Quantum Mechanics and the Large Reference Limit

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Plan

- ► Why QRFs?
- What are quantum states relativized to?
- ► Exploring internal reference frames
- QRFs and Wigner's Friend

A Worry About Emergence

- ► There are good reasons to think that (at least some) quantum descriptions are necessarily relativized.
- ► Theoretical results suggest that we only get the full quantum formalism in the limit as the reference becomes large (quasi-classical).
- ▶ If that is right, then there is a problem with trying to use the quantum formalism to describe the emergence of classical spacetime and the classical world - such descriptions are implicitly presupposing a pre-existing classical reference frame.
- ▶ What does it look like to describe the emergence of these things *without* presupposing a classical reference frame?

Why QRFs: Wigner's Friend

- Put Chidi in a box and have him perform a measurement on a system S in the state $a|0\rangle + b|1\rangle^1$.
- Alice has complete coherent control over the whole contents of the box.
- ▶ Applying unitary quantum mechanics from Alice's point of view, Chidi and *S* are now in the the state:

$$\psi_{SC} = a|0\rangle_S|0\rangle_C + b|1\rangle_S|1\rangle_C$$

▶ Alice could in principle confirm that they are in this state by performing tomographic measurements.

¹Eugene P. Wigner. "The Unreasonable Effectiveness of Mathematics in the Natural Sciences. Richard Courant lecture in mathematical sciences delivered at New York University, May 11, 1959". In: Communications on Pure and Applied Mathematics 13.1 (1960), pp. 1–14. ISSN: 1097-0312. DOI: 10.1002/cpa.3160130102. URL: http://dx.doi.org/10.1002/cpa.3160130102.

Why QRFs: Wigner's Friend

- Suppose unitary quantum mechanics is always correct for both Alice and Chidi (the wavefunction doesn't collapse for Alice except when Alice performs a measurement, and the same for Chidi)
- ► The state of *S* which correctly predicts Chidi's measurements is:

Either $|0\rangle_{\mathcal{S}}$ or $|1\rangle_{\mathcal{S}},$ depending on the measurement outcome.

► The state of *S* which correctly predicts Alice's measurements is:

$$\psi_{\mathcal{S}} = |a|^2 |0\rangle \langle 0|_{\mathcal{S}} + |b|^2 |1\rangle \langle 1|_{\mathcal{S}}$$



Why QRFs: Wigner's Friend

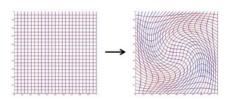
- ▶ If unitary quantum mechanics is always correct for all observers, some relativization of states is inevitable.
- In Everett, we have 'relative states.'
- ▶ In de Broglie-Bohm, we have conditional wavefunctions².
- Relational quantum mechanics relativizes states in a more radical way - there are no observer-independent states³

²Detlef Durr, Sheldon Goldstein, and Nino Zanghi. "Quantum equilibrium and the origin of absolute uncertainty". In: Journal of Statistical Physics 67.5-6 (1992), 843–907. ISSN: 1572-9613. DOI: 10.1007/bf01049004. URL: http://dx.doi.org/10.1007/bF01049004, Travis Norsen. "Bohmian Conditional Wave Functions (and the status of the quantum state)". In: Journal of Physics: Conference Series 701 (Mar. 2016), p. 012003. DOI: 10.1088/1742-6596/701/1/012003.

³Carlo Rovelli. "Relational quantum mechanics". In: International Journal of Theoretical Physics 35.8 (1996), 1637–1678. ISSN: 1572-9575. DOI: 10.1007/bf02302261. URL: http://dx.doicorg/1091007/bf02302261.

- ► In classical physics, physical systems are in effect described relative to a fixed spacetime background.
- In a relativistic context, we no longer have a fixed spacetime background.

- The standard interpretation of General Relativity says that histories related by a diffeomorphism are physically identical.
- ➤ This means that variables defined at individual coordinates, like 'the momentum of the particle at time t' are not physically real
- Descriptions of things relative to spacetime points are not meaningful



- ► Two classes of variables which *are* physically real:
 - ▶ Highly nonlocal quantities defined over the whole of spacetime
 - Relational quantities built out of correlations between field values
- Most things we observe in our ordinary lives will be relational observables.
- ▶ 'The momentum of the particle at time $t' \rightarrow$ 'the momentum of the particle, relative to the clock reading time t.'

