Many-body chirality of topological stabilizer states

(Ce n’est pas une talk)

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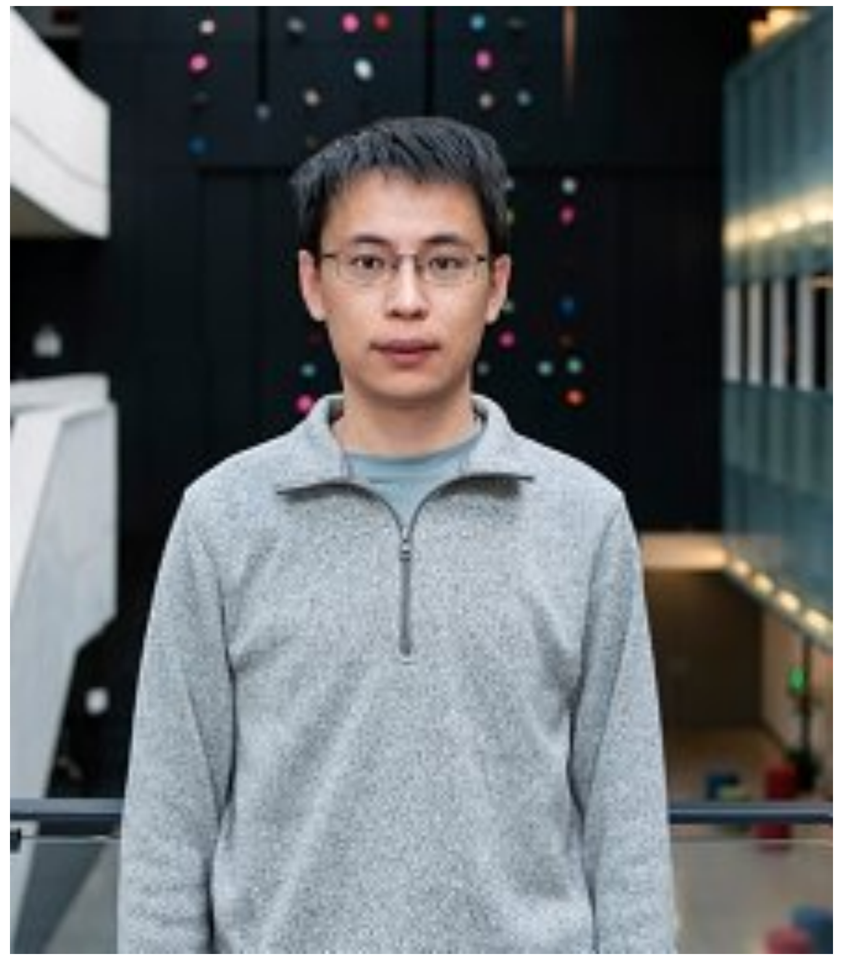

