Emergent Gravity from Matrix Quantum Mechanics

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Inaugural Workshop on Emergent Geometries

November 18, 2025

Olle Engkvist Foundation

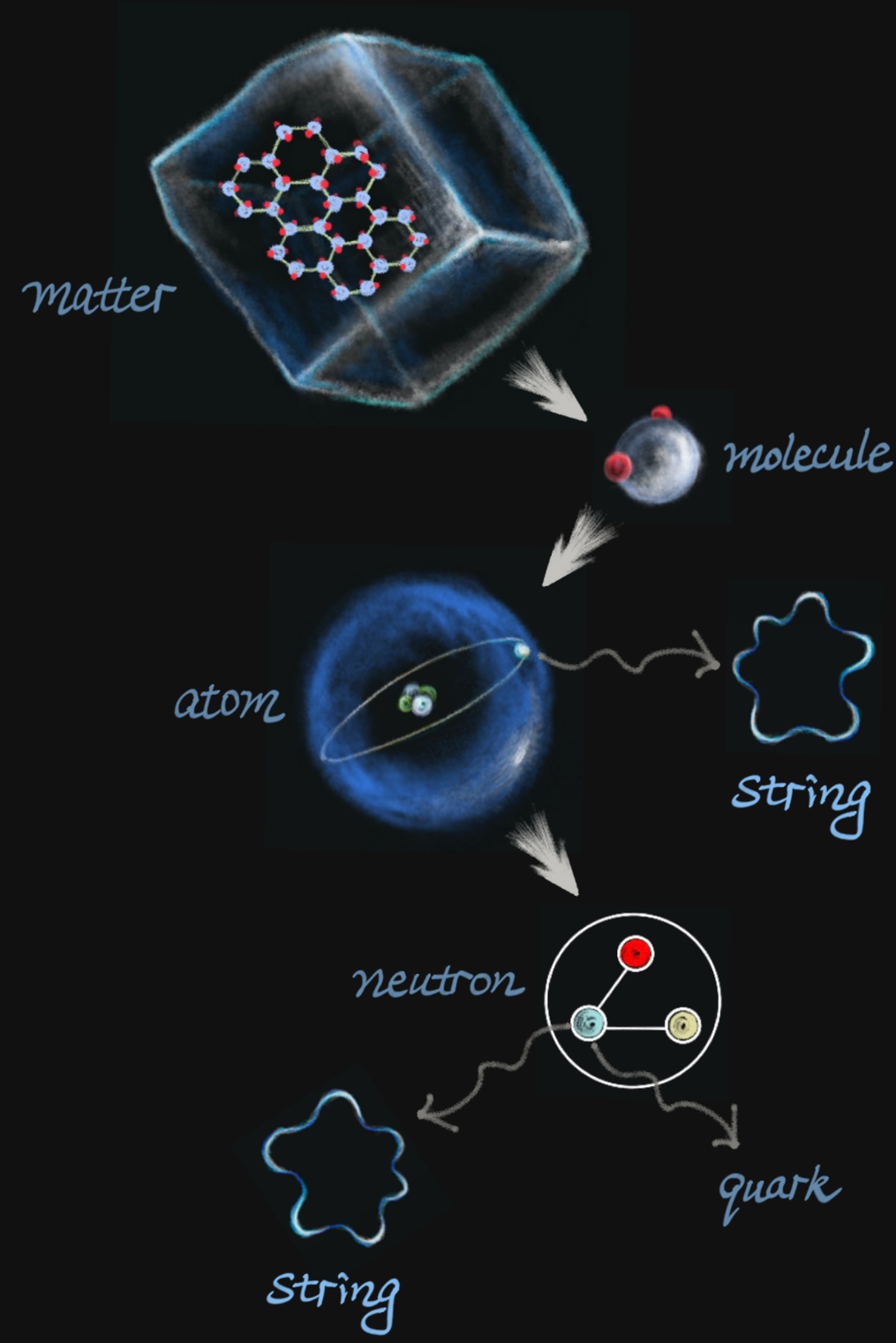
Funded by Villum Young Investigator Program & Olle Engkvists Stiftelse

VILLUM FONDEN

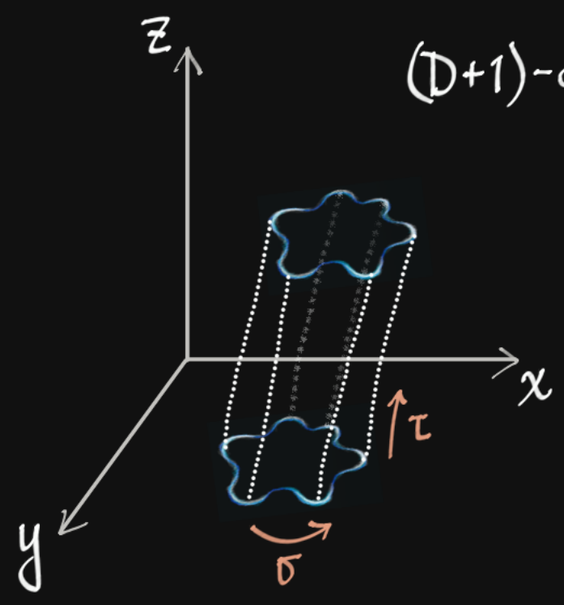
String theory

mathematically self-consistent framework
for testing ideas of quantum gravity

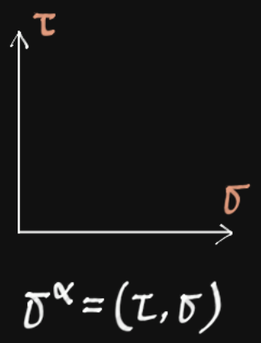
quantum gravity as
a soup of strings



Emergent Gravity from String Theory



(D+1)-dim.



