Emergence of Spacetime from Fluctuations & Dynamical Wick Rotation

Barbara Šoda, assistant prof. @ University of Zagreb, Croatia

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- Time and space are similar enough to be mixed in Lorentz transformations, and yet dissimilar enough to be clearly separated in the Lorentz signature.

- Nothing a priori wrong with that. But can we try to explain it on a deeper level of analysis?

Outline

- **Part 1: a quantum gravity path integral based on spectral geometry.**
  - Gravity + free fermions and free bosons,
  - Computable observables: effective dimension, expected volume and number of degrees of freedom of the expected spacetime.
- **Part 2: added interactions,**
  - Gravity + interacting fermions and bosons,
  - Change of effective signature via a field fluctuations-induced disorder for the geometry,
  - —> Anderson localization of space into time, dynamical Wick Rotation.

Part 1:
A QG path integral based on spectral geometry

Based on: M. Reitz, B. Šoda, and A. Kempf, Phys. Rev. Lett. 131, 211501 (2023).

- In the low-energy picture, a spacetime manifold is hosting matter, in the form of quantum fields.

- Could this picture emerge from a “pre-geometric” regime?

- Could it emerge multiple times, at different energy scales, with different spacetime dimensions?

- Challenge: to find a mathematical framework that can describe both regimes: geometric and pre-geometric.

- We choose path integral: need to make it independent of position basis (with the goal of having a chance at leaving the geometric regime)

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- What is the action? Gravity, free fermions, free bosons: $S = S_g + S_f + S_b$

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P. B. Gilkey, J. Diff. Geom. 10, 601 (1975)

- Gravitational action: $N = \dfrac{1}{16\pi^2}\displaystyle\int d^4x\sqrt{g}\left(\frac{\bar{\Lambda}^2}{2} + \frac{\bar{\Lambda}}{6}R + O(R^2)\right)$

Number of eigenvalues of the Laplacian operator below the UV cutoff Λ

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Number of eigenvalues of the Laplacian operator below the UV cutoff Λ

$\boxed{\mu = \frac{6\pi}{\bar{\Lambda}}}$

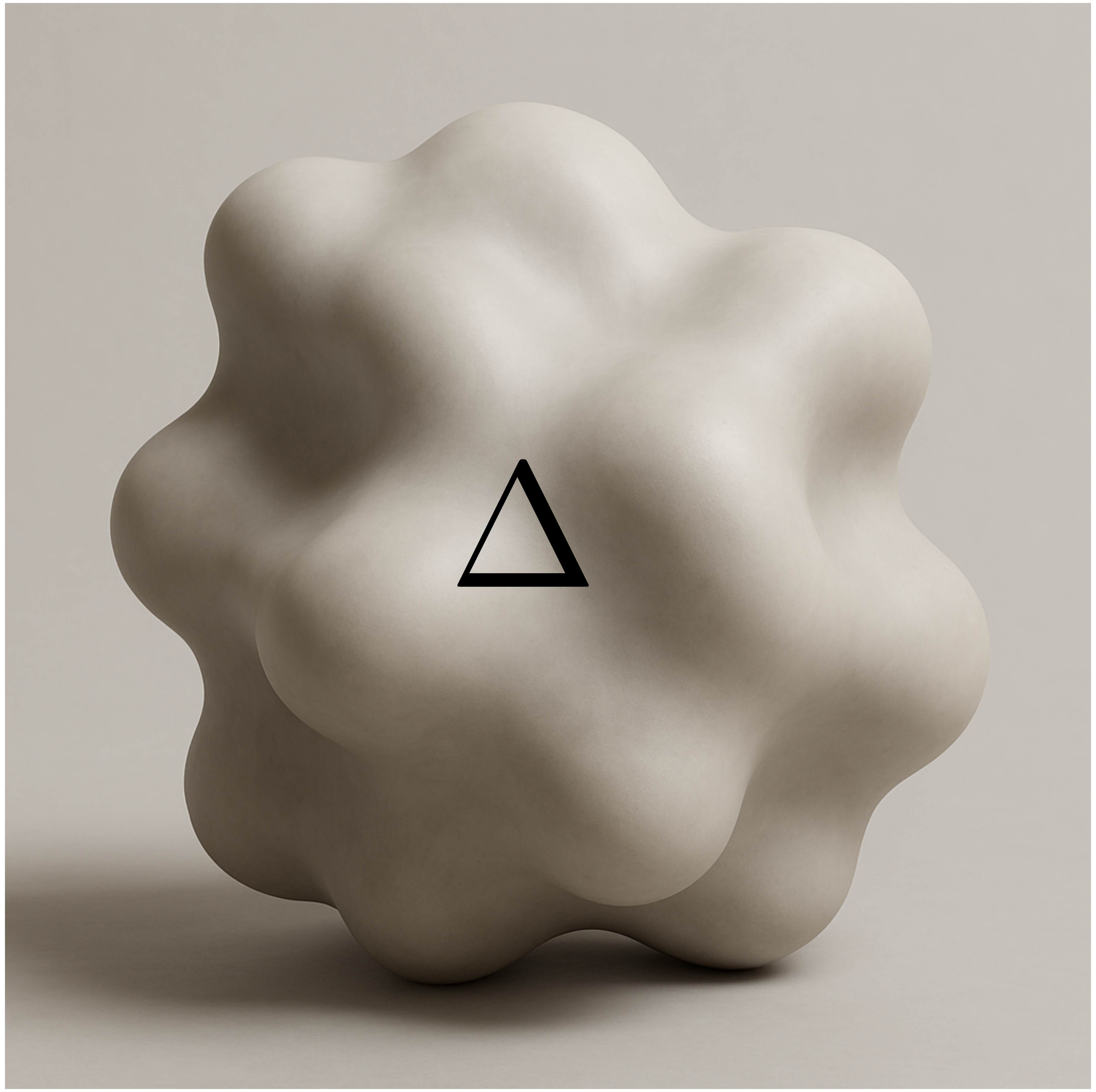
- Take Gilkey's formula as the action for gravity: $S_g = \mu N$

