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Italiadomani
PIANO NAZIONALE DI RIPRESA E RESILIENZA

UNIVERSITÀ DI PAVIA

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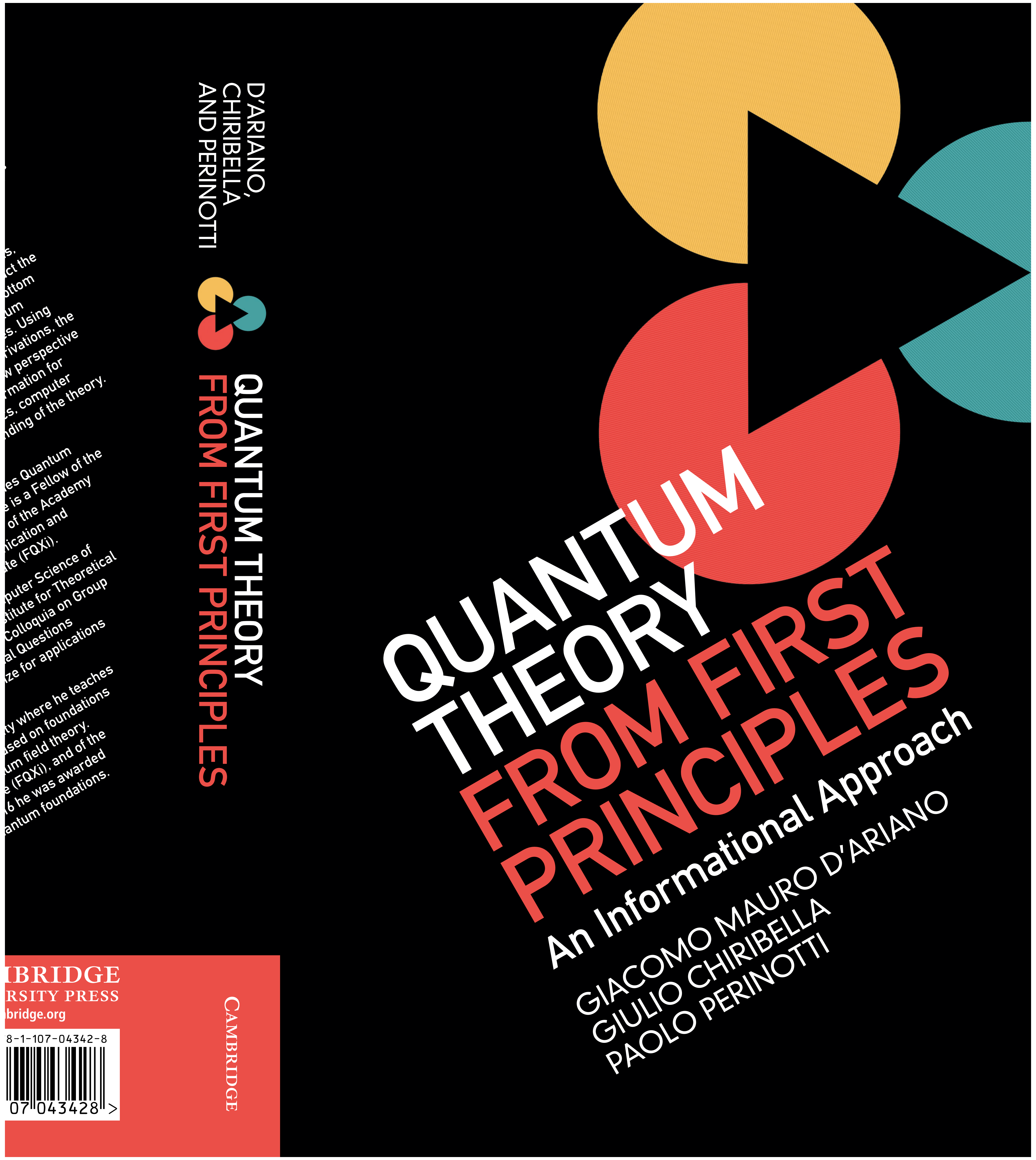
QUIT quantum information theory group

INFN Istituto Nazionale di Fisica Nucleare

Quantum physics as quantum information (and just nothing more)

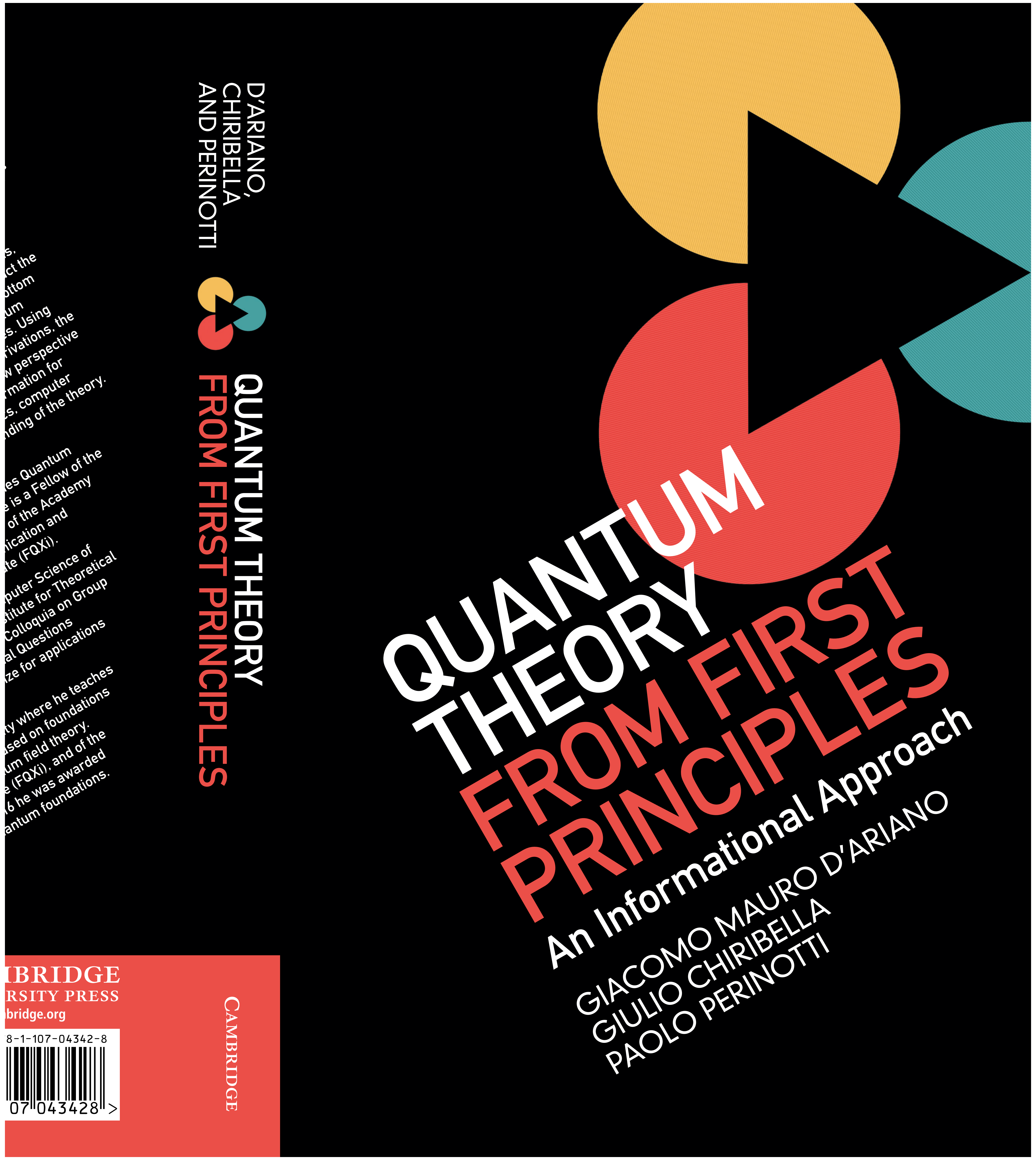
STOCKHOLM, NOVEMBER 19TH 2025- INAUGURAL WORKSHOP ON EMERGENT GEOMETRIES



Quantum information

From operational axioms

- The mathematical language of quantum mechanics expresses the features of a theory of information processing



Quantum information
From operational axioms

- The mathematical language of quantum mechanics expresses the features of a theory of information processing
- Systems are information carriers in the first place rather than being elementary constituents of matter

Mechanical semantics

- We obtain quantum theory as an information theory
- No mechanical semantics
  - Missing: notions of space and time, mass, energy, momentum…
  - Can we recover mechanical concepts?
  - Can we recover physical laws?

Laplace’s universe

- “An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past could be present before its eyes.”

  Pierre Simon de Laplace, “Essai philosophique sur les probabilités”, 1814.

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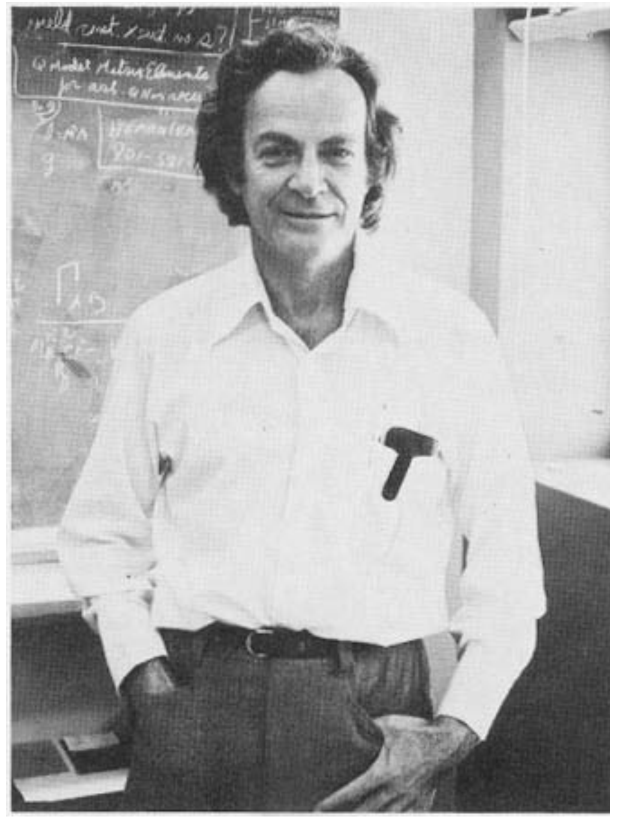
- Laplace’s idea has some modern elements in its bones: physical law as an algorithm

Algorithmic universe



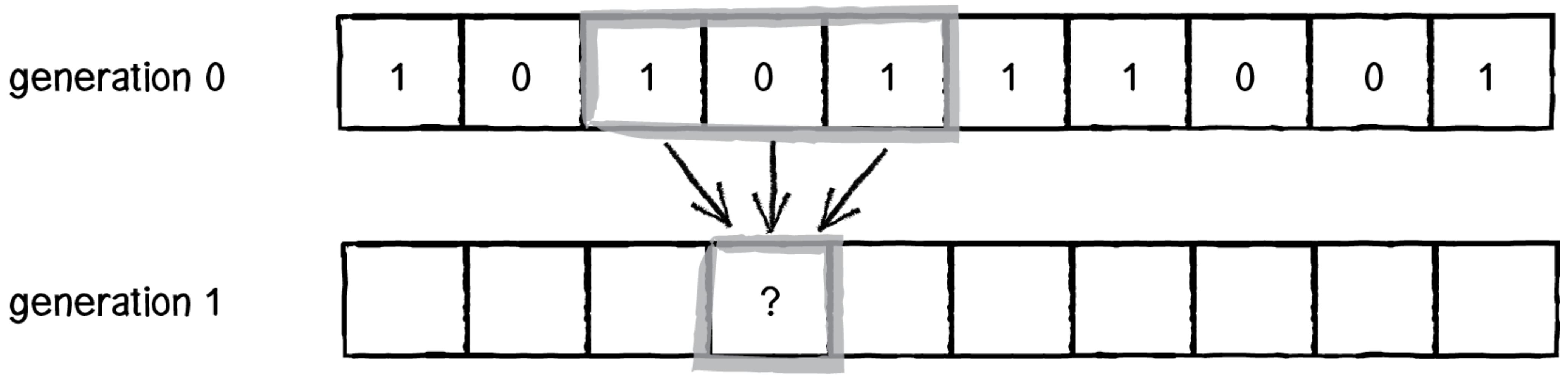
I want to talk about the possibility that there is to be an exact simulation, that the computer will do exactly the same as nature. If this is to be proved and the type of computer is as I've already explained, then it's going to be necessary that everything that happens in a finite volume of space and time would have to be exactly analyzable with a finite number of logical operations. The present theory of physics is not that way, apparently. It allows space to go down into infinitesimal distances, wavelengths to get infinitely great, terms to be summed in infinite order, and so forth; and therefore, if this proposition is right, physical law is wrong.

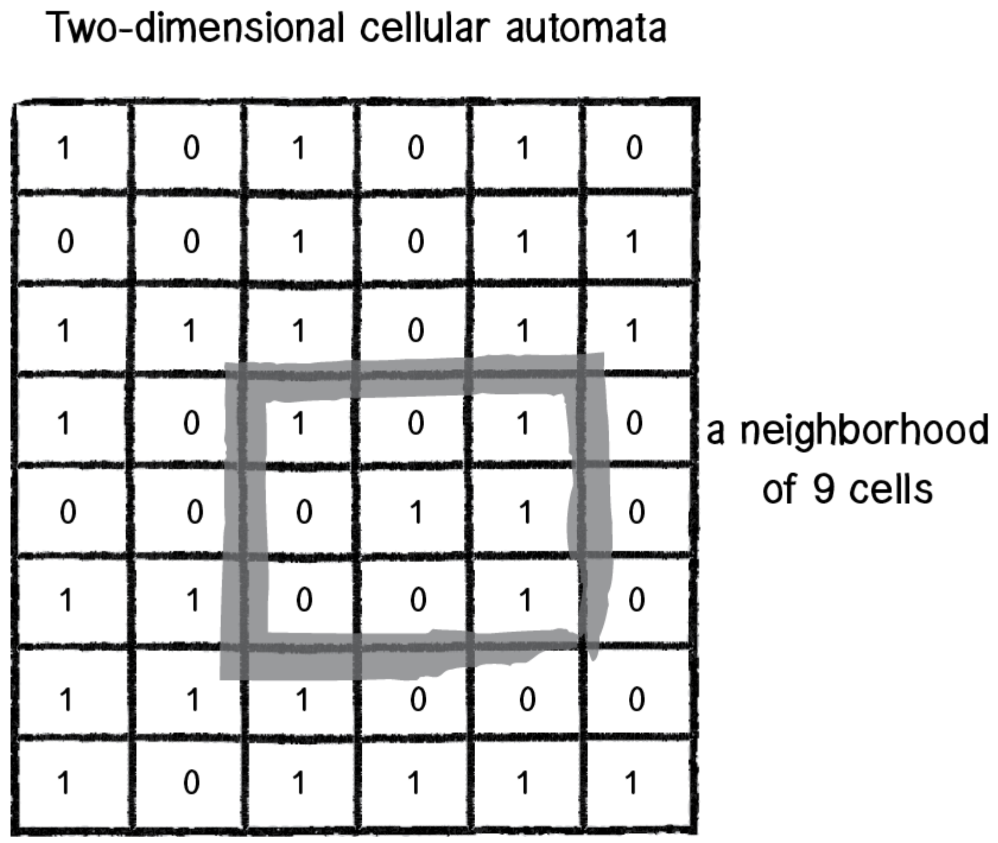
R. Feynman, Int. J. Theo. Ph. **21**, 467 (1982)

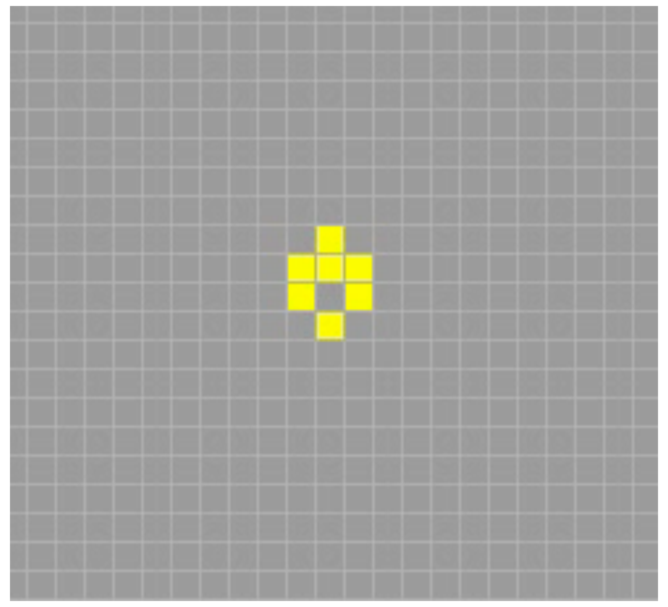
Cellular automaton

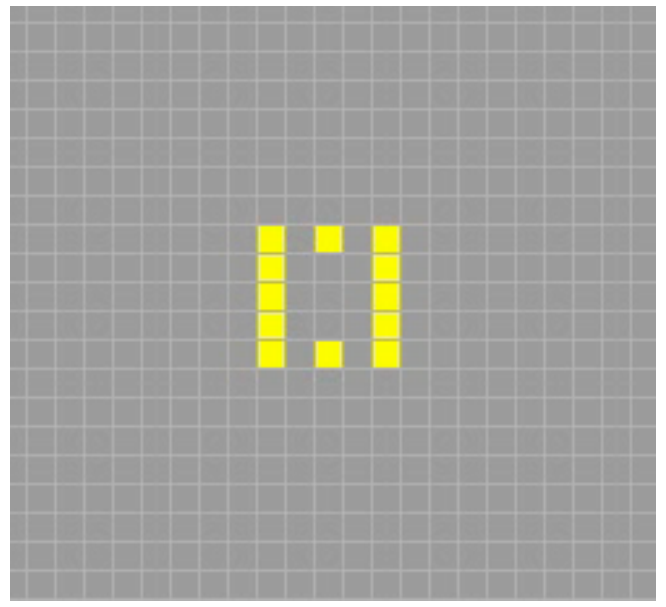
J. Von Neumann and A. W. Burks, “Theory of self-reproducing automata” 1966

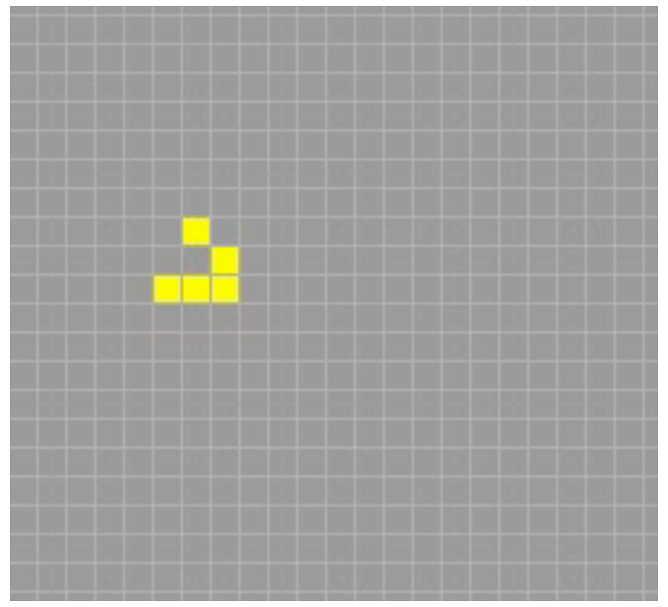
What is a cellular automaton?











http://web.stanford.edu/~cdebs/GameOfLife/

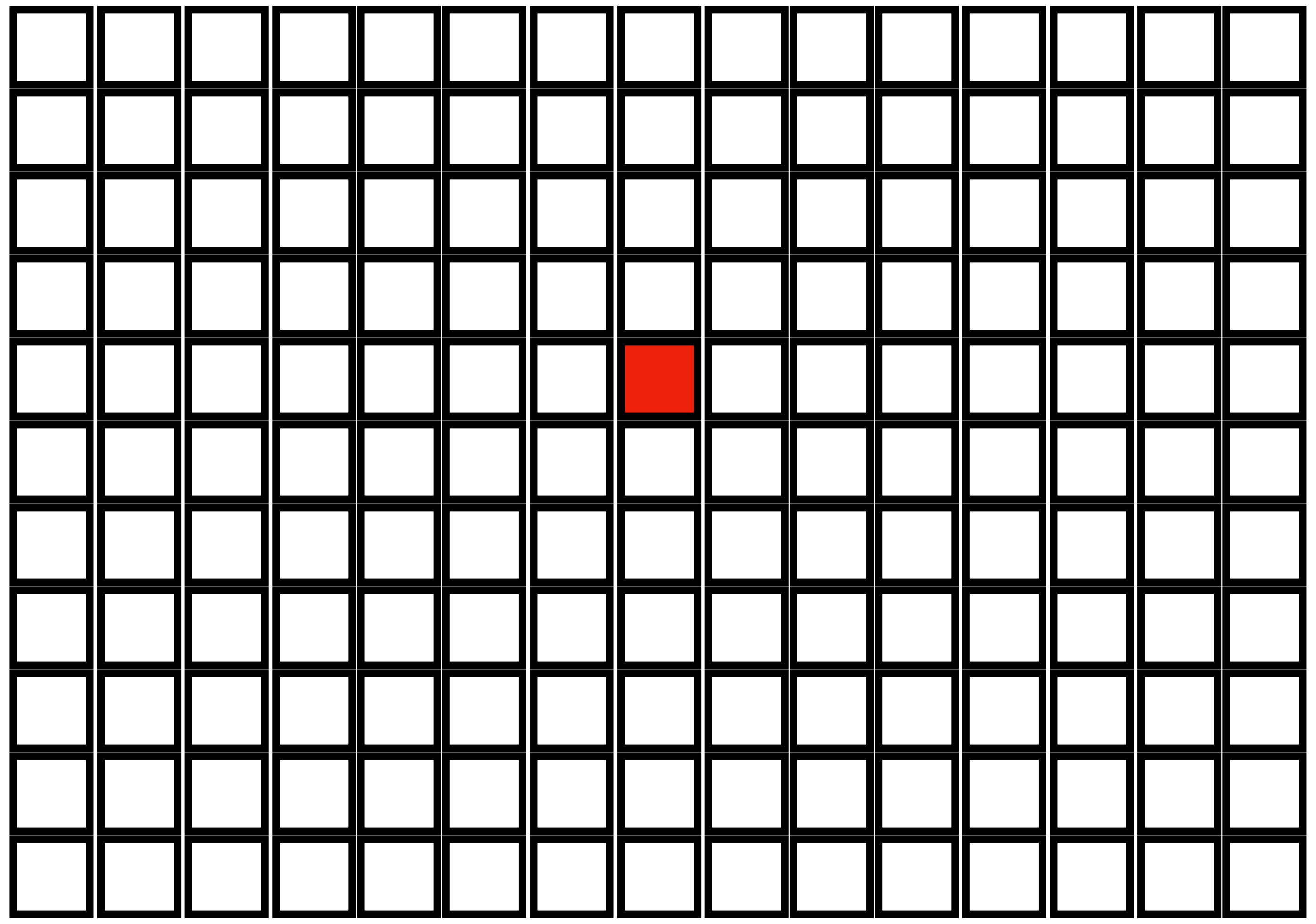
Quantum cellular automata

Quantum cellular automata

Neighbourhood of a cell

■ $= \mathcal{H}_x \leftrightarrow \mathsf{A}_x$

C.A.: $U : \bigotimes_x \mathcal{H}_x \to \bigotimes_x \mathcal{H}_x$



B. Schumacher and R.F. Werner, arXiv:quant-ph/040517

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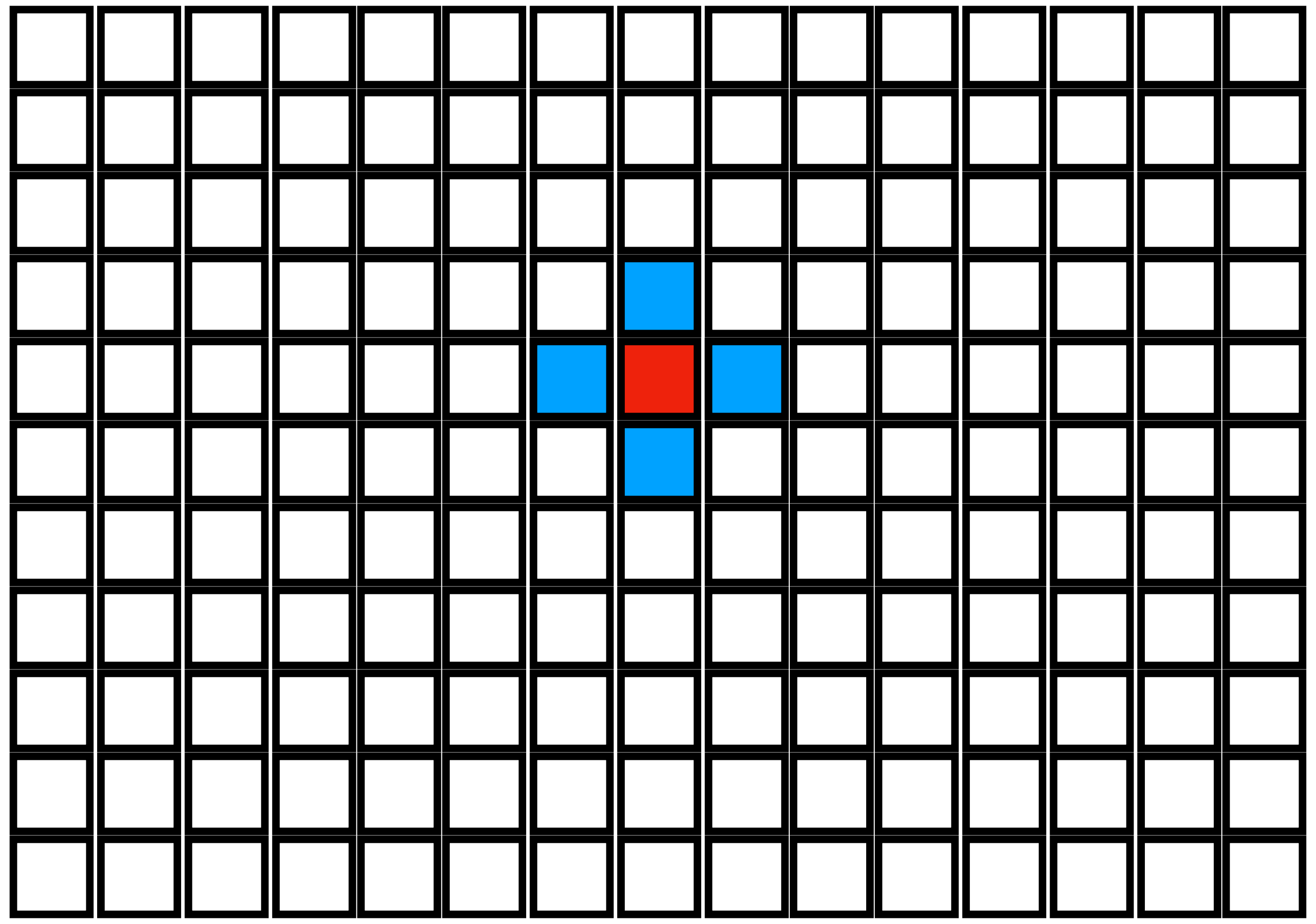
Backward neighbourhood of the cell x_0

$$U^{-1}(\mathrm{A}_{x_0} \otimes I_{\bar{x}_0})U = \mathrm{A}_{\mathrm{N}(x_0)} \otimes I_{\bar{N}(x_0)}$$



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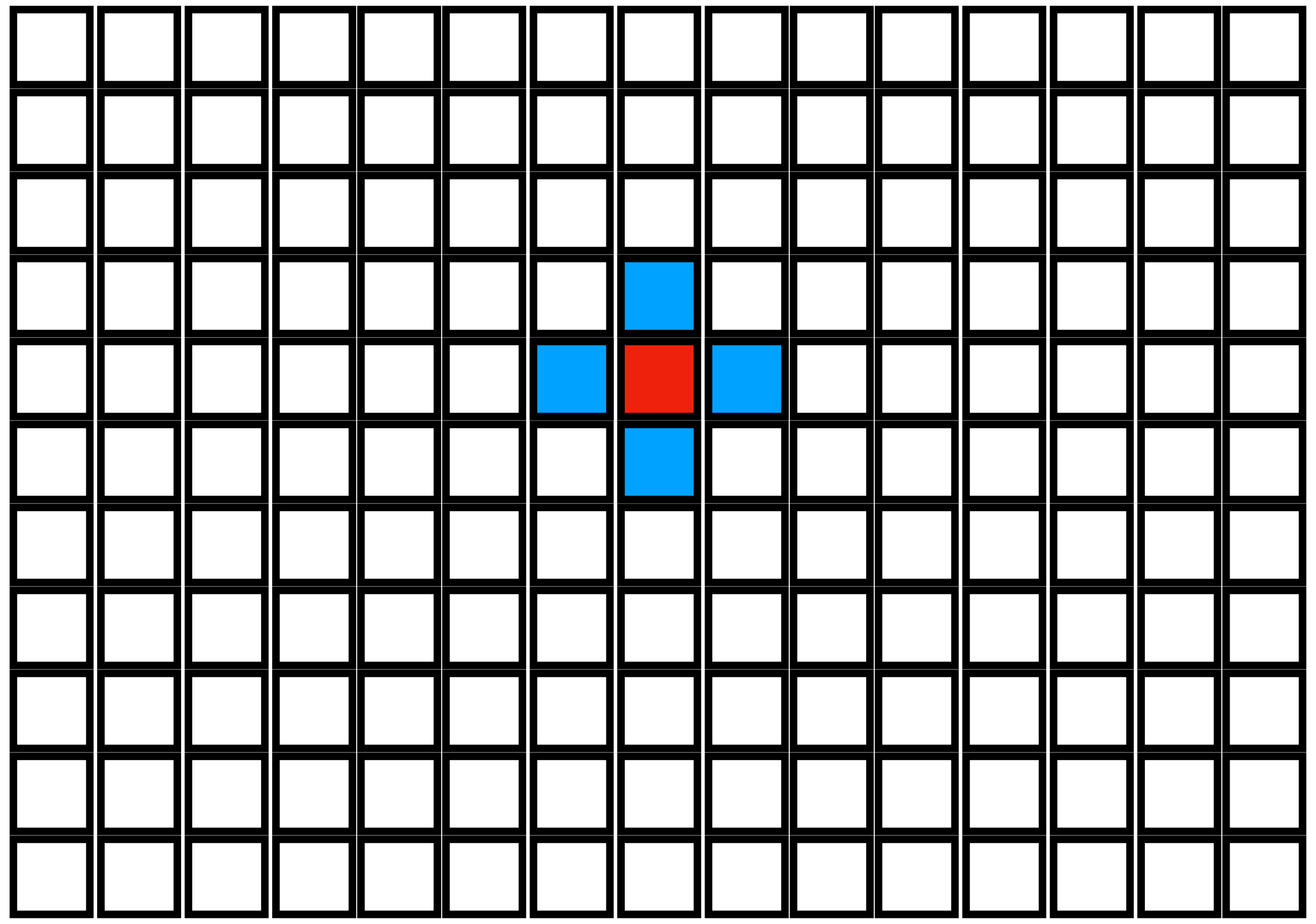

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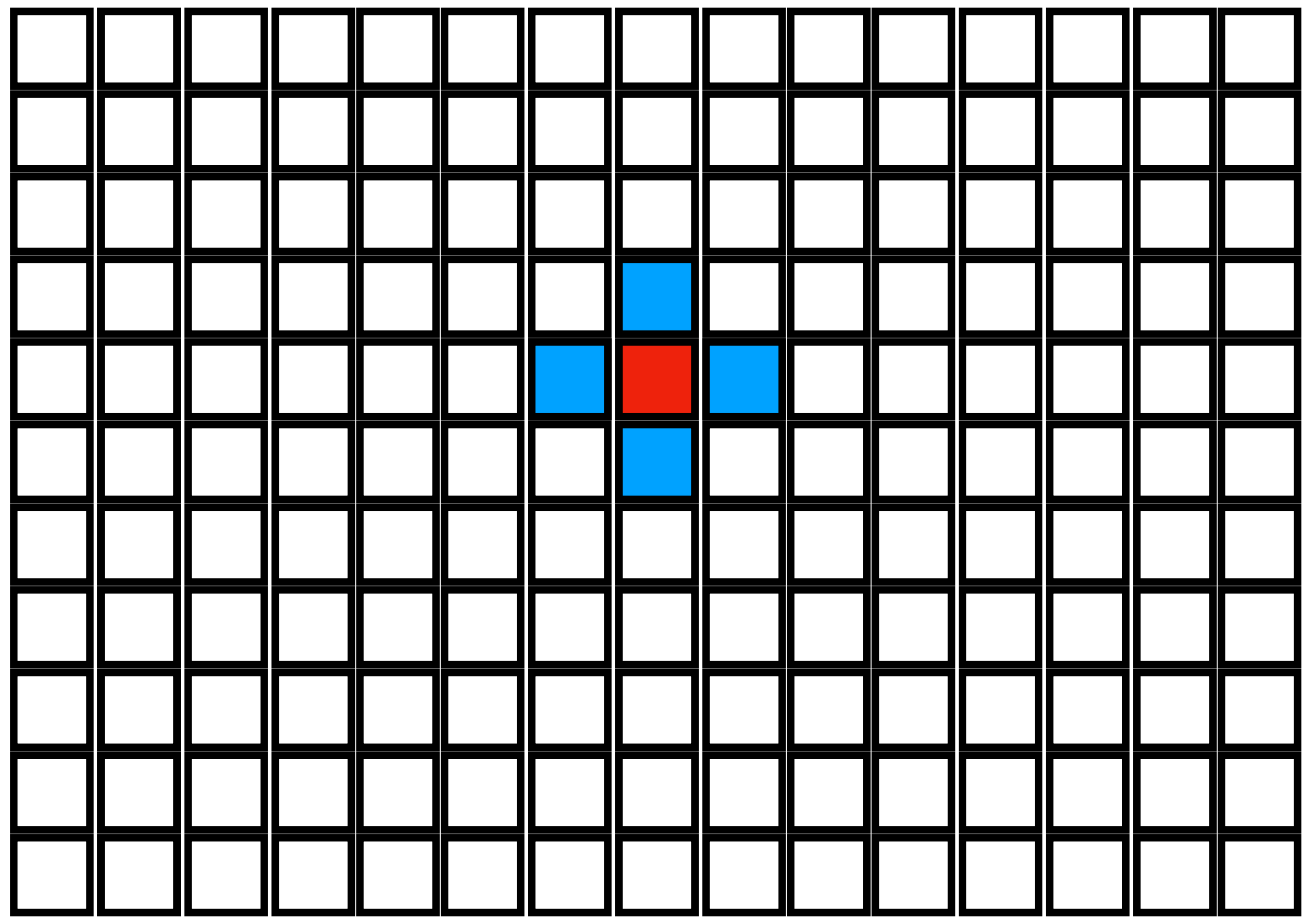

