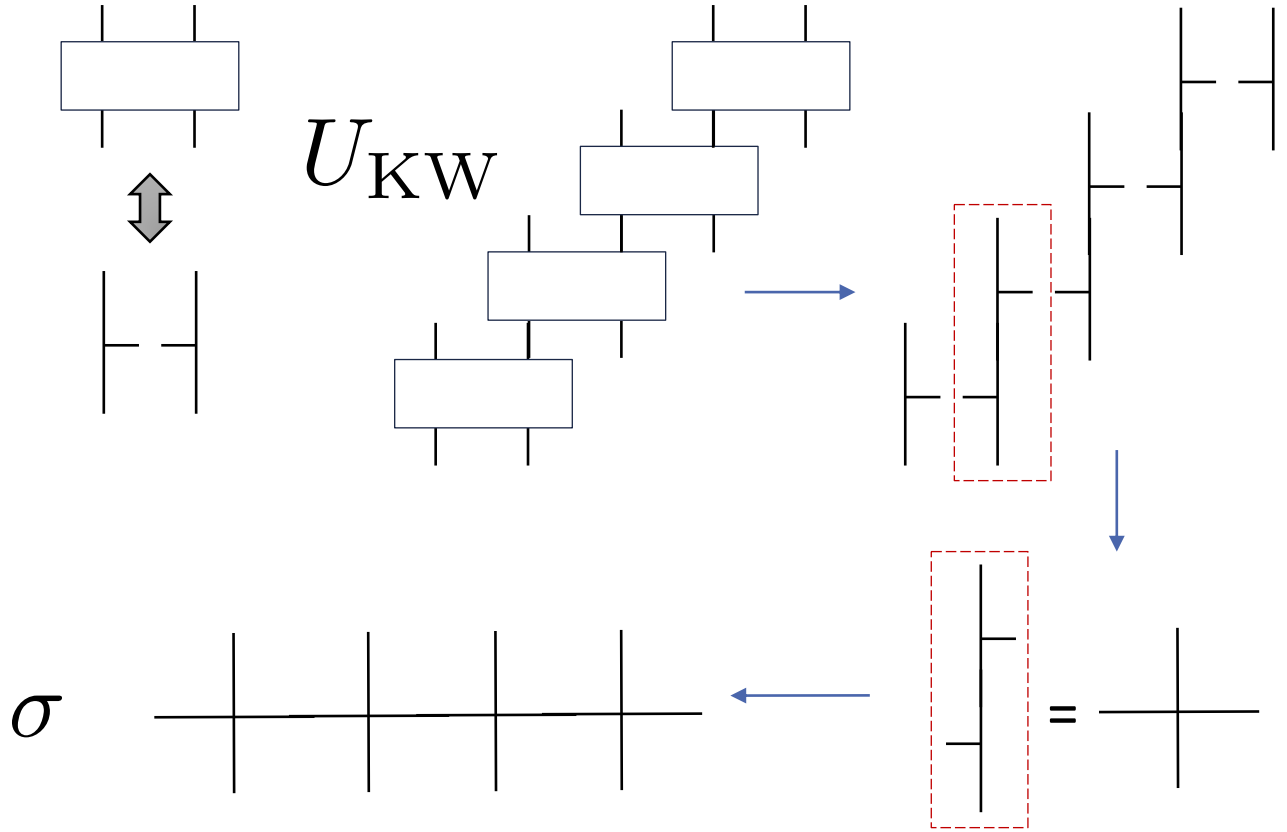
$$
\begin{aligned}
X_1 &\rightarrow X_1 X_2 ... X_{N-1} X_N \cdot Z_1 Z_N \\
X_i &\rightarrow Z_{i-1} Z_i, \; i = 2, ..., N \\
Z_i Z_{i+1} &\rightarrow X_i, i = 2 ... N \\
Z_1 Z_2 &\rightarrow X_2 ... X_N
\end{aligned}
$$

not translation invariant

and invertible



Kramers-Wannier:
matrix product operator representation



Kramers-Wannier

$$\sigma^\dagger \times \sigma = I + \eta$$

$$\sigma^\dagger X_i \sigma = Z_{i-1} Z_i \left(I + \eta\right)$$

$$\sigma^\dagger Z_{i-1} Z_i \sigma = X_{i-1} \left(I + \eta\right)$$

1D generalized symmetry

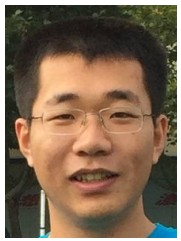
- “sequential” Matrix product operator
- Implemented with the sequential circuit in the bulk, together with non-unitary operations at the end

1D generalized symmetry

- Forms a fusion category
- $\mathcal{D}^\dagger \times \mathcal{D} = I + \ldots$ 'weakly' non-invertible
- 'anti'-symmetry is unique
- $\mathcal{D}_\beta \times \mathcal{D}_\alpha = \sum_\gamma N_{\alpha\beta}^\gamma \mathcal{D}_\gamma, \;\; N_{\alpha\beta} \in \mathbb{Z}$

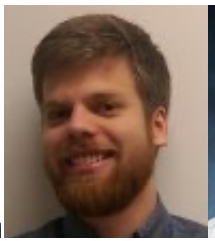
- Sequential Quantum Circuit as map between gapped phases
  Phys. Rev. B 109, 075116 (2024)
- Renormalization of Ising cage-net model and generalized foliation
  Phys. Rev. B 108, 035148 (2023)
- String operator for Cheshire strings in topological phases
  Phys. Rev. B 109, 165149 (2024)
- Sequential Adiabatic Generation of Chiral Topological Stat
  SciPost Phys. 19, 107 (2025)

Xinyu Liu



Xiuqi Ma

Zongyuan Wang



David Stephen

Mike Hermele

Nathanan Tantivasadakarn

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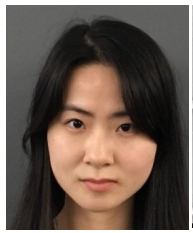
Arpit Dua

Jingyu Zhao

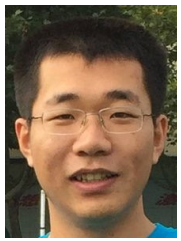


Wenjie Ji

Michael Levin

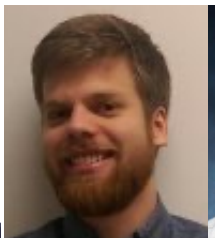
- Fusion of one-dimensional gapped phases and their domain walls
  Phys. Rev. B 110, 165144 (2024)
- Topological defects of 2+1D systems from line excitations in 3+1D bulk
  arXiv:2407.02488
- Fusion of Low-Entanglement Excitations in 2D Toric Code
  SciPost Phys. 18, 106 (2025)
- Sequential Circuit as Generalized Symmetry on Lattice
  arXiv:2507.22394

Xinyu Liu



Xiuqi Ma

Zongyuan Wang



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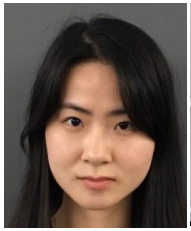

Wenjie Ji

Michael Levin