Seismic inversions: a sophisticated method for stellar characterisation in PLATO

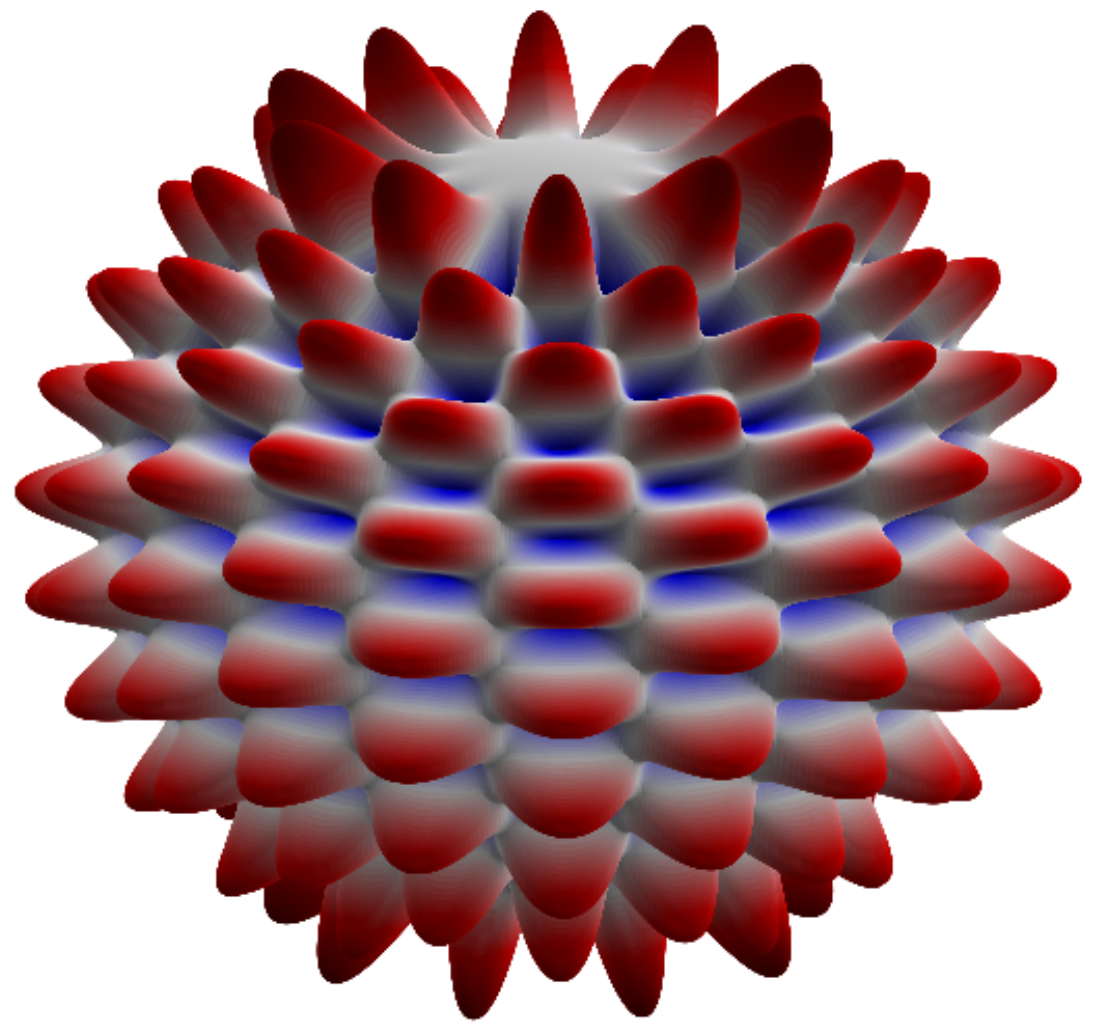
Jérôme Bétrisey

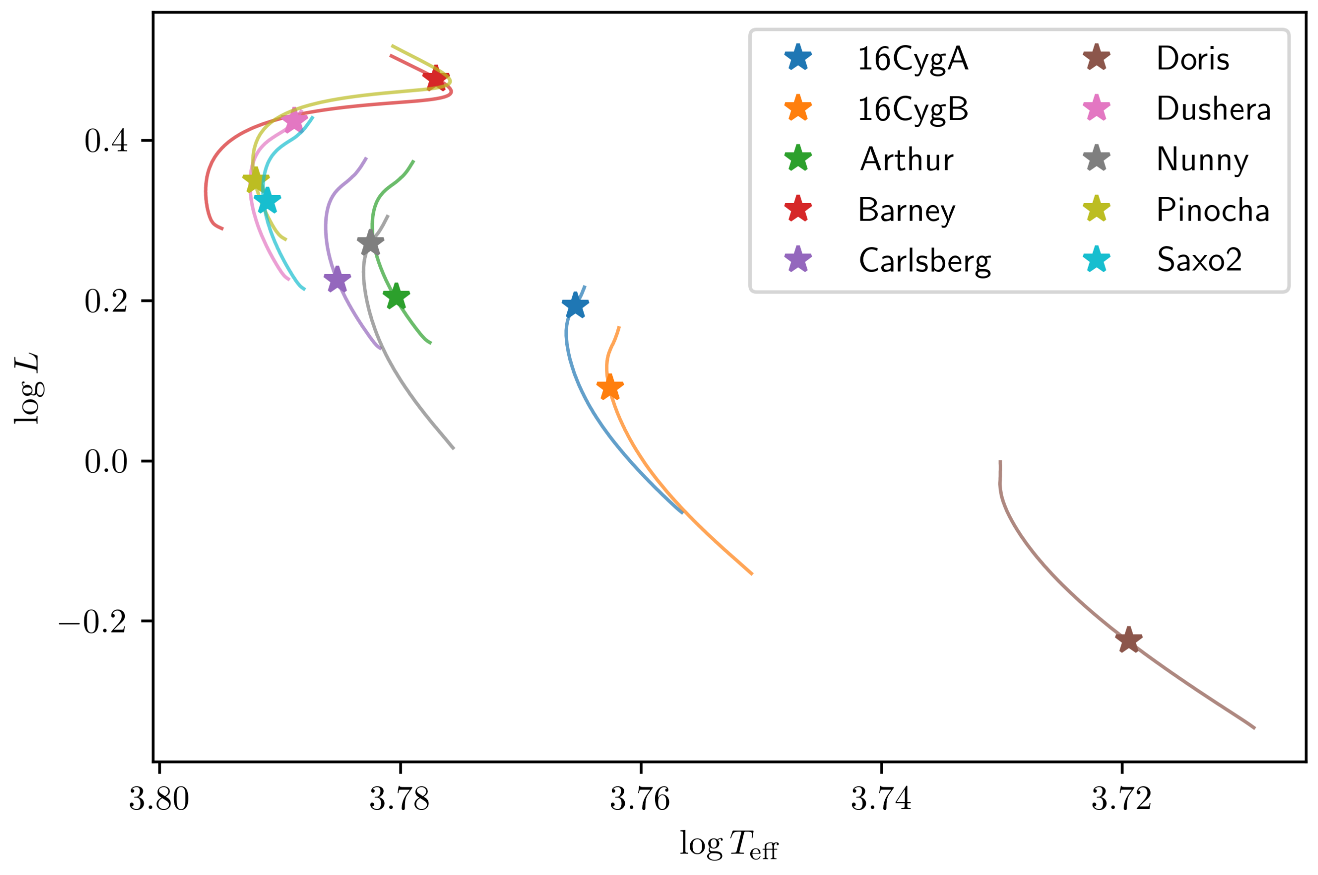
jerome.betrisey@physics.uu.se, Department of Physics and Astronomy, Uppsala University, Box 516, SE-751 20 Uppsala, Sweden