"gravitational action is the number of Laplacian's eigenvalues on the manifold below a UV cutoff"

- Basis independently: $\boxed{S_g = \mu \mathrm{Tr}\left(\mathbf{1}\right)}$

Visual aid for the Gilkey formula

- N-dimensional compact manifold,
- Second order differential operator,
- There is a UV cutoff ,
- -> The formula is valid.
- (In Euclidean signature)



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Action for free fermions and bosons:

- Gravity action: dimension of Laplacian operator’s Hilbert space on a compact Riemannian manifold with a UV cutoff:

$$\boxed{S_g \propto \mathrm{Tr}\left(\mathbf{1}\right)}$$

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$$S_b = \frac{1}{2}\sum_{i=1}^{N_b} \mathrm{Tr}\left((\Delta + m^2)|\phi\rangle_i\langle\phi|_i\right)$$

- In the Laplacian eigenbasis:

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- Free fermions: Dirac fields

$$S_f = \int d^4x \sqrt{g}\bar{\Psi}\left(i\Gamma^\mu D_\mu\right)\Psi$$

$$\boxed{S_f = \sum_{i=1}^{N_f}\sum_{n=1}^{N} \sqrt{\lambda_i}\theta_n^i\bar{\theta}_n^i}$$

- Total action: $\boxed{S = S_g + S_f + S_b}$

- Full path integral: $$Z = \sum_{N=1}^{\infty} \int_{m^2}^{\Lambda} \mathcal{D}\lambda \int \mathcal{D}\phi \int \mathcal{D}\theta \mathcal{D}\bar{\theta} e^{-\beta S} \frac{\Lambda^{N(\frac{N_f}{2}-1)}}{(N-1)!}$$

Laplacian's eigenvalues

Field components in the Laplacian eigenbasis

- Summing over: a) all possible Laplacian spectra (some are representable as manifolds, some are not),

  b) all possible field values, given in the Laplacian eigenbasis,

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- Relatively easy to evaluate analytically (Gaussian fields, though weakly interacting via gravity):

$$\boxed{Z = C\ m^{d-2} \exp\left[2\ C\ \frac{\Lambda^{d/2} - m^d}{d}\right]}$$

where

$$C := (2\pi)^{\frac{N_b}{2}} \frac{e^{-\beta\mu}}{\Lambda^{1-\frac{N_f}{2}}} \beta^{\mu\beta_{max}}$$

Effective dimension of emergent spacetimes

- How do we get the effective dimension? From the probability distribution of eigenvalues:

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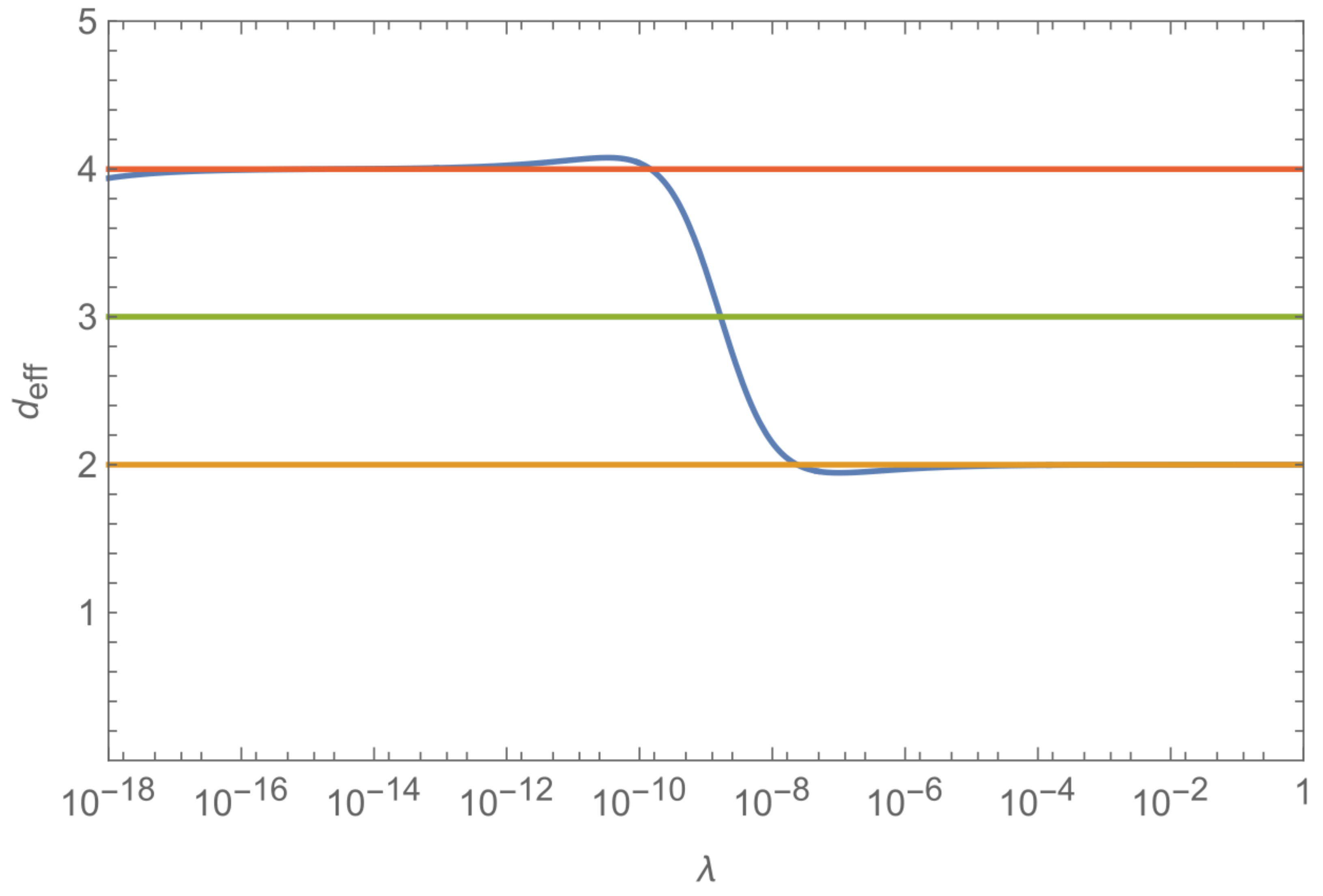
Different effective dimension at different energies for non-trivial mass spectra.



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- Valid for trivial mass spectrum. For nontrivial spectrum: see image, $d_{eff}(\lambda) = -2\lambda \frac{\partial \log(p(\lambda))}{\partial \lambda} + 2$

Effective volume and number of DOF of emergent spacetimes

- The **expected number** of eigenfunctions evaluates to: $\langle N \rangle = \frac{-Z^{-1}}{\beta}\frac{\partial Z}{\partial \mu} = 1 + 2C\ \frac{\Lambda^{d/2} - m^d}{d}.$

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- Recall: Spectral gap of the Laplacian is closely related to ℓ^{-2}, with ℓ its largest geodesic distance and, roughly, the volume $V \approx \ell^d$.

- Therefore, the **effective volume** is roughly $V_{eff} \equiv \underline{\langle g \rangle}^{-d/2}$

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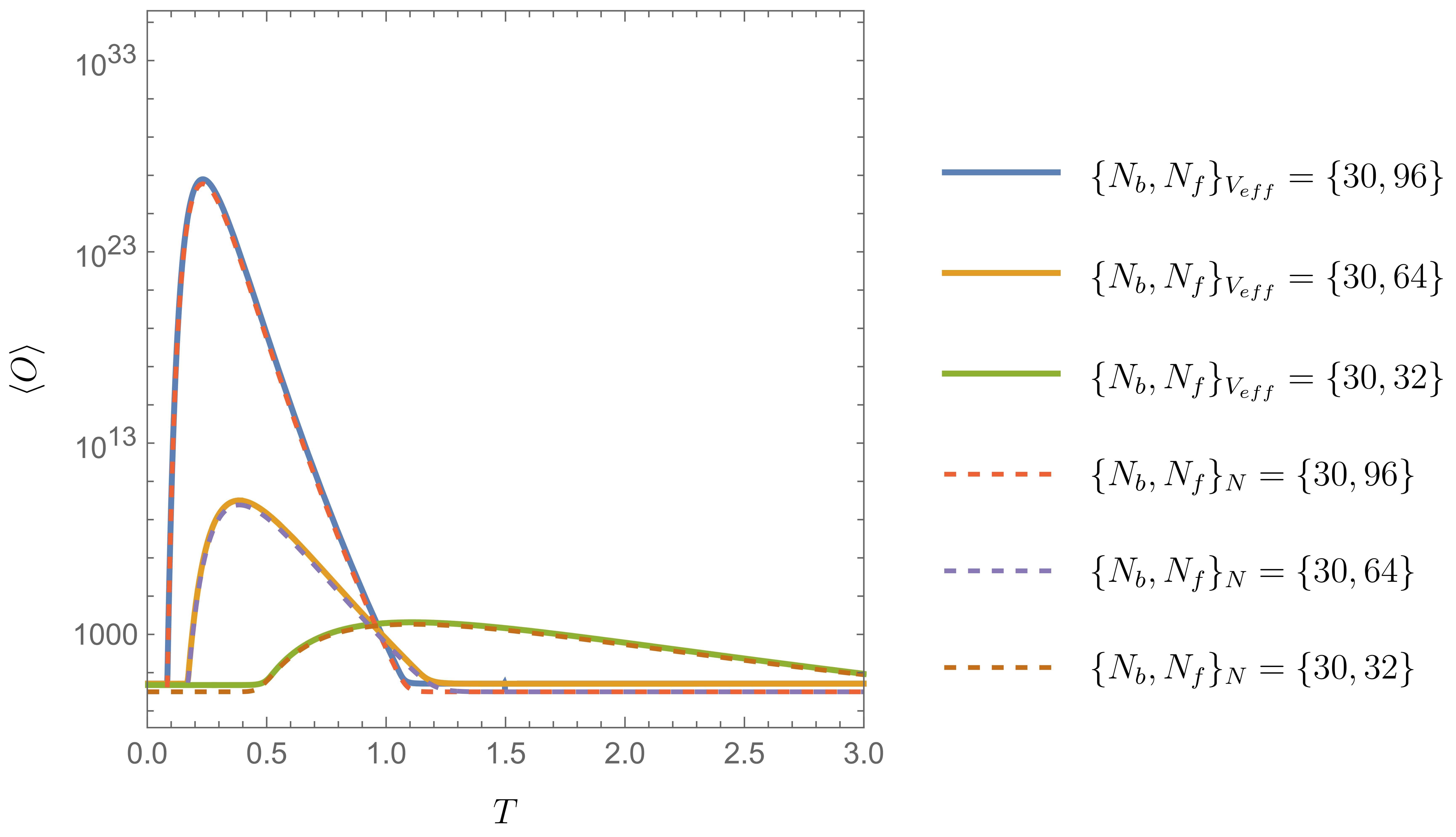
- The expected spectral gap is calculable: fix first eigenvalue to mass and $\langle \lambda_2 \rangle = \int_m^\Lambda d\lambda_2\ \lambda_2 P(\lambda_2 | N \geq 2)$.