- So we must assign properties relative to other physical systems - rods and clocks, etc.
- Quantum states describe properties of systems, so they must be relativized too.
- But if quantum mechanics is universal, then all possible reference systems are quantum.
- We must learn to describe quantum systems relative to other quantum systems.

Why QRFs? Combining gravity + quantum

Undertanding quantum reference frames and relational quantum descriptions is likely to help with attempts at formulating a quantum gravity theory.

Vidotto:

The relationality that characterizes the relational interpretation of quantum mechanics is in fact not so unconventional after all. Rather, it characterizes modern physics ... relationality is present, perhaps in a transversal way, in virtually all aspects of contemporary physics.⁴

What are quantum states relativized to?

Wigner's Friend suggests they are sometimes relativized to observers. But what else?

- Anthropocentric: quantum states are relativized to (human) observers
 Neo-Copenhagen approaches⁵, maybe QBism⁶.
- Quantum universalism: quantum states can be relativized to any physical system Relational quantum mechanics⁷, perspectival modal approaches⁸, quantum reference frame formalism⁹

⁵Anton Zeilinger. "A Foundational Principle for Quantum Mechanics". In: Foundations of Physics 29.4 (1999), pp. 631–643. DOI: 10.1023/A:1018820410908, Časlav Brukner. "On the Quantum Measurement Problem". In: Quantum [Un]Speakables II: Half a Century of Bell's Theorem. Ed. by Reinhold Bertlmann and Anton Zeilinger. Cham: Springer International Publishing, 2017, pp. 95–117.

⁶C. A. Fuchs, N. D. Mermin, and R. Schack. "An introduction to QBism with an application to the locality of quantum mechanics". In: American Journal of Physics 82 (Aug. 2014), pp. 749–754. DOI: 10.1119/1.4874855. arXiv: 1311.5253 [quant-ph].

⁷A. Di Biagio and C. Rovelli. "Relational Quantum Mechanics is about Facts, not States: A reply to Pienaar and Brukner". In: Foundations of Physics 62.52 (2022). URL: https://doi.org/10.1007/s10701-022-00579-5.

⁸Gyula Bene and Dennis Dieks. "A Perspectival Version of the Modal Interpretation of Quantum Mechanics and the Origin of Macroscopic Behavior". In: Foundations of Physics 32.5 (2001), pp. 645–671. DOI: 10.1023/a:1016014008418, Simon Kochen et al. "A new interpretation of quantum mechanics". In: Symposium on the foundations of modern physics. Vol. 50. 1985, pp. 151–169.

⁹Marius Krumm, Philipp A. Höhn, and Markus P. Müller. "Quantum reference frame transformations as symmetries and the paradox of the third particle". In: *Quantum* 5 (2021), p. 530. ISSN: 5221-327X. DOI: 10. 22331/q-2021-08-27-530. URL: http://dx.doi.org/10.22331/q-2021-08-27-530. ⊌ ₱ ♭ ⟨ ₱ ♭ ⟩

What are states relativized *to*?

- Anthropocentric: not appealing for realists, not a good fit for other relational ideas in physics.
- Universalism: most physical systems don't have enough resources to meaningfully define quantum states

Brukner: Qubits are not observers¹⁰

- ▶ In RQM, when two systems interact, an 'observation' is made - a variable of one system becomes definite relative to the other system
- ▶ But in what basis?
- When two quantum systems interact and become entangled, there is always a basis (the 'Schmidt basis') in which the resulting state looks particularly simple:

Schmidt basis:

$$\alpha |0\rangle |0\rangle + \beta |1\rangle |1\rangle$$

Not Schmidt basis:

$$\alpha |0\rangle |0\rangle + \gamma |0\rangle |1\rangle + \eta |1\rangle |0\rangle + \beta |1\rangle |1\rangle$$

Brukner: Qubits are not observers

- ▶ The Schmidt basis is not unique if there is degeneracy.
- For large systems, assuming only a small number of coefficients are degenerate, the Schmidt basis can still be approximately unique - or we can just appeal to the basis favoured by decoherence.
- But for a pair of maximally entangled qubits the Schmidt basis is not even approximately unique.