Collaborators: Daniel Reese (PSM) and Gaël Buldgen (PDC)

UPPSALA UNIVERSITET

General context



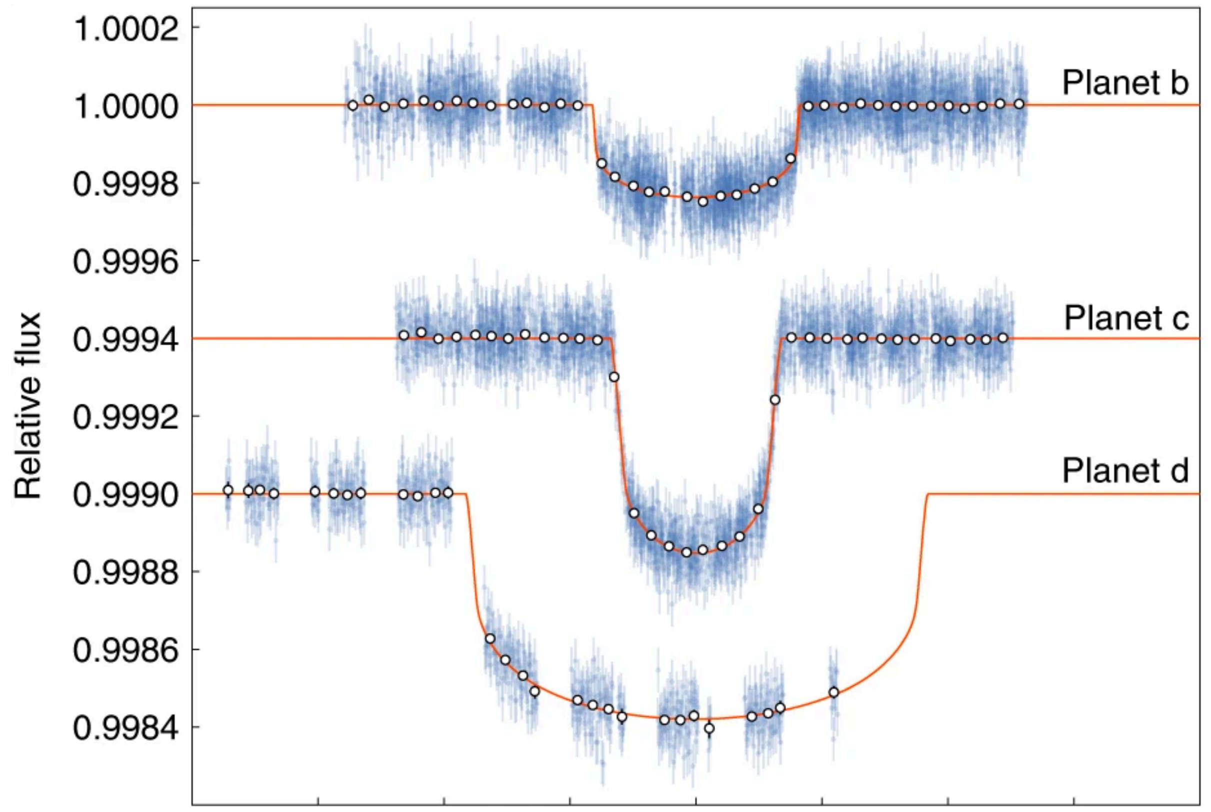


Constrain precisely and accurately the stellar parameters (mass, radius, age, etc.)