$\sigma^\alpha = (\tau, \sigma)$

$X^\mu(\tau,\sigma)$

(D+1)-dim.

$G_{\mu\nu}$

$\mu = 0, 1, \cdots, D$

$X^n \partial X \partial X$ is marginal!

2D quantum field theory

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma \left(\partial_\alpha X^\mu \partial^\alpha X_\mu + \gamma_{\mu\nu\rho} X^\mu \partial_\alpha X^\nu \partial^\alpha X^\rho + \cdots\right)$$

$$= -\frac{1}{4\pi\alpha'}\int d^2\sigma\ \partial_\alpha X^\mu \partial^\alpha X^\nu G_{\mu\nu}(X) \qquad \alpha' \sim l_s^2$$

renormalization group flow

$\beta(\gamma_{\mu\nu\rho})$, $\beta(\gamma_{\mu\nu\rho\sigma})$, $\cdots$ for infinite many couplings

collectively, $\beta(G_{\mu\nu}) = \alpha' R_{\mu\nu}(G) + \mathcal{O}(\alpha'^2)$

Emergent Gravity from String Theory

emergent metric $G_{\mu\nu}$: coherent state of strings

one-loop quantum corrections: $\beta(G_{\mu\nu}) = \alpha' R_{\mu\nu}(G) = 0$ ← scale invariance

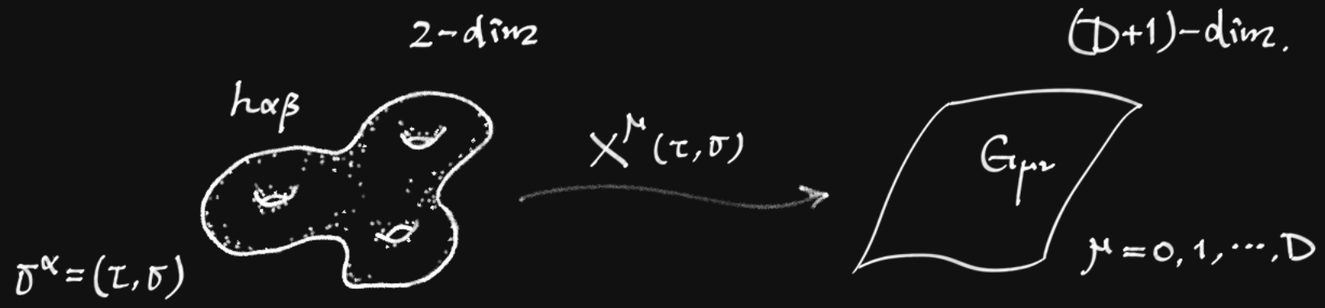
emergent Einstein's equation in vacuum: $R_{\mu\nu} = 0$

path integral $Z = \int \mathcal{D}x\, e^{\frac{i}{\hbar} S_{string}[x]}$ defines a Quantum Gravity

Sum over all Riemann surfaces

Emergent Gravity from String Theory

$$S = -\frac{1}{2}\int d^2\sigma \left(-\partial_\alpha X^0 \partial^\alpha X^0 + \partial_\alpha X^i \partial^\alpha X^i + \text{interactions}\right)$$ ghost from X^0?



metric — string winding

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma \left[\sqrt{-h}\, h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}(X) + \epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu}(X)\right]$$

$$+\frac{1}{4\pi}\int d^2\sigma \sqrt{-h}\, R(h)\, \Phi(X) + \text{worldsheet fermions}$$

dilaton

no ghost : worldsheet is conformal

no tachyon : worldsheet is supersymmetric

quantum consistency

no renormalization group flow

$$\beta(G) = \beta(B) = \beta(\Phi) = 0$$

emergent 10D Quantum Gravity

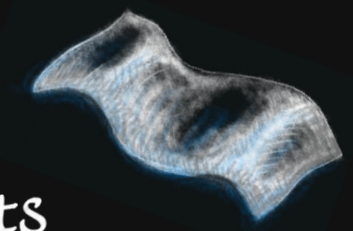
What are Emergent?

string theory as a duality

2D worldsheet quantum field theory		10D quantum gravity
global transformations		diffeomorphisms & gauge symmetries
worldsheet fields		embedding spacetime coordinates
correlation functions		scattering amplitudes
vanishing β-functions		classical equations of motion