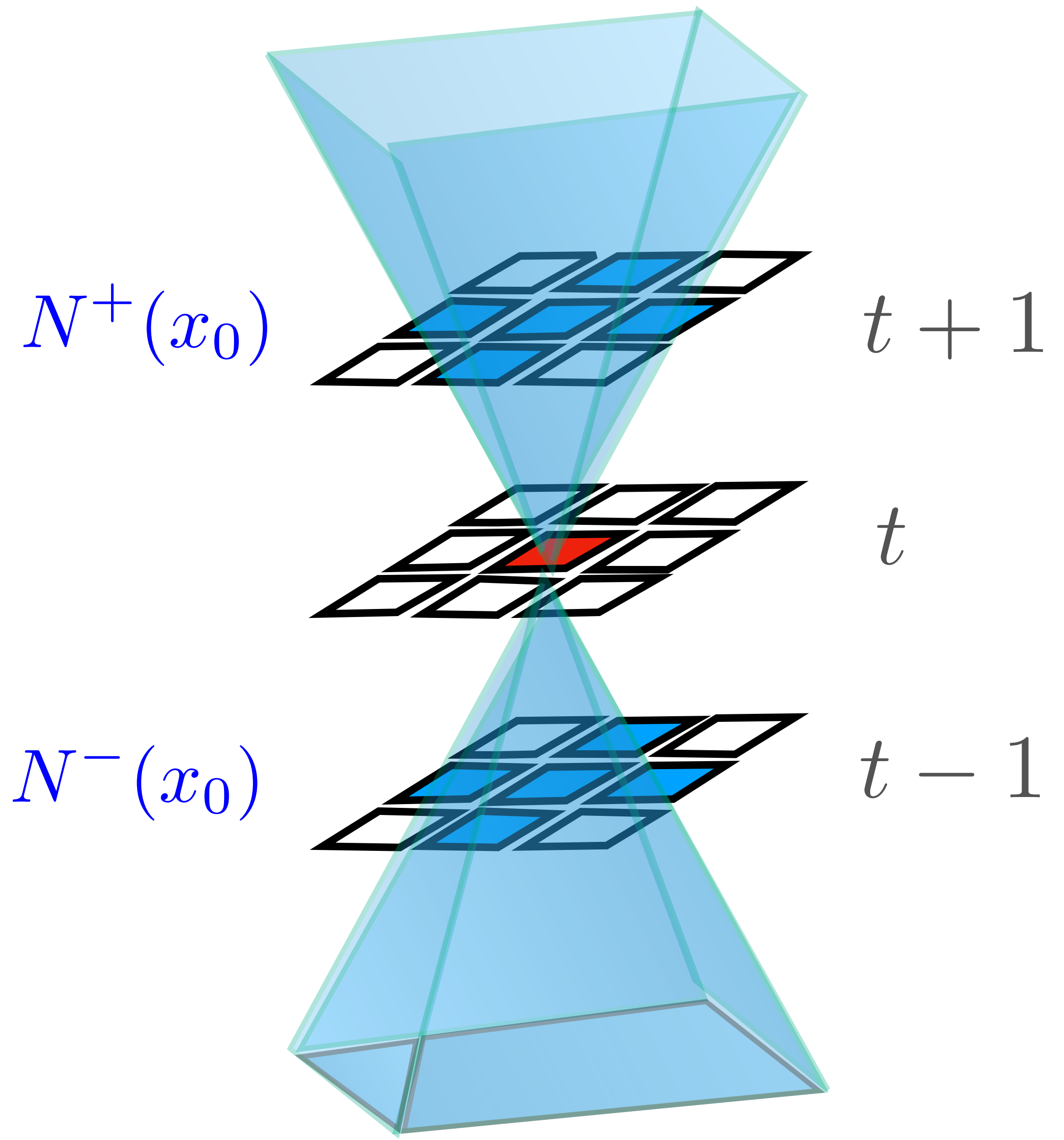

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Quantum cellular automata

Infinite case

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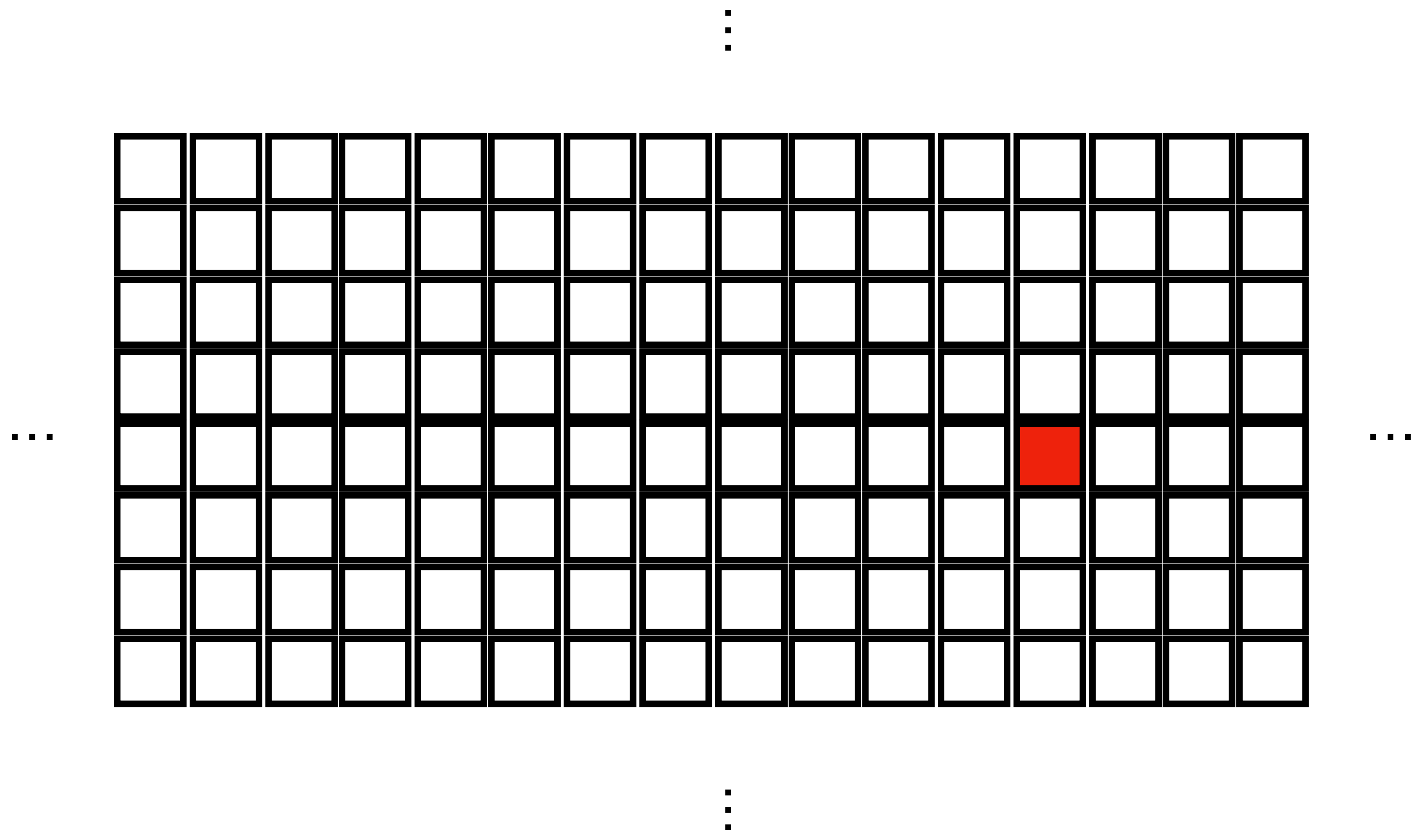

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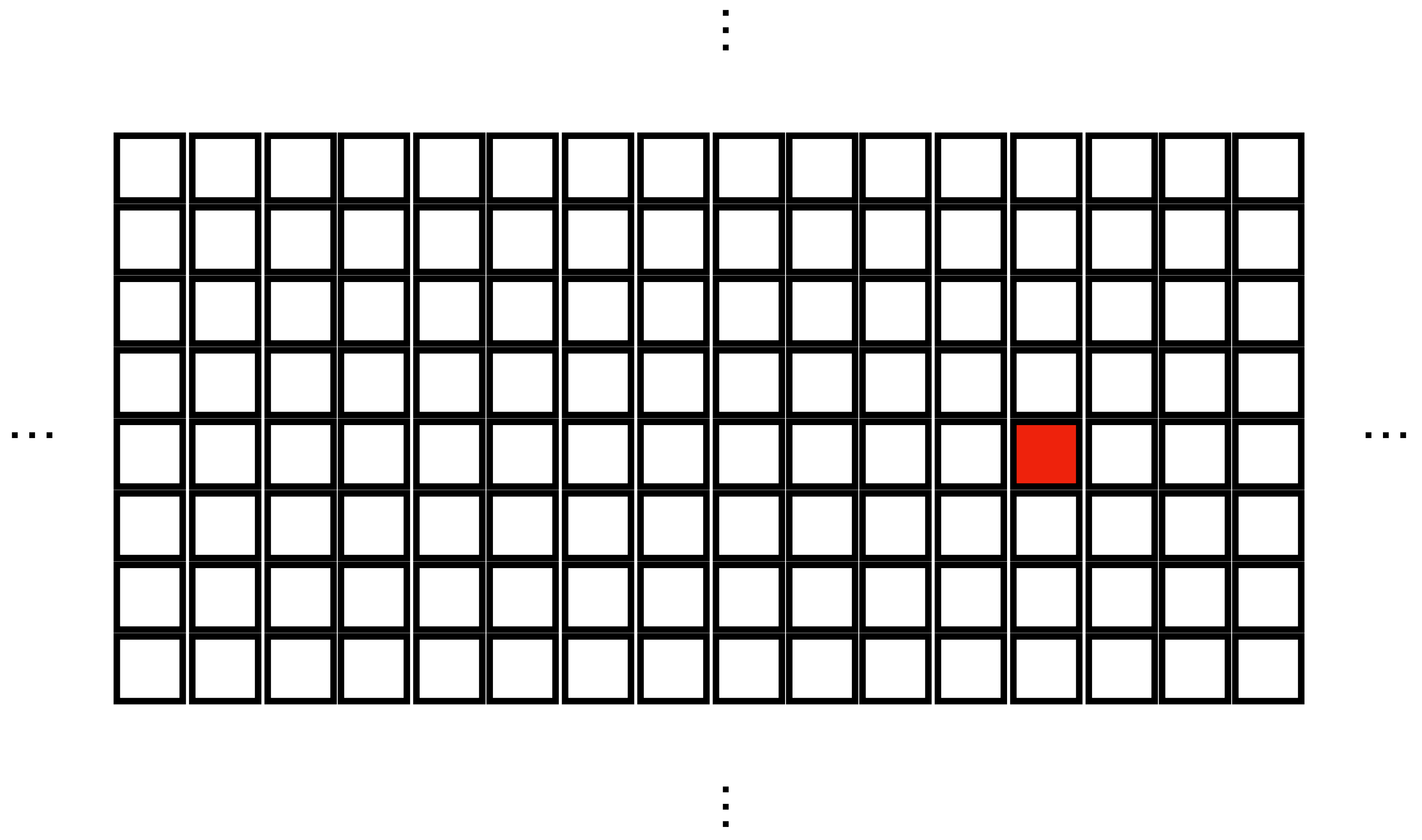

$= \mathcal{H}_x \leftrightarrow \mathsf{A}_x$

$$\boxed{\mathsf{A} = \text{`}\bigotimes_x\text{'}\,\mathsf{A}_x}$$

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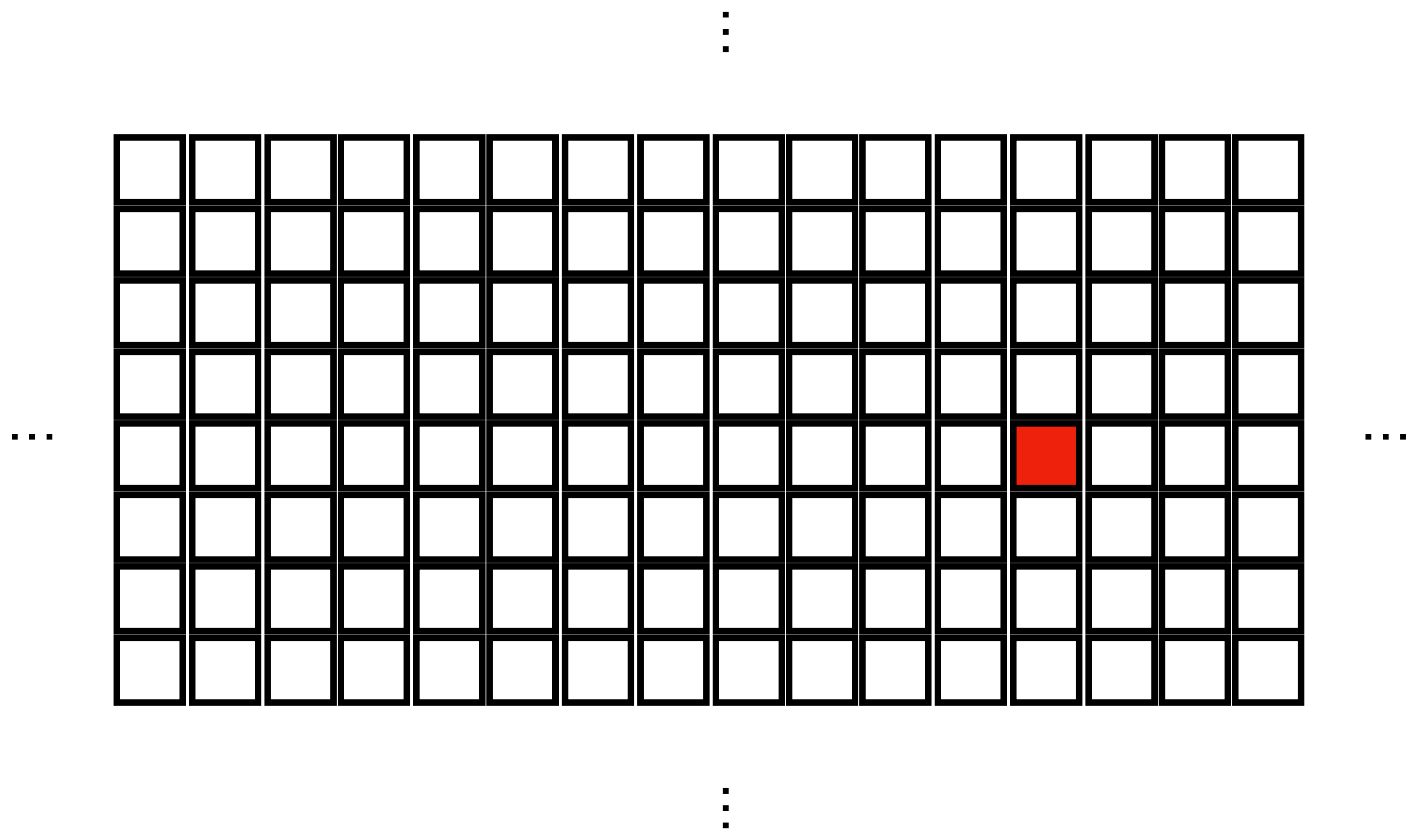
Infinite system

Inductive limit

 $= \mathcal{H}_x \leftrightarrow \mathsf{A}_x$



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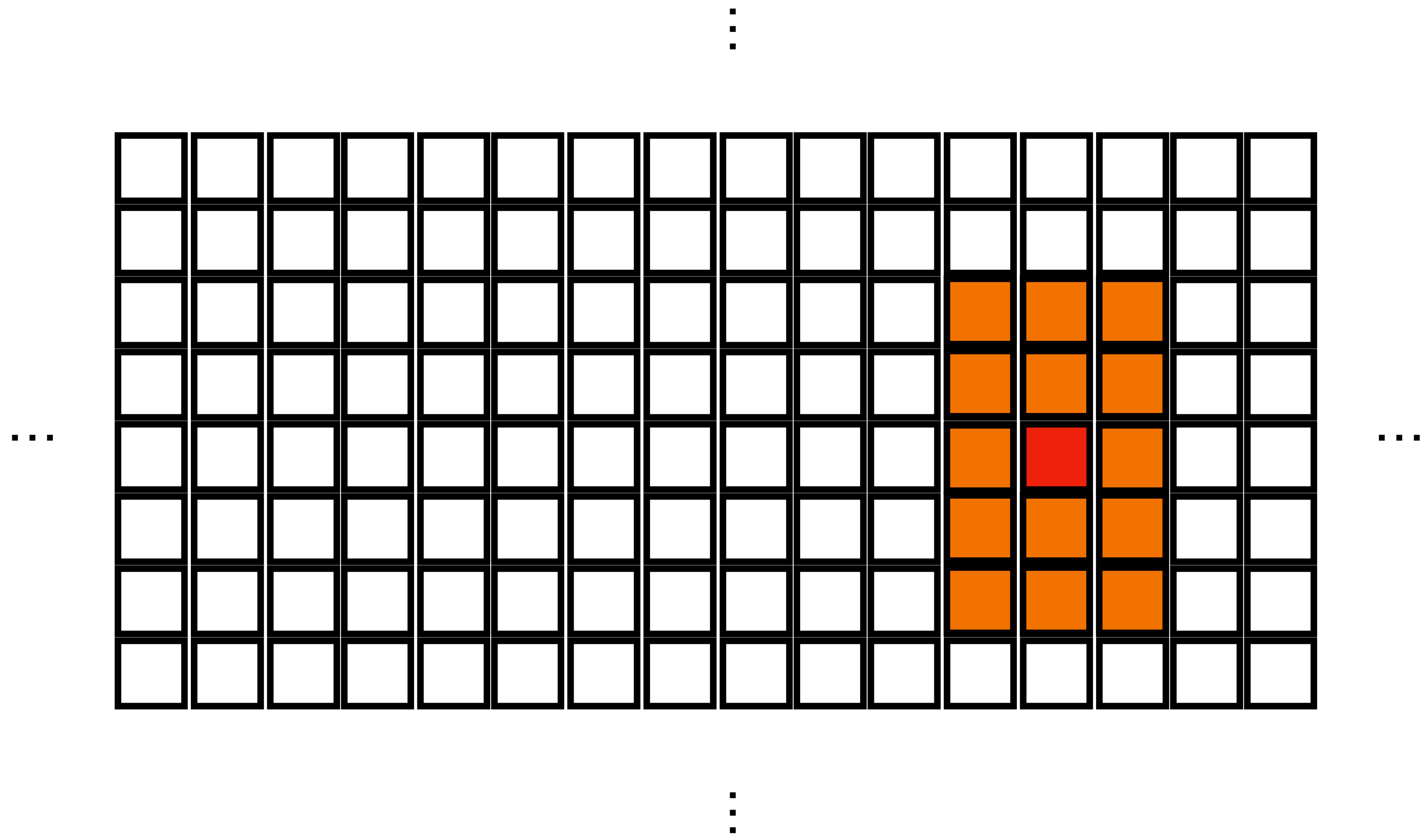

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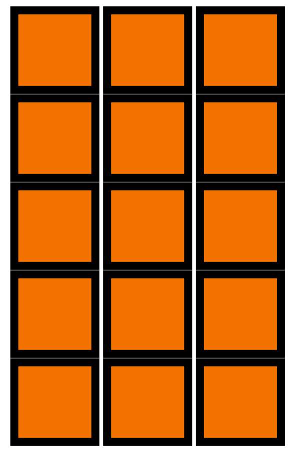
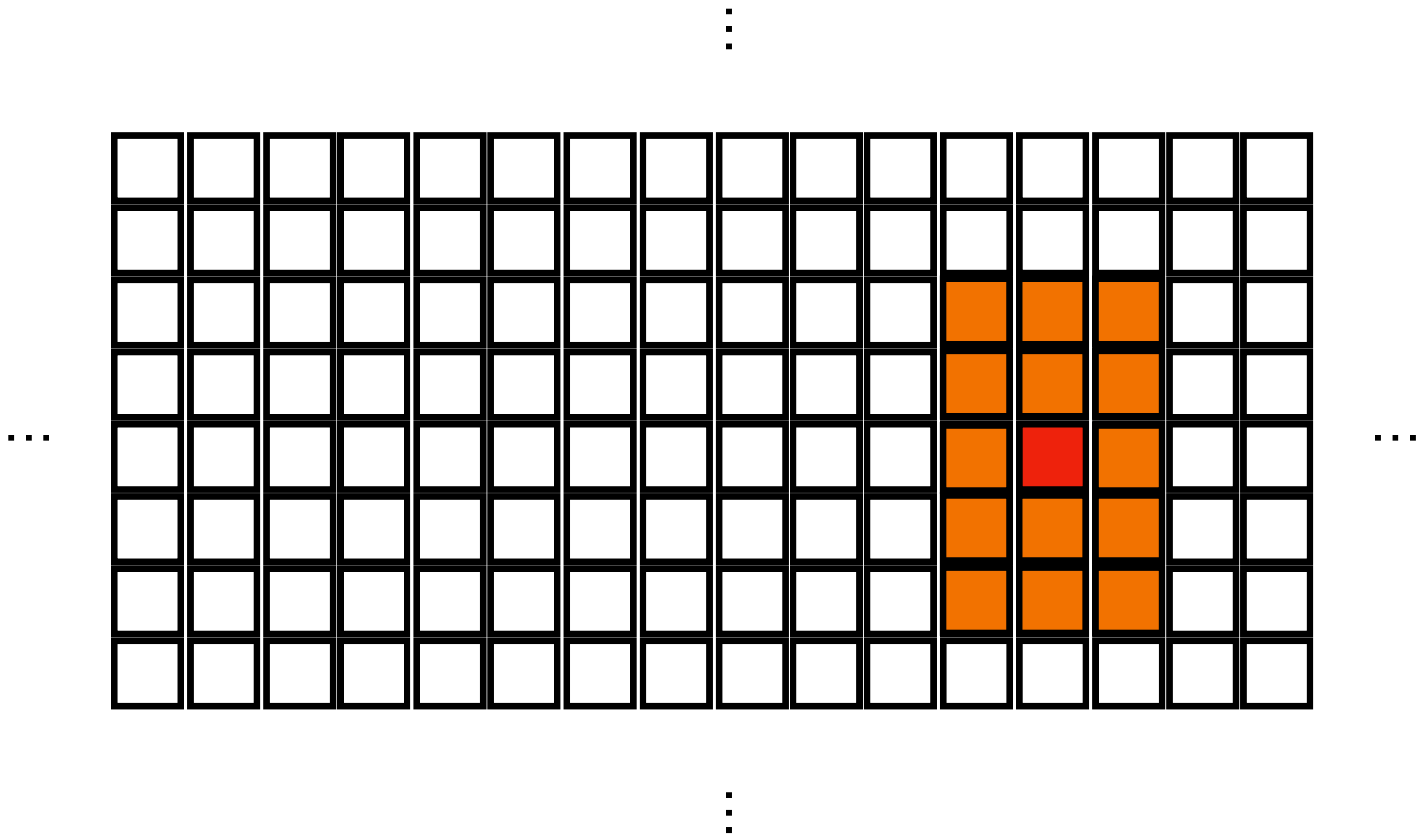


$$\mathsf{A}_R = \bigotimes_{x \in R} \mathsf{A}_x$$



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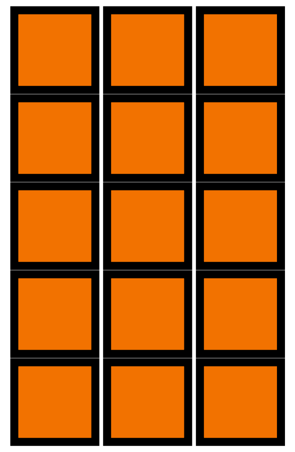

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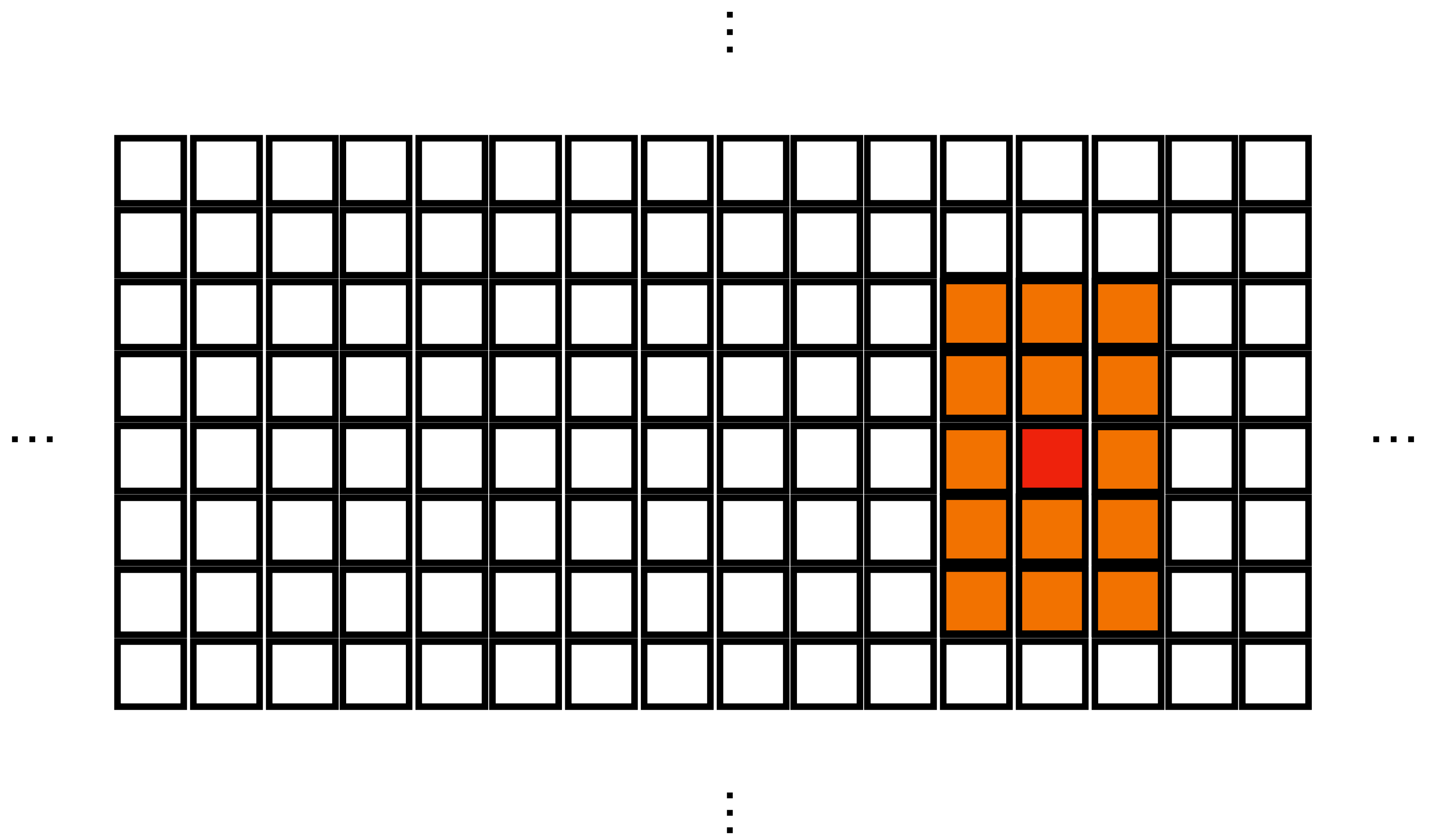

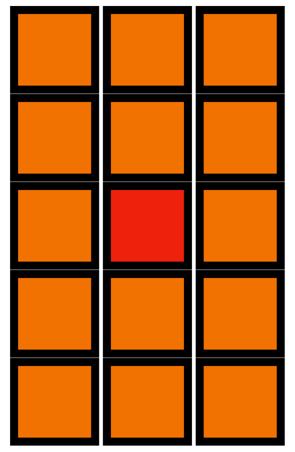

A_{x_0} ‘$\subseteq$’ A_R

Infinite system

Inductive limit

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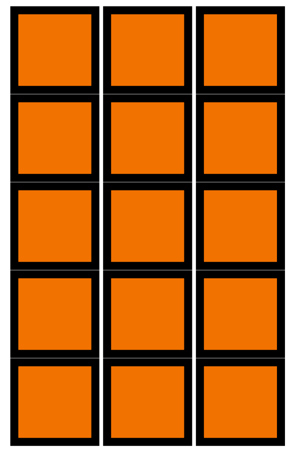

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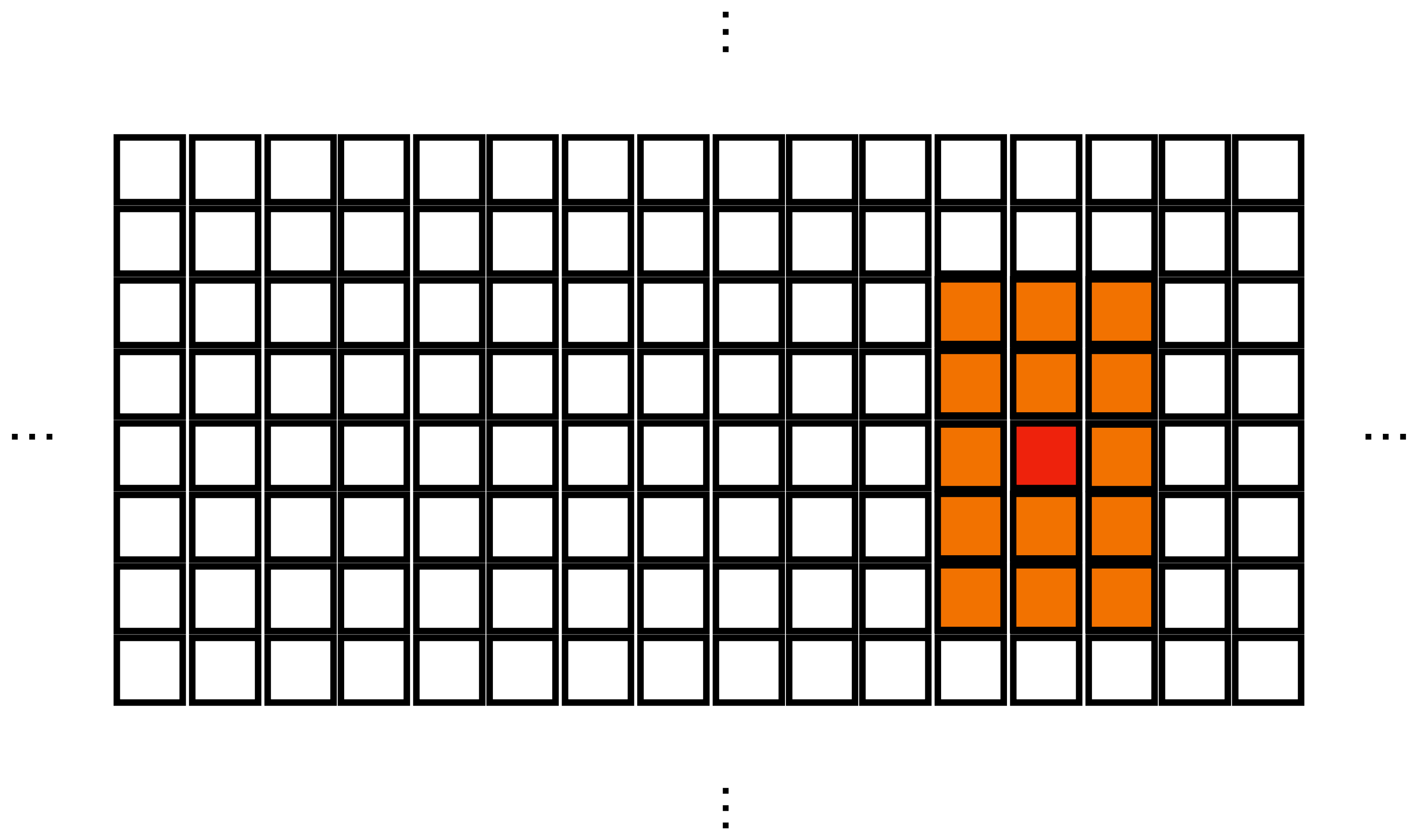
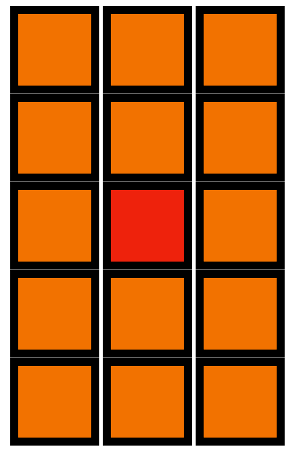


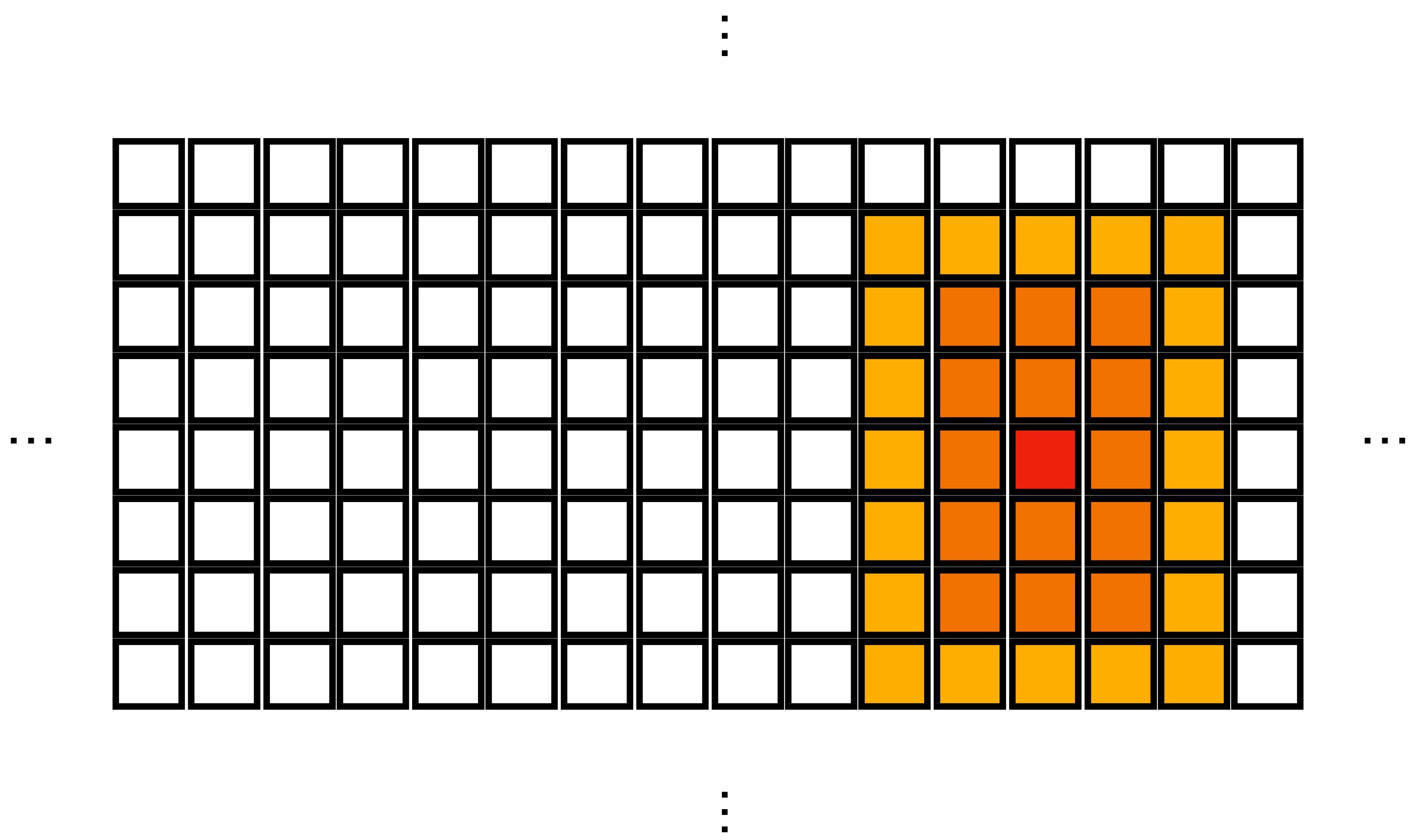


A_{x_0} ‘ $\subseteq$ ’A_R

$$f_{x_0,R} : \mathsf{A}_{x_0} \to \mathsf{A}_R$$
$$f_{x_0,R}(B) = B \otimes I_{R \setminus \{x_0\}}$$