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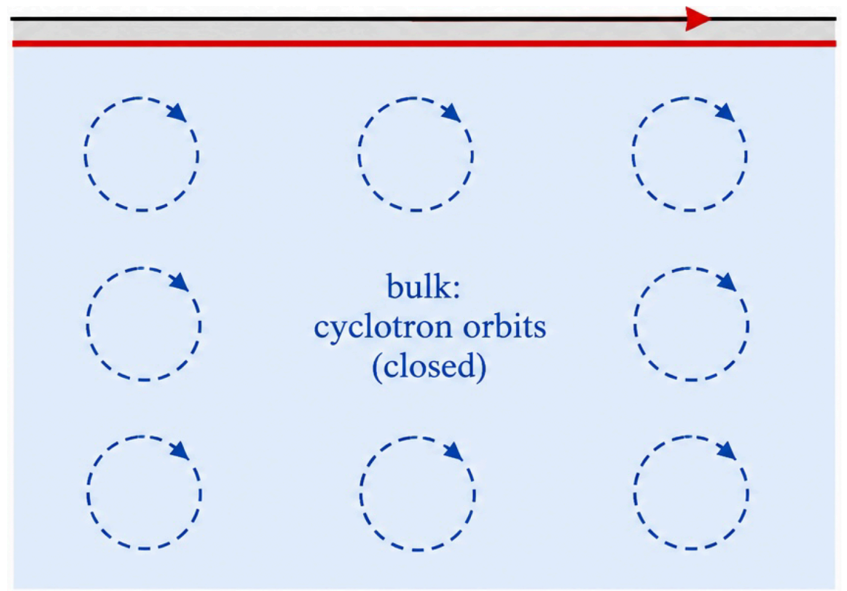
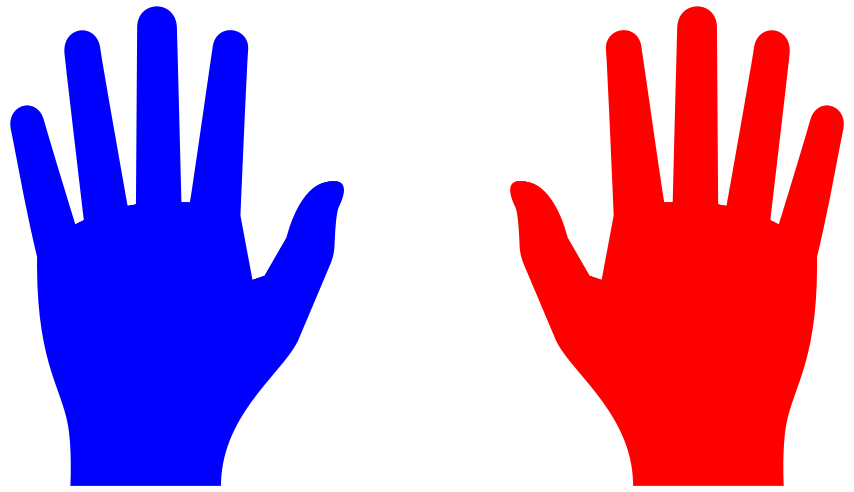
Motivation

- Often, chirality = chiral edge modes
- More generally, it is a handedness





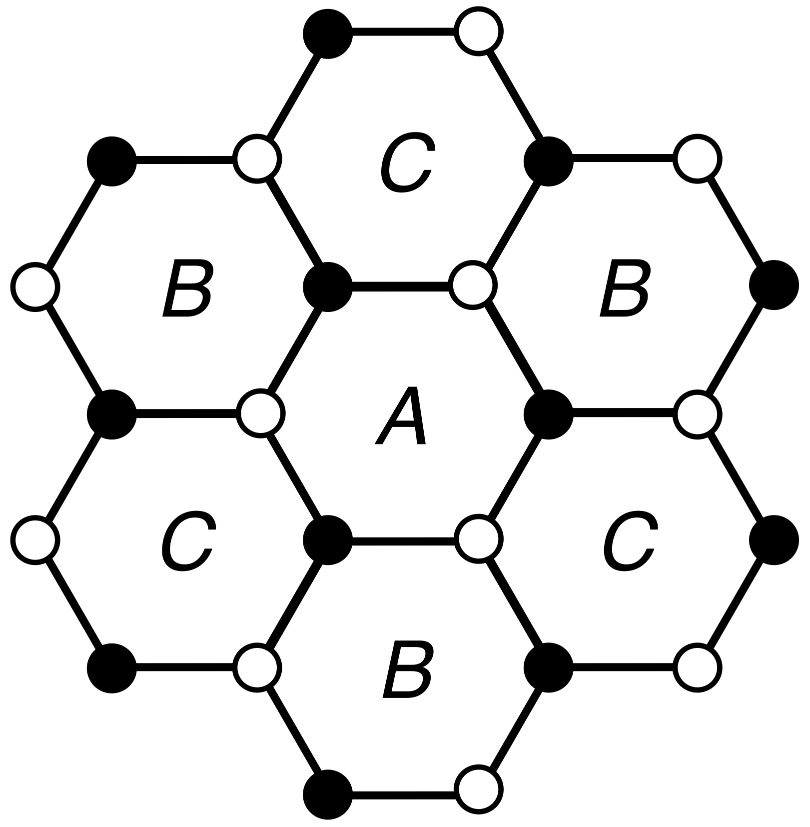
- How should we define chirality in many-body systems (including mixed states)?
  - Want a quantum-information-theoretic definition, at the level of states

Modular commutator: $J(A, B, C) = i\mathrm{Tr}(\rho_{ABC}[\log\rho_{AB}, \log\rho_{BC}]) = \frac{\pi}{3}c_-$