from QFT perspective: only 2D spacetime & gravity

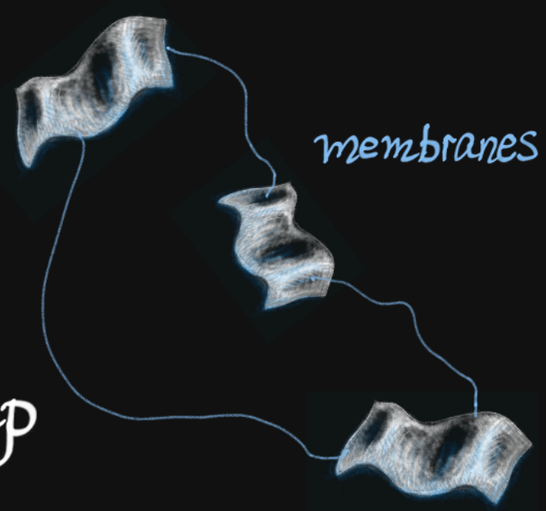
Towards M-Theory: a Theory of Membranes



2D extended objects

string

nonperturbative regime:
strong quantum gravity

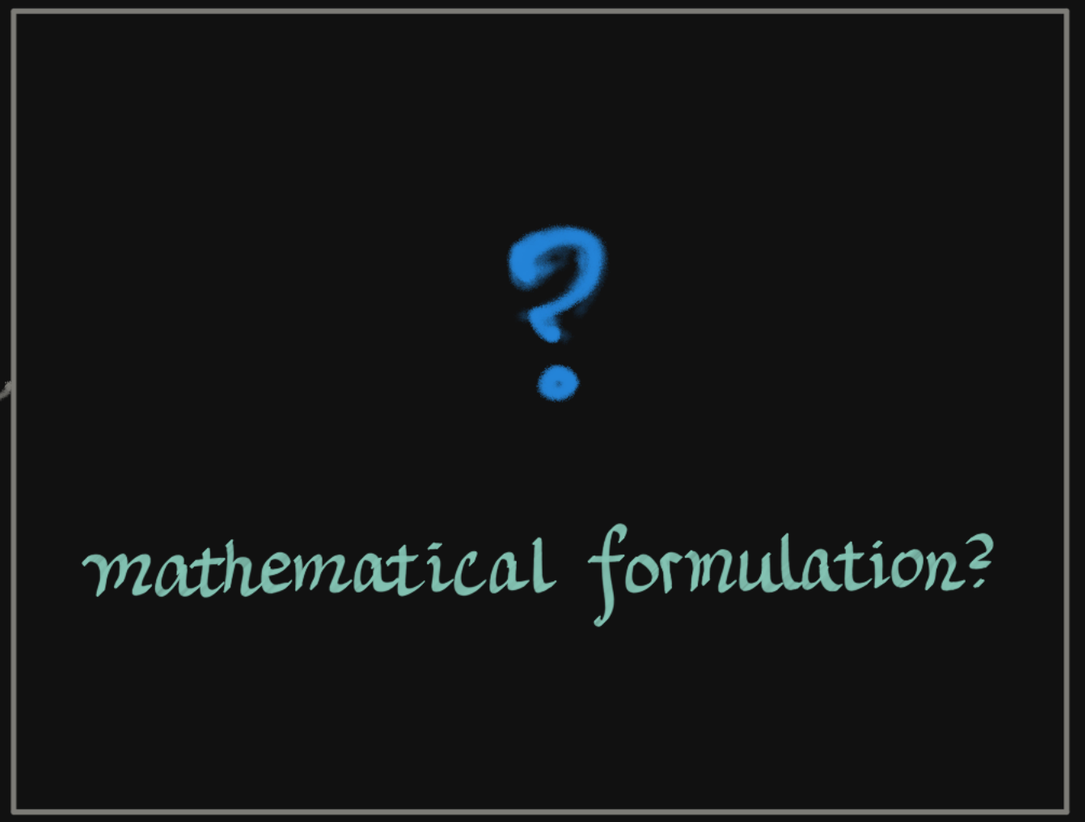


membranes

M-theory

membrane soup

Emergent Universe?



Towards M-Theory: a Theory of Membranes

nonperturbative regime:
strong quantum gravity

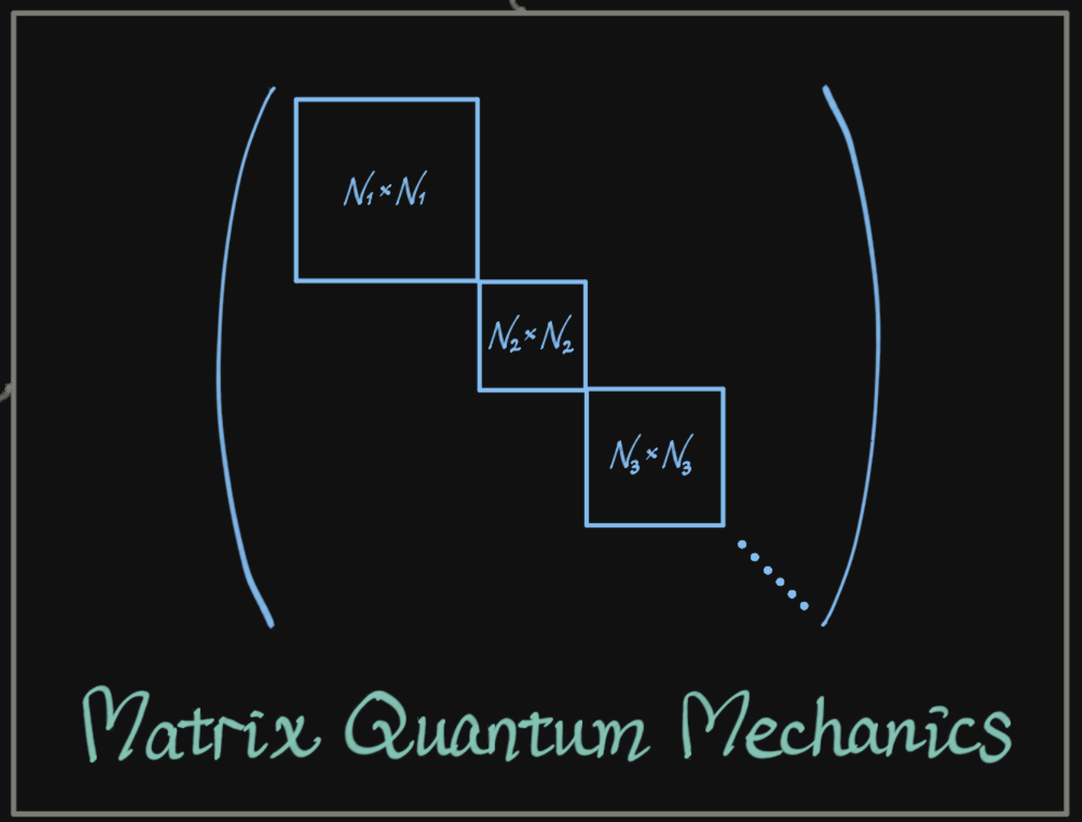
M-theory

membrane soup

Emergent Universe?

large N

future: emergent black holes?
quantum simulation/computing?