Infinite system

Inductive limit



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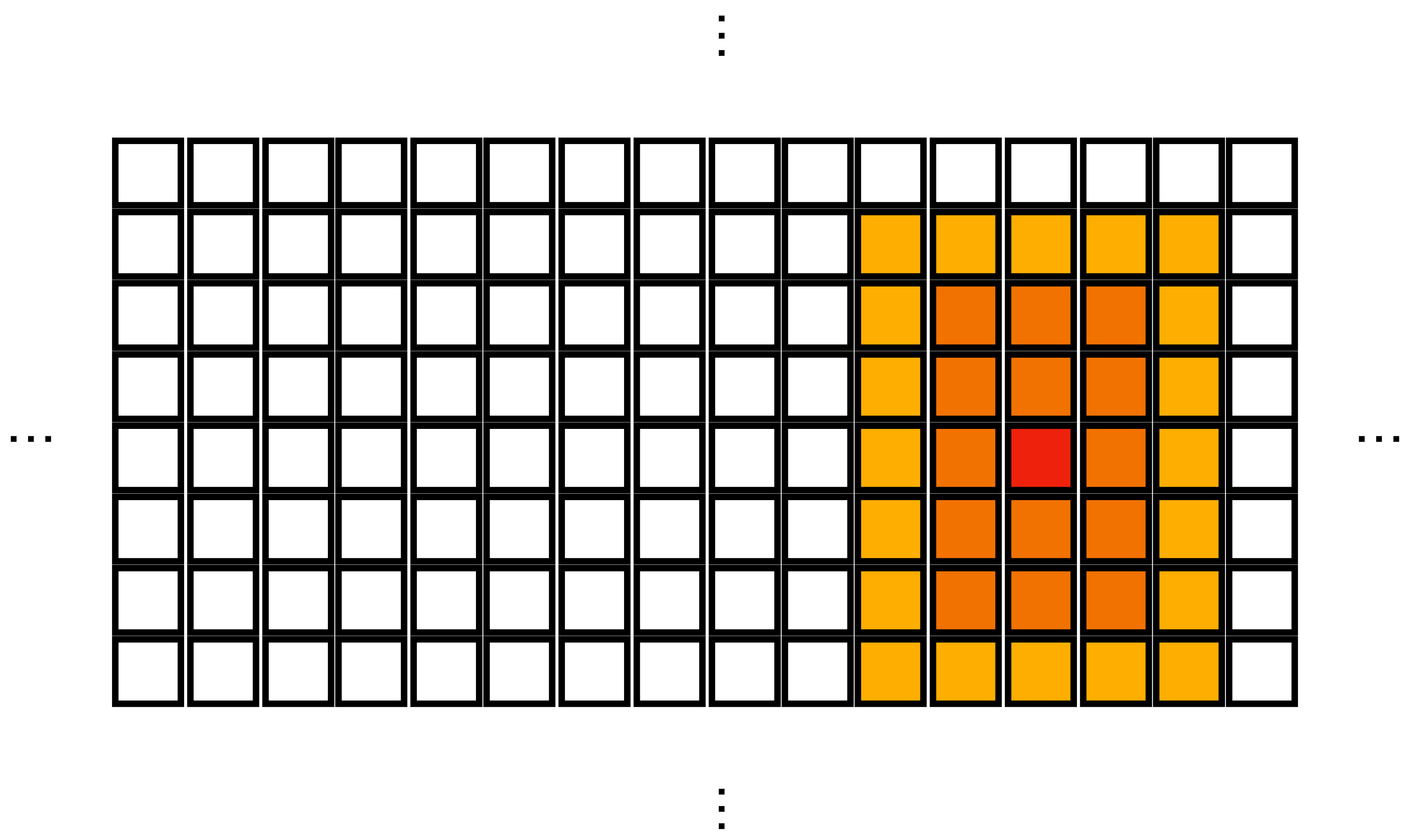

$R_0 \subseteq R_1 \subseteq R_2 \subseteq \dots$

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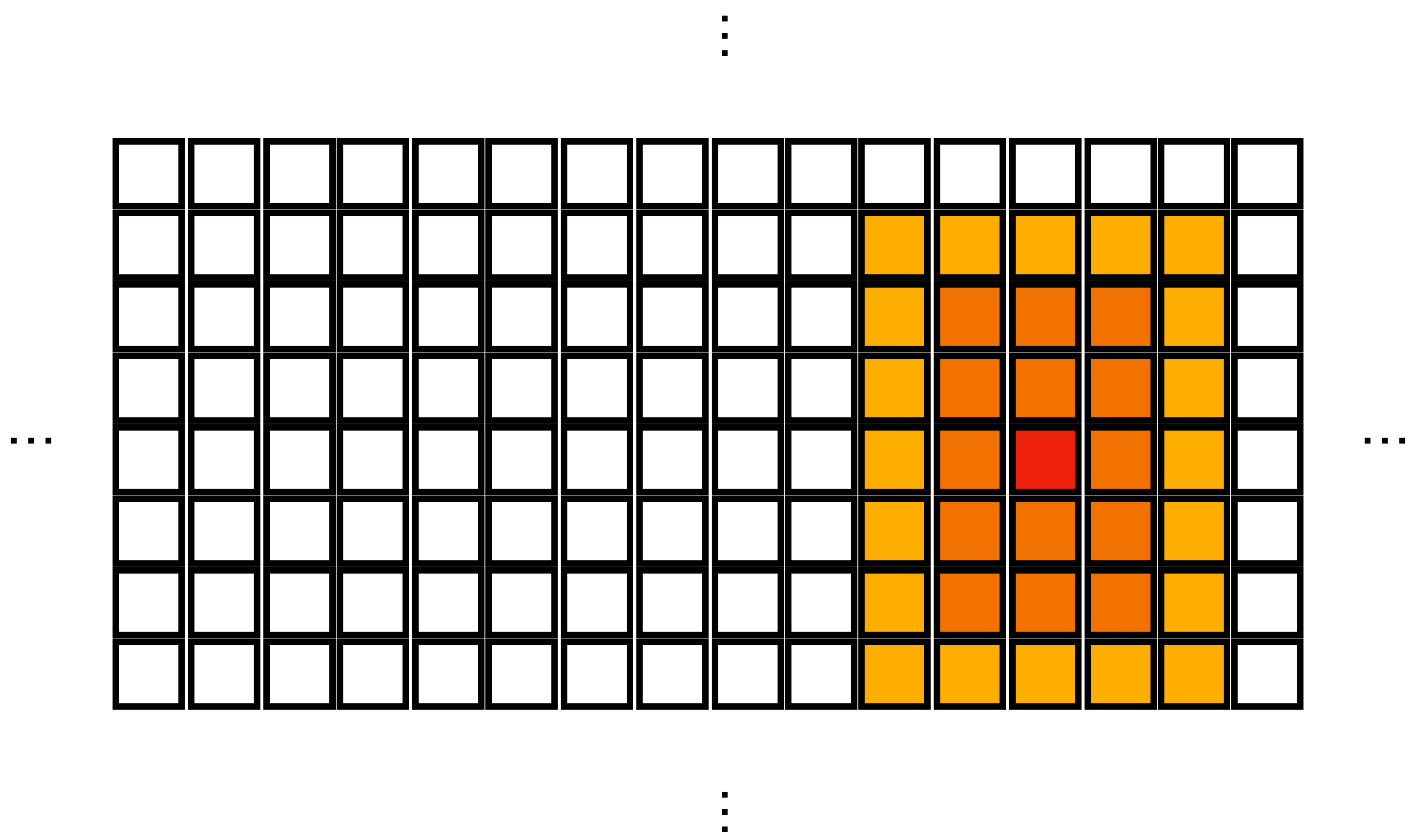

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$$f_{ii}(B) = B$$
$$f_{jk} \circ f_{ij} = f_{ik}$$

Infinite system

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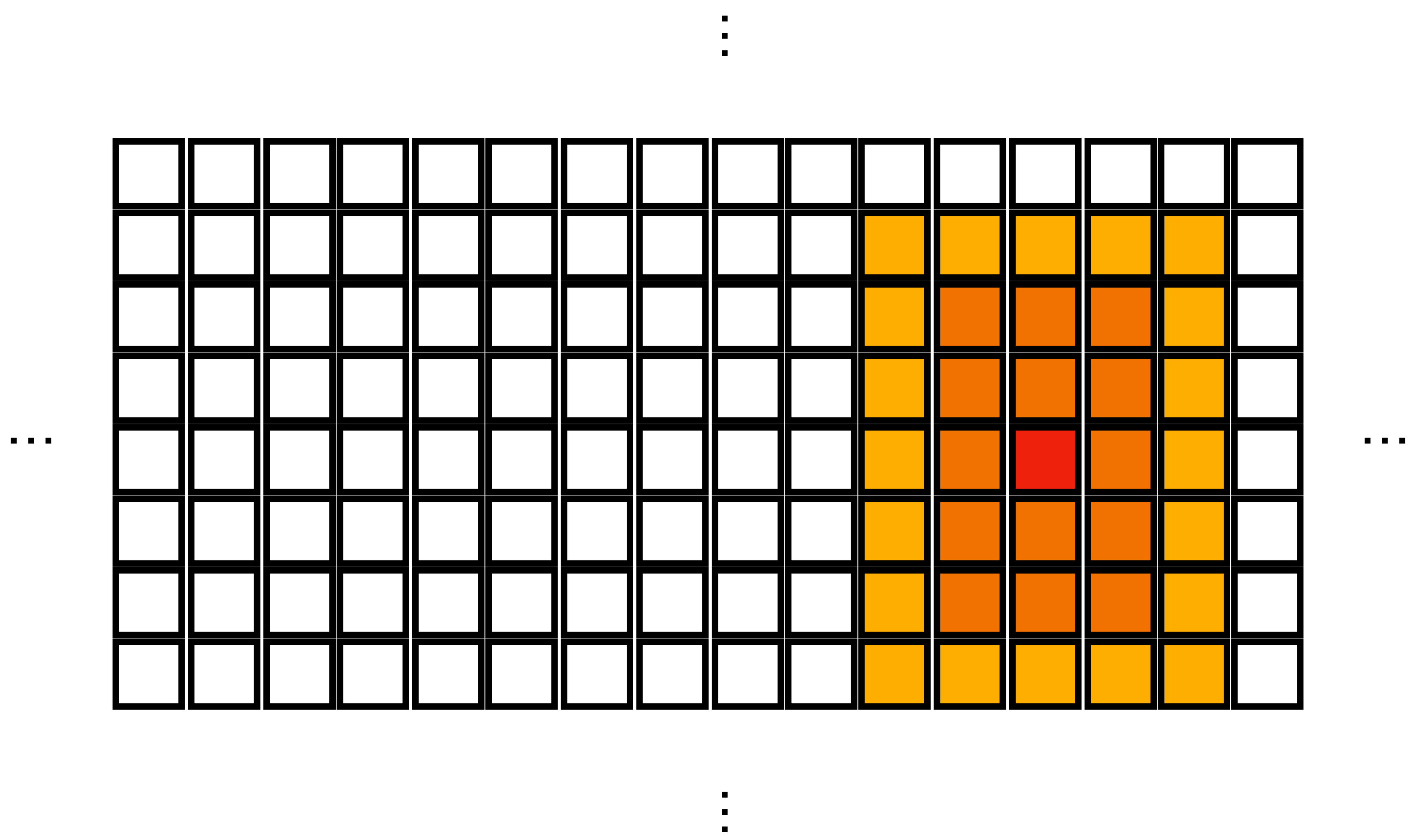

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$B_j \sim B_i \;$ if $\; \exists R_k \supseteq R_i, R_j \;$ s.t. $\; f_{ik}(B_i) = f_{jk}(B_j)$

$$\mathsf{A}_L := \bigsqcup_i \mathsf{A}_{R_i} / \sim$$

Infinite system

Topological limit

- Inductive limit: all local operators on arbitrarily large but finite regions

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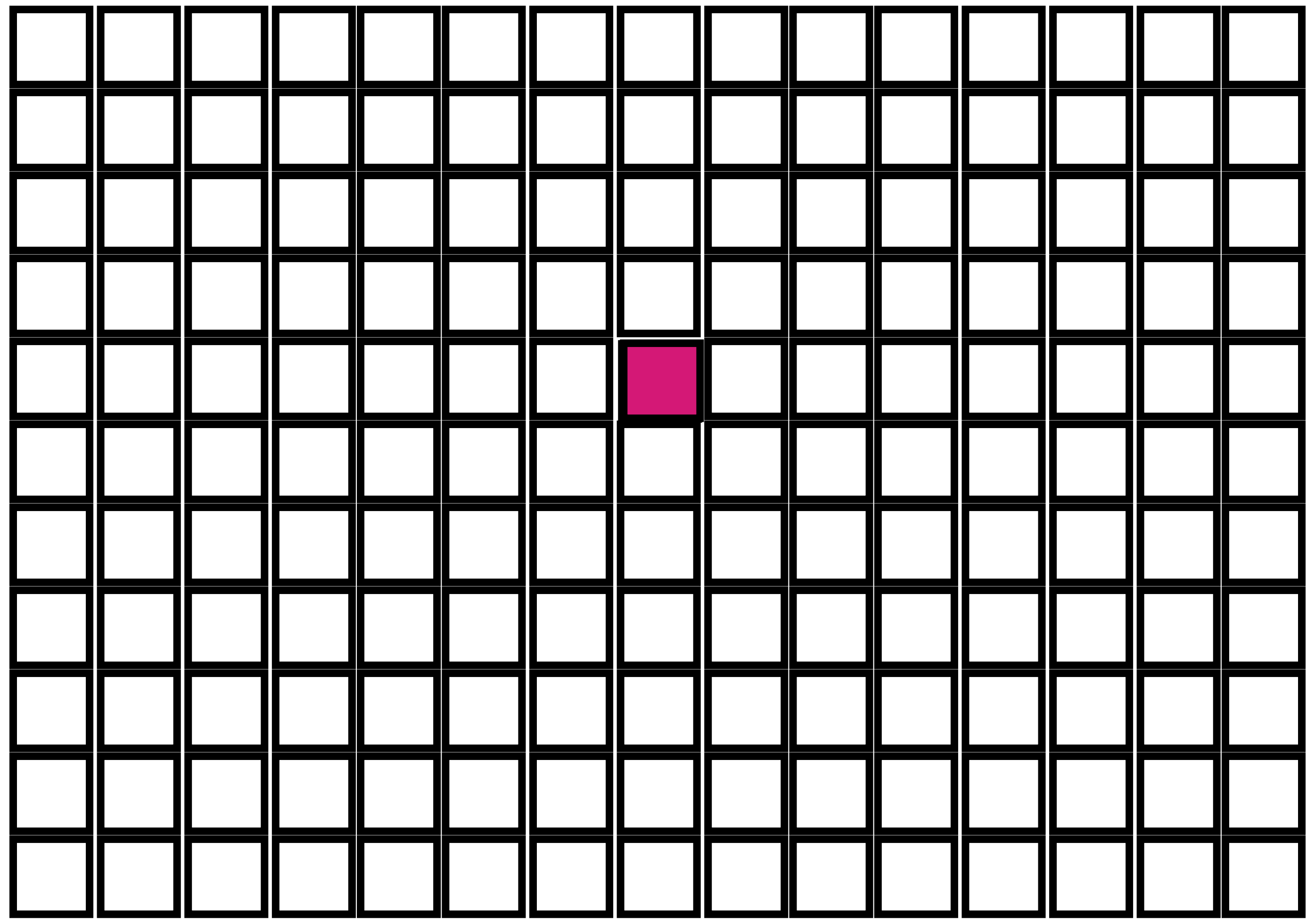
- A is the quasi-local algebra

Quantum Cellular Automaton

Infinite case

- The following definition was given for QCA on $\mathbb{Z}^d$



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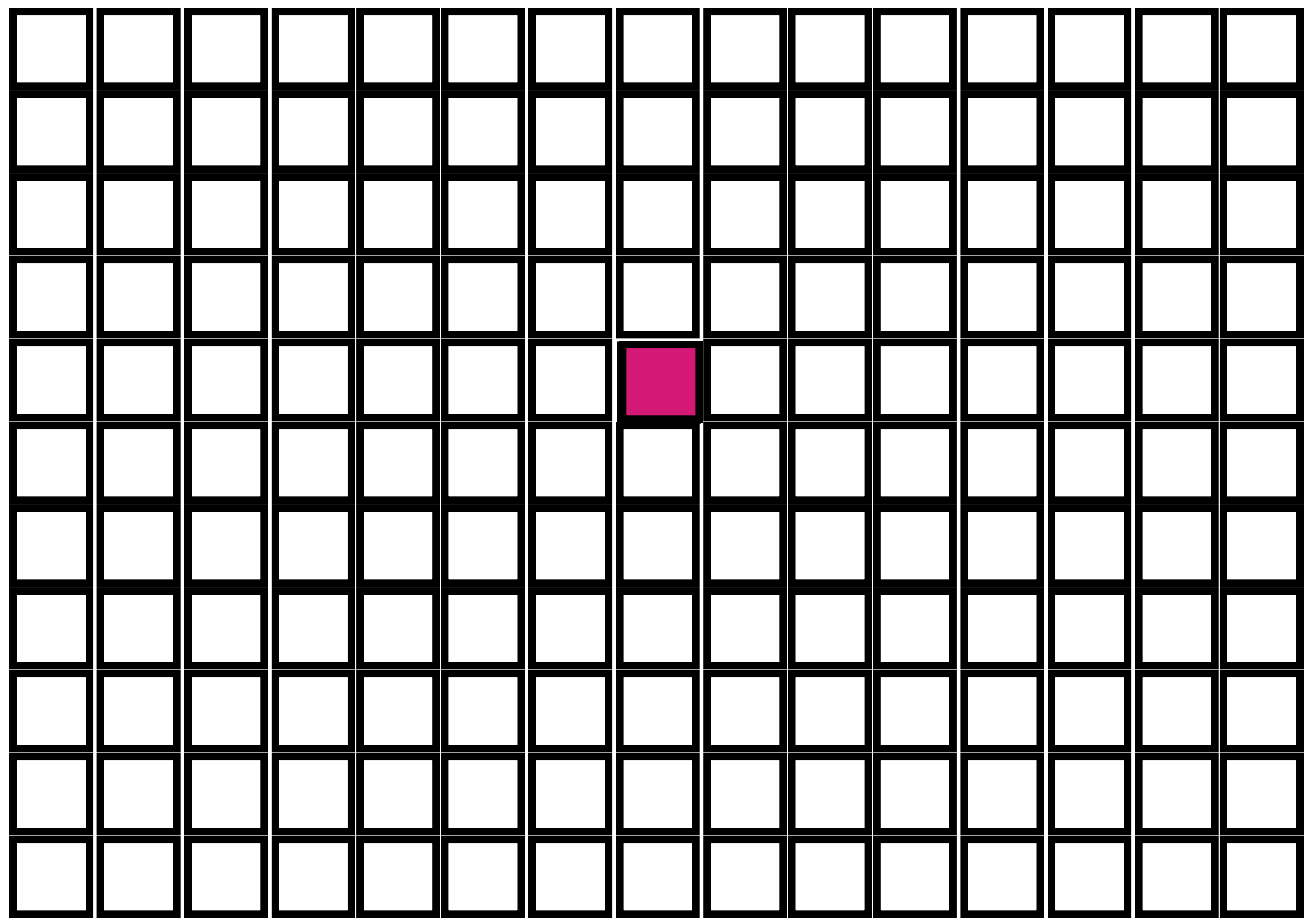

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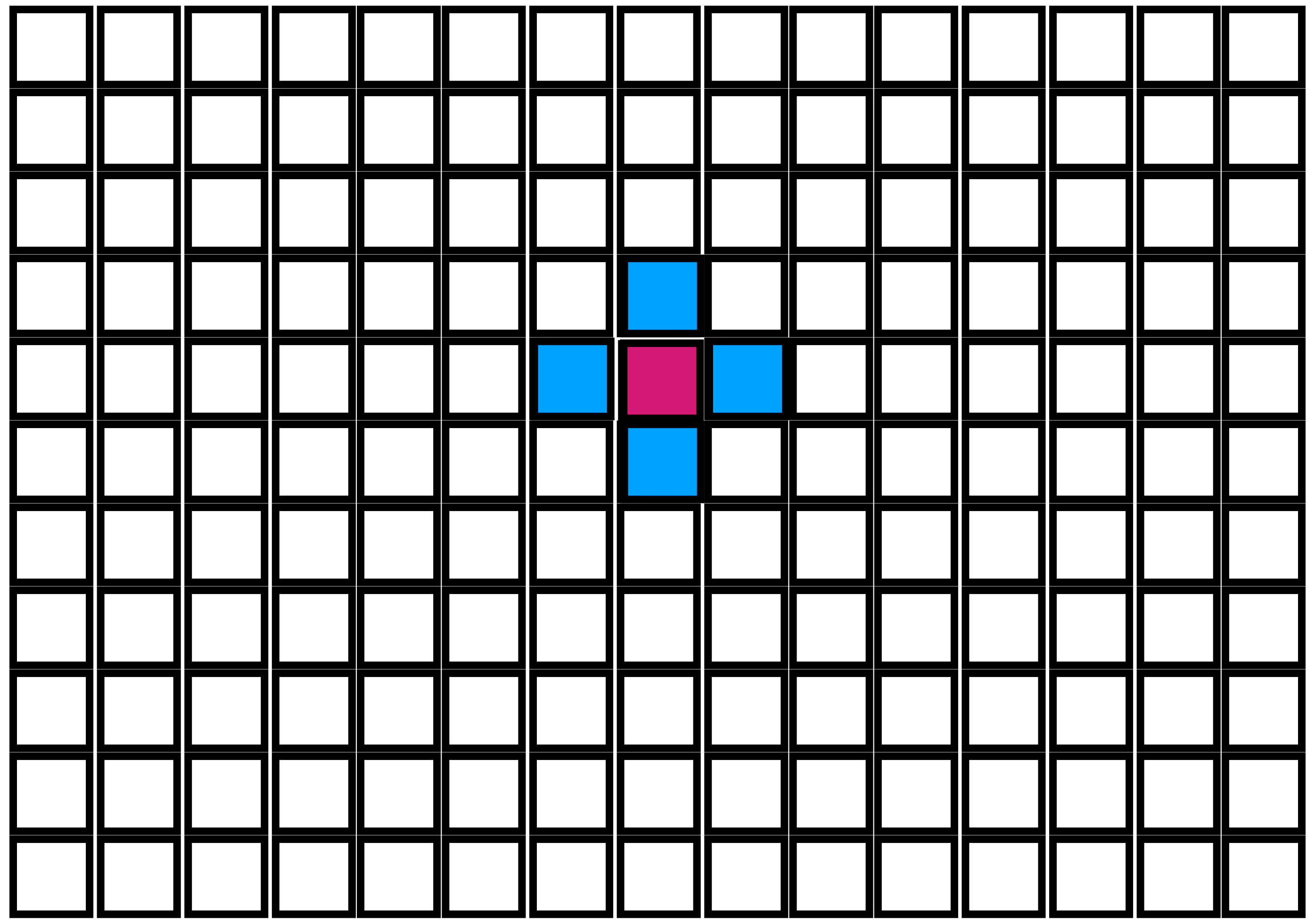

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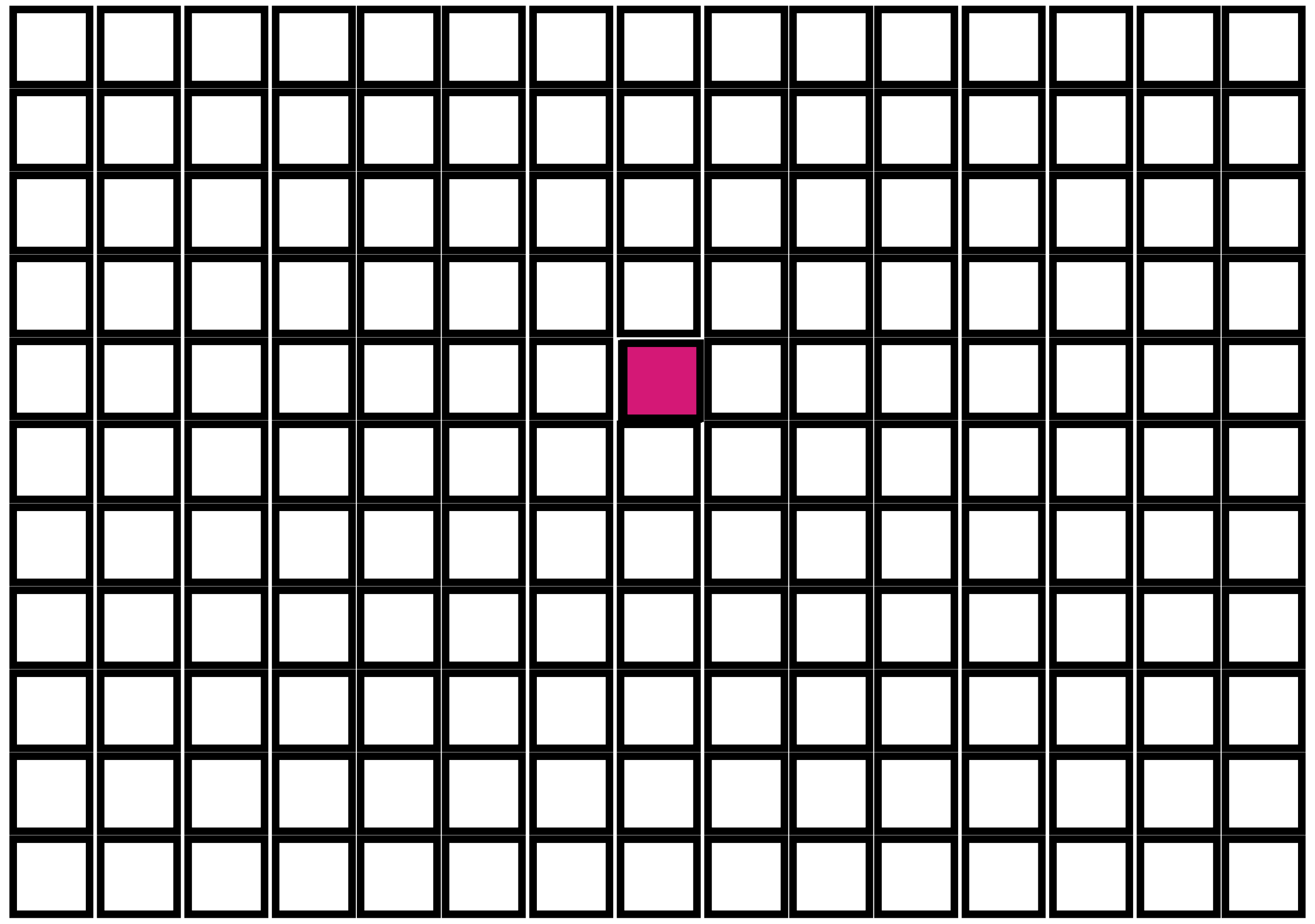
$$\mathcal{V}[\mathcal{T}_z(A_x)] = \mathcal{T}_z[\mathcal{V}(A_x)]$$



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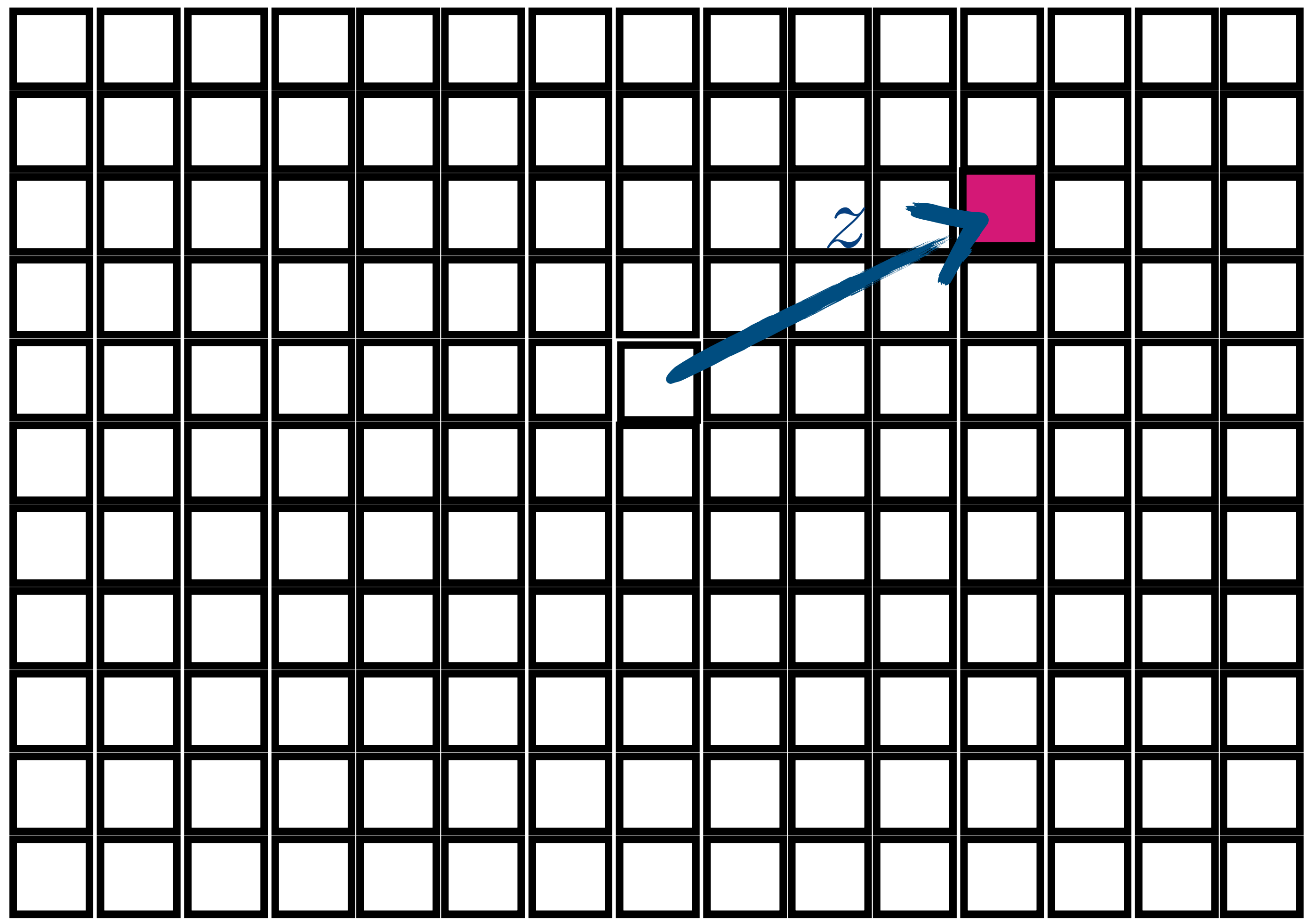

$$\mathcal{T}_z(A_x) = A_{z+x}$$

B. Schumacher and R.F. Werner, arXiv:quant-ph/040517

Local update rule

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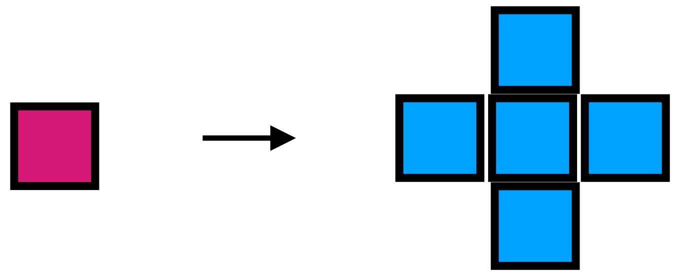