Not a full characterization of chirality, spoofed

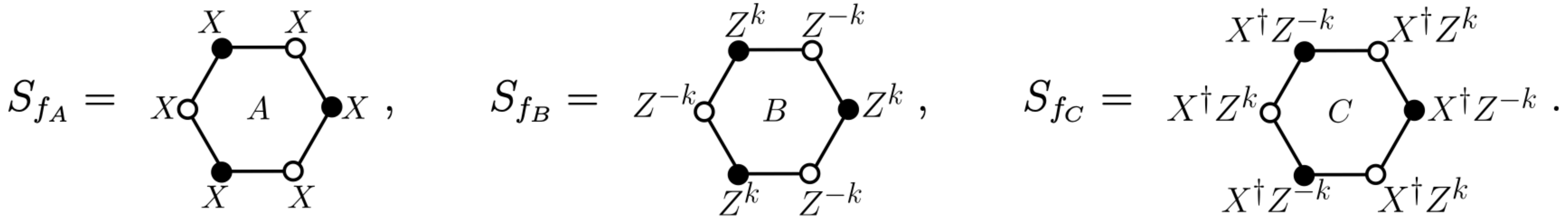
- Is there a faithful, local diagnostic of chirality?

Lattice models

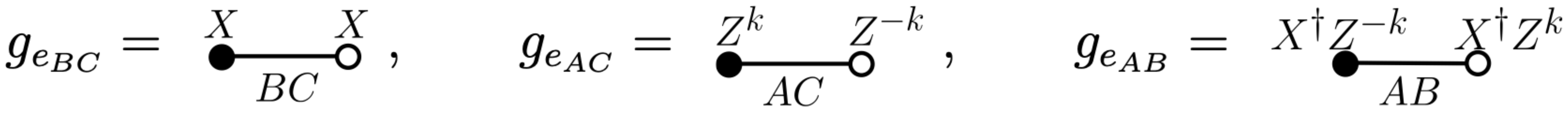


d-dimensional qudit on each vertex

