UNIVERSITÉ DE GENÈVE

Acoustic fluid perturbations in first-order phase transitions

Antonino Salvino Midiri

Work (*in preparation*) in collaboration with:
Chiara Caprini, Simona Procacci & Alberto Roper Pol

Nordita – January 22nd 2026

Swiss National Science Foundation *«Exploring the early Universe with Gravitational Waves and Primordial Magnetic Fields»*

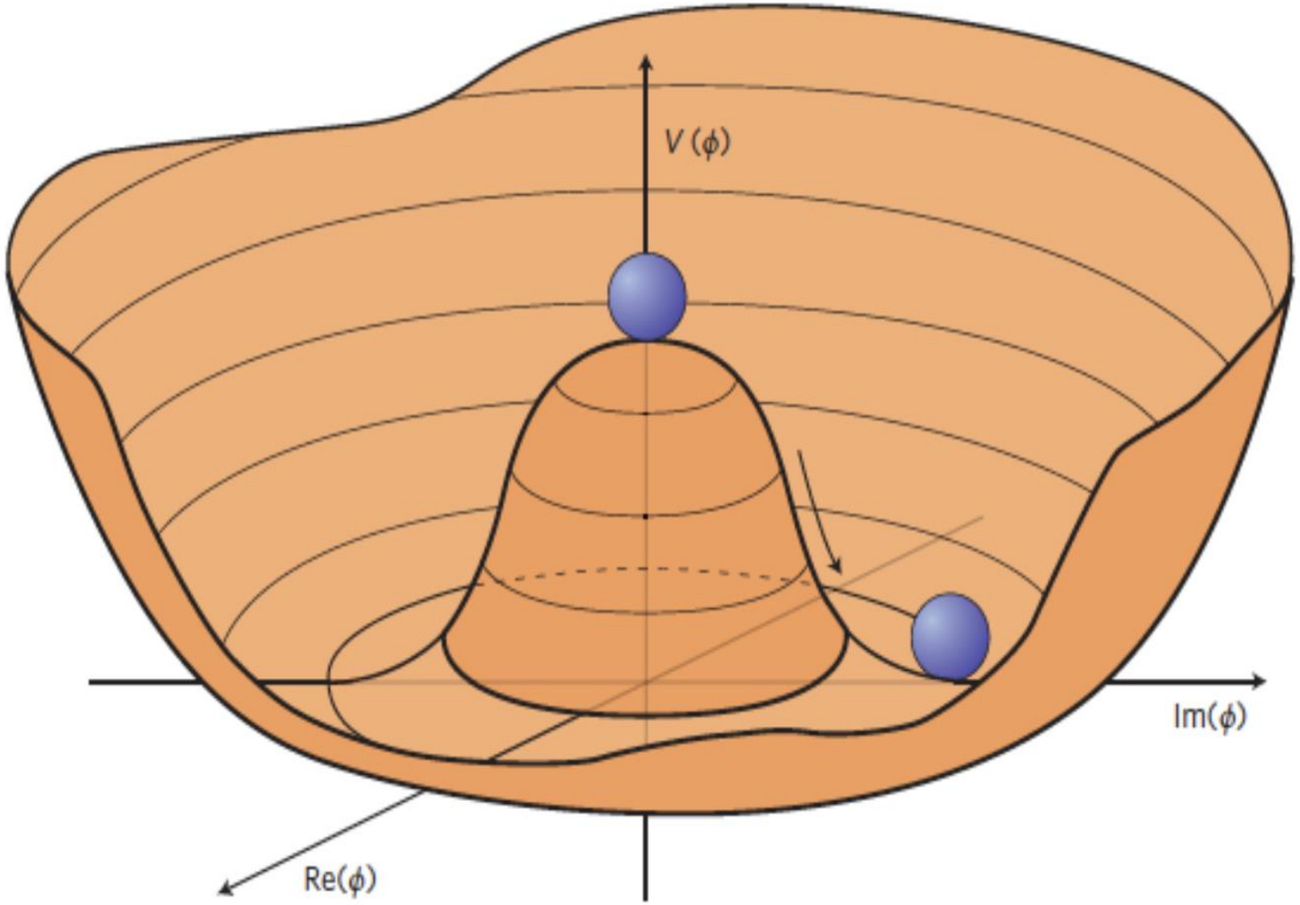
Introduction: phase transitions in the early Universe

LHC $\longrightarrow$ Standard Model Higgs (ϕ) has nonzero vev at $T = 0$

Introduction: phase transitions in the early Universe

LHC $\longrightarrow$ Standard Model Higgs (ϕ) has nonzero vev at $T = 0$

$$V_0(|\phi|) = -\frac{\mu^2}{2}|\phi|^2 + \frac{\lambda^2}{4}|\phi|^4$$



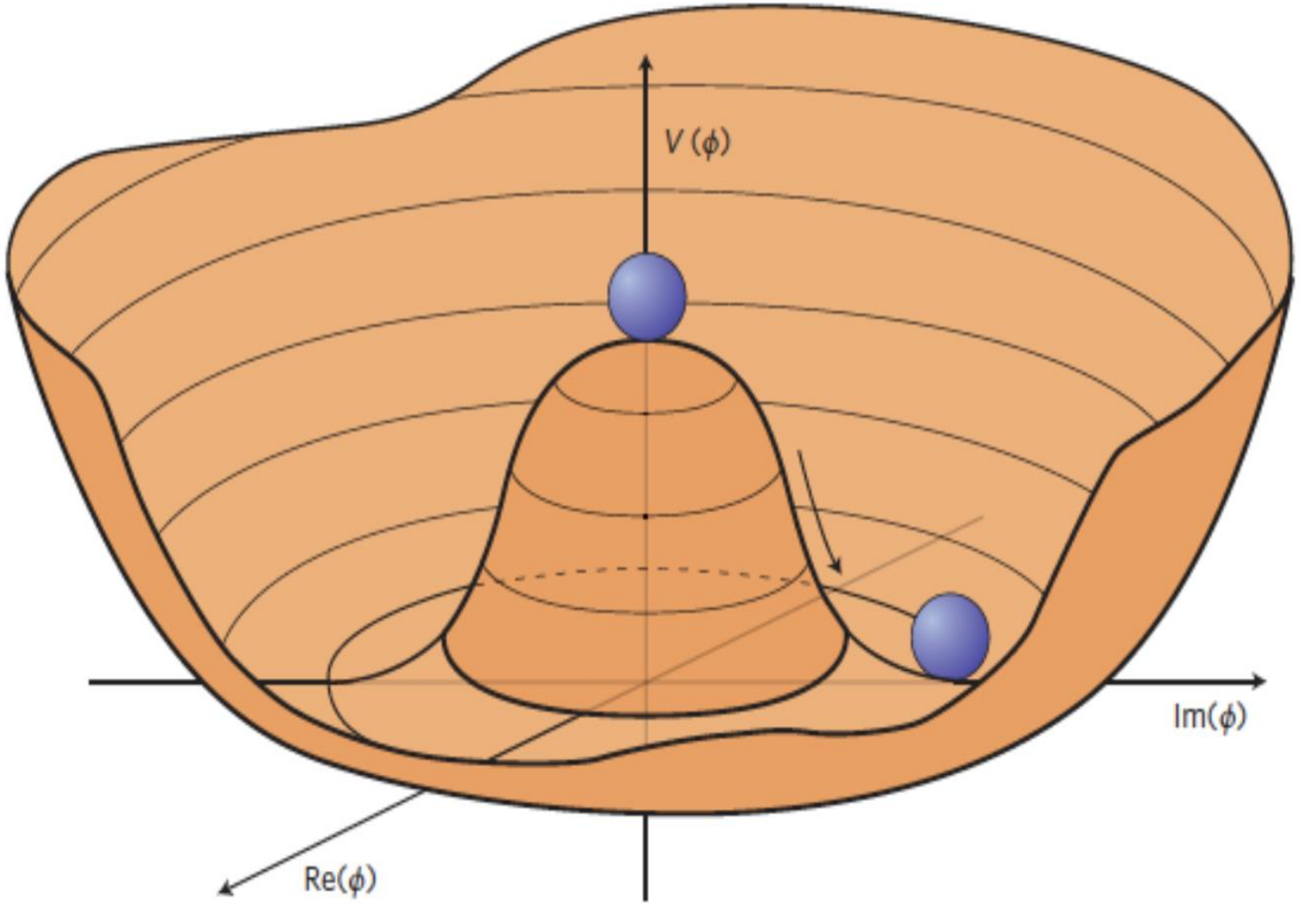
Introduction: phase transitions in the early Universe

LHC ⟶ Standard Model Higgs (ϕ) has nonzero vev at $T = 0$

$$V_0(|\phi|) = -\frac{\mu^2}{2}|\phi|^2 + \frac{\lambda^2}{4}|\phi|^4$$

Minimum of the potential at $T = 0$

$$\frac{\partial V_0(|\phi|)}{\partial |\phi|} = 0$$



Introduction: phase transitions in the early Universe

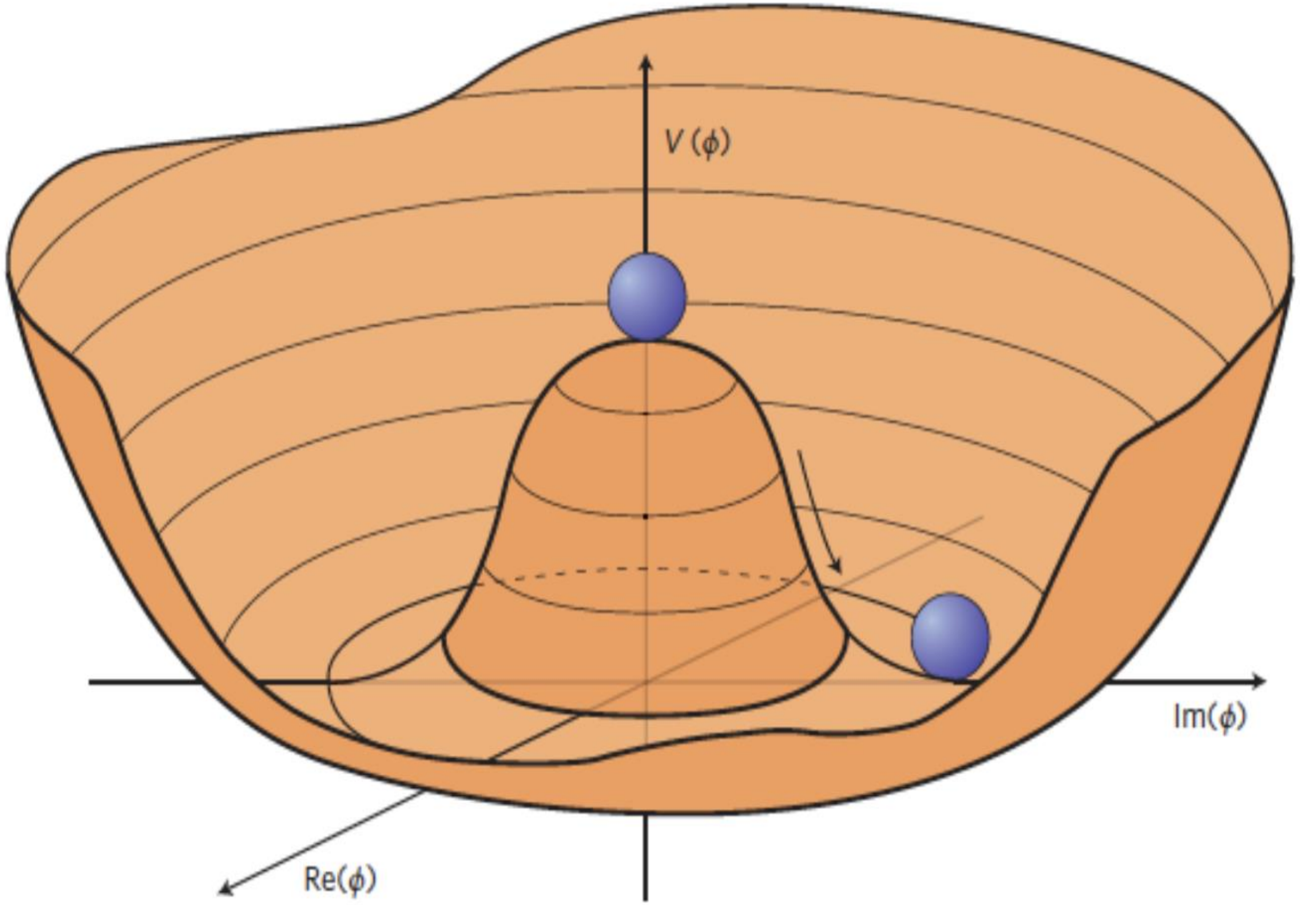
LHC $\longrightarrow$ Standard Model Higgs (ϕ) has nonzero vev at $T=0$

$$V_0(|\phi|) = -\frac{\mu^2}{2}|\phi|^2 + \frac{\lambda^2}{4}|\phi|^4$$

Minimum of the potential at $T=0$

$$\frac{\partial V_0(|\phi|)}{\partial|\phi|} = 0$$

$\xrightarrow{\text{vev}}$ $|\phi| = \sqrt{\mu^2/\lambda^2} \equiv v \neq 0$



Introduction: phase transitions in the early Universe

LHC $\longrightarrow$ Standard Model Higgs (ϕ) has nonzero vev at $T = 0$

In the primordial plasma
at finite temperature?

Introduction: phase transitions in the early Universe

LHC $\longrightarrow$ Standard Model Higgs (ϕ) has nonzero vev at $T = 0$

In the primordial plasma at finite temperature $\xrightarrow{\text{Thermal QFT}}$ $V_{eff} = V_0(|\phi|) + D^2|\phi|^2T^2 + \cdots$

Introduction: phase transitions in the early Universe

LHC → Standard Model Higgs (ϕ) has nonzero vev at $T = 0$

In the primordial plasma at finite temperature $\xrightarrow{\text{Thermal QFT}}$ $V_{eff} = V_0(|\phi|) + D^2|\phi|^2T^2 + \cdots$

dominant contribution at high temperature

Introduction: phase transitions in the early Universe

LHC $\longrightarrow$ Standard Model Higgs (ϕ) has nonzero vev at $T = 0$

In the primordial plasma at finite temperature $\xrightarrow{\text{Thermal QFT}}$ $V_{eff} = V_0(|\phi|) + D^2|\phi|^2T^2 + \cdots$

dominant contribution at high temperature

Minimum of the potential at high T $\longrightarrow$ $\dfrac{\partial V_{eff}(|\phi|, T)}{\partial |\phi|} = 0$ $\longrightarrow$ $|\phi| = 0$

Introduction: phase transitions in the early Universe

LHC $\longrightarrow$ Standard Model Higgs (ϕ) has nonzero vev at $T = 0$

In the primordial plasma at finite temperature $\xrightarrow{\text{Thermal QFT}}$ $V_{eff} = V_0(|\phi|) + D^2|\phi|^2T^2 + \cdots$

dominant contribution at high temperature

Minimum of the potential at high T $\longrightarrow$ $\dfrac{\partial V_{eff}(|\phi|, T)}{\partial|\phi|} = 0$ $\longrightarrow$ $|\phi| = 0$

As temperature decreases the Higgs vev goes from zero to $v \neq 0$

Introduction: phase transitions in the early Universe

LHC $\longrightarrow$ Standard Model Higgs (ϕ) has nonzero vev at $T = 0$

In the primordial plasma at finite temperature $\xrightarrow{\text{Thermal QFT}}$ $V_{eff} = V_0(|\phi|) + D^2|\phi|^2T^2 + \cdots$

dominant contribution at high temperature

Minimum of the potential at high T $\longrightarrow$ $\dfrac{\partial V_{eff}(|\phi|, T)}{\partial|\phi|} = 0$ $\longrightarrow$ $|\phi| = 0$

As temperature decreases the Higgs vev goes from zero to $v \neq 0$ $\longrightarrow$ Electroweak Spontaneous Symmetry Breaking (EWSSB)

$SU(3)_C \bigotimes SU(2)_L \bigotimes U(1)_Y \to SU(3)_C \bigotimes U(1)_{em}$

Introduction: phase transitions in the early Universe

The way in which the transition from the symmetric phase (*zero* vev) to the broken phase (*nonzero* vev) occurs depends on V_{eff}

Introduction: phase transitions in the early Universe

The way in which the transition from the symmetric phase (*zero* vev) to the broken phase (*nonzero* vev) occurs depends on V_{eff}

Let us consider the Standard Model case [9203203]

$$V_{eff}(|\phi|, \mathrm{T}) = D^2(\mathrm{T}^2 - \mathrm{T}_0^2)|\phi|^2 - E^2\,\mathrm{T}\,|\phi|^3 + \frac{\lambda^2}{4}\,|\phi|^4$$

Introduction: phase transitions in the early Universe

The way in which the transition from the symmetric phase (*zero* vev) to the broken phase (*nonzero* vev) occurs depends on V_{eff}

Let us consider the Standard Model case [9203203]

$$V_{eff}(|\phi|, \mathrm{T}) = D^2(\mathrm{T}^2 - \mathrm{T}_0^2)|\phi|^2 - E^2\,\mathrm{T}\,|\phi|^3 + \frac{\lambda^2}{4}\,|\phi|^4$$

If $E = 0$ the minimum is given by

$T \geq T_0$ $\qquad\qquad |\phi| = 0$

Introduction: phase transitions in the early Universe

The way in which the transition from the symmetric phase (*zero* vev) to the broken phase (*nonzero* vev) occurs depends on V_{eff}

Let us consider the Standard Model case [9203203]

$$V_{eff}(|\phi|, \mathrm{T}) = D^2(\mathrm{T}^2 - \mathrm{T}_0^2)|\phi|^2 - E^2\,\mathrm{T}\,|\phi|^3 + \frac{\lambda^2}{4}\,|\phi|^4$$

If $E = 0$ the minimum is given by

$T \geq T_0$ $\qquad |\phi| = 0$

$T \leq T_0$ $\qquad |\phi| = \sqrt{\frac{2D^2}{\lambda^2}(T_0^2 - T^2)}$

Introduction: phase transitions in the early Universe

The way in which the transition from the symmetric phase (*zero* vev) to the broken phase (*nonzero* vev) occurs depends on V_{eff}

Let us consider the Standard Model case [9203203]

$$V_{eff}(|\phi|,\mathrm{T}) = D^2(\mathrm{T}^2 - \mathrm{T}_0^2)|\phi|^2 - E^2\,\mathrm{T}\,|\phi|^3 + \frac{\lambda^2}{4}\,|\phi|^4$$

If $E = 0$ the minimum is given by

$T \geq T_0$ $\quad |\phi| = 0$

$T \leq T_0$ $\quad |\phi| = \sqrt{\frac{2D^2}{\lambda^2}(T_0^2 - T^2)}$

Across the transition (at $T = T_0$)

$|\phi|$ (*order parameter*) is continuous

Introduction: phase transitions in the early Universe

The way in which the transition from the symmetric phase (*zero* vev) to the broken phase (*nonzero* vev) occurs depends on V_{eff}

Let us consider the Standard Model case [9203203]

$$V_{eff}(|\phi|,\mathrm{T}) = D^2(\mathrm{T}^2 - \mathrm{T}_0^2)|\phi|^2 - E^2\,\mathrm{T}\,|\phi|^3 + \frac{\lambda^2}{4}\,|\phi|^4$$

If $E = 0$ the minimum is given by

$T \geq T_0$ $\qquad |\phi| = 0$

$T \leq T_0$ $\qquad |\phi| = \sqrt{\frac{2D^2}{\lambda^2}(T_0^2 - T^2)}$

Across the transition (at $T = T_0$)

$|\phi|$ (*order parameter*) is continuous

$\partial_T|\phi|$ is discontinuous

Introduction: phase transitions in the early Universe

The way in which the transition from the symmetric phase (*zero* vev) to the broken phase (*nonzero* vev) occurs depends on V_{eff}

Let us consider the Standard Model case [9203203]

$$V_{eff}(|\phi|, \mathrm{T}) = D^2(\mathrm{T}^2 - \mathrm{T}_0^2)|\phi|^2 - E^2\,\mathrm{T}\,|\phi|^3 + \frac{\lambda^2}{4}\,|\phi|^4$$

If $E = 0$ the minimum is given by

$$T \geq T_0 \qquad |\phi| = 0$$

$$T \leq T_0 \qquad |\phi| = \sqrt{\frac{2D^2}{\lambda^2}(T_0^2 - T^2)}$$

Across the transition (at $T = T_0$)

$|\phi|$ (*order parameter*) is continuous

$\partial_T|\phi|$ is discontinuous

$\longrightarrow$ Second-Order Phase Transition

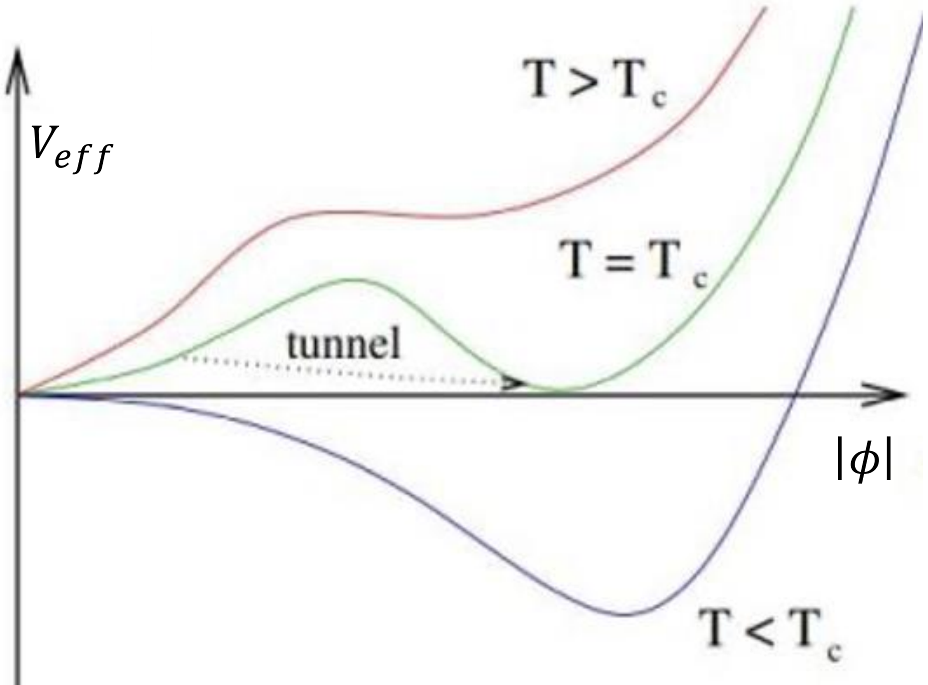
Introduction: phase transitions in the early Universe

The way in which the transition from the symmetric phase (*zero* vev) to the broken phase (*nonzero* vev) occurs depends on V_{eff}

Let us consider the Standard Model case [9203203]

$$V_{eff}(|\phi|, \mathrm{T}) = D^2(\mathrm{T}^2 - \mathrm{T}_0^2)|\phi|^2 - E^2\,\mathrm{T}\,|\phi|^3 + \frac{\lambda^2}{4}\,|\phi|^4$$

If $E \neq 0$



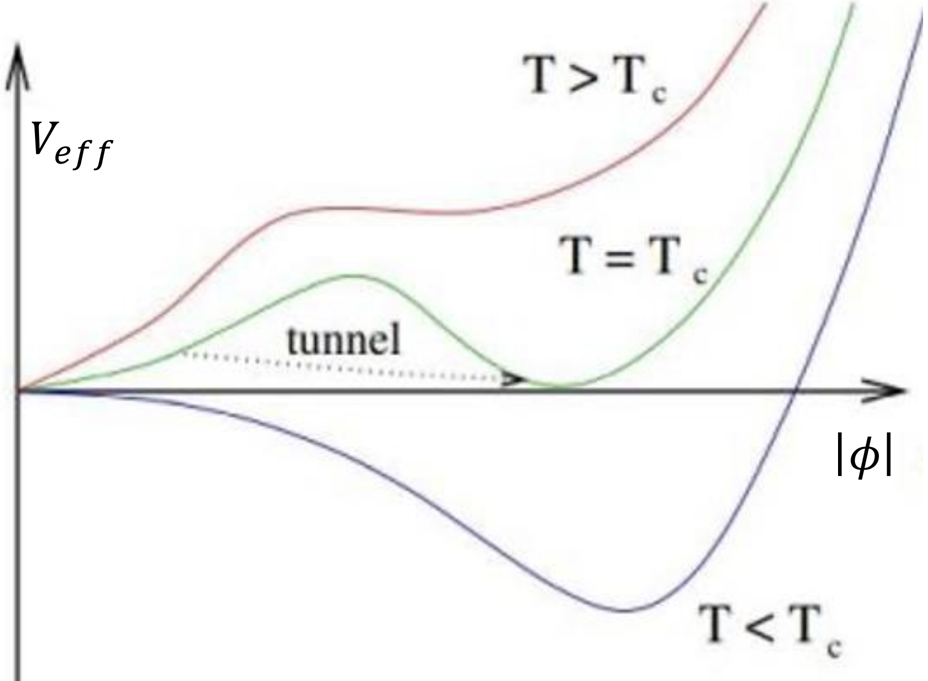
Introduction: phase transitions in the early Universe

The way in which the transition from the symmetric phase (*zero* vev) to the broken phase (*nonzero* vev) occurs depends on V_{eff}

Let us consider the Standard Model case [9203203]

$$V_{eff}(|\phi|, \mathrm{T}) = D^2(\mathrm{T}^2 - \mathrm{T}_0^2)|\phi|^2 - E^2\, \mathrm{T}\, |\phi|^3 + \frac{\lambda^2}{4}\, |\phi|^4$$

If $E \neq 0$



Across the transition

$|\phi|$ (*order parameter*) is discontinuous

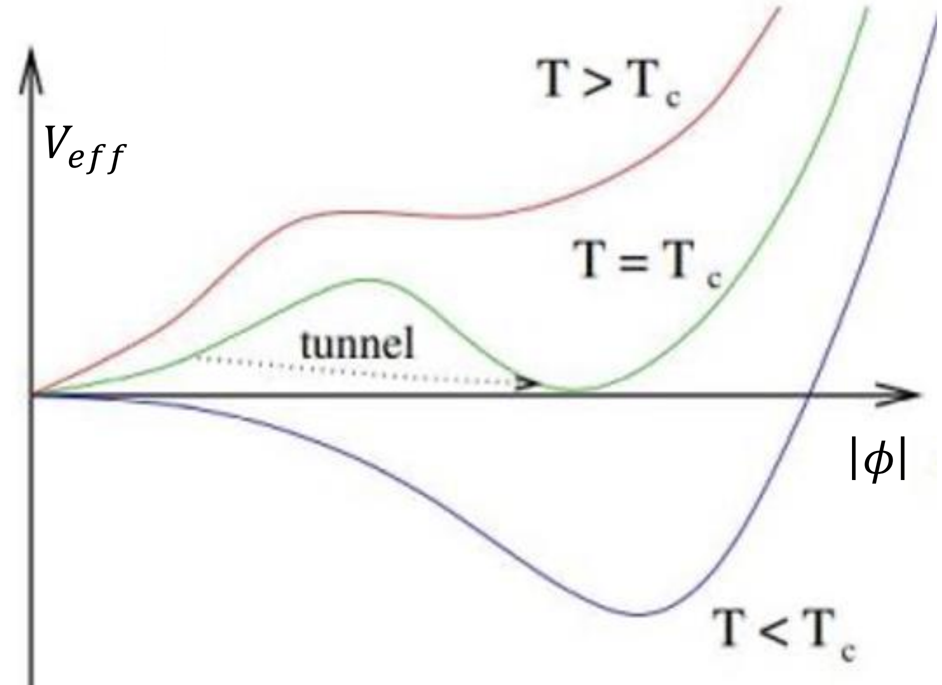
Introduction: phase transitions in the early Universe

The way in which the transition from the symmetric phase (*zero* vev) to the broken phase (*nonzero* vev) occurs depends on V_{eff}

Let us consider the Standard Model case [9203203]

$$V_{eff}(|\phi|, \mathrm{T}) = D^2(\mathrm{T}^2 - \mathrm{T}_0^2)|\phi|^2 - E^2\,\mathrm{T}\,|\phi|^3 + \frac{\lambda^2}{4}\,|\phi|^4$$

If $E \neq 0$



Across the transition

$|\phi|$ (*order parameter*) is discontinuous

→ First-Order Phase Transition

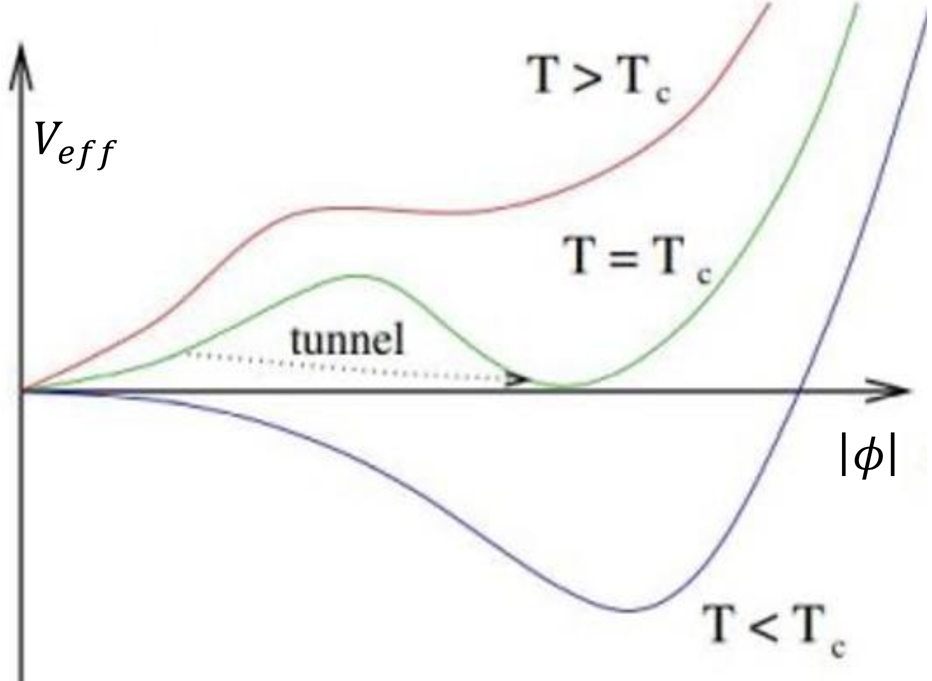
Introduction: phase transitions in the early Universe

The way in which the transition from the symmetric phase (*zero* vev) to the broken phase (*nonzero* vev) occurs depends on V_{eff}

Let us consider the Standard Model case [9203203]

$$V_{eff}(|\phi|, \mathrm{T}) = D^2(\mathrm{T}^2 - \mathrm{T}_0^2)|\phi|^2 - E^2\,\mathrm{T}\,|\phi|^3 + \frac{\lambda^2}{4}\,|\phi|^4$$

If $E \neq 0$



In the Standard Model the EW phase transition is a crossover ($E \neq 0$ but small)

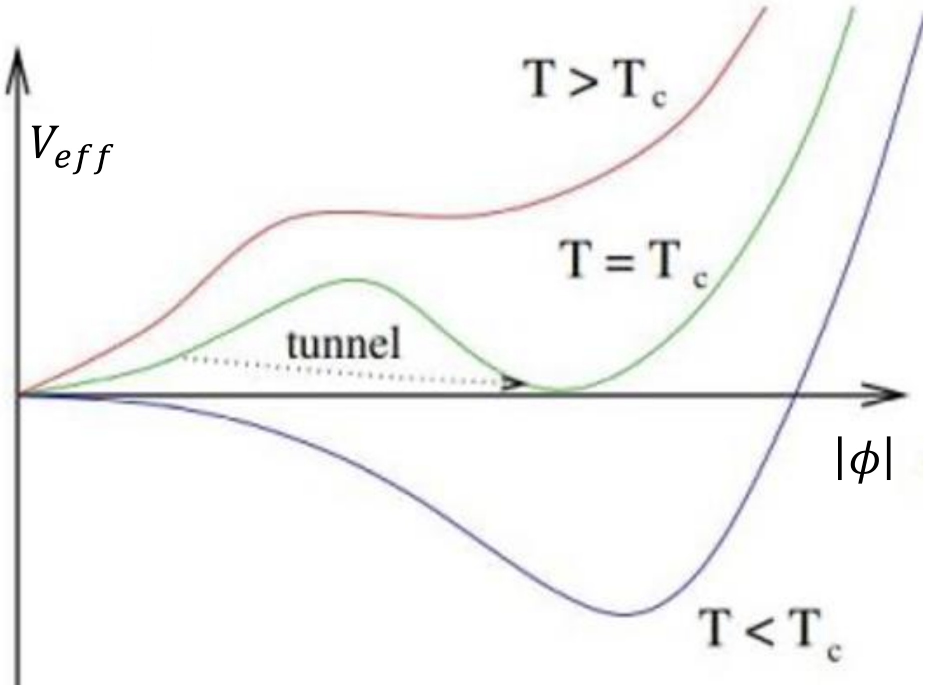
Introduction: phase transitions in the early Universe

The way in which the transition from the symmetric phase (*zero* vev) to the broken phase (*nonzero* vev) occurs depends on V_{eff}

Let us consider the Standard Model case [9203203]

$$V_{eff}(|\phi|, \mathrm{T}) = D^2(\mathrm{T}^2 - \mathrm{T}_0^2)|\phi|^2 - E^2\,\mathrm{T}\,|\phi|^3 + \frac{\lambda^2}{4}\,|\phi|^4$$

