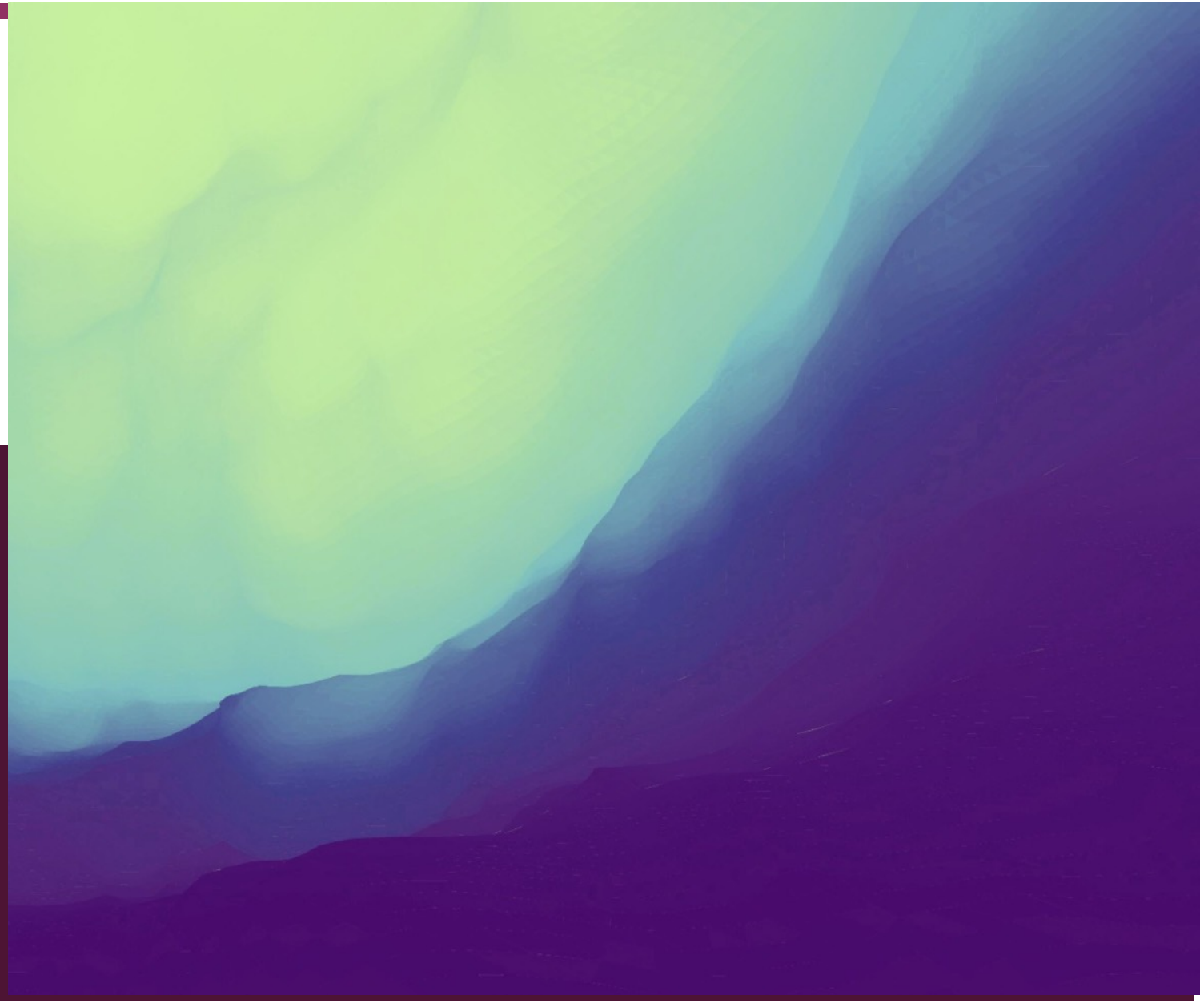
A ROAD MAP FOR GRAVITIZING QUANTUM MECHANICS



CHUNJUN (CHARLES) CAO
VIRGINIA TECH
WORKSHOP ON EMERGENT GEOMETRIES
NORDITA NOV 2025

A PUNCHY NAME?

- Space from Hilbert space?
- Bulk entanglement gravity → BEG

STARTING POINT

- Common approaches to create a quantum theory: Quantizing a classical action!
- Assumes some structures from classical theory
  - e.g. phase space, spacetime geometry, gauge symmetry, locality etc
- Instead: posit QM as the fundamental theory

$$|\psi\rangle \in \mathcal{H} \quad \text{and} \quad \hat{\mathrm{H}}$$

- Backout geometry and gravity from the complex quantum system

STARTING POINT

- Common appro
- Assumes some
  - e.g. phase sp
- Instead: posit Q
- Recover geometry and gravity from the complex quantum system

"Gravitize" Quantum Mechanics

$$\mathrm{GR} \subset \mathrm{QM}$$

ROADMAP

	State	Quantum Hamilltonian
QM	$\|\psi\rangle \in \mathcal{H}$	$\hat{H}$

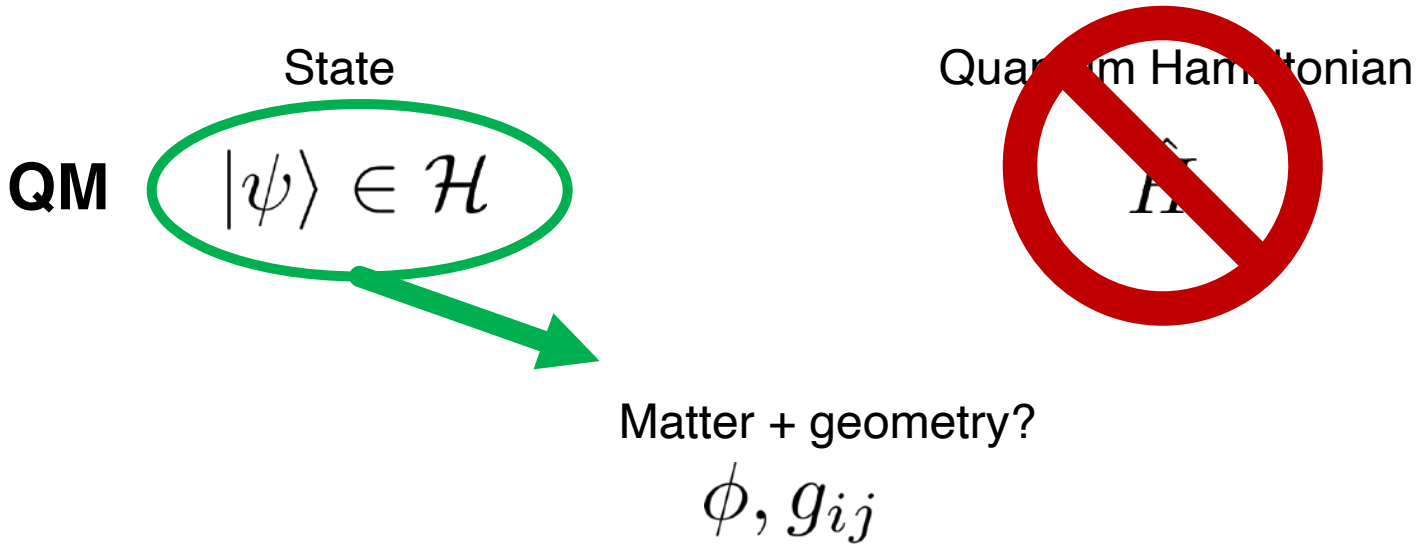
ROADMAP

State

QM $|\psi\rangle \in \mathcal{H}$

Quantum Hamiltonian

ROADMAP


State
Quantum Hamiltonian
QM
$|\psi\rangle \in \mathcal{H}$
$\hat{H}$
Matter + geometry?
ϕ, g_{ij}

ROADMAP

State

QM $|\psi\rangle \in \mathcal{H}$

Quantum Hamiltonian

Matter + geometry?

ϕ, g_{ij}

Features of gravity?

GR $G_{\mu\nu}t^\mu t^\nu = 8\pi G_N T_{\mu\nu}t^\mu t^\nu$

3 PARTS

- Rough shape and dimension
- Matter and geometry through QECC (a separation of “scales”)
- Towards emergent gravity

- Aspirational work in progress

GEOMETRIZING A QUANTUM STATE

How to get the parts? Quantum mereology?