For reviews, see e.g. Chaplin & Miglio 2013; García & Ballot 2019

Multidisciplinary applications

Exoplanetology

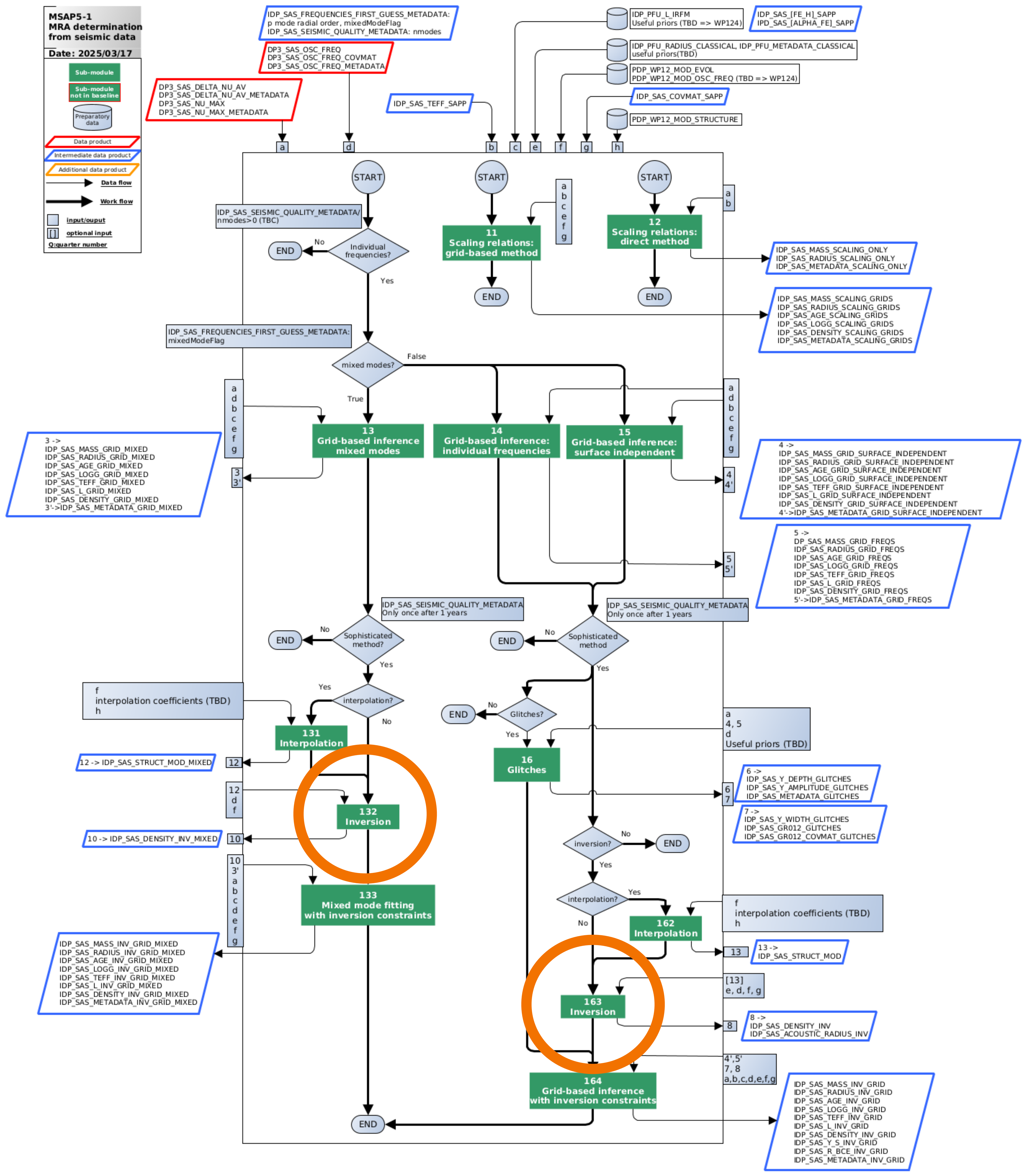


From Delrez+ 2021

Galactic archeology

From eso.org

MSAP5 and WP124 200 (Inverse methods)



Scaling relations

Grid-based inferences (« Forward modelling »)

Sophisticated methods:
seismic inversions and helium glitch

Combination of forward and inverse methods:
grid-based inference with inversion constraints

Forward modelling

Observations

Frequencies
+
Classical constraints

Stellar physics

Hypotheses*
+
Free variables**

Minimisation

Adjustment of a cost function

$$\chi^2 = \sum_i \frac{(O_i - M_i)^2}{\sigma_i^2}$$

Stellar model

Hypotheses*
+
Optimized variables**

*__on the physical ingredients__ (abundances, opacities, atmosphere, fixed parameters such as overshooting or/and α_{MLT}, …), and **on the pulsation model** (adiabaticity, small amplitudes, effects of rotation, …)

e.g. **mass, **age**, **initial chemical composition** $(X_0 \text{ and/or } Z_0)$, **mixing-length parameter**, **overshooting**, …

Seismic inversions

Stellar model

Forward modelling $\longrightarrow$

$\longleftarrow$ Inverse approaches

Oscillation frequencies

Perturbative analysis of stellar oscillations at linear order

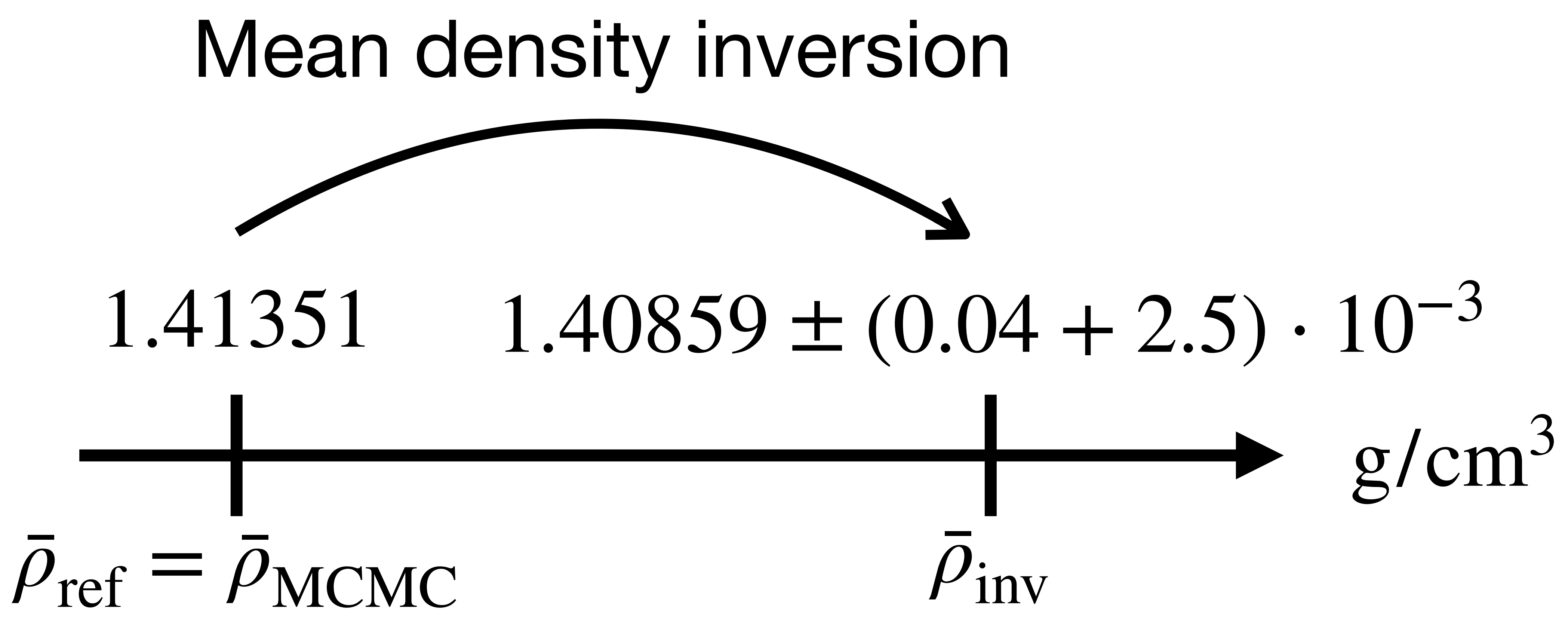
- **Equation of motion fulfills a variational principle** (Chandrasekhar 1964; Chandrasekhar & Lebovitz 1964; Clement 1964; Lynden-Bell & Ostriker 1967)
- In the case of individual frequencies, the **frequency perturbation is directly related to the structural perturbation** (Dziembowski+ 1990):