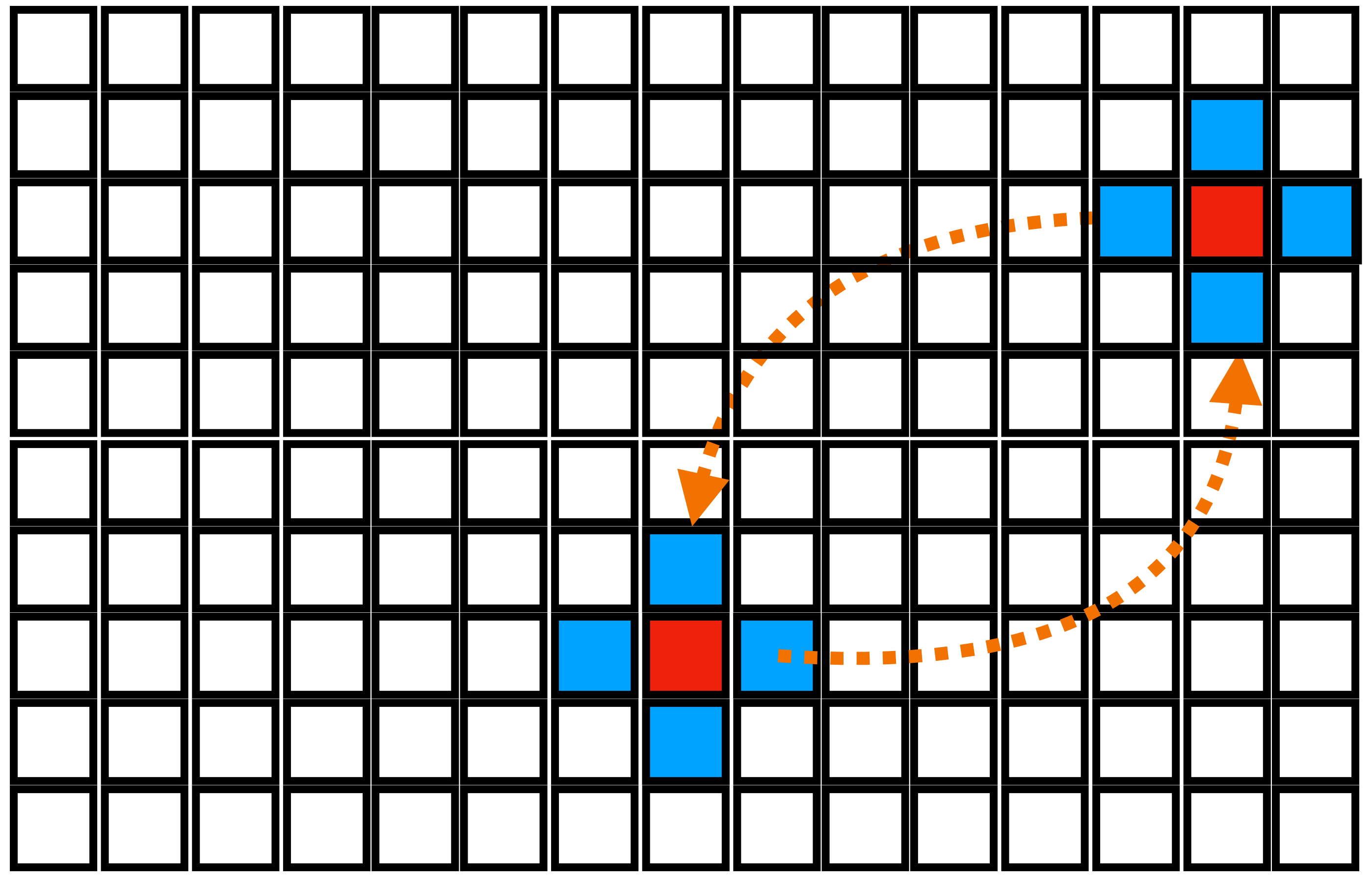

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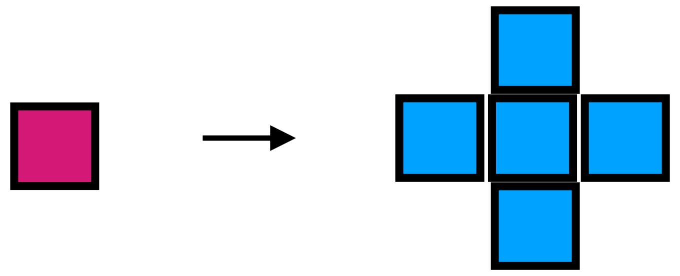
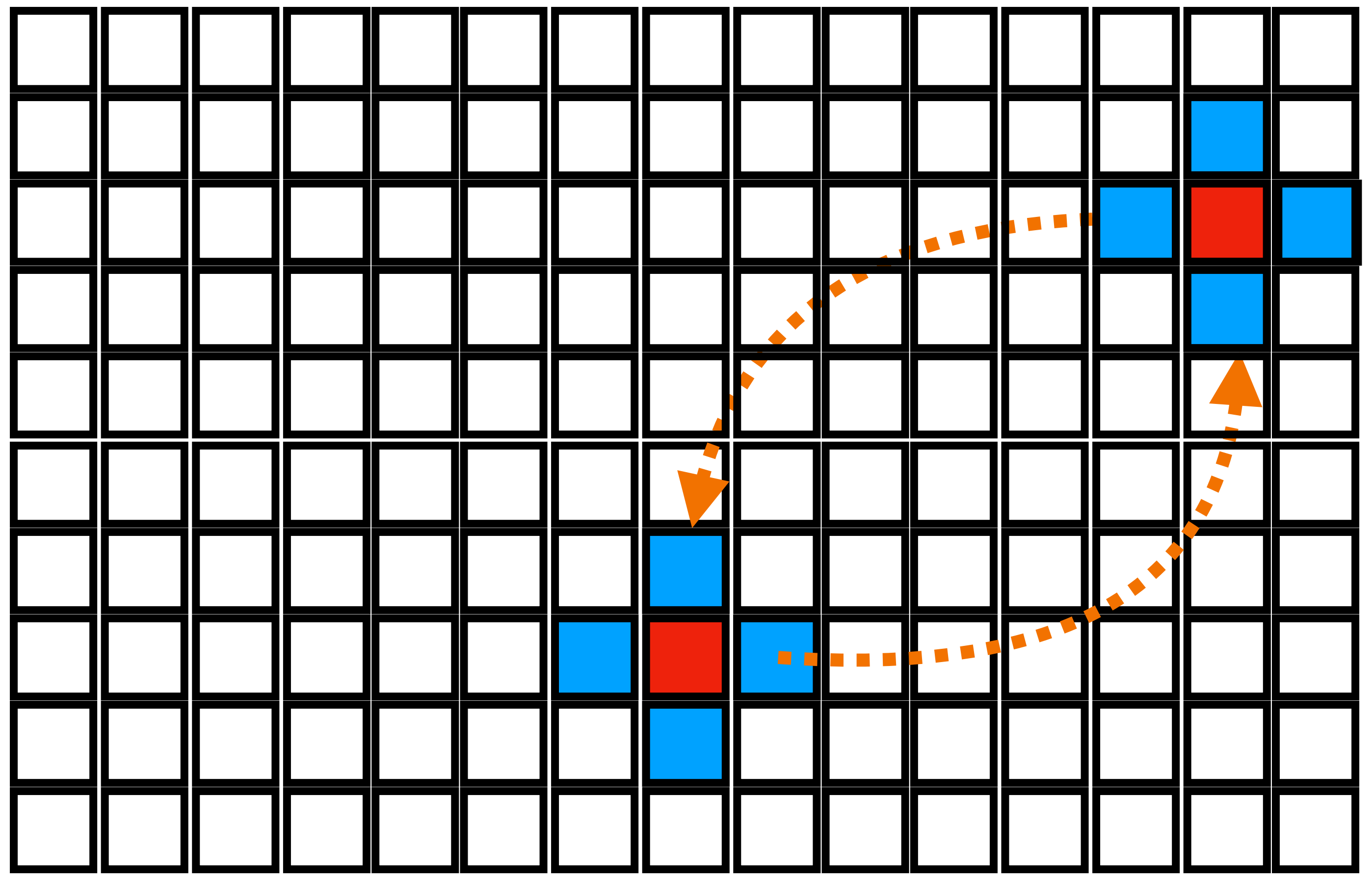


- The local rule at x=0 completely specifies the QCA

$$\mathcal{V}(B_x) = \mathcal{T}_x\{\mathcal{V}[\mathcal{T}_x^{-1}(B_x)]\}$$
$$= \mathcal{T}_x\{\mathcal{V}_0[\mathcal{T}_x^{-1}(B_x)]\}$$



B. Schumacher and R.F. Werner, arXiv:quant-ph/040517

Local update rule

- Theorem: every homomorphism of a full matrix algebra has the form

$$\mathcal{V}(A) = U(I \otimes A)U^\dagger$$

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$$\mathscr{V}(B_R) = \sum_i \prod_{x\in R} \mathcal{V}_x(B_x^{(i)})$$

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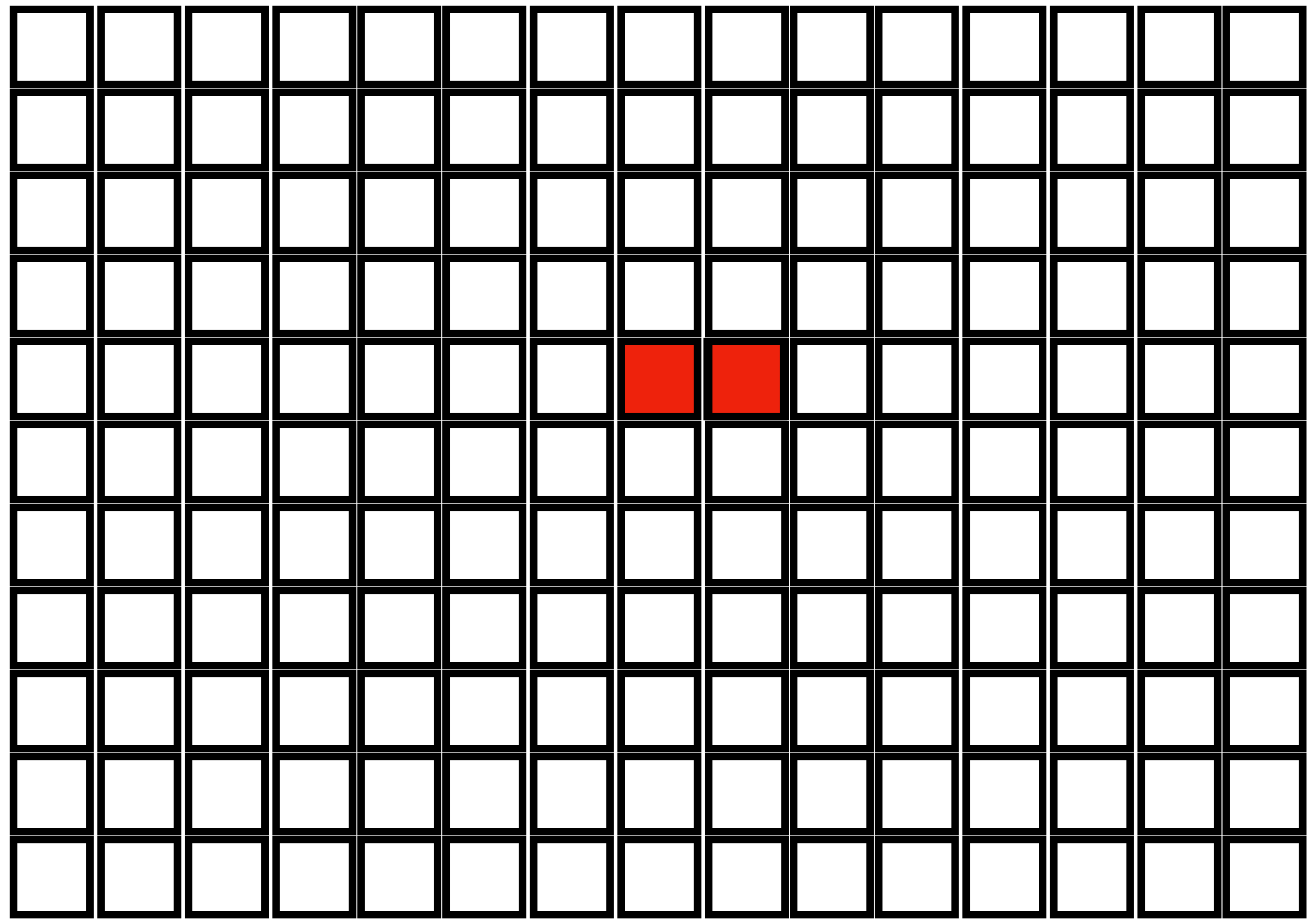
- Isometry: image of limits (of the quasi-local algebra) is determined

B. Schumacher and R.F. Werner, arXiv:quant-ph/040517

Characterisation of local rules

- Necessarily $[\mathcal{V}_x(B_x), \mathcal{V}_y(B_y)] = 0$



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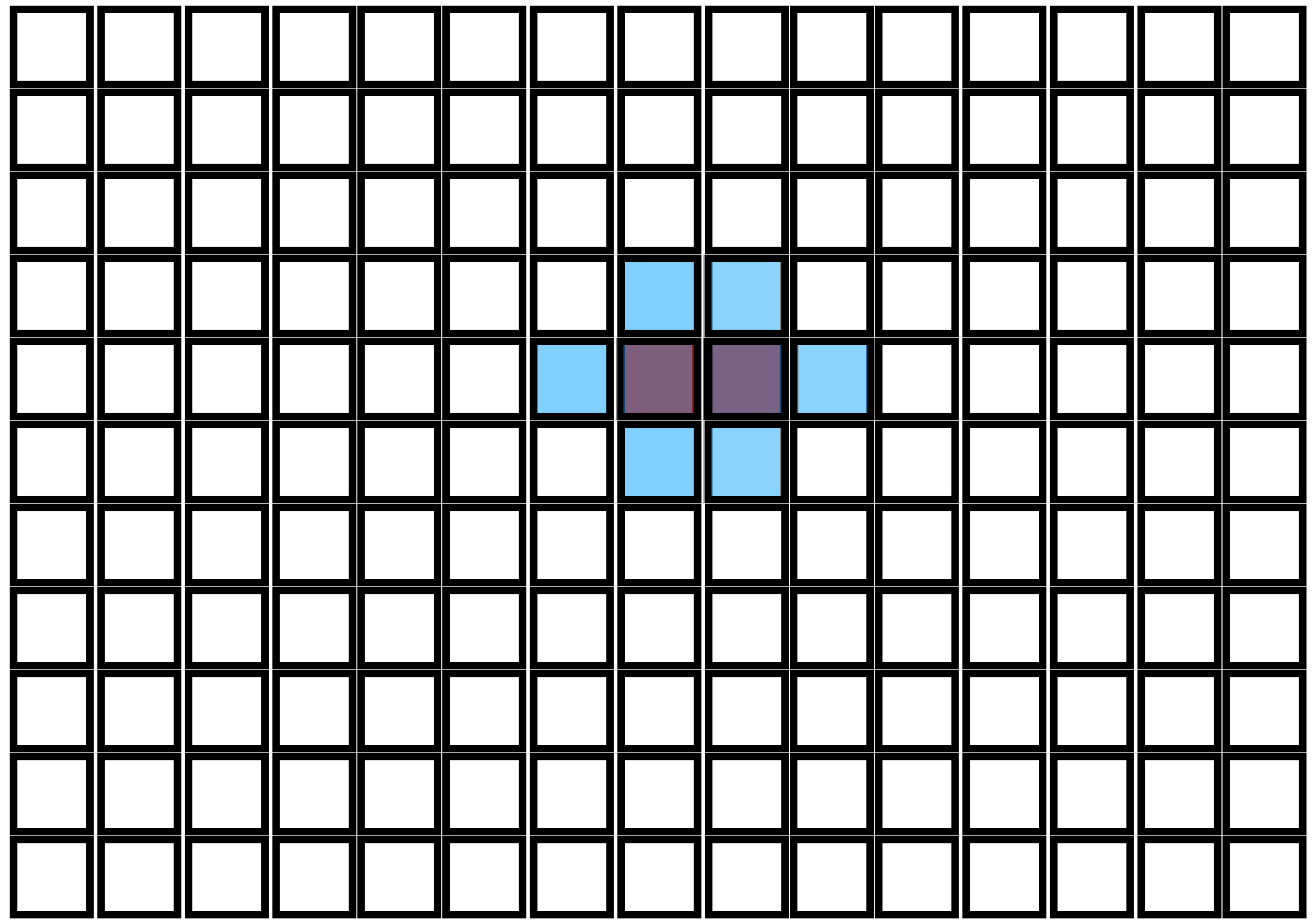

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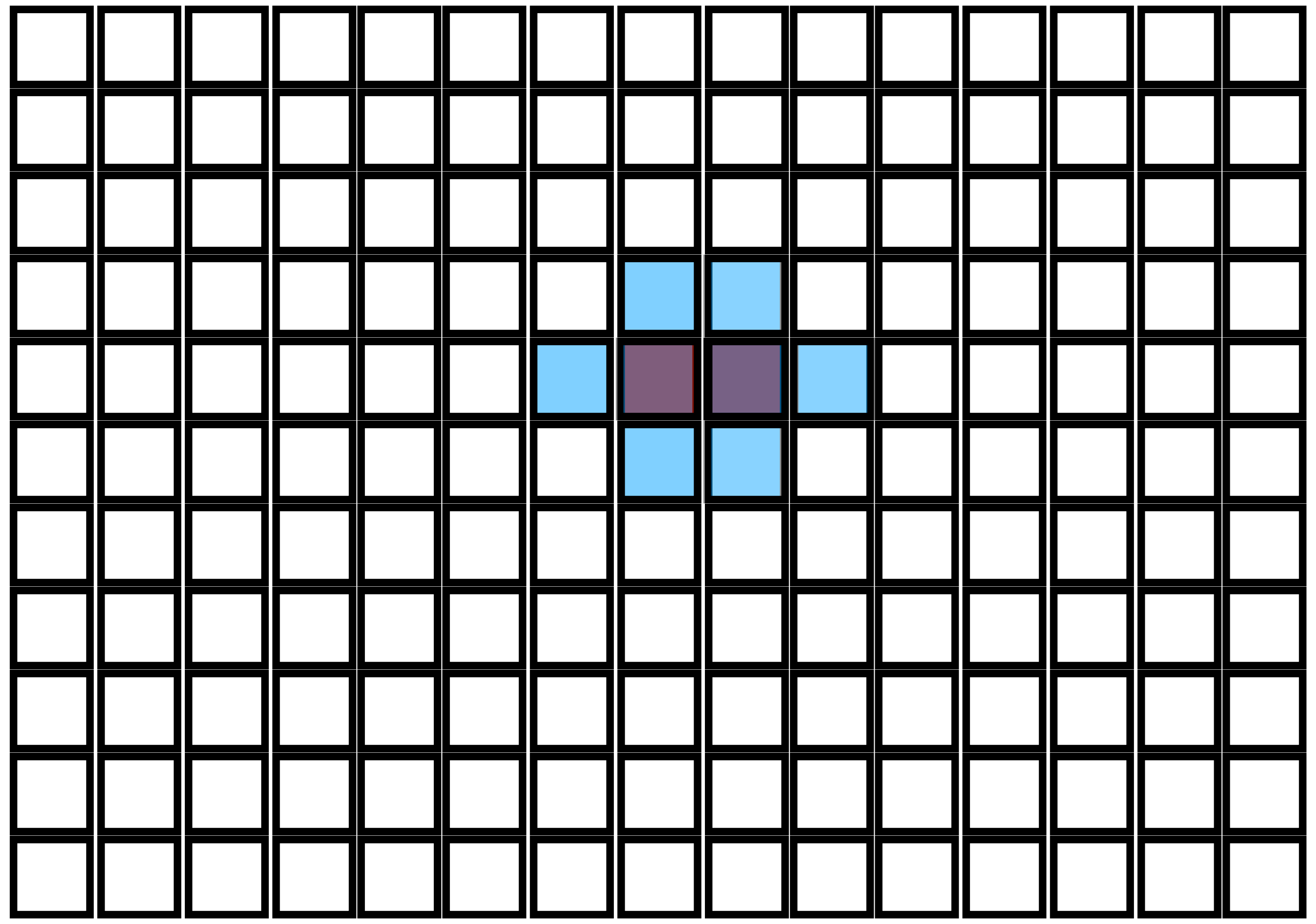
Characterisation of local rules

- Necessarily $[\mathcal{V}_x(B_x), \mathcal{V}_y(B_y)] = 0$
- Also the converse is true



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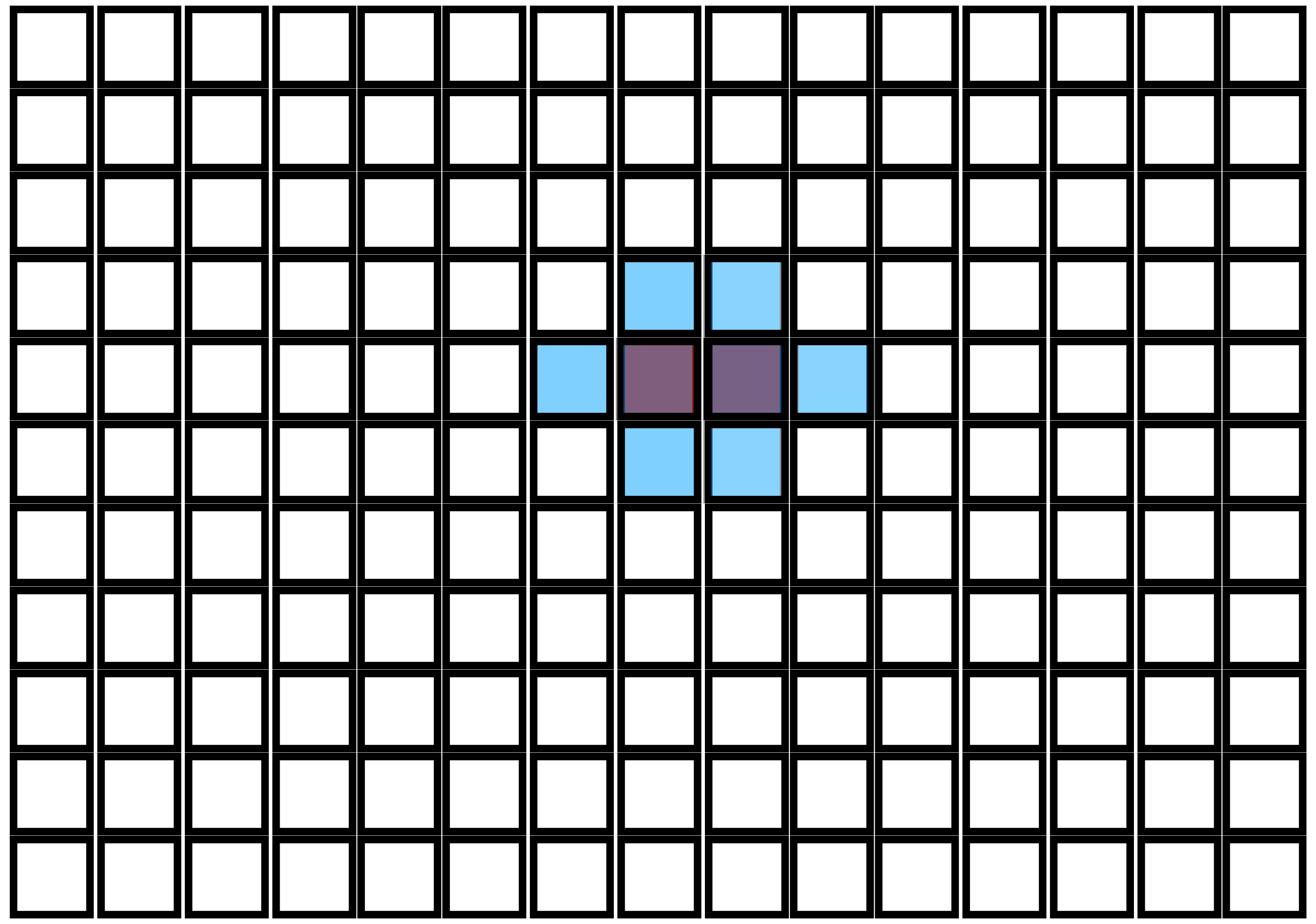
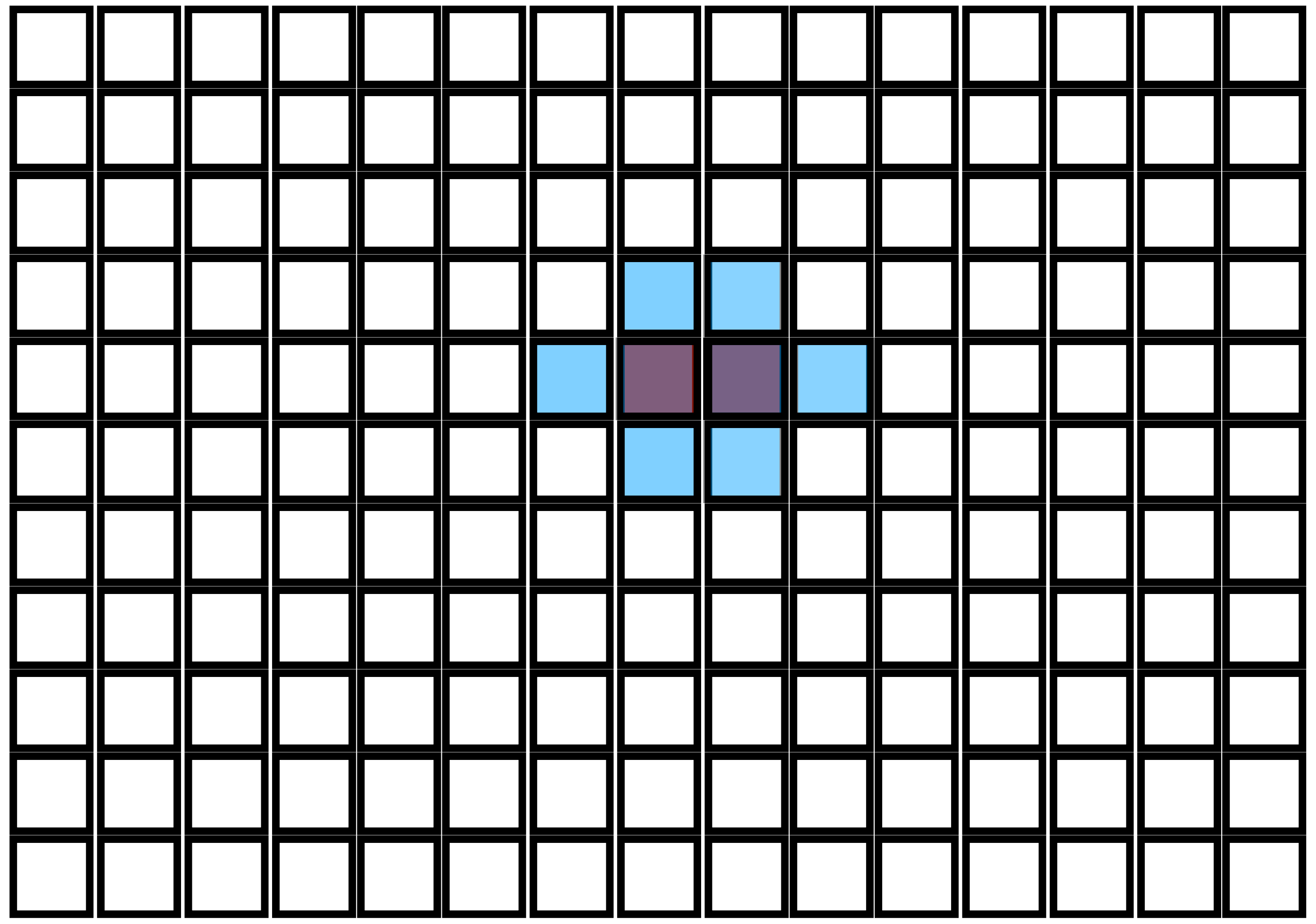


B. Schumacher and R.F. Werner, arXiv:quant-ph/040517

Wrapping lemma

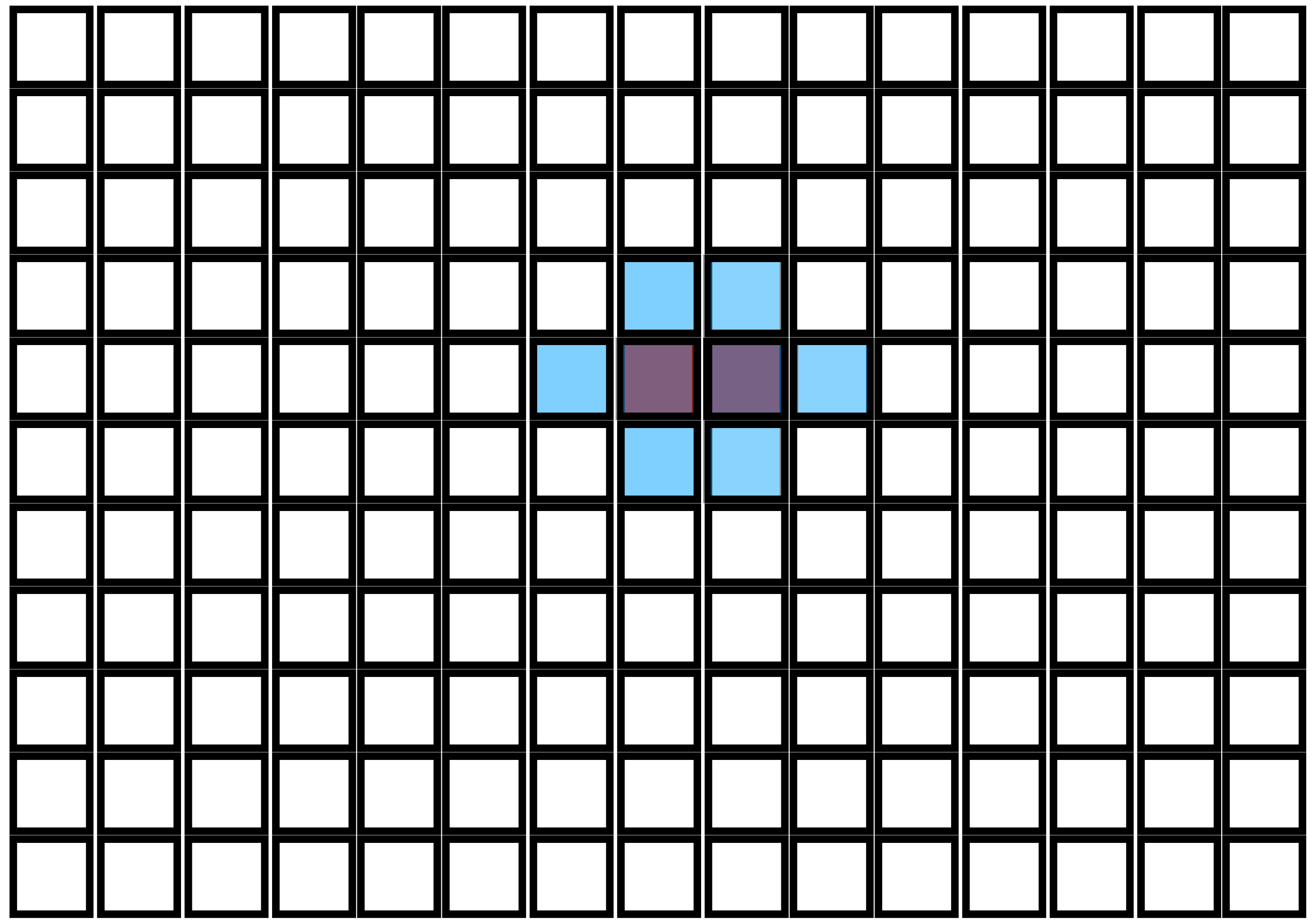
- The commutation condition is local

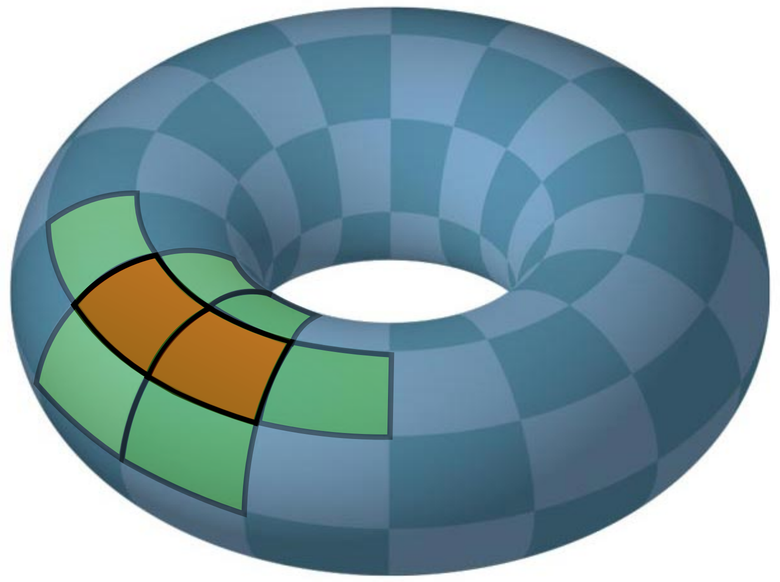


B. Schumacher and R.F. Werner, arXiv:quant-ph/040517

Wrapping lemma

- The commutation condition is local
  - The same for infinite/finite QCA





B. Schumacher and R.F. Werner, arXiv:quant-ph/040517

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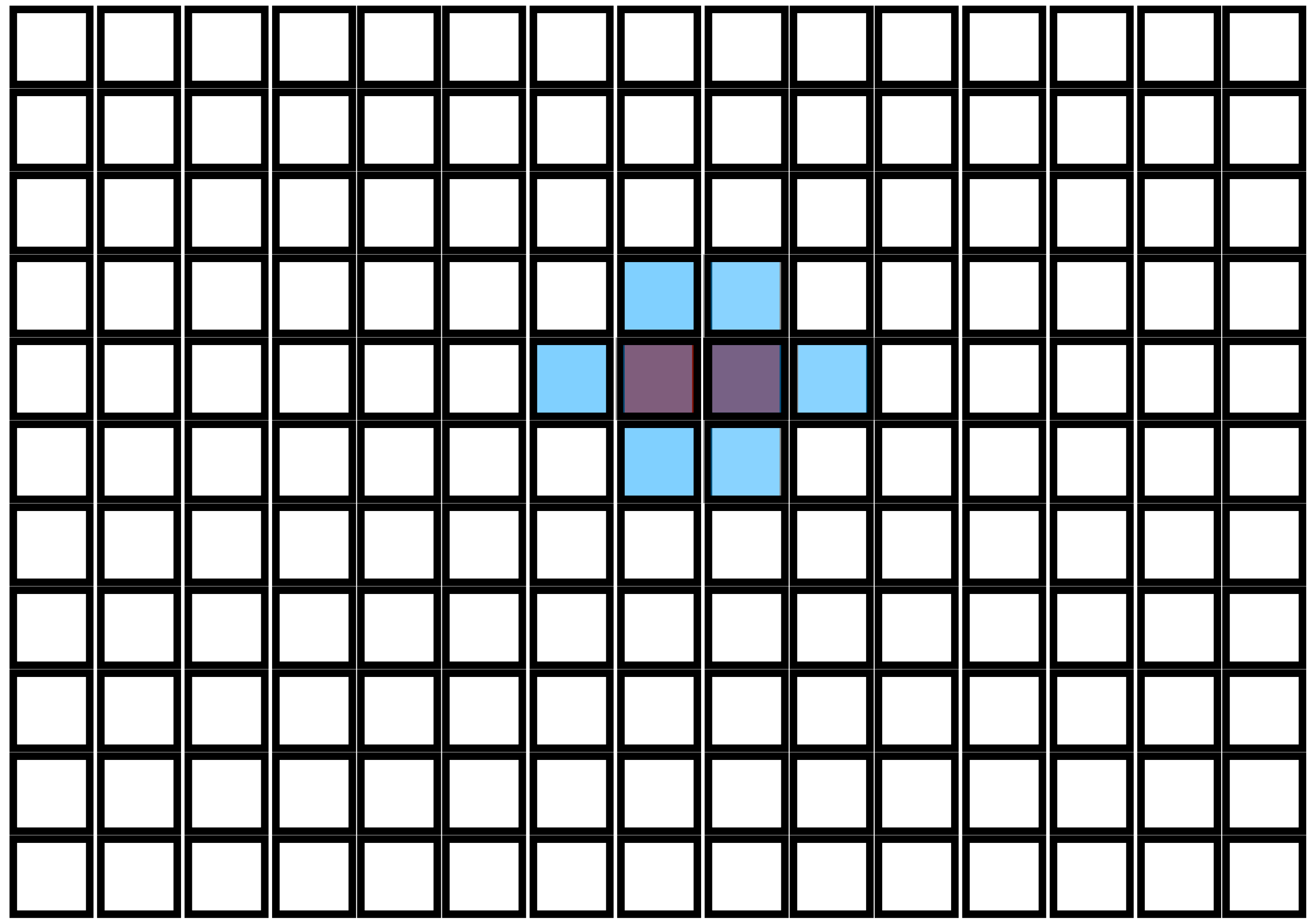
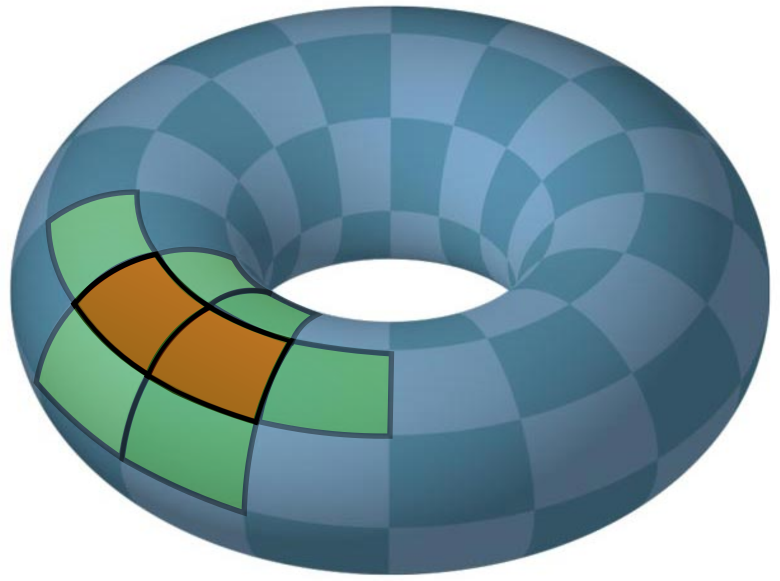
- Legitimate local rules on $\mathbb{Z}^d$ are legitimate on $\mathbb{Z}_k^d$