If $E \neq 0$

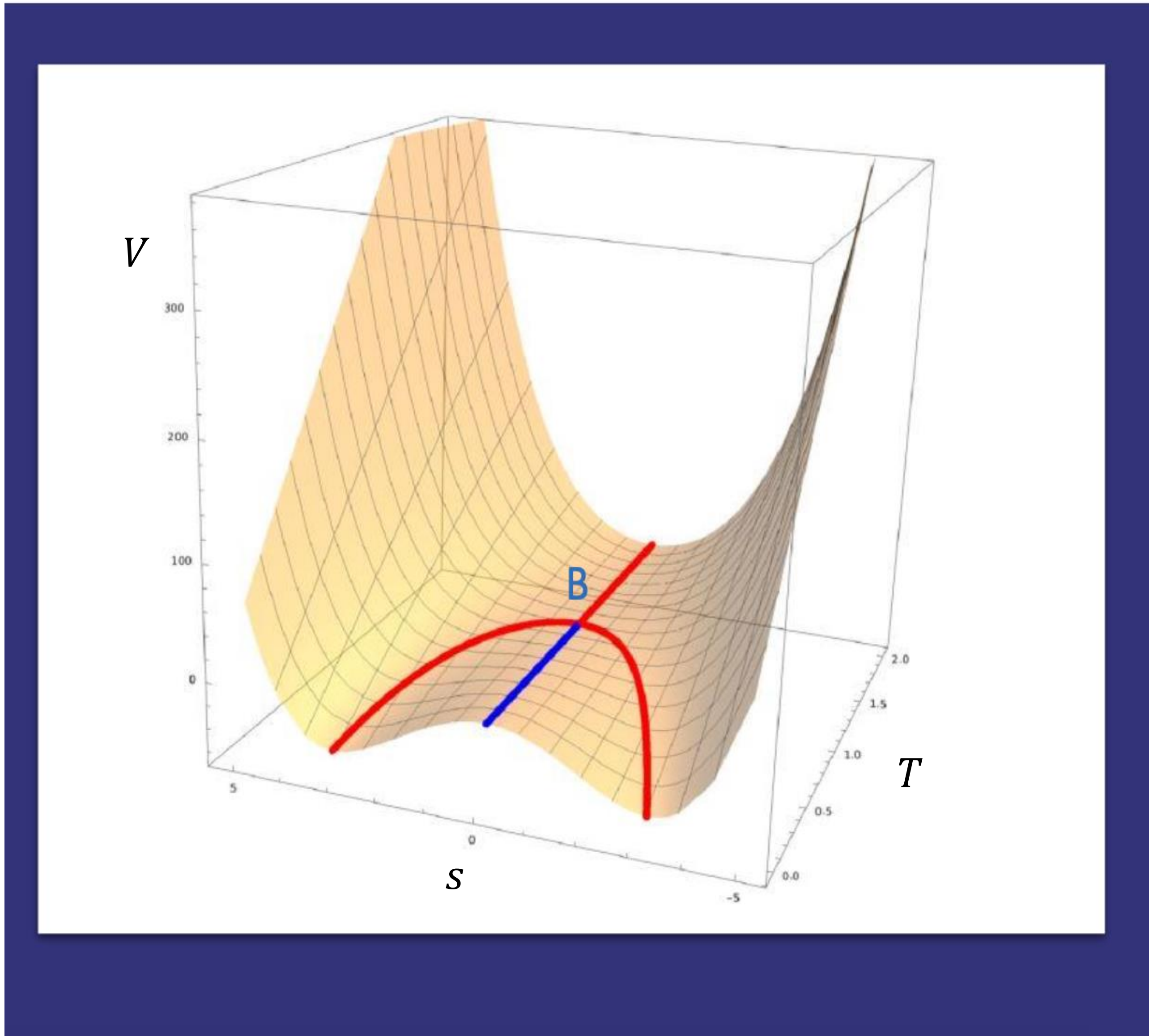


In the Standard Model the EW phase transition is a crossover ($E \neq 0$ but small)

However in BSM theories we can easily have first-order phase transitions (e. g. in SUSY already at tree level)

Second-Order Phase Transition

$$V(T,s) = 10(T-1)s^2 + \frac{s^4}{2}$$

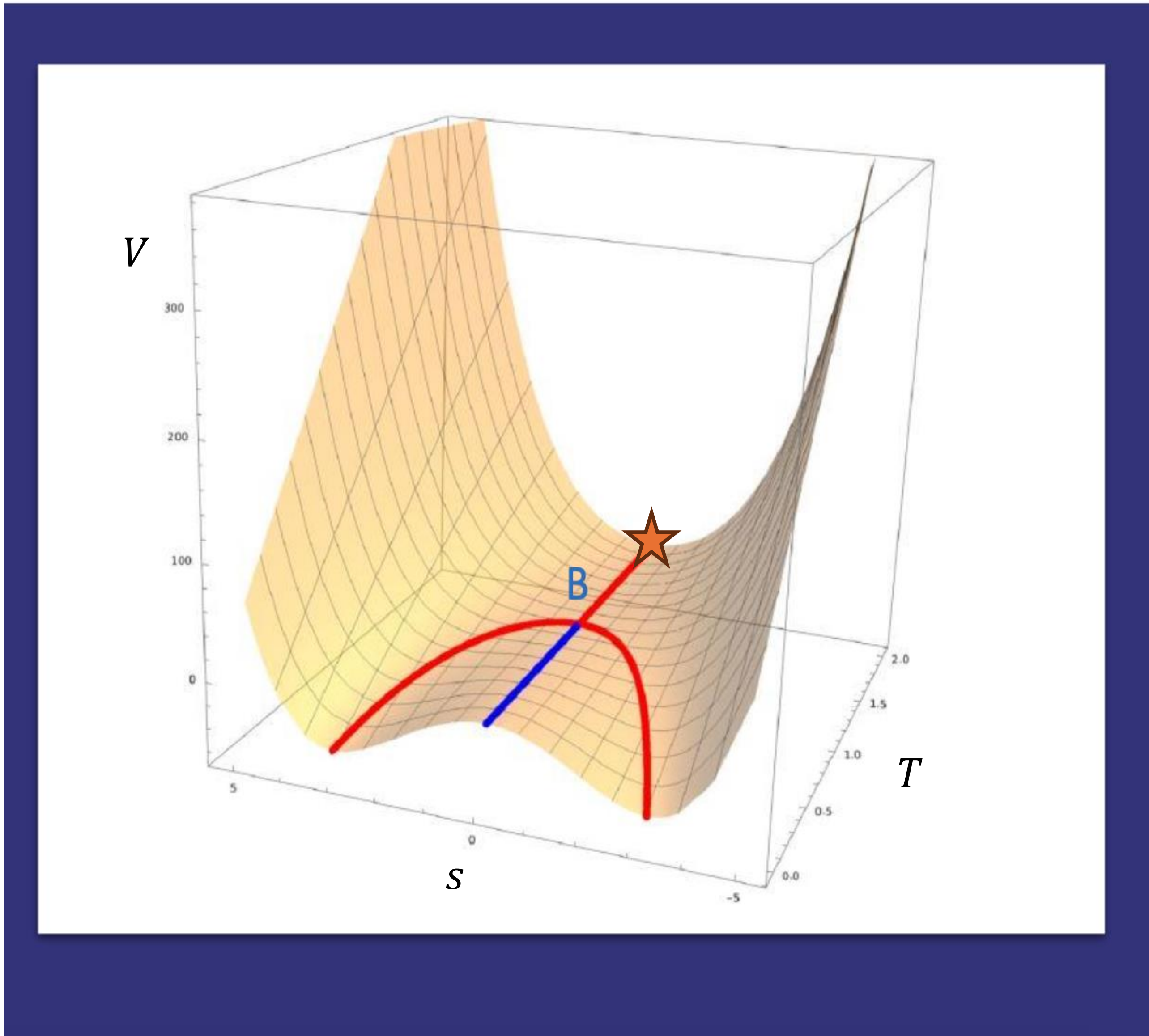


Second-Order Phase Transition

$$V(T,s) = 10(T-1)s^2 + \frac{s^4}{2}$$

phase transition temperature

$\boxed{T > T_0}$



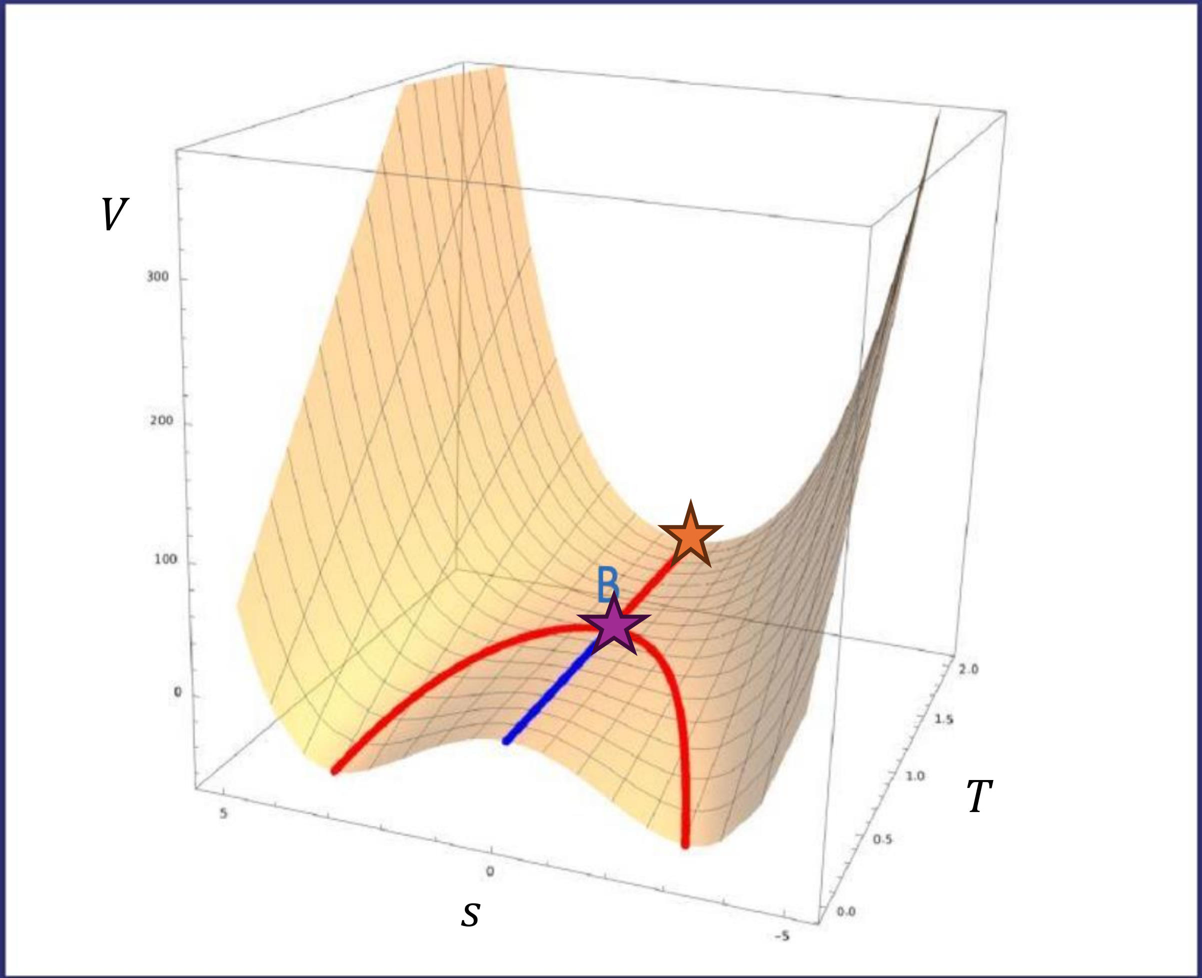
Second-Order Phase Transition

$$V(T,s) = 10(T-1)s^2 + \frac{s^4}{2}$$

phase transition temperature

$\boxed{T > T_0}$

$\boxed{T = T_0}$



Second-Order Phase Transition

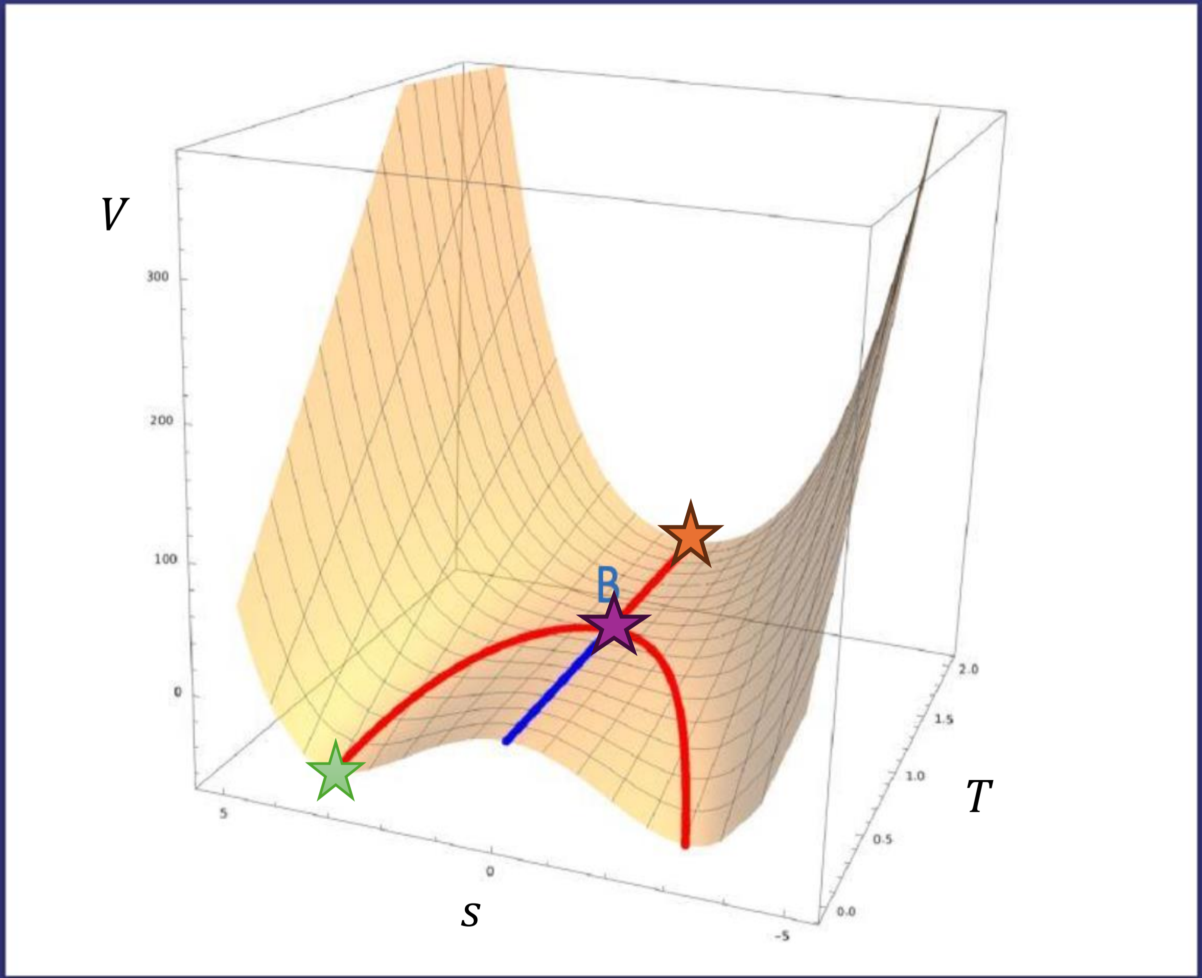
$$V(T,s) = 10(T-1)s^2 + \frac{s^4}{2}$$

phase transition temperature

$T > T_0$

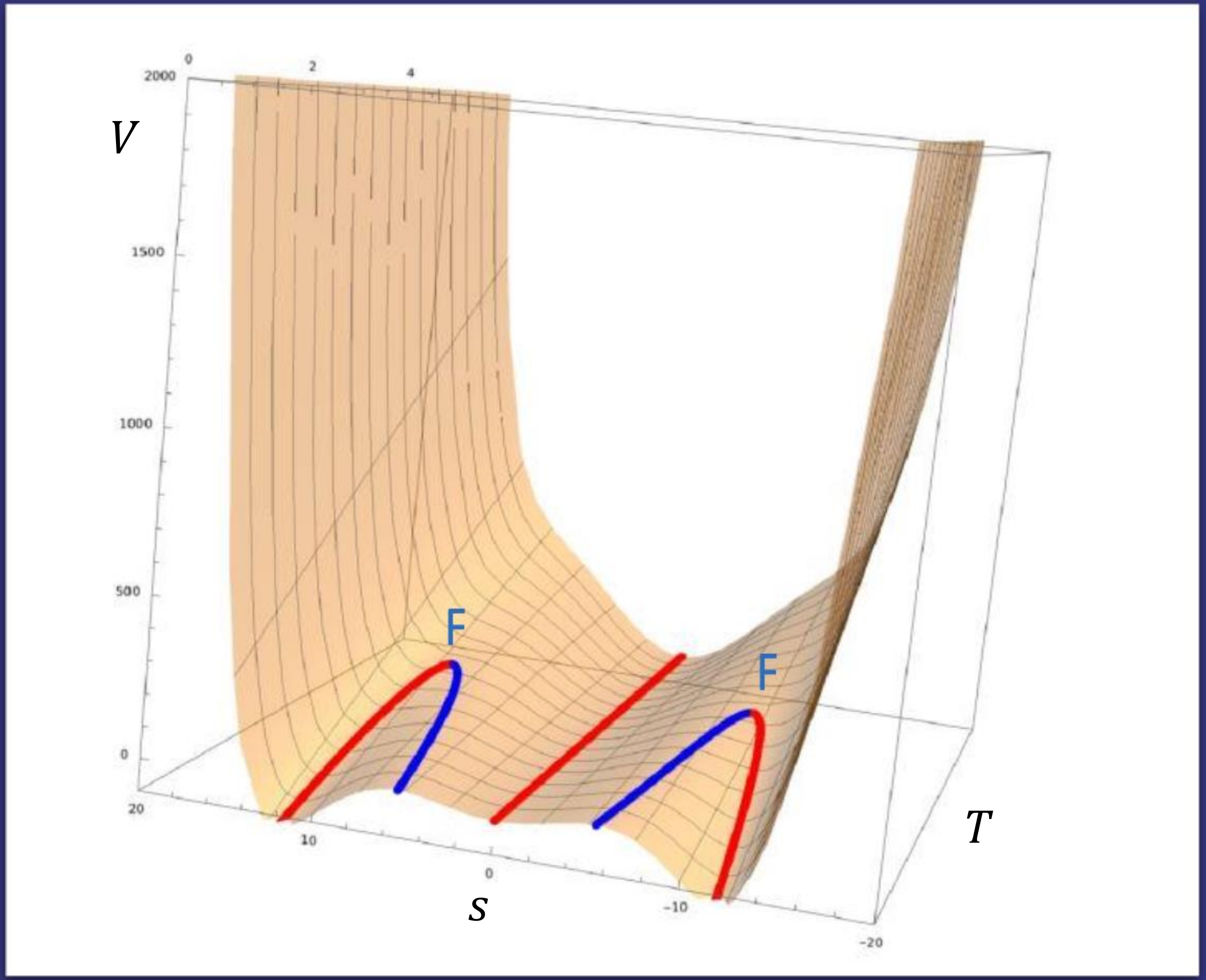
$T = T_0$

$T = 0$



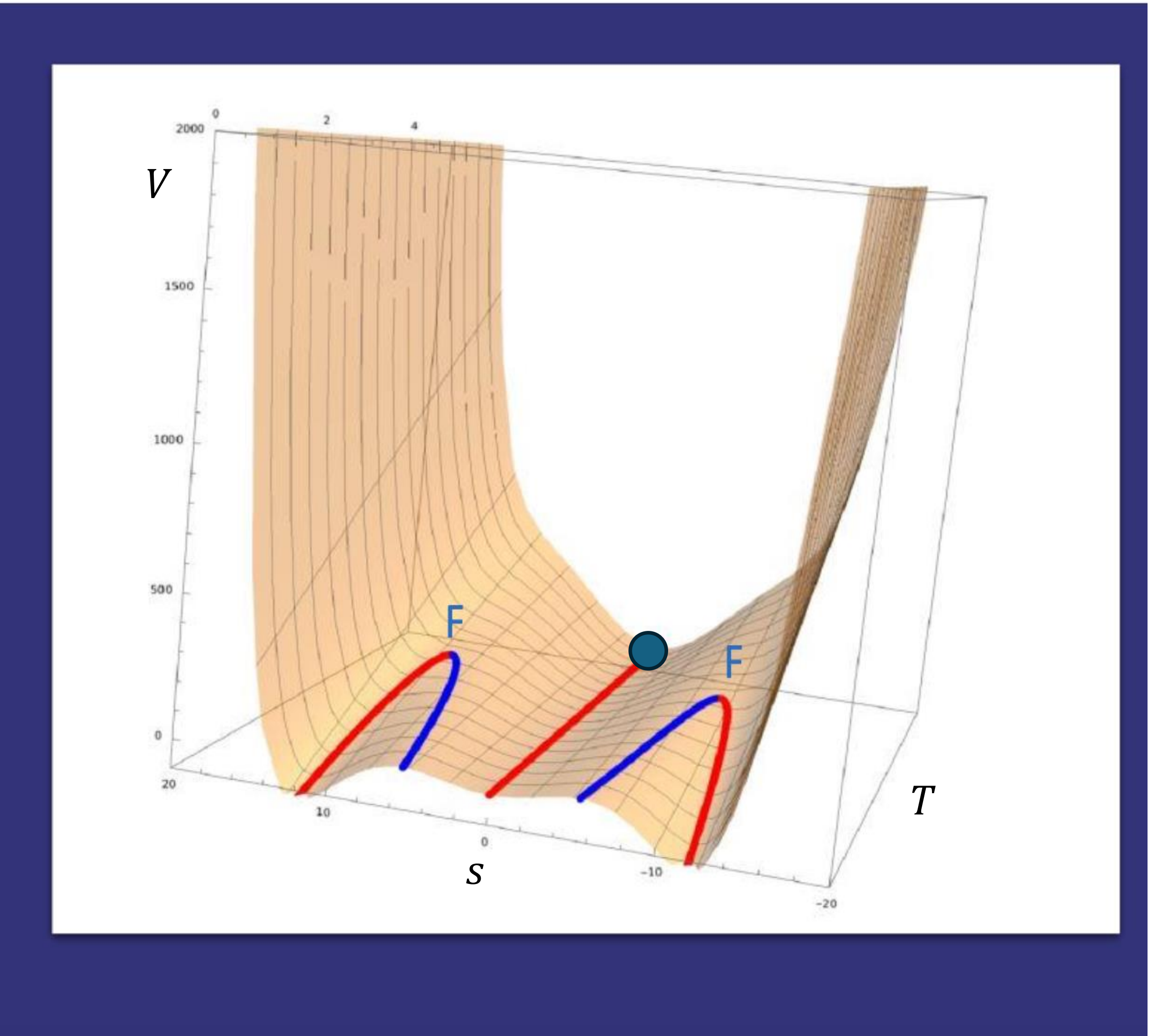
First-Order Phase Transition

$$V(T,s) = (T+3)s^2 - \left(\frac{s}{2}\right)^4 + \left(\frac{s}{4}\right)^6$$



First-Order Phase Transition

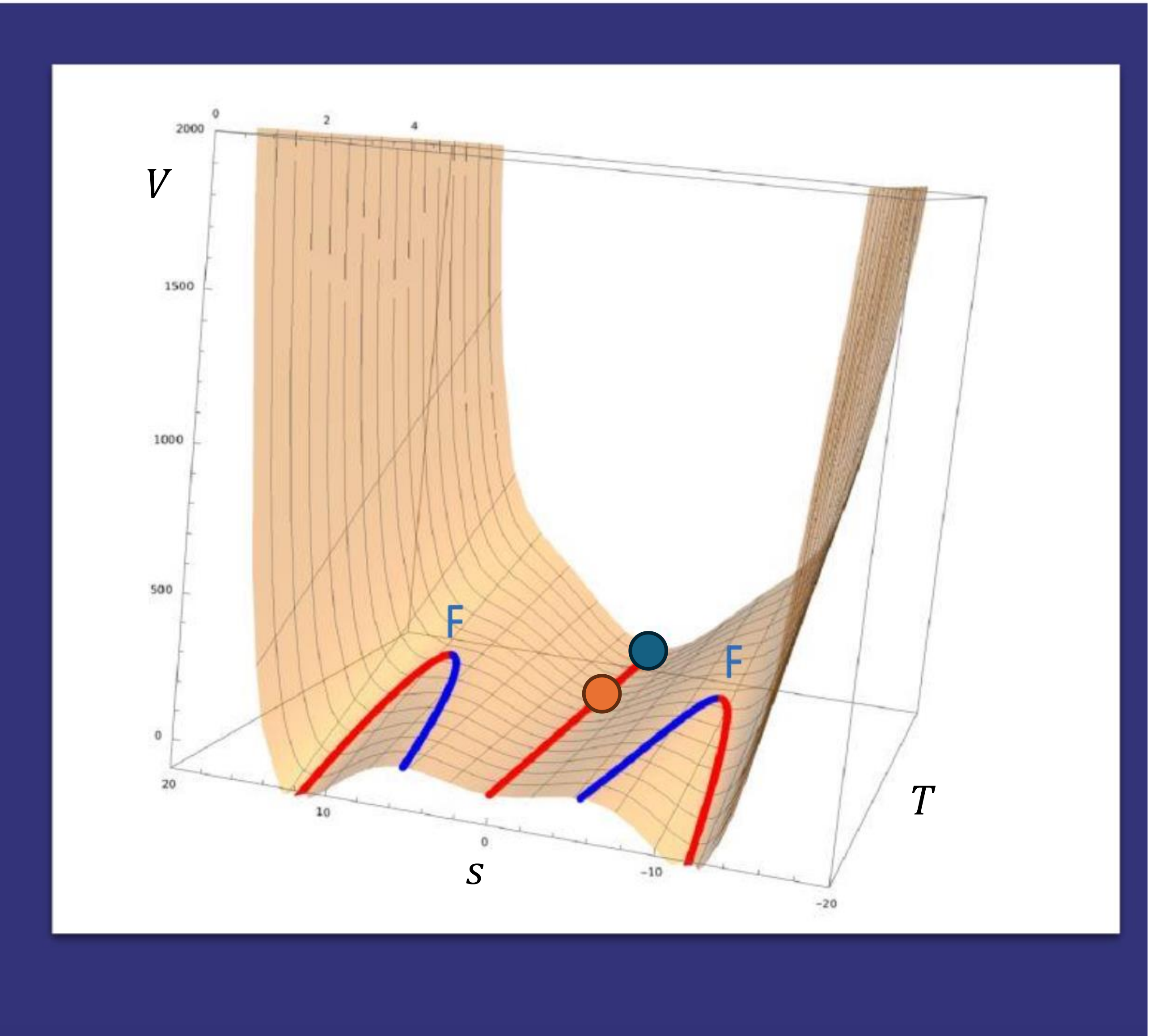
$$V(T,s) = (T+3)s^2 - \left(\frac{s}{2}\right)^4 + \left(\frac{s}{4}\right)^6$$



High T

First-Order Phase Transition

$$V(T,s) = (T+3)s^2 - \left(\frac{s}{2}\right)^4 + \left(\frac{s}{4}\right)^6$$



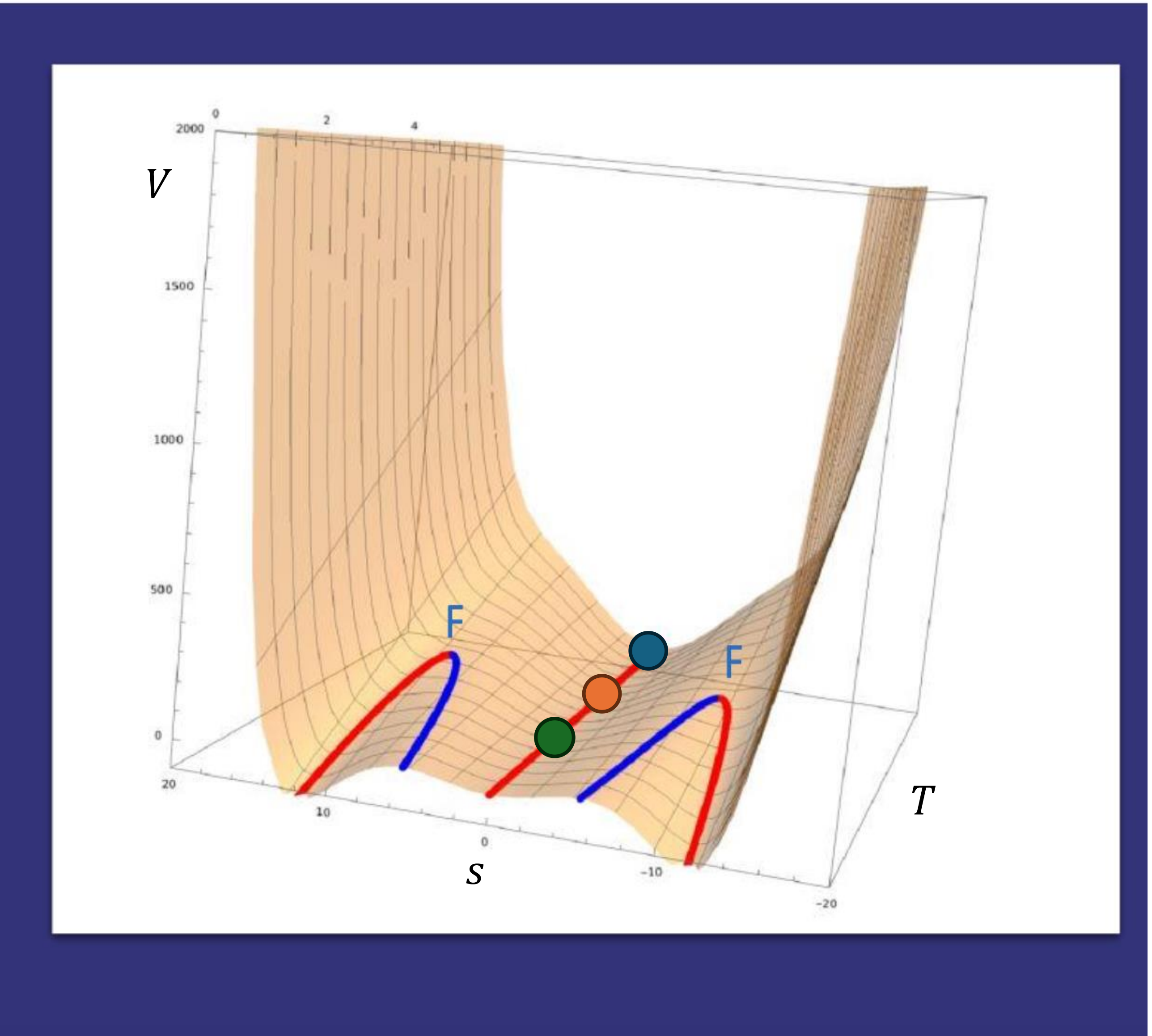
High T

$T = T^*$

Temperature at which local minima appear outside the origin

First-Order Phase Transition

$$V(T, s) = (T+3)s^2 - \left(\frac{s}{2}\right)^4 + \left(\frac{s}{4}\right)^6$$



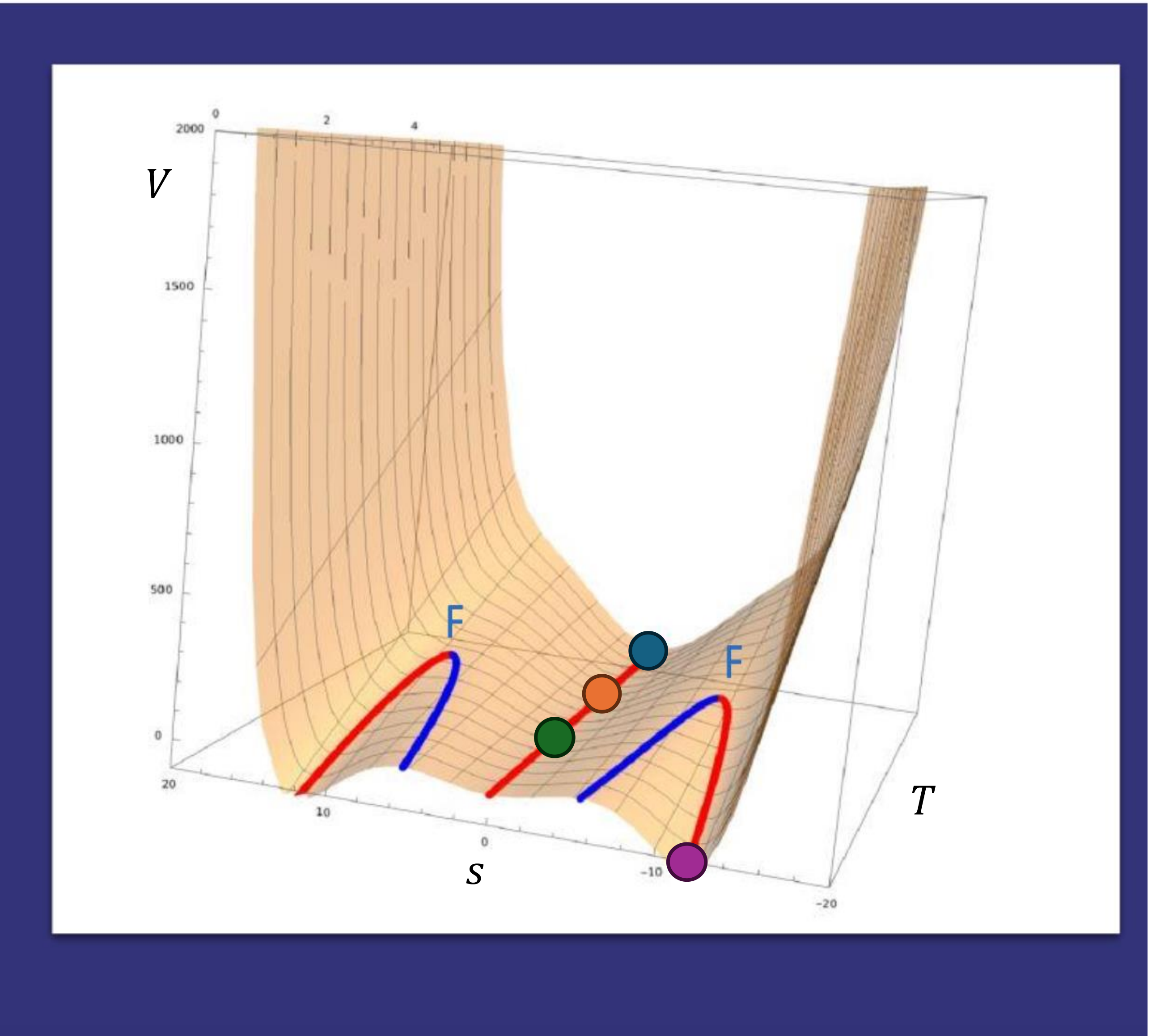
$\boxed{\text{High T}}$

$\boxed{T = T^*}$ Temperature at which local minima appear outside the origin

$\boxed{T = T_C}$ Critical temperature (minima outside and at the origin are degenerate)

First-Order Phase Transition

$$V(T,s) = (T+3)s^2 - \left(\frac{s}{2}\right)^4 + \left(\frac{s}{4}\right)^6$$



High T

$T = T^*$ Temperature at which local minima appear outside the origin

$T = T_C$ Critical temperature (minima outside and at the origin are degenerate)