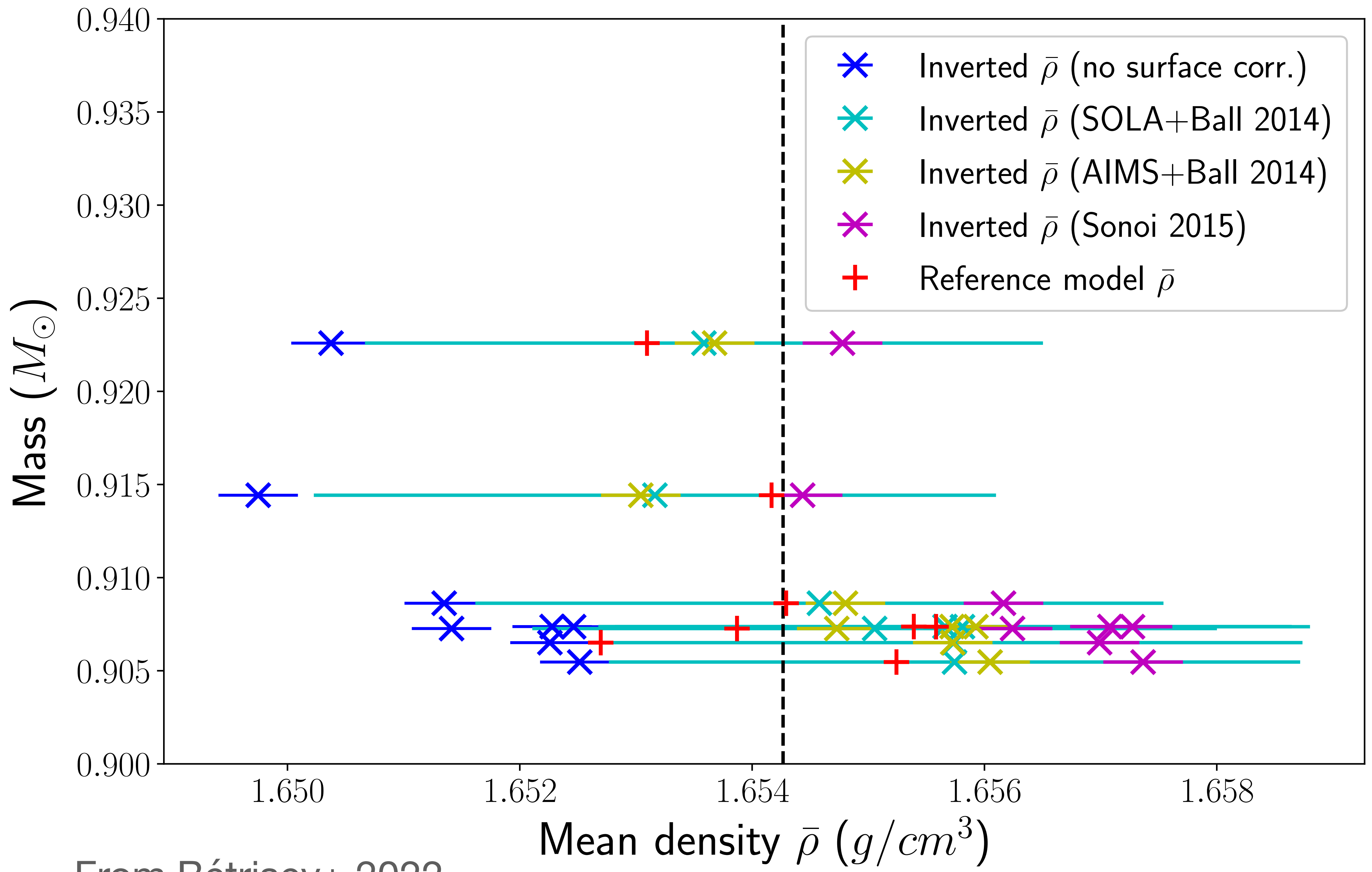
$$\frac{\delta\nu^{n,l}}{\nu^{n,l}} = \int_0^R K^{n,l}_{\rho,c^2}(r)\frac{\delta\rho}{\rho}dr + \int_0^R K^{n,l}_{c^2,\rho}(r)\frac{\delta c^2}{c^2}dr + \mathcal{O}(\delta^2) \quad (1)$$

Can be solved in a quasi model-independent way

- Idea: **combine the equations (1) to compute a structural correction of a reference model** (e.g. best fit model from the forward modeling) **based on the observed frequency differences**
- **Several methods to solve the structure inversion equation:** RLS (Phillips 1962; Tikhonov 1963), MOLA (Backus & Gilbert 1968, 1970), and SOLA (Pijpers & Thompson 1992, 1994)