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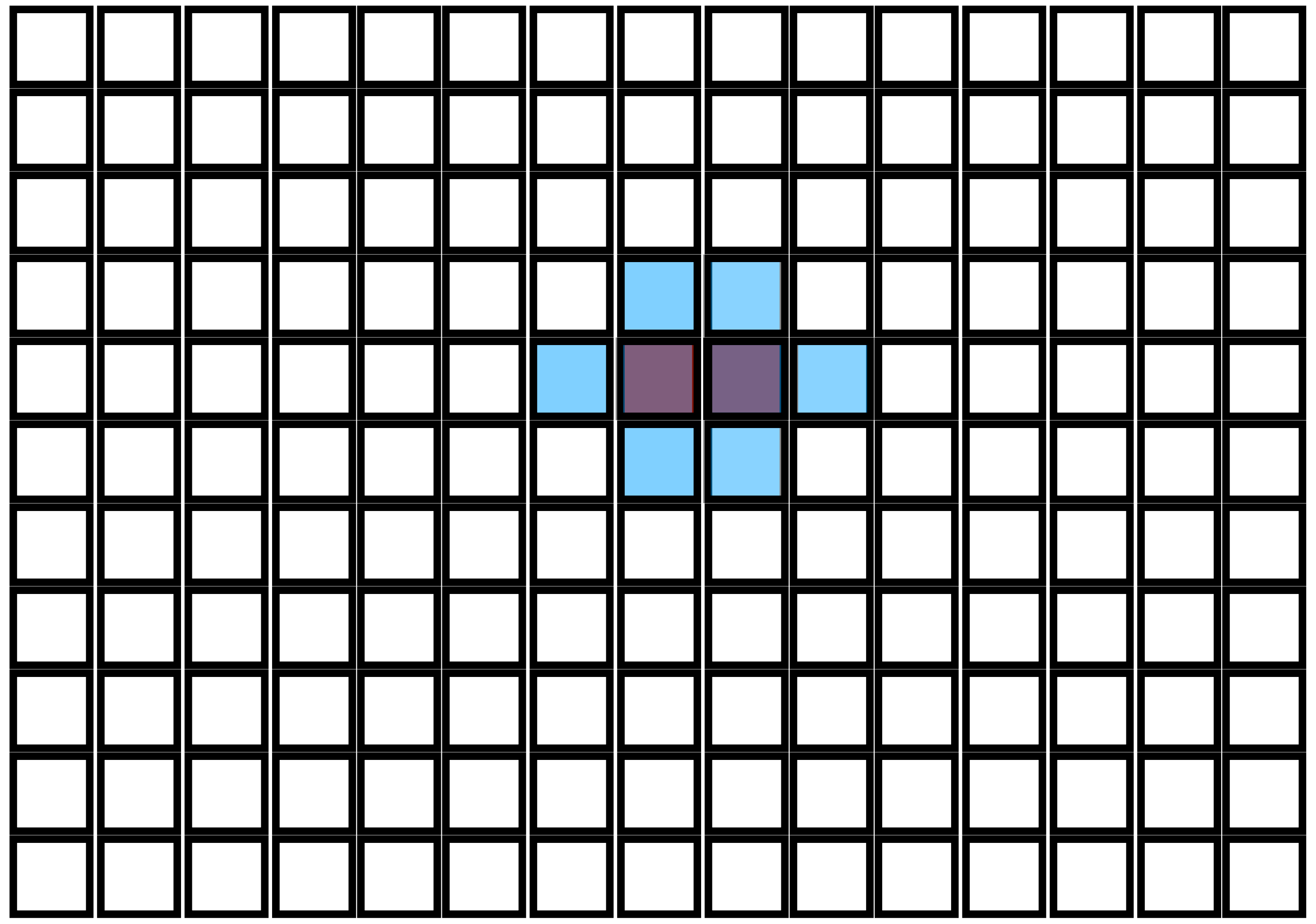

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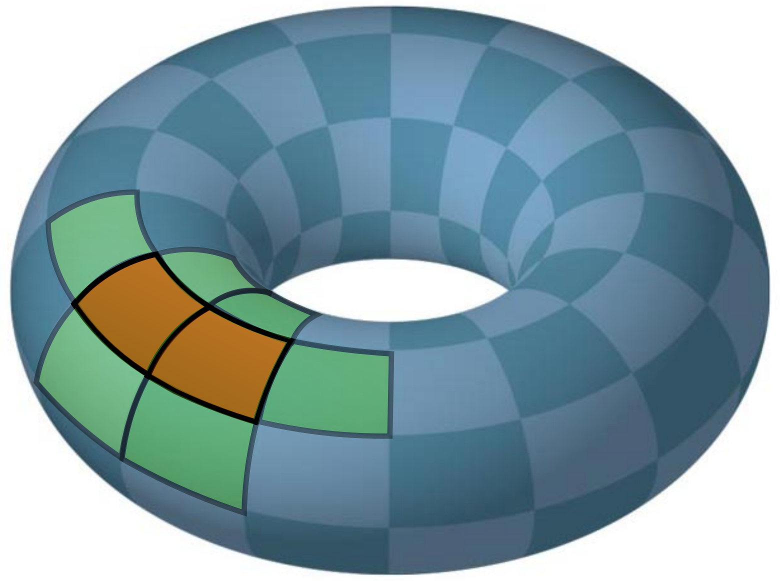
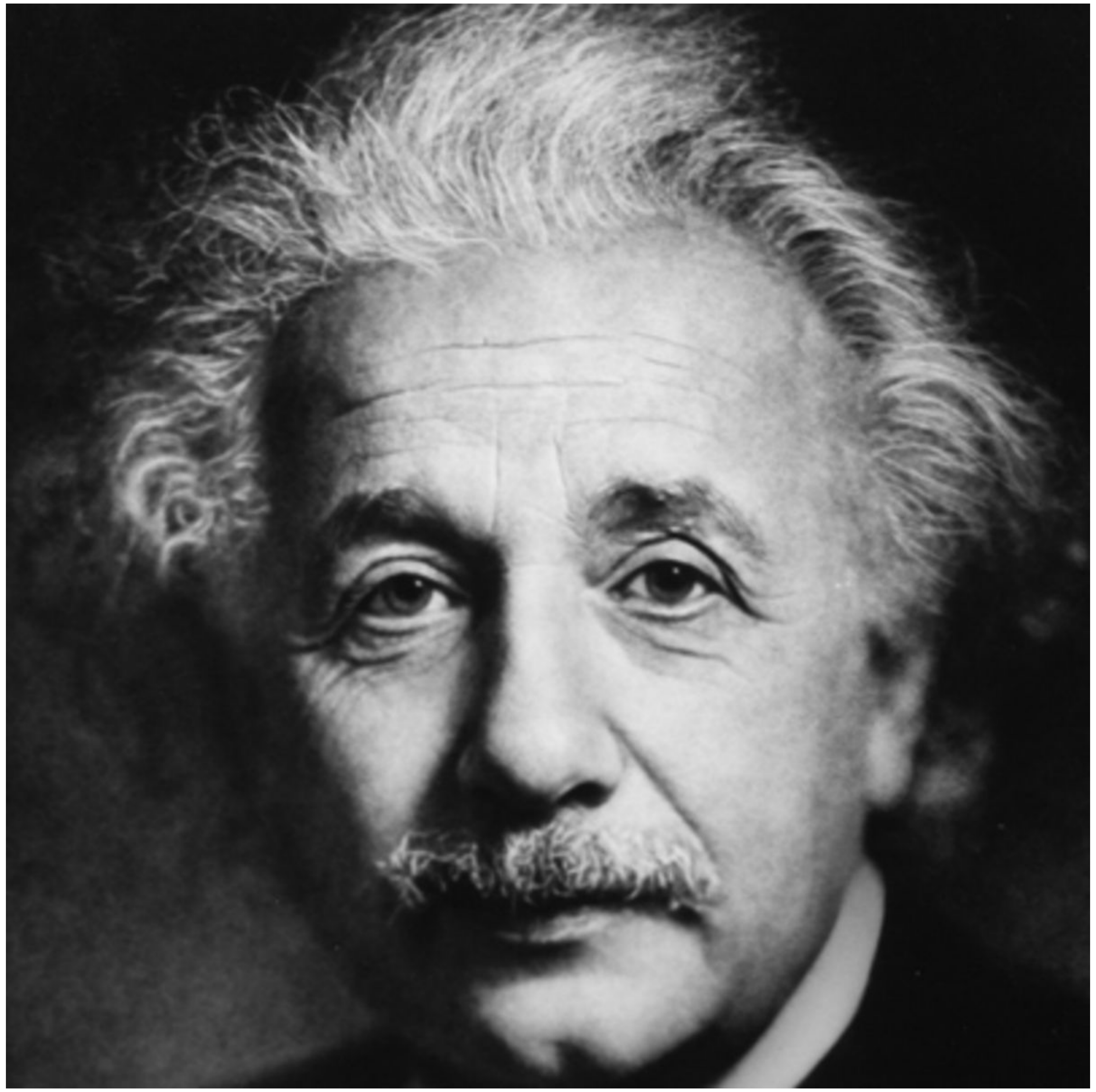


B. Schumacher and R.F. Werner, arXiv:quant-ph/040517

Emergent physical laws

"The question of the validity of the hypotheses of geometry in the infinitely small is bound up with the question of the ground of the metric relations of space. In this last question, which we may still regard as belonging to the doctrine of space, is found the application of the remark made above; that in a discrete manifoldness, the ground of its metric relations is given in the notion of it, while in a continuous manifoldness, this ground must come from outside. Either therefore the reality which underlies space must form a discrete manifoldness, or we must seek the ground of its metric relations outside it, in binding forces which act upon it."

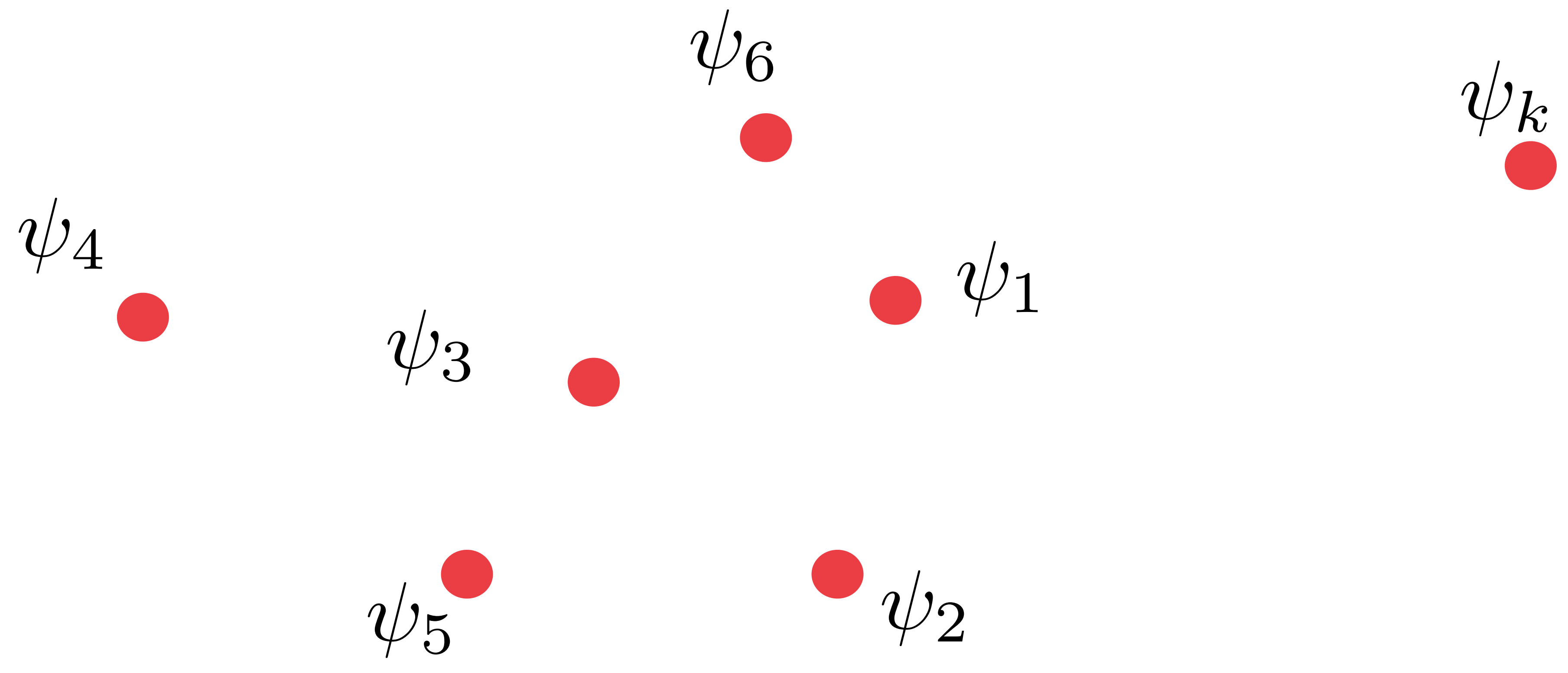
Riemann, "On the Hypotheses which lie at the Bases of Geometry", 1854.



"But you have correctly grasped <u>the drawback that the continuum brings</u>. If the molecular view of matter is the correct (appropriate) one, i.e., if a part of the universe is to be represented by a finite number of moving points, then the continuum of the present theory contains too great a manifold of possibilities. I also believe that this too great is responsible for the fact that our present means of description <u>miscarry with the quantum theory</u>. The problem seems to me how one can formulate statements about a discontinuum without calling upon a continuum (space-time) as an aid; the latter should be banned from the theory as a supplementary construction not justified by the essence of the problem, which corresponds to nothing "real". <u>But we still lack the mathematical structure unfortunately.</u> How much have I already plagued myself in this way!"

Einstein, letter to Walter Dällenbach, November 1916.

Fermionic cellular automata



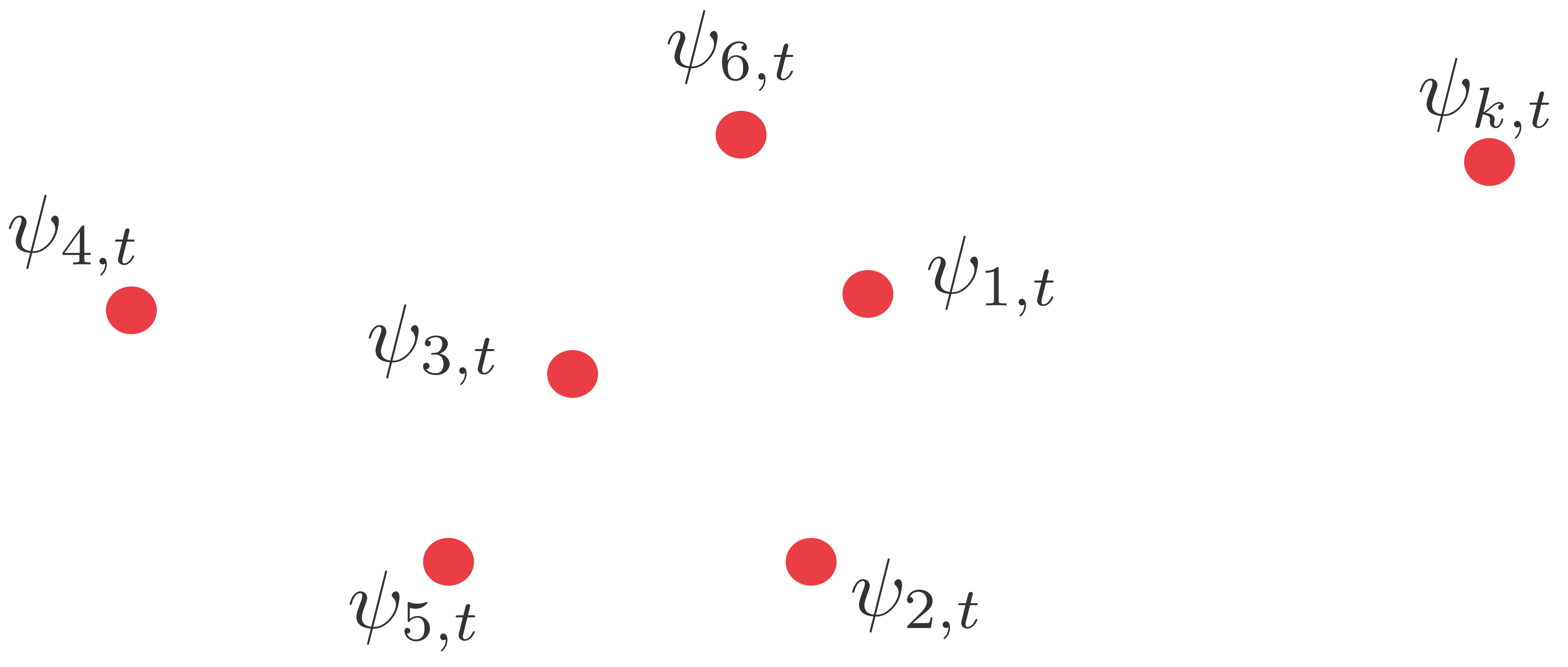
$$\psi_g = \begin{pmatrix} \psi_g^1 \\ \psi_g^2 \\ \vdots \\ \psi_g^{s_g} \end{pmatrix}$$

- Fermionic field

$$\{\psi_i^r, \psi_j^{r'}\} = 0,$$
$$\{\psi_i^r, \psi_j^{r'\dagger}\} = \delta_{ij}\delta_{rr'} I,$$

G. M. D'Ariano and PP, Phys. Rev. A 90, 062106 (2014).

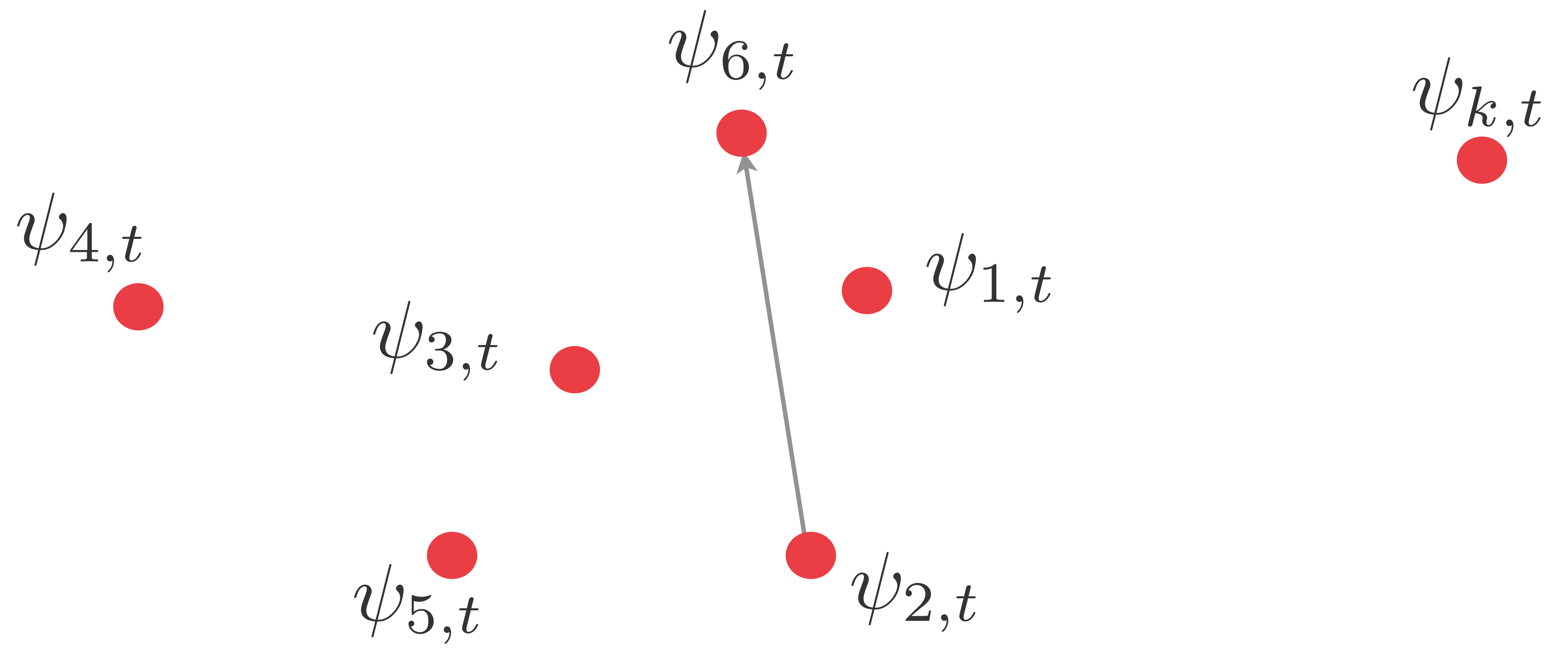
Fermionic cellular automata



- Homomorphism $\mathscr{V}(\psi_{i,t}) \subseteq \mathsf{A}_{S_i}$

G. M. D'Ariano and PP, Phys. Rev. A 90, 062106 (2014).

Fermionic cellular automata



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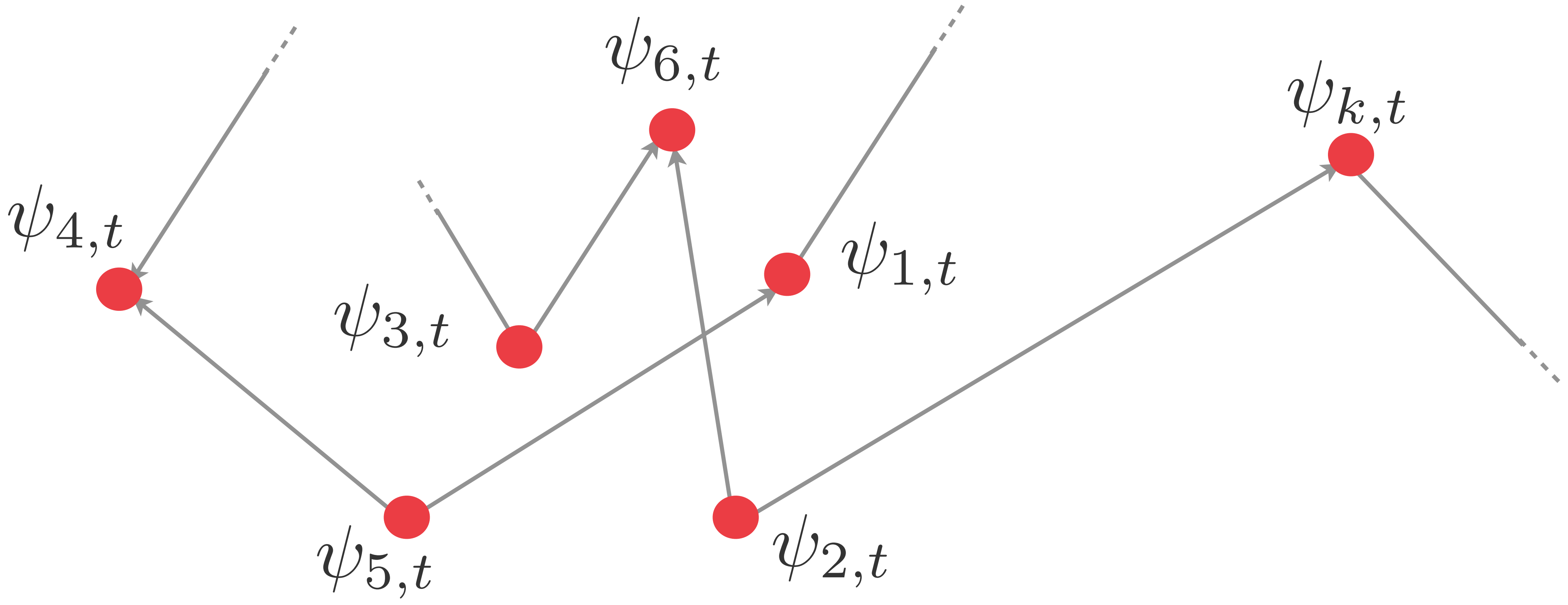

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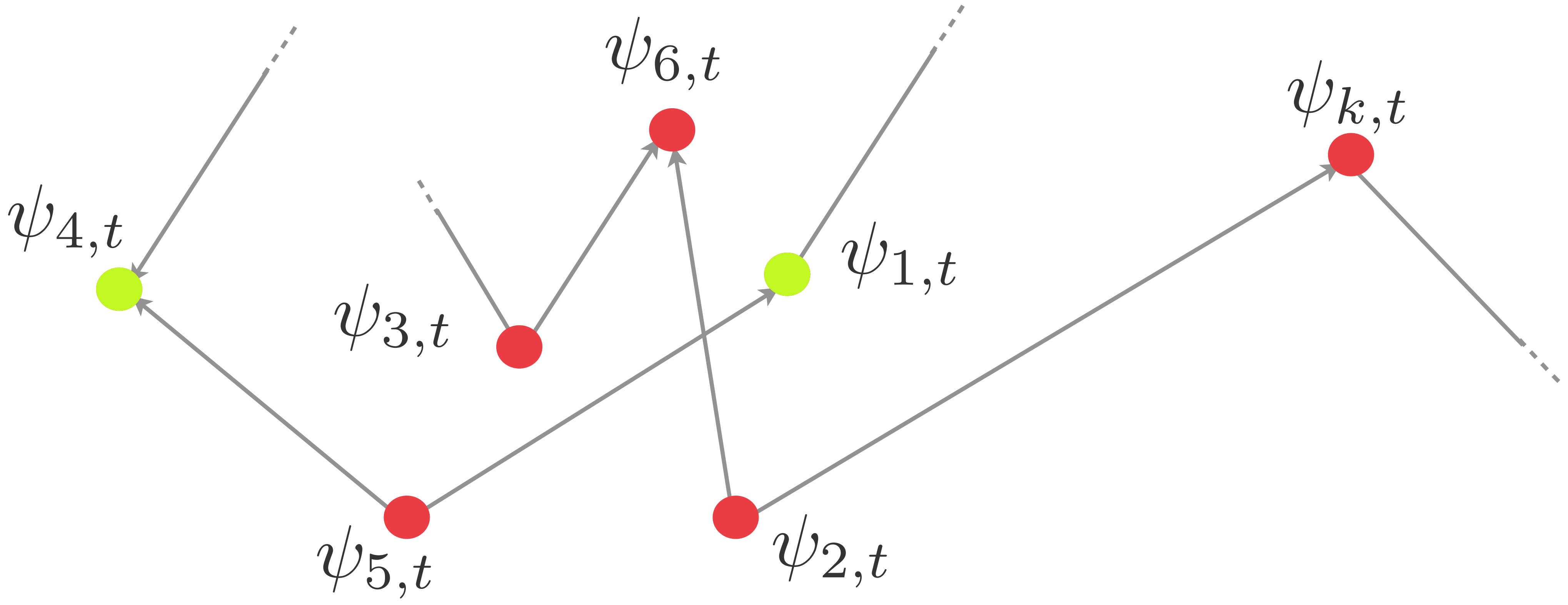

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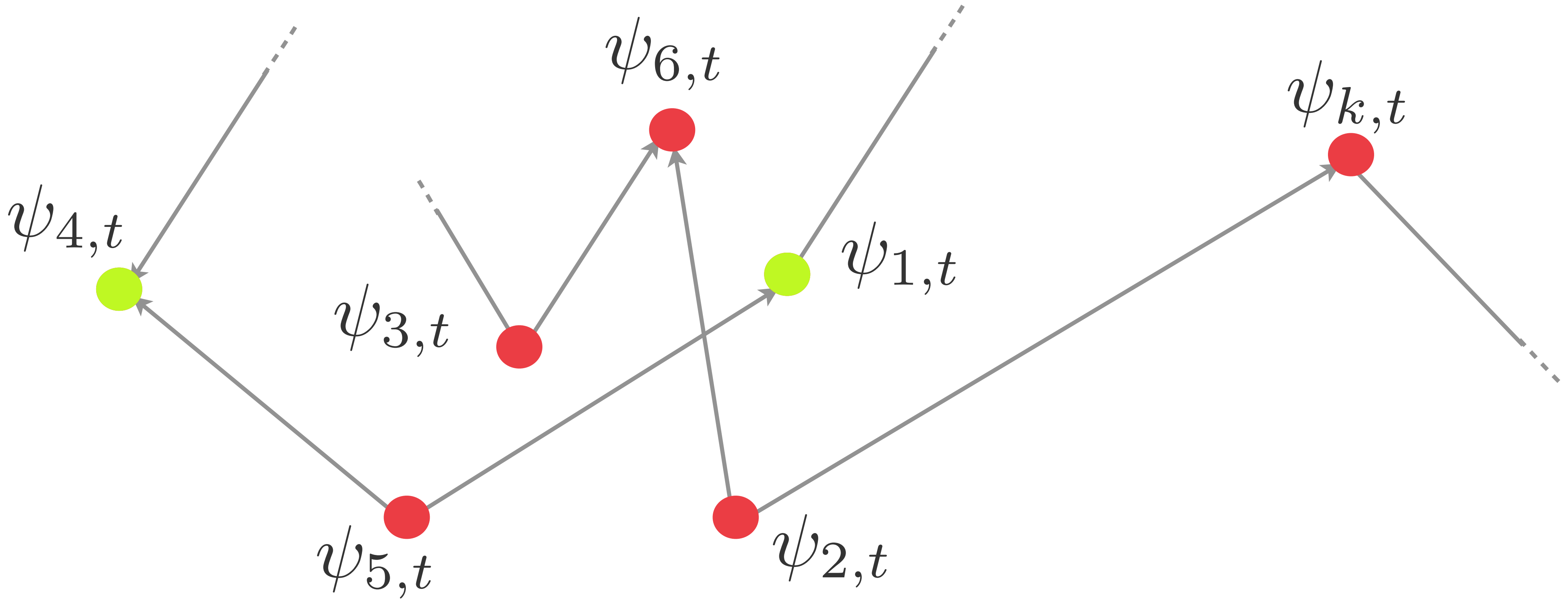

- Homomorphism $\mathscr{V}(\psi_{i,t}) \subseteq \mathsf{A}_{S_i}$
- Reversibility

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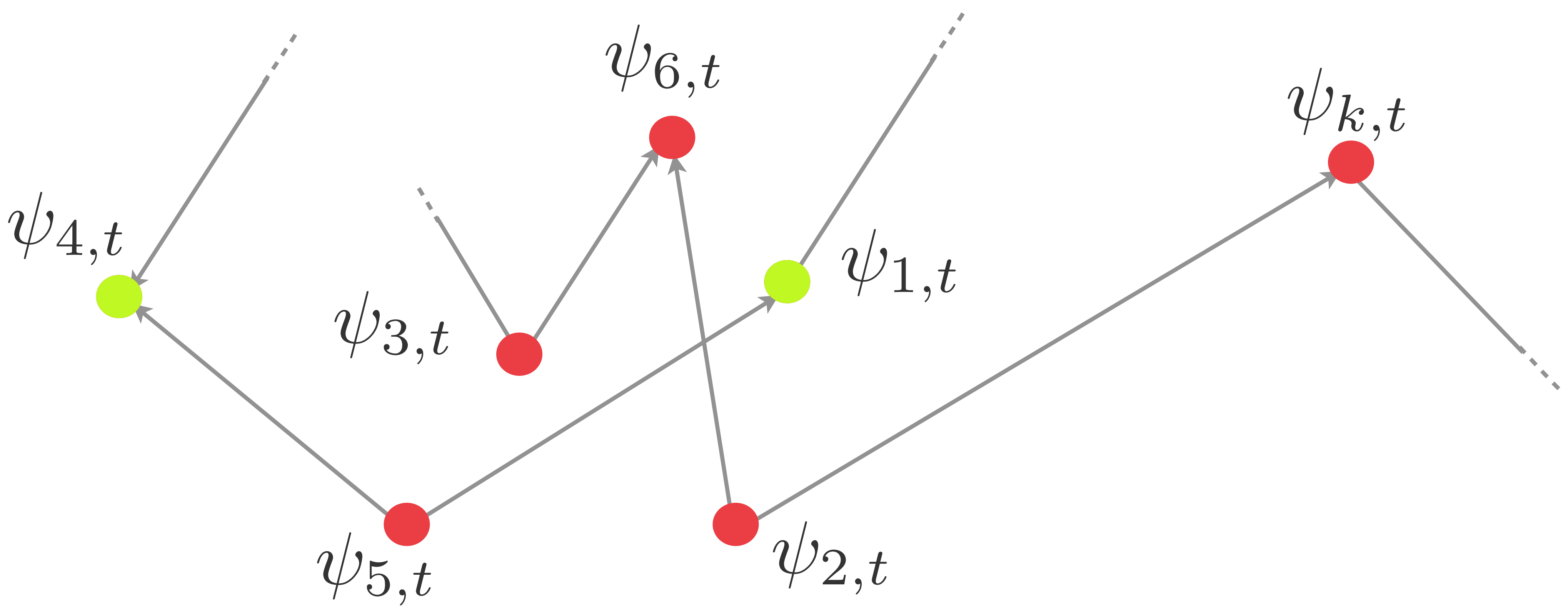
Fermionic cellular automata



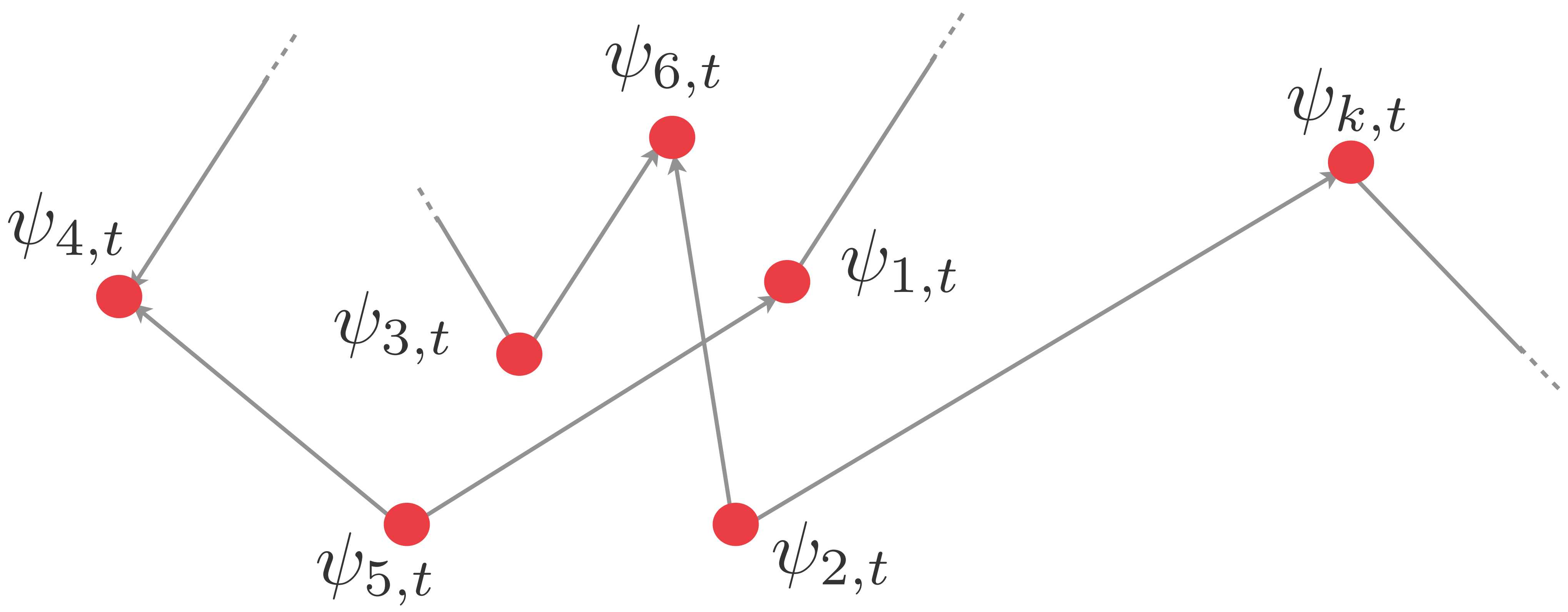
- Homomorphism $\mathscr{V}(\psi_{i,t}) \subseteq \mathsf{A}_{S_i}$