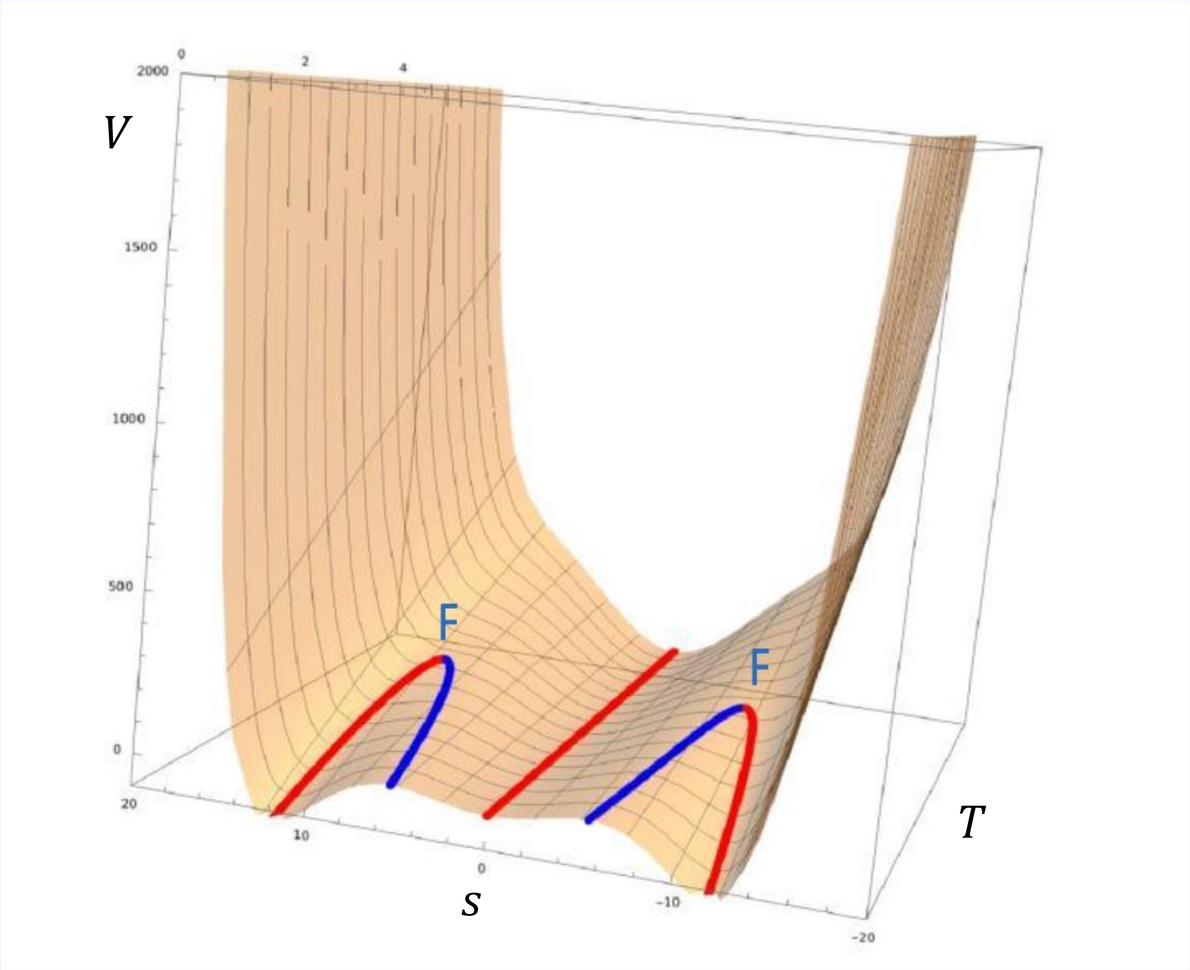
$T = T_N$ Nucleation temperature (at which the phase transition occurs)

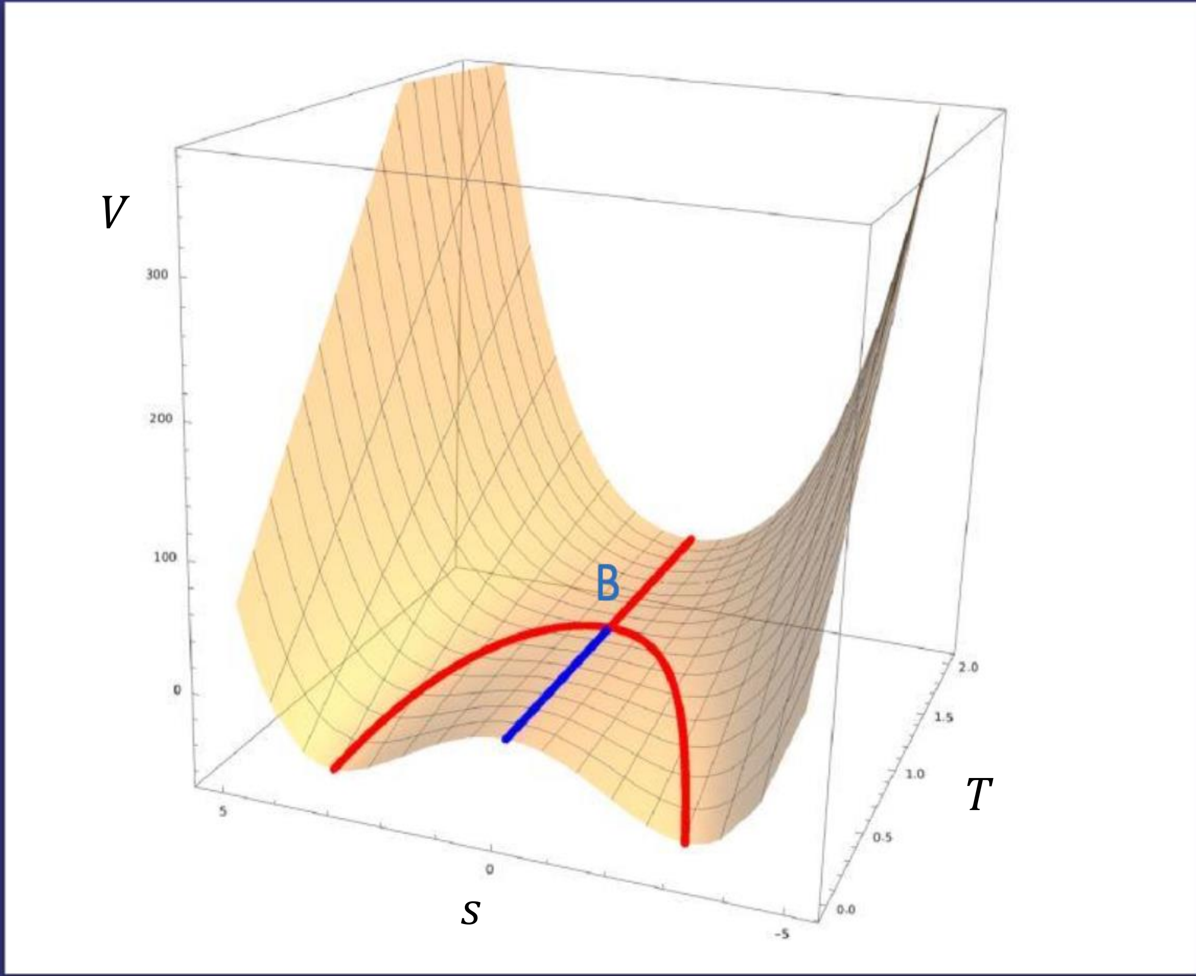
First-Order Phase Transition

$$V(T,s) = (T+3)s^2 - \left(\frac{s}{2}\right)^4 + \left(\frac{s}{4}\right)^6$$

Second-Order Phase Transition

$$V(T,s) = 10(T-1)s^2 + \frac{s^4}{2}$$





Introduction: first-order phase transitions and baryogenesis

Introduction: first-order phase transitions and baryogenesis

Explaining matter excess over antimatter requires baryon asymmetry (BAU problem)

$$\frac{n_b - \bar{n}_b}{s} = \frac{1}{7.04}\frac{n_b - \bar{n}_b}{n_\gamma} = \begin{cases} 8.2 - 9.4 \times 10^{-11}, & \text{(BBN)}, \\ 8.65 \pm 0.09 \times 10^{-11}, & \text{(CMB)}. \end{cases}$$

Introduction: first-order phase transitions and baryogenesis

Explaining matter excess over antimatter requires baryon asymmetry (BAU problem)

$$\frac{n_b - \bar{n}_b}{s} = \frac{1}{7.04}\frac{n_b - \bar{n}_b}{n_\gamma} = \begin{cases} 8.2 - 9.4 \times 10^{-11}, & \text{(BBN)}, \\ 8.65 \pm 0.09 \times 10^{-11}, & \text{(CMB)}. \end{cases}$$

A. Sakharov (1967) → dynamical baryogenesis mechanism requires three conditions:

1. Baryon number violation.
2. Charge (C) and charge-parity (CP) violation.
3. Departure from thermal equilibrium.

Introduction: first-order phase transitions and baryogenesis

Explaining matter excess over antimatter requires baryon asymmetry (BAU problem)

$$\frac{n_b - \bar{n}_b}{s} = \frac{1}{7.04}\frac{n_b - \bar{n}_b}{n_\gamma} = \begin{cases} 8.2 - 9.4 \times 10^{-11}, & \text{(BBN)}, \\ 8.65 \pm 0.09 \times 10^{-11}, & \text{(CMB)}. \end{cases}$$

A. Sakharov (1967) → dynamical baryogenesis mechanism requires three conditions:

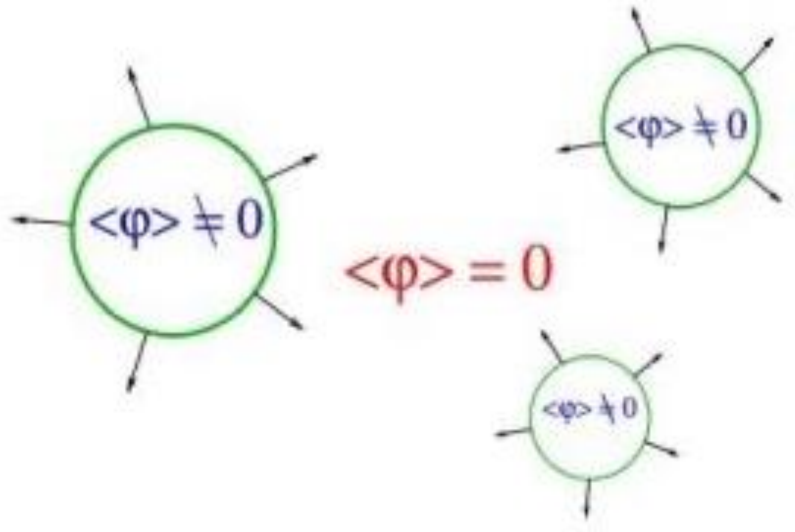
1. Baryon number violation.
2. Charge (C) and charge-parity (CP) violation.
3. Departure from thermal equilibrium.

A possible solution → EW baryogenesis

See recent review by Jorinde van de Vis et al. [2508.09989]

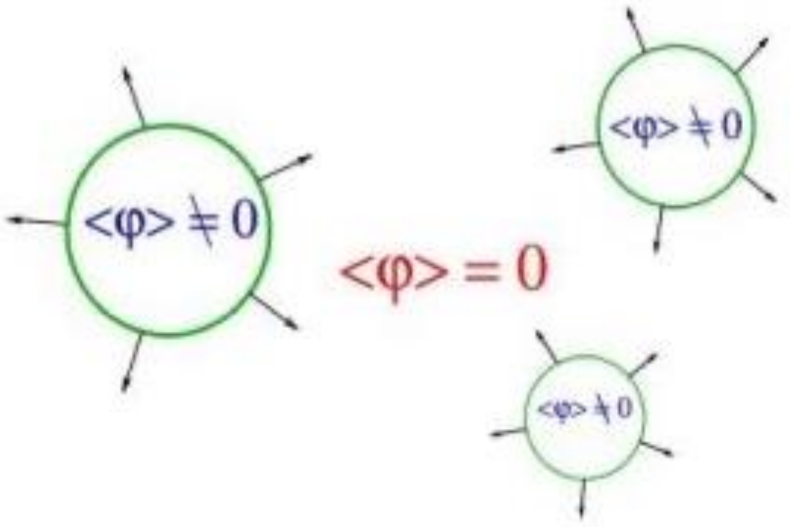
Introduction: first-order phase transitions and baryogenesis

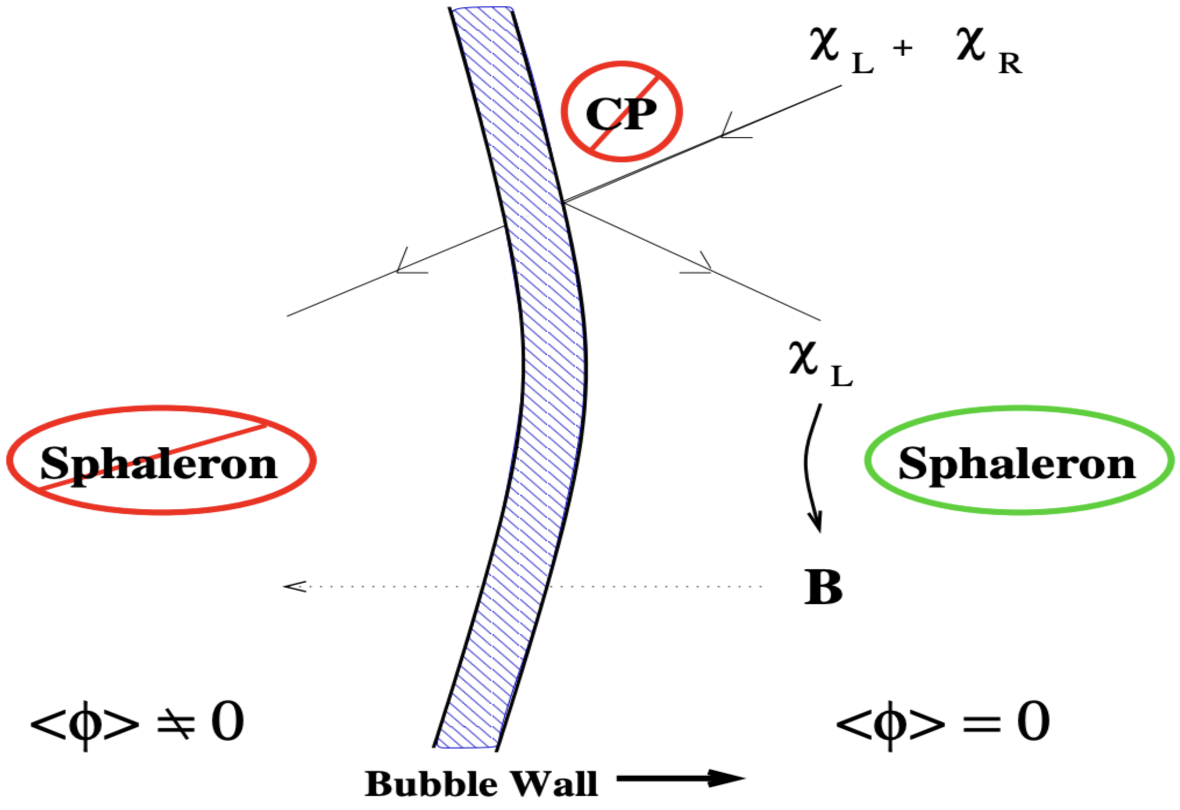
First-Order Phase Transitions occur through the nucleation of broken phase bubbles



Introduction: first-order phase transitions and baryogenesis

First-Order Phase Transitions occur through the nucleation of broken phase bubbles

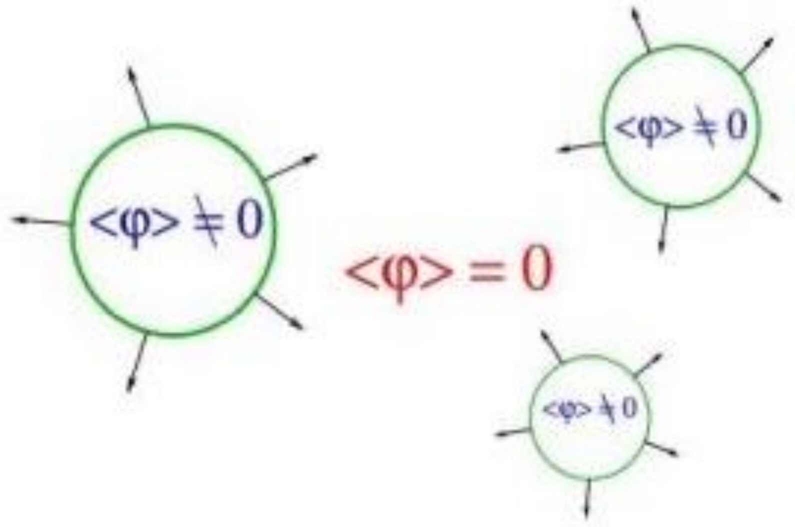




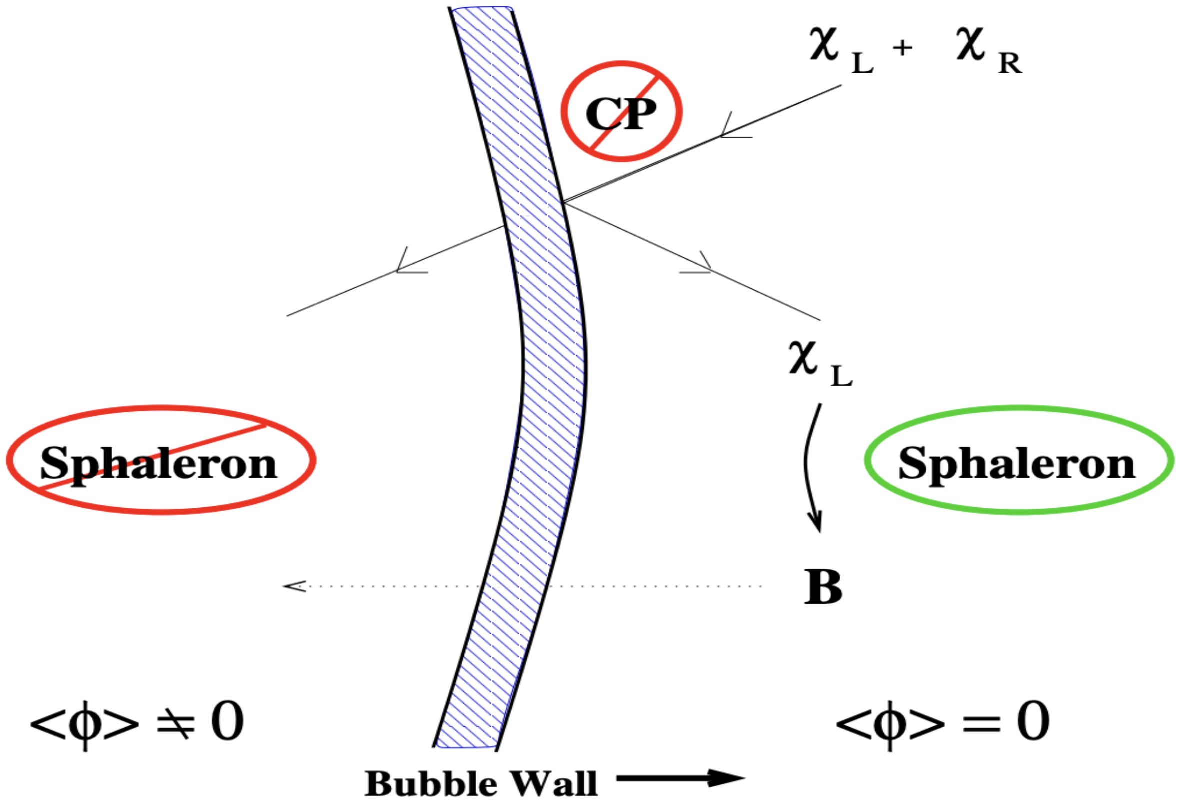
Morrissey & Ramsey-Musolf [1206.2942]

Introduction: first-order phase transitions and baryogenesis

First-Order Phase Transitions occur through the nucleation of broken phase bubbles



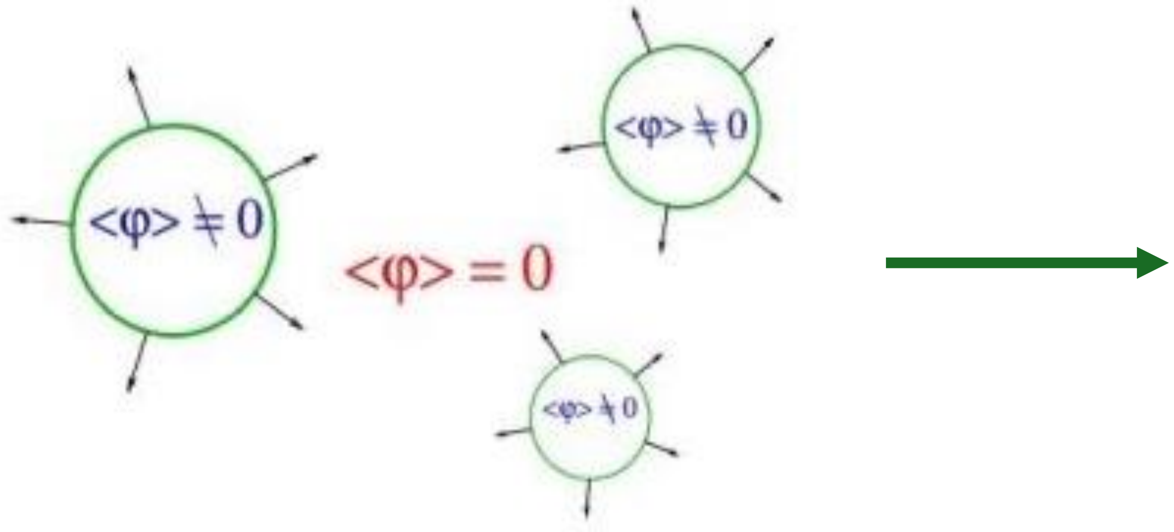
EW sphalerons → Baryon number violation

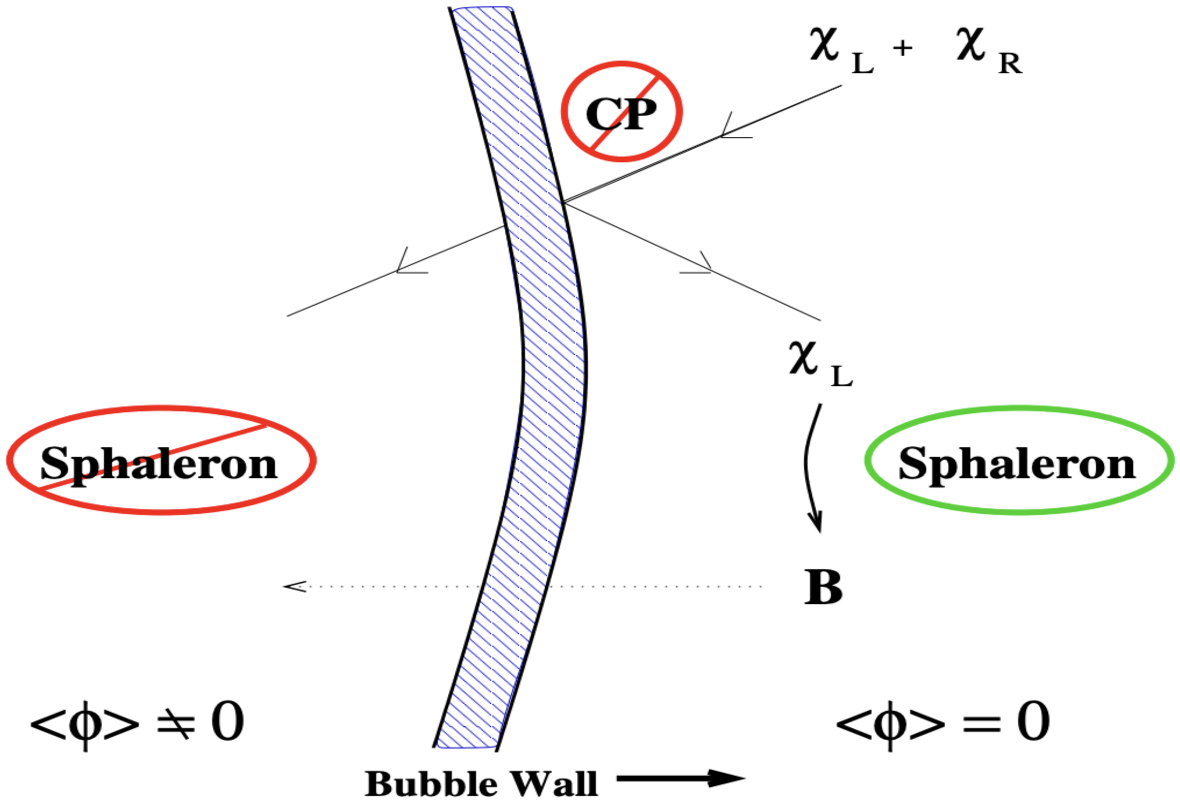


Morrissey & Ramsey-Musolf [1206.2942]

Introduction: first-order phase transitions and baryogenesis

First-Order Phase Transitions occur through the nucleation of broken phase bubbles





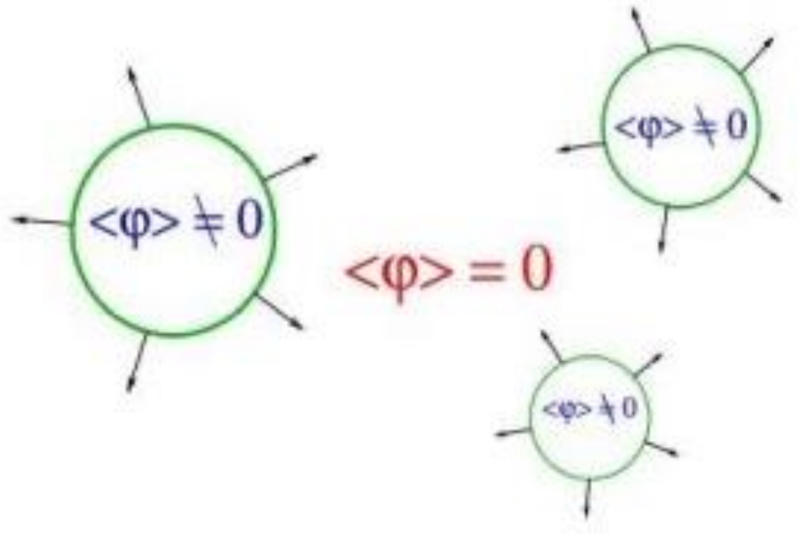
EW sphalerons → Baryon number violation

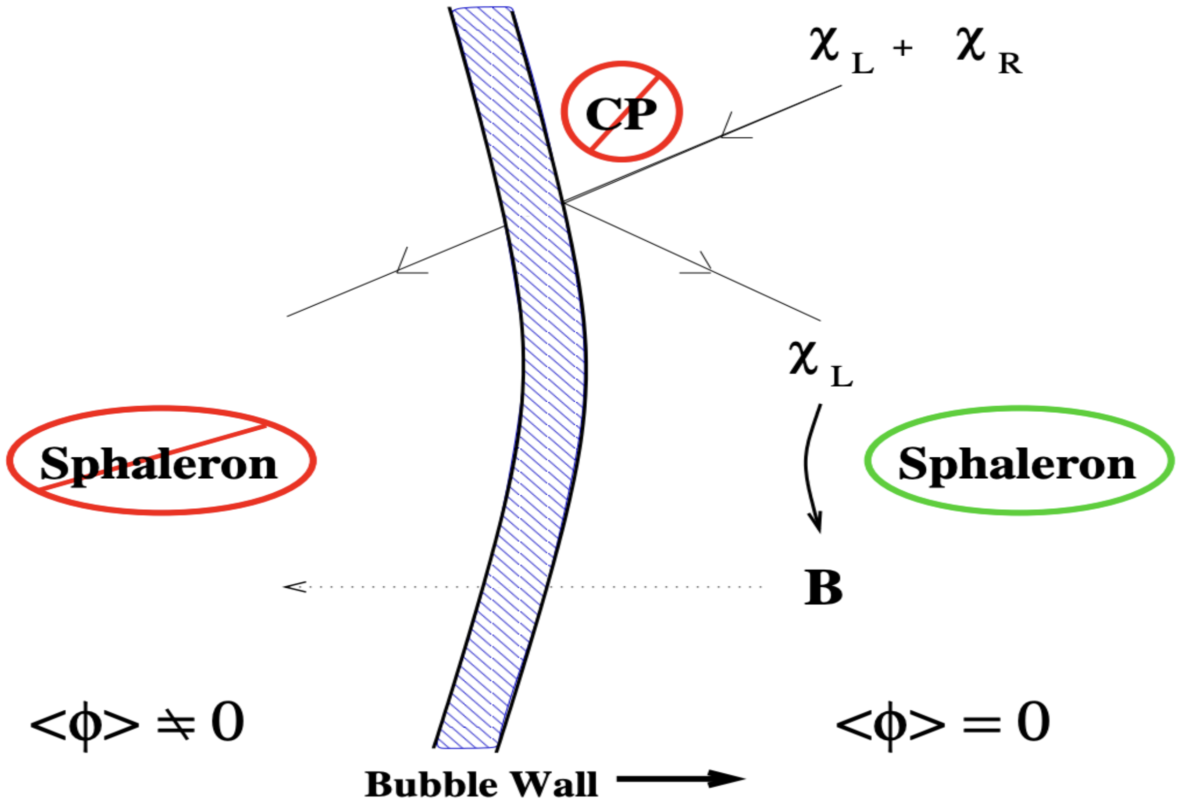
CKM matrix (or BSM physics) → C and CP violation

Morrissey & Ramsey-Musolf [1206.2942]

Introduction: first-order phase transitions and baryogenesis

First-Order Phase Transitions occur through the nucleation of broken phase bubbles





EW sphalerons → Baryon number violation

CKM matrix (or BSM physics) → C and CP violation

Bubble wall motion → departure from thermal equilibrium

Morrissey & Ramsey-Musolf [1206.2942]

Introduction: phase transitions and primordial magnetic fields

$10^{-16}G < B < 10^{-9}G$ on Mpc scales
(lower bounds from blazars and upper from CMB)

Introduction: phase transitions and primordial magnetic fields

$10^{-16}G < B < 10^{-9}G$ on Mpc scales
(lower bounds from blazars and upper from CMB)

EW Magnetogenesis: Kibble Mechanism

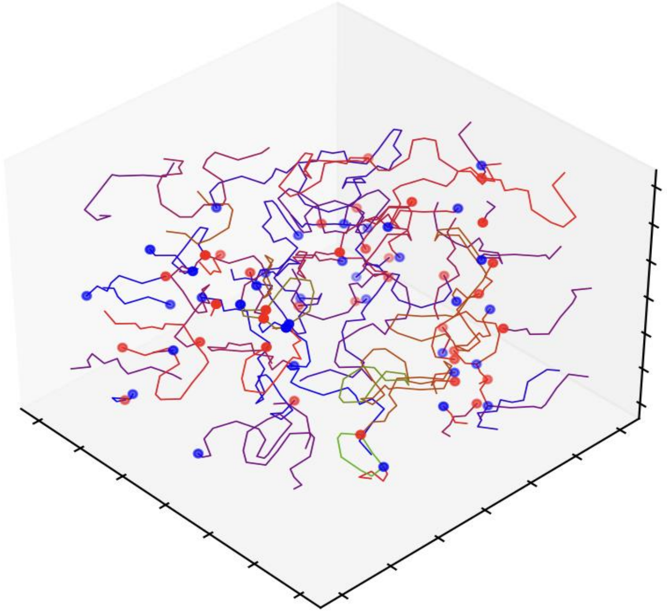
EWSSB $\rightarrow |\phi|^2 = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = \eta^2$

Higgs takes different values in causally disconnected zones
→ Vacuum Manifold $S^2 \times S^1$

[2010.10525, 2108.05357, 2302.00512]

Introduction: phase transitions and primordial magnetic fields

$10^{-16}G < B < 10^{-9}G$ on Mpc scales
(lower bounds from blazars and upper from CMB)



EW Magnetogenesis: Kibble Mechanism

EWSSB → $|\phi|^2 = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = \eta^2$

Higgs takes different values in causally disconnected zones
→ Vacuum Manifold $S^2 \times S^1$

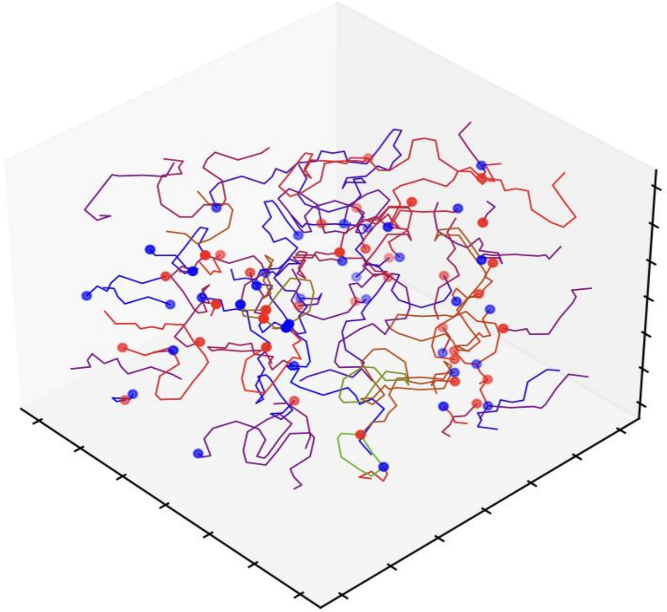
Monopoles and Strings → $\vec{\nabla} \cdot \vec{B} \neq 0$

[2010.10525, 2108.05357, 2302.00512]

Introduction: phase transitions and primordial magnetic fields

$10^{-16}G < B < 10^{-9}G$ on Mpc scales
(lower bounds from blazars and upper from CMB)

EW Magnetogenesis: Kibble Mechanism



EWSSB → $|\phi|^2 = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = \eta^2$

Higgs takes different values in causally disconnected zones
→ Vacuum Manifold $S^2 \times S^1$

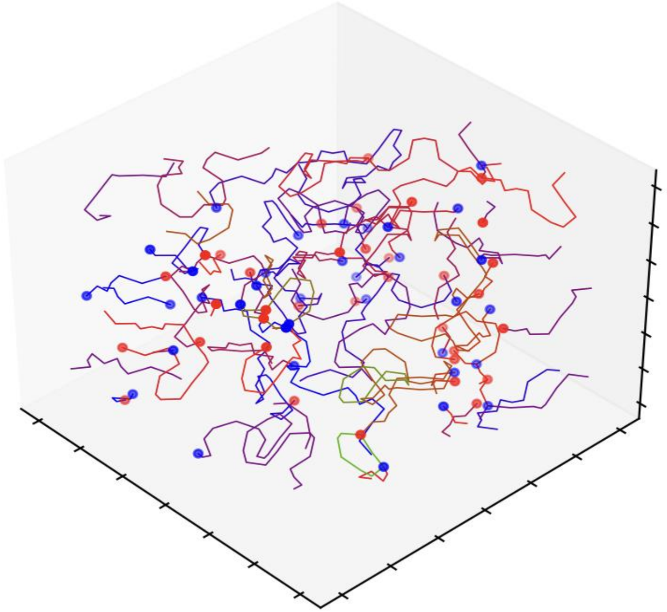
Monopoles and Strings → $\vec{\nabla} \cdot \vec{B} \neq 0$

't Hooft, Vachaspati *et al.* → $A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\frac{2\sin\theta_w}{g}(\partial_\mu\hat{\Phi}^\dagger\partial_\nu\hat{\Phi} - \partial_\nu\hat{\Phi}^\dagger\partial_\mu\hat{\Phi})$

[Nucl. Phys. B 79 (1974) 276, 2010.10525, 2108.05357, 2302.00512]