Effective volume and number of DOF of emergent spacetimes



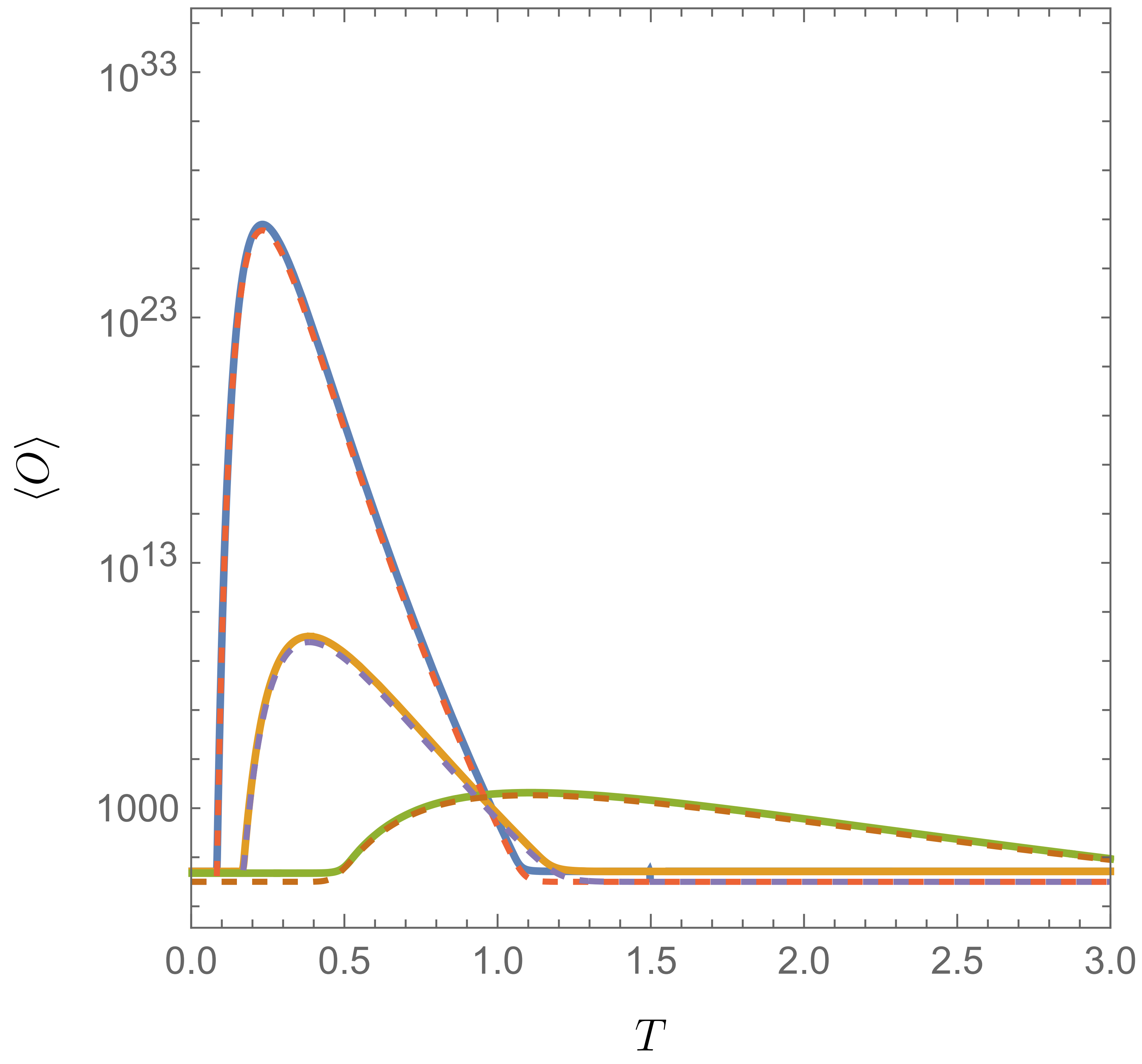
Dashed line = expected number of DOF, solid line = expected volume