Carroll, Singh 2020 + follow up

$$|\psi\rangle \in \bigotimes_i \mathcal{H}_i$$

GEOMETRIZING A QUANTUM STATE

$$I(i:j) = S(i) + S(j) - S(i \cup j)$$

- where

$$\rho = |\psi\rangle\langle\psi|$$

$$\rho_i = \mathrm{Tr}_{V\setminus\{i\}}\,\rho$$

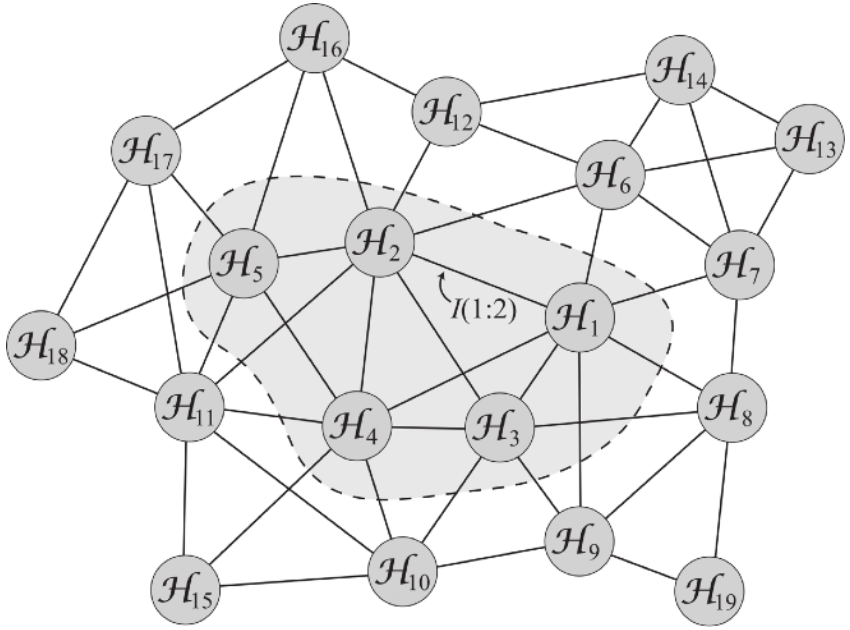
$$\rho_{ij} = \mathrm{Tr}_{V\setminus\{i,j\}}\,\rho$$

- And the von Neumann entropy

$$S(i) = -\,\mathrm{Tr}\,\rho_i \log \rho_i$$

$$S(i \cup j) = -\,\mathrm{Tr}\,\rho_{ij} \log \rho_{ij}$$

GEOMETRIZING A QUANTUM STATE

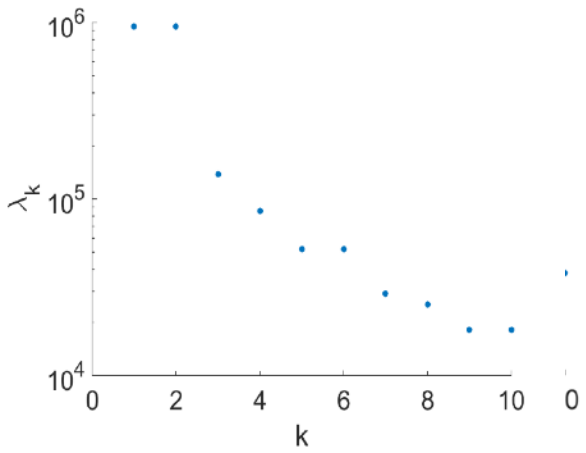


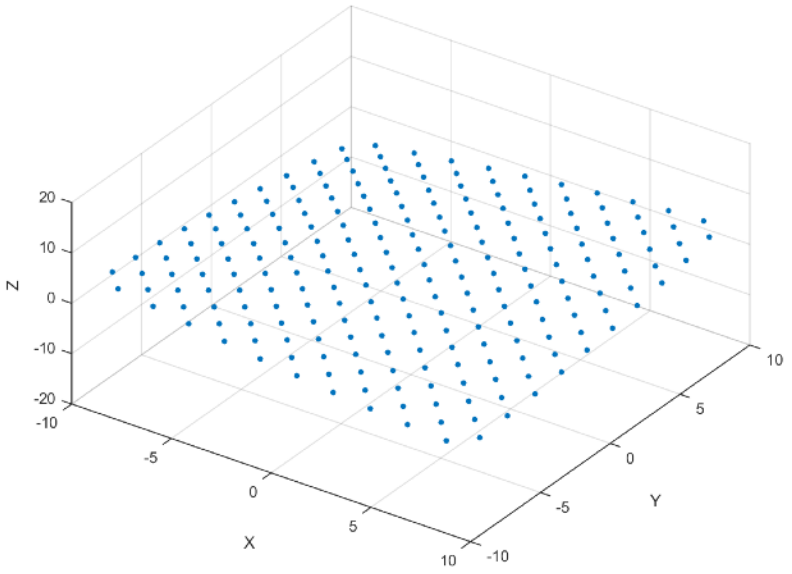
Large mutual information → higher correlation → closer

Small mutual information → low correlation → farther

GEOMETRIZING A QUANTUM STATE

- A set of procedures involving multidimensional scaling $\{I(i:j)\} \rightarrow \{\lambda_k, \vec{v}_k\}$
- Examples

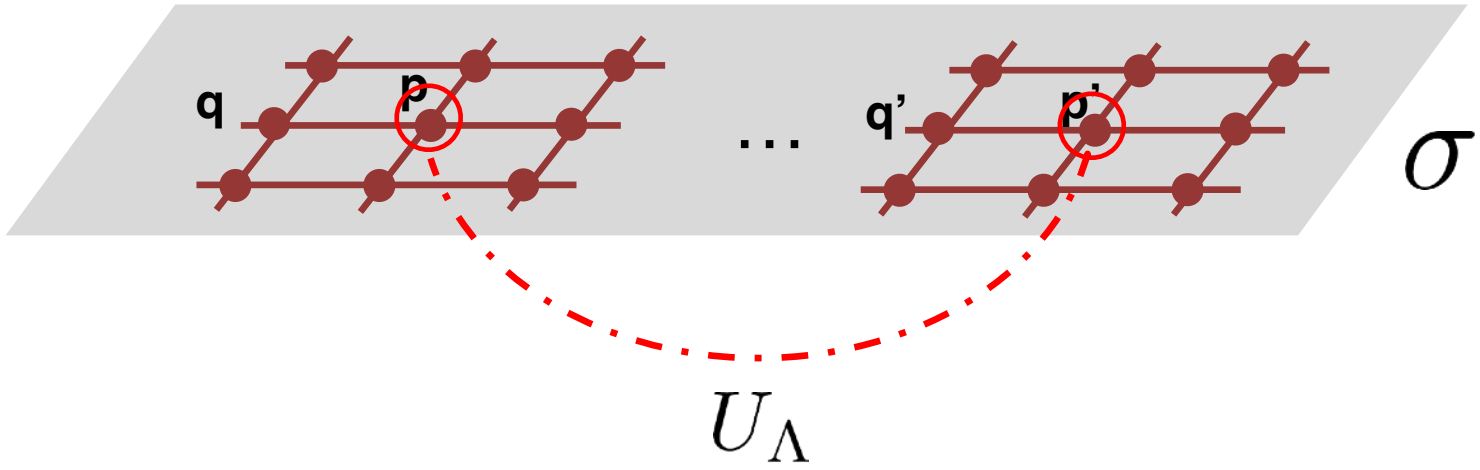




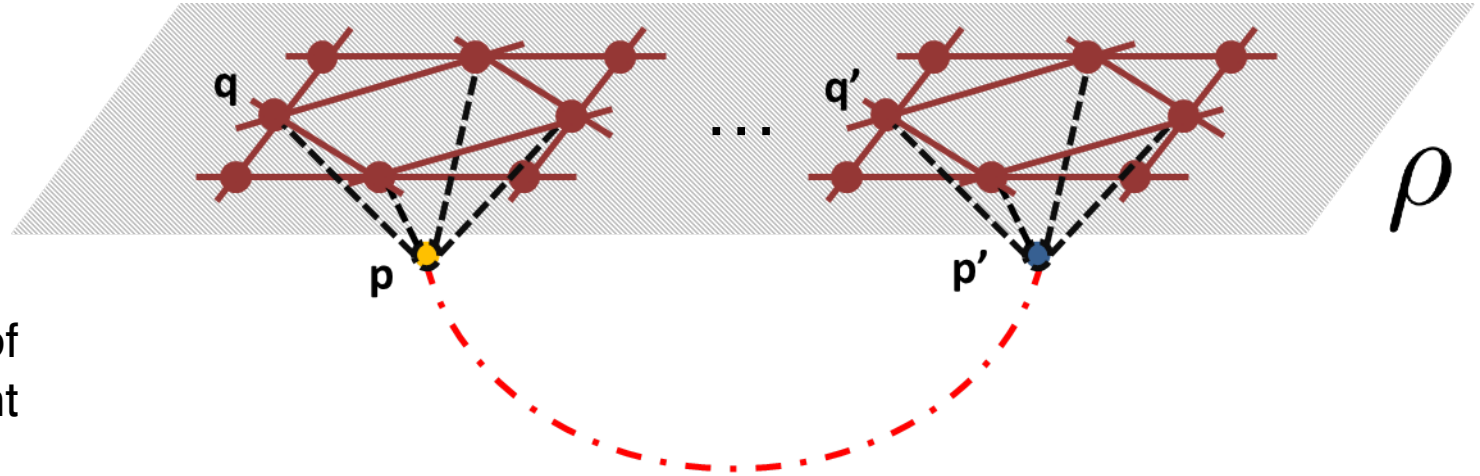
C.C., Sean Carroll, Spyridon Michalakis 1606.08444