Introduction: phase transitions and primordial magnetic fields

$10^{-16}G < B < 10^{-9}G$ on Mpc scales
(lower bounds from blazars and upper from CMB)



EW Magnetogenesis: Kibble Mechanism

EWSSB → $|\phi|^2 = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = \eta^2$

Higgs takes different values in causally disconnected zones
→ Vacuum Manifold $S^2 \times S^1$

Monopoles and Strings → $\vec{\nabla} \cdot \vec{B} \neq 0$

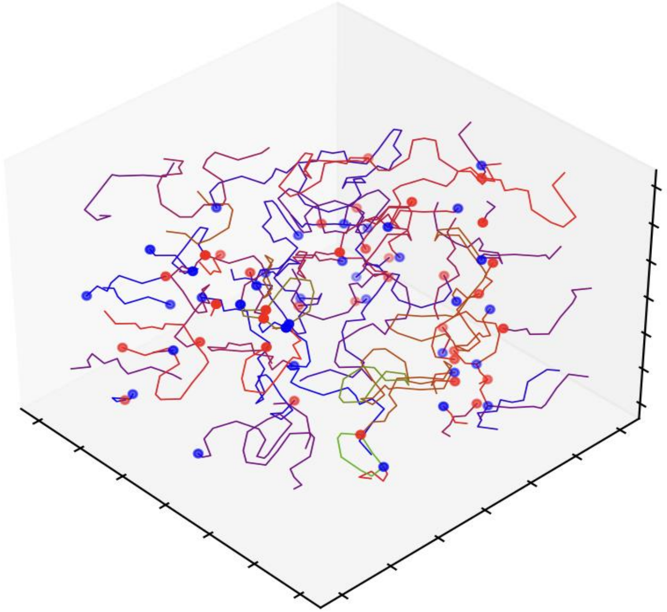
't Hooft, Vachaspati *et al.* → $A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\frac{2\sin\theta_w}{g}(\partial_\mu \hat{\Phi}^\dagger \partial_\nu \hat{\Phi} - \partial_\nu \hat{\Phi}^\dagger \partial_\mu \hat{\Phi})$

Annihilation of monopoles-antimonopoles pairs with residual $\vec{B} \neq 0$

[Nucl. Phys. B 79 (1974) 276, 2010.10525, 2108.05357, 2302.00512]

Introduction: *first-order* phase transitions and primordial magnetic fields

$10^{-16}G < B < 10^{-9}G$ on Mpc scales
(lower bounds from blazars and upper from CMB)



EW Magnetogenesis: Kibble Mechanism

EWSSB → $|\phi|^2 = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = \eta^2$

Higgs takes different values in *different broken phase bubbles*
→ Vacuum Manifold $S^2 \times S^1$

Monopoles and Strings → $\vec{\nabla} \cdot \vec{B} \neq 0$

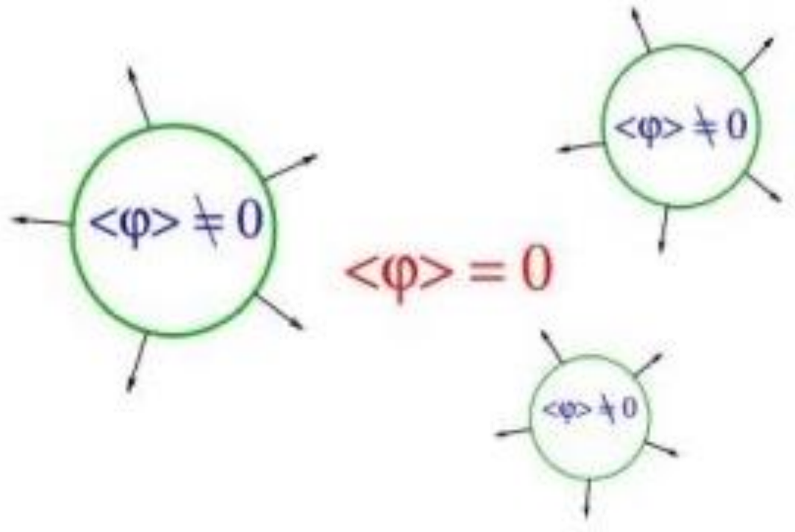
‘t Hooft, Vachaspati *et al.* → $A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\frac{2\sin\theta_w}{g}(\partial_\mu\hat{\Phi}^\dagger\partial_\nu\hat{\Phi} - \partial_\nu\hat{\Phi}^\dagger\partial_\mu\hat{\Phi})$

Annihilation of monopoles-antimonopoles pairs with residual $\vec{B} \neq 0$

[Nucl. Phys. B 79 (1974) 276, 2010.10525, 2108.05357, 2302.00512]

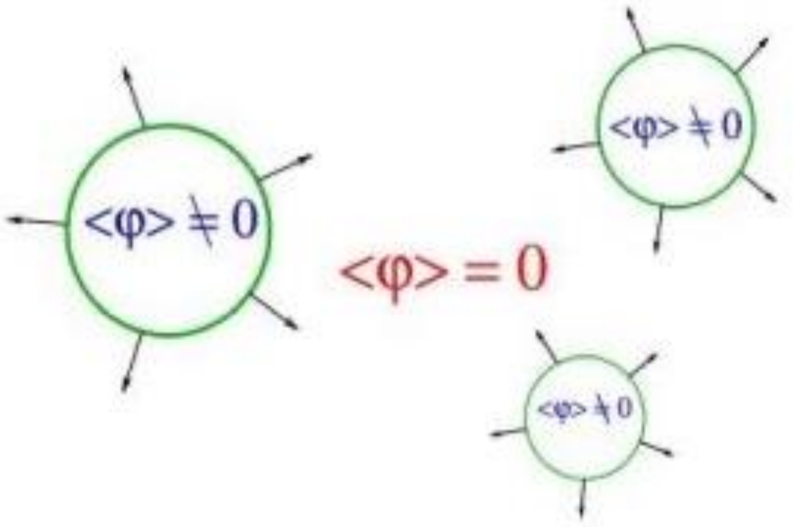
Introduction: first-order phase transitions and gravitational waves

First-Order Phase Transitions occur through the nucleation of broken phase bubbles

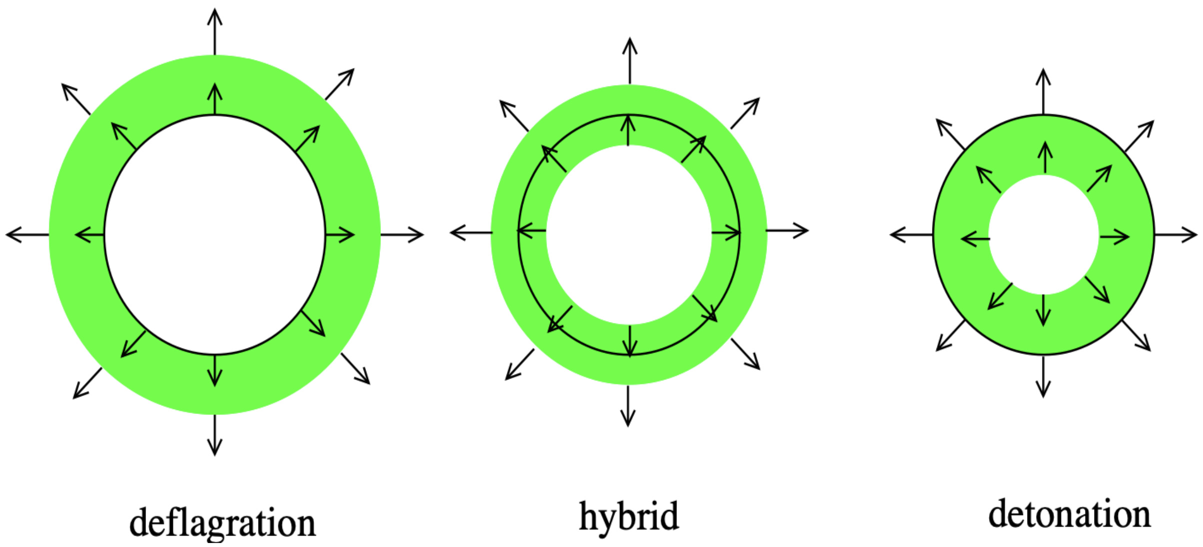


Introduction: first-order phase transitions and gravitational waves

First-Order Phase Transitions occur through the nucleation of broken phase bubbles



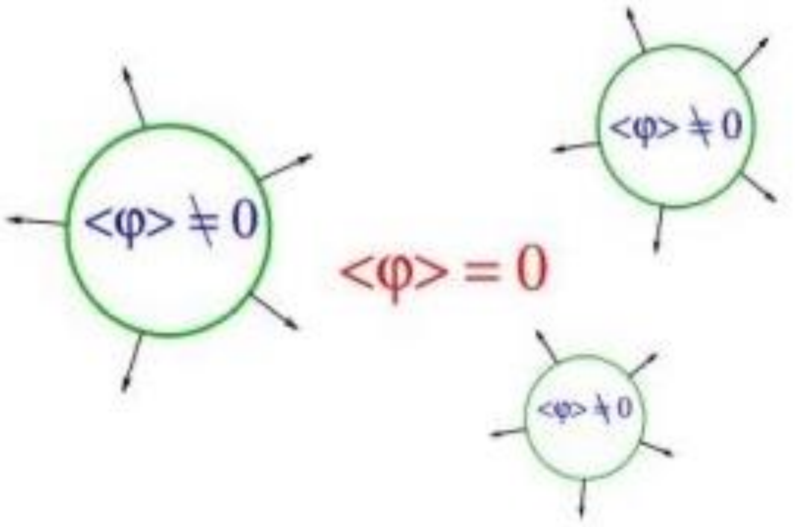
friction between scalar and plasma



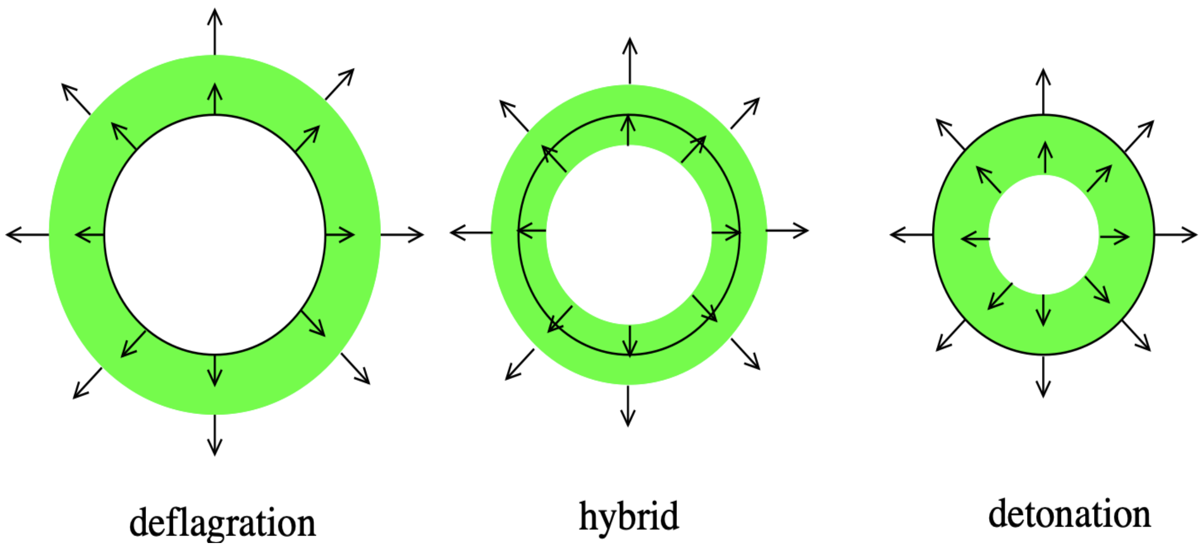
Espinosa et al. [1004.4187]

Introduction: first-order phase transitions and gravitational waves

First-Order Phase Transitions occur through the nucleation of broken phase bubbles



friction between scalar and plasma

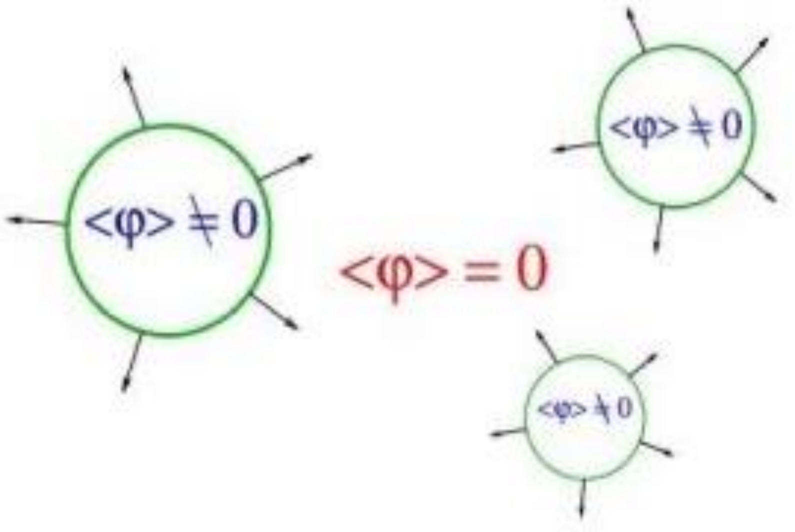


Espinosa et al. [1004.4187]

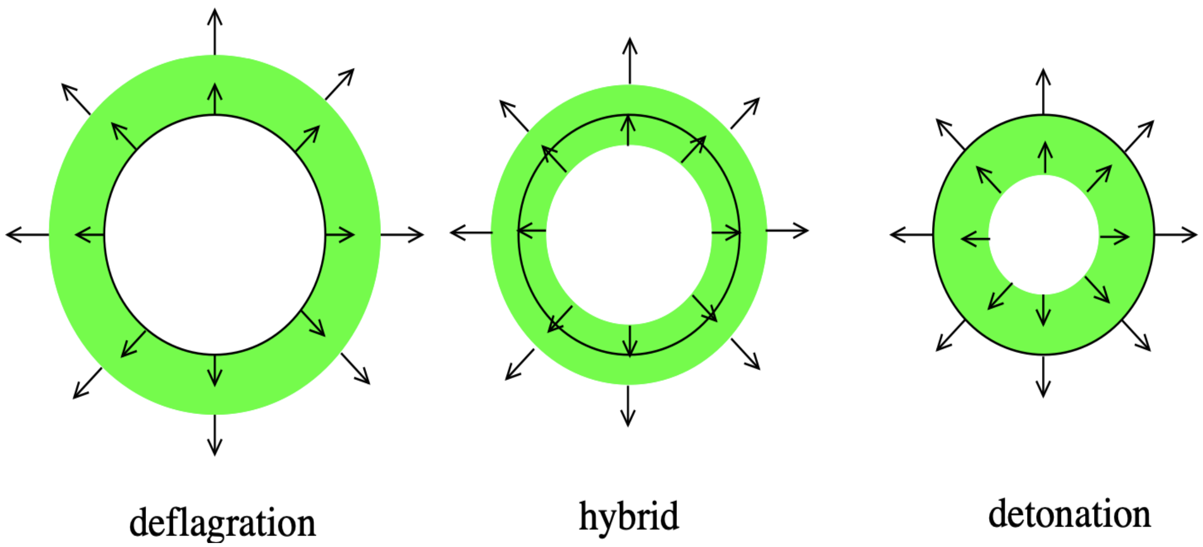
Bubble expansion phase → scalar and fluid profiles are spherically symmetric

Introduction: first-order phase transitions and gravitational waves

First-Order Phase Transitions occur through the nucleation of broken phase bubbles



friction between scalar and plasma



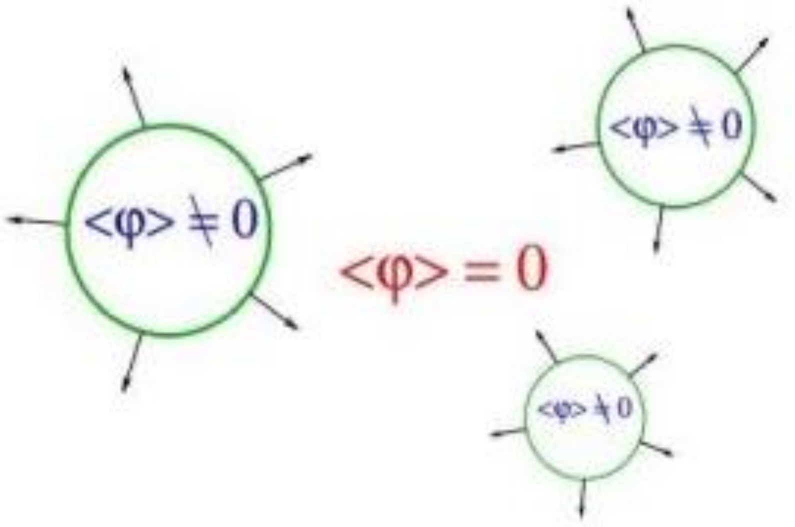
Espinosa et al. [1004.4187]

Bubble expansion phase → scalar and fluid profiles are spherically symmetric

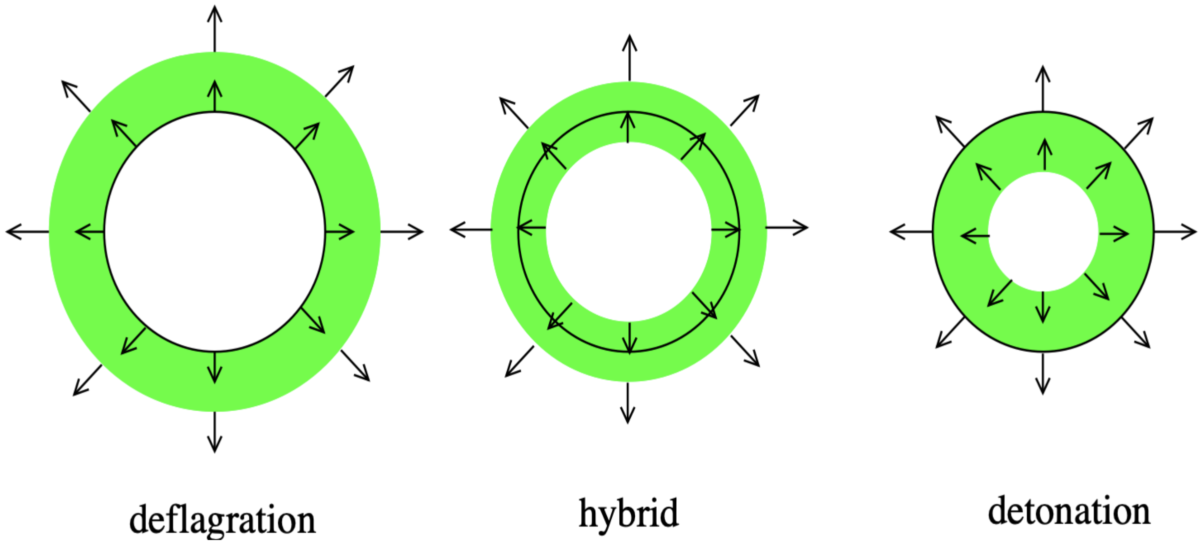
No anisotropic stresses → No gravitational wave production

Introduction: first-order phase transitions and gravitational waves

First-Order Phase Transitions occur through the nucleation of broken phase bubbles



friction between scalar and plasma

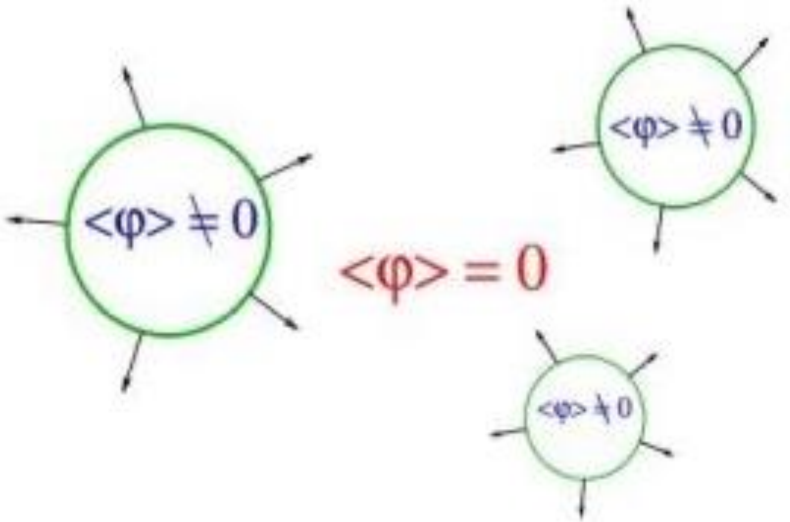


Espinosa et al. [1004.4187]

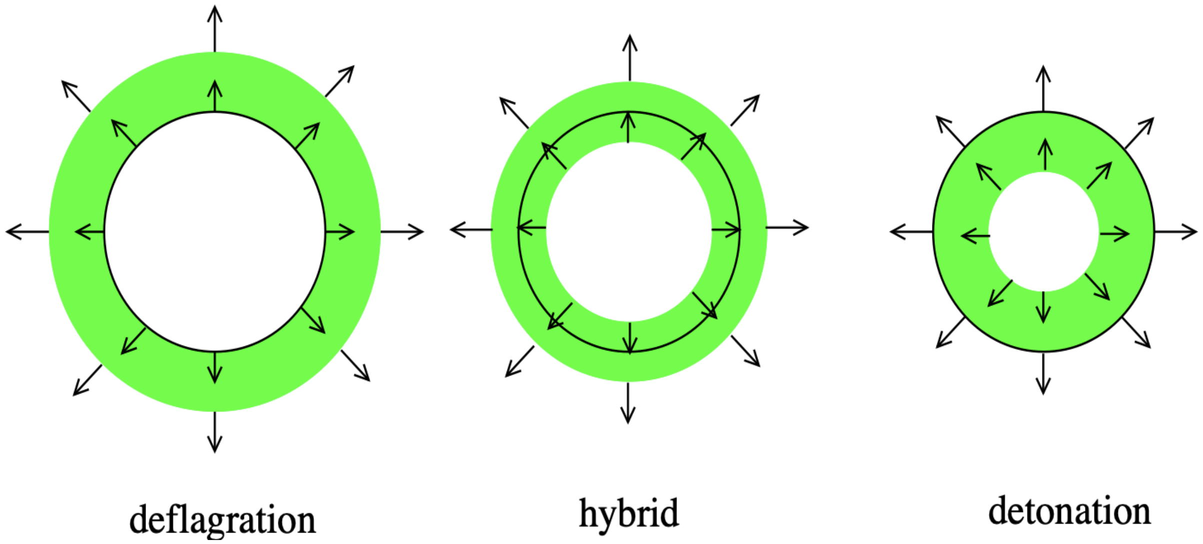
Bubble collisions break spherical symmetry

Introduction: first-order phase transitions and gravitational waves

First-Order Phase Transitions occur through the nucleation of broken phase bubbles



friction between scalar and plasma

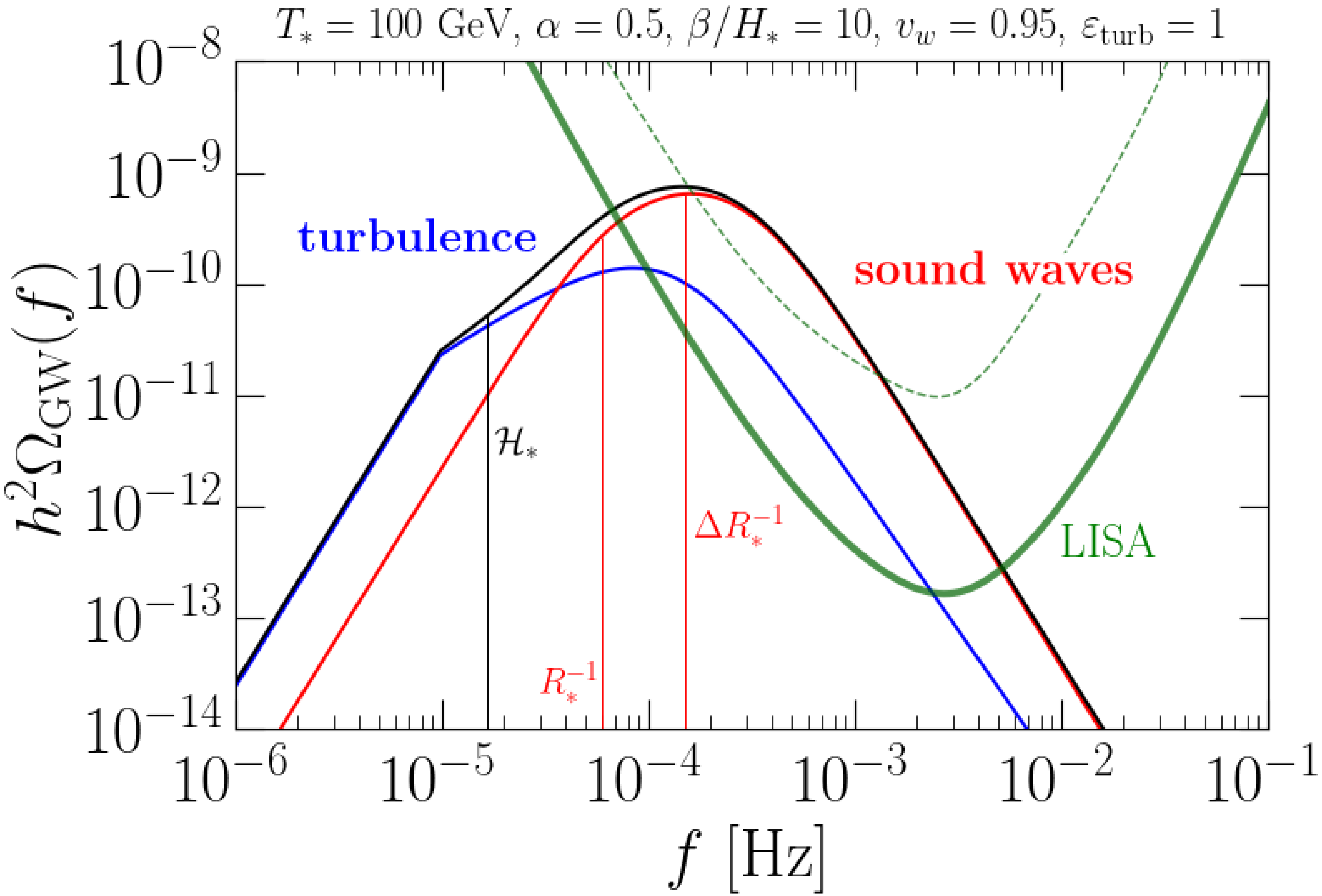


Espinosa et al. [1004.4187]

Bubble collisions break spherical symmetry

Nonzero anisotropic stresses → scalar and fluid can produce gravitational waves

Introduction: first-order phase transitions and gravitational waves

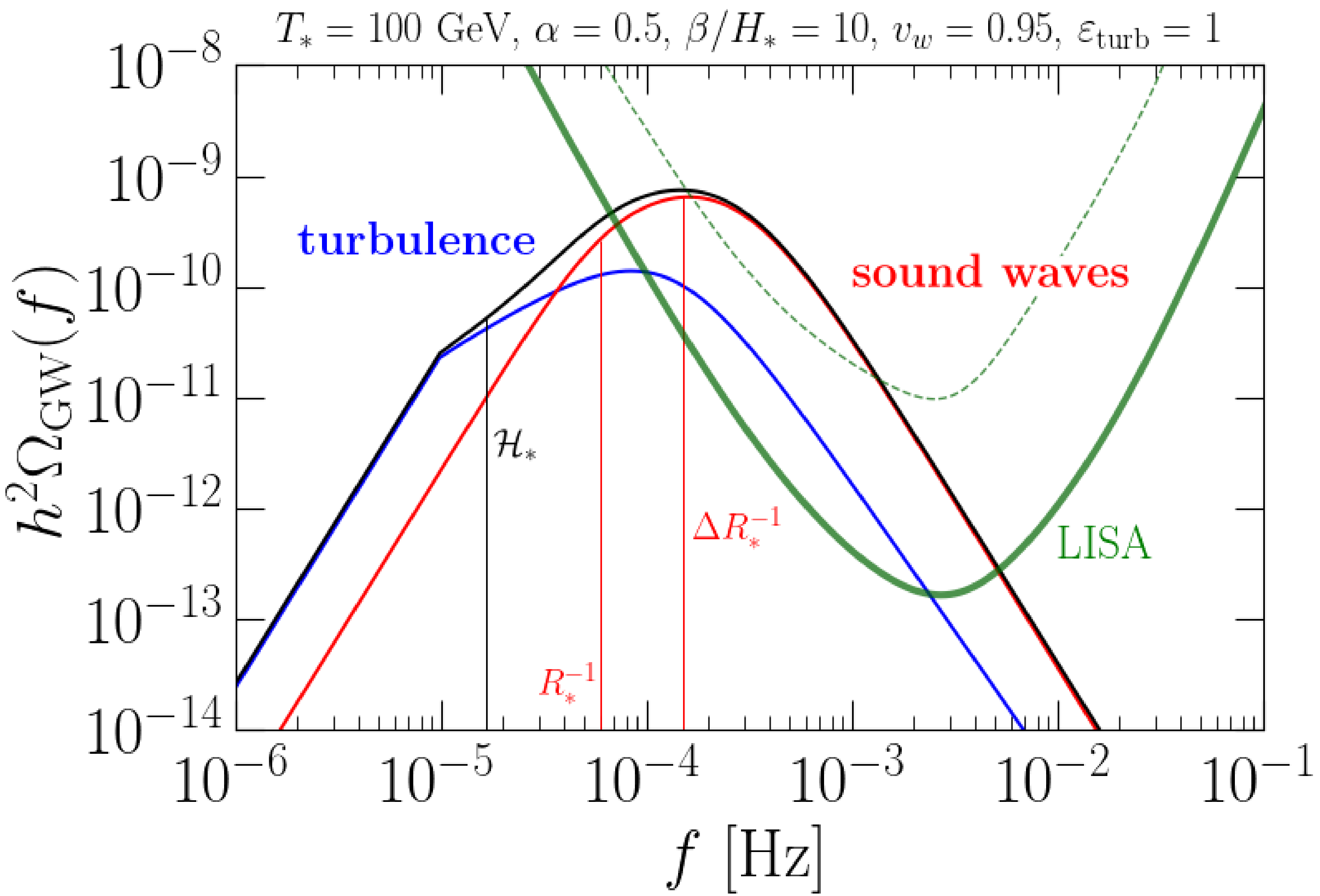


GW background from EW phase transition in the LISA sensitivity band!

Credits: Alberto Roper Pol

Lisa Cosmology Working Group [2403.03723]

Introduction: first-order phase transitions and gravitational waves



Sound-shell model

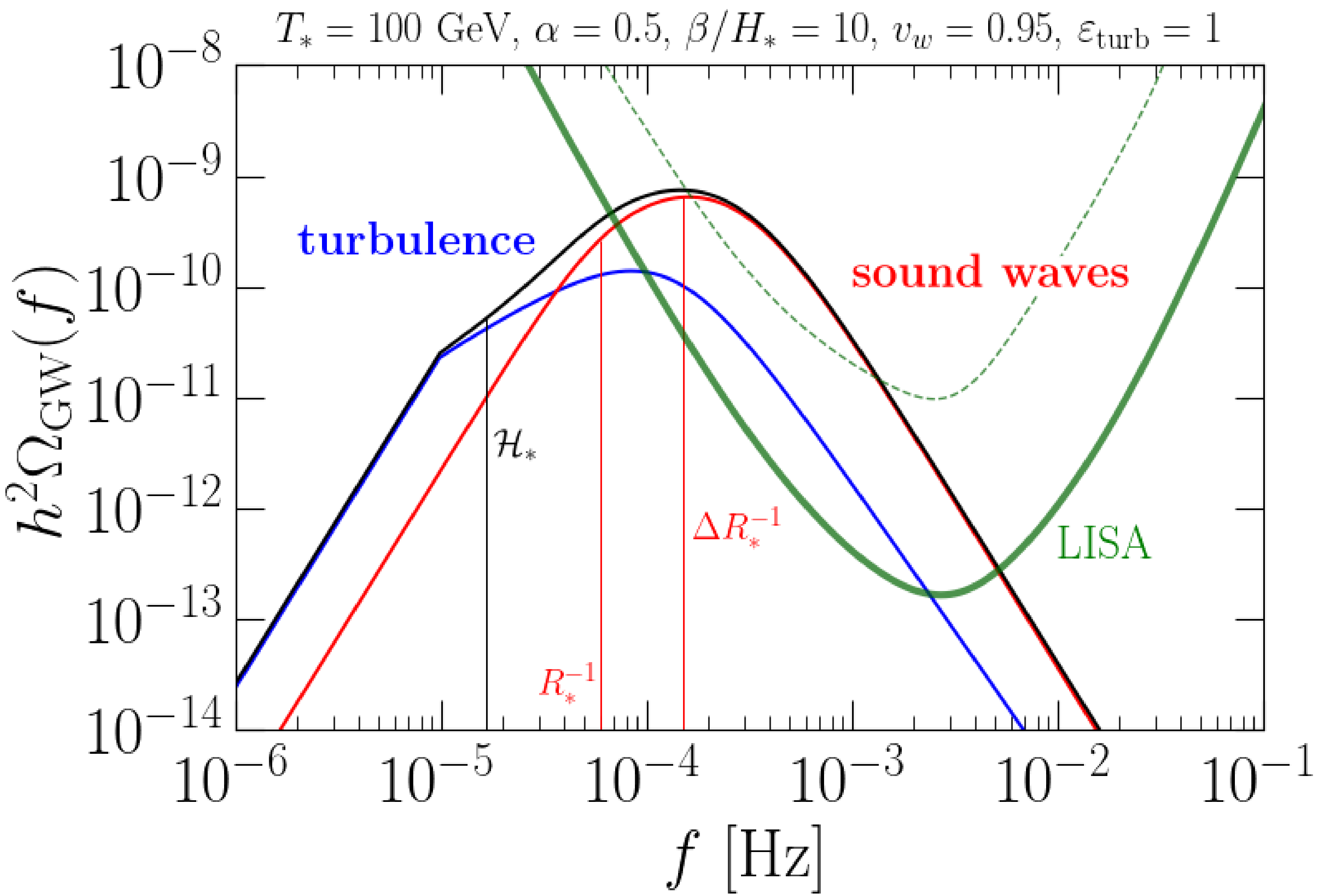
Hindmarsh & Hijazi [1909.10040]

GW background from EW phase transition in the LISA sensitivity band!