Consistency check:

- Hawking & Gilkey formula showed that classically: $$N = \frac{1}{16\pi^2}\int d^4x\sqrt{g}\left(\frac{\bar{\Lambda}^2}{2} + \frac{\bar{\Lambda}}{6}R + O(R^2)\right)$$

- Classically, the density of degrees of freedom is constant up to corrections: $$\frac{N}{V} = \frac{\bar{\Lambda}^2}{32\pi^2} + \int dV\ O(R)$$

- Consistency Check: N/V still constant after quantization + corrections



Results:

- Obtained dimensions and volumes of emergent spacetimes
-> depending on gravity & boson pull vs. fermion pressure,

- That balance depends on energy scale through mass spectrum,
yields energy-dependent spacetime dimensions & emergence,

- Consistency check OK.

The low-energy picture of spacetime as a manifold, hosting matter in the form of quantum fields emerges from a pregeometric regime:

- At different energy scales,
- With spacetime changing dimension.

Part 2:
Field interactions added to the QG path integral, Anderson localization of Euclidean dimension(s)

Based on: B. Šoda, In preparation

Interacting fermions and bosons in the path integral:

- Add a Yukawa coupling to the action: $S_{\mathrm{Y}} = g \int d^d x \; \bar{\psi}(x) \psi(x) \, \phi(x)$
- Total action: $S = \underbrace{S_g}_{\substack{\text{Gravity} \\ \text{Gilkey action}}} + \underbrace{S_f + S_b}_{\text{Free fields}} + \underbrace{S_Y}_{\substack{\text{Yukawa} \\ \text{Interaction}}}$

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- We write the interactions in the eigenbasis of the Laplacian: $S_{\mathrm{Y}} = g \sum_{\alpha=1}^{N_f} \sum_{a=1}^{N_b} \sum_{k,m,\ell=1}^{N} C_{km\ell}\, \bar{\theta}_k^\alpha\, \theta_m^\alpha \; \phi_\ell^a$

- With C coefficients being overlaps between Laplacian eigenstates: $C_{km\ell} = \int_{\mathcal{M}} u_k^*(x) \; u_m(x) \; u_\ell(x) \; d\mu(x).$

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- We now proceed to integrate out the fields' degrees of freedom in the full path integral with interacting fields.

Interacting fermions and bosons in the path integral:

- Integrating out the fields in the path integral serves as a source of disorder for the background geometry:
  - Weak coupling: **does not change the sign** of the probability distribution for eigenvalues.

$$Z = \sum_{N=1}^{\infty} \frac{1}{N!} \int_0^{\Lambda} \cdots \int_0^{\Lambda} \prod_{k=1}^{N} d\lambda_k \ \left(\lambda_k + M_b^2\right)^{-N_b/2} \ \left(\sqrt{\lambda_k} + M_f\right)^{N_f} \times$$

$$\times \exp\left[\Delta S_{\text{int}}(\{\lambda_k\})\right], \quad \Delta S_{\text{int}}(\{\lambda_k\}) \in \mathbb{R}$$

  - Strong coupling: **changes the sign** of the probability distribution for eigenvalues.
    - As long as the coupling g is greater than a critical coupling.

Change of sign of eigenvalues indicates change of signature.
Is it Lorentz or a different combination of - and + signs?

Note: it is not surprising that critical amount
of disorder is necessary.

Anderson Localization

Perimeter bistro, picture credit: E. Livine

P. Anderson, 1958: disorder localizes wavefunctions: $\left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\text{rand}}(\vec{r})\right)\Psi = E\Psi$