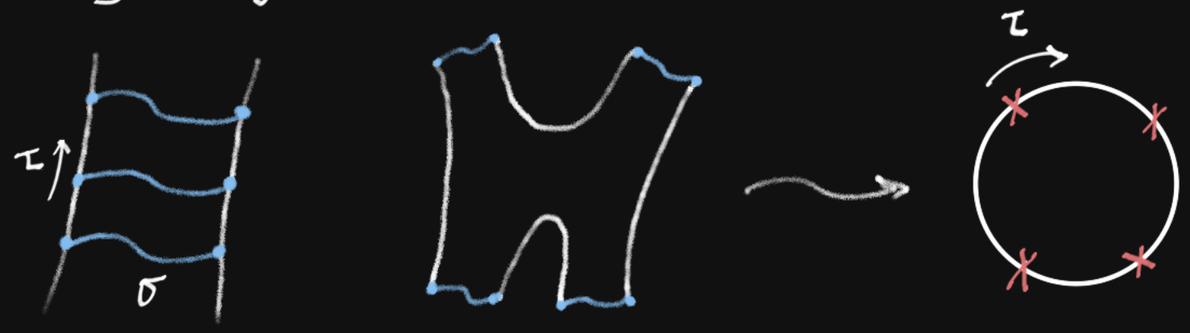
[De Wit, Hoppe, Nicolai '88]
[Banks, Fischer, Shenker, Susskind '96]

Why Branes?

STRINGS AREN'T ALWAYS FUNDAMENTAL!

so far, closed string only.

open strings

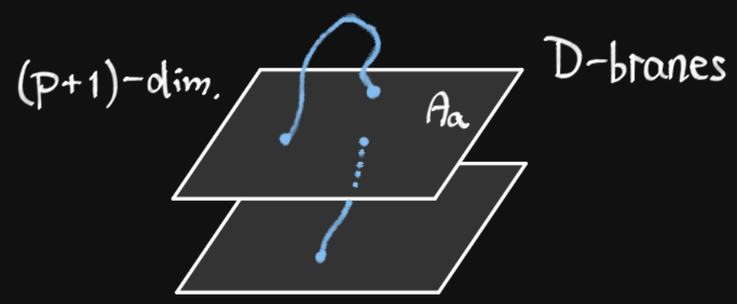


$$S_{\text{boundary}} = \frac{1}{2\pi\alpha'}\int d\tau \left[\partial_\tau X^a A_a(X) + \partial_\tau X^i \pi_i(X)\right]$$

$a = 0, \cdots, p \qquad i = p+1, \cdots, 9$

vanishing renormalization group flows $\beta(A_a) = \beta(\pi_i) = 0$

Yang-Mills brane-bending



strong string coupling implies light branes!

$$S_{Dp} = -T_p \int d^{p+1}\sigma \sqrt{-\det\left(\eta_{ab} + \partial_a \pi^i \partial_b \pi^i + F_{ab}\right)}$$

$$T_p \sim \frac{1}{\text{string coupling}} \qquad F_{ab} = \partial_a A_b - \partial_b A_a$$

Strongly-Coupled Gauge Theories?

a simple example: 4D Maxwell theory

$$S = -\frac{1}{4e^2}\int d^4\sigma\, F_{ab}F^{ab} \qquad F_{ab} = \partial_a A_b - \partial_b A_a$$

equivalently,

electromagnetic duality

$$S = -\frac{1}{4}\int d^4\sigma \left[\frac{1}{e^2}F_{ab}F^{ab} + \epsilon^{abcd}\widetilde{F}_{cd}(F_{ab} - \partial_a A_b + \partial_b A_a)\right]$$

integrate out A_a: $d\widetilde{F} = 0 \Rightarrow \widetilde{F}_{ab} = \partial_a \widetilde{A}_b - \partial_b \widetilde{A}_a$

now F_{ab} is non-dynamical.

integrate out F_{ab}: $\widetilde{S} = -\frac{1}{4\tilde{e}^2}\int d^4\sigma\, \widetilde{F}_{ab}\widetilde{F}^{ab} \qquad \tilde{e} \sim \frac{1}{e}$

strong-weak duality

D-Branes at Large String Coupling

* D2-brane

$$S = -T_2 \int d^3\sigma \sqrt{-\det\left(\partial_a X^\mu \partial_b X_\mu + F_{ab}\right)} \qquad T_2 \sim \frac{1}{g_s} \leftarrow \text{string coupling}$$

$$a = 0, 1, 2 \qquad \mu = 0, 1, \cdots, 9 \qquad \hookrightarrow F = dA$$

equivalently,

$$S \sim -\int d^3\sigma \left[\frac{1}{g_s} \sqrt{-\det\left(\partial_a X^\mu \partial_b X_\mu + F_{ab}\right)} + \epsilon^{abc} \widetilde{F}_a \left(F_{bc} - \partial_b A_c + \partial_c A_b\right) \right]$$

integrate out A_a: $d\widetilde{F} = 0 \Rightarrow \widetilde{F}_a = \partial_a X^{11}$

$$\widetilde{S} \sim -\int d^3\sigma \sqrt{-\det\left(\partial_a X^\mu \partial_b X_\mu + g_s^2\, \partial_a X^{11} \partial_b X^{11}\right)}$$

g_s^2 ↑ controls the size of emergent 11th dimension

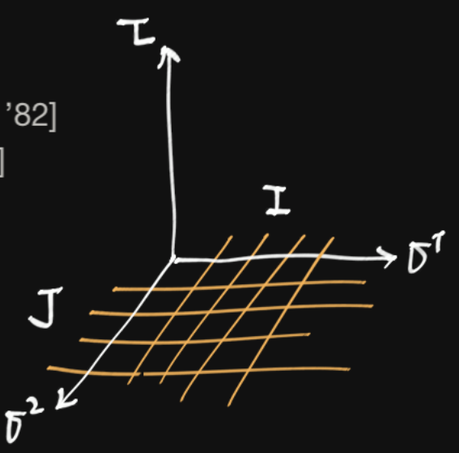
$g_s \to \infty \Rightarrow$ membranes in 11D M-theory!