- Reversibility

- Locality $|S_i| \leq N < \infty$

G. M. D'Ariano and PP, Phys. Rev. A 90, 062106 (2014).

Free evolution: linear case



- Linearity

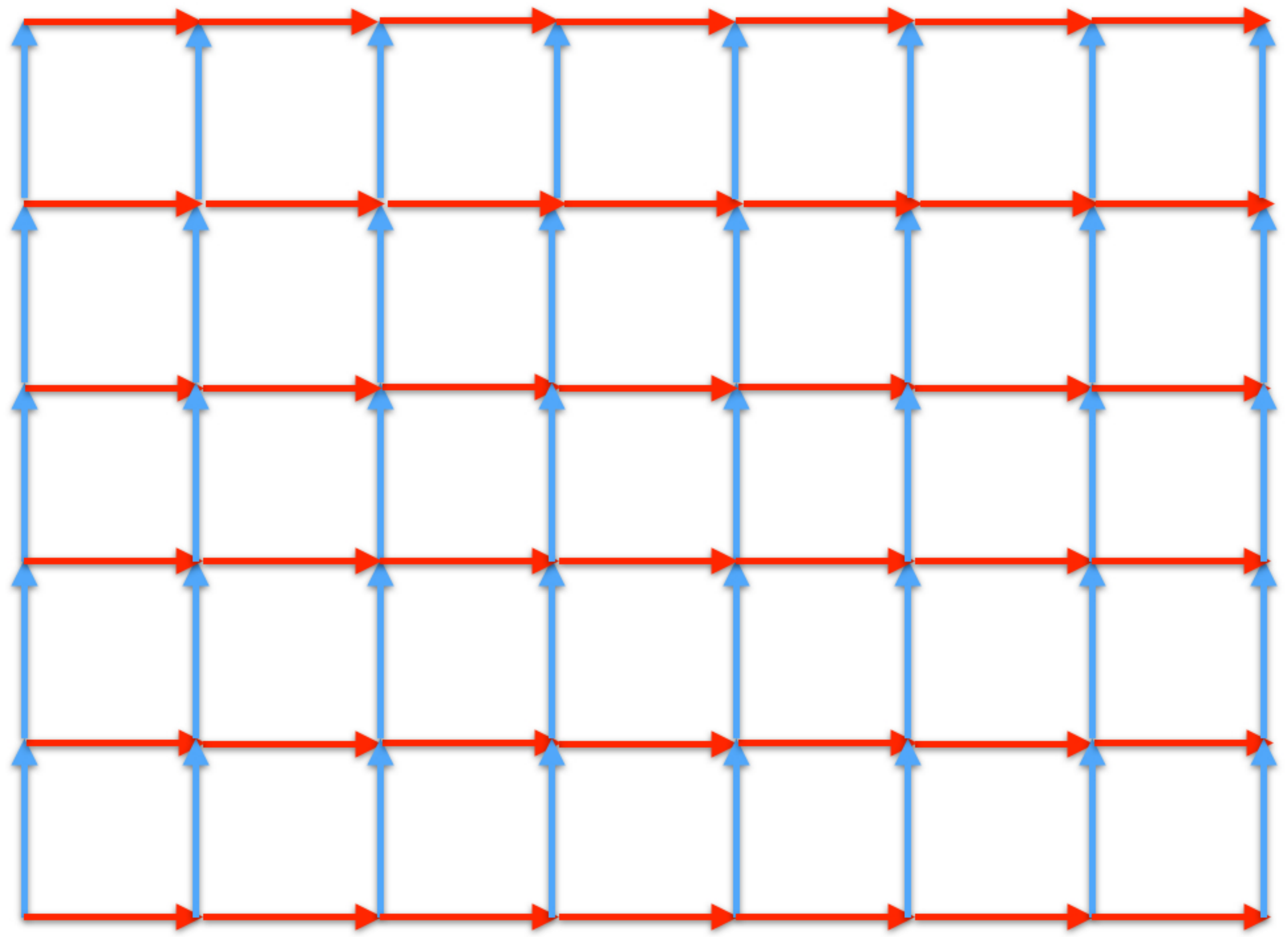
$$\psi_{i,t+1} = \sum_j A_{i,j} \psi_{j,t}$$

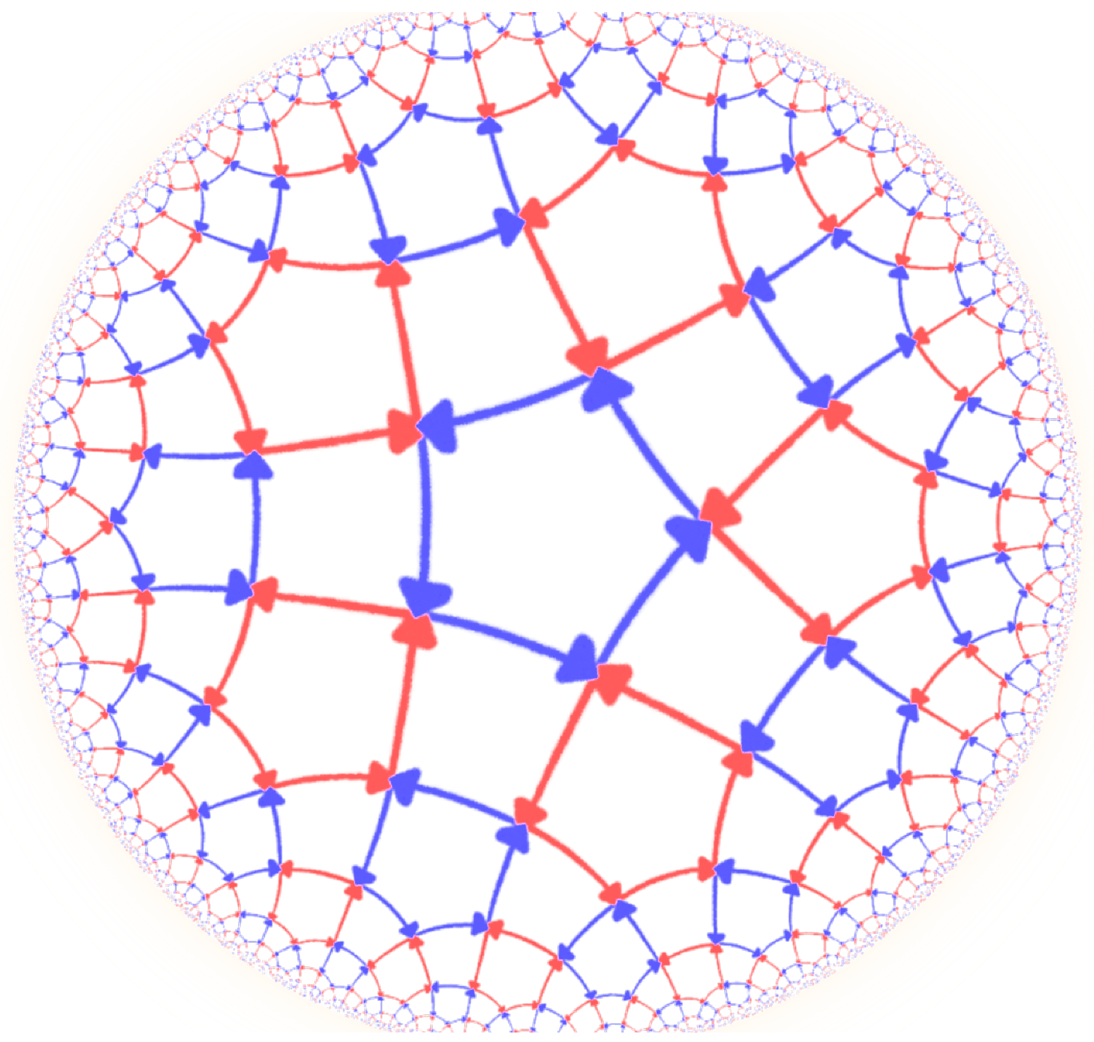
$$S_i = \{A_{ij} \neq 0\}$$

G. M. D'Ariano and PP, Phys. Rev. A 90, 062106 (2014).

Homogeneity

"One can distinguish the role of two different systems only with reference to any third one"





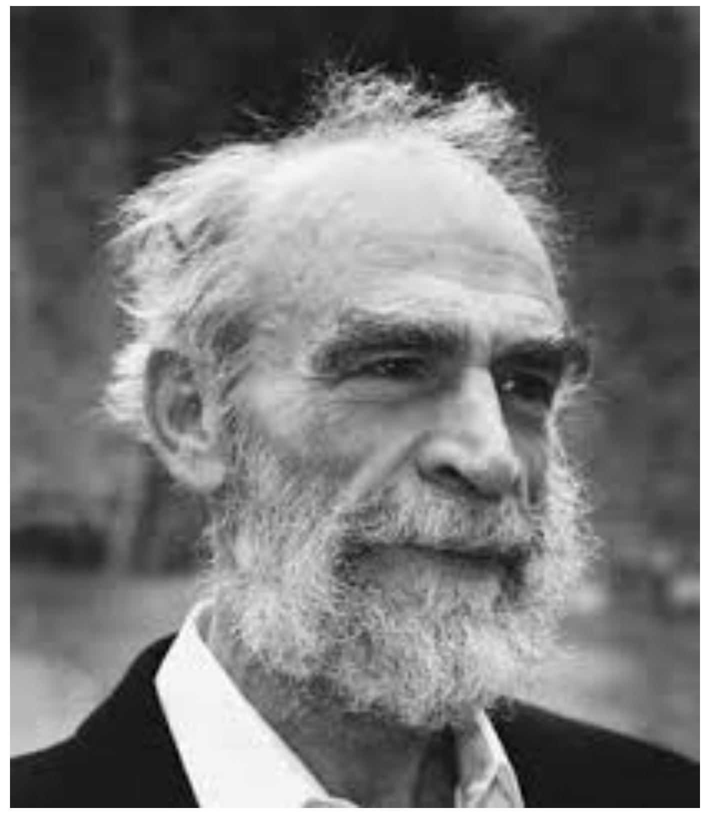
- The graph of causal connections is a group
- The cellular automaton is translation invariant $\mathcal{V}[\mathcal{T}_z(A_x) = \mathcal{T}_z[\mathcal{V}(A_x)]$

G. M. D'Ariano and PP, Phys. Rev. A 90, 062106 (2014).
P.P., Quantum 4, 294 (2020)

Geometric group theory

Studies algebraic properties of groups

in connection with geometric ones



Mikhail Gromov

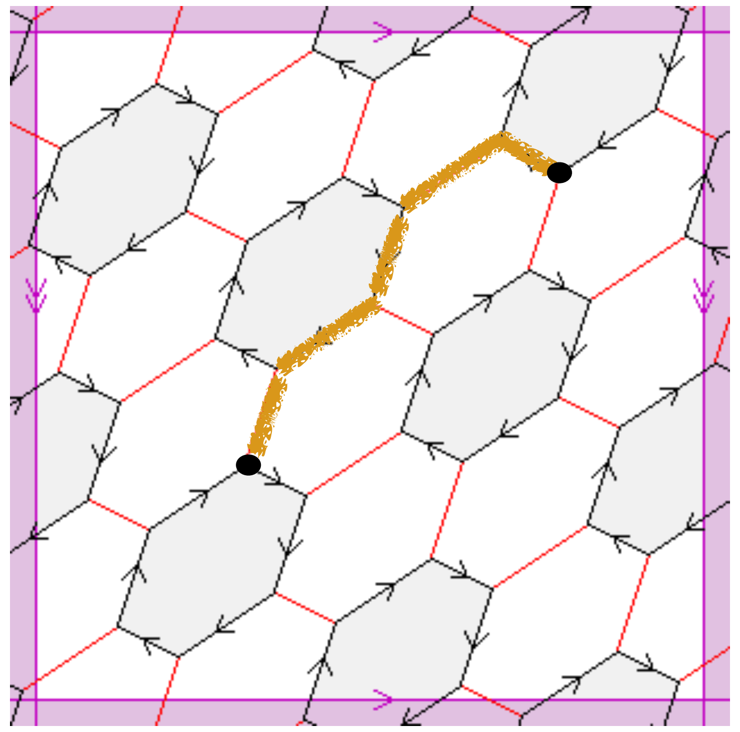
Quasi-isometry

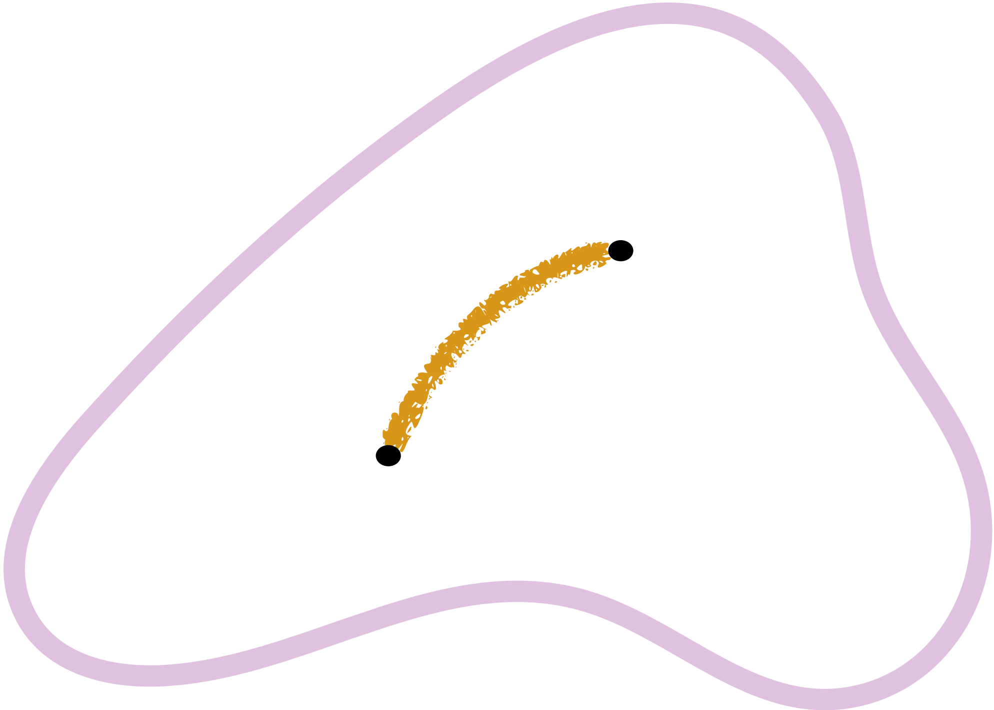
$$bd(\mathsf{E}(g), \mathsf{E}(g'))_R - a \leq d(g, g')_G \leq \tfrac{1}{b} d(\mathsf{E}(g), \mathsf{E}(g'))_R + a$$

$$\forall g \in G \quad \exists z \in R \text{ s.t. } d(z, \mathsf{E}(g))_R < a$$

Emergent space(-time)

Homogeneity → Uniquely identified geometry





G. M. D'Ariano and PP, Phys. Rev. A 90, 062106 (2014).

Walk operator

- Right-regular representation of G $\qquad T_g|f\rangle := |fg^{-1}\rangle \qquad |f\rangle \in \ell^2(G)$

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Walk operator

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- Walk operator $W : \ell^2(G) \otimes \mathbb{C}^s \to \ell^2(G) \otimes \mathbb{C}^s$

$$W = \sum_{h \in S} T_h \otimes A_h$$

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- The automaton is reversible iff the walk is unitary

G. M. D'Ariano and PP, Phys. Rev. A 90, 062106 (2014).

Unitarity

- Unitarity conditions

$$\sum_{h\in S} A_h^\dagger A_h = \sum_{h\in S} A_h A_h^\dagger = I$$

$$\sum_{h^{-1}h'=g} A_h^\dagger A_{h'} = \sum_{hh'^{-1}=g} A_h A_{h'}^\dagger = 0$$

- The conditions involve length-four relators

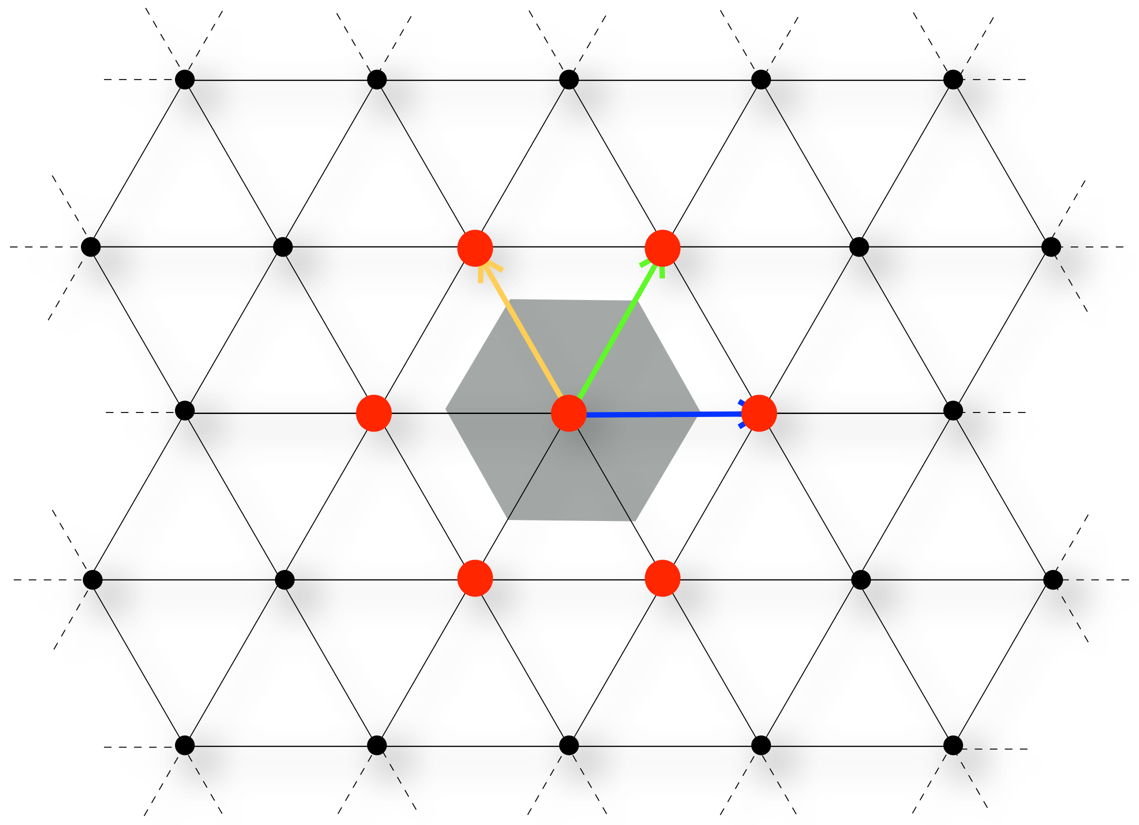
$$h^{-1}h' = f^{-1}f' \;\Leftrightarrow\; h^{-1}h'f'^{-1}f = e$$

Diagonalising the walk

- From translation invariance $[T_{\mathbf{h}} \otimes I, W] = 0$

$$W = \int_B \mathrm{d}^d\,\mathbf{k}\ |\mathbf{k}\rangle\langle\mathbf{k}| \otimes W_{\mathbf{k}}$$

$$W_{\mathbf{k}} := \sum_{\mathbf{h}\in S} e^{-i\mathbf{k}\cdot\mathbf{h}} A_{\mathbf{h}}$$

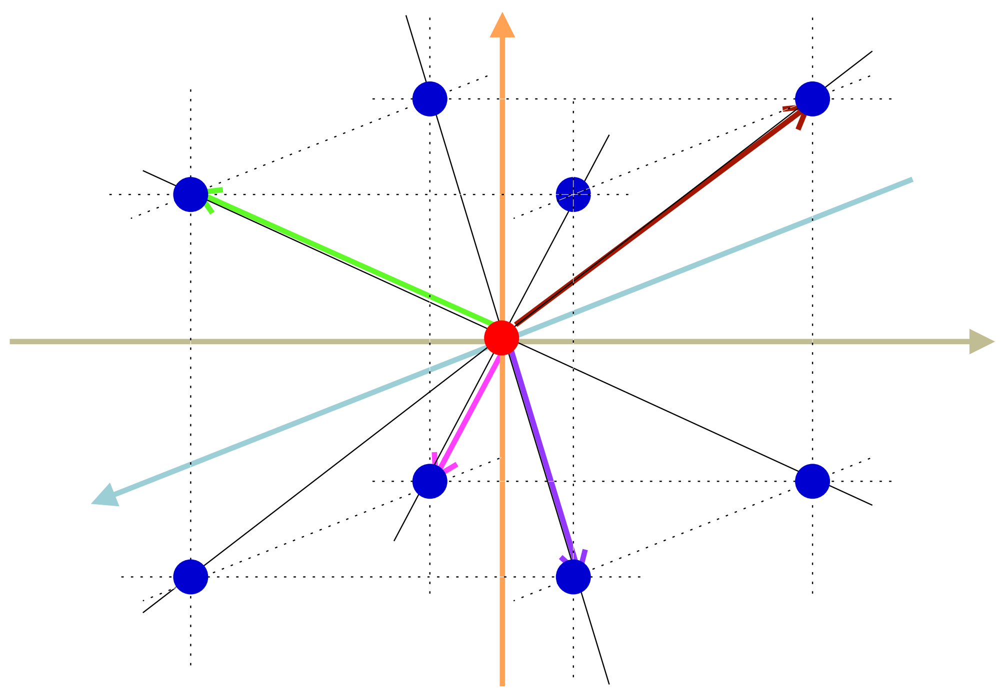


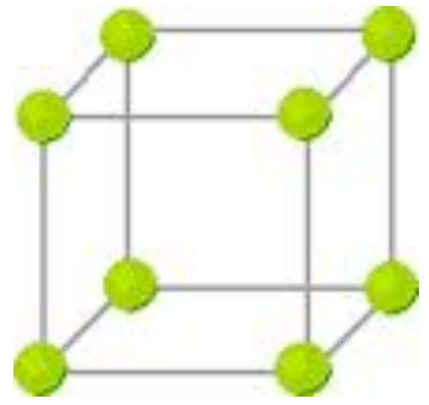
Brillouin zone

G. M. D'Ariano and PP, Phys. Rev. A 90, 062106 (2014).

Non-interacting QCA

$$G = \mathbb{Z}^3$$





G. M. D'Ariano and PP, Phys. Rev. A 90, 062106 (2014)
G. M. D'Ariano, M. Erba and P.P., Phys. Rev. A 96, 062101 (2017)

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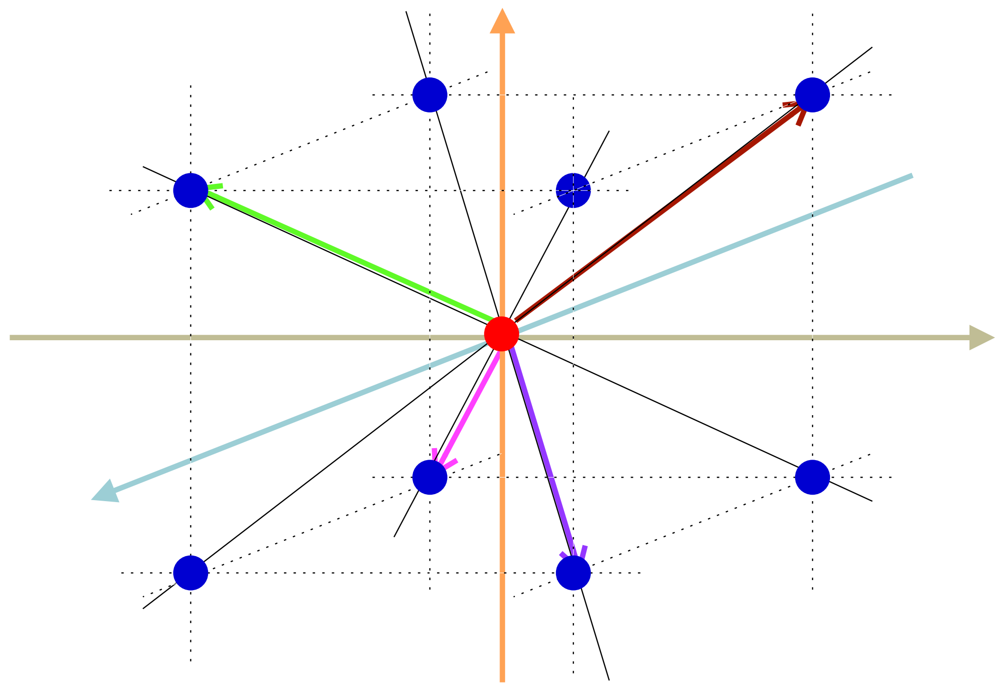

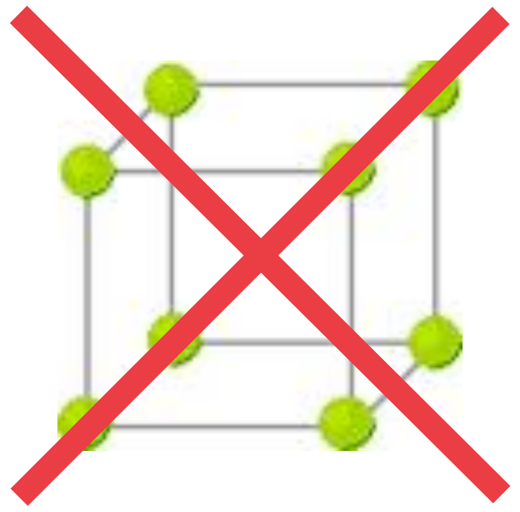

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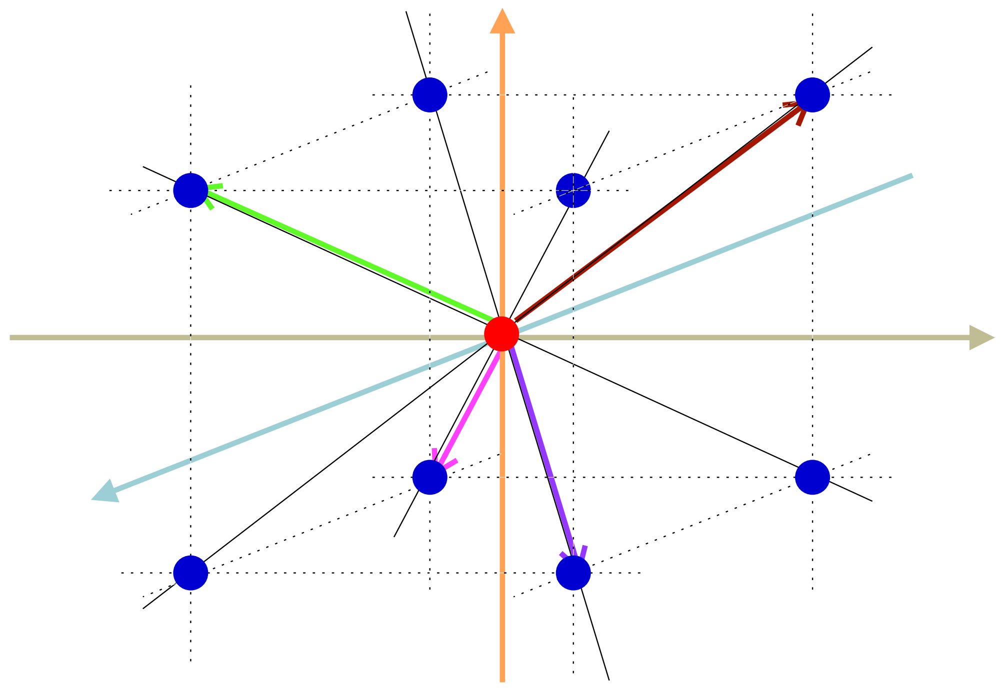

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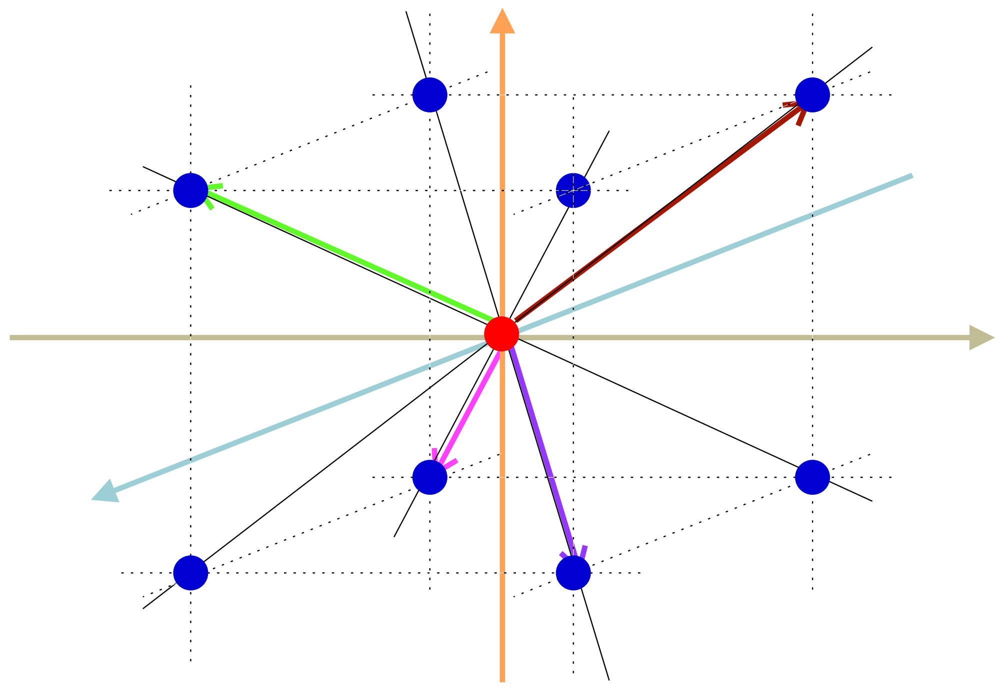

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$$\Downarrow$$

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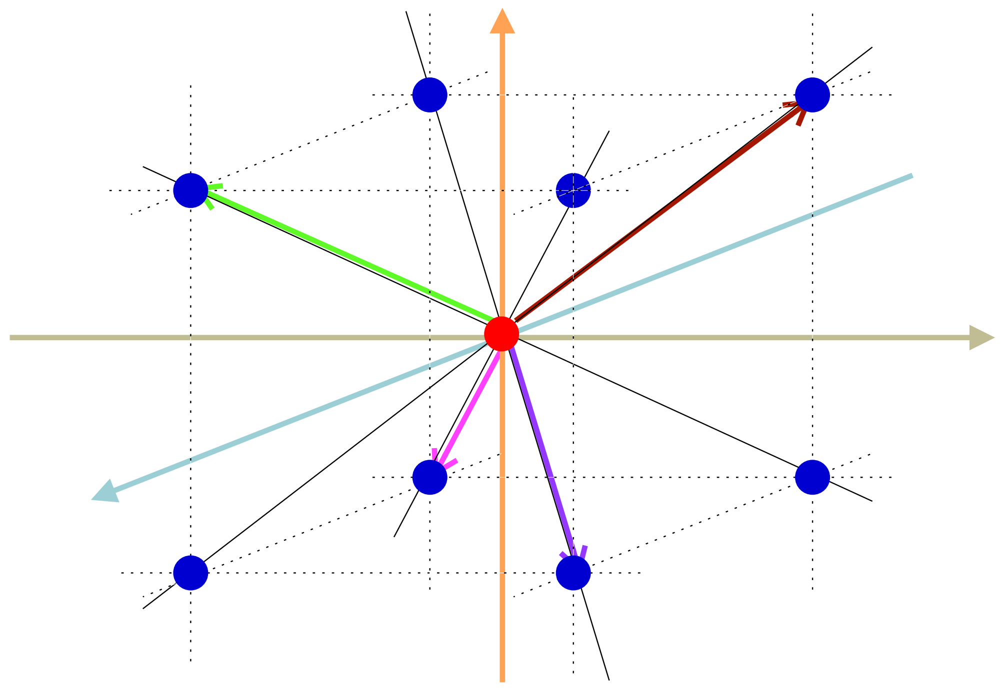

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Weyl's equations

G. M. D'Ariano and PP, Phys. Rev. A 90, 062106 (2014)
G. M. D'Ariano, M. Erba and P.P., Phys. Rev. A 96, 062101 (2017)

Free quantum field theory

- Composing Weyl automata
  - Dirac's equation

$$Z_{\mathbf{k}}^{\pm} = \begin{pmatrix} nW_{\mathbf{k}}^{\pm} & imI \\ imI & nW_{\mathbf{k}}^{\pm\dagger} \end{pmatrix} \qquad m \leq 1$$

$$i\partial_t \psi_{\mathbf{k},t} = (s\boldsymbol{\alpha} \cdot \mathbf{k} + c\beta)\psi_{\mathbf{k},t}$$

- Entangled states
  - Maxwell's equations

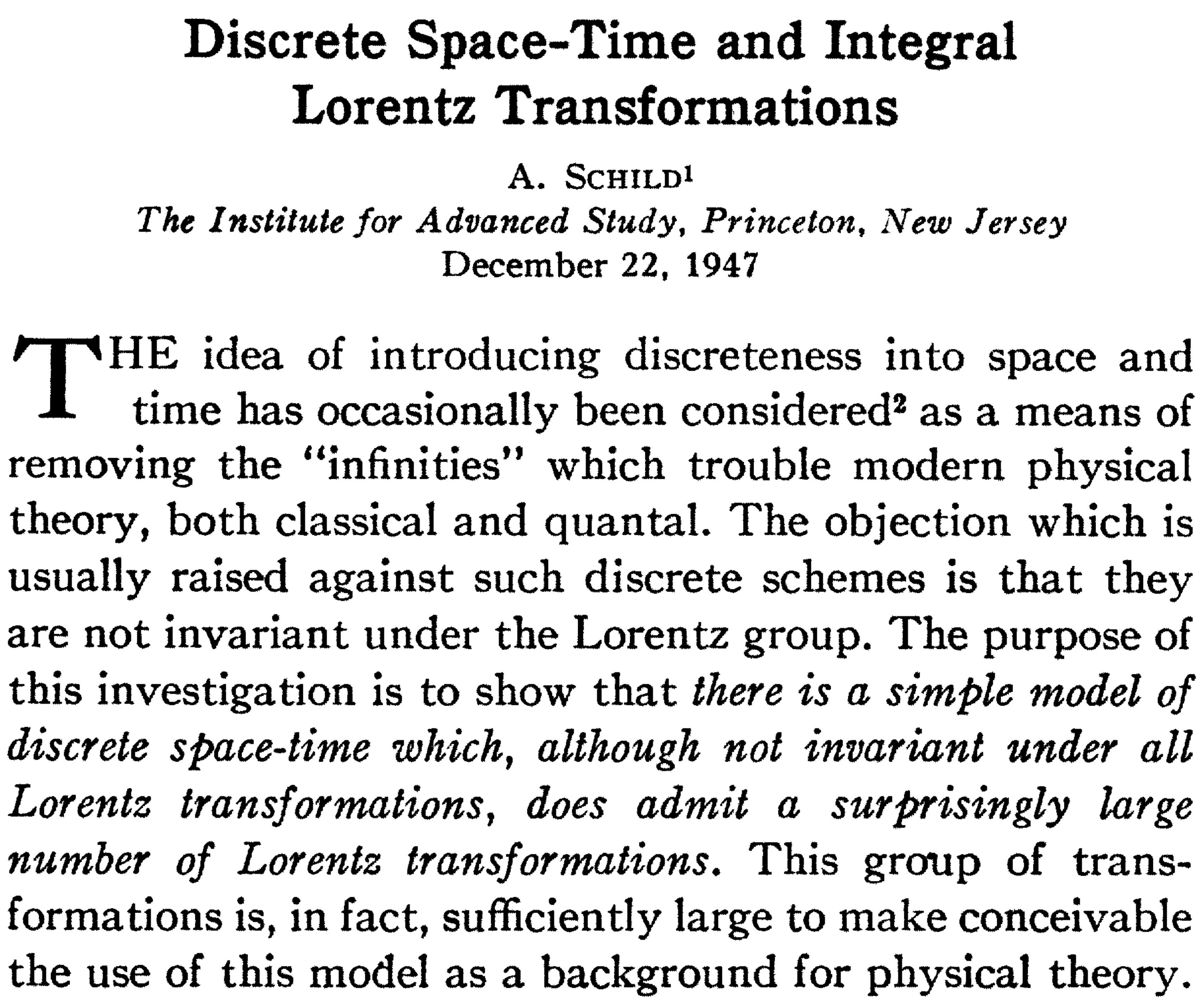
$$M_{\mathbf{k}}^{\pm} = W_{\mathbf{k}}^{\pm} \otimes W^{\pm}{}_{\mathbf{k}}^{*}$$

$$\partial_t \operatorname{Re}\mathbf{H}(\mathbf{x},t) = \nabla \times \operatorname{Im}\mathbf{H}(\mathbf{x},t),$$

$$\partial_t \operatorname{Im}\mathbf{H}(\mathbf{x},t) = -\nabla \times \operatorname{Re}\mathbf{H}(\mathbf{x},t),$$

G. M. D'Ariano and PP, Phys. Rev. A 90, 062106 (2014),
G. M. D'Ariano, M. Erba and P.P., Phys. Rev. A 96, 062101 (2017)
A. Bisio, G. M. D'Ariano and PP, Ann. Phys. 368, 177 (2016).