Stabilizers: Strong symmetries



Large degeneracy! Logical operators: Weak symmetries

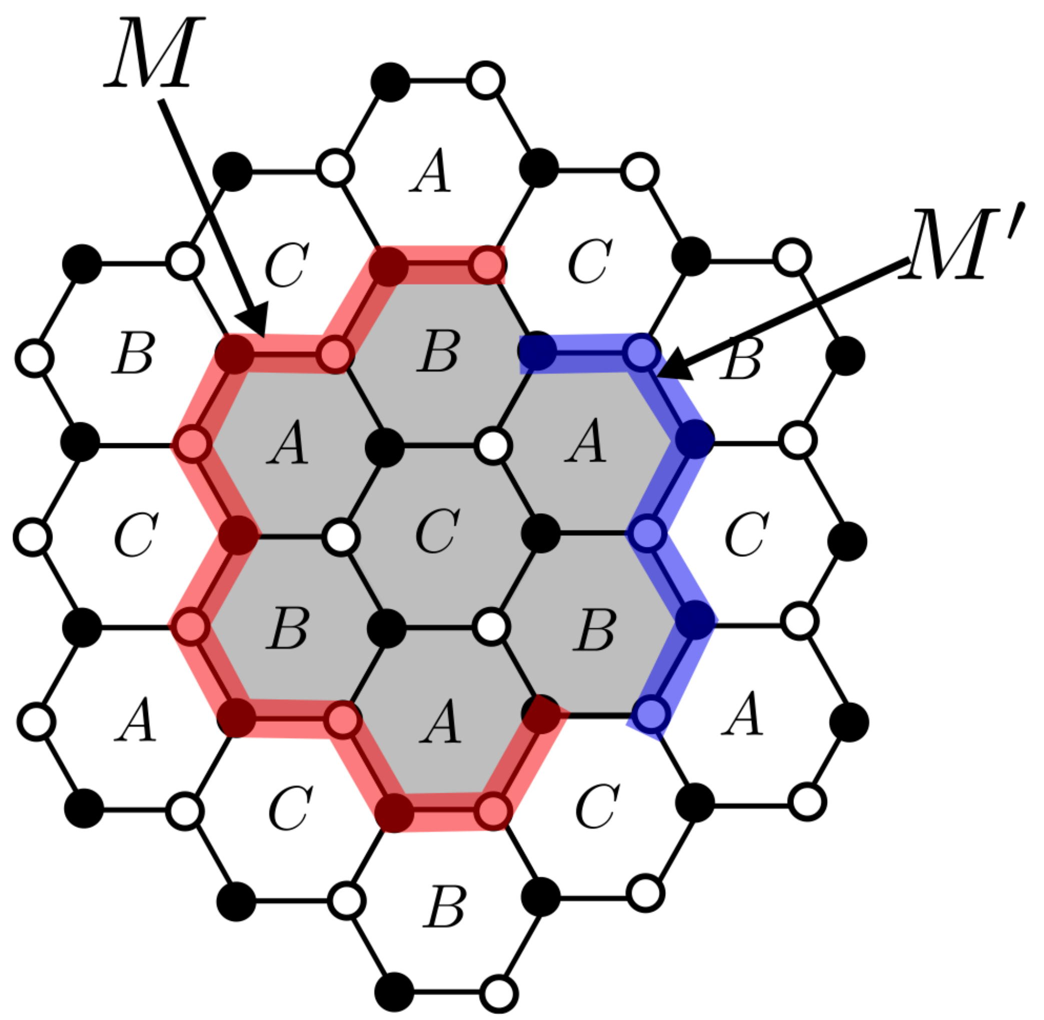


Maximally mixed in the logical space:

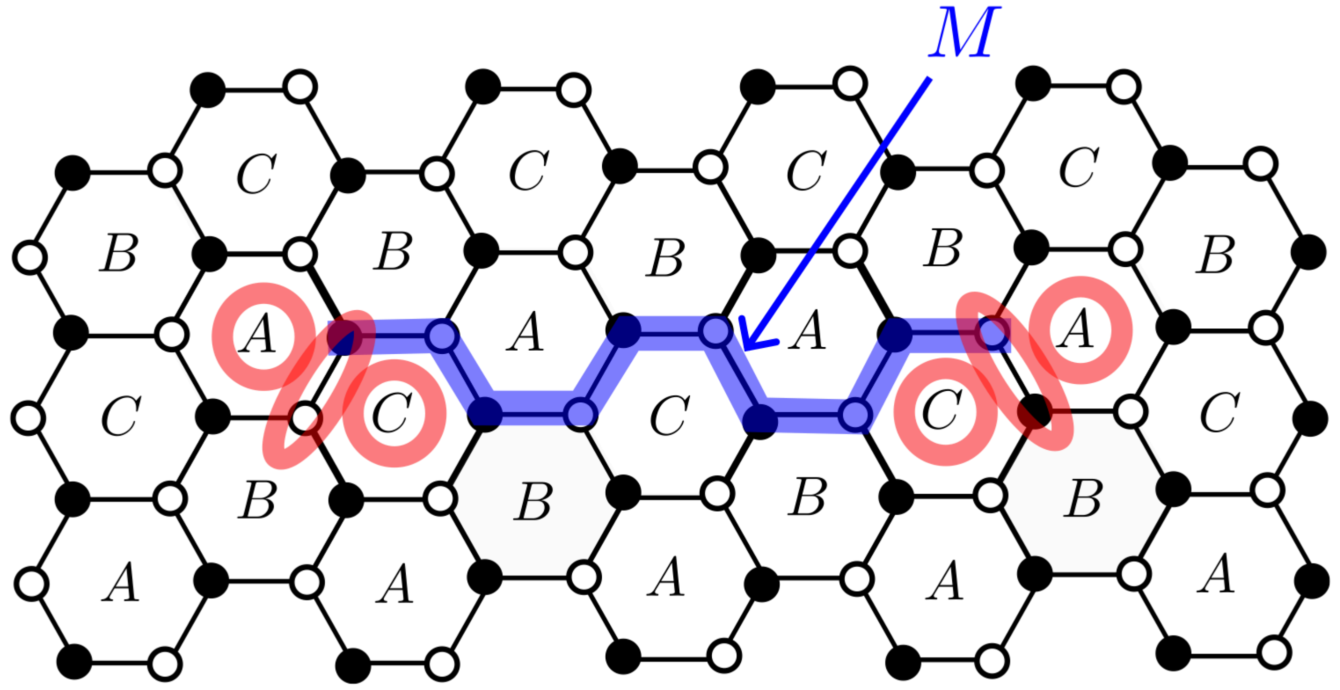
$$\rho_d^{(k)} = \frac{1}{d^n} \sum_{S \in \mathcal{S}} S$$