Quantization of Membranes?

Notoriously difficult:

- nonrenormalizable 3D QFT
- 3D worldvolume gravity?
- 3D topologies: no perturbative coupling

non-perturbative method?



[Goldstone '82]
[Hoppe '82]

$X^i_{IJ}(\tau)$ $\quad I,J=1,\cdots,N$

$X^i(\tau,\sigma^1,\sigma^2)$

$i=1,\cdots,9$

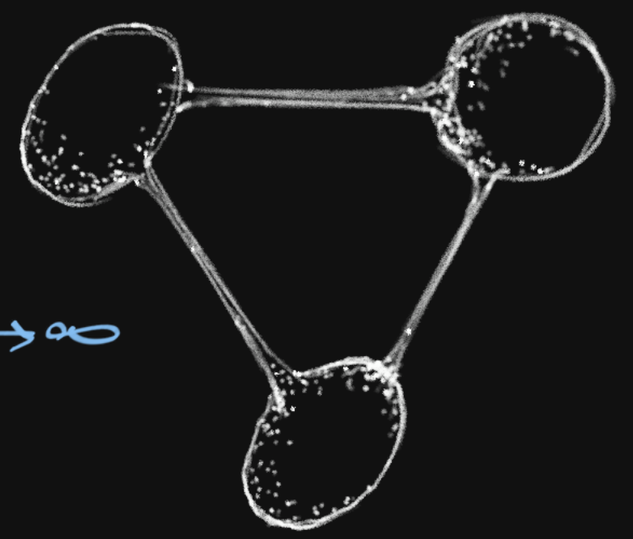
matrix quantum mechanics $\quad S\sim\int d\tau\,\mathrm{tr}\left(\dot{X}^i\dot{X}^i-\frac{1}{4}[X^i,X^j]^2+\text{fermions}\right)$

[De Wit, Hoppe, Nicolai '88]

UV-theory:

quantum critical membrane?

$N\to\infty$



membrane field theory

[Banks, Fischer, Shenker, Susskind '96]

M-theory?

String Theoretic Perspective

* D0-brane : relativistic particle

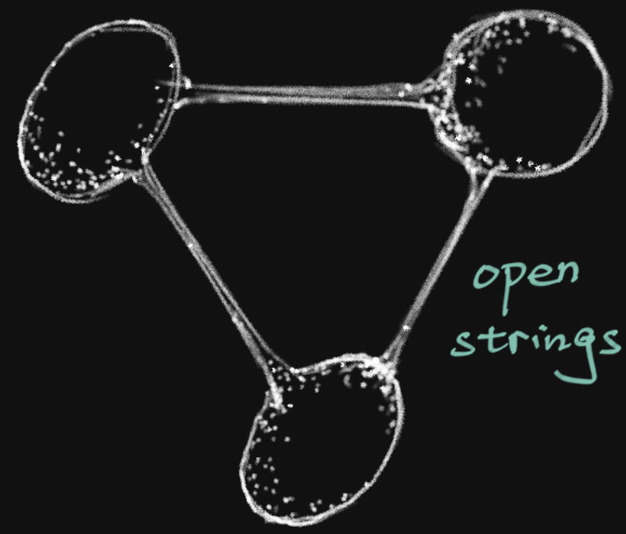
$$S = -mc\int d\tau \sqrt{c^2\dot{X}_0^2 - \dot{X}^i\dot{X}^i} + \frac{1}{l_s}\int d\tau\, \dot{X}^\mu A_\mu \qquad m = \frac{1}{g_s l_s}$$

electromagnetic field

BPS decoupling limit : $c \to \infty$ with $A_1 = \frac{c^2}{g_s}$

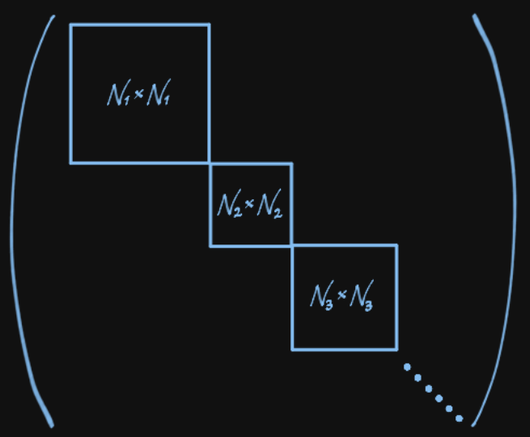
$$S \to -\frac{1}{2}m\int d\tau\, \dot{X}^i\dot{X}^i$$ non-relativistic particle

D0-brane bound states

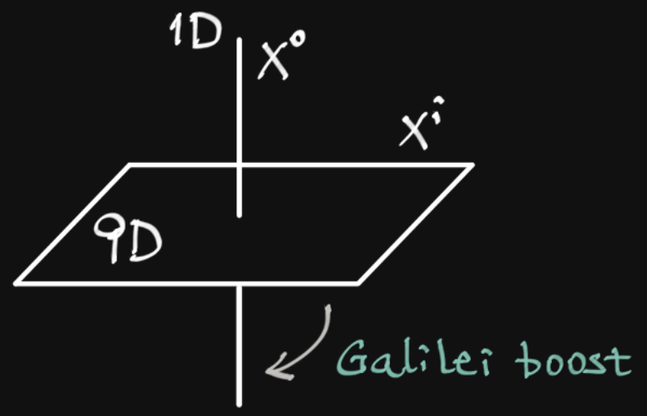


a stack of D0-branes ⇒ matrix quantum mechanics

$$S \sim \int d\tau\, \mathrm{tr}\left(\dot{X}^i\dot{X}^i - \frac{1}{4}[X^i, X^j]^2 + \text{fermions}\right) \qquad i = 1, \cdots, 9$$



non-Lorentzian geometry in 10D



Non-Lorentzian Quantum Gravity?