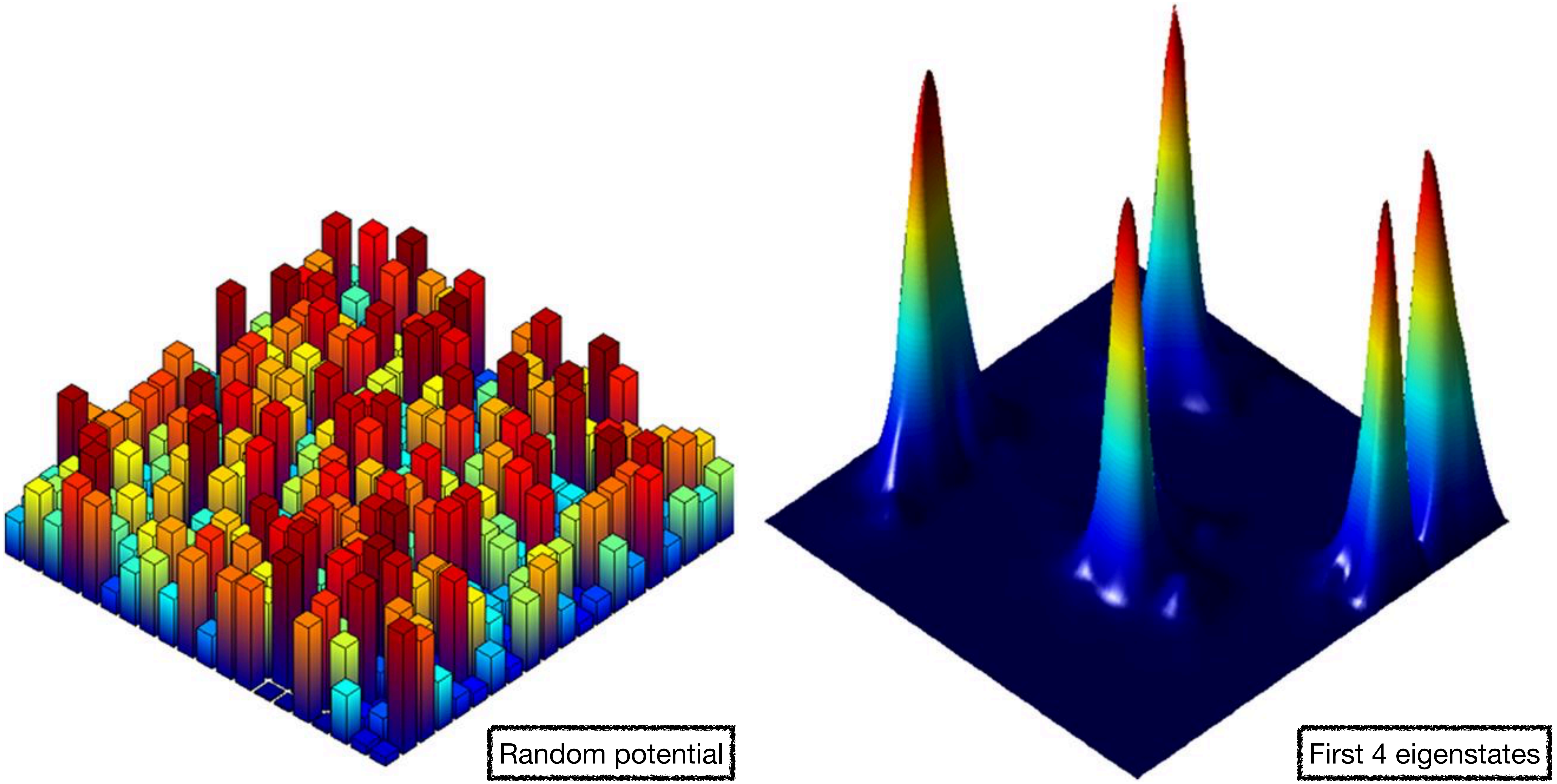
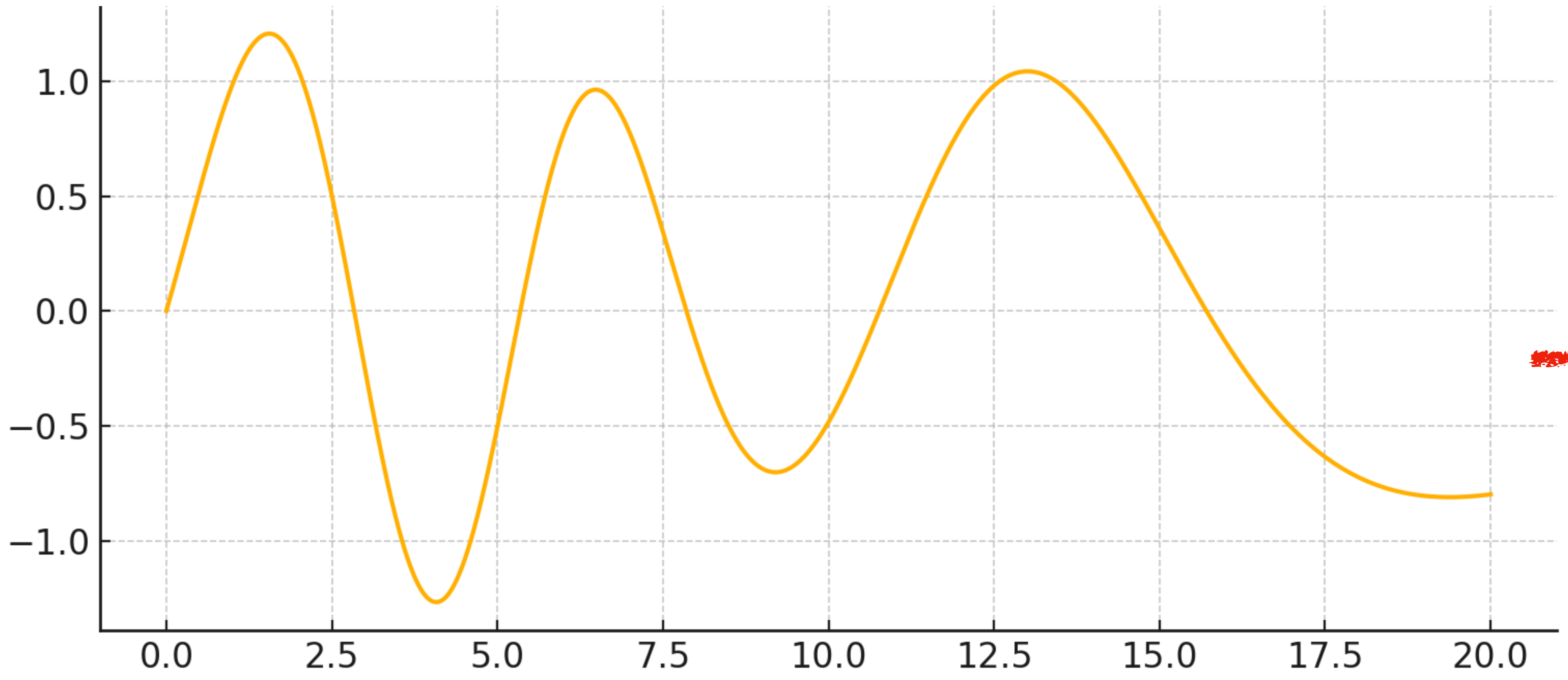