Alternative: Emergence

- We can always give a description of one physical system relative to another physical system, but this approximates the full quantum formalism only in the limit as at least one of the systems becomes large.
- ▶ The quantum formalism is the right way to describe the physics of microscopic particles relative to a conscious human observer, but it is not necessarily the right way to characterize microscopic particles relative to other microscopic systems
- ▶ The state of a system S_1 relative to another system S_2 should reflect the actual physical resources available to S_2 .
- ➤ A qubit does not have many resources, so the state relative to a qubit should be simple.

Alternative: Emergence

- Shares with anthropocentricism the idea that at least some characteristics of the quantum formalism reflect special features of human observers rather than intrinsic features of the systems we are trying to describe.
- ► Shares with universalism the idea that human observers shouldn't be privileged in the formulation of the theory

QRFs

- ► The structural approach aims to break the redundancy that arises when we are trying to describe the quantum state of a system *S* without external reference frames¹¹, ¹².
- ► Choose a subsystem of *S* and use symmetry transformations to write down the state of the rest of *S* relative to the subsystem, thus fixing a unique representation for this state.
- ► These are coordinate transformations, not attempts to write down a realistic 'perspective' for the relevant system.
- They do not depend on the available resources.
- One may finally ask whether observers who are part of the theory may in fact have good operational access to that chosen frame of reference, but this is an additional (though important) question that we here regard as secondary'¹³

¹³Marius Krumm, Philipp A. Höhn, and Markus P. Müller. "Quantum reference frame transformations as symmetries and the paradox of the third particle". In: Quantum 5 (2021), p. 530, ISSN: 2521-327X. DOIL: 10.2231/q-2021-08-27-530. ISSN: 2521-327X. DOIL: 10.2231/q-2021-08-27-530.



¹¹Flaminia Giacomini, Esteban Castro-Ruiz, and Caslav Brukner. "Quantum mechanics and the covariance of physical laws in quantum reference frames". In: Nature Communications 10 (Jan. 2019). DOI: 10.1038/s41467-018-08155-0.

¹²Augustin Vanrietvelde et al. "A change of perspective: switching quantum reference frames via a perspective-neutral framework". In: Quantum 4 (2020), p. 225. ISSN: 2521-327X. DOI: 10.22331/q-2020-01-27-225. URL: http://dx.doi.org/10.22331/q-2020-01-27-225.

QRFs

- ► The quantum information (QI) approach is primarily applied in scenarios where an observer does not have access to a reference frame for some system S and the goal is to understand what operational tasks the observer can complete, e.g. what information they can extract from S¹⁴, ¹⁵.
- Does seem to be describing a perspective rather than merely applying a coordinate transformation.
- However, 'in the QI approach, it is usually not necessary to take the extra step to internal frame choices and to ask how a system is described relative to one of it'16.

¹⁴Stephen D. Bartlett, Terry Rudolph, and Robert W. Spekkens. "Reference frames, superselection rules, and quantum information". In: Reviews of Modern Physics 79.2 (Apr. 2007), 555–609. ISSN: 1539-0756. DOI: 10.1103/revmodphys.79.555. URL: http://dx.doi.org/10.1103/RevModPhys.79.555.

¹⁵Alexander R. H. Smith. "Communicating without shared reference frames". In: *Phys. Rev. A* 99 (5 2019), p. 052315. DOI: 10.1103/PhysRevA.99.052315. URL: https://link.aps.org/doi/10.1103/PhysRevA.99.052315.

¹⁶ Marius Krumm, Philipp A. Höhn, and Markus P. Müller. "Quantum reference frame transformations as symmetries and the paradox of the third particle". In: Quantum 5 (2021), p. 530. ISSN: 2521-327X. DOI: 10.22331/q-2021-08-27-530. URL: http://dx.doi.org/10.22331/q-2021-08-27-580. □ ♣ ♣ ♣ ♣ ♣ ♣ ♣

Principles

- ► What does the 'internal' state of one system relative to another look like?
- Intrinsicality: the internal state of S relative to R should contain only information about the relationship between R and S by themselves, i.e. it should be definable without reference to any external reference frame.
- ▶ Resource-relativity: the internal state of *S* relative to *R* should scale with the resources available to *R*, i.e. the amount of information encoded in the internal state of *S* relative to *R* cannot be greater than the information that could be extracted about the relation between *S* and *R* by an external classical observer.