← Credits: Alberto Roper Pol

Lisa Cosmology Working Group [2403.03723]

Introduction: first-order phase transitions and gravitational waves



Sound-shell model

Hindmarsh & Hijazi [1909.10040]

Constant-in-time model

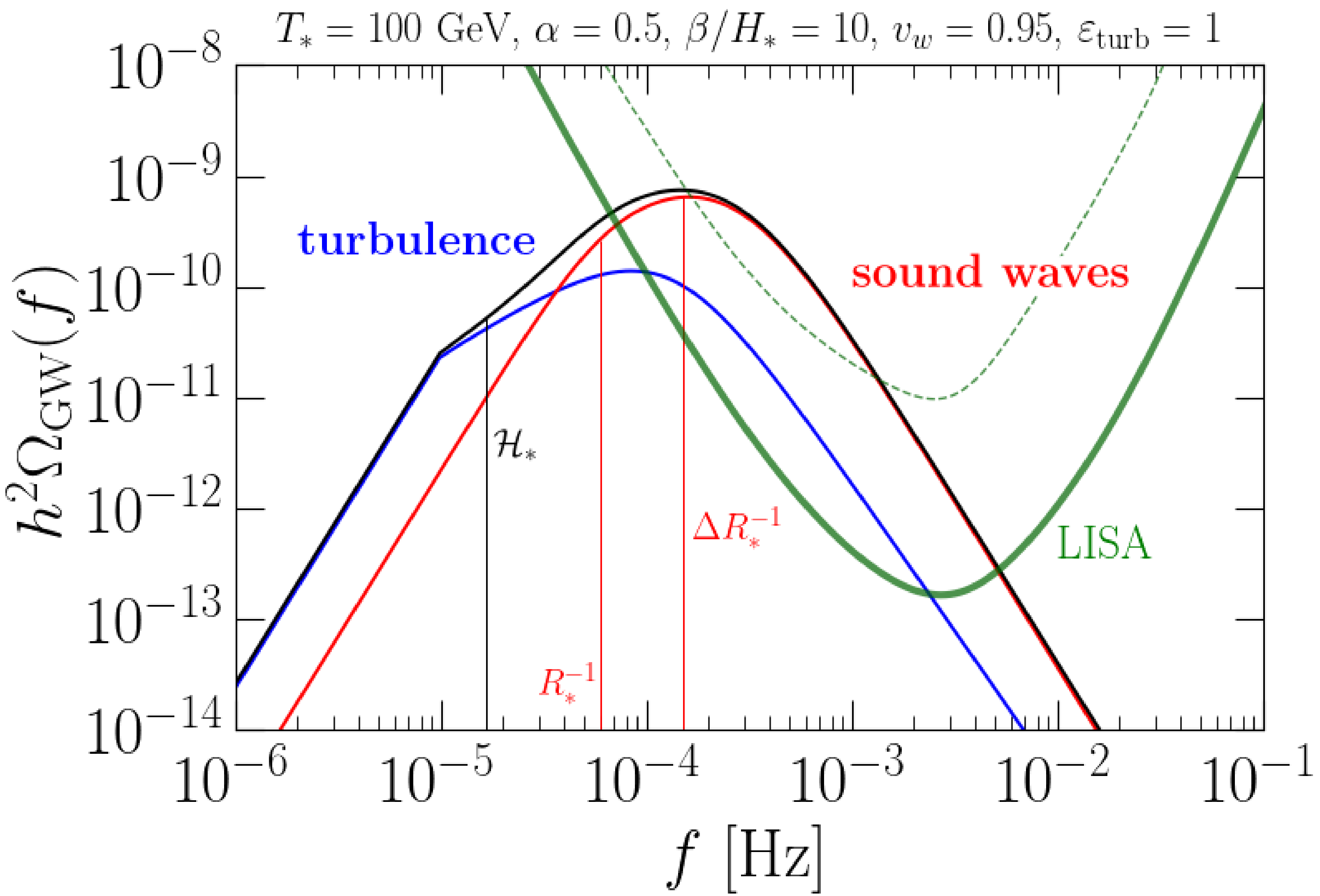
Roper Pol, Caprini et al. [2201.05630]

GW background from EW phase transition in the LISA sensitivity band!

← Credits: Alberto Roper Pol

Lisa Cosmology Working Group [2403.03723]

Introduction: first-order phase transitions and gravitational waves

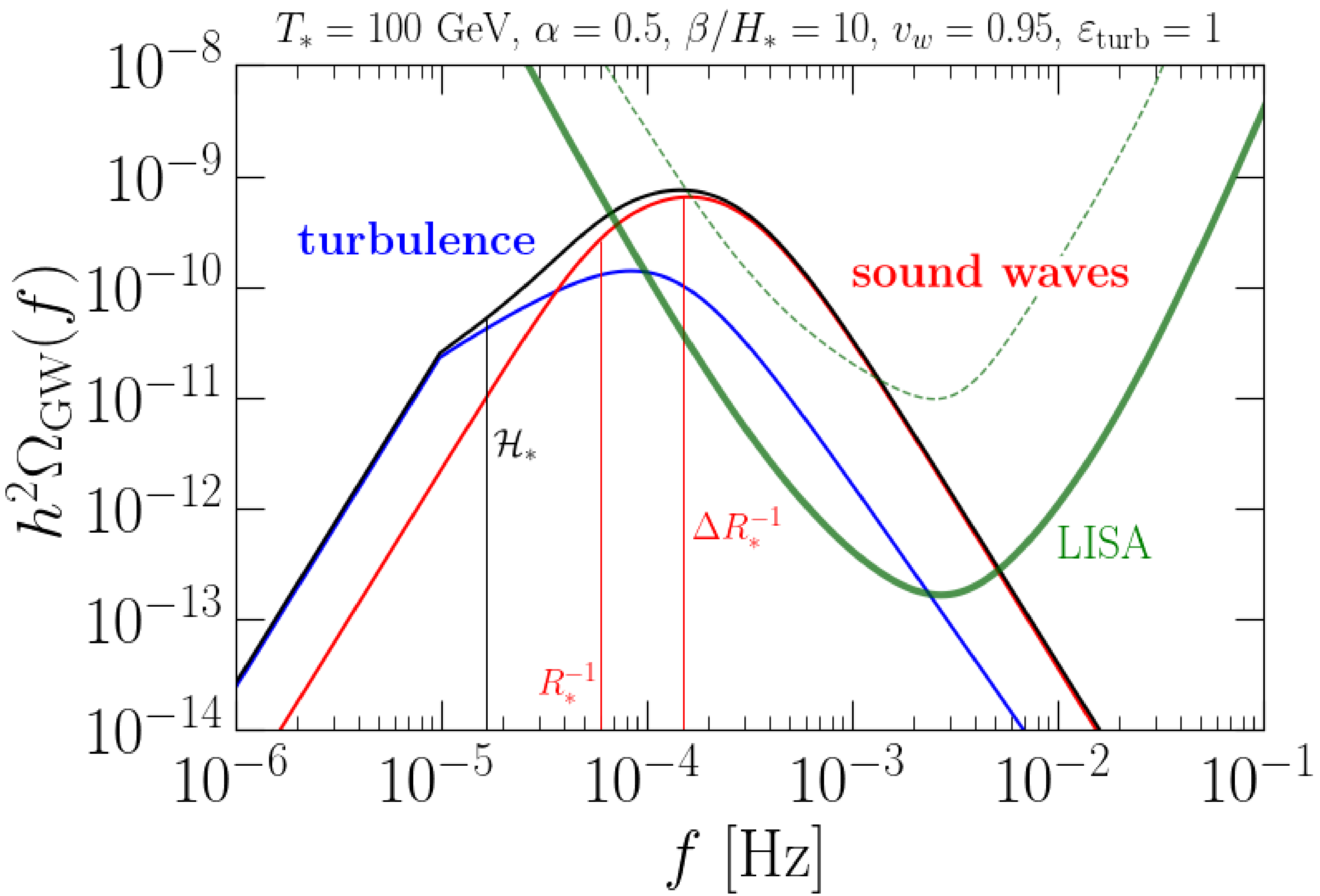


What is the origin of the peak scales in the GW spectrum from sound waves?

← Credits: Alberto Roper Pol

Lisa Cosmology Working Group [2403.03723]

Introduction: first-order phase transitions and gravitational waves



What is the origin of the peak scales in the GW spectrum from sound waves?

Are they actually related to R_* & ΔR_*?

Credits: Alberto Roper Pol

Lisa Cosmology Working Group [2403.03723]

Outline

- Fluid perturbations from expanding scalar bubbles
- Evolution of the fluid perturbations:
  before, *across* and *after* bubble collisions
- Consequences for the gravitational wave spectrum

Fluid perturbations from expanding scalar bubbles

$$T_{\mu\nu}^{\rm tot} = w_{\rm tot}\, u_\mu u_\nu + p_{\rm tot}\, g_{\mu\nu} + \partial_\mu \phi\, \partial_\nu \phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma \phi\, \partial^\sigma \phi\right)$$

$$w_{\rm tot} = w - T\frac{\partial V_{\rm eff}(\phi, T)}{\partial T}$$

$$p_{\rm tot} = p - V_{\rm eff}(\phi, T)$$

Fluid perturbations from expanding scalar bubbles

$$T^{tot}_{\mu\nu} = w_{tot}\, u_\mu u_\nu + p_{tot}\, g_{\mu\nu} + \partial_\mu\phi\, \partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma\phi\, \partial^\sigma\phi\right)$$

$$w_{tot} = w - T\frac{\partial V_{eff}(\phi, T)}{\partial T}$$

$$p_{tot} = p - V_{eff}(\phi, T)$$

$$\begin{cases} \nabla_\mu T^{\mu\nu}_{tot} = 0 \\ \nabla_\sigma(\partial^\sigma\phi) - \dfrac{\partial V}{\partial \phi} = \delta_{friction} \end{cases}$$

Fluid perturbations from expanding scalar bubbles

$$T_{\mu\nu}^{tot} = w_{tot}\, u_\mu u_\nu + p_{tot}\, g_{\mu\nu} + \partial_\mu\phi\, \partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma\phi\, \partial^\sigma\phi\right)$$

$$w_{tot} = w - T\frac{\partial V_{eff}(\phi, T)}{\partial T}$$

$$p_{tot} = p - V_{eff}(\phi, T)$$

$$\begin{cases} \nabla_\mu T_{tot}^{\mu\nu} = 0 \\ \nabla_\sigma(\partial^\sigma\phi) - \dfrac{\partial V}{\partial\phi} = \delta_{friction} \end{cases}$$

$$\eta\, u^\mu \partial_\mu \phi$$

Fluid perturbations from expanding scalar bubbles

$$T_{\mu\nu}^{tot} = w_{tot}\, u_\mu u_\nu + p_{tot}\, g_{\mu\nu} + \partial_\mu\phi\, \partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma\phi\, \partial^\sigma\phi\right)$$

$$w_{tot} = w - T\frac{\partial V_{eff}(\phi, T)}{\partial T}$$

$$p_{tot} = p - V_{eff}(\phi, T)$$

$$\begin{cases} \nabla_\mu T_{tot}^{\mu\nu} = 0 \\ \nabla_\sigma(\partial^\sigma\phi) - \dfrac{\partial V}{\partial\phi} = \delta_{friction} \end{cases}$$

$$\eta\, u^\mu \partial_\mu\phi$$

Understanding the full picture requires lattice simulations

[2407.05826] [1504.03291] [2409.03651][2505.17824]

But how far can we go analytically?

Fluid perturbations from expanding scalar bubbles

$$T_{\mu\nu}^{tot} = w_{tot}\, u_\mu u_\nu + p_{tot}\, g_{\mu\nu} + \partial_\mu\phi\, \partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma\phi\, \partial^\sigma\phi\right)$$

Simplifying assumptions:

Fluid perturbations from expanding scalar bubbles

$$T_{\mu\nu}^{tot} = w_{tot}\, u_\mu u_\nu + p_{tot}\, g_{\mu\nu} + \partial_\mu \phi\, \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} \partial_\sigma \phi\, \partial^\sigma \phi \right)$$

Simplifying assumptions:

- Flat spacetime $g_{\mu\nu} \to \eta_{\mu\nu}$

Beyond? See Giombi et al. [2504.08037]

Fluid perturbations from expanding scalar bubbles

$$T_{\mu\nu}^{\rm tot} = w_{\rm tot}\, u_\mu u_\nu + p_{\rm tot}\, g_{\mu\nu} + \partial_\mu\phi\, \partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma\phi\, \partial^\sigma\phi\right)$$

Simplifying assumptions:

- Flat spacetime $\quad g_{\mu\nu} \to \eta_{\mu\nu}$
- Bag equation of state $\longrightarrow$ (+) Symmetric phase, (–) Broken phase $\longrightarrow$

$$p_{\rm tot}^{\pm} = \frac{1}{3} a_\pm T_\pm^4 - \epsilon_\pm$$

$$e_{\rm tot}^{\pm} = a_\pm T_\pm^4 + \epsilon_\pm$$

$$w_{\rm tot}^{\pm} = e_{\rm tot}^{\pm} + p_{\rm tot}^{\pm} = \frac{4}{3} a_\pm T_\pm^4$$

Beyond? See Giombi et al. [2504.08037] [2409.01426]

Fluid perturbations from expanding scalar bubbles

$$T_{\mu\nu}^{tot} = w_{tot}\, u_\mu u_\nu + p_{tot}\, g_{\mu\nu} + \cancel{\partial_\mu \phi\, \partial_\nu \phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma \phi\, \partial^\sigma \phi\right)}$$

Simplifying assumptions:

- Flat spacetime $g_{\mu\nu} \to \eta_{\mu\nu}$
- Bag equation of state $\longrightarrow$ (+) Symmetric phase (−) Broken phase $\longrightarrow$

$$p_{tot}^{\pm} = \frac{1}{3} a_\pm T_\pm^4 - \epsilon_\pm$$

$$e_{tot}^{\pm} = a_\pm T_\pm^4 + \epsilon_\pm$$

$$w_{tot}^{\pm} = e_{tot}^{\pm} + p_{tot}^{\pm} = \frac{4}{3} a_\pm T_\pm^4$$

- Neglect scalar field profiles

Beyond? See Giombi et al. [2504.08037] [2409.01426]

Fluid perturbations from expanding scalar bubbles

$$T_{\mu\nu}^{tot} = w_{tot}\, u_\mu u_\nu + p_{tot}\, \eta_{\mu\nu}$$

Perfect fluid

$$\partial_\mu T^{\mu\nu}_{tot} = 0$$

$$p^{\pm}_{tot} = \frac{1}{3} a_\pm T_\pm^4 - \epsilon_\pm \,, \quad w^{\pm}_{tot} = \frac{4}{3} a_\pm T_\pm^4$$

$$\epsilon_+ = \epsilon > 0$$

$$\epsilon_- = 0$$

Fluid perturbations from expanding scalar bubbles

$$T^{tot}_{\mu\nu} = w_{tot}\, u_\mu u_\nu + p_{tot}\, \eta_{\mu\nu}$$

$$p^{\pm}_{tot} = \frac{1}{3} a_\pm T^4_\pm - \epsilon_\pm \,, \quad w^{\pm}_{tot} = \frac{4}{3} a_\pm T^4_\pm$$

$$\epsilon_+ = \epsilon > 0$$

$$\epsilon_- = 0$$

Perfect fluid

$$\partial_\mu T^{\mu\nu}_{tot} = 0$$

$$\gamma^2 = 1/(1-v^2)$$

- Solutions with spherical fluid velocity profile $u^\mu = \gamma(1, v\,\hat{r})$

Fluid perturbations from expanding scalar bubbles

$$T^{tot}_{\mu\nu} = w_{tot}\, u_\mu u_\nu + p_{tot}\, \eta_{\mu\nu}$$

$$p^{\pm}_{tot} = \frac{1}{3} a_\pm T^4_\pm - \epsilon_\pm \,, \quad w^{\pm}_{tot} = \frac{4}{3} a_\pm T^4_\pm$$

$$\epsilon_+ = \epsilon > 0$$

$$\epsilon_- = 0$$

Perfect fluid

$$\partial_\mu T^{\mu\nu}_{tot} = 0$$

$\gamma^2 = 1/(1-v^2)$

- Solutions with spherical fluid velocity profile $u^\mu = \gamma(1, v\,\hat{r})$

- Be $r^{(n)} = |\mathbf{r} - \mathbf{x}_0^{(n)}|$ the distance to the nucleation center of the n-th bubble $\mathbf{x}_0^{(n)}$, $t^{(n)} = t - t_0^{(n)}$ the time since it nucleated at $t_0^{(n)}$ and $\xi = r^{(n)}/t^{(n)}$

Fluid perturbations from expanding scalar bubbles

$$T^{tot}_{\mu\nu} = w_{tot}\, u_\mu u_\nu + p_{tot}\, \eta_{\mu\nu}$$

$$p^{\pm}_{tot} = \frac{1}{3} a_\pm T_\pm^4 - \epsilon_\pm \,, \; w^{\pm}_{tot} = \frac{4}{3} a_\pm T_\pm^4$$

$$\epsilon_+ = \epsilon > 0$$
$$\epsilon_- = 0$$

Perfect fluid

$$\partial_\mu T^{\mu\nu}_{tot} = 0$$

$\gamma^2 = 1/(1 - v^2)$

- Solutions with spherical fluid velocity profile $u^\mu = \gamma(1, v\, \hat{r})$

- Be $r^{(n)} = |\mathbf{r} - \mathbf{x}_0^{(n)}|$ the distance to the nucleation center of the n-th bubble $\mathbf{x}_0^{(n)}$, $t^{(n)} = t - t_0^{(n)}$ the time since it nucleated at $t_0^{(n)}$ and $\xi = r^{(n)}/t^{(n)}$

- For a superposition of bubbles we have $\mathbf{v} = \sum_{n=1}^{N_b} v_{ip}(\xi)\, \hat{r}^{(n)}$

Fluid perturbations from expanding scalar bubbles

$$T^{\mathrm{tot}}_{\mu\nu} = w_{\mathrm{tot}}\, u_\mu u_\nu + p_{\mathrm{tot}}\, \eta_{\mu\nu}$$

$$p^{\pm}_{\mathrm{tot}} = \frac{1}{3} a_\pm T_\pm^4 - \epsilon_\pm \,,\ w^{\pm}_{\mathrm{tot}} = \frac{4}{3} a_\pm T_\pm^4$$

Perfect fluid

$$\epsilon_+ = \epsilon > 0$$
$$\epsilon_- = 0$$

$$\partial_\mu T^{\mu\nu}_{\mathrm{tot}} = 0$$

- Impose boundary conditions across the wall using the bag equation of state

$$\mathbf{v} = \sum_{n=1}^{N_b} v_{ip}(\xi)\, \hat{r}^{(n)}$$

Fluid perturbations from expanding scalar bubbles

$$T^{\rm tot}_{\mu\nu} = w_{\rm tot}\, u_\mu u_\nu + p_{\rm tot}\, \eta_{\mu\nu}$$

Perfect fluid

$$p^{\pm}_{\rm tot} = \frac{1}{3} a_\pm T^4_\pm - \epsilon_\pm \,, \quad w^{\pm}_{\rm tot} = \frac{4}{3} a_\pm T^4_\pm$$

$$\epsilon_+ = \epsilon > 0$$
$$\epsilon_- = 0$$

$$\partial_\mu T^{\mu\nu}_{\rm tot} = 0$$

- Impose boundary conditions across the wall using the bag equation of state

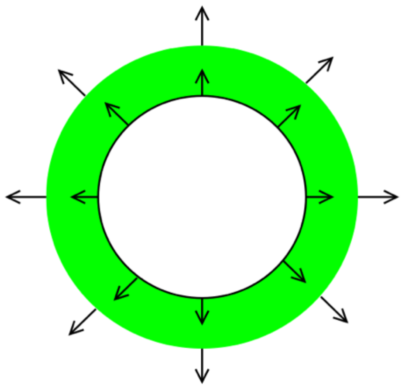
$$\mathbf{v} = \sum_{n=1}^{N_b} v_{ip}(\xi)\, \hat{r}^{(n)}$$

- Depending on the wall velocity ξ_w and the phase transition strenght $\alpha = \epsilon/e_n$ we find three types of solution

Fluid perturbations from expanding scalar bubbles

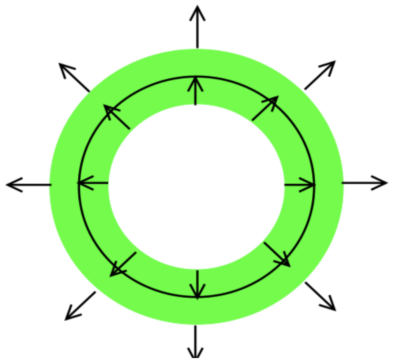
DEFLAGRATIONS

$\xi_w < c_s$



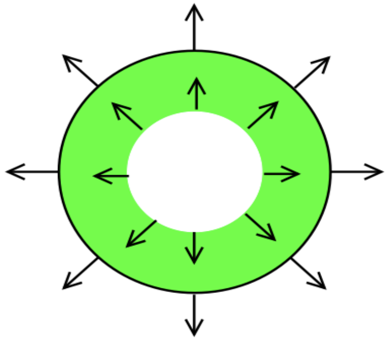
HYBRIDS

$c_s < \xi_w < v_{CJ}(\alpha)$



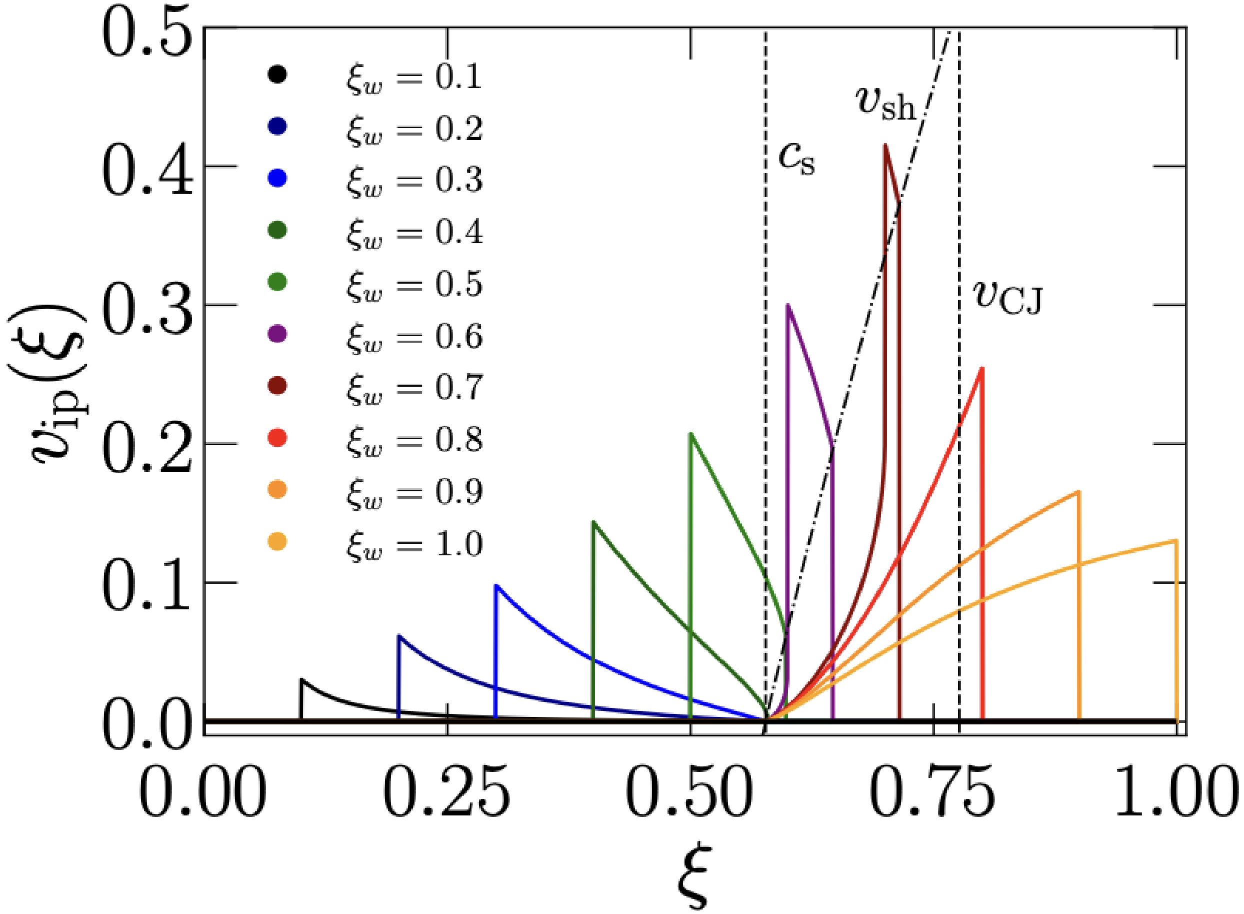
DETONATIONS

$\xi_w > v_{CJ}(\alpha)$



pip install cosmoGW

$$v_{CJ}(\alpha) = \frac{1 + \sqrt{\alpha(2 + 3\alpha)}}{\sqrt{3}(1 + \alpha)}$$



Espinosa et al. [1004.4187]

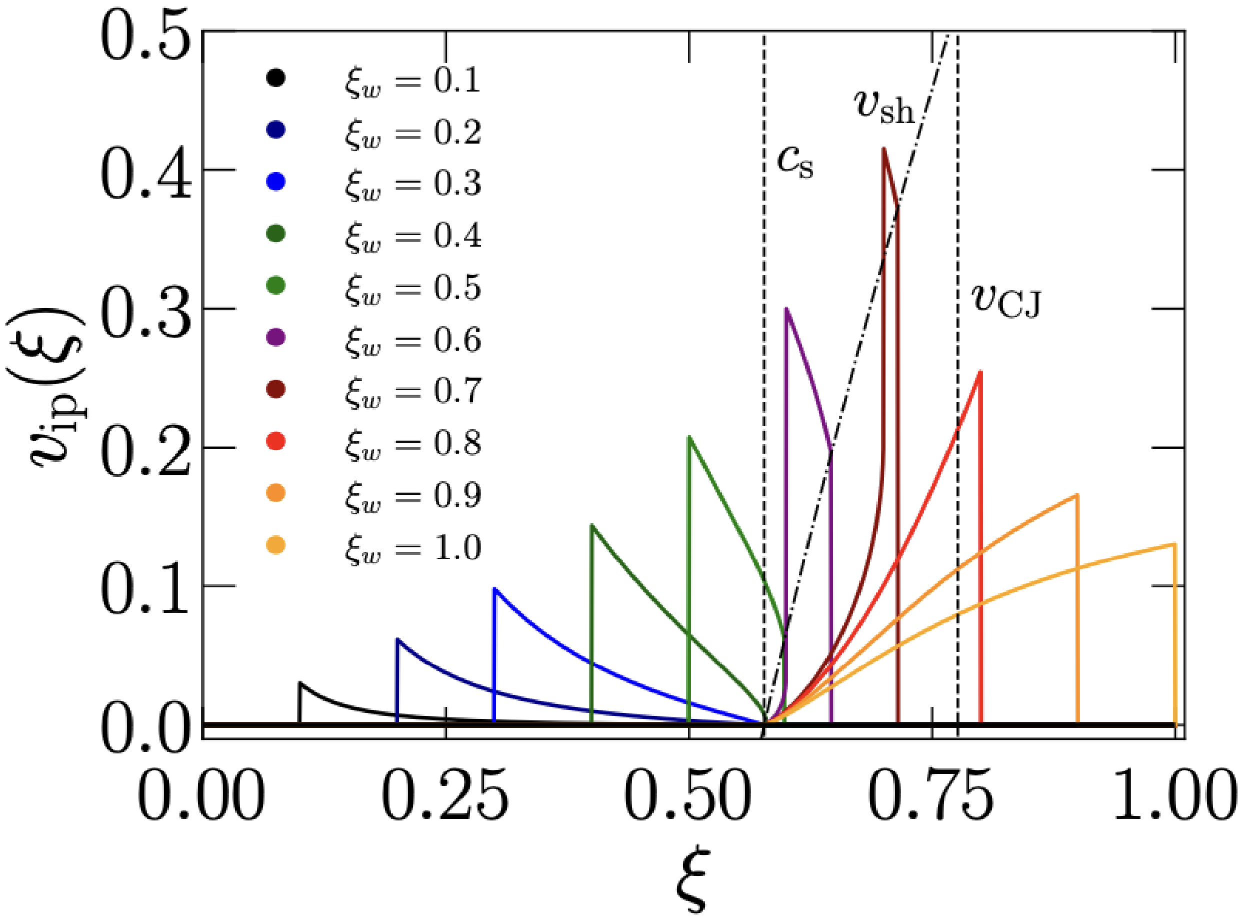
Fluid perturbations from expanding scalar bubbles

Properties of the profiles:

- Compact support
  $v_{ip}(\xi) \neq 0 \quad for \;\; \xi_b < \xi < \xi_f$

pip install cosmoGW

$$v_{CJ}(\alpha) = \frac{1 + \sqrt{\alpha(2 + 3\alpha)}}{\sqrt{3}(1 + \alpha)}$$



Fluid perturbations from expanding scalar bubbles

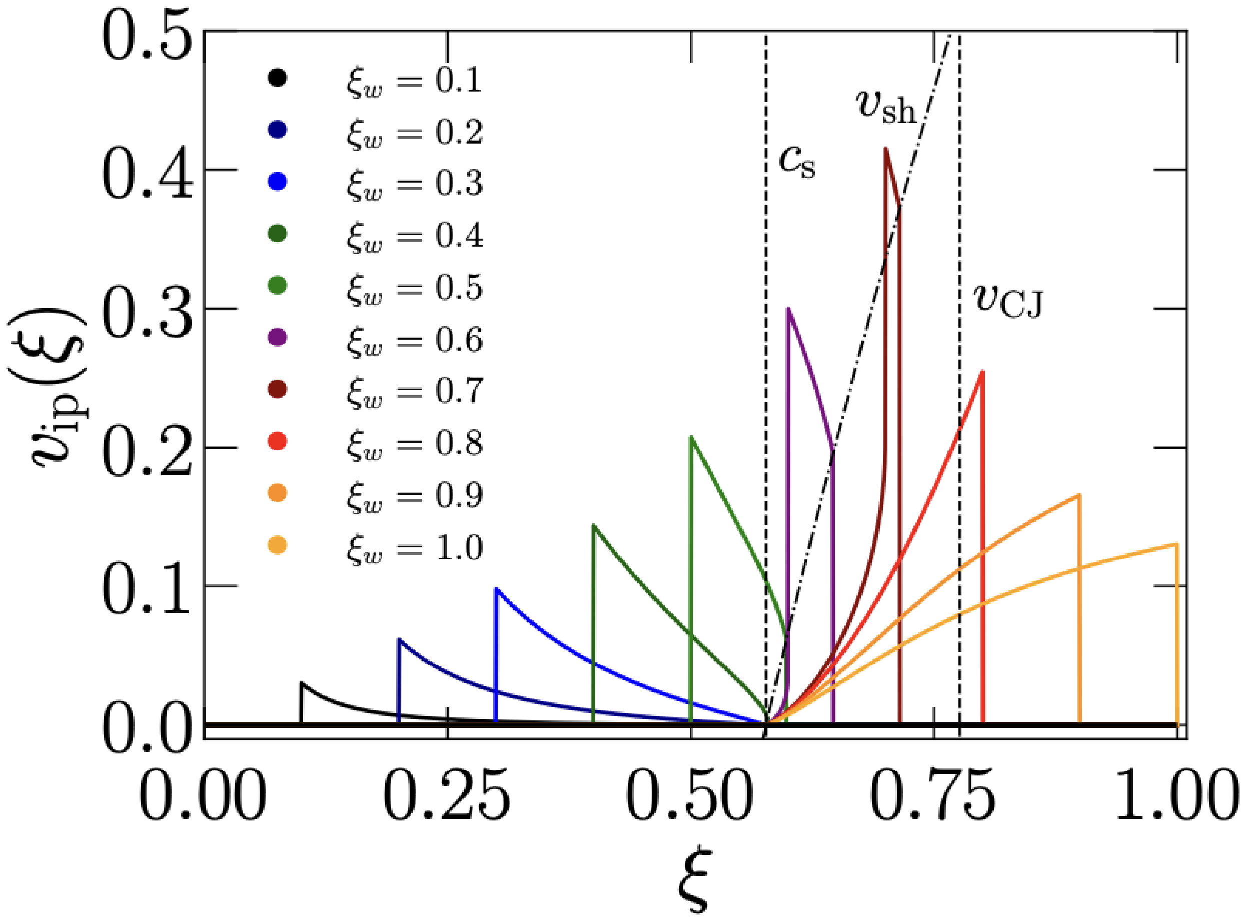
Properties of the profiles:

- Compact support
  $v_{ip}(\xi) \neq 0 \quad for \;\; \xi_b < \xi < \xi_f$

- Discontinuity at ξ_w

pip install cosmoGW

$$v_{CJ}(\alpha) = \frac{1+\sqrt{\alpha(2+3\alpha)}}{\sqrt{3}(1+\alpha)}$$



Fluid perturbations from expanding scalar bubbles

Properties of the profiles:

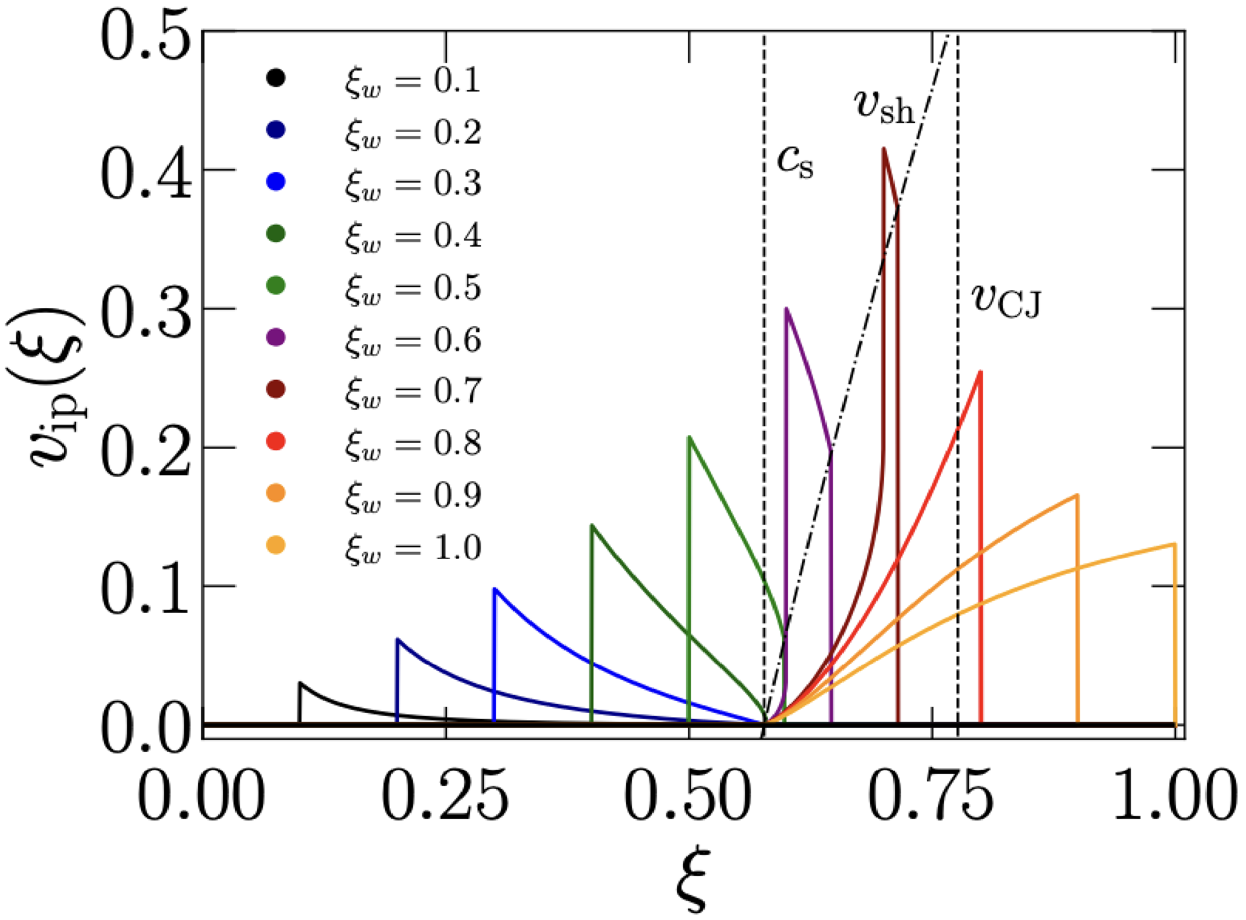
- Compact support
  $v_{ip}(\xi) \neq 0 \quad for \ \ \xi_b < \xi < \xi_f$