Anyon theory (using doubled space)

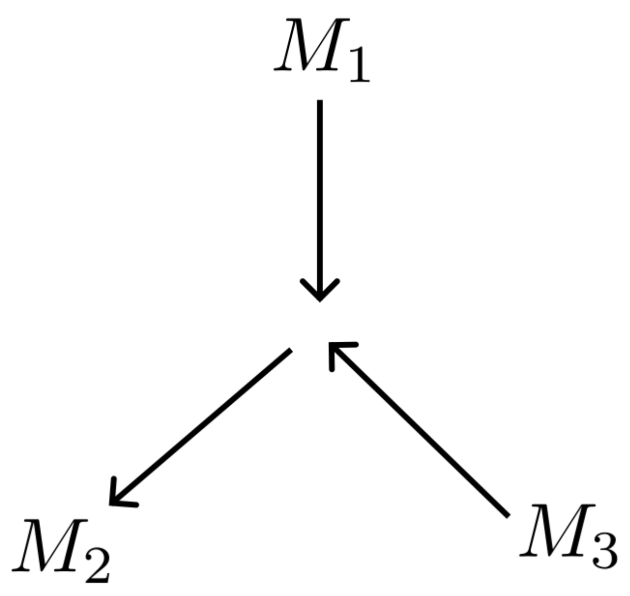
Product of stabilizers over region is a loop-like strong symmetry



Truncation violates stabilizers at the endpoints



T-junction calculation of exchange statistics:



<u>Anyon theory is $\mathbb{Z}_d^{(k)}$</u>

- Subtheory of $\mathbb{Z}_d$ toric code generated by em^k
- Example: $\mathbb{Z}_3^{(1)} = \{1,\ em,\ e^2m^2\}$

Mirror invariance

a^j	$\mathbb{Z}_3^{(1)}$
1	1
a	ω
a^2	ω

$\omega = e^{2\pi i/3}$

Not mirror invariant

a^j	$\mathbb{Z}_5^{(1)}$	$\mathbb{Z}_5^{(-1)}$
1	1	1
a	ω	ω^4
a^2	ω^4	ω
a^3	ω^4	ω
a^4	ω	ω^4

$\omega = e^{2\pi i/5}$

Mirror invariant

a^j	$\mathbb{Z}_9^{(1)}$	$\mathbb{Z}_9^{(-1)}$
1	1	1
a	ω	ω^8
a^2	ω^4	ω^5
a^3	1	1
a^4	ω^7	ω^2
a^5	ω^7	ω^2
a^6	1	1
a^7	ω^4	ω^5
a^8	ω	ω^8

$\omega = e^{2\pi i/9}$

Not mirror invariant

Chirality under local operations

Definition

A many-body state ρ is LO-chiral if there does not exist a finite-depth local operation Φ such that:

$$\Phi(\rho) = \rho^*$$

Chirality under local unitaries

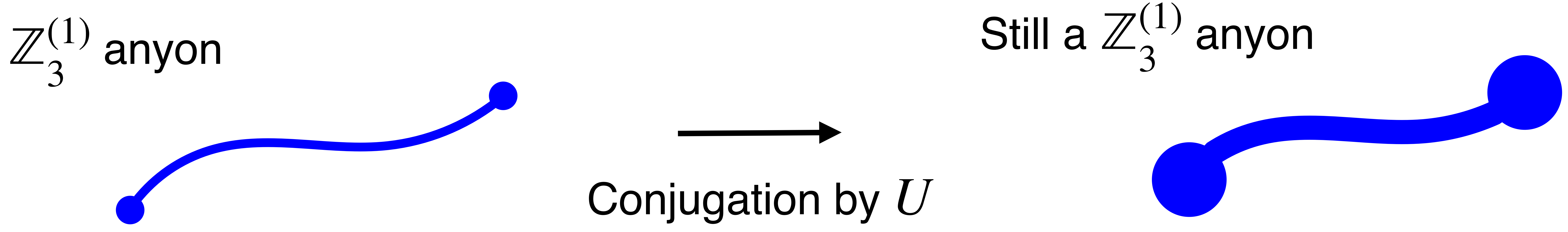
Definition

A many-body state ρ is LU-chiral if there does not exist a finite-depth circuit U such that:

$$U\rho U^\dagger = \rho^*$$

- Naively: anyon theory does not change under U
- Therefore: non-mirror invariant models, like $\mathbb{Z}_3^{(1)}$, should be LU-chiral