ER=EPR

entanglement perturbation



ER=EPR



$$\rho = U\sigma U^\dagger$$

Monogamy of entanglement

$$R(p) \propto \epsilon_p$$

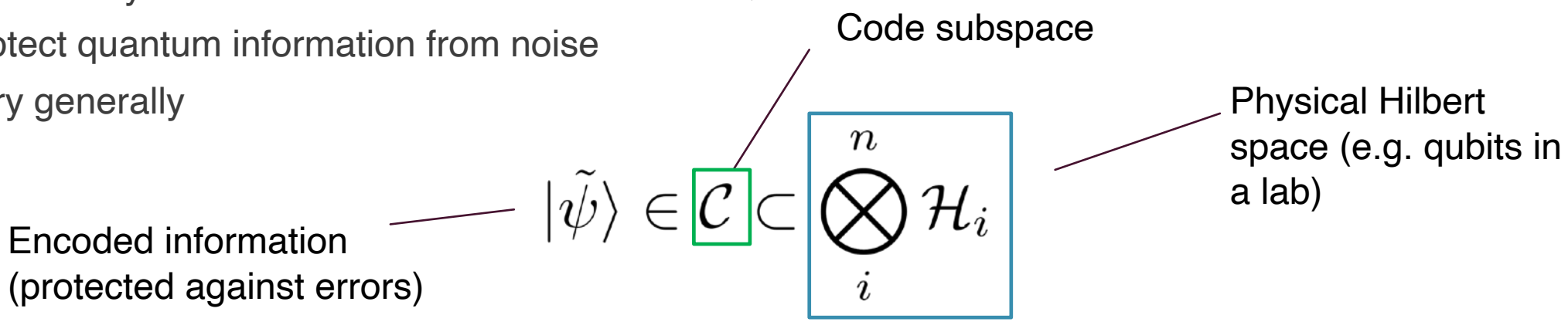
$$H_p = -\log \sigma_p, \delta\langle H_p\rangle_\rho = \epsilon_p$$

DO BETTER? → PART 2

- We can get a very rough idea of where things are, entanglement perturbations can lead to gravity-like behaviours
- We can do better and make these connections even more precise.
- We'd like to
  - separate matter from (space-time) geometry
  - Recover metric tensor up to gauge
- How do we do it by only looking at quantum states?

MORE STRUCTURES THROUGH QECC

- We can say more if the code has structure of a QECC
- Protect quantum information from noise
- Very generally



QUANTUM ERROR CORRECTION CODE: EXAMPLE

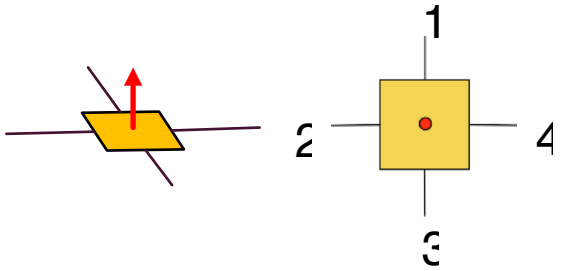
- Smear QI non-locally over many physical qubits so that the protected information is not damaged with few errors

$$V : \mathcal{H}_{\text{logical}} \rightarrow \mathcal{H}_{\text{phys}}$$ (V=encoding map)

- e.g. 1 qb erasure error correction code

$$|0\rangle \xrightarrow{V} |0000\rangle + |1111\rangle = |\tilde{0}\rangle$$

$$|1\rangle \xrightarrow{V} |1100\rangle + |0011\rangle = |\tilde{1}\rangle$$



$$\mathcal{C} = span\{|\tilde{0}\rangle, |\tilde{1}\rangle\} \subset \mathcal{H}_2^{\otimes 4}$$

- Can recover info even if qubit 4 is lost

$$|\tilde{\psi}\rangle = a|\tilde{0}\rangle + b|\tilde{1}\rangle, \qquad U_{123}|\tilde{\psi}\rangle_{1234} = |\psi\rangle_1 |\text{GHZ}\rangle_{234}$$

THE IDENTIFICATIONS

- If we stare at the QECC, we notice that it has two ingredients of interest

$$|\psi\rangle = a|0\rangle + b|1\rangle, \qquad U_{123}|\psi\rangle_{1234} = |\psi\rangle_1 |\mathrm{GHZ}\rangle_{234}$$

The message (logical information)

Entanglement which help protect the msg against error

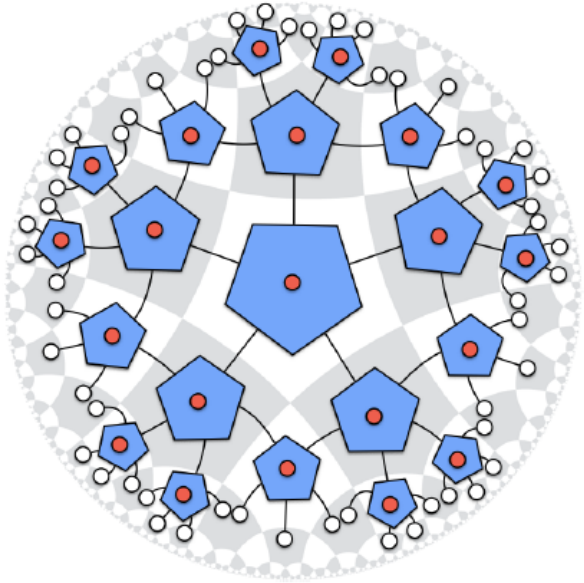
Identify with:

Builds Matter Fields

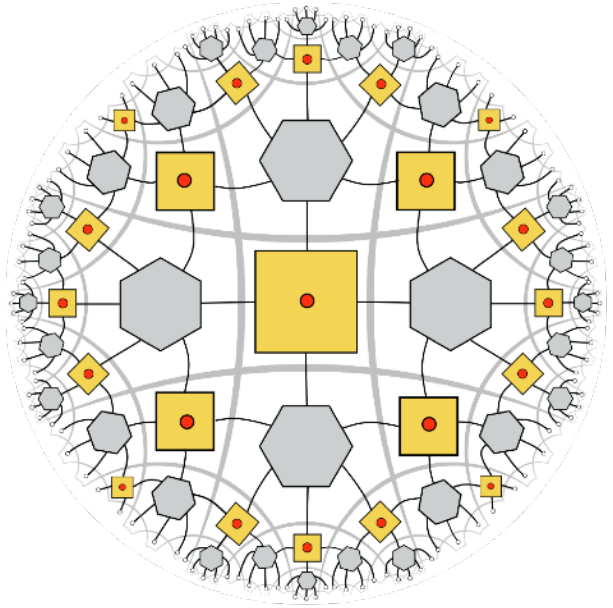
Builds Background Geometry

Harlow 2016; Cao, Carroll, 2017

BUILD SOME INTUITION FROM ADS/CFT



Pastawski, Yoshida, Harlow, Preskill 2015



Cao, Lackey 2020

Both the physical and the logical qubits can be arranged geometrically based on the entanglement patterns of the state

$$V_0 : \mathcal{H}_{\text{logical}} \rightarrow \mathcal{H}_{\text{physical}}$$

RECALL THE IDENTIFICATIONS

- Recall the two pieces of information we had before from QECCs

$$|\psi\rangle = a|0\rangle + b|1\rangle, \qquad U_{123}|\psi\rangle_{1234} = |\psi\rangle_1 |\mathrm{GHZ}\rangle_{234}$$

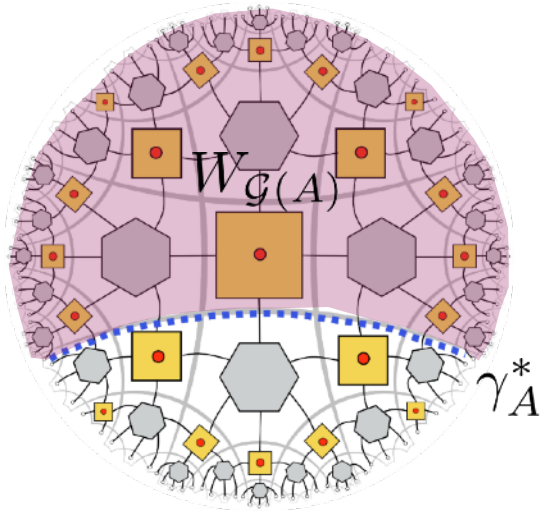
The encoded message

Entanglement which help protect the msg against error

- More generally

$$U_A \tilde{\rho}_A U_A^\dagger = \rho \otimes \chi_A$$

- E.g.