- Discontinuity at ξ_w

- Deflagrations with ξ_w close to c_s and hybrids have an additional discontinuity at $\xi = v_{sh}$

pip install cosmoGW

$$v_{CJ}(\alpha) = \frac{1+\sqrt{\alpha(2+3\alpha)}}{\sqrt{3}(1+\alpha)}$$



Fluid perturbations from expanding scalar bubbles

Self-similar profiles in Fourier space

$$\mathbf{v} = \sum_{n=1}^{N_b} \mathbf{v}^{(n)} = \sum_{n=1}^{N_b} v_{ip}(\xi)\, \hat{r}^{(n)}$$

$$\mathbf{v}^{(n)}(t, \mathbf{k}) = -i\left[t^{(n)}\right]^3 e^{i\mathbf{k}\cdot\mathbf{x}_0^{(n)}}\, \hat{\mathbf{k}}\, f'(z)$$

$$f\left(z \equiv k\, t^{(n)}\right) = 4\pi \int_0^\infty j_0(z\xi)\, \xi\, v_{ip}(\xi)\, d\xi$$

$$j_0(x) = \sin x / x$$

Kinetic spectrum in Fourier space $\propto \langle v_i(t, \boldsymbol{k}) v_i(t, \boldsymbol{k}') \rangle \propto |f'(z)|^2$

Fluid perturbations from expanding scalar bubbles

Self-similar profiles in Fourier space

$$\mathbf{v} = \sum_{n=1}^{N_b} \mathbf{v}^{(n)} = \sum_{n=1}^{N_b} v_{ip}(\xi)\, \hat{r}^{(n)}$$

$$\mathbf{v}^{(n)}(t, \mathbf{k}) = -i \left[t^{(n)}\right]^3 e^{i\mathbf{k}\cdot\mathbf{x}_0^{(n)}} \hat{\mathbf{k}}\, f'(z)$$

$$f\left(z \equiv k\, t^{(n)}\right) = 4\pi \int_0^\infty j_0(z\xi)\, \xi\, v_{ip}(\xi)\, d\xi$$

$$j_0(x) = \sin x / x$$

Kinetic spectrum in Fourier space $\propto \langle v_i(t, \boldsymbol{k}) v_i(t, \boldsymbol{k}') \rangle \propto |f'(z)|^2$

Self-similar profiles $\rightarrow |f'(z)|^2 \rightarrow$ Kinetic spectrum in the bubble expansion phase

Average over stochastic realizations

Fluid perturbations from expanding scalar bubbles

$$\mathbf{v}^{(\mathrm{n})}(\mathrm{t},\mathbf{k}) = -\mathrm{i}\left[\mathrm{t}^{(\mathrm{n})}\right]^{3} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}_0^{(\mathrm{n})}}\,\hat{\mathbf{k}}\, f'(z)$$

$$f'(\mathrm{z} \equiv \mathrm{k}\,\mathrm{t}^{(\mathrm{n})}) = -4\pi \int_0^\infty \mathrm{j}_1(\mathrm{z}\xi)\,\xi^2\,\mathrm{v}_{\mathrm{ip}}(\xi)\,\mathrm{d}\xi$$

Properties of $|f'(z)|^2$

Fluid perturbations from expanding scalar bubbles

$$\mathbf{v}^{(n)}(t,\mathbf{k}) = -i\left[t^{(n)}\right]^3 e^{i\mathbf{k}\cdot\mathbf{x}_0^{(n)}}\hat{\mathbf{k}}\, f'(z)$$

$$f'(z \equiv k\, t^{(n)}) = -4\pi\int_0^\infty j_1(z\xi)\,\xi^2\, v_{ip}(\xi)\, d\xi$$

Properties of $|f'(z)|^2$

Large scales $k = z/t^{(n)} \to 0$

$$f'(z) \to z\left[-\frac{4\pi}{3}\int_{\xi_b}^{\xi_f}\xi^3\, v_{ip}(\xi)\, d\xi\right] \longrightarrow |f'(z)|^2 \to |f_0'|^2\, z^2$$

Related to causality

Compact support of $v_{ip}(\xi)$

Fluid perturbations from expanding scalar bubbles

$$\mathbf{v}^{(\mathrm{n})}(\mathrm{t},\mathbf{k}) = -\mathrm{i}\left[\mathrm{t}^{(\mathrm{n})}\right]^3 e^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}_0^{(\mathrm{n})}}\hat{\mathbf{k}}\, f'(z)$$

$$f'(\mathrm{z} \equiv \mathrm{k}\,\mathrm{t}^{(\mathrm{n})}) = -4\pi \int_0^\infty \mathrm{j}_1(\mathrm{z}\xi)\, \xi^2\, \mathrm{v}_{\mathrm{ip}}(\xi)\, \mathrm{d}\xi$$

Properties of $|f'(z)|^2$

Small scales $k = z/t^{(n)} \to \infty$

$$f'(z) \to z^{-2}\left\{-4\pi\left[\xi_{sh} \sin z\xi_{sh}\, \Delta v_{ip}(\xi_{sh}) + \xi_w \sin z\xi_w\, \Delta v_{ip}(\xi_w)\right]\right\}$$

From the discontinuities of $v_{ip}(\xi)$

$$\longrightarrow \quad |f'(z)|^2 \to |f'_\infty|^2\, z^{-4}$$

$$\longrightarrow \quad {|f'(z)|_{env}}^2 \to |f'_{env}|^2\, z^{-4}$$

$$f'_{env} = 4\pi\left[\xi_{sh} v_{sh}^- + \xi_w |\Delta v_{ip}(\xi_w)|\right]$$

Fluid perturbations from expanding scalar bubbles

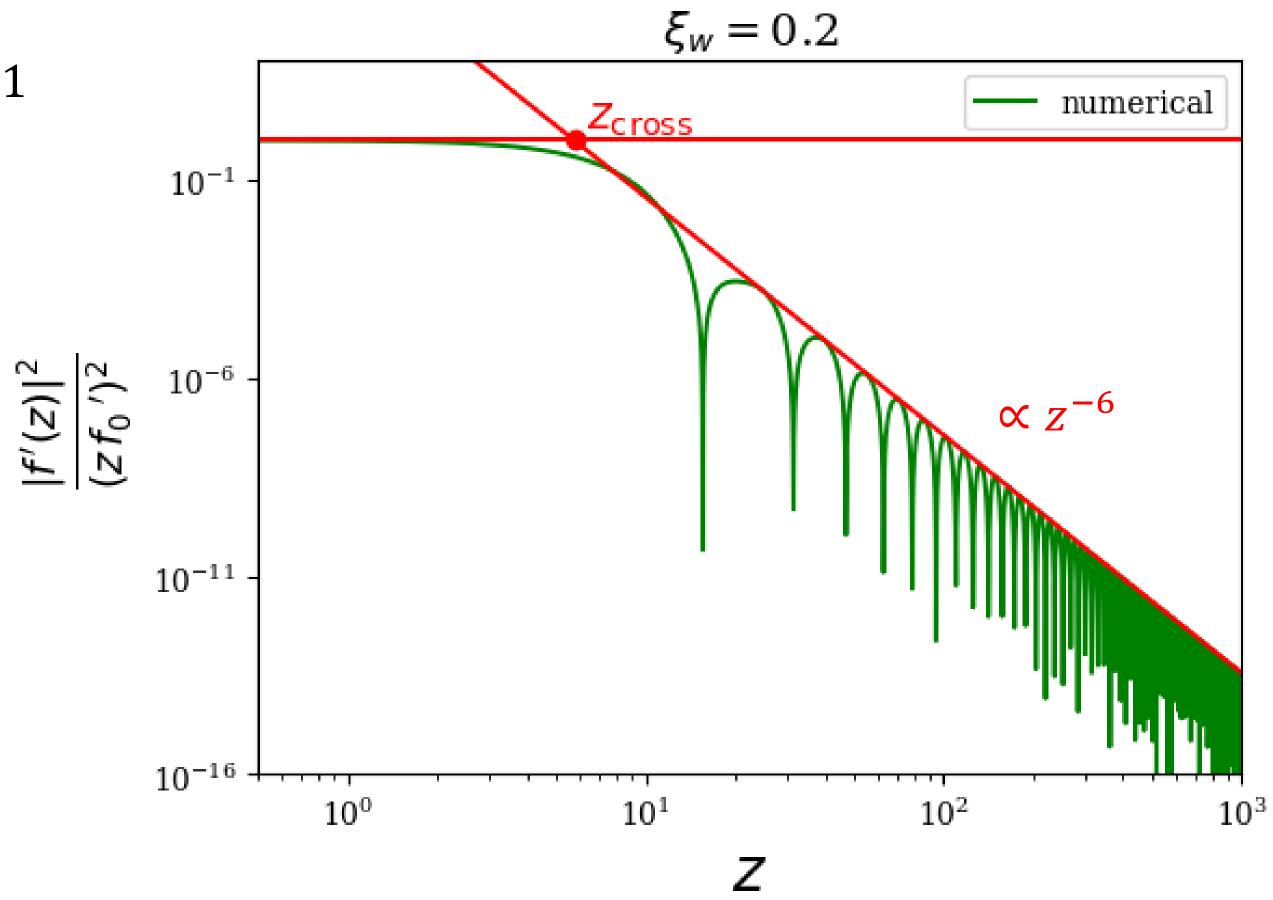
Properties of $|f'(z)|^2$

$$\mathbf{v}^{(n)}(t,\mathbf{k}) = -i\left[t^{(n)}\right]^3 e^{i\mathbf{k}\cdot\mathbf{x}_0^{(n)}}\hat{\mathbf{k}}\, f'(z)$$

$$f'(z) = -4\pi\int_0^\infty j_1(z\xi)\,\xi^2\, v_{ip}(\xi)\, d\xi$$

$$z_{cross} = \left|\frac{f'_{env}}{f'_0}\right|^{1/3}$$

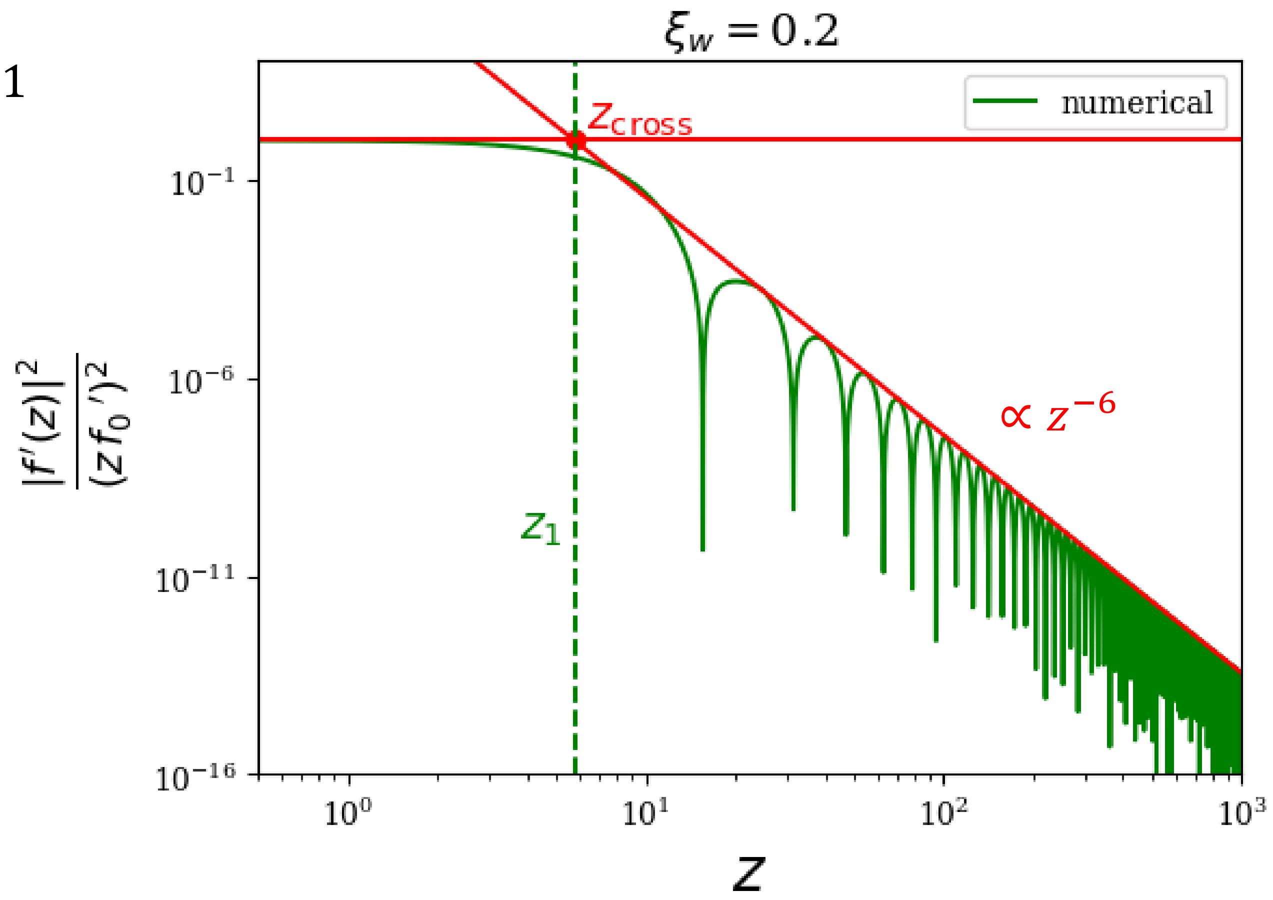
$\alpha = 0.1$



Fluid perturbations from expanding scalar bubbles

Properties of $|f'(z)|^2$

$\alpha = 0.1$



$$\mathbf{v}^{(n)}(t,\mathbf{k}) = -i\left[t^{(n)}\right]^3 e^{i\mathbf{k}\cdot\mathbf{x}_0^{(n)}}\,\hat{\mathbf{k}}\,f'(z)$$

$$f'(z) = -4\pi\int_0^\infty j_1(z\xi)\,\xi^2\,v_{ip}(\xi)\,d\xi$$

$$z_{cross} = \left|\frac{f'_{env}}{f'_0}\right|^{1/3}$$

Significant deviations from $\sim z^2$ begin around

$$z_1 \approx \frac{3\pi}{2}\left(\xi_f + \xi_b\right)^{-1}$$

(for generic α & ξ_w)

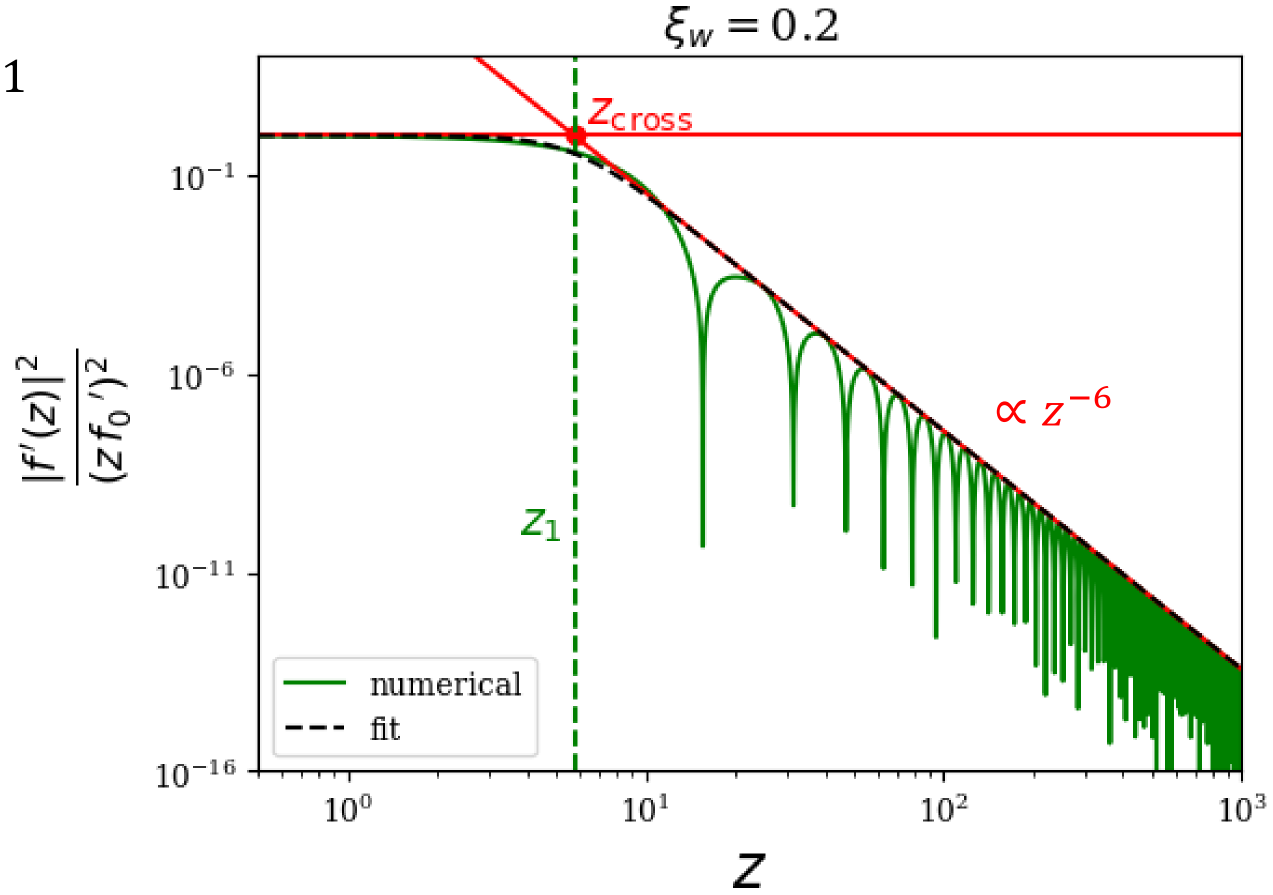
Fluid perturbations from expanding scalar bubbles

Properties of $|f'(z)|^2$

$$\mathbf{v}^{(n)}(t,\mathbf{k}) = -i\left[t^{(n)}\right]^3 e^{i\mathbf{k}\cdot\mathbf{x}_0^{(n)}}\,\hat{\mathbf{k}}\, f'(z)$$

$$f'(z) = -4\pi \int_0^\infty j_1(z\xi)\,\xi^2\, v_{ip}(\xi)\, d\xi$$

$\alpha = 0.1$



$$z_1 \approx \frac{3\pi}{2}\left(\xi_f + \xi_b\right)^{-1}$$

$$|f'(z)|^2_{env} \approx |f_0'|^2\, z^2 \left[1 + \left(\frac{z}{z_1}\right)^{a_1}\right]^{-\frac{6}{a_1}}$$

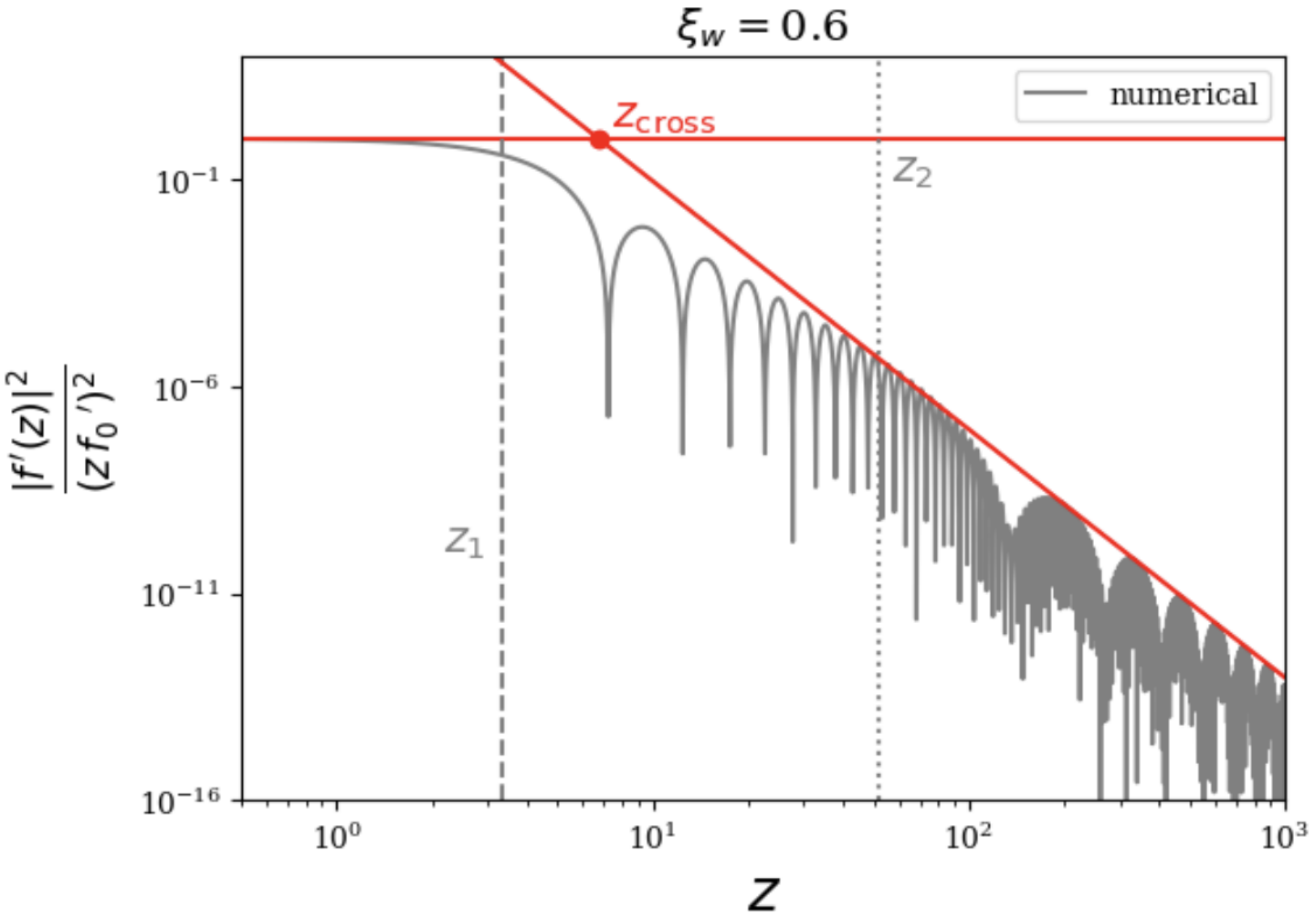
$$(\xi_w \lesssim v_{CJ}(\alpha)/2)$$

$a_1 = 4$

Fluid perturbations from expanding scalar bubbles

Properties of $|f'(z)|^2$

$\alpha = 0.1$



$$z_1 \approx \frac{3\pi}{2}\left(\xi_f + \xi_b\right)^{-1}$$

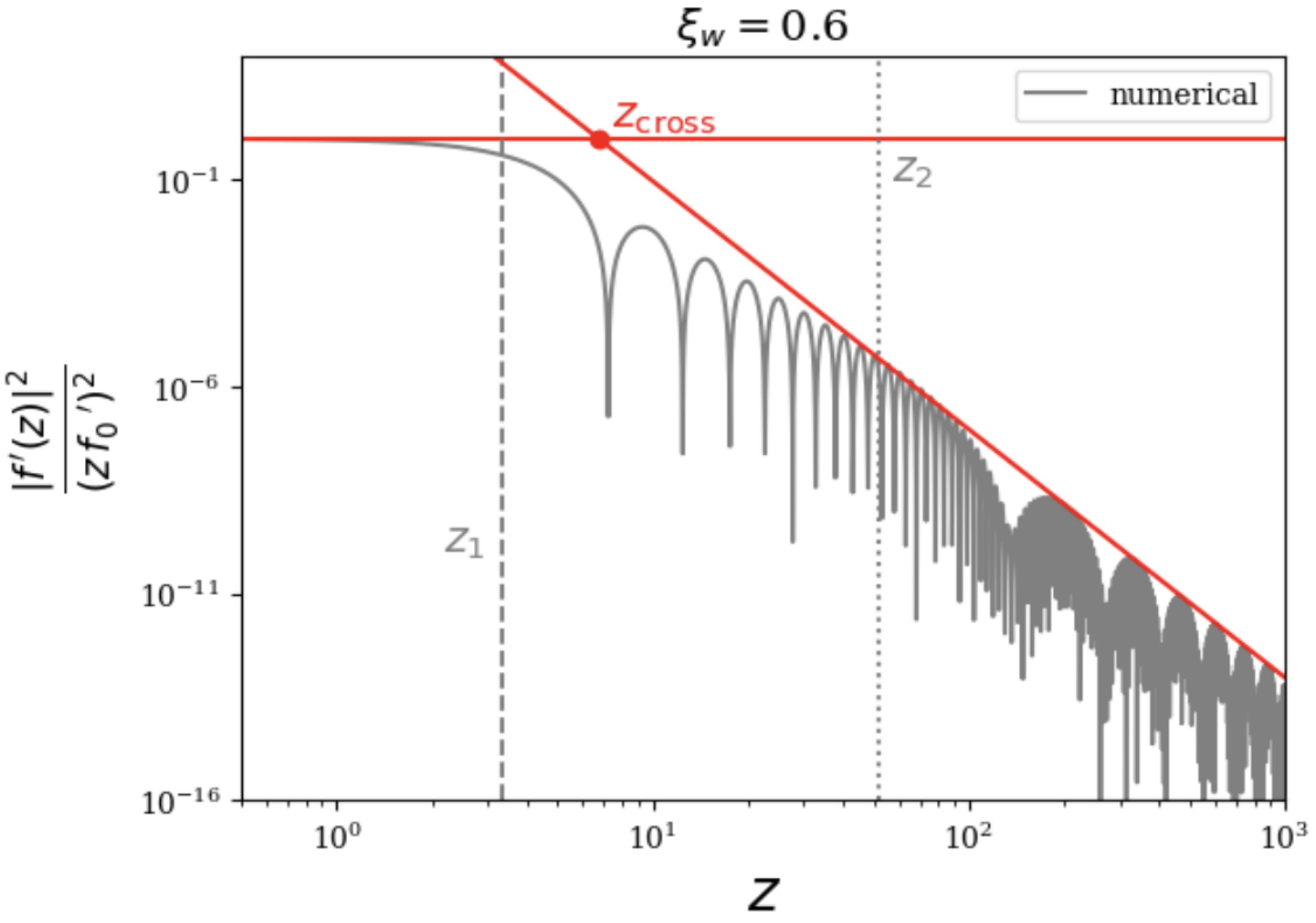
The $\sim z^{-4}$ begins around

$$z_2 \approx \pi \times \begin{cases} \left(\xi_f - \xi_b\right)^{-1} & (\xi_w < c_s) \\ \left(\xi_f - \xi_w\right)^{-1} & (c_s < \xi_w < v_{CJ}) \\ \left(\xi_f - \xi_b\right)^{-1} & (\xi_w > v_{CJ}) \end{cases}$$

Fluid perturbations from expanding scalar bubbles

Properties of $|f'(z)|^2$

$\alpha = 0.1$



$$z_1 \approx \frac{3\pi}{2}\left(\xi_f + \xi_b\right)^{-1}$$

The $\sim z^{-4}$ begins around

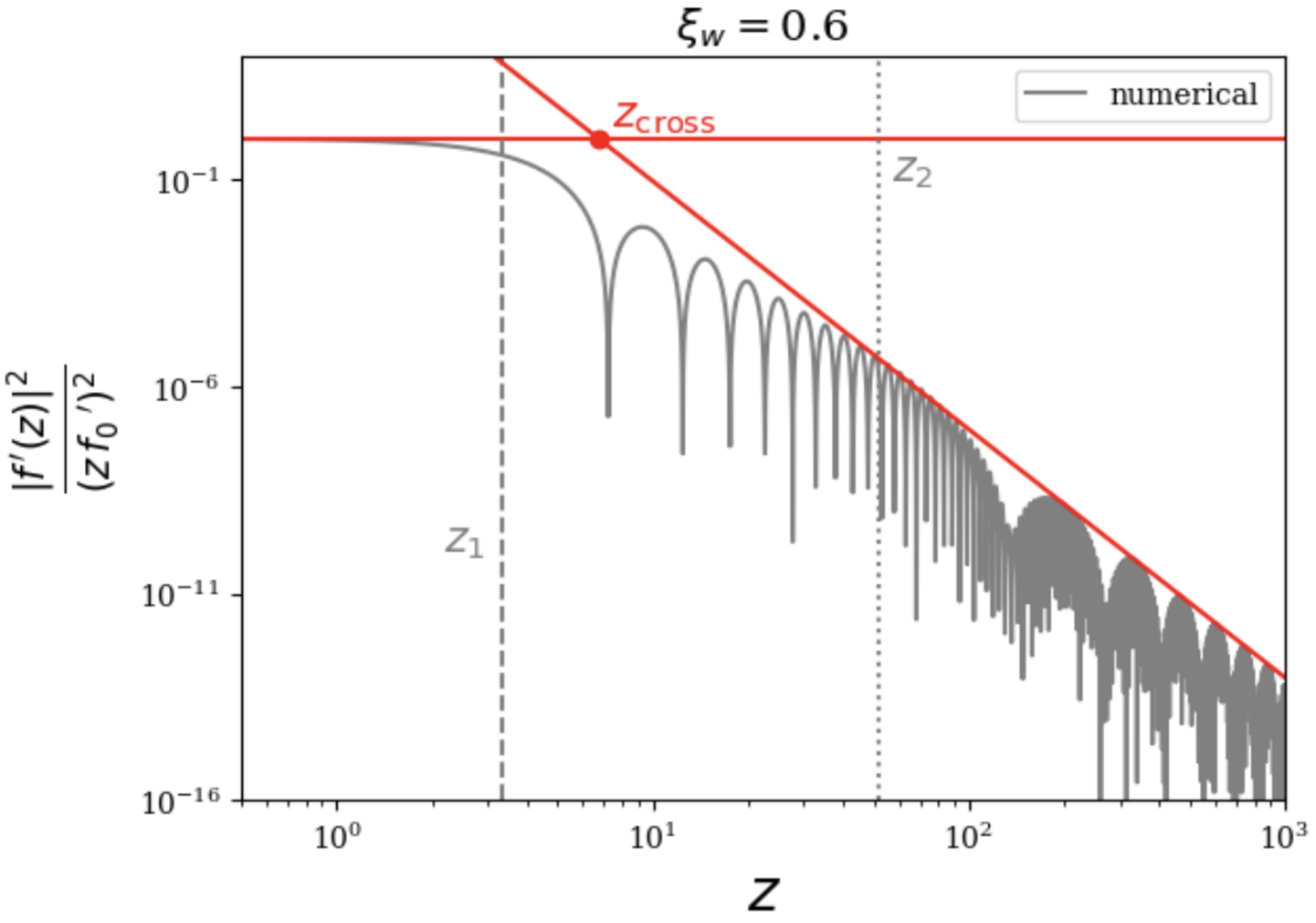
$$z_2 \approx \pi \times \begin{cases} \left(\xi_f - \xi_b\right)^{-1} & (\xi_w < c_s) \\ \left(\xi_f - \xi_w\right)^{-1} & (c_s < \xi_w < v_{CJ}) \\ \left(\xi_f - \xi_b\right)^{-1} & (\xi_w > v_{CJ}) \end{cases}$$

$\xi_f - \xi_b \propto \Delta R_*$ (sound shell thickness)

Fluid perturbations from expanding scalar bubbles

Properties of $|f'(z)|^2$

$\alpha = 0.1$



$$z_1 \approx \frac{3\pi}{2}\left(\xi_f + \xi_b\right)^{-1}$$

The $\sim z^{-4}$ begins around

$$z_2 \approx \pi \times \begin{cases} \left(\xi_f - \xi_b\right)^{-1} & (\xi_w < c_s) \\ \left(\xi_f - \xi_w\right)^{-1} & (c_s < \xi_w < v_{CJ}) \\ \left(\xi_f - \xi_b\right)^{-1} & (\xi_w > v_{CJ}) \end{cases}$$

$\xi_f - \xi_b \propto \Delta R_*$ (sound shell thickness)

$\xi_f - \xi_w = \xi_{sh} - \xi_w$ distance between discontinuities (for hybrids)