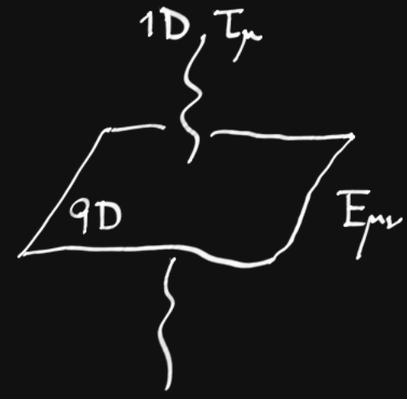
$$ds_{10}^2 = -c(\tau_\mu dx^\mu)^2 + \frac{1}{c} E_{\mu\nu}\, dx^\mu dx^\nu$$

(rank-9)

$$A_1 = c^2 g_s^{-1} \tau_\mu dx^\mu$$

$$e^{\Phi} = c^{-\frac{3}{2}} g_s$$

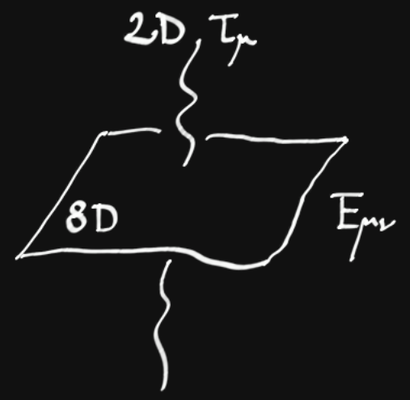
$c \to \infty$



strings are non-perturbative

dual to BPS limit for fundamental string

- matrix string theory [Motl '97] [Dijkgraaf, Verlinde, Verlinde '97]
- first quantized string: first-principles worldsheet approach [Gomis, Ooguri '00] [Danielsson, Guijosa, Kruczenski '00] [Bergshoeff, Gomis, ZY '18]
- β-functions: stringy generalization of non-Lorentzian gravity [Gomis, Oh, ZY '19] [Gallegos, Gürsoy, Zinnato '19] [Gomis, ZY, Yu '19]
- c-deformation is $T\bar{T}$-deformation: flow to full string theory [Blair '22] [Blair, Lahnsteiner, Obers, ZY '24]



Non-Perturbative String

[Blair, Lahnsteiner, Obers, ZY '23]
[Gomis, ZY '23]
[Maskalaniec, ZY, Utku, WIP]

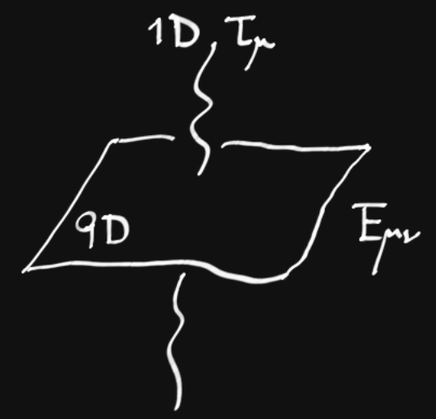
$$ds_{10}^2 = -c(\tau_\mu dx^\mu)^2 + \frac{1}{c} E_{\mu\nu} dx^\mu dx^\nu$$

rank-9

$$A_1 = c^2 g_s^{-1} \tau_\mu dx^\mu$$

$$e^\Phi = c^{-\frac{3}{2}} g_s$$

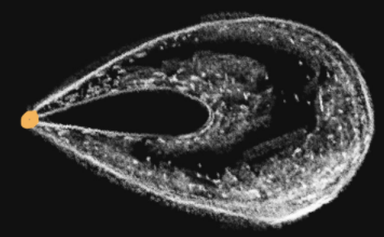
$c \to \infty$



can we derive spacetime dynamics from fundamental string? [Gomis, ZY '23]

$$S = -\frac{T}{2}\int d^2\sigma\, \partial_\alpha X^\mu \partial^\alpha X^\nu G_{\mu\nu}$$

$$\xrightarrow{c\to\infty} -\frac{T}{2}\int d^2\sigma \left[\partial_\sigma X^\mu \tau_\mu + (\partial_\tau X^\mu \tau_\mu)^2 + \partial_\sigma X^\mu \partial_\sigma X^\nu E_{\mu\nu}\right]$$



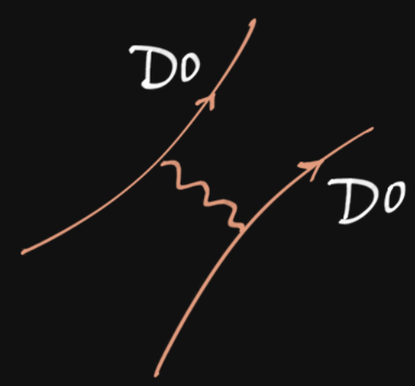
worldsheet is also Galilean, no string vibration

analog of vanishing β-function for spacetime dynamics?