Relativistic covariance and discreteness



Discrete Space-Time and Integral Lorentz Transformations

A. SCHILD[1]
The Institute for Advanced Study, Princeton, New Jersey
December 22, 1947

THE idea of introducing discreteness into space and time has occasionally been considered[2] as a means of removing the "infinities" which trouble modern physical theory, both classical and quantal. The objection which is usually raised against such discrete schemes is that they are not invariant under the Lorentz group. The purpose of this investigation is to show that *there is a simple model of discrete space-time which, although not invariant under all Lorentz transformations, does admit a surprisingly large number of Lorentz transformations*. This group of transformations is, in fact, sufficiently large to make conceivable the use of this model as a background for physical theory.

Relativistic covariance and discreteness

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$v=(n^2-1)^{\frac{1}{2}}/n$, where n is any positive integer. The smallest non-zero velocity is $\frac{1}{2}3^{\frac{1}{2}}=$ 0.866 times the velocity of light.

Relativistic covariance

- Relativity principle turned on its head

A. Bisio, G. M. D'Ariano and PP, Phys. Rev. A 94, 042120 (2016)

Relativistic covariance

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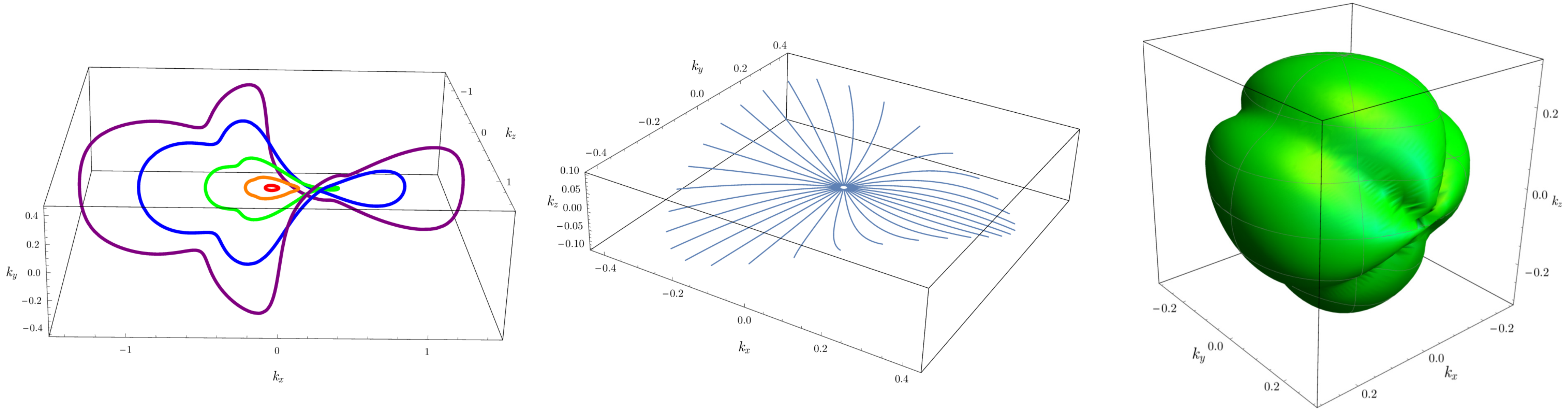
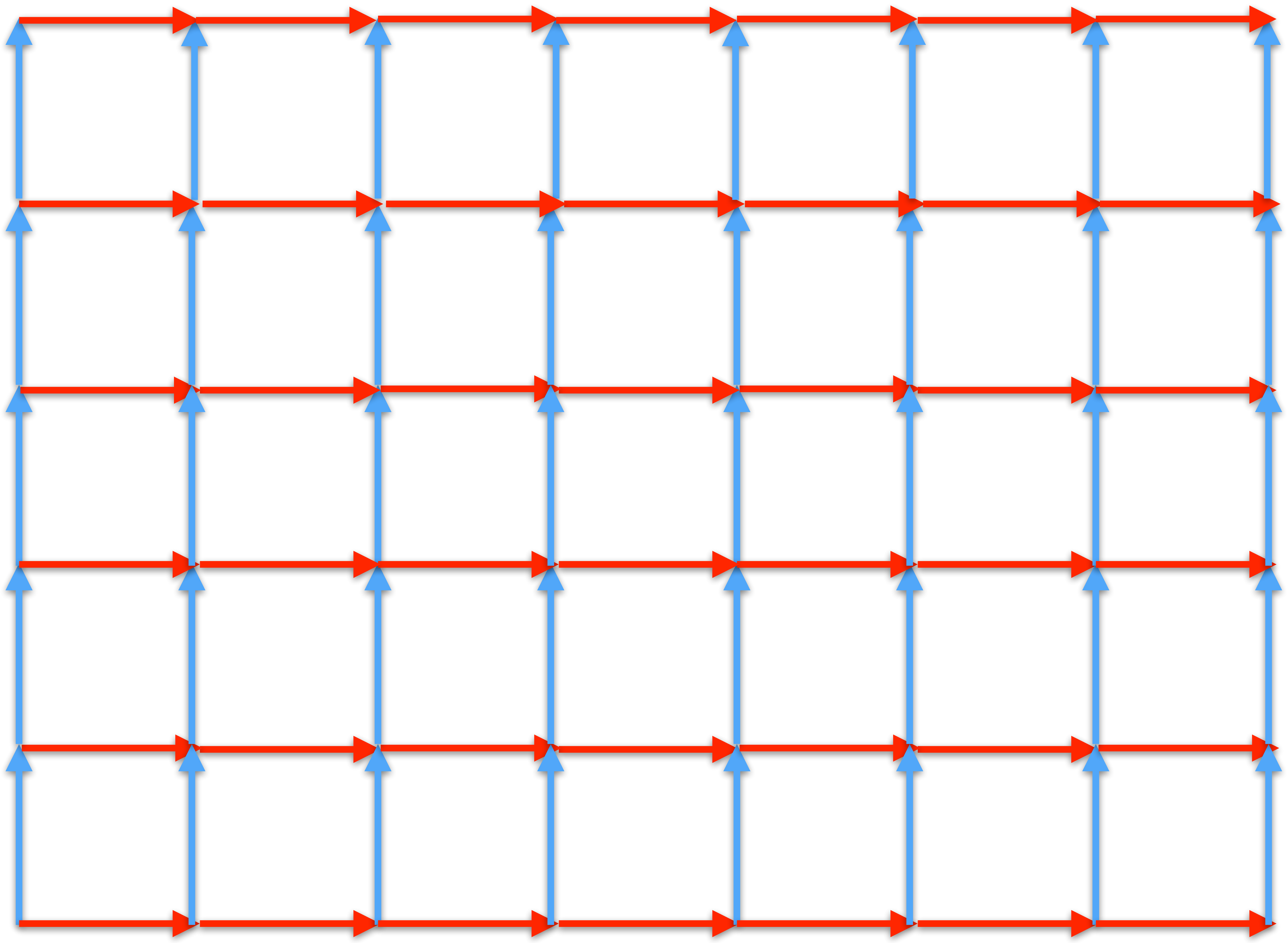


A. Bisio, G. M. D'Ariano and PP, Phys. Rev. A 94, 042120 (2016)

Renormalisation

Renormalisation: CA



- Hypothesis: FCA describe “fundamental” theories

N. Israeli and N. Goldenfeld, Phys. Rev. E 73, 026203 (2006)

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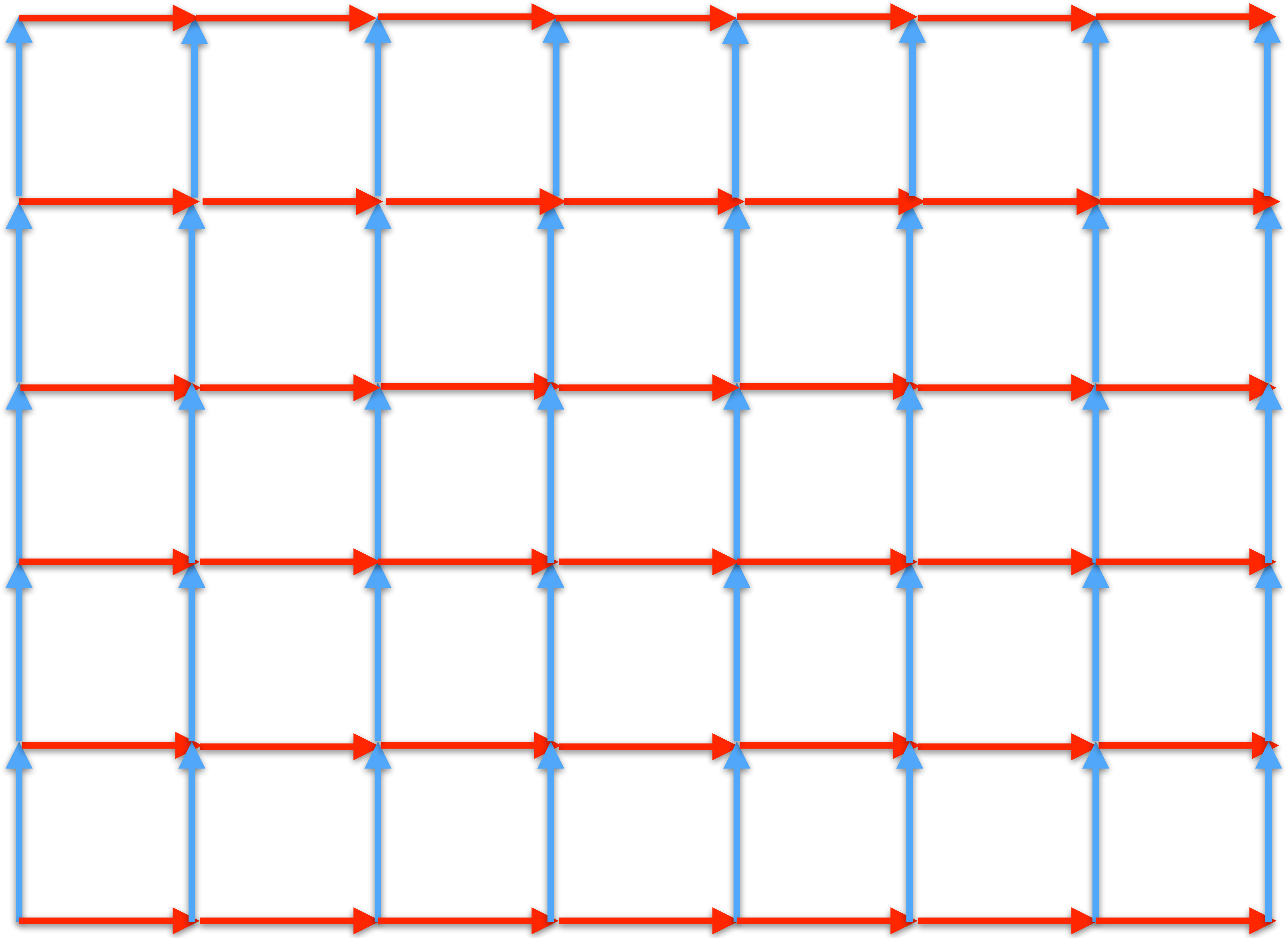

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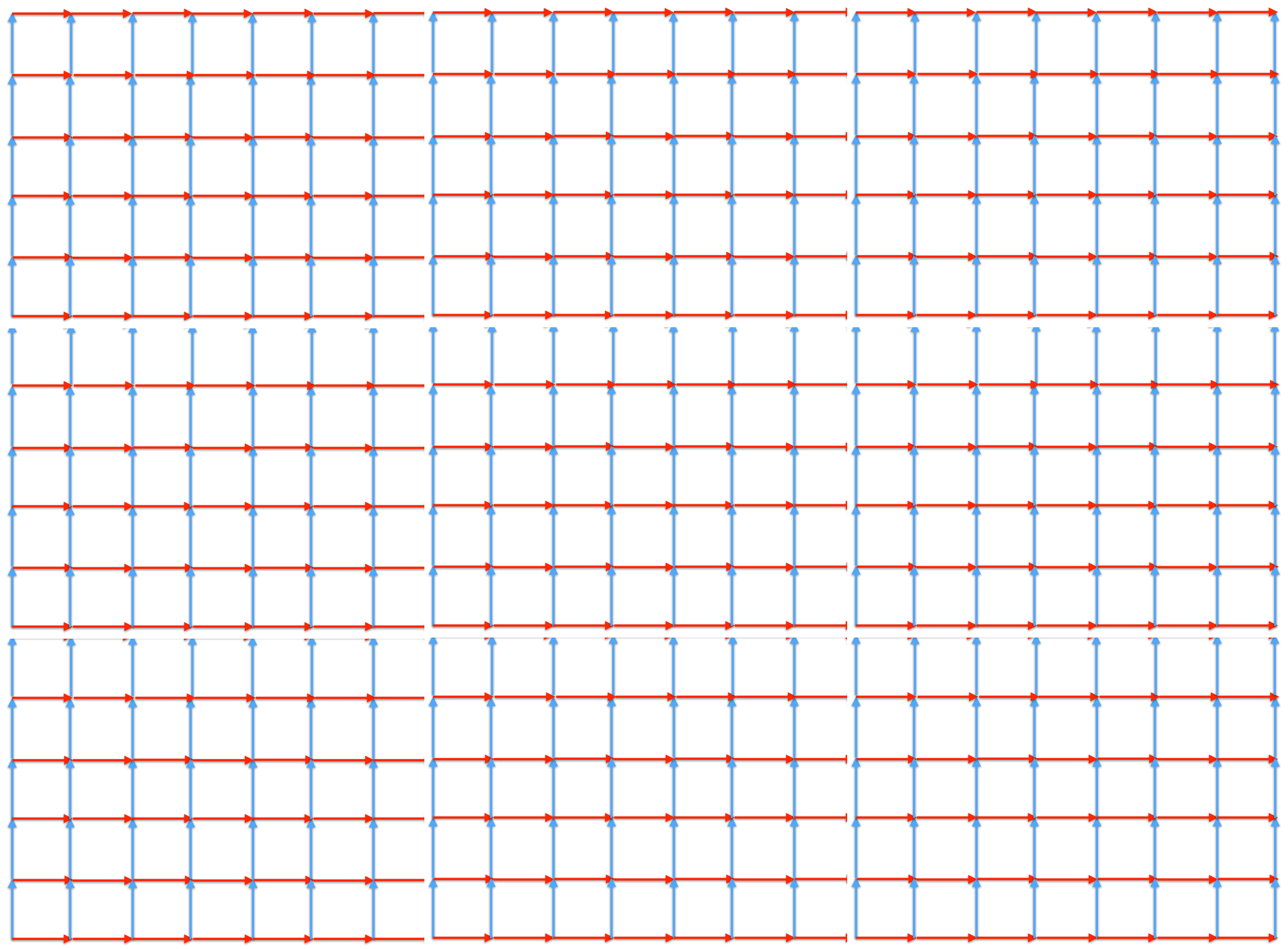

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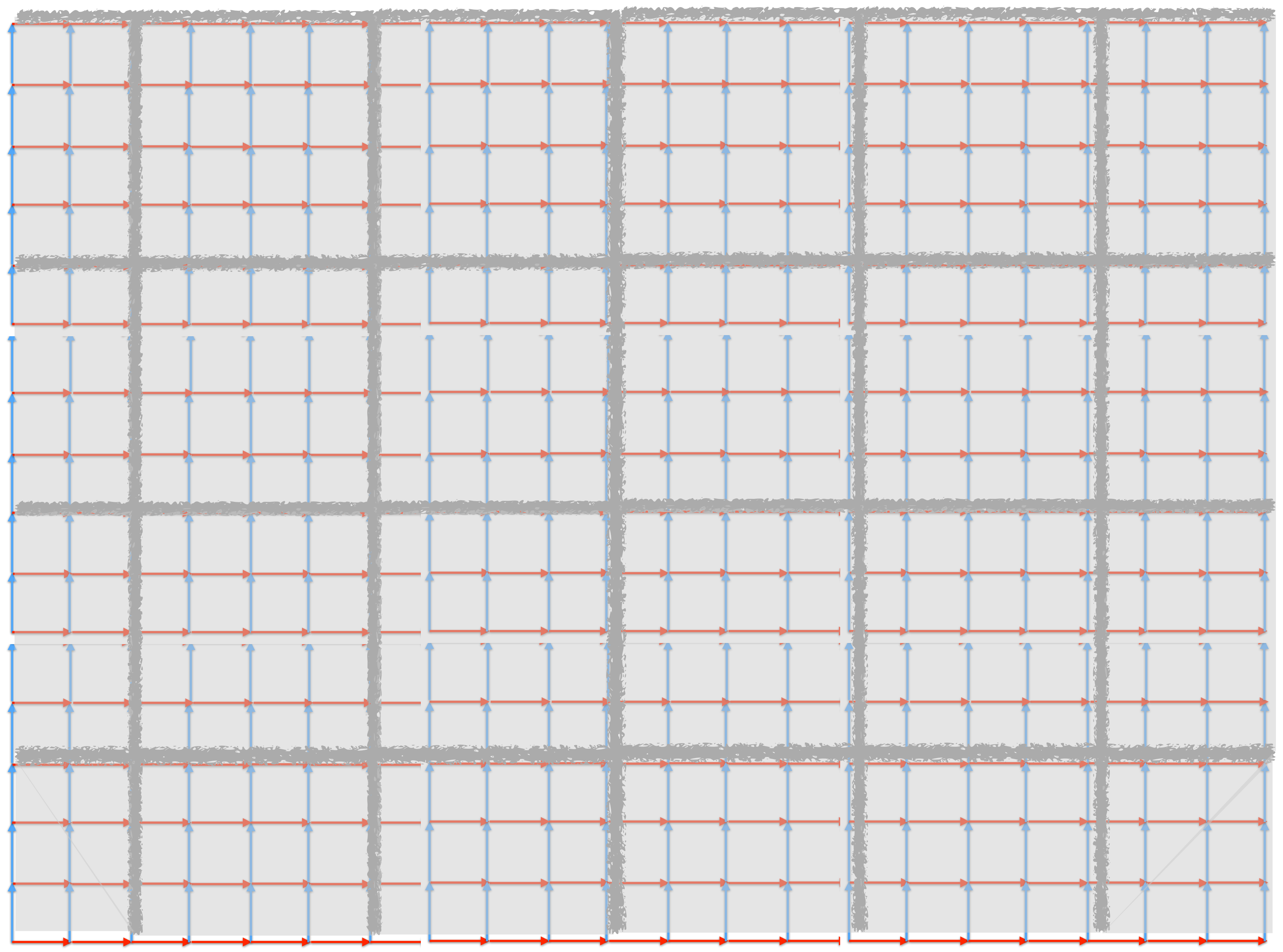

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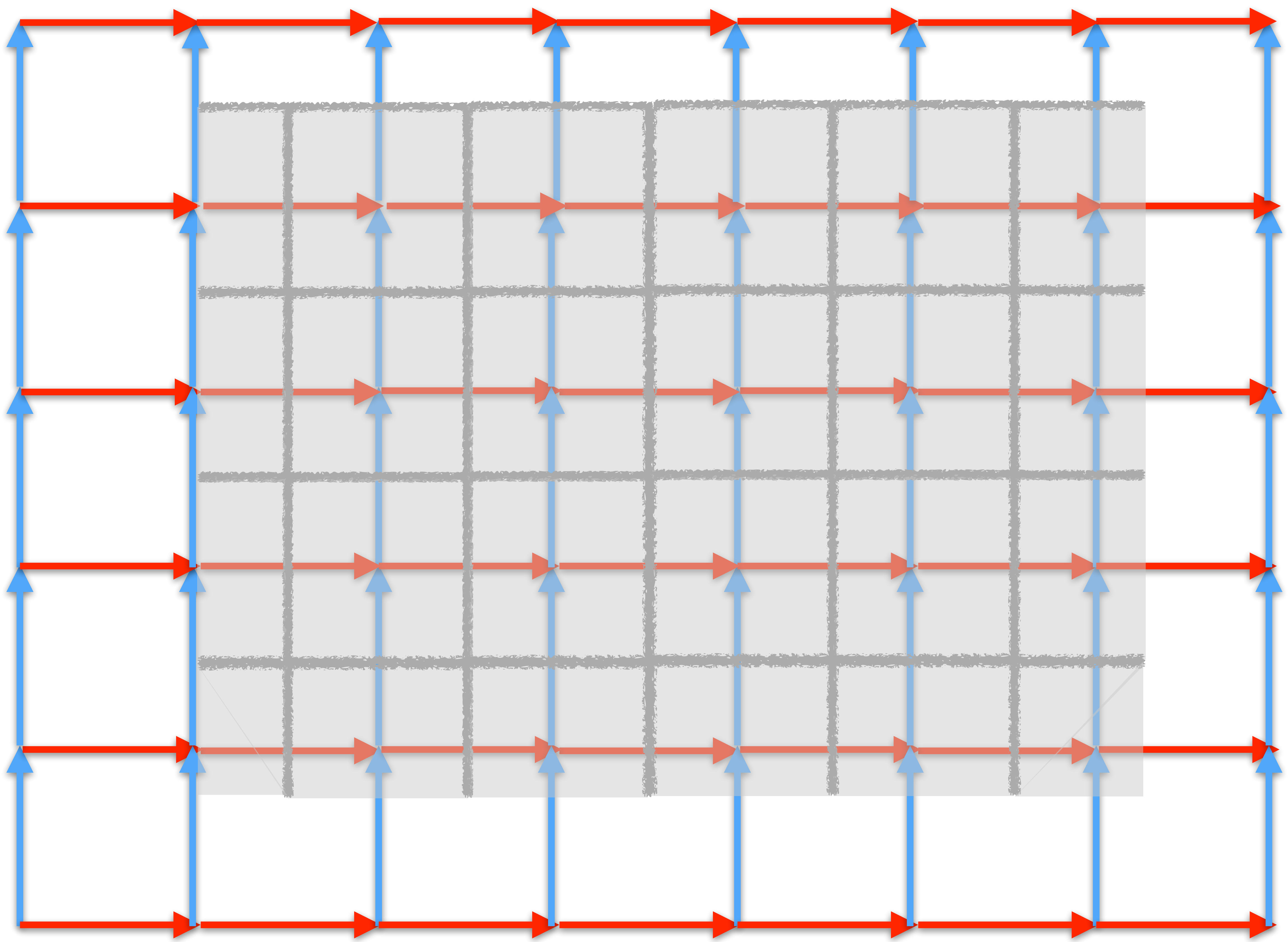

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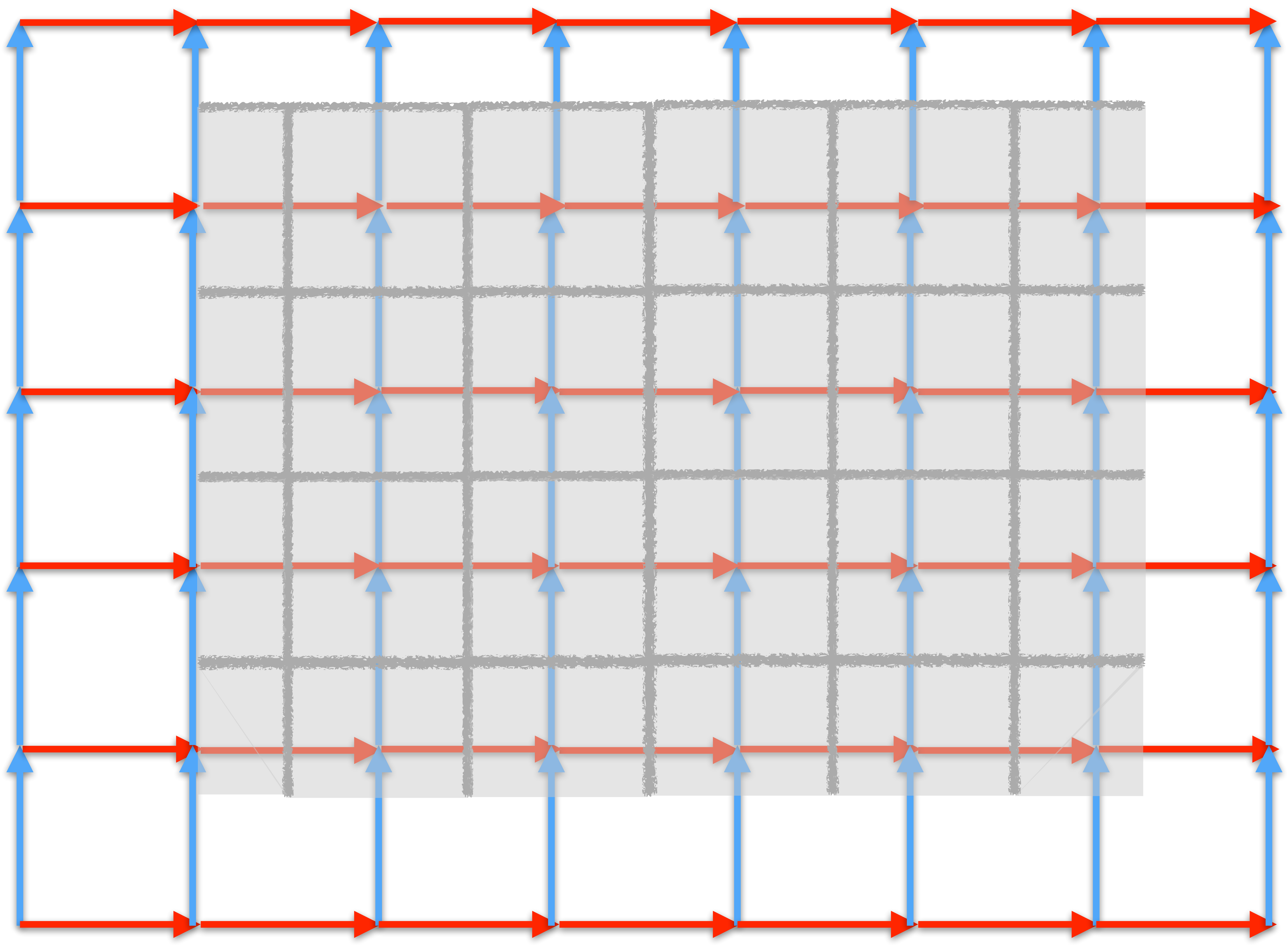

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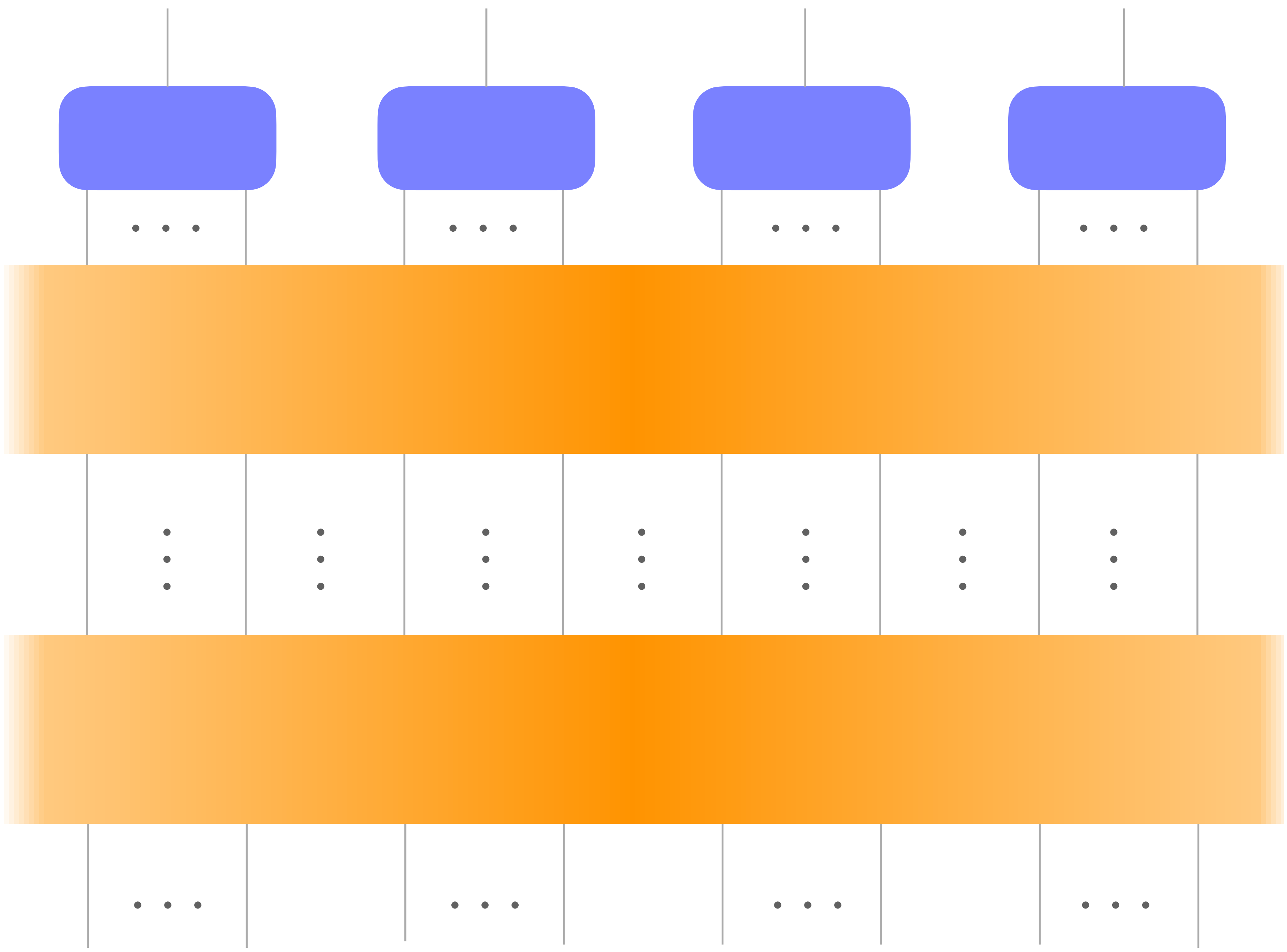
Renormalisation: QCA

$$\mathscr{J} \circ \mathscr{T}^N = \mathscr{S} \circ \mathscr{J}$$



- Intuitive idea:
  - Perform N steps of $\mathscr{T}$
  - Discard degrees of freedom (N cells are mapped to 1 cell)

L. S. Trezzini, A. Bisio, PP, Quantum 9, 1756 (2025)

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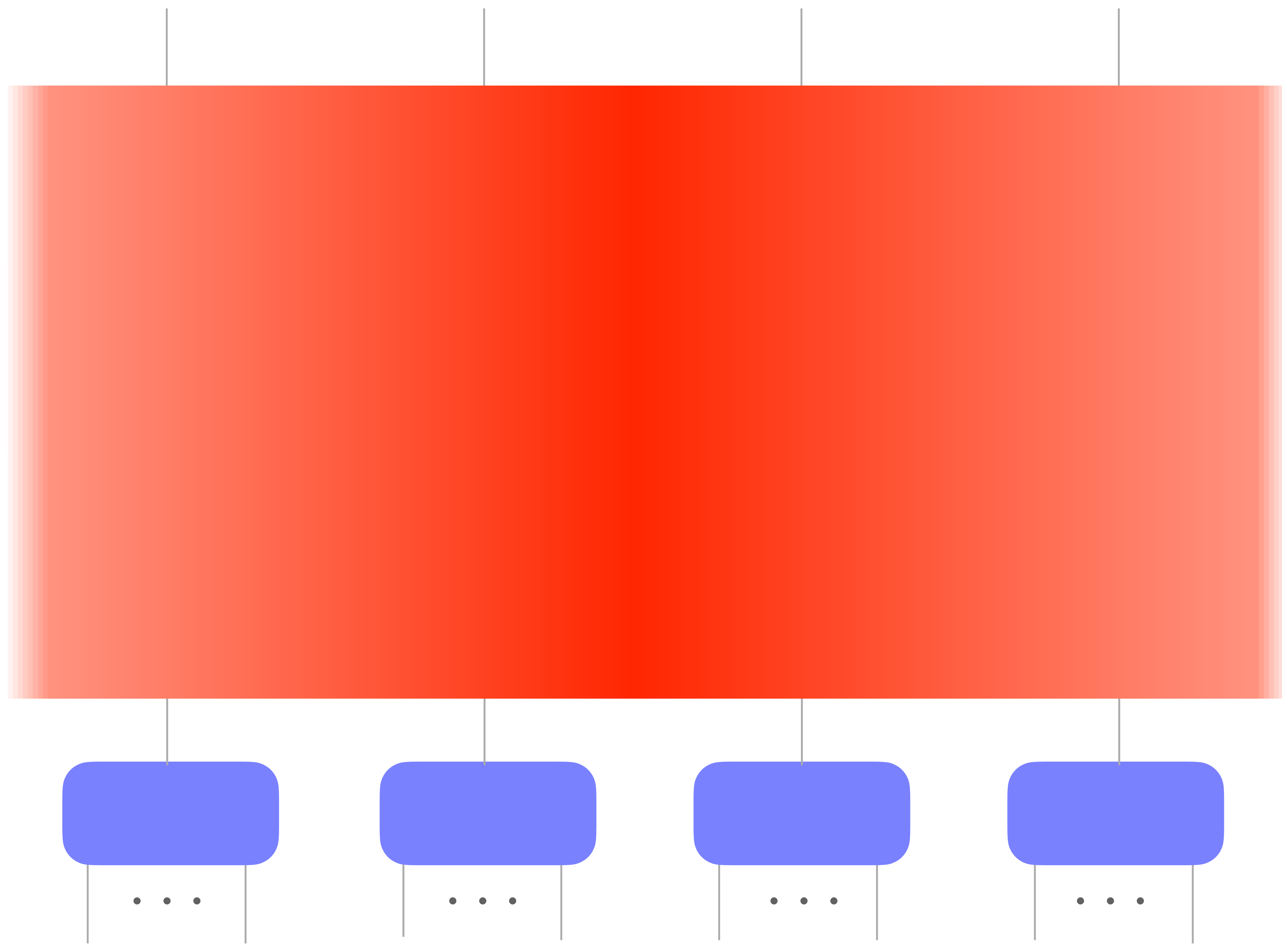
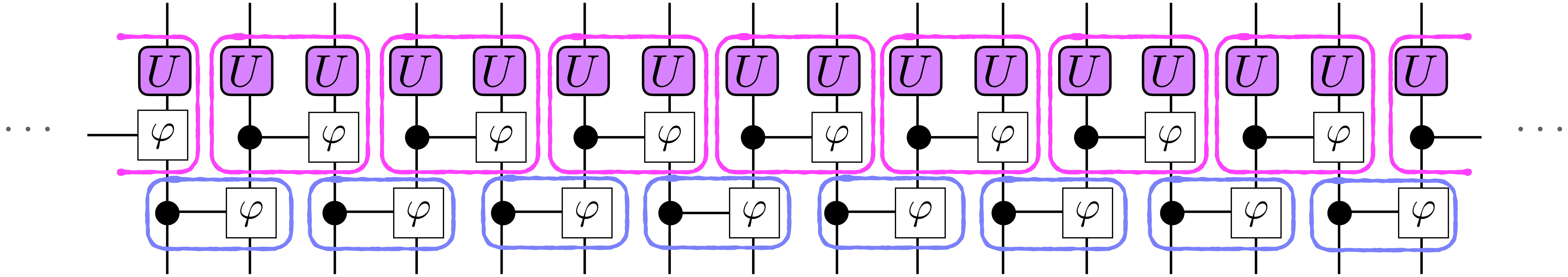