Chirality under local unitaries



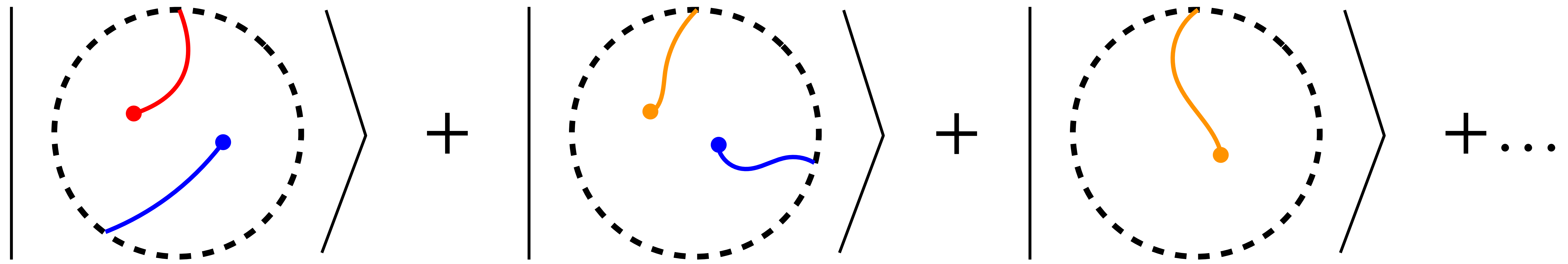
There are no $\mathbb{Z}_3^{(1)}$ anyons in $\mathbb{Z}_3^{(-1)}$, so we are done??

Need to prove that $\mathbb{Z}_3^{(-1)}$ mixed state does not have any $\mathbb{Z}_3^{(1)}$ anyons…

Chirality under local unitaries

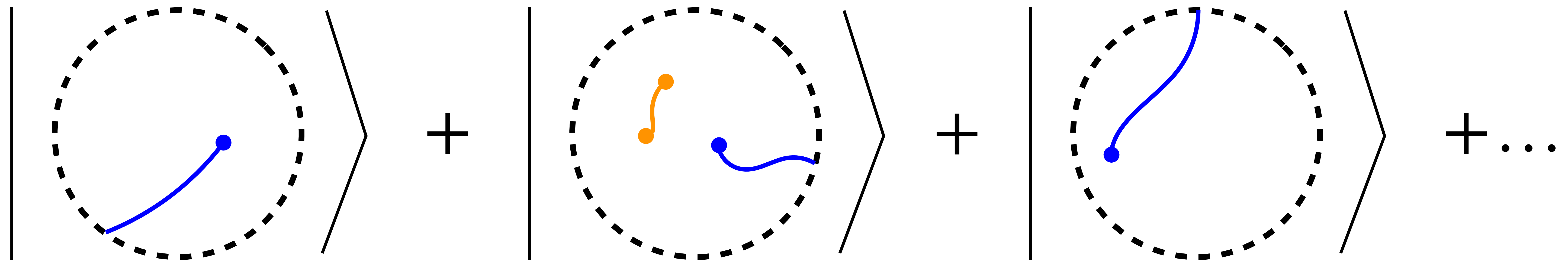
$\mathbb{Z}_3^{(1)}$ anyons could be spoofed by:

(1) Superpositions of different anyon types



Proved that fattened string operator creates definite $\mathbb{Z}_3^{(-1)}$ anyon type