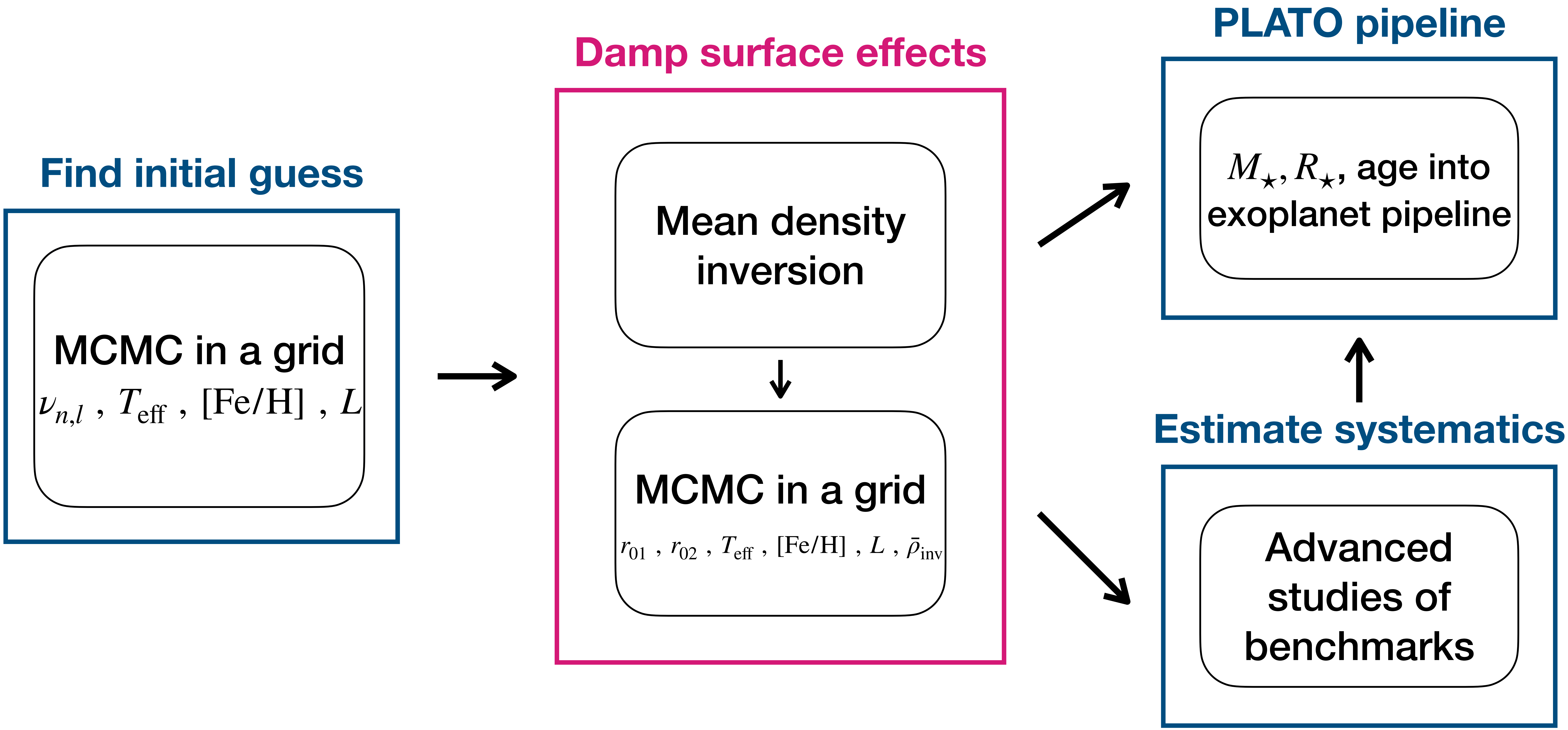
Mean density inversion





From Bétrisey+ 2022

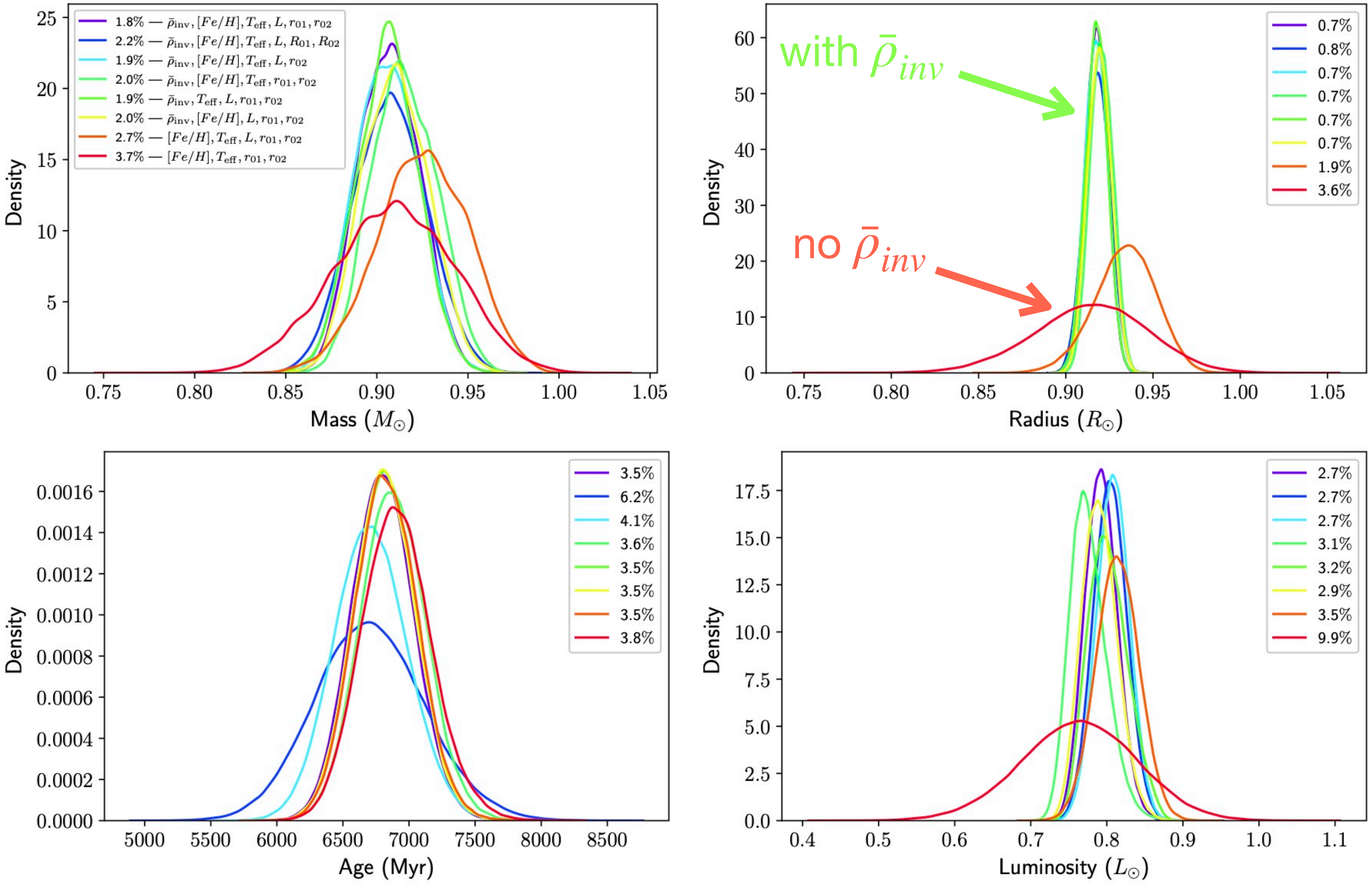
Combining forward and inverse methods



See also: e.g. Reese+ 2012; Buldgen+ 2019; Bétrisey 2024

See e.g. Buldgen+ 2019, Bétrisey+ 2022

Inverted mean density: benefits

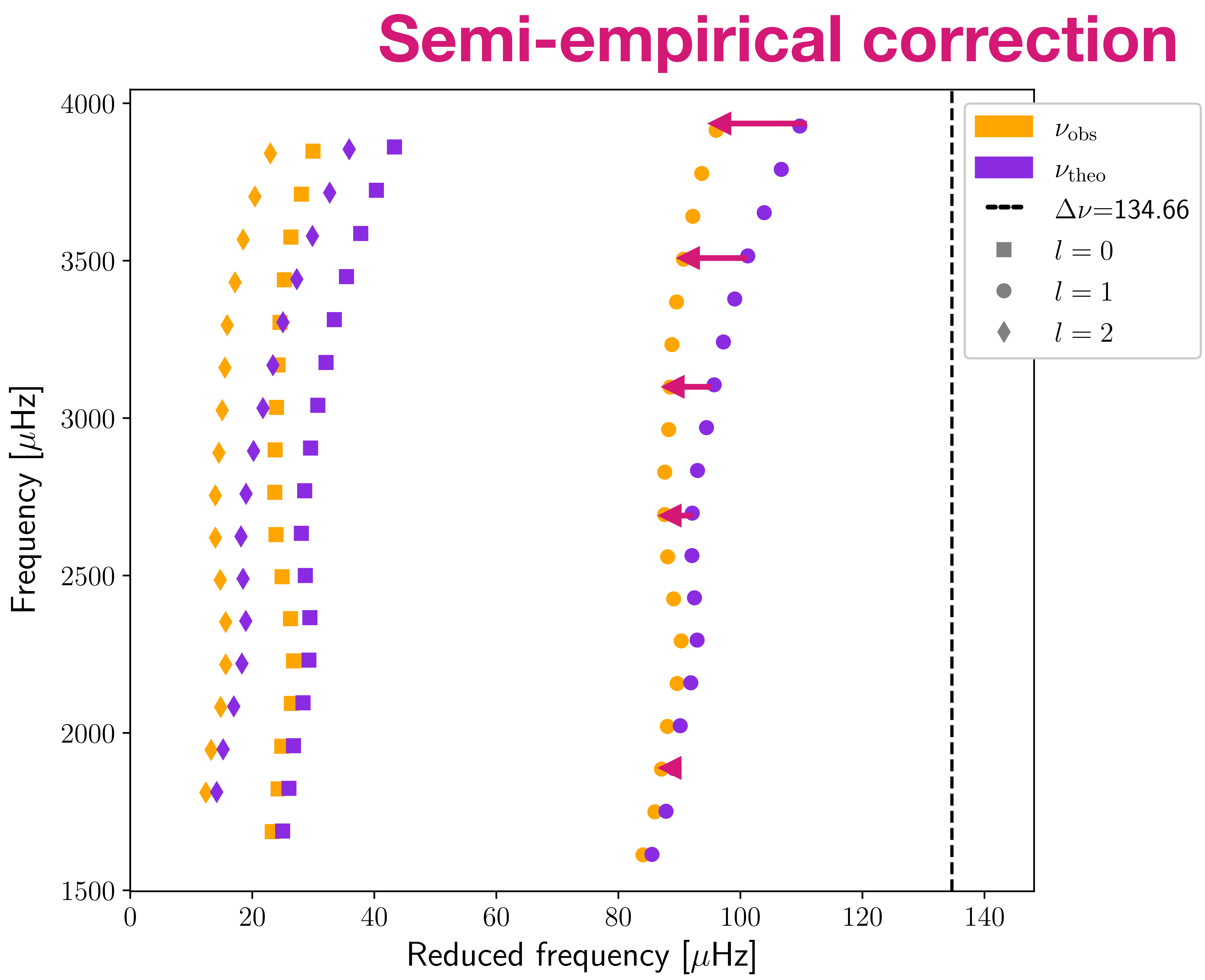


$\bar{\rho}_{inv}$ in the constraints

- **Increases stability** of the minimization
- **Reintroduces robust info** (lost with ratios) **about mean density**