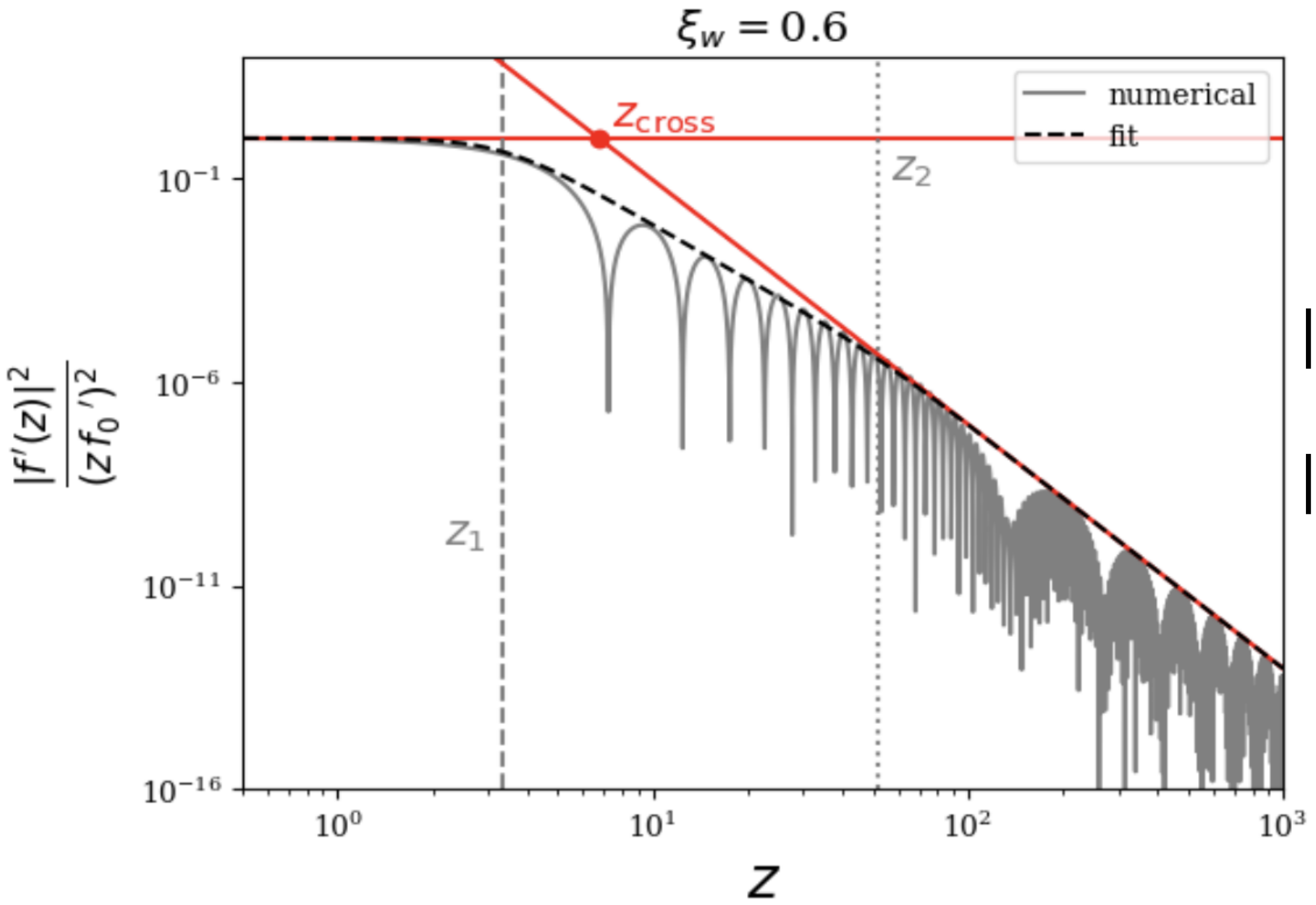
Fluid perturbations from expanding scalar bubbles

Properties of $|f'(z)|^2$

$\alpha = 0.1$

$$z_1 \approx \frac{3\pi}{2}\left(\xi_f + \xi_b\right)^{-1}$$

$$z_2 \approx \pi \times \begin{cases} \left(\xi_f - \xi_b\right)^{-1} & (\xi_w < c_s) \\ \left(\xi_f - \xi_w\right)^{-1} & (c_s < \xi_w < v_{CJ}) \\ \left(\xi_f - \xi_b\right)^{-1} & (\xi_w > v_{CJ}) \end{cases}$$



$$|f'(z)|^2_{env} \approx |f_0'|^2\, z^2 \left[1 + \left(\frac{z}{z_1}\right)^{a_1}\right]^{\frac{\gamma-2}{a_1}} \left[1 + \left(\frac{z}{z_2}\right)^{a_2}\right]^{\frac{-\gamma-4}{a_2}}$$

$(\xi_w \gtrsim v_{CJ}(\alpha)/2)$

$$\gamma = 2\left[1 - 3\frac{\log(z_2/z_{cross})}{\log(z_2/z_1)}\right]$$

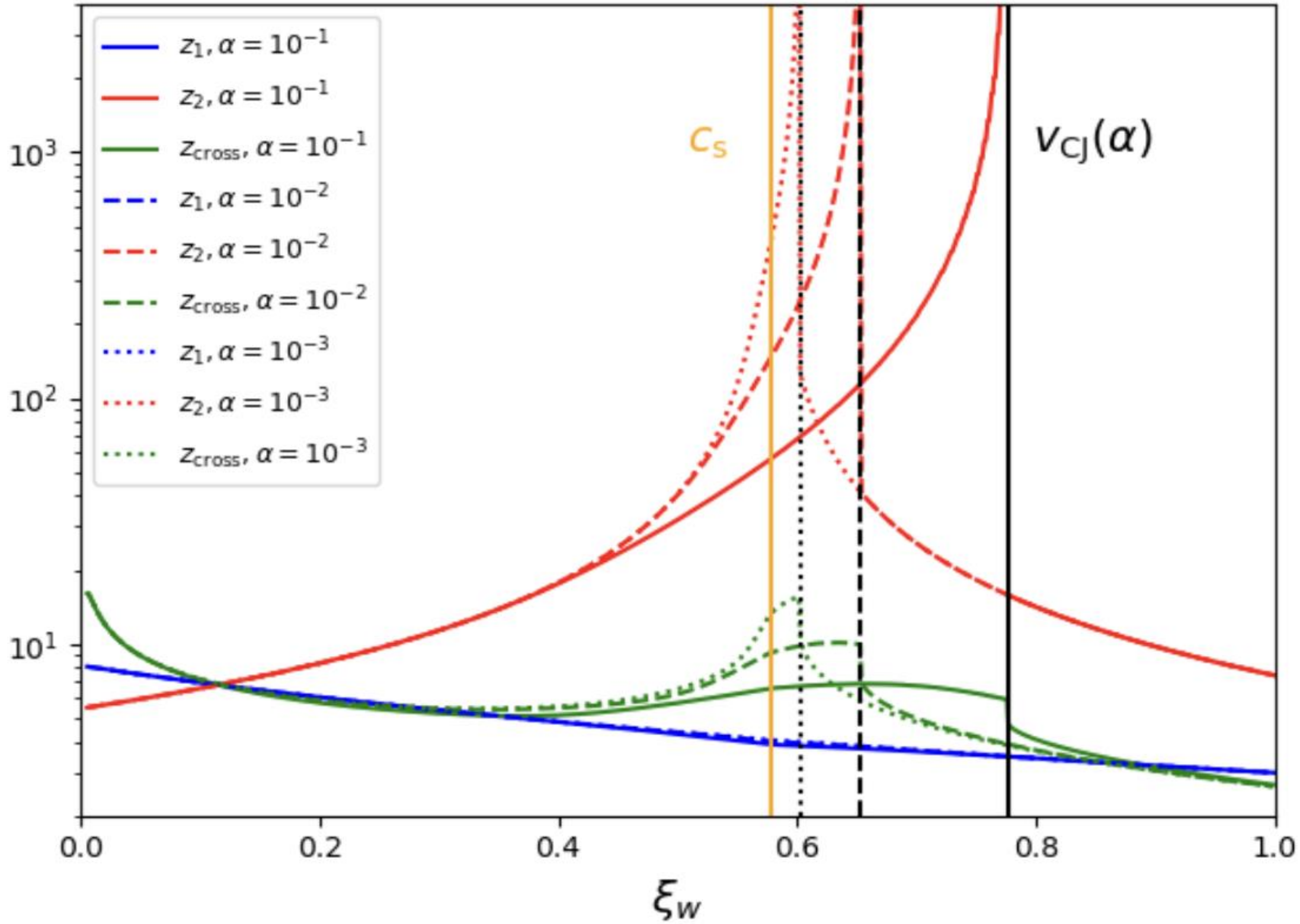
$a_1 = a_2 = 4$

Fluid perturbations from expanding scalar bubbles

Scales of $|f'(z)|^2$

$$z_1 \approx \frac{3\pi}{2}\left(\xi_f + \xi_b\right)^{-1}$$

$$z_2 \approx \pi \times \begin{cases} \left(\xi_f - \xi_b\right)^{-1} & (\xi_w < c_s) \\ \left(\xi_f - \xi_w\right)^{-1} & (c_s < \xi_w < v_{CJ}) \\ \left(\xi_f - \xi_b\right)^{-1} & (\xi_w > v_{CJ}) \end{cases}$$



$$|f'(z)|^2_{env} \approx |f_0'|^2 z^2 \left[1 + \left(\frac{z}{z_1}\right)^{a_1}\right]^{\frac{\gamma-2}{a_1}} \left[1 + \left(\frac{z}{z_2}\right)^{a_2}\right]^{\frac{-\gamma-4}{a_2}}$$

$(\xi_w \gtrsim v_{CJ}(\alpha)/2)$

$$\gamma = 2\left[1 - 3\frac{\log(z_2/z_{cross})}{\log(z_2/z_1)}\right]$$

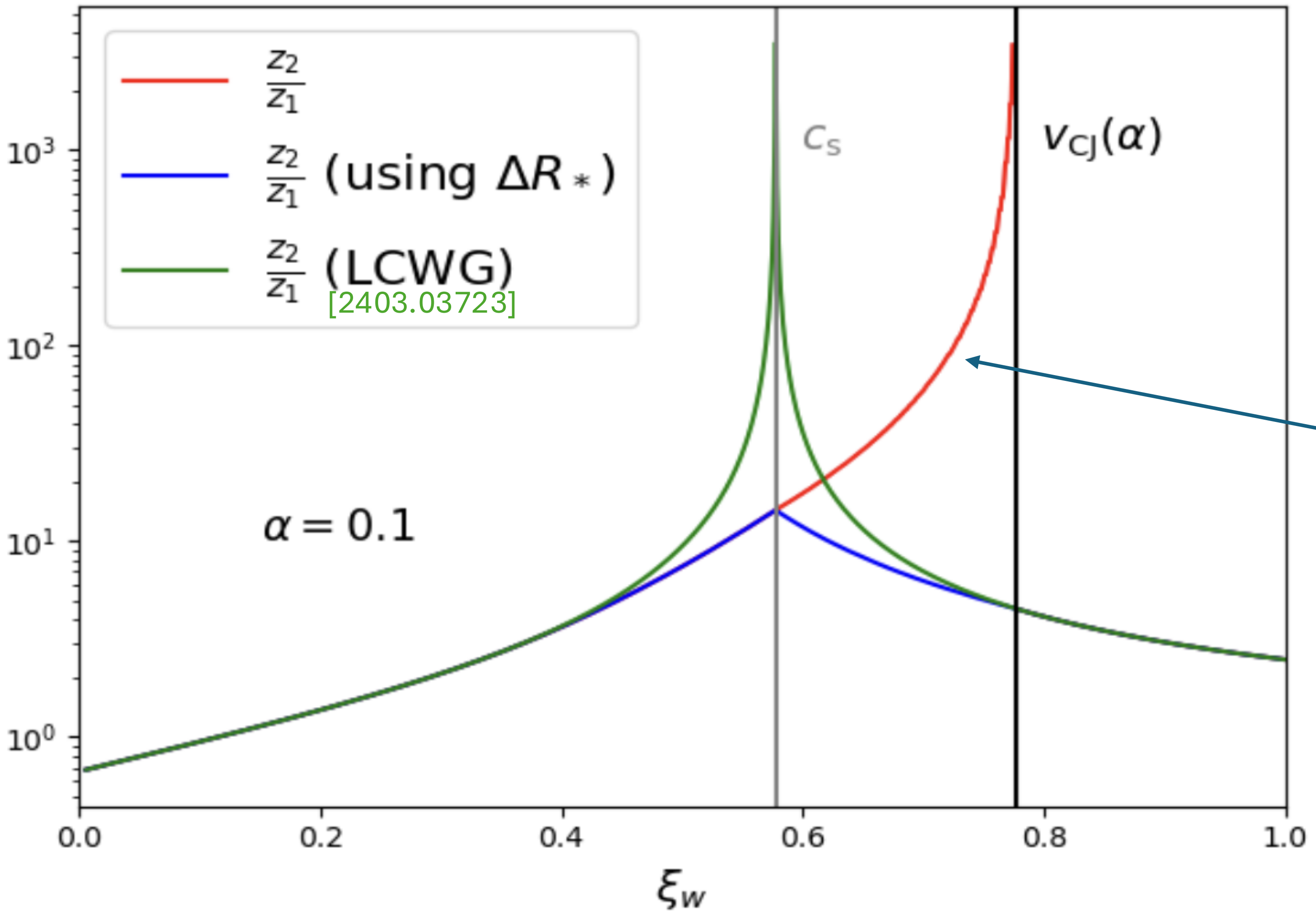
$a_1 = a_2 = 4$

Fluid perturbations from expanding scalar bubbles

Scales of $|f'(z)|^2$

$$z_1 \approx \frac{3\pi}{2}\left(\xi_f + \xi_b\right)^{-1}$$

$$z_2 \approx \pi \times \begin{cases} \left(\xi_f - \xi_b\right)^{-1} & (\xi_w < c_s) \\ \left(\xi_f - \xi_w\right)^{-1} & (c_s < \xi_w < v_{CJ}) \\ \left(\xi_f - \xi_b\right)^{-1} & (\xi_w > v_{CJ}) \end{cases}$$



Much broader spectrum for hybrids than using

$$z_2 = \pi \times \left(\xi_f - \xi_b\right)^{-1} \propto \Delta R_*^{-1}$$

$$z_2 = \pi \times |c_s - \xi_w|^{-1}$$ (Lisa Cosmology Working Group)

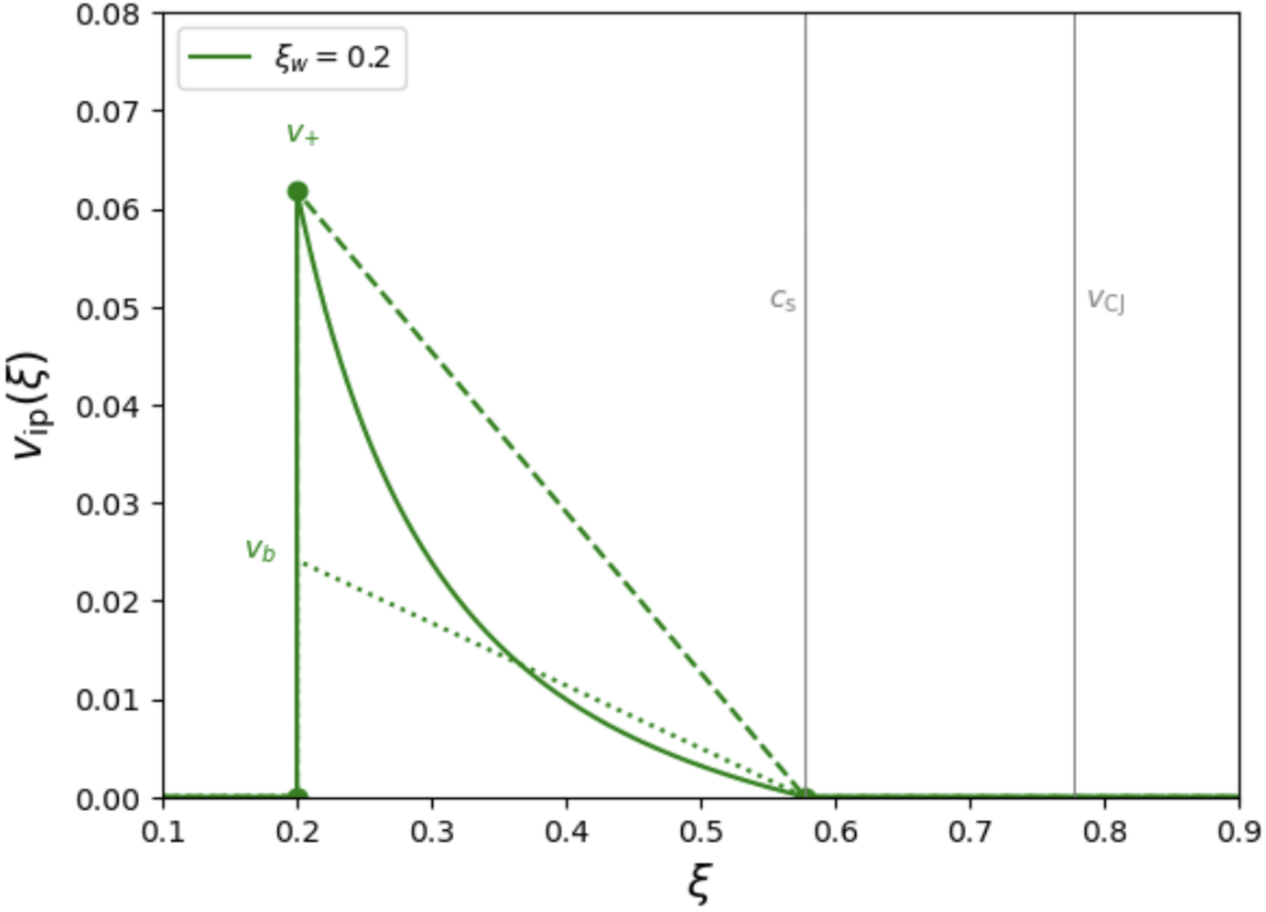
Fluid perturbations from expanding scalar bubbles

Scales of $|f'(z)|^2$

$$z_1 \approx \frac{3\pi}{2}\left(\xi_f + \xi_b\right)^{-1}$$

$$z_2 \approx \pi \times \begin{cases} \left(\xi_f - \xi_b\right)^{-1} & (\xi_w < c_s) \\ \left(\xi_f - \xi_w\right)^{-1} & (c_s < \xi_w < v_{CJ}) \\ \left(\xi_f - \xi_b\right)^{-1} & (\xi_w > v_{CJ}) \end{cases}$$

→ Toy models



Approximate the deflagrations with a linearly decreasing velocity profile

$$v(\xi) = v_{const}\,\frac{\xi_f - \xi}{\xi_f - \xi_b} \qquad (\xi_b < \xi < \xi_f)$$

v_{const} can be chosen in order to reproduce either the small scales (v_+) or the large scales (v_b) limit of $|f'(z)|^2$

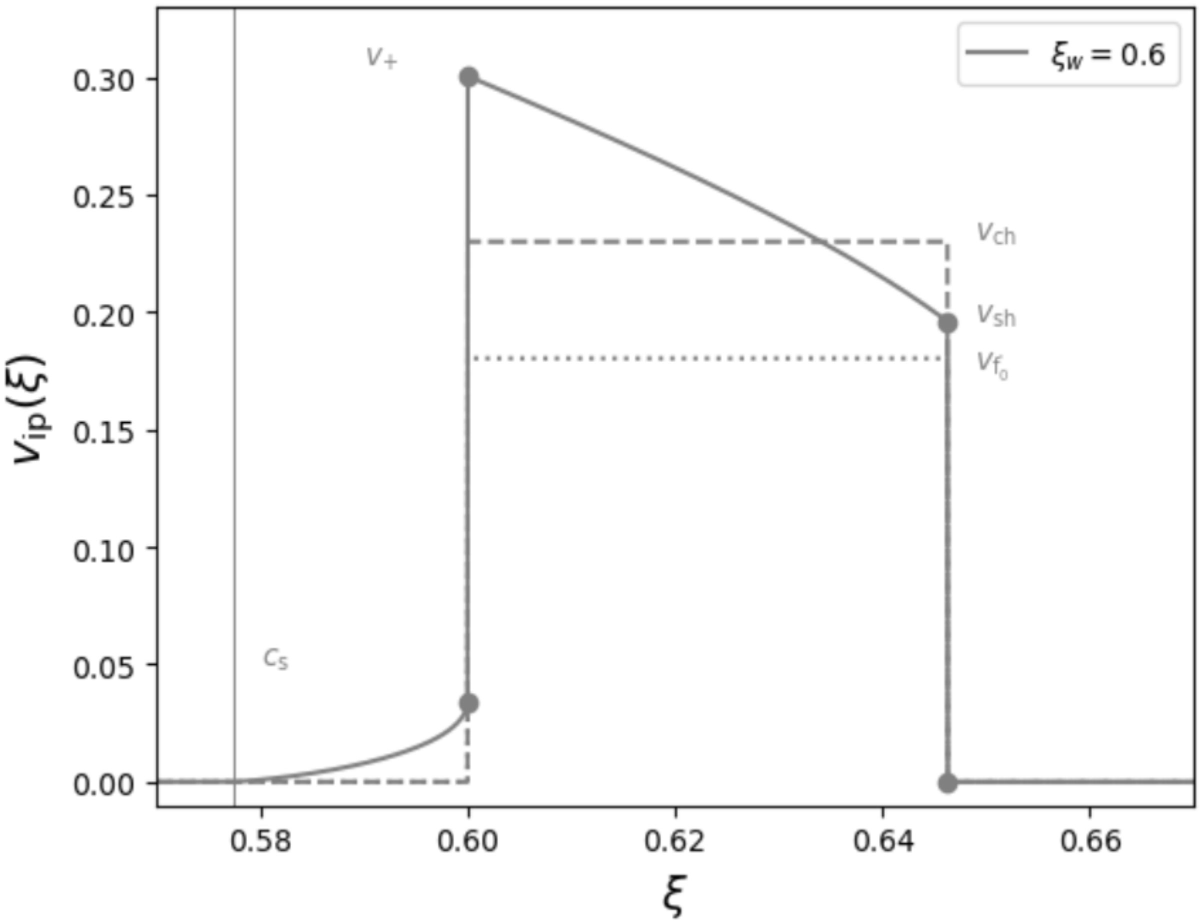
Fluid perturbations from expanding scalar bubbles

Scales of $|f'(z)|^2$

$$z_1 \approx \frac{3\pi}{2}\left(\xi_f + \xi_b\right)^{-1}$$

$$z_2 \approx \pi \times \begin{cases} \left(\xi_f - \xi_b\right)^{-1} & (\xi_w < c_s) \\ \left(\xi_f - \xi_w\right)^{-1} & (c_s < \xi_w < v_{CJ}) \\ \left(\xi_f - \xi_b\right)^{-1} & (\xi_w > v_{CJ}) \end{cases}$$

→ Toy models



Approximate the hybrids with a constant velocity profile only between discontinuities

$$v(\xi) = v_{const} \qquad (\xi_w < \xi < \xi_f)$$

v_{const} can be chosen in order to reproduce either the small scales (v_{ch}) or the large scales $\left(v_{f_0'}\right)$ limit of $|f'(z)|^2$

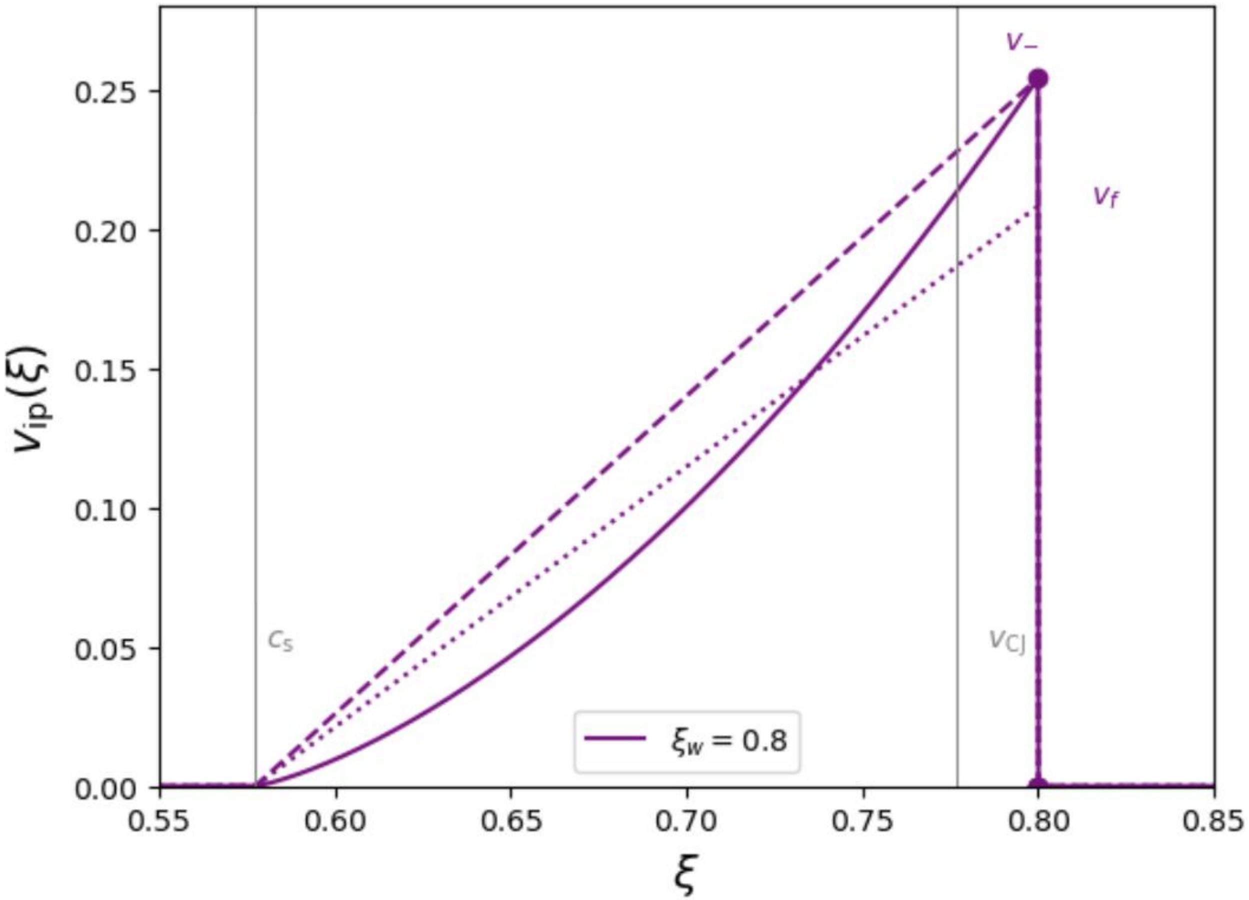
Fluid perturbations from expanding scalar bubbles

Scales of $|f'(z)|^2$

$$z_1 \approx \frac{3\pi}{2}\left(\xi_f + \xi_b\right)^{-1}$$

$$z_2 \approx \pi \times \begin{cases} \left(\xi_f - \xi_b\right)^{-1} & (\xi_w < c_s) \\ \left(\xi_f - \xi_w\right)^{-1} & (c_s < \xi_w < v_{CJ}) \\ \left(\xi_f - \xi_b\right)^{-1} & (\xi_w > v_{CJ}) \end{cases}$$

$\longrightarrow$ Toy models

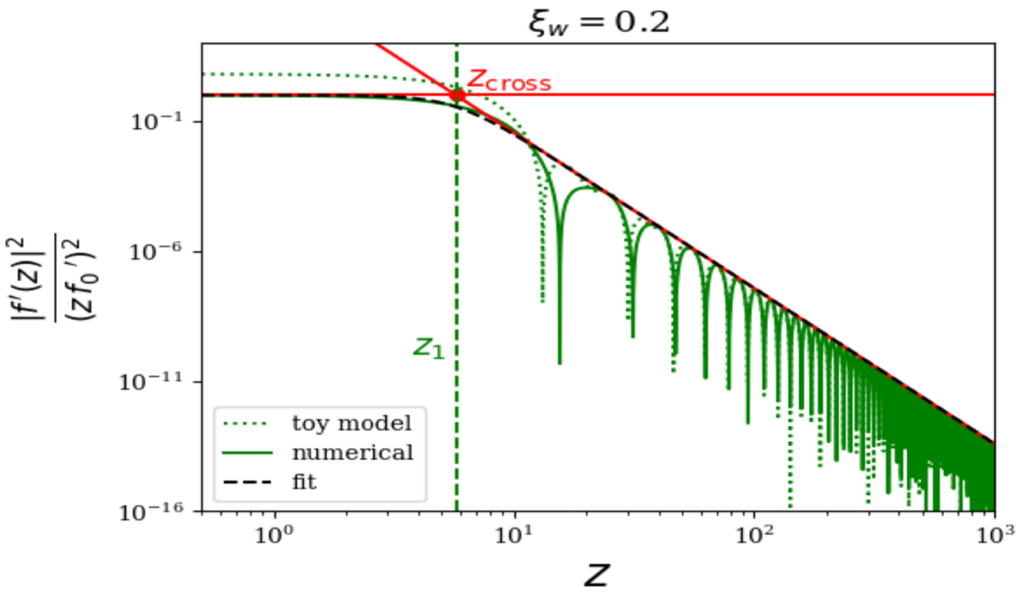


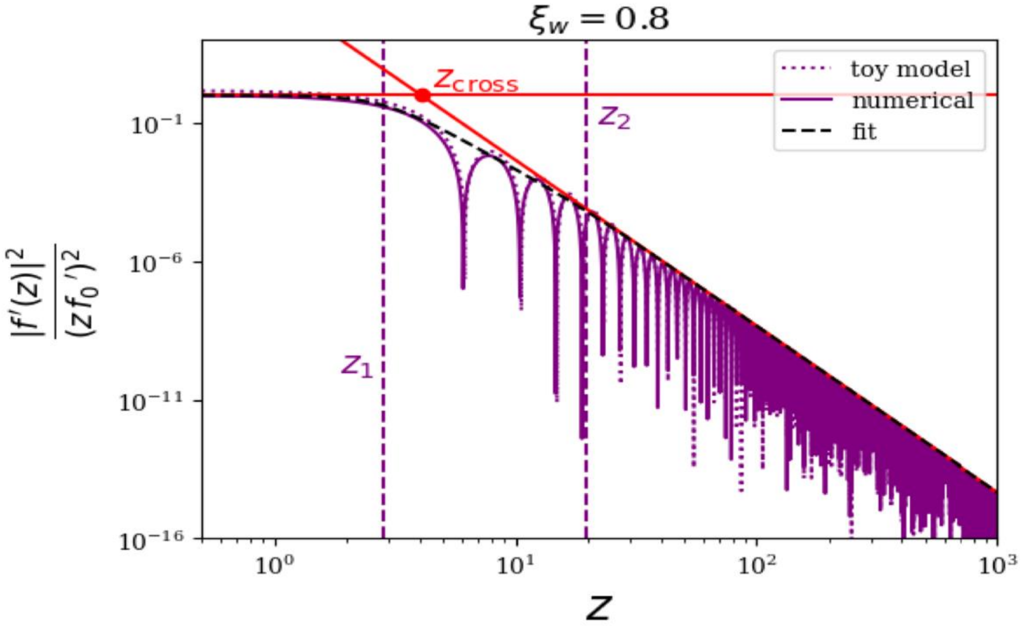
Approximate the detonations with a linearly increasing velocity profile

$$v(\xi) = v_{const}\,\frac{\xi - \xi_b}{\xi_f - \xi_b} \qquad (\xi_b < \xi < \xi_f)$$

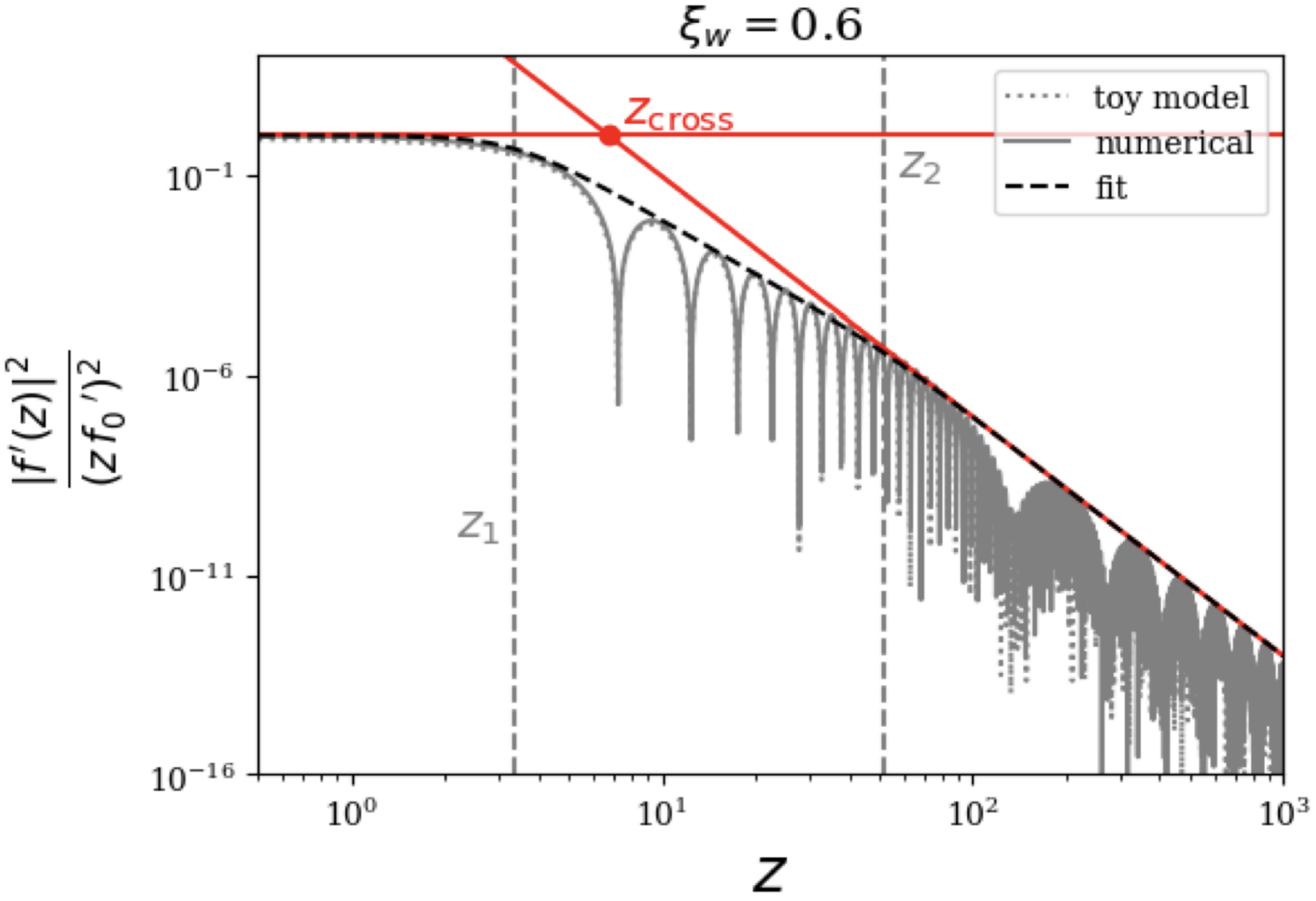
v_{const} can be chosen in order to reproduce either the small scales (v_-) or the large scales (v_f) limit of $|f'(z)|^2$

Fluid perturbations from expanding scalar bubbles





Numerical vs Fits vs Toy models



Evolution of the fluid perturbations: *before* collisions

Computing the kinetic spectrum in the bubble expansion phase requires averaging over stochastic realizations

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}') \right\rangle_{\boldsymbol{x}_0^{(n)},\, t_0^{(n)}}$$

$$\mathbf{v}^{(\mathrm{n})}(\mathrm{t},\mathbf{k}) = -\mathrm{i}\left[\mathrm{t}^{(\mathrm{n})}\right]^3 \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}_0^{(\mathrm{n})}}\, \hat{\mathbf{k}}\, f'(z)$$

$$f'(z) = -4\pi \int_0^\infty \mathrm{j}_1(\mathrm{z}\xi)\, \xi^2\, \mathrm{v}_{\mathrm{ip}}(\xi)\, \mathrm{d}\xi$$

Evolution of the fluid perturbations: *before* collisions

Computing the kinetic spectrum in the bubble expansion phase requires averaging over stochastic realizations