Emergent Newton-Like Quantum Gravity

finite-N matrix quantum mechanics

Newton-like interaction between non-rel. particles



fundamental string in $c \to \infty$

worldsheet supersymmetry is key! [Adamo, Casali, Skinner '18]

techniques from ambitwistor string theory (timelike T-dual)

- first-order formalism (curved $\beta\gamma$-systems)
- suitable gauge choice: worldsheet theory is chiral
- quantum consistency from supersymmetry: 10D Newton-like gravity?

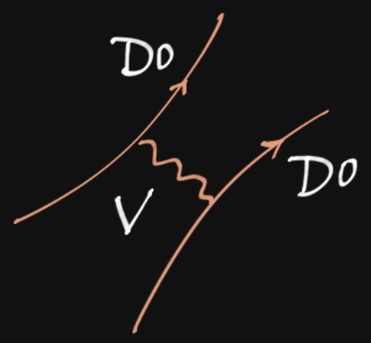
[work in progress]

Holographic Perspective

[Blair, Lahnsteiner, Obers, ZY '24]
[Harmark, Lahnsteiner, Obers '25]
[Guijosa '25]

Newton-like potential ?

$$V \sim \frac{v^4}{r^7}$$



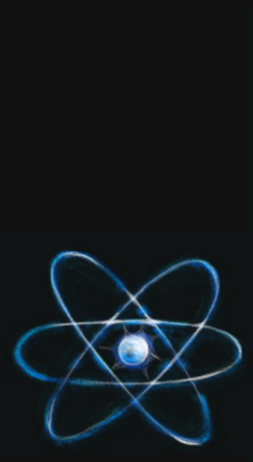
backreact D0 :

deform from Newton-like geometry

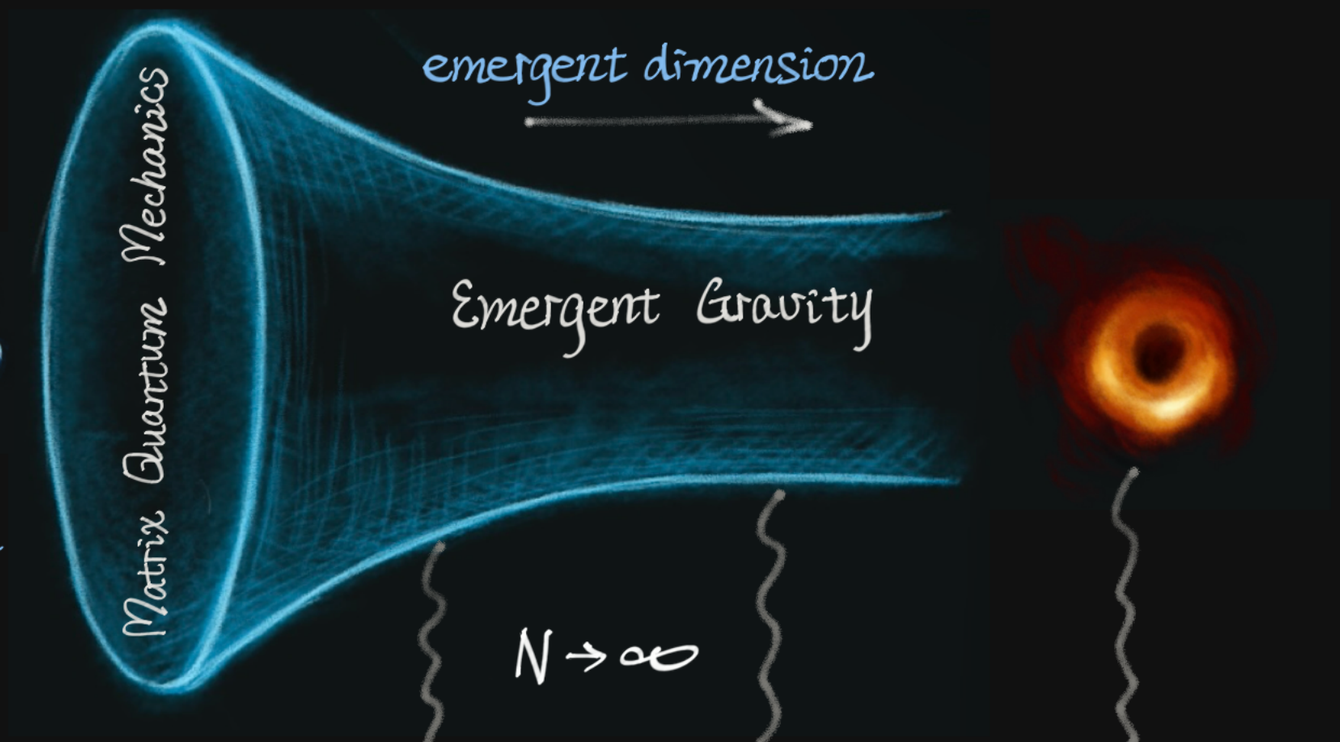
to relativistic geometry

(dual to $T\bar{T}$-deformation)