- **Better constraints** on stellar **masses and radii**

→ **enhanced precision on stellar mass and radius**

Surface effects



1D evolutionary models

- Mixing-length theory (MLT)
- Issue: 3D turbulence closure problem with $t_{convection} \sim t_{oscillations}$
- **Frequencies are shifted**

Treatment of surface effects

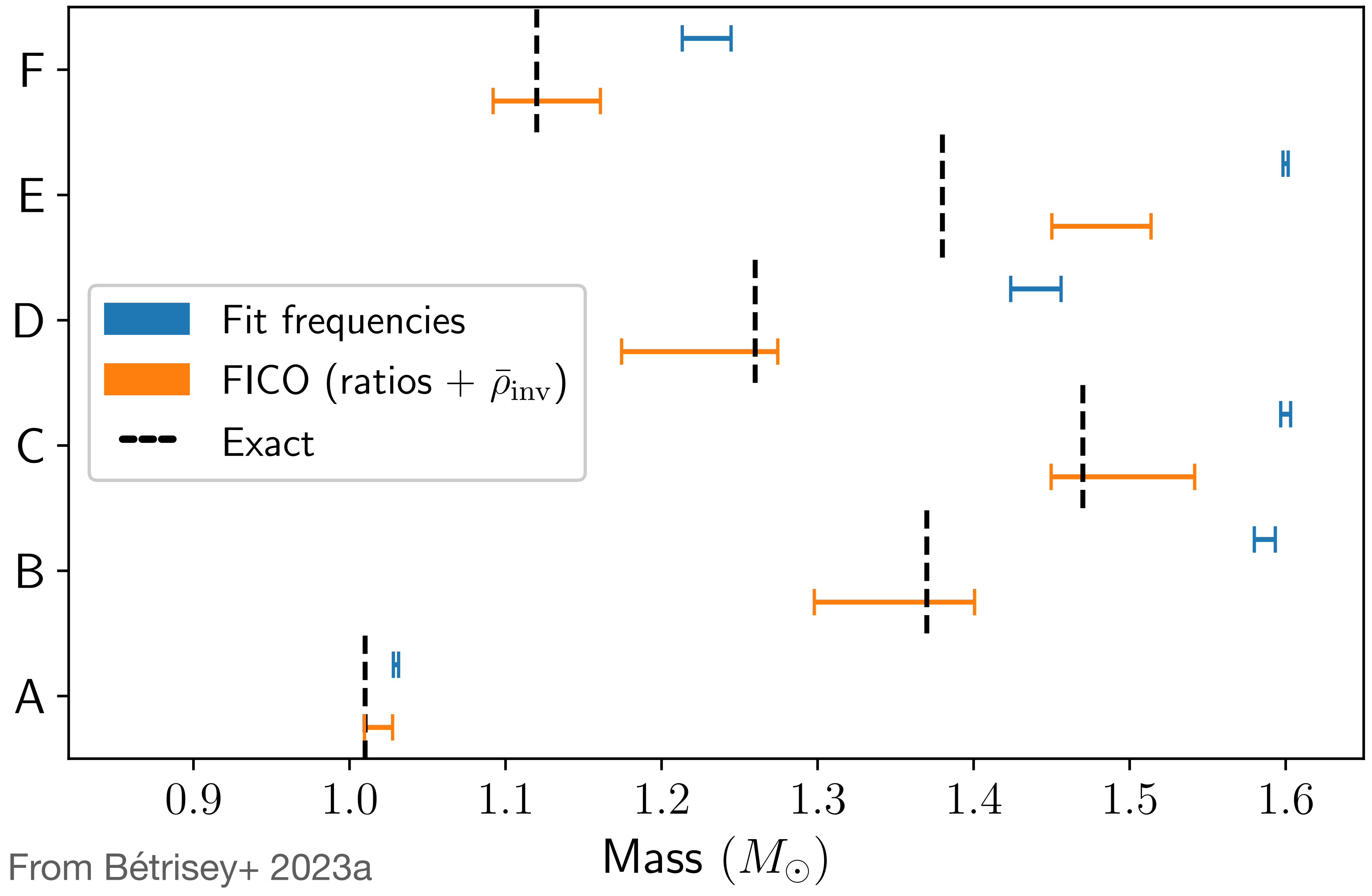
- **Semi-empirical corrections**
- **Surface-independent methods**