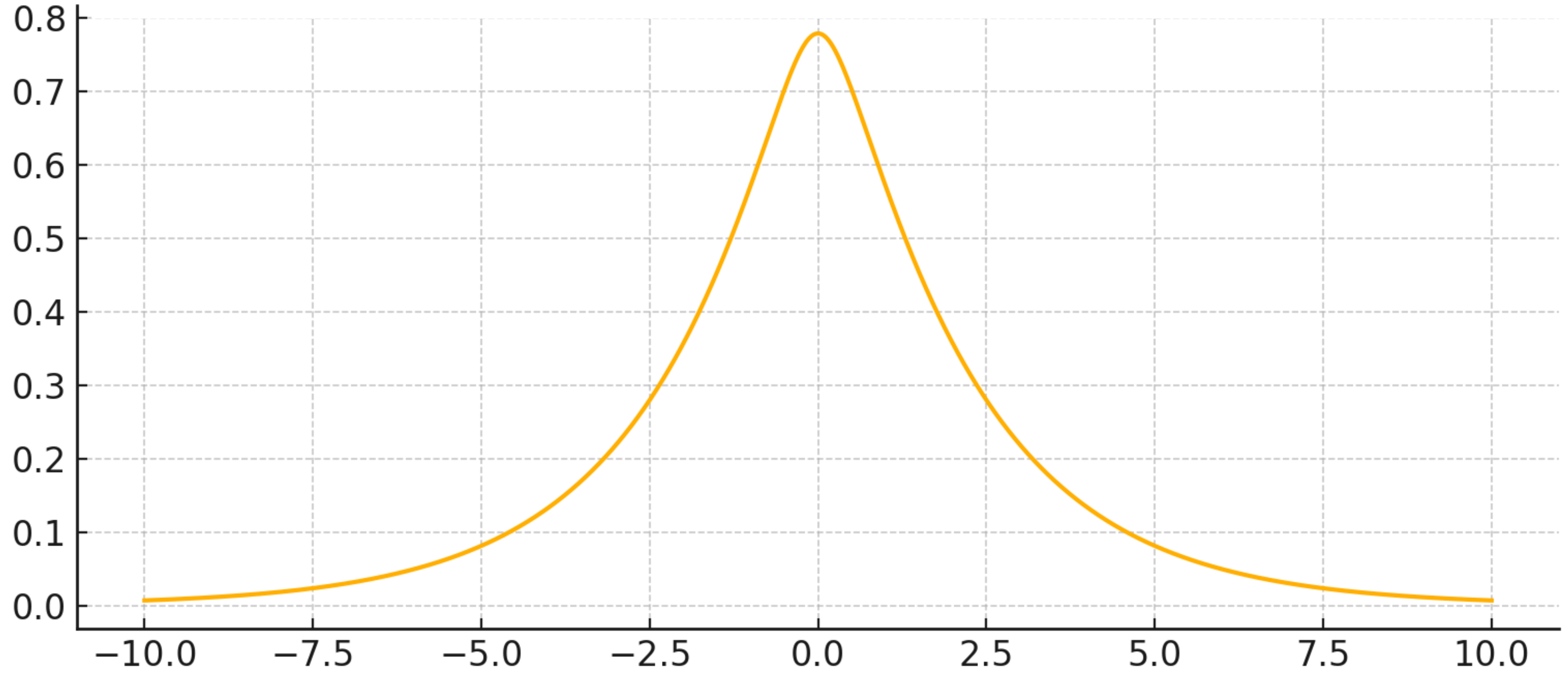
Image from: M. Filoche, S. Mayboroda, PNAS, *Universal mechanism for Anderson and weak localization, 2012*

In 1+1 and 2+1 dimensional systems any small disorder will do, in 3 and higher, a critical amount of disorder is necessary.