a consequence of BPS nature



non-Lorentzia
10D geometry

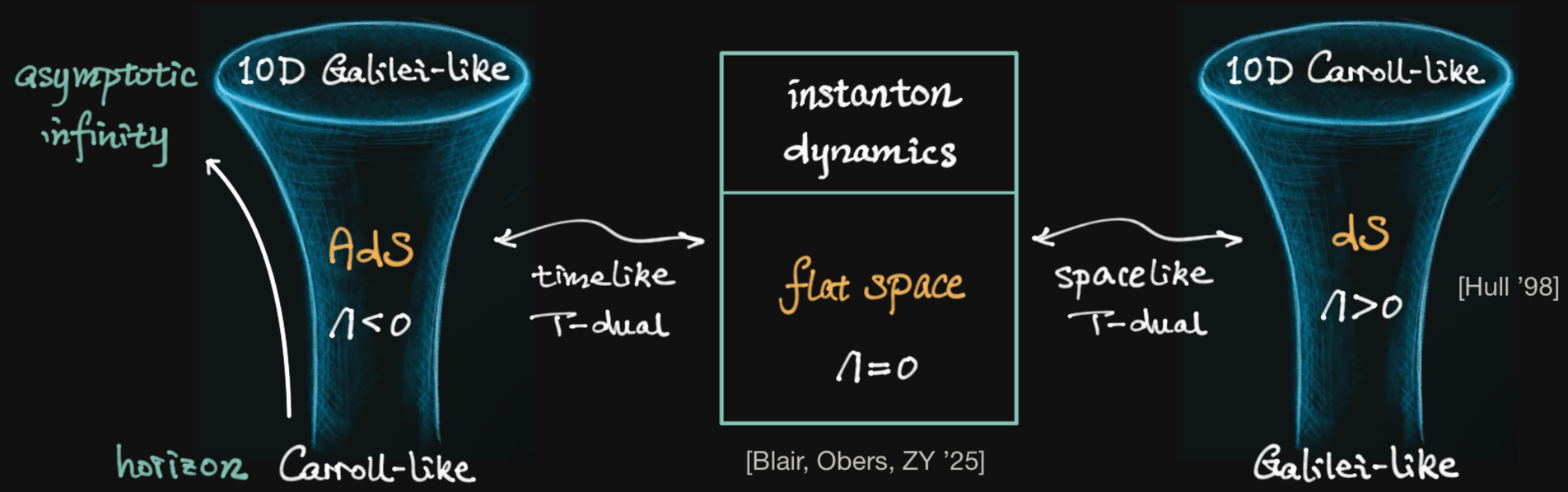


T-dual on 3-torus

AdS_5/CFT_4

[Itzhaki, Maldacena,
Sonnenschein, Yankielowicz '98]

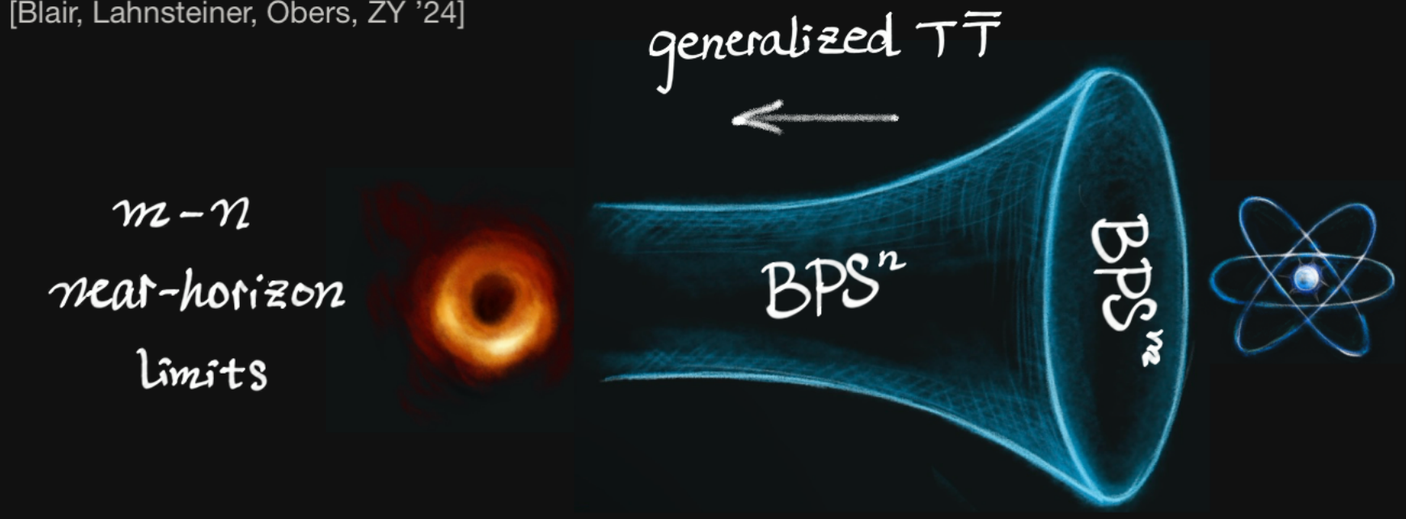
Construct Holography from BPS Limits



[Blair, Obers, ZY '25]

[Hull '98]

[De Boer, Dijkgraaf, Harmark, Obers]
[Blair, Lahnsteiner, Obers, ZY '24]



non-Lorentzian holography from top-down?

[Lambert, Smith '24] [Blair, Lahnsteiner, Obers, ZY '24 '25] [Harmark, Lahnsteiner, Obers '25] ···

But, What is M-Theory?

large-N matrix quantum mechanics:

M-theory in 11D asymptotically flat spacetime

this is a non-perturbative quantization of supermembrane

↓

3D nonrenormalizable QFT

Wilsonian UV-completion?

classical flow

quantum critical membrane : emergent bimetric geometry

[Hořava '08] [Hořava, Melby-Thompson, Randall] [ZY '22]

worldvolume scalars
+
3D Hořava supergravity

$t \to bt \quad x^i \to b^{\frac{1}{z}} x^i$

- renormalizable worldvolume QFT
- relativistic target space geometry
- quantum flow to e.g. matrix quantum mechanics?

Summary

perturbative string : 10D quantum gravity from 2D QFT

BPS limits : quantum gravity on (conformally) AdS

from large-N super Yang-Mills

- D-brane dynamics : worldvolume formalism at finite-N?
  - embedding coordinates are matrices

    non-commutative nature of quantum gravity?

M-theory : first principles approach?

- quantum critical membrane: is M-theory fundamentally non-rel.?

Thank You!