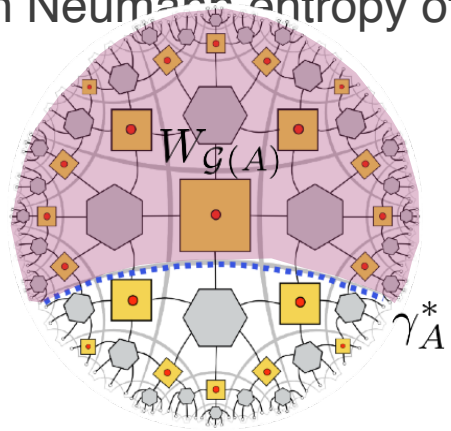


$$|\tilde{\psi}\rangle \in \mathcal{C} \subset \bigotimes_i \mathcal{H}_i$$

$$\tilde{\rho}_A = \mathrm{Tr}_{A^c}[|\tilde{\psi}\rangle\langle\tilde{\psi}|]$$

ENTANGLEMENT ENTROPY

- The von Neumann entropy of all QECCs naturally decomposes into 2 parts



$$S(\tilde{\rho}_A) = S(\rho_{W_{\mathcal{G}}(A)}) + S(\chi_A)$$

Entropy of logical/ bulk info

Entropy of residue → min cut

- This decomposition has an interpretation in AdS/CFT as the RT / FLM formula

$$S(A) = S(\Sigma_A) + \frac{|\gamma_A^*|}{4G_N}$$

Entropy of bulk matter

min surface area

"continuum limit"

Faulkner, Lewkowycz, Maldacena 2013,
Harlow 2016

GENERALIZATION

- The properties we used so far holds for any QECC (Harlow 2016)
- In fact, we use AdS/CFT very minimally except using it to draw analogies

- Therefore, we can build codes that have very different emergent geometries

Generalized entropy

$$|\tilde{\Omega}\rangle \in \mathcal{C} \subset \bigotimes_i \mathcal{H}_i \qquad |\tilde{\Omega}\rangle \to \tilde{\rho}_A \to S(A) = S_{\text{matter}}(A) + S_{\text{geometry}}(A)$$

$$\{S_{\text{geometry}}(A), \forall A\} \to \text{protoarea of interface between regions } A,\ \bar{A}$$

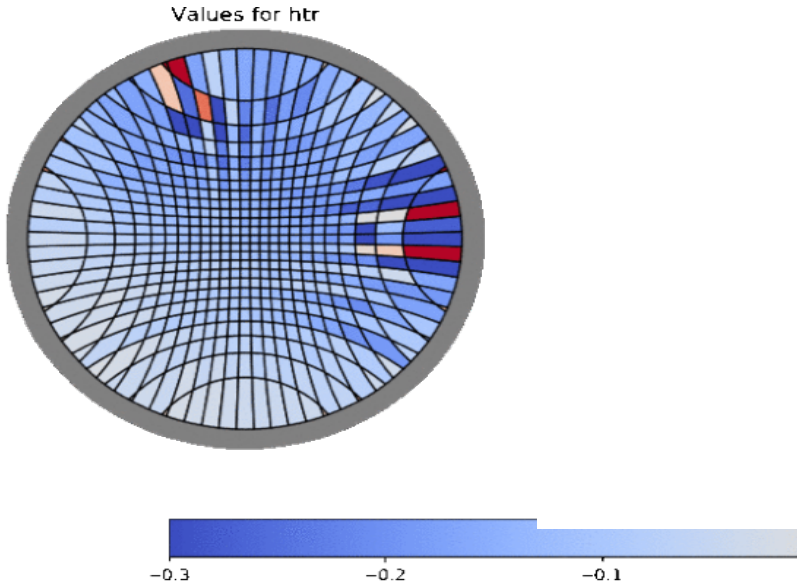
- For example, w/ inverse tensor Radon transform

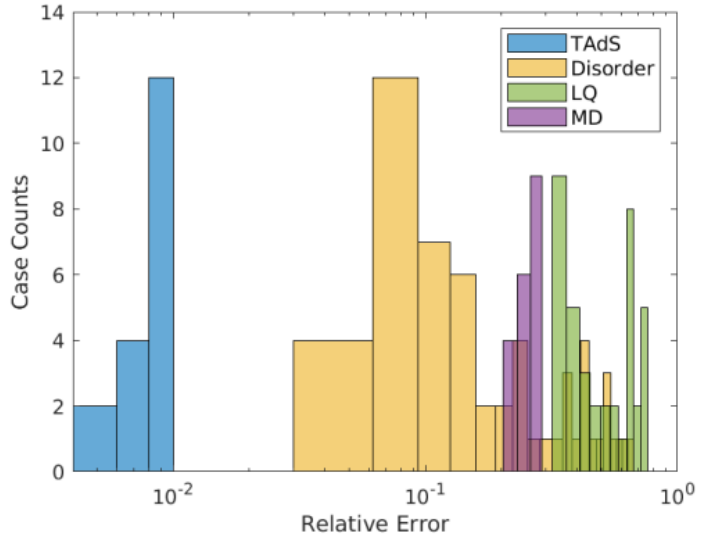
$$\{S_{\text{geometry}}(A), \forall A\} \xrightarrow{\text{is geometric?}} \begin{cases} \text{yes} \to \delta_{ij} + h_{ij} \\ \text{no} \to \text{how bad} \end{cases}$$