- Intuitive idea:
  - Perform N steps of $\mathscr{T}$
  - Discard degrees of freedom (N cells are mapped to 1 cell)
- This must be equal to
  - Discard degrees of freedom
  - Perform 1 step of $\mathscr{S}$

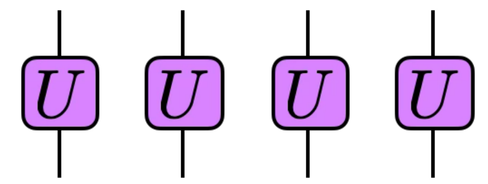
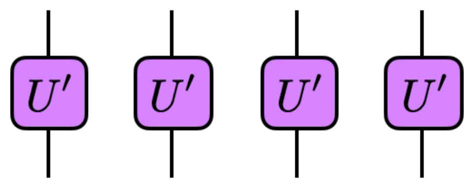
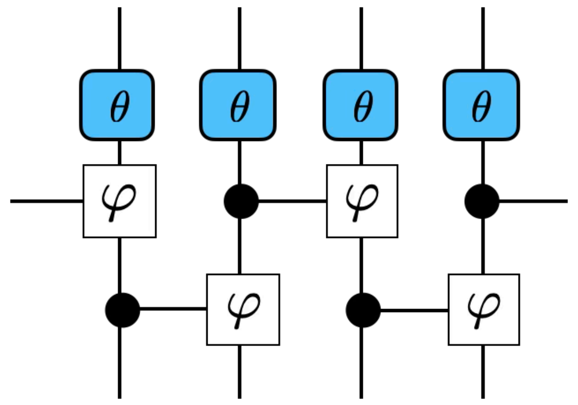
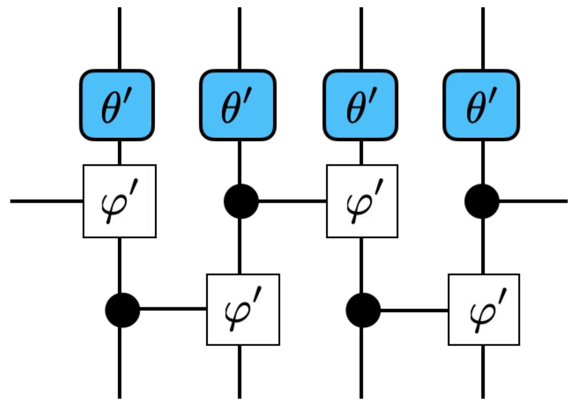
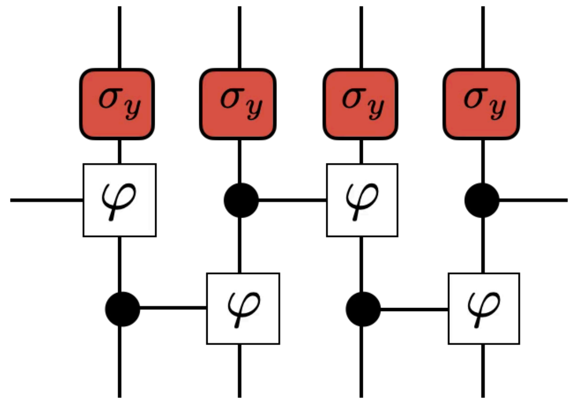
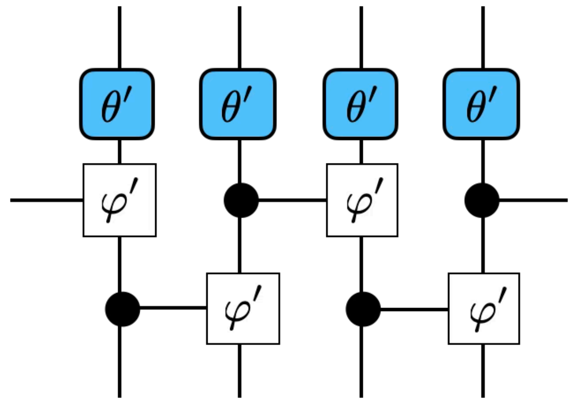
L. S. Trezzini, A. Bisio, PP, Quantum 9, 1756 (2025)

Case-study: qubit FDQCs



B. Schumacher and R.F. Werner, arXiv:quant-ph/040517

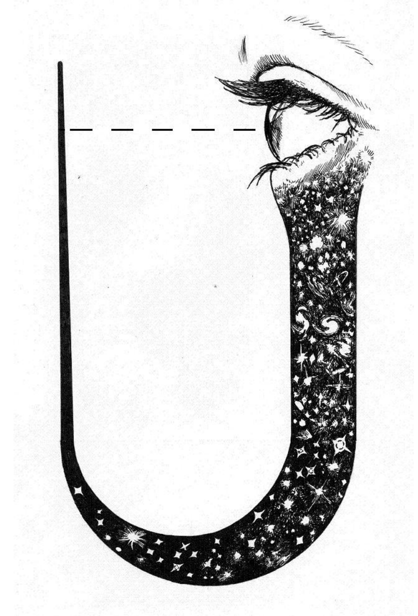
The renormalisation flow

Starting QCA	Renormalised QCA	Renormalisation Flow		
		$P = \|0\rangle\langle 0\| \otimes \|0\rangle\langle 0\| + \|1\rangle\langle 1\| \otimes \|1\rangle\langle 1\|$	$P = \|0\rangle\langle 0\| \otimes \|1\rangle\langle 1\| + \|1\rangle\langle 1\| \otimes \|0\rangle\langle 0\|$	$P = I \otimes \|c\rangle\langle c\|$
		$U' = U^2$	$U' = I$	$U' = U$
		$\begin{cases} \varphi' = 2\varphi \\ \theta' = \theta - 4\varphi \end{cases}$ $\begin{cases} \varphi' = 2\varphi \\ \theta' = 4\theta - 3\varphi \end{cases}$	$\begin{cases} \varphi' = -2\varphi \\ \theta' = \varphi \end{cases}$	$\begin{cases} \varphi' = 0 \\ \theta' = \pm(\theta + \delta_{c,0}\varphi) \end{cases}$
		$\begin{cases} \varphi' = 2\varphi \\ \theta' = -\varphi \end{cases}$	$\begin{cases} \varphi' = -2\varphi \\ \theta' = \varphi \end{cases}$	$\begin{cases} \varphi' = 0 \\ \theta' = \pm(2c - 1)\varphi \end{cases}$

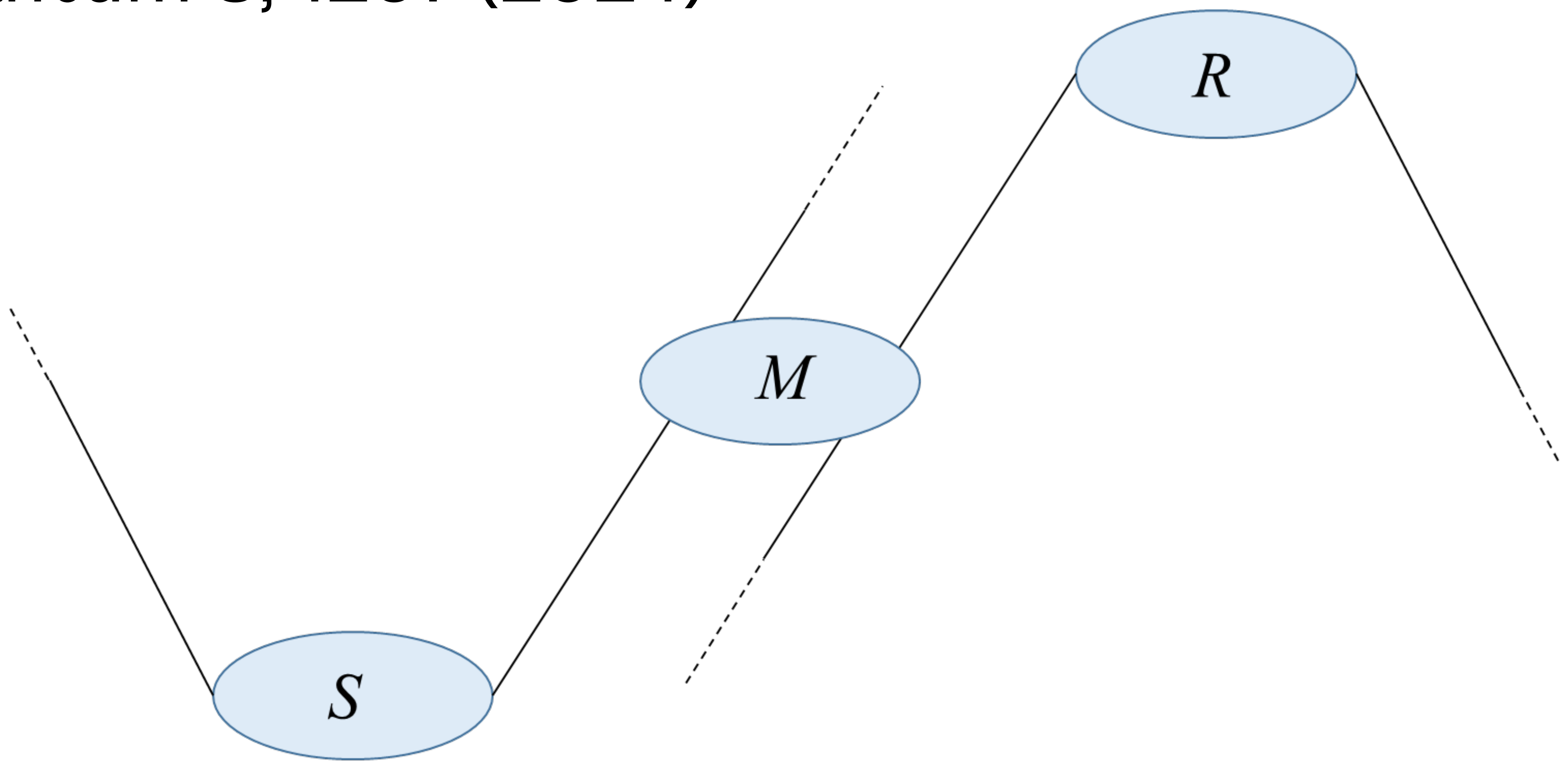
L. S. Trezzini, A. Bisio, PP, Quantum 9, 1756 (2025)

Local measurements

The observer

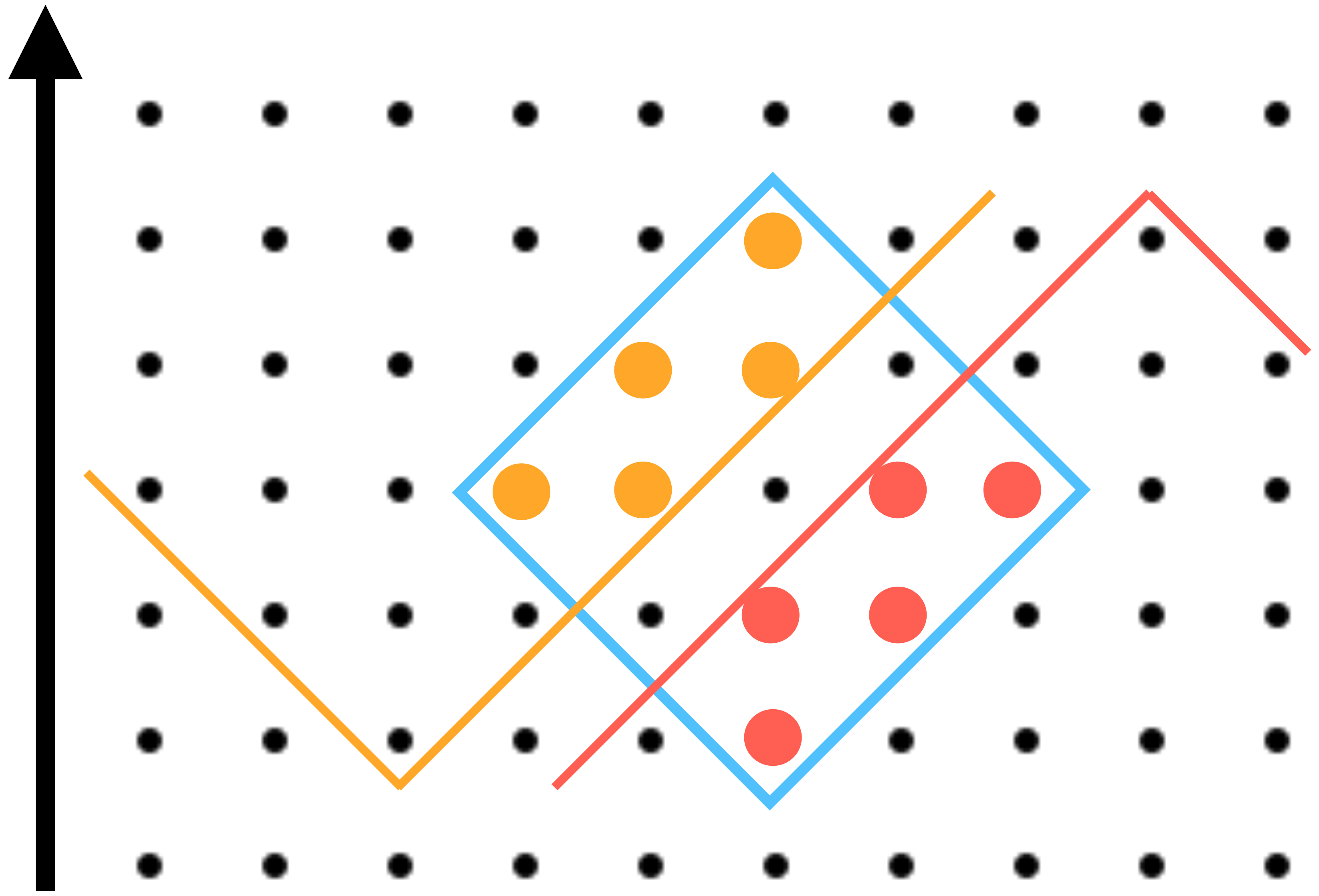


- Measurements in QFT produce paradoxical situations
  Y. Aharonov, D. Z. Albert, Phys. Rev. D 24, 359 (1981)
  S. Popescu, L. Vaidman, Phys. Rev. A 49, 4331 (1994)
  R. D. Sorkin, In “Directions in general relativity: Proc. 1993 Int Symp.” 293–305. (1993)
  N. Gisin, F. Del Santo, Quantum 8, 1267 (2024)



Measurement: a proposal

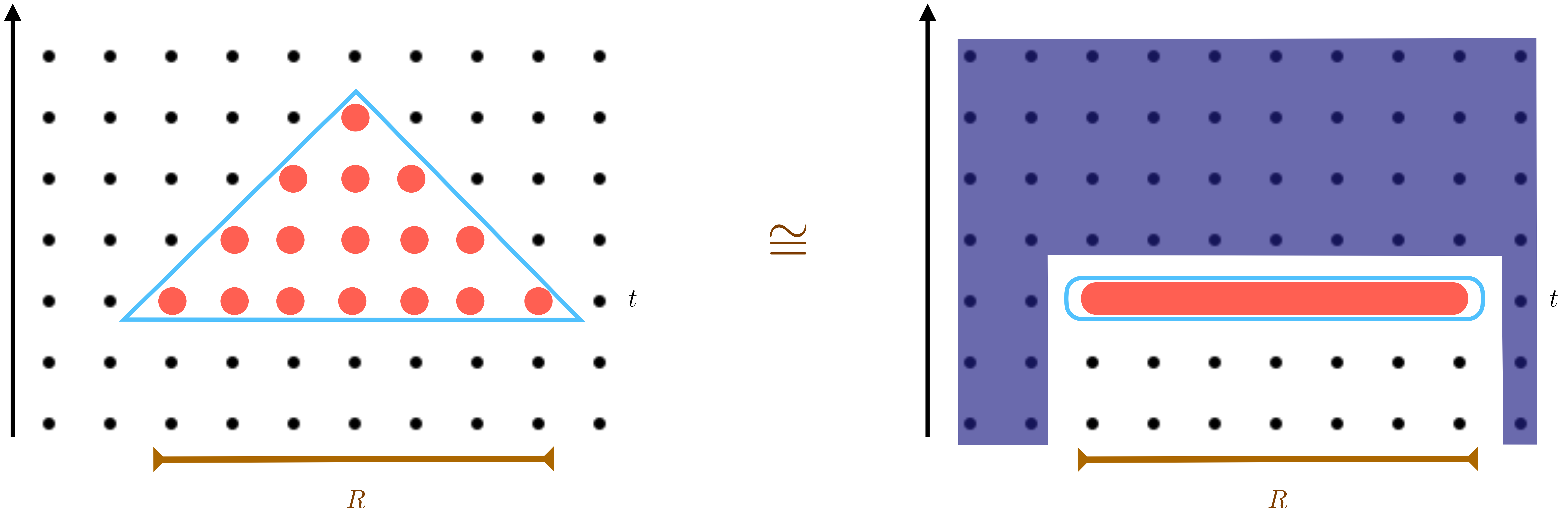
- The measurement process is supposed to be
  - external to the picture, local, arbitrary



T. Brambilla, A. Tosini, PP, in preparation

Question 1

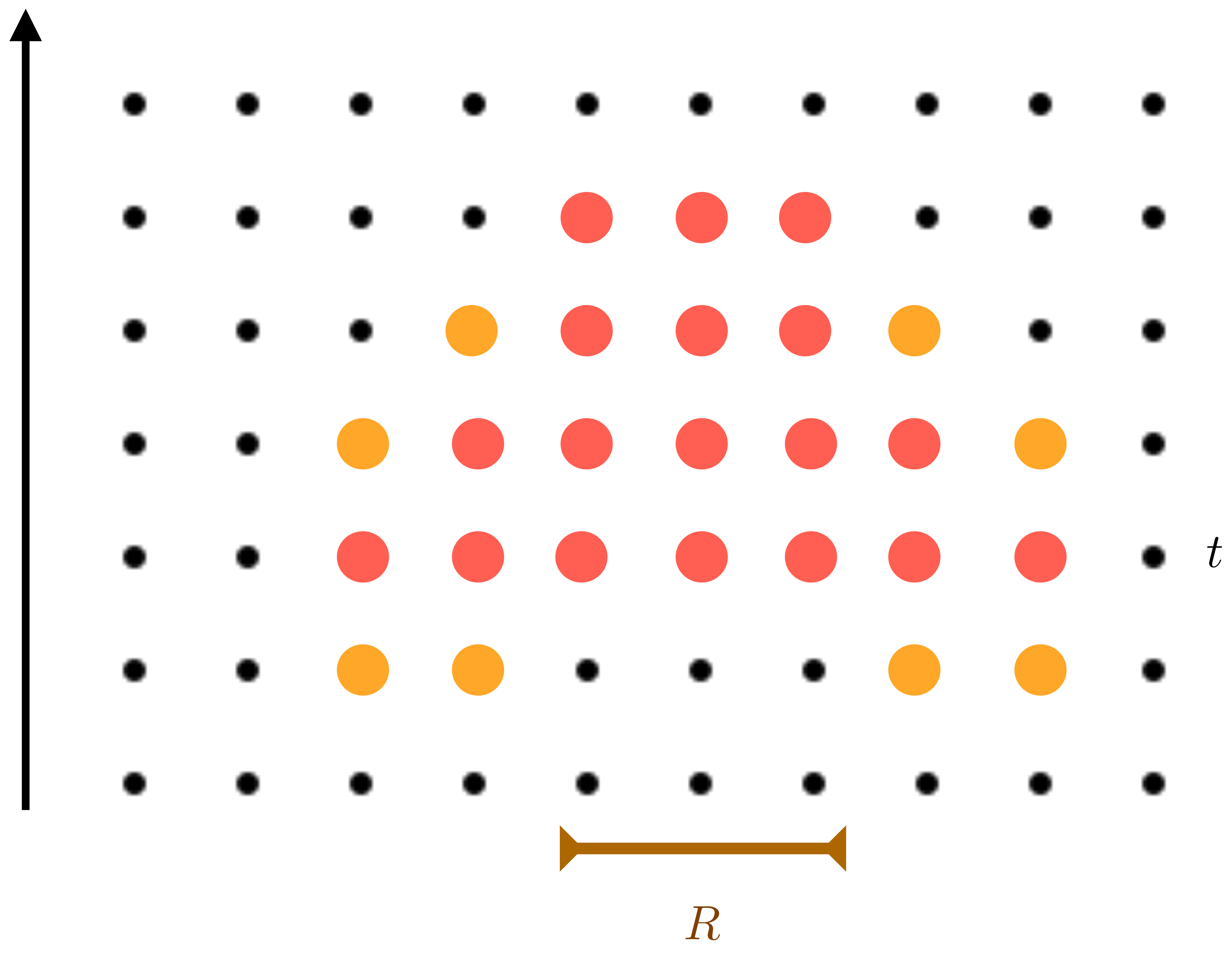
- What POVMs can be achieved on a region R at step t?



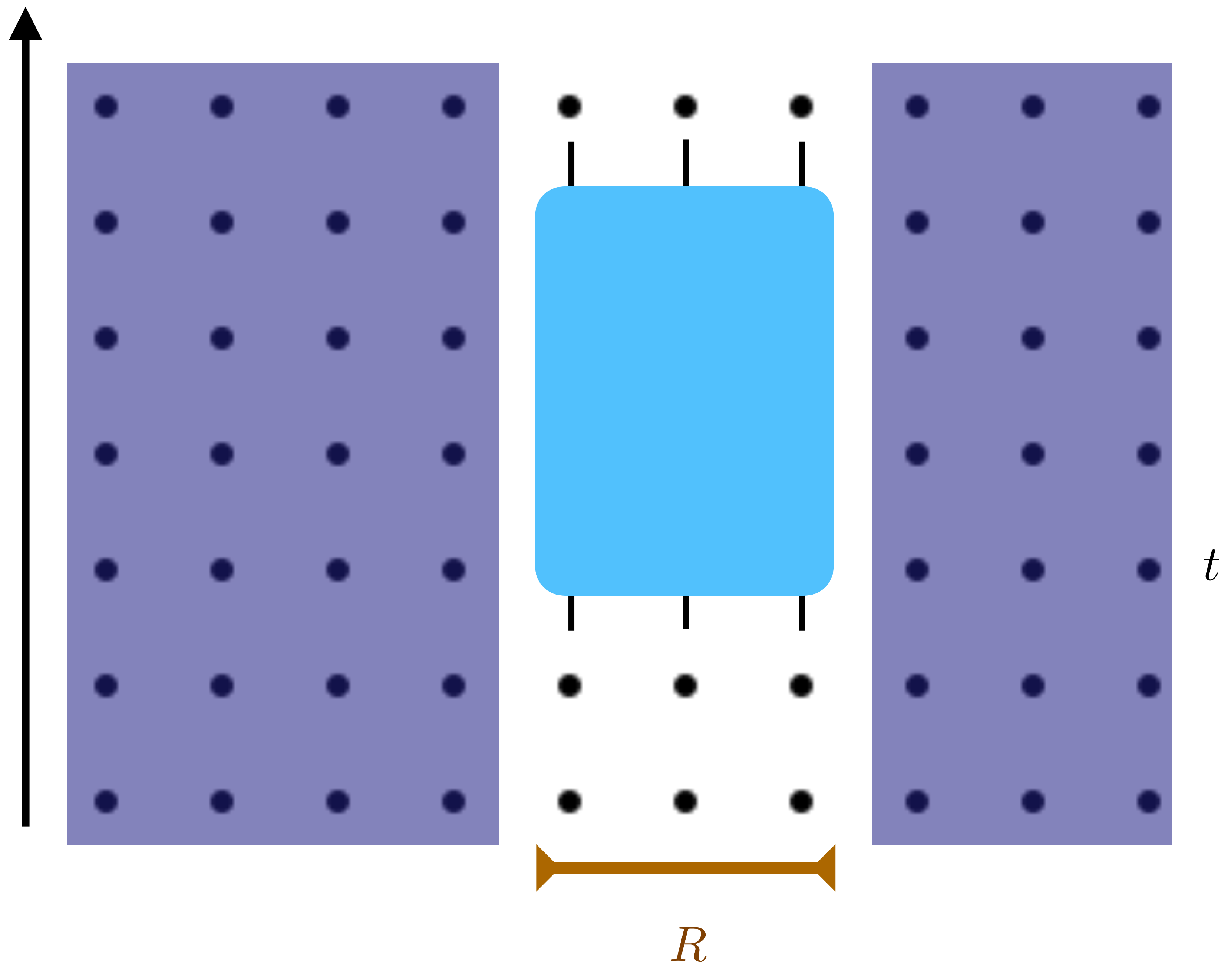
T. Brambilla, A. Tosini, PP, in preparation

Question 2

- What instruments can be achieved on a region R at step t?



$\cong$



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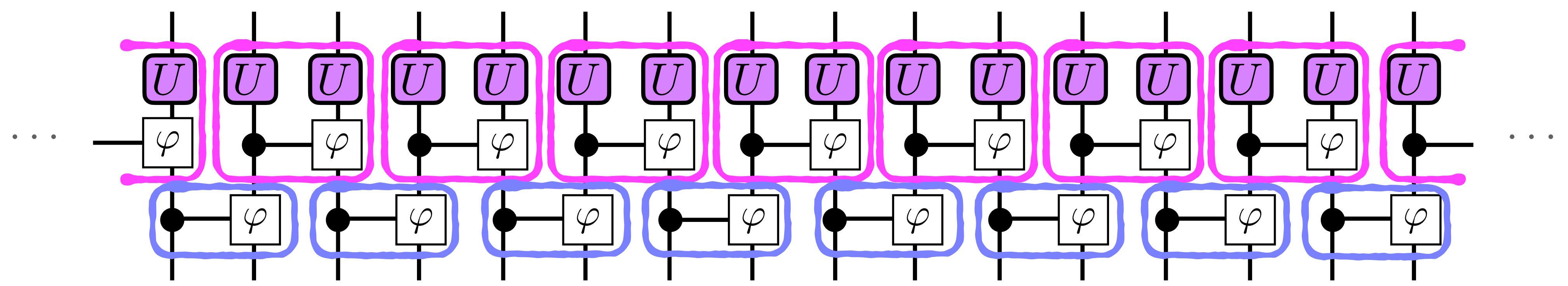
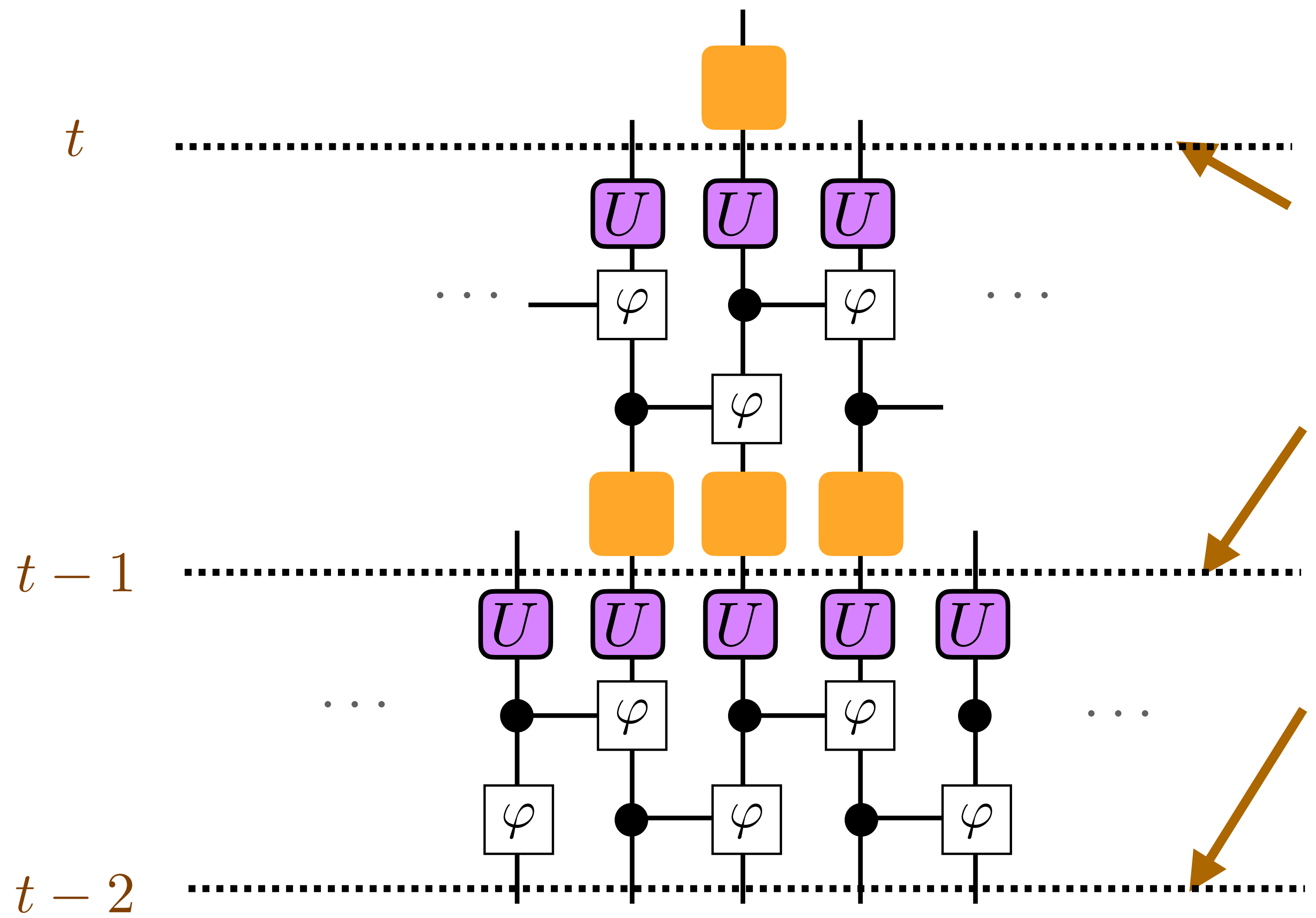
Results

- Case study: 1+1-d nearest neighbour qubit QCA
- Architecture:

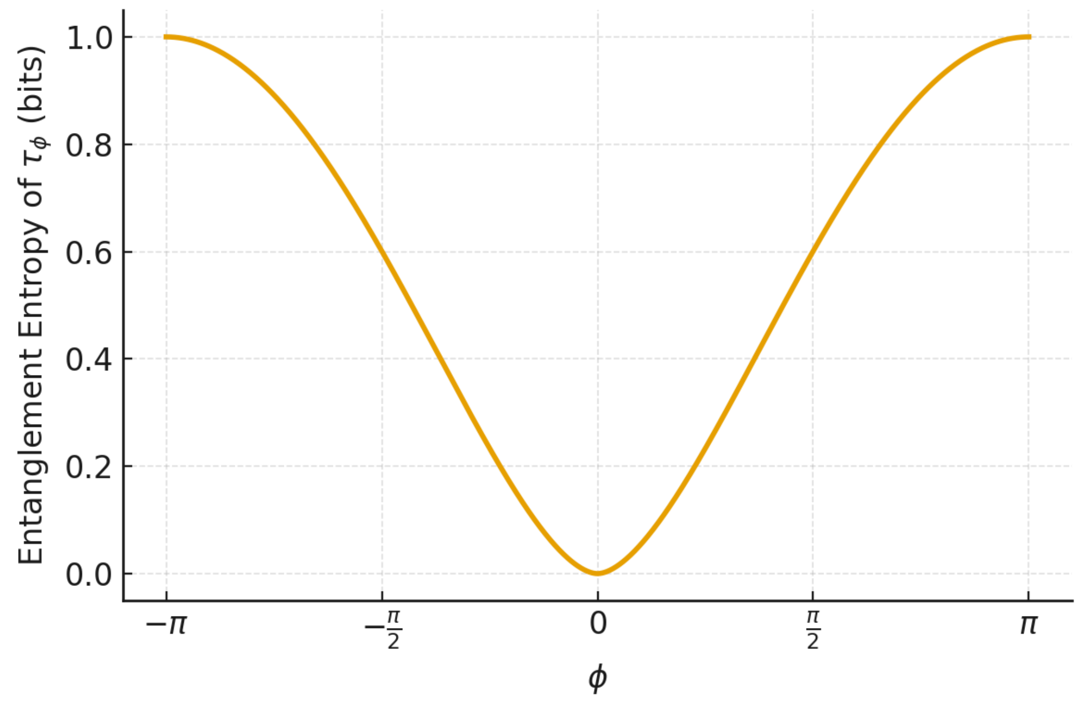


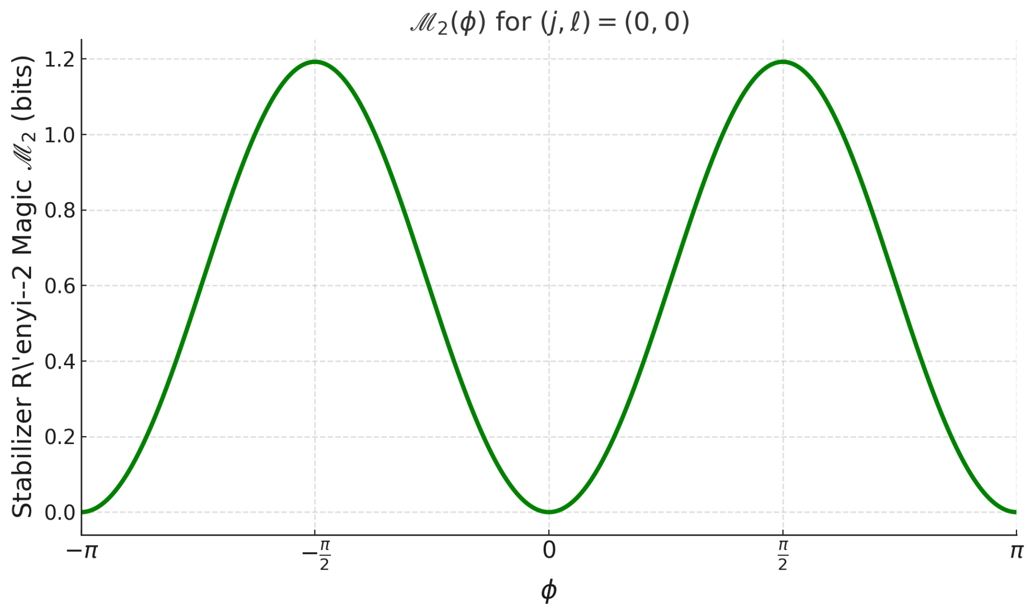


Local POVMs

LOCC POVMs

Entanglement/magicness





T. Brambilla, A. Tosini, PP, in preparation

The missing pieces

- How to reconstruct the dynamical laws in the emergent manifold?
  - Few simple cases worked out
  - We need a general technique
- Main ingredients
  - Classification of QCAs on a given graph
  - Representations of the (quasi-)local algebra
    (statistical mechanics/choice of vacuum)
  - Renormalisation
    (reconstruction of large-scale dynamics)

Conclusions and outlook

- Proof of concept
  - Mechanical notions can be recovered in an information processing context
  - Problem: appropriate notion of a “continuum limit”
  - Role of the observer:
    - local measurements
    - phrase the emergent laws in geometric and mechanical terms