(2) Superpositions of same anyon type, different configurations



Proved that there exists local unitary that “cleans” up the anyons

Chirality under local operations

Definition

A many-body state ρ is LO-chiral if there does not exist a finite-depth local operation Φ such that:

$$\Phi(\rho) = \rho^*$$

LO-chirality

A stabilizer mixed/pure state supporting abelian anyons $\mathcal{A}$ is LO-chiral iff the anyon theory is not mirror invariant:

$$\mathcal{A} \not\simeq \mathcal{A}^*.$$

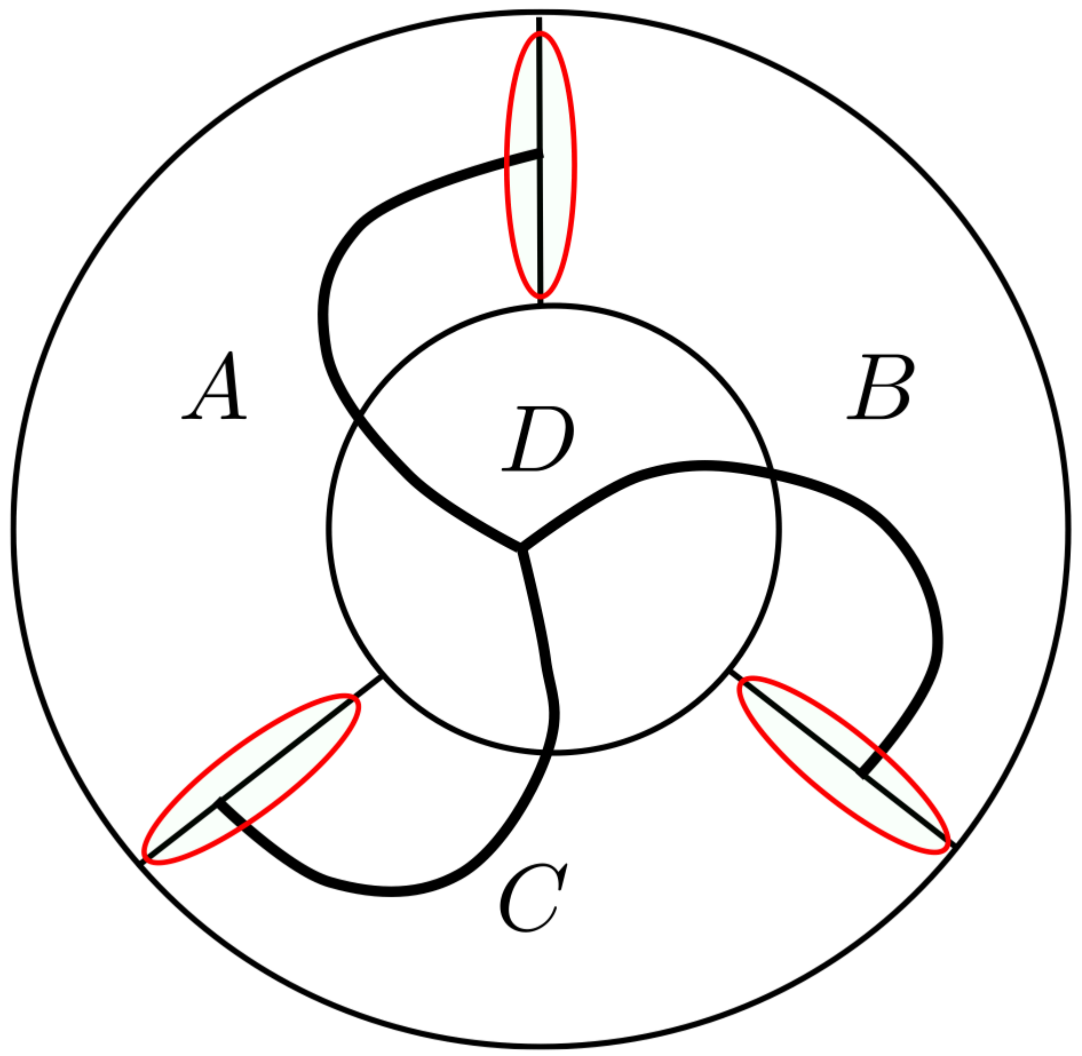
4-partite chirality

Definition

A state ρ is n-partite chiral if there does not exist an n-partite unitary $U = U_1 \otimes U_2 \otimes \cdots \otimes U_n$ such that:

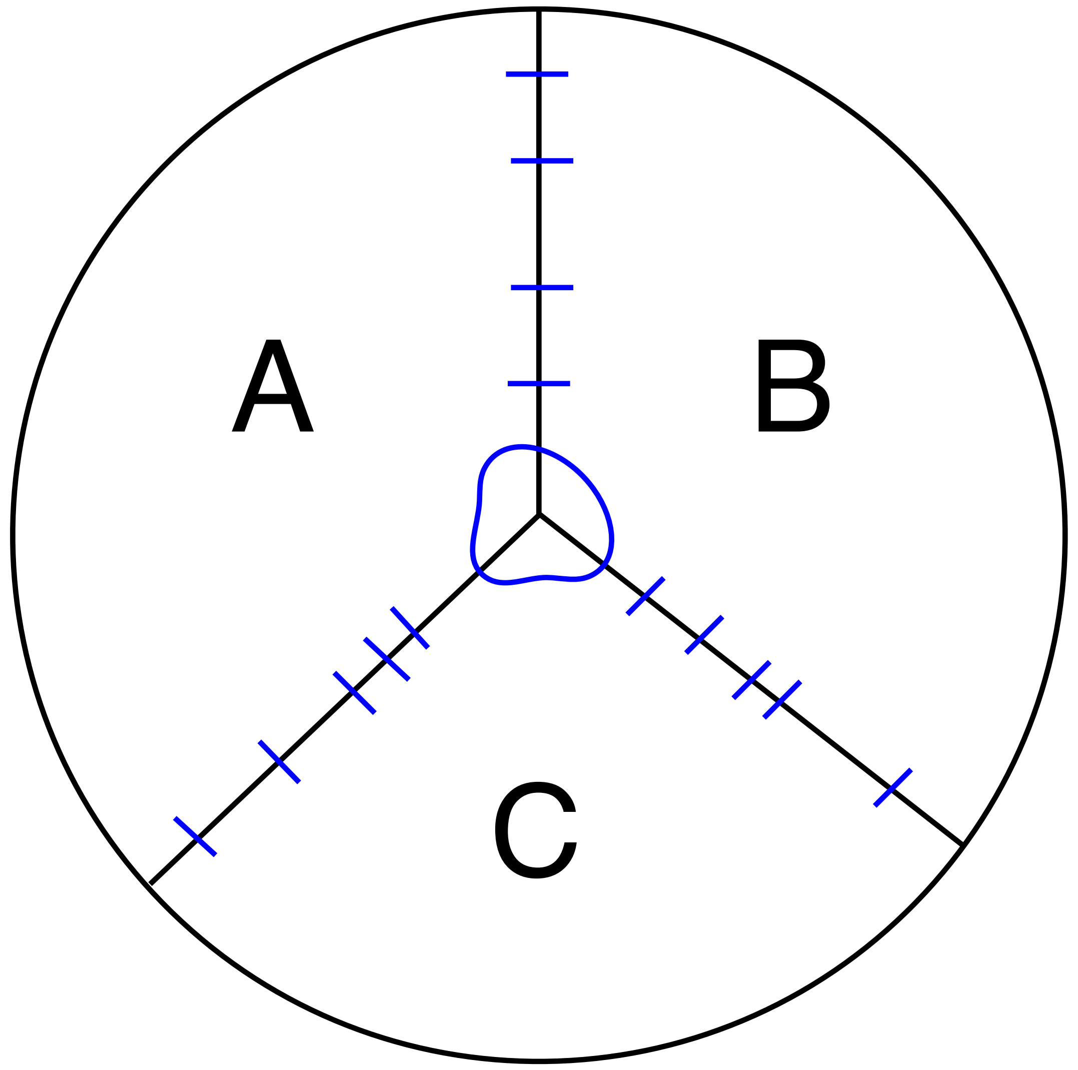
$$U\rho U^\dagger = \rho^*$$

4-partite chiral for non-mirror invariant models



T-junction is obstruction

Not 3-partite chiral



Reduces to GHZ and Bell pairs

Implies entanglement diagnostic needs to be 4-partite, at least

Open questions

- Is there a local diagnostic for chirality?
- Can we design explicit, controllably solvable lattice models for a wider class of chiral phases?
  - Kitaev's honeycomb model
  - Continuous variable models
- Are the $\mathbb{Z}_3^{(1)}$ mixed state and a $\mathbb{Z}_3^{(1)}$ pure state in the same phase?
- Can we extend our notion of chirality to an obstruction to parity symmetry?
  - Three fermion theory
- Is it valuable to discuss chirality in the context of mixed states?
- Close connections between chirality in: (1) pure states, (2) constrained pure states (invertible subalgebras), (3) mixed states, and (4) boundaries of 3D models. Can we fill in the details of (2)?