Cao, Carroll 2017
Cao, Qi, Swingle, Tang 2020
Bao, Cao, Fischetti, Keeler 2020

GEOMETRIC RECONSTRUCTION

CC. X. Qi, B. Swingle, E. Tang (2020)

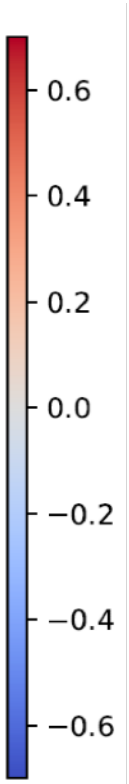


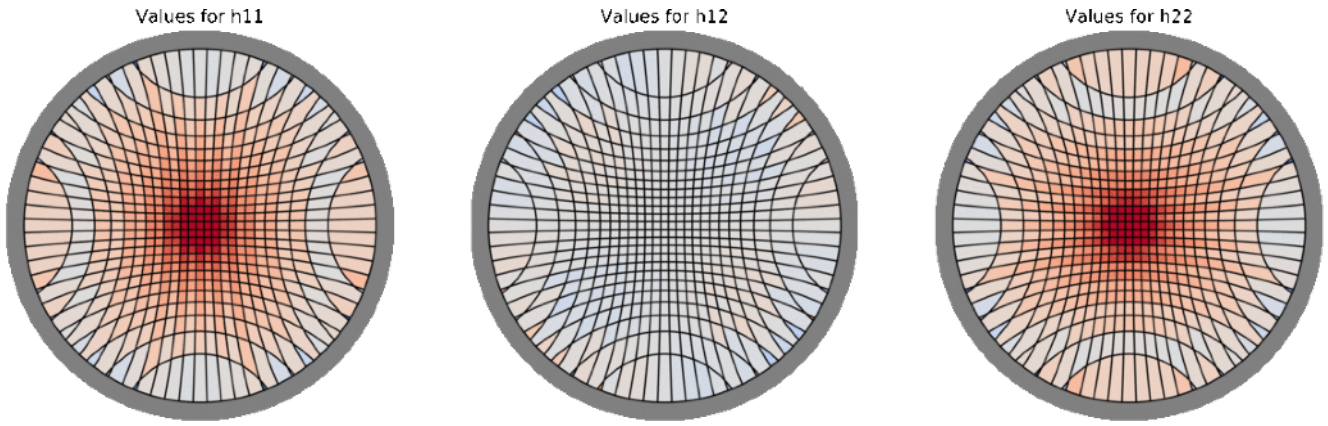


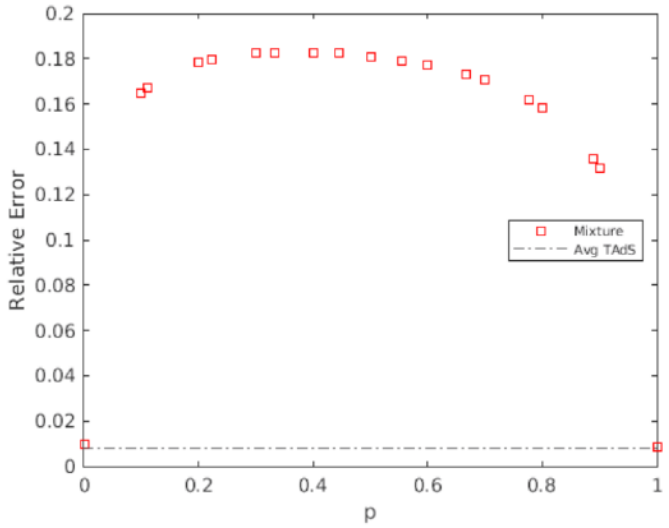
Geometrical (small error)

Non -geometrical (large error)

NON-GEOMETRIC RECON







Superpose black hole at different temperatures

WHAT ABOUT GRAVITY? → PART 3

- Geometry is nice, but not good enough

- It would be nice to also say something about gravity

LINEARIZED GRAVITY (CONSTRAIN QM SYSTEM?)

- Work backwards from linearized EE (or rather Hamiltonian constraint) around the flat background

$$\delta G_{\mu\nu} t^\mu t^\nu = 8\pi G_N \delta T_{\mu\nu} t^\mu t^\nu$$

- It actually becomes an entropy constraint through Radon transforms

$$\mathcal{R}[\delta R] = 16\pi G_N \mathcal{R}[\delta T_{tt}]$$

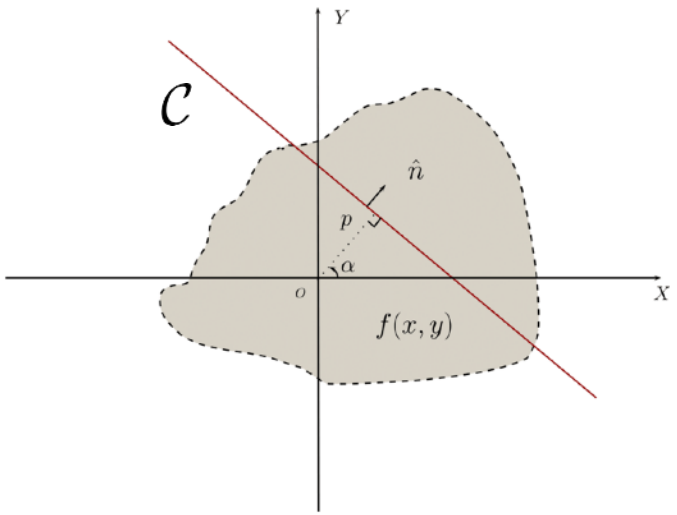
$$\updownarrow$$

$$\delta A/4G + \delta\langle H_{QFT}\rangle = 0$$

Proto-area in QECC

Entanglement first law

$$\delta S_{\text{geo/UV}}(\mathcal{C}) + \delta S_{\text{matter/IR}}(\mathcal{C}) = \delta S_{\partial/\text{total}}(=0)$$



Cao, Carroll 2017

LINEARIZED GRAVITY

- Compare

$$\delta S(\mathcal{C})_{\text{geom}} + \delta S(\mathcal{C})_{\text{matter}} = 0$$

Flat space

Compare with the RT formula with sub-leading correction (AdS/CFT)

$$\delta S_{\text{geom}}(\mathcal{A}_B) + \delta S_{\text{bulk}}(\Sigma_B) = \delta S_{\text{boundary}}(B)$$

Compare with entanglement equilibrium, T. Jacobson 1505.04753

$$\delta S_{\text{UV}} + \delta S_{\text{IR}} = 0$$

FINDING GRAVITY IN QECC

- Need to find QECCs that satisfy such entropic relations

$$\delta S_{\text{emerg geom}} + \delta S_{\text{encoded}} = 0$$

or generalizations

- How about the codes we've looked at so far?

PHYSICS INTERPRETATION

Exact erasure correction codes we discussed	⟷	QFT on background
Logical Information	⟷	Matter field configurations
Entanglement need to build a good code	⟷	Geometry of the background space(time)

PHYSICS INTERPRETATION

Exact erasure

cor

v

Log

Enta

bui

$$\delta S_{\text{geom}} = 0, \quad \delta S_{\text{matter}} \neq 0$$

$$\delta S_{\text{geom}} = 0, \quad \text{no matter what } |\tilde{\psi}\rangle \text{ is}$$

Matter is decoupled from gravity!

field

ed

igurations

he

ce(time)

THE ZEROTH STEP

- At the very least, we need codes where

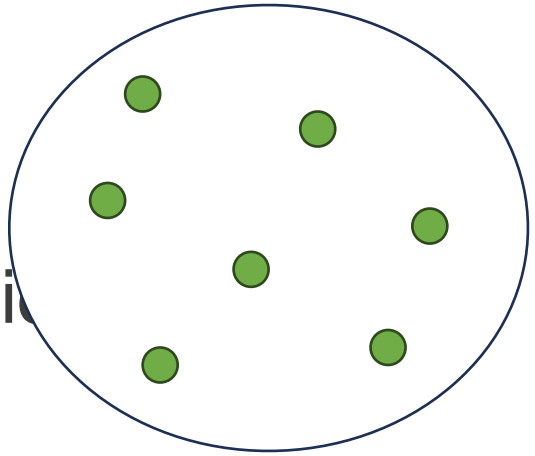
$$\Delta|\tilde{\psi}\rangle \to \Delta S_{\text{geom}}$$

- Is it possible? AdS/CFT was able to do it

Ok, what's missing?

ALL WE NEED IS A BIT OF MAGIC?