Group-averaging

- Suppose our system and reference are subject to a symmetry group *G* − e.g. spatial translations.
- We can get rid of redundant information by averaging over states in such a way that all states related by symmetry transformations are mapped to a single state.
- Incoherent group-averaging (or the 'G-twirl') maps a density matrix ρ to another density matrix ρ' , producing a mixture over all eigenspaces of the generator of the group, as follows:

$$\rho' = \int_{G} U(g) \rho U(g)^{\dagger} dg$$

► (There is also a coherent version; I'm using incoherent here since it is generally accepted as the right approach for the scenario of an external observer without a reference frame).

Principles

Suppose G is the symmetry group of $S \otimes R$, with unitary representation U(g). Let P be a property of S, and let p(P) be the probability distribution over the possible values for P predicted by the internal state of S relative to R. Then there is some group-averaged state $\rho' = \int_G U(g)\rho U(g)^{\dagger}$, and a possible POVM $\{O_i\}$ on $S \otimes R$, such that p(P) is isomorphic to the probability distribution $\{Tr(\rho'M_i)\}$.

- ► Spin-1/2 *S*, and a reference *R* consisting of a single spin-*g* system symmetry group the spatial rotations.
- ▶ Relative to the external reference frame, S is in state $\psi = \alpha |0\rangle + \beta |1\rangle$ and R is in an eigenstate of maximal spin along the $|0\rangle$ axis.
- The probability distribution over a value of spin encoded in the internal state of S relative to R must be isomorphic to $\{Tr(\rho'M_i)\}$ for some state $\rho' = \int_G U(g)\rho U(g)^{\dagger}$, and some POVM $\{O_i\}$ on $S\otimes R$. To get something nontrivial, we need the POVM operators to commute with the rotation group.
- ► Total angular momentum commutes with incoherent group averaging and can be thought of as measuring whether they are parallel or antiparallel.

▶ They are found to be parallel with probability:

$$|\alpha|^2 + \frac{|\beta|^2}{2g+1}$$

and antiparallel:

$$\frac{2g|\beta|^2}{2g+1}$$

▶ For small g, probabilities diverge from $|\alpha|^2$ and $|\beta|^2$

So what is the state?

$$\sqrt{|\alpha|^2+\frac{|\beta|^2}{2\mathsf{g}+1}}|0\rangle+\sqrt{\frac{2\mathsf{g}|\beta|^2}{2\mathsf{g}+1}}|1\rangle$$

- No possibility of a relative phase or a mixed state.
- General form

$$\rho = \frac{1}{2} \begin{bmatrix} 1 - \frac{2g|\beta|^2}{2g+1} & a_x - ia_y \\ a_x + ia_y & 1 + \frac{2g|\beta|^2}{2g+1} \end{bmatrix}$$

- There are no other measurements we can do here to get nontrivial probabilities.
- ▶ There is no way to fix a_x or a_y . We can't really get a quantum state at all just a classical probability distribution over the outcomes of one measurement.

- ► We can add some more spins to our reference system suppose we add two orthogonal spins, also of spin *g*.
- ► Then we have three measurements, checking if *S* is parallel to each of the three spins.
- ► Using these to fix the parameters, we get a density matrix with Bloch vector:

$$\left[1 - \frac{2g}{2g+1}|\alpha - \beta|^2, 1 - \frac{2g}{2g+1}|\alpha + i\beta|^2, 1 - \frac{4g|\beta|^2}{2g+1}\right]$$

- Still not a quantum state, since its magnitude is greater than 1.
- ▶ But magnitude approaches 1 as g goes to infinity, so as our three spins become large we do get a quantum state.

General Case

► Looking at features of the group-averaged state:

$$ho_{SR}' = \int_G U^S(g) U^R(g)
ho_{SR} U^R(g)^\dagger U^S(g)^\dagger dg$$

- ▶ The information content of the internal state of S relative to R should be equal to $I(S:R)_{\rho'_{SR}}$, the mutual information between R and S in some group-averaged state ρ'_{SR} .
- ► Following the quantum convention, the maximum information content of a state in the internal state space is the log of the dimension of the internal state space.