For further details about the treatment of convection in 1D stellar models, see e.g. Maeder+ 2009, Kippenhahn+ 2012, Kupka & Muthsam 2017, Kupka 2020, and Joyce & Tayar 2023

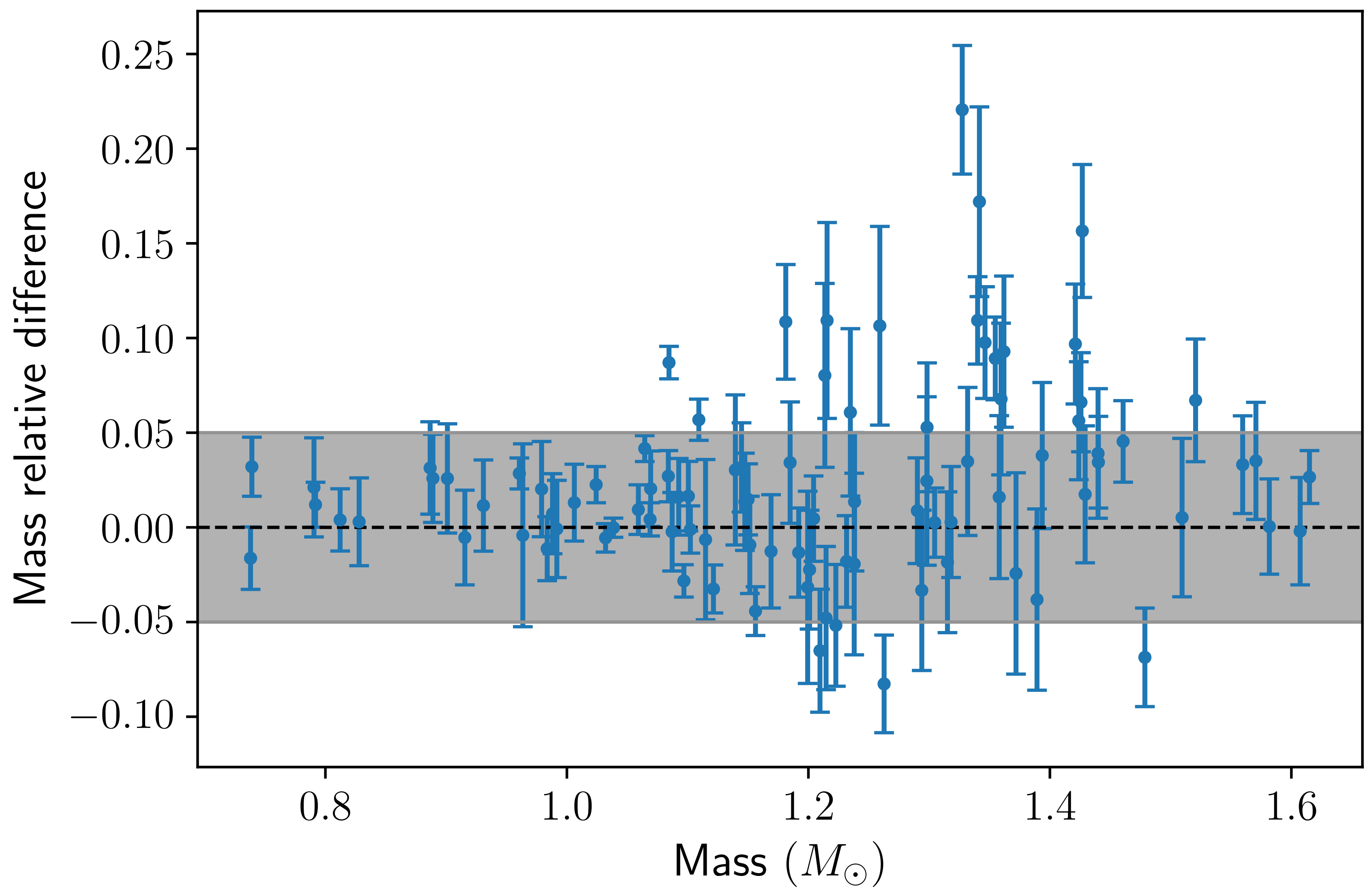
Surface effects in *Kepler* data

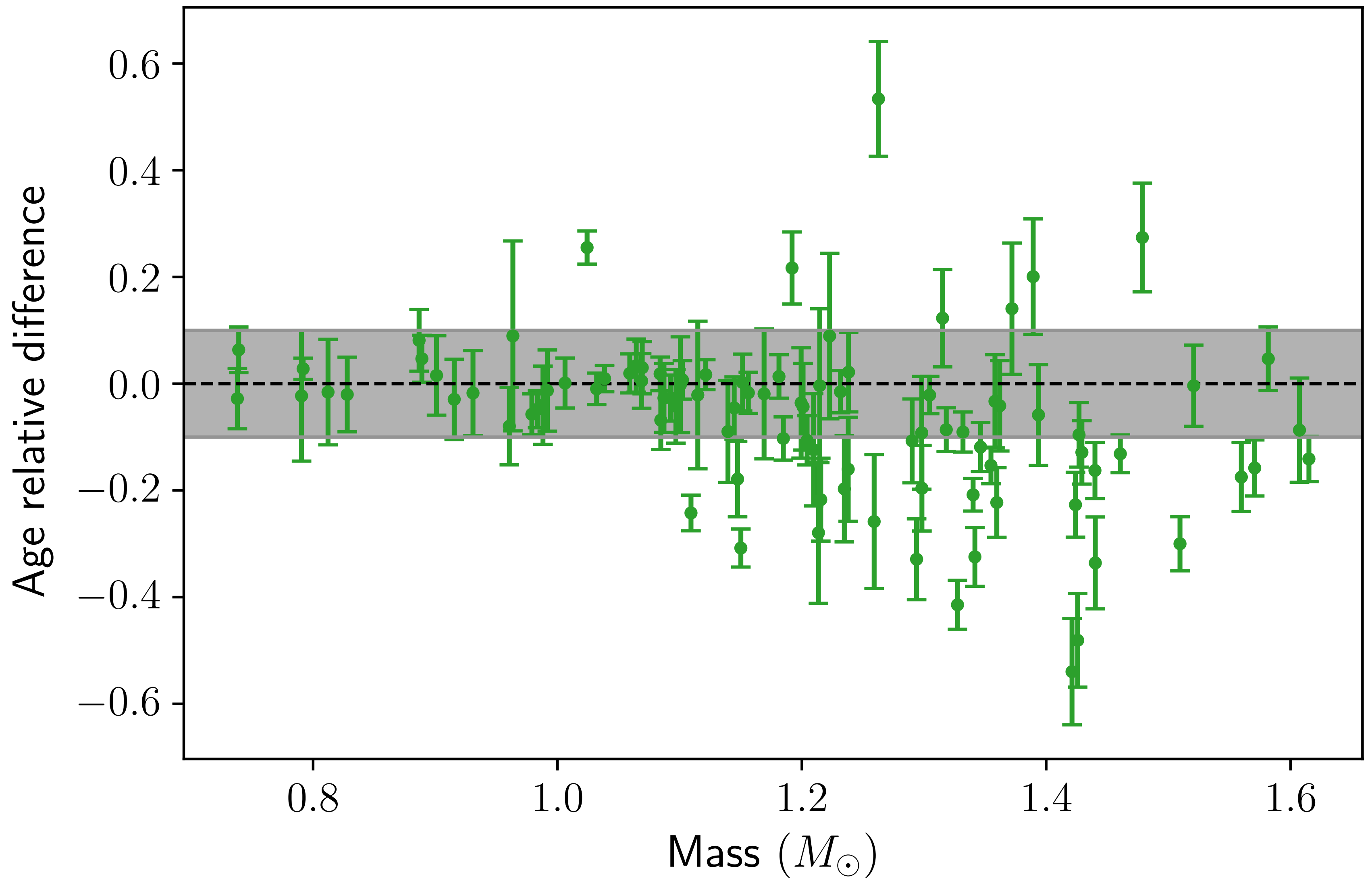
Systematic bias due to surface effects

- Mass and radius are overestimated
- Age is underestimated



From Bétrisey+ 2023a





See e.g. Ball & Gizon 2017; Nsamba+ 2018; Jorgensen+ 2020, 2021; Cunha+ 2021; Bétrisey+ 2023a

Automation of seismic inversions

State of the art

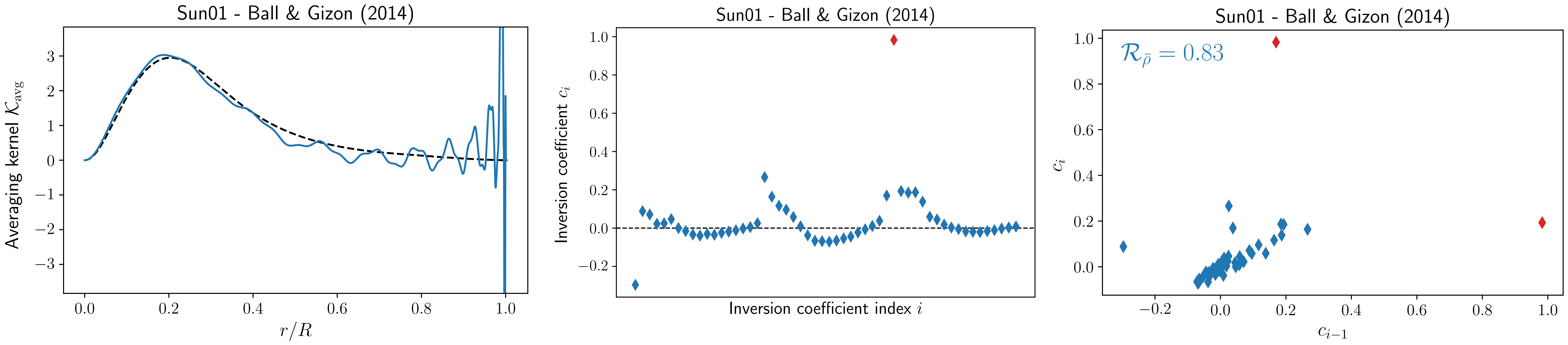
- **Seismic inversions were developed for individual modelling** (e.g. Buldgen+ 2022a; for a review)
- **Synthetic models with known structures have been extensively employed to validate and establish the reliability of inversions** (in particular in Reese et al. 2012; Buldgen et al. 2015b, 2018 for the inversions considered in Bétrisey+ 2024a)
- **Numerical stability of inversions is assessed by manual inspection of diagnostic plots**
- **NB:** numerical stability can be compromised by factors such as data quality or unaccounted non-linearities

My contributions