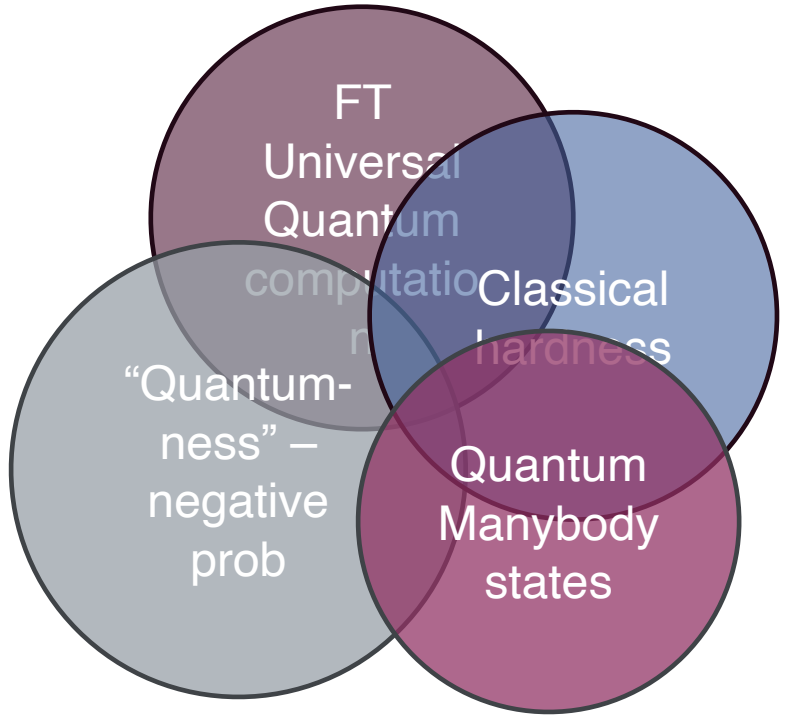
- Entanglement > classical correlation



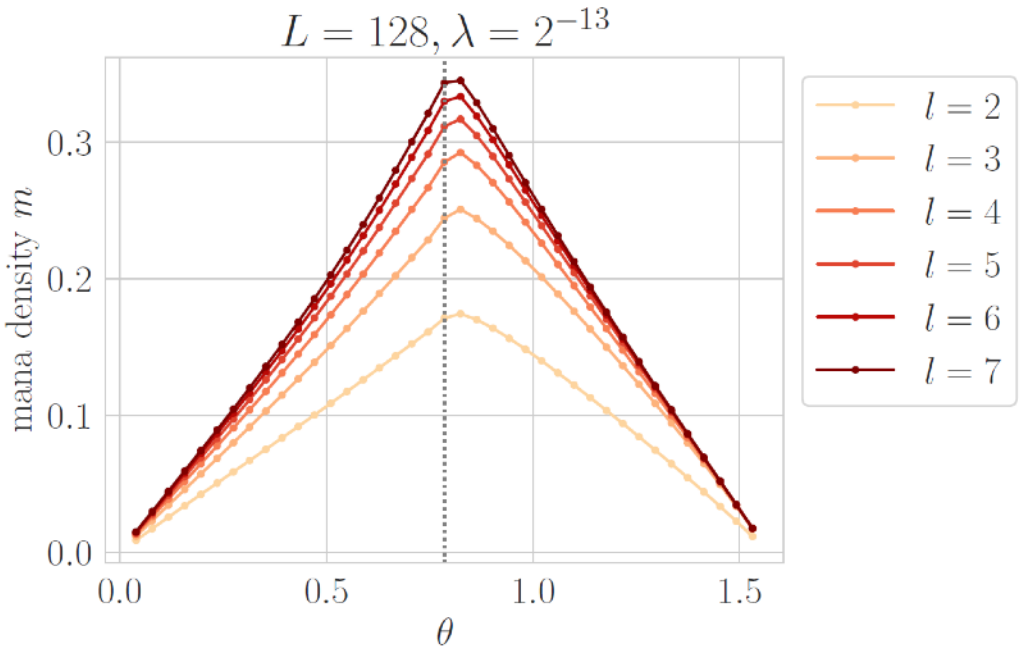
- Magic (non-stabilizerness) “>” classical computati
  - Each oasis can contain highly entangled state
  - Distance to the nearest “oasis”
- The codes we are dealing with so far have zero magic

INTUITIVELY, WHAT DOES IT DO?

- Necessary for quantum advantage, but insufficient by itself
- Related to Wigner negativity
- Empirically correlate with hardness of classical simulation
  - Monte Carlo convergence
  - Computational complexity in stabilizer simulation
  - Tensor networks (non-local magic)
- Lots of it in physical quantum systems
  - “Orthogonal” to entanglement



CFT AND GRAVITY ARE MAGICAL

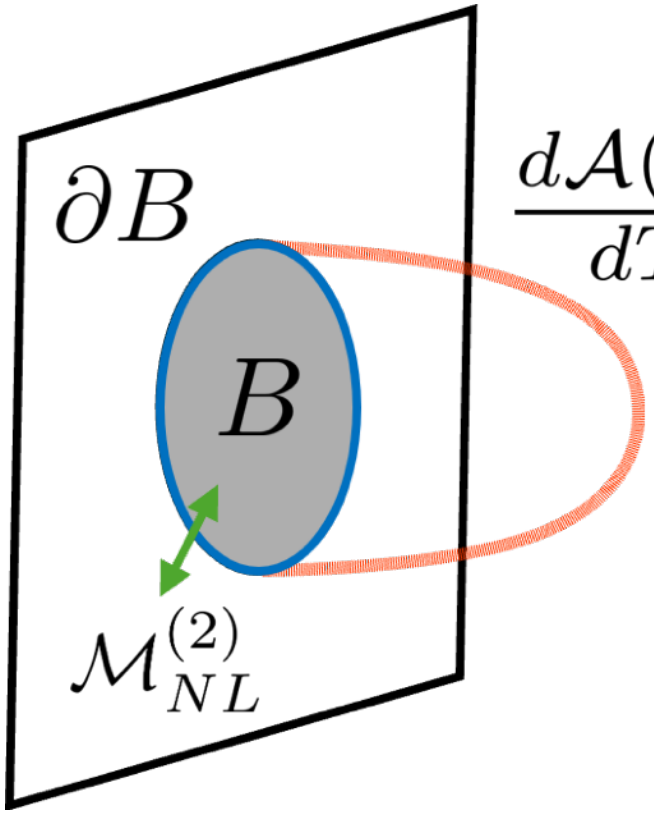


White, C.C., Swingle (2021)

$$H_{POTTS} = -\cos\theta \sum_i [Z_i^\dagger Z_{i+1} + h.c.] - \sin\theta \sum_i [X_i^\dagger + X_i]$$

Criticality at $\pi/4$

CFT AND GRAVITY ARE MAGICAL



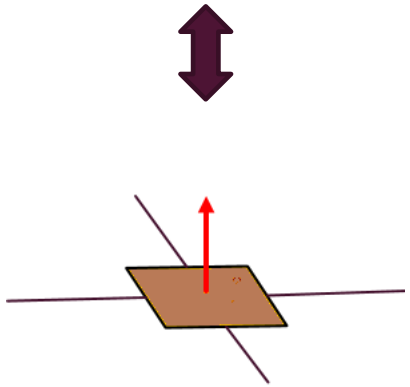
$$\frac{d\mathcal{A}(T)}{dT}\Big|_{T=0} \qquad \frac{d\mathcal{A}}{dT}\Big|_{T=0} \sim \mathcal{M}_{NL}$$

- Magic is found in holographic CFTs
- Provably required for gravitational backreaction in QECCs to have the right algebraic structure

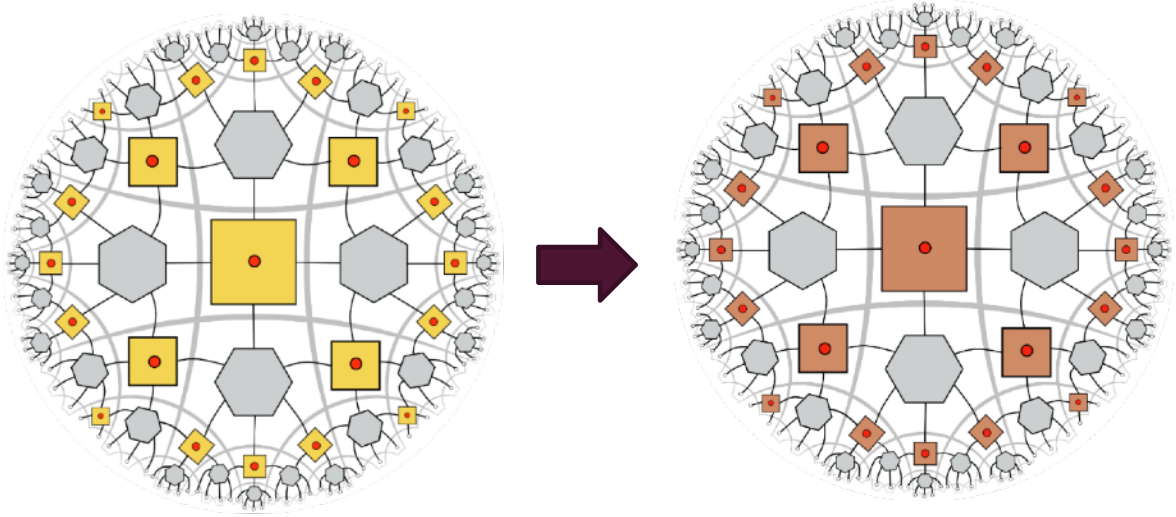
C.C. 2023
C.C., Cheng, Hamma, Leone, Munizzi, Oliviero 2024

MAGIC DUST ON QUANTUM CODES

$$|0000\rangle + |1111\rangle \to |\tilde{0}\rangle$$
$$\alpha|1100\rangle + \beta|0011\rangle \to |\tilde{1}\rangle$$