General Case

- ► The group-averaged state is block-diagonal, with each block corresponding to one of the invariant subspaces¹⁷. The von Neumann entropy of this matrix is simply the sum of the von Neumann entropies of the blocks.
- If there are no non-trivial invariant subspaces, $I(S:R)_{
 ho'_{SR}}=0$.
- In the case where S and R do not individually have any nontrivial invariant subspaces but $S \otimes R$ has one nontrivial invariant subspace of dimension n_{SR} we can show that the mutual information obeys:
 - $\max_{\rho_{SR}} I(S:R)_{\rho_{SR}'} = \max_{\rho_{SR}} (\ln(n_{SR}) S(\xi_{\rho_{SR}'})) = \ln(n_{SR})$
- ▶ So the dimension of the internal state space of S relative to R is upper bounded by the dimension of the invariant subspace of $S \otimes R$.

¹⁷Stephen D. Bartlett, Terry Rudolph, and Robert W. Spekkens. "Reference frames, superselection rules, and quantum information". In: Reviews of Modern Physics 79.2 (Apr. 2007), 555–609. ISSN: 1539-0756. DOI: 10.1103/reymodphys.79.555. URL: http://dx.doi.org/10.1103/RevModPhys.79.5558 → 4 ラ → 4 ラ → 5 ⇒

General Case

- ► There is no invariant subspace for 1, 2, or 3 qubits. But there is a subspace of dimension 2 for 4 qubits.
- ➤ This matches the prior result we need a reference made up of three independent qubits to get a full quantum state space for one qubit, i.e. 4 qubits in total to get a single qubit internal state space.
- For an array of N qubits, if X is the number of qubits we can encode in the invariant subspace, then the ratio of X to N behaves as $1-\frac{3}{2}\frac{\log(N)}{N}$ in the limit of large N
- ➤ So for a large enough number of qubits the internal state space will fit the whole state and we recover the original quantum description.

- ▶ We can straightforwardly solve Brukner's problem.
- ► The internal state of one qubit relative to another is just a classical probability distribution over two outcomes.
- ► There is only one possible measurement no preferred basis problem.

- This poses problems for emergence involving quantum mechanics.
- The emergence of the classical world, and the emergence of spacetime.
- If quantum states must be relativized to a macroscopic system, then quantum states of spacetime must also be relativized to a macroscopic system.
- A 'macroscopic system' necessarily presupposes a background spacetime.
- ➤ So there is a potential circularity in attempts to get spacetime to emerge from a quantum description.

- Of course this varies framework by framework.
- We may start with some substratum and then go directly from that substratum to appropriate relational descriptions, in which case perhaps the substratum itself does not need to be relativized to a macroscopic system.
- ▶ But the points above give us reason to be careful about the assumptions we make about the nature of that substratum.
- ▶ If the state of one qubit relative to another is not 'quantum' in the ordinary sense, the intrinsic description of two qubits is not 'quantum' the intrinsic description of things much smaller than qubits might not be 'quantum' either.

- ► We should keep in mind the possibility that spacetime might emerge relative to reference frames, and may not be well-defined in the absence of a reference frame.
- ➤ This can be illustrated by considering a Wigner's Friend scenario where the variable measured in spatiotemporal - e.g. spin, associated with direction in spacetime and the rotation group.

- ► The Bong et al paradox¹⁸.
- ▶ Alice and Chidi perform measurements on two halves of an entangled two-qubit state, and then Bob keeps Alice's laboratory in a coherent state and performs a measurement on it, while a fourth observer Divya keeps Chidi's laboratory in a coherent state and performs a measurement on it.
- For certain choices of measurements for the four observers, there is no possible probability distribution over the four measurement outcomes A, B, C, D which reproduces all the pairwise predictions of quantum mechanics for the pairs $\{(B, D), (B, C), (A, D), (A, C)\}$, assuming locality, no-retrocausality, no-superdeterminism.

- ► Of course we can straightforwardly resolve this paradox by just allowing non-locality or retrocausality.
- ▶ But suppose we want a more specifically 'relational' solution.