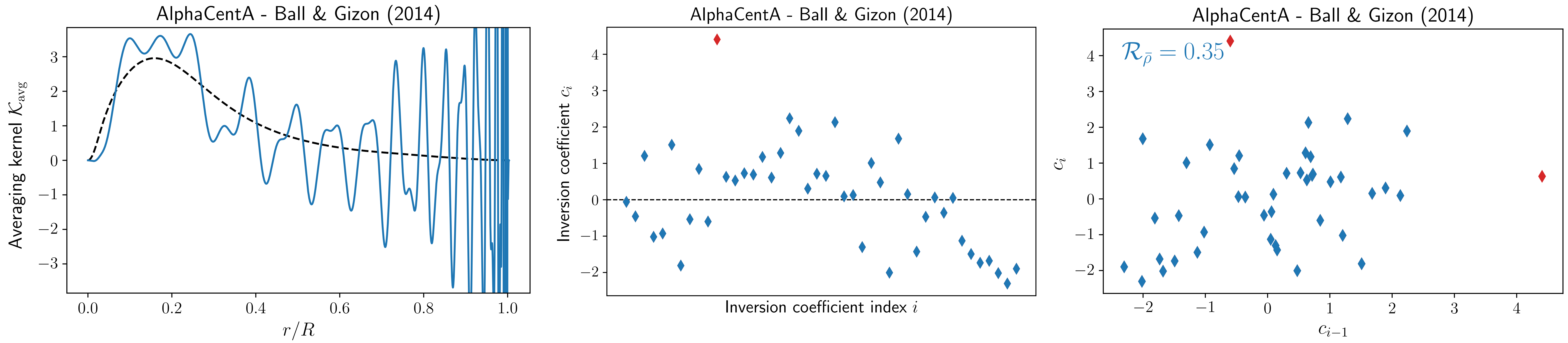
- **Demonstration of the scalability of mean density and acoustic radius inversions** (Bétrisey+ 2024a)
- **VATES :** module to automatically assess the numerical stability of the inversion
  - **Performs as well as a human modeler** based on testes on a sample of about 100 *Kepler* targets
  - **Very fast** (~ 4 ms) : because it is based on byproducts of the inversion

VATES

Stable case :



Unstable case :



Conclusions

Inverse methods

- **Grid-based inferences have demonstrated very good performances** but they do not perfectly reproduce the observed oscillation frequencies
- **Seismic inversions exploit these frequency discrepancies to extract additional insights** from the frequency spectrum
- **Seismic inversions serve as a valuable complement to grid-based inferences**, enabling more accurate and comprehensive stellar characterisations

VATES

- **Module to automatically assess the numerical stability of the inversion**
- **Assessment procedure performs as well as a human modeler :** successfully tested on a realistic sample of about 100 *Kepler* targets
- **Based on byproducts of the inversion → very fast : ~ 4 ms**

Thank you for your attention !