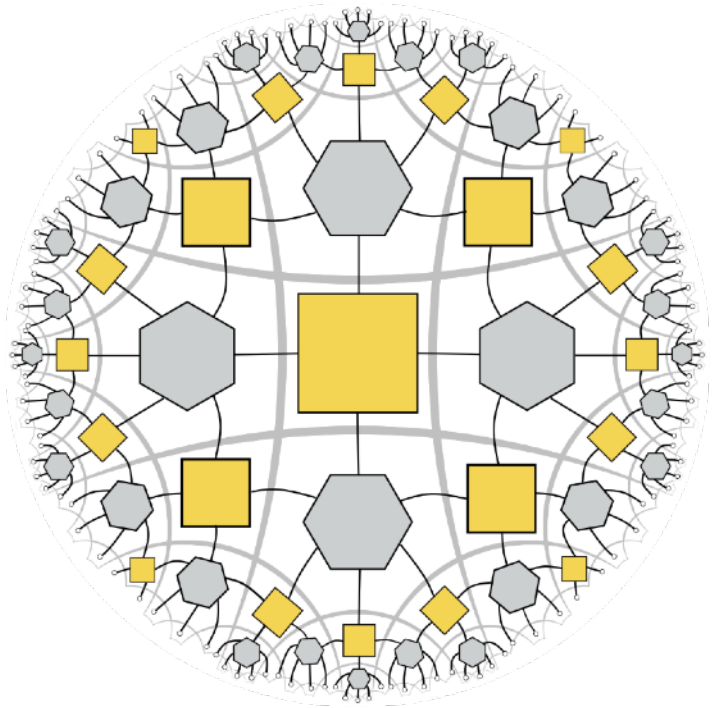


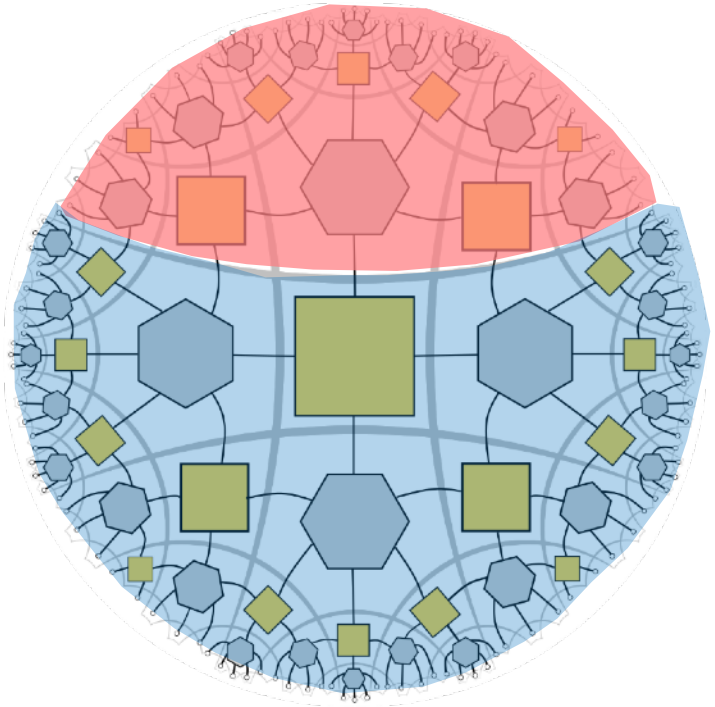
C.C, B. Lackey (2020)
C.C, (2023)



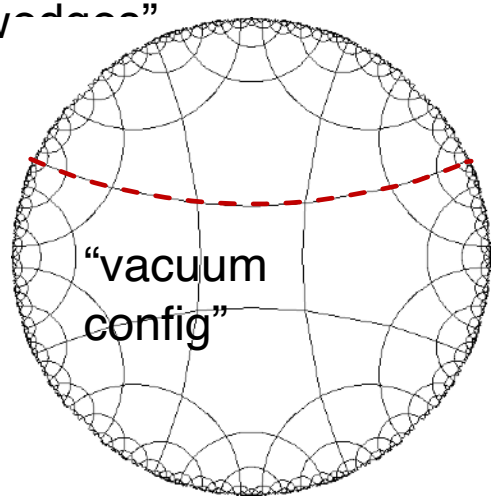
ALGEBRAIC VIEW: THE “VACUUM STATE”

- Example: all bulk states are 0 states





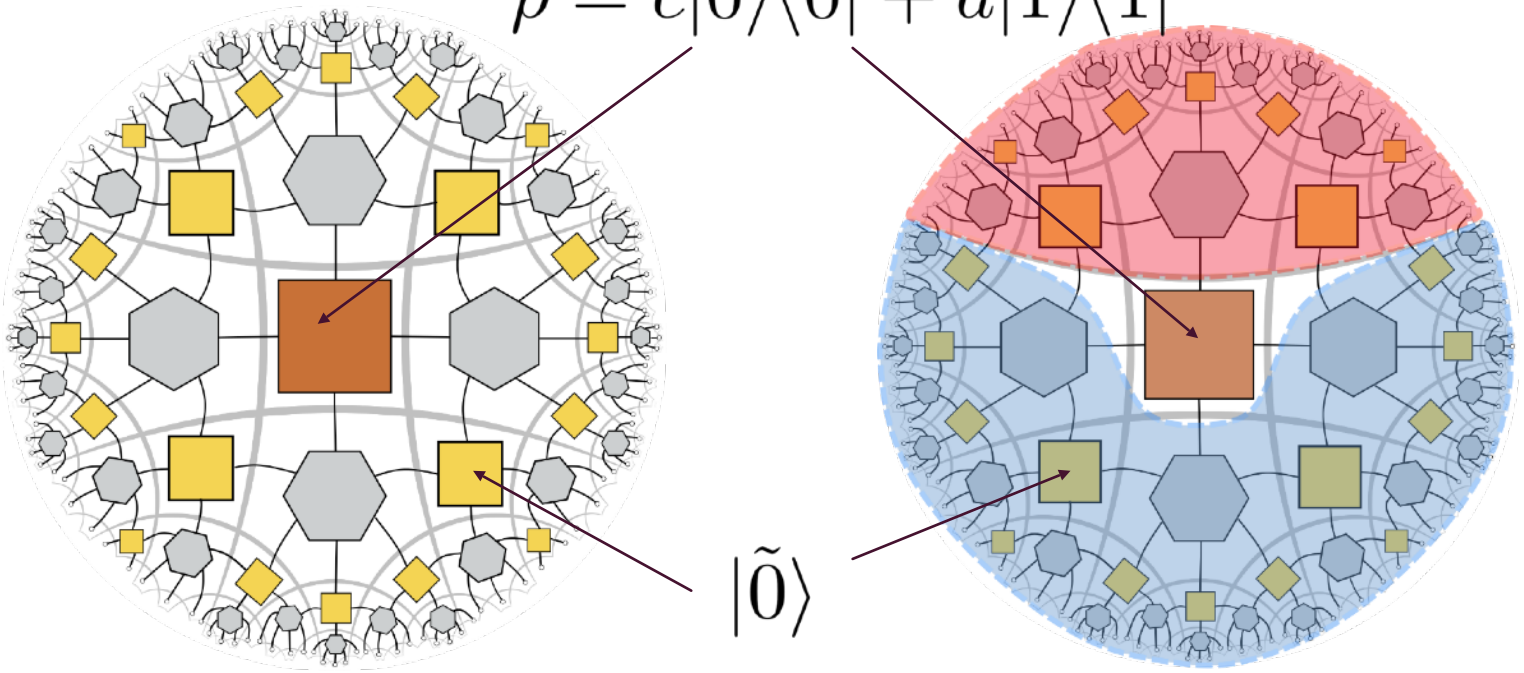
Again, we can recover the state from subregions with their respective “entanglement wedges”