- We could deny 'absoluteness of observed events,' i.e. there is no fact of the matter about the relation between Alice's observation and Bob's observation so no probability distribution over all four together.
- ▶ But this leads to some severe epistemic problems.

- ▶ Alternative: there is no fact of the matter about the orientation of Alice's axis relative to Bob's, but there is a fact of the matter about what Alice has seen.
- ► Either she saw that the spin is parallel with her axis, or she saw that it is antiparallel to her axis.

- Looking at the Bong et al paradox:
- ➤ A could be the variable defined by Alice's own internal observation, A_I: 'Alice saw 'down' relative to her axis' or 'Alice saw 'up' relative to her axis.'
- ▶ Or A could be the variable A_E, defined within Bob's reference frame, which combines the facts about what Alice saw with the facts about what direction her axis is found to point in relative to Bob's axis.
- Suppose Alice's axis comes to have a well-defined orientation relative to Bob's axis if Bob actually checks (he talks to Alice instead of measuring to find B).

- ▶ 'Absoluteness of observed events' requires a well-defined probability distribution across A_I , B, C_I , D. But this distribution can't be compared to the quantum predictions, since QM only makes predictions for spins measured relative to a common spacetime reference frame.
- ▶ We can compare A_E , B, C_E , D to the predictions of QM. But there is no well-defined probability distribution across A_E , B, C_E , D since A_E exists only on runs when Bob does not obtain the value B.
- No quantum predictions are violated, but we get to keep absoluteness of observed events.
- ► There is a fact of the matter about what Alice saw, and if Bob asks her he will find out.

- ▶ A theory is said to have a symmetry if there is a class of transformations { T} which leave its equations of motion and solutions unchanged. It is natural to say that these transformations do not produce physically real change.
- ▶ Applying a transformation in { T} to a proper subsystem will not change anything about the internal state of the system, but the transformation may still be detectable when we compare the subsystem to its environment.
- ▶ A variable *V* of the subsystem which is changed by the transformations { *T* } is not meaningful when we consider the subsystem in isolation but can still be meaningful when compared to an external reference frame.

- ► The theory is subject to a rotation symmetry so rotations do not change the internal state of Alice and her laboratory
- 'Alice has seen the spin parallel to her axis' lives in the invariant subspace with respect to the rotation group.
- But rotations of Alice's laboratory are meaningful with respect to Bob's laboratory.
- So we can imagine a superposition:

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle_{S}|P\rangle_{A}+|\downarrow\rangle_{S}|P'\rangle_{A})$$

 $|P\rangle_A$ and $|P'\rangle_A$ represent states of Alice where she has observed the particle to be parallel to her own axis, but they have different total orientation relative to Bob.

Standard Wigner's Friend description:

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle_{\mathcal{S}} + |\downarrow\rangle_{\mathcal{S}}) \otimes |\psi\rangle_{\mathcal{A}} \ \rightarrow \ \frac{1}{\sqrt{2}}(|\uparrow\rangle_{\mathcal{S}}|\uparrow\rangle_{\mathcal{A}} + |\downarrow\rangle_{\mathcal{S}}|\downarrow\rangle_{\mathcal{A}})$$

Different proposal - for the case where Alice observed the particle to be up relative to her axis, the overall state relative to Bob should be represented as follows:

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle_S|P\rangle_A + |\downarrow\rangle_S|P'\rangle_A)$$

There is a well-defined fact about what Alice saw and if Bob measures the invariant subspace he will find out that Alice is in the state P, i.e. he learns the actual result of her measurement.

But if he performs quantum experiments on the orientation variable, he will see quantum interference and reproduce the usual quantum predictions.

- ► If we buy this, then the 'paradox' arises precisely from assuming a fixed, shared spacetime background.
- Once we note that each reference frame has its own 'spacetime' and there is not in general a shared meaning of 'orientation,' the paradox is resolved.
- We may think of this as a manifestation of the idea that spacetime emerges only relative to reference frames.

Conclusion

- ► There are good reasons to think that (at least some) quantum descriptions are necessarily relativized (to something!)
- ► This is important to remember when we try to understand the emergence of the classical world and classical spacetime.
- We should think carefully about what kinds of descriptions are meaningful prior to the emergence of classical reference frames.
- We should be careful about the attribution of properties to systems in the absence of an appropriate reference frame.