Presence of disorder changes the behaviour of wavefunctions:

from oscillatory to exponentially decaying.





$$\nabla^2\psi = -k^2\psi,\ \ k \in \mathbb{R}$$

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Sign flip!

A toy model

Assume we add disorder to only one dimension: $$-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial z^2}+V_{\mathrm{rand}}(x)$$

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Schematically: $$-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial z^2}+V_{\mathrm{rand}}(x)\equiv+\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial z^2}.$$

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Or, if we are in 4 dimensions, we can have an effective flip of the signature from Euclidean to Lorentzian:

$$\boxed{\nabla^2+V_{\mathrm{rand}}(t)\equiv\Box^2.}$$

Laplacian op. -> D'Alembertian op.

Time is the localized dimension, while spatial dimensions are extended.

Yukawa is isotropic in the Euclidean signature, how can it single out one or more dimensions for localization?

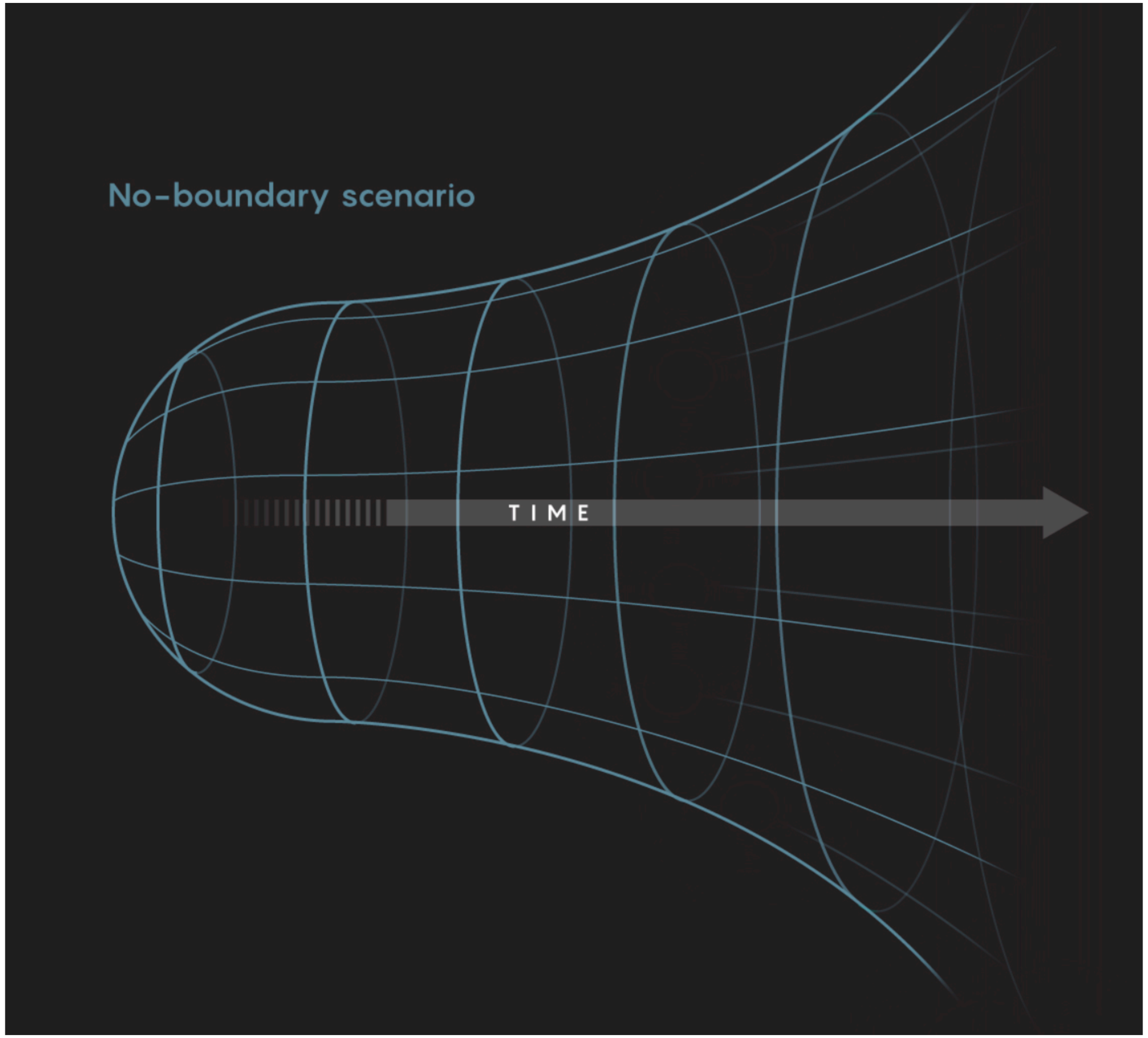
At weak disorder, still isotropic, no localized dimension.

When we increase the amount of disorder, i.e. increase coupling g, a localization happens anisotropically.

Conjecture: the time-like direction is selected as a spontaneously broken symmetry.

If we increase disorder further, possibly more dimensions become localized.

On the Hartle-Hawking no-boundary proposal:



Picture credit: Quanta

Thank you for your attention!

St Mark Church, Zagreb