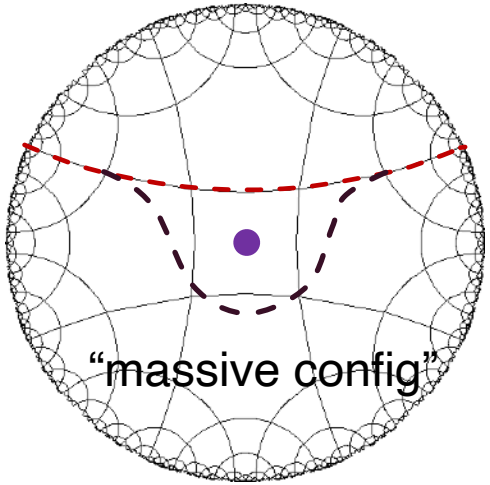
ALGEBRAIC VIEW: THE “MASSIVE” STATE

- Suppose we change one of the bulk states to a nonzero state

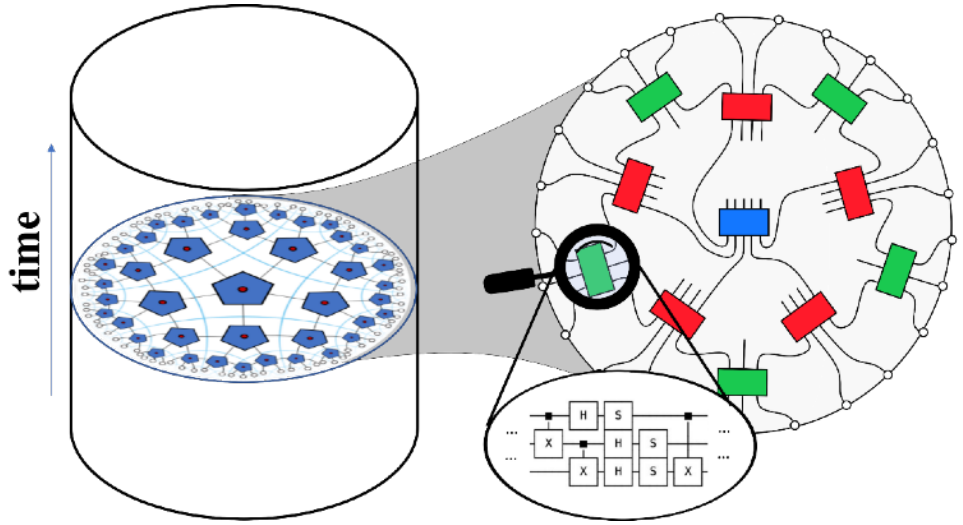
$$\tilde{\rho} = c|\tilde{0}\rangle\langle\tilde{0}| + d|\tilde{1}\rangle\langle\tilde{1}|$$

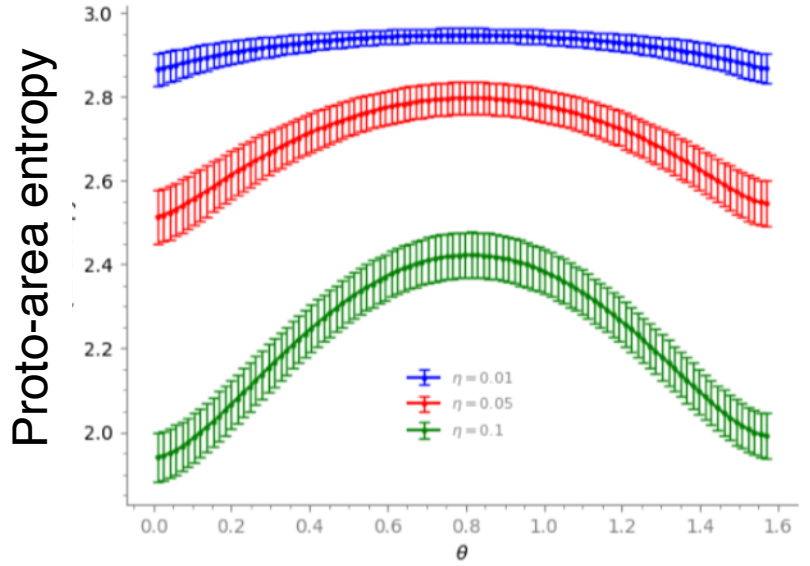


$$|\bar{0}\rangle \leftrightarrow \text{“vacuum config”}$$
$$\rho \leftrightarrow \text{“mass inserted”}$$



ENTROPIC VIEW





$$\Delta|\tilde{\psi}\rangle \rightarrow \Delta S_{\text{matter}} \rightarrow \Delta S_{\text{geom}}$$

Nair, Cheng, Biswas, Su, Gharibayan, Li, Noel, Preskill,
C.C. WIP

HOLDS FOR GENERAL ERASURE CORRECTION CODES

For any exact erasure correction code satisfying complementary recovery with trivial area operator, consider a perturbation away from this code

Theorem 1 (Heuristic): to leading order in perturbation increasing the bulk entropy almost always leads to increase on the area entropy

$$\Delta S_{\text{geom}} \sim c f(\Delta S_{\text{matter}})$$

Theorem 2 (Heuristic): the coupling constant is given by the tripartite non-local magic in the system

$$\Delta S_{\text{geom}} \sim \mathcal{M}_{NL} f(\Delta S_{\text{matter}})$$

Definition (Heuristic): tripartite non-local magic is magic that cannot be removed by unitaries that act on fewer than 3 parties at one time.

C.C., Cheng, Li, Nair, Preskill
WIP

SO HOW ABOUT GRAVITY?

Analog **gravitational features** are connected to **(non-local) magic** of a code!

From a state perspective:

$$U_{Recovery}|\tilde{\psi}\rangle = |\psi\rangle|\chi\rangle \rightarrow U_{Recovery}|\tilde{\psi}\rangle = \sum_i c_i|\psi_i\rangle_{\text{matter}}|\chi_i\rangle_{geom}$$

SUMMARY

Item (Starting from a QECC/quantum states)	Feasibility
Emergent dimensionality	
Separate matter from background geometry (c.f. RG)	
Generalizability to different geometries	
Recover metric tensor for emergent geometry	
Find QECC where matter entropy change triggers geometric entropy change	

TOWARDS LINEARIZED EINSTEIN'S EQUATION

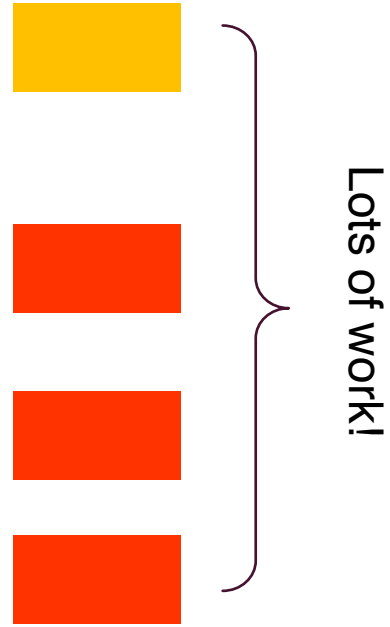
- Now S_{geom} can vary, but it still needs to vary in a specific w

- Need time evolution to generate a space-time geometry

- Need Lorentz invariance

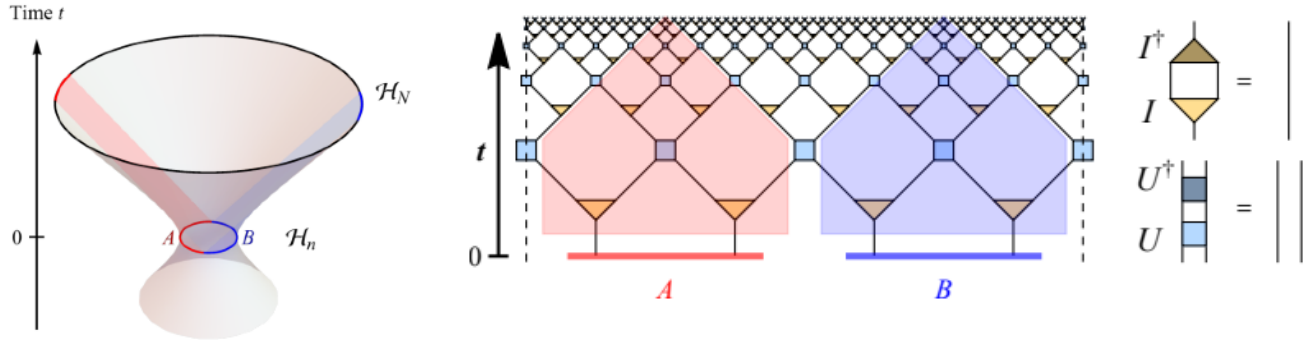
- Suppose the linearized Hamiltonian constraint holds f all t^μ

$$\delta G_{\mu\nu} = 8\pi G_N \delta T_{\mu\nu}$$



SOME SPECULATIONS

Can spacetime emerge from the entanglement patterns as a quantum circuit?



Bao, Cao, Carroll, Chatwin-Davies 2017
Cao, Chemissany, Jahn, Zimboras 2023

Concatenation → RG? (c.f. MERA)

$$\mathcal{C}^{(0)} \subset \mathcal{C}^{(1)} \subset \cdots \subset \mathcal{H}$$

IR ⟵ UV

Connection with QRF

$$\mathcal{C} \leftrightarrow \mathcal{H}_P, \ \mathcal{H} \leftrightarrow \mathcal{H}_{Kin}, \ |\tilde{\psi}\rangle = \sum_i c_i |\psi_i\rangle_S |\chi_i\rangle_C$$

Carrozza, Chatwin-Davies, Hoehn, Mele 2024

FUTURE OUTLOOK

- We have a roadmap to go from entanglement to geometry/gravity
- Specific cases and examples
- Many more questions
- Experimental connections?

FUTURE OUTLOOK

THANK YOU!