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}') \right\rangle_{\boldsymbol{x}_0^{(n)},\, t_0^{(n)}}$$

$$\mathbf{v}^{(\mathrm{n})}(\mathrm{t},\mathbf{k}) = -\mathrm{i}\left[\mathrm{t}^{(\mathrm{n})}\right]^3 \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}_0^{(\mathrm{n})}}\,\hat{\mathbf{k}}\, f'(z)$$

$$f'(z) = -4\pi \int_0^\infty \mathrm{j}_1(\mathrm{z}\xi)\,\xi^2\,\mathrm{v}_{\mathrm{ip}}(\xi)\,\mathrm{d}\xi$$

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}') \right\rangle_{\boldsymbol{x}_0^{(n)}} = \widehat{\boldsymbol{k}}_i\, \widehat{\boldsymbol{k}}_j\, \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\, n_b(t)(t-t_0)^6 |f'(z)|^2$$

Average over nucleation locations (homogeneously distributed)

Evolution of the fluid perturbations: *before* collisions

Computing the kinetic spectrum in the bubble expansion phase requires averaging over stochastic realizations

$$\mathbf{v}^{(\mathrm{n})}(\mathrm{t},\mathbf{k}) = -\mathrm{i}\left[\mathrm{t}^{(\mathrm{n})}\right]^3 \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}_0^{(\mathrm{n})}}\,\hat{\mathbf{k}}\, f'(z)$$

$$f'(z) = -4\pi \int_0^\infty \mathrm{j}_1(\mathrm{z}\xi)\,\xi^2\,\mathrm{v}_{\mathrm{ip}}(\xi)\,\mathrm{d}\xi$$

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)},\,t_0^{(n)}}$$

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)},\,t_0^{(n)}} = \widehat{\boldsymbol{k}}_i\,\widehat{\boldsymbol{k}}_j\,\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\,n_b(t)\;\times$$

$$\times \int_{t_c}^{t} dt_0\,\Gamma(t_0)\,(t-t_0)^6\,|f'(z)|^2$$

Average over nucleation times

nucleation rate

$z = k(t - t_0)$

Evolution of the fluid perturbations: *before* collisions

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}') \right\rangle_{\boldsymbol{x}_0^{(n)},t_0^{(n)}} = \widehat{\boldsymbol{k}}_i\,\widehat{\boldsymbol{k}}_j\,\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\,n_b(t)\int_{t_c}^{t} dt_0\,\Gamma(t_0)\,(t-t_0)^6\,|f'(z)|^2$$

$$\Gamma(t_0) = p(t_0)\,h(t_0)$$

volume fraction in the symmetric phase

$$h(t) = \exp\left[-\frac{4\pi}{3}\int_{t_c}^{t} dt_0\,p(t_0)\,\xi_w^3\,(t-t_0)^3\right]$$

nucleation probability per unit volume

$$p(t) \sim e^{-S(t)}$$

Evolution of the fluid perturbations: *before* collisions

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)},t_0^{(n)}} = \widehat{\boldsymbol{k}}_i\,\widehat{\boldsymbol{k}}_j\,\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\,n_b(t)\int_{t_c}^{t} dt_0\,\Gamma(t_0)\,(t-t_0)^6\,|f'(z)|^2$$

$$\Gamma(t_0) = p(t_0)\,h(t_0)$$

volume fraction in the symmetric phase

$$h(t) = \exp\left[-\frac{4\pi}{3}\int_{t_c}^{t} dt_0\,p(t_0)\,\xi_w^3\,(t-t_0)^3\right]$$

nucleation probability per unit volume

$$p(t) \sim e^{-S(t)}$$

Exponential nucleation $\quad S(t) \simeq S(t_*) - \beta(t-t_*) \rightarrow p(t) \simeq p_* e^{\beta(t-t_*)}$

Gaussian nucleation $\quad S(t) \simeq S(t_*) + \frac{1}{2}\beta^2(t-t_*)^2 \rightarrow p(t) \simeq p_* e^{-\frac{\beta^2}{2}(t-t_*)^2}$

Evolution of the fluid perturbations: *across* collisions

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)},t_0^{(n)}} = \widehat{\boldsymbol{k}}_i\,\widehat{\boldsymbol{k}}_j\,\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\underbrace{\frac{n_b(\tilde{t})}{\beta^6}\int_{\tilde{t}_c}^{\tilde{t}} d\tilde{t}_0\,\Gamma(\tilde{t}_0)\,(\tilde{t}-\tilde{t}_0)^6\,|f'(z)|^2}_{F_L(\tilde{t}<\tilde{t}_{sw},\tilde{k})}$$

$$\tilde{t} = \beta t, \qquad \tilde{k} = k/\beta$$

- Need to average over nucleation and collision times

- We can introduce a normalized lifetime distribution $\nu(\tilde{T})$

Evolution of the fluid perturbations: *across* collisions

$$\left\langle v_i(t,\boldsymbol{k})v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)},t_0^{(n)}} = \hat{\boldsymbol{k}}_i\,\hat{\boldsymbol{k}}_j\,\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\underbrace{\frac{n_b(\tilde{t})}{\beta^6}\int_{\tilde{t}_c}^{\tilde{t}} d\tilde{t}_0\,\Gamma(\tilde{t}_0)\,(\tilde{t}-\tilde{t}_0)^6\,|f'(z)|^2}_{F_L(\tilde{t}<\tilde{t}_{sw},\tilde{k})}$$

$$\tilde{t} = \beta t, \qquad \tilde{k} = k/\beta$$

- Need to average over nucleation and collision times

- We can introduce a normalized lifetime distribution $\nu(\tilde{T})$

$$F_L(\tilde{t}_{sw}^{+},\tilde{k}) = \frac{n_b(\tilde{t})}{\beta^6}\int_0^\infty d\tilde{T}\,\nu(\tilde{T})\tilde{T}^6\left|f'(\tilde{k}\tilde{T})\right|^2$$

Hindmarsh & Hijazi [1909.10040] $\longrightarrow$ $\nu(\tilde{T}) = -\frac{1}{\beta\,n_b}\int_{\tilde{t}_c}^{\infty} d\tilde{t}\,p(\tilde{t})\frac{dh}{d\tilde{t}}(\tilde{t}+\tilde{T})$

Probability for a bubble to nucleate at $\tilde{t}$ and disappear at $\tilde{t}+\tilde{T}$

Evolution of the fluid perturbations: *across* collisions

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)},\, t_0^{(n)}} = \widehat{\boldsymbol{k}}_i\, \widehat{\boldsymbol{k}}_j\, \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\, F_L(t,k)$$

Hindmarsh & Hijazi [1909.10040]

$$F_L\left(\tilde t < \tilde t_{sw}, \tilde k\right) = \frac{n_b(\tilde t)}{\beta^6} \int_{\tilde t_c}^{\tilde t} d\tilde t_0\, \Gamma(\tilde t_0)\, (\tilde t - \tilde t_0)^6\, \left|f'(\tilde k \tilde T)\right|^2$$

$$F_L\left(\tilde t_{sw}^{+}, \tilde k\right) = \frac{n_b(\tilde t_{sw})}{\beta^6} \int_0^{\infty} d\tilde T\, \nu(\tilde T)\, \tilde T^6 \left|f'(\tilde k \tilde T)\right|^2$$

$$\nu(\tilde T) = -\frac{1}{\beta\, n_b} \int_{\tilde t_c}^{\infty} d\tilde t\, p(\tilde t) \frac{dh}{d\tilde t}(\tilde t + \tilde T)$$

Evolution of the fluid perturbations: *across* collisions

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)},\, t_0^{(n)}} = \widehat{\boldsymbol{k}}_i\, \widehat{\boldsymbol{k}}_j\, \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\, F_L(t,k)$$

Hindmarsh & Hijazi [1909.10040]

$$F_L(\tilde{t} < \tilde{t}_{sw}, \tilde{k}) = \frac{n_b(\tilde{t})}{\beta^6} \int_{\tilde{t}_c}^{\tilde{t}} d\tilde{t}_0\, \Gamma(\tilde{t}_0)\, (\tilde{t}-\tilde{t}_0)^6\, \left|f'(\tilde{k}\tilde{T})\right|^2$$

$$\nu(\tilde{T}) = -\frac{1}{\beta\, n_b} \int_{\tilde{t}_c}^{\infty} d\tilde{t}\, p(\tilde{t}) \frac{dh}{d\tilde{t}}(\tilde{t}+\tilde{T})$$

$$F_L(\tilde{t}_{sw}^{+}, \tilde{k}) = \frac{n_b(\tilde{t}_{sw})}{\beta^6} \int_0^{\infty} d\tilde{T}\, \nu(\tilde{T}) \tilde{T}^6 \left|f'(\tilde{k}\tilde{T})\right|^2$$

$$\int_0^{\infty} d\tilde{T}\, \tilde{T}^3\, \nu(\tilde{T}) = -\frac{3\beta^3}{4\pi \xi_w^3 n_b(\tilde{t}_{sw})}$$

For any $p(t)$!

Evolution of the fluid perturbations: *across* collisions

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)},\, t_0^{(n)}} = \widehat{\boldsymbol{k}}_i\, \widehat{\boldsymbol{k}}_j\, \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\, F_L(t,k)$$

Hindmarsh & Hijazi [1909.10040]

$$F_L(\tilde{t} < \tilde{t}_{sw}, \tilde{k}) = \frac{n_b(\tilde{t})}{\beta^6} \int_{\tilde{t}_c}^{\tilde{t}} d\tilde{t}_0\, \Gamma(\tilde{t}_0)\, (\tilde{t}-\tilde{t}_0)^6\, \left|f'(\tilde{k}\tilde{T})\right|^2$$

$$F_L(\tilde{t}_{sw}^+, \tilde{k}) = \frac{n_b(\tilde{t}_{sw})}{\beta^6} \int_0^\infty d\tilde{T}\, \nu(\tilde{T}) \tilde{T}^6 \left|f'(\tilde{k}\tilde{T})\right|^2$$

$$\nu(\tilde{T}) = -\frac{1}{\beta\, n_b} \int_{\tilde{t}_c}^{\infty} d\tilde{t}\, p(\tilde{t}) \frac{dh}{d\tilde{t}}(\tilde{t}+\tilde{T})$$

$$v_{rms}^2(\tilde{t}_{sw}^+) = \int_0^\infty dk\, \frac{k^2}{2\pi^2} F_L(\tilde{t}_{sw}^+, \tilde{k}) = \underbrace{\frac{4\pi}{\beta^3 V_b} \int_0^\infty \xi^2 v_{ip}^2(\xi) d\xi}_{v_{rms}^2 \text{ of a single velocity profile}}$$

$$\int_0^\infty d\tilde{T}\, \tilde{T}^3\, \nu(\tilde{T}) = -\frac{3\beta^3}{4\pi \xi_w^3 n_b(\tilde{t}_{sw})}$$

$$V_b = \frac{4\pi}{3}\left(\frac{\xi_w}{\beta}\right)^3$$

For any $p(t)$!

Evolution of the fluid perturbations: *across* collisions

$$\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}')\rangle_{\boldsymbol{x}_0^{(n)},\, t_0^{(n)}} = \hat{\boldsymbol{k}}_i\, \hat{\boldsymbol{k}}_j\, \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\, F_L(t,k)$$

$$V_b = \frac{4\pi}{3}\left(\frac{\xi_w}{\beta}\right)^3$$

$$F_L(\tilde{t} < \tilde{t}_{sw}, \tilde{k}) = \frac{n_b(\tilde{t})}{\beta^6}\int_{\tilde{t}_c}^{\tilde{t}} d\tilde{t}_0\, \Gamma(\tilde{t}_0)\, (\tilde{t}-\tilde{t}_0)^6\, \left|f'(\tilde{k}\tilde{T})\right|^2$$

$$F_L(\tilde{t}_{sw}^{+}, \tilde{k}) = \frac{n_b(\tilde{t}_{sw})}{\beta^6}\int_0^{\infty} d\tilde{T}\, \nu(\tilde{T})\tilde{T}^6 \left|f'(\tilde{k}\tilde{T})\right|^2$$

$$v_{rms}^2(\tilde{t}_{sw}^{+}) = \overbrace{\frac{4\pi}{\beta^3 V_b}\int_0^{\infty} \xi^2 v_{ip}^2(\xi)\, d\xi}^{v_{rms}^2 \text{ of a single velocity profile}}$$

$$v_{rms}^2(\tilde{t}_{sw}^{-}) = 4\pi \int_0^{\infty} \xi^2 v_{ip}^2(\xi)\, d\xi \int_{\tilde{t}_c}^{\tilde{t}_{sw}^{-}} d\tilde{t}_0\, \Gamma(\tilde{t}_0)\, (\tilde{t}_{sw}^{-} - \tilde{t}_0)^3 = v_{rms}^2(\tilde{t}_{sw}^{+})$$

Conservation of v_{rms}^2 across collisions?

Evolution of the fluid perturbations: *across* collisions

$$v_{rms}^2(\tilde{t}_{sw}^+) = \frac{4\pi}{\beta^3 V_b}\int_0^\infty \xi^2 v_{ip}^2(\xi)\,d\xi$$

$$V_b = \frac{4\pi}{3}\left(\frac{\xi_w}{\beta}\right)^3$$

$$v_{rms}^2(\tilde{t}_{sw}^-) = 4\pi\int_0^\infty \xi^2 v_{ip}^2(\xi)\,d\xi \int_{\tilde{t}_c}^{\tilde{t}_{sw}^-} d\tilde{t}_0\,\Gamma(\tilde{t}_0)\,(\tilde{t}_{sw}^- - \tilde{t}_0)^3 = v_{rms}^2(\tilde{t}_{sw}^+)$$

Evolution of the fluid perturbations: *across* collisions

$$v_{rms}^2(\tilde{t}_{sw}^+) = \frac{4\pi}{\beta^3 V_b}\int_0^\infty \xi^2 v_{ip}^2(\xi)d\xi$$

$$V_b = \frac{4\pi}{3}\left(\frac{\xi_w}{\beta}\right)^3$$

$$v_{rms}^2(\tilde{t}_{sw}^-) = 4\pi\int_0^\infty \xi^2 v_{ip}^2(\xi)d\xi \int_{\tilde{t}_c}^{\tilde{t}_{sw}^-} d\tilde{t}_0\, \Gamma(\tilde{t}_0)\,(\tilde{t}_{sw}^- - \tilde{t}_0)^3 = v_{rms}^2(\tilde{t}_{sw}^+)$$

The lifetime distribution average implies conservation of v_{rms}^2 across collisions (as the value after collisions is the same as in the single velocity profile). We can also in general define an effective initial time of the sound wave phase t_{sw} at which v_{rms}^2 reaches the single-profile value as

Evolution of the fluid perturbations: *across* collisions

$$v_{rms}^2(\tilde{t}_{sw}^+) = \frac{4\pi}{\beta^3 V_b}\int_0^\infty \xi^2 v_{ip}^2(\xi)d\xi$$

$$V_b = \frac{4\pi}{3}\left(\frac{\xi_w}{\beta}\right)^3$$

$$v_{rms}^2(\tilde{t}_{sw}^-) = 4\pi\int_0^\infty \xi^2 v_{ip}^2(\xi)d\xi \int_{\tilde{t}_c}^{\tilde{t}_{sw}^-} d\tilde{t}_0\, \Gamma(\tilde{t}_0)\,(\tilde{t}_{sw}^- - \tilde{t}_0)^3 = v_{rms}^2(\tilde{t}_{sw}^+)$$

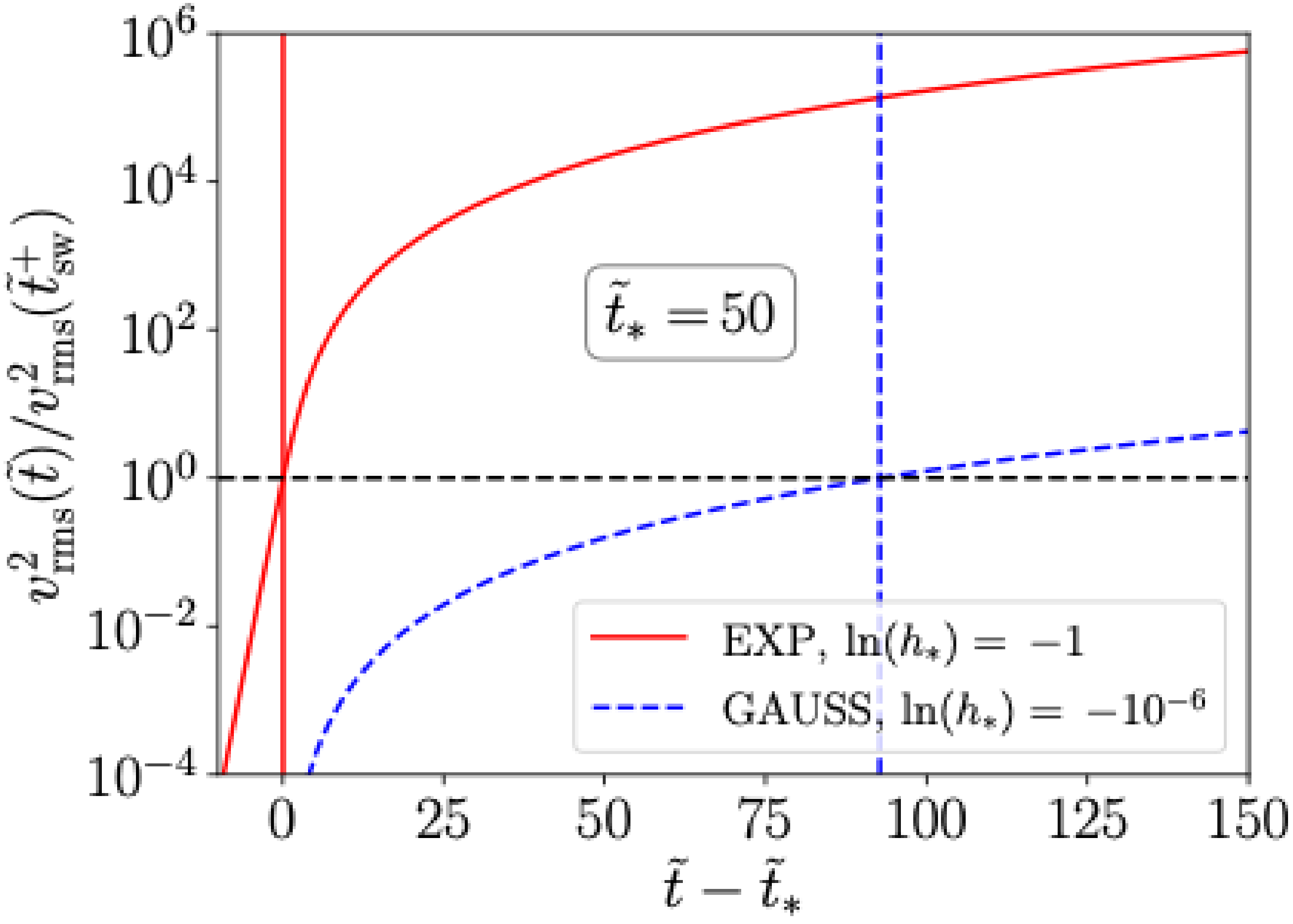
The lifetime distribution average implies conservation of v_{rms}^2 across collisions (as the value after collisions is the same as in the single velocity profile). We can also in general define an effective initial time of the sound wave phase t_{sw} at which v_{rms}^2 reaches the single-profile value as

$$\frac{4\pi\xi_w^3}{3}\int_{\tilde{t}_c}^{\tilde{t}_{sw}} d\tilde{t}_0\, \Gamma(\tilde{t}_0)\,(\tilde{t}_{sw} - \tilde{t}_0)^3 = 1 \longleftrightarrow \frac{v_{rms}^2(\tilde{t}_{sw}^-)}{v_{rms}^2(\tilde{t}_{sw}^+)} = 1$$

Evolution of the fluid perturbations: *across* collisions

$$\frac{4\pi\xi_w^3}{3}\int_{\tilde{t}_c}^{\tilde{t}_{sw}} d\tilde{t}_0\,\Gamma(\tilde{t}_0)\,(\tilde{t}_{sw}-\tilde{t}_0)^3 = 1$$

Effective initial time of the sound wave phase



Numerically we see that in the limit $\tilde{t}_* - \tilde{t}_c \gg 1$

$$(\tilde{t}_{sw}-\tilde{t}_c)/(\tilde{t}_*-\tilde{t}_c) \to 1$$

Evolution of the fluid perturbations: *across* collisions

Properties of the kinetic spectrum at the time of collisions

$$F_L(\tilde{t}_{sw}, \tilde{k}) = \frac{n_b(\tilde{t}_{sw})}{\beta^6} \int_0^\infty d\tilde{T}\, \nu(\tilde{T}) \tilde{T}^6 \left| f'(\tilde{k}\tilde{T}) \right|^2$$

Large scales $k \to 0$ $\quad F_L \to k^2 F_L^{(0)}$

Small scales $k \to \infty$ $\quad F_L \to k^{-4} F_L^{(env)}$

$F_L^{(0)}$ & $F_L^{(env)}$ can be computed from $|f_0'|^2, |f_{env}'|^2, p(t)$

Evolution of the fluid perturbations: *across* collisions

Properties of the kinetic spectrum at the time of collisions

$$k_{cross} = \left(\frac{F_L^{(env)}}{F_L^{(0)}}\right)^{1/6}$$

$$F_L(\tilde{t}_{sw}, \tilde{k}) = \frac{n_b(\tilde{t}_{sw})}{\beta^6} \int_0^\infty d\tilde{T}\, \nu(\tilde{T}) \tilde{T}^6 \left|f'(\tilde{k}\tilde{T})\right|^2$$

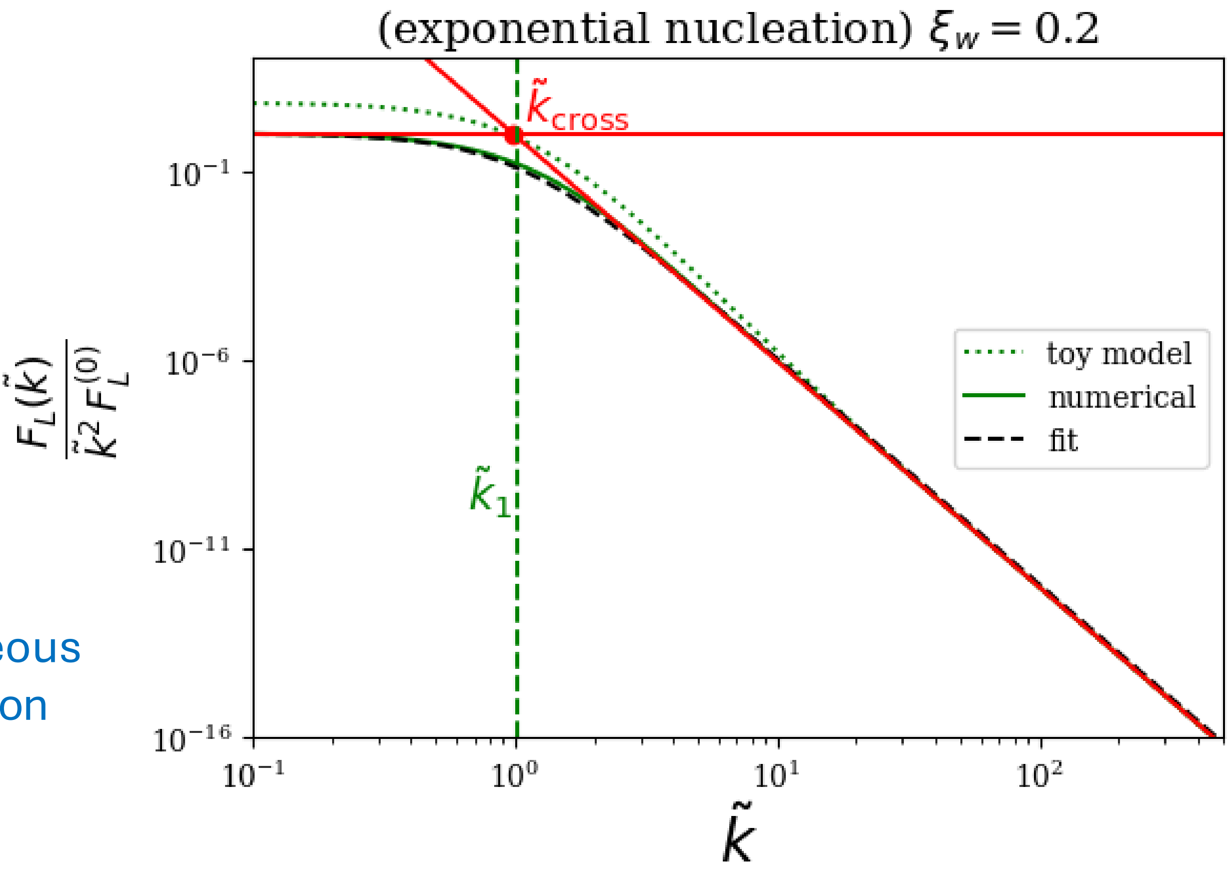
Large scales $k \to 0$ $\quad F_L \to k^2 F_L^{(0)}$

Small scales $k \to \infty$ $\quad F_L \to k^{-4} F_L^{(env)}$

$F_L^{(0)}$ & $F_L^{(env)}$ can be computed from $|f_0'|^2, |f_{env}'|^2, p(t)$

$\tilde{k}_1 \simeq \dfrac{z_1}{5.7}$ exponential nucleation

$\tilde{k}_1 \simeq \dfrac{z_1}{2.5}$ simultaneous nucleation



Evolution of the fluid perturbations: *across* collisions

Properties of the kinetic spectrum at the time of collisions

$$k_{cross} = \left(\frac{F_L^{(env)}}{F_L^{(0)}}\right)^{1/6}$$

$$F_L(\tilde{t}_{sw}, \tilde{k}) = \frac{n_b(\tilde{t}_{sw})}{\beta^6}\int_0^\infty d\tilde{T}\, \nu(\tilde{T})\tilde{T}^6 \left|f'(\tilde{k}\tilde{T})\right|^2$$

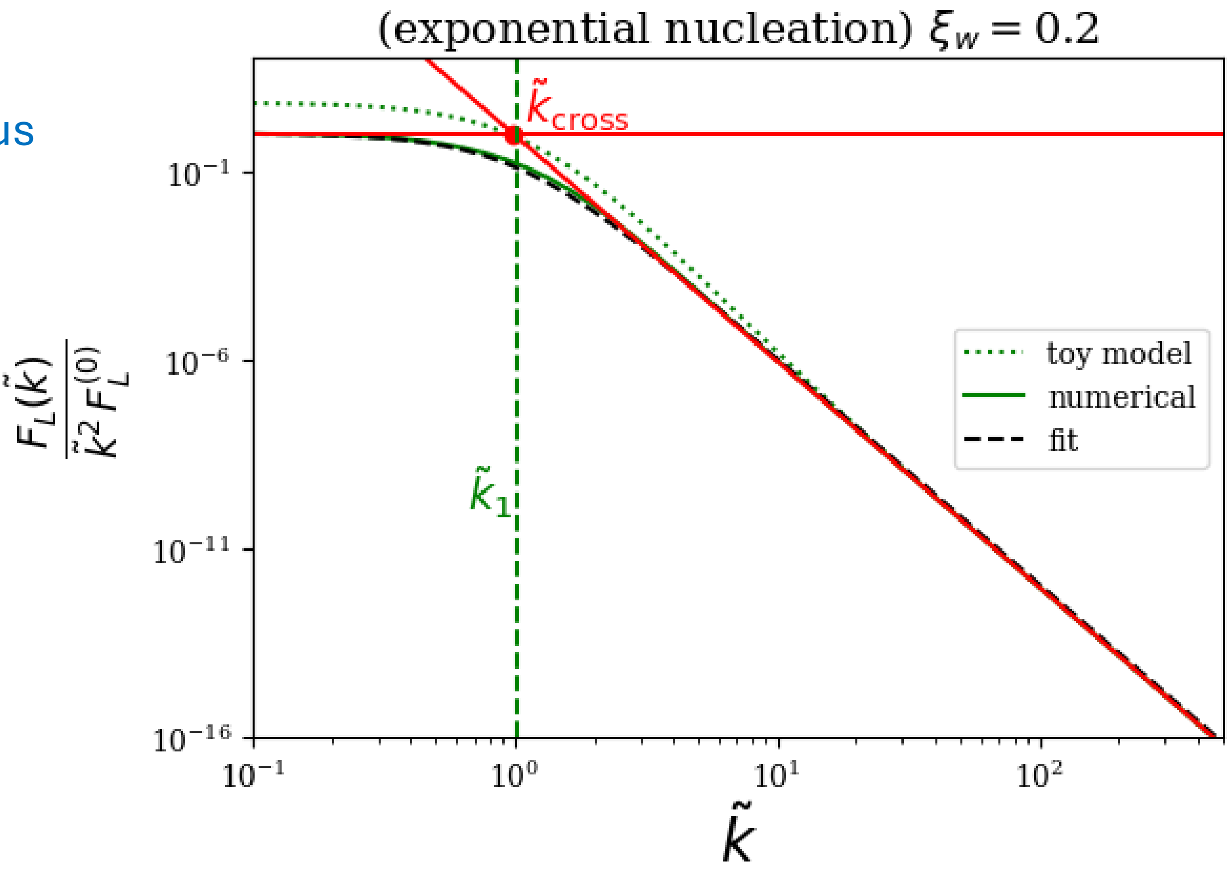
$\tilde{k}_1 \simeq \frac{z_1}{5.7}$ exponential nucleation

$\tilde{k}_1 \simeq \frac{z_1}{2.5}$ simultaneous nucleation

$$F_L \approx F_L^{(0)}\, k^2 \left[1 + \left(\frac{k}{k_1}\right)^{b_1}\right]^{-\frac{6}{b_1}}$$

$(\xi_w \lesssim v_{CJ}/2)$

$b_1 = 2$



Evolution of the fluid perturbations: *across* collisions

Properties of the kinetic spectrum at the time of collisions

$$k_{cross} = \left(\frac{F_L^{(env)}}{F_L^{(0)}}\right)^{1/6}$$

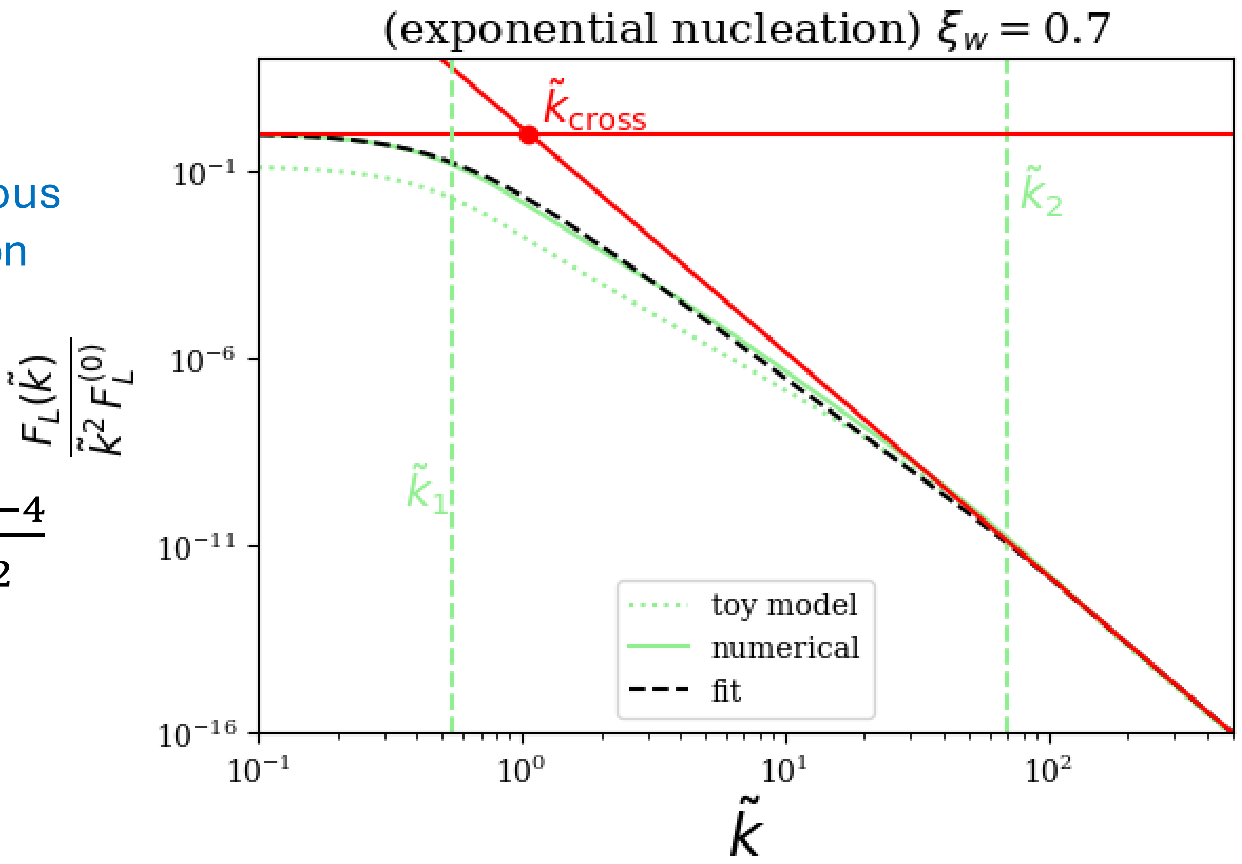
$$F_L(\tilde{t}_{sw}, \tilde{k}) = \frac{n_b(\tilde{t}_{sw})}{\beta^6} \int_0^\infty d\tilde{T}\, \nu(\tilde{T}) \tilde{T}^6 \left|f'(\tilde{k}\tilde{T})\right|^2$$

$$\tilde{k}_1 \simeq \frac{z_1}{5.7} \qquad \tilde{k}_2 \simeq \frac{z_2}{2.4}$$

exponential nucleation

$$\tilde{k}_1 \simeq \frac{z_1}{2.5} \qquad \tilde{k}_2 \simeq \frac{z_2}{1.3}$$

simultaneous nucleation



$$F_L \approx F_L^{(0)} k^2 \left[1 + \left(\frac{k}{k_1}\right)^{b_1}\right]^{\frac{\sigma-2}{b_1}} \left[1 + \left(\frac{k}{k_2}\right)^{b_2}\right]^{\frac{-\sigma-4}{b_2}}$$

$(\xi_w \gtrsim v_{CJ}/2)$

$$\sigma = 2\left[1 - 3\frac{\log(k_2/k_{cross})}{\log(k_2/k_1)}\right]$$

$b_1 = 2, b_2 = 4$

Consequences for the gravitational wave spectrum

$$\Omega_{GW}(\tau_0, k) = 3\,\mathcal{T}_{GW} \iint_{\tau_*}^{\min[\tau_0, \tau_{fin}]} \frac{d\tau_1}{\tau_1}\frac{d\tau_2}{\tau_2} \cos k(\tau_0 - \tau_1) \cos k(\tau_0 - \tau_2)\, E_\Pi(k, \tau_1, \tau_2)$$

UETC for sound-waves with full $F_L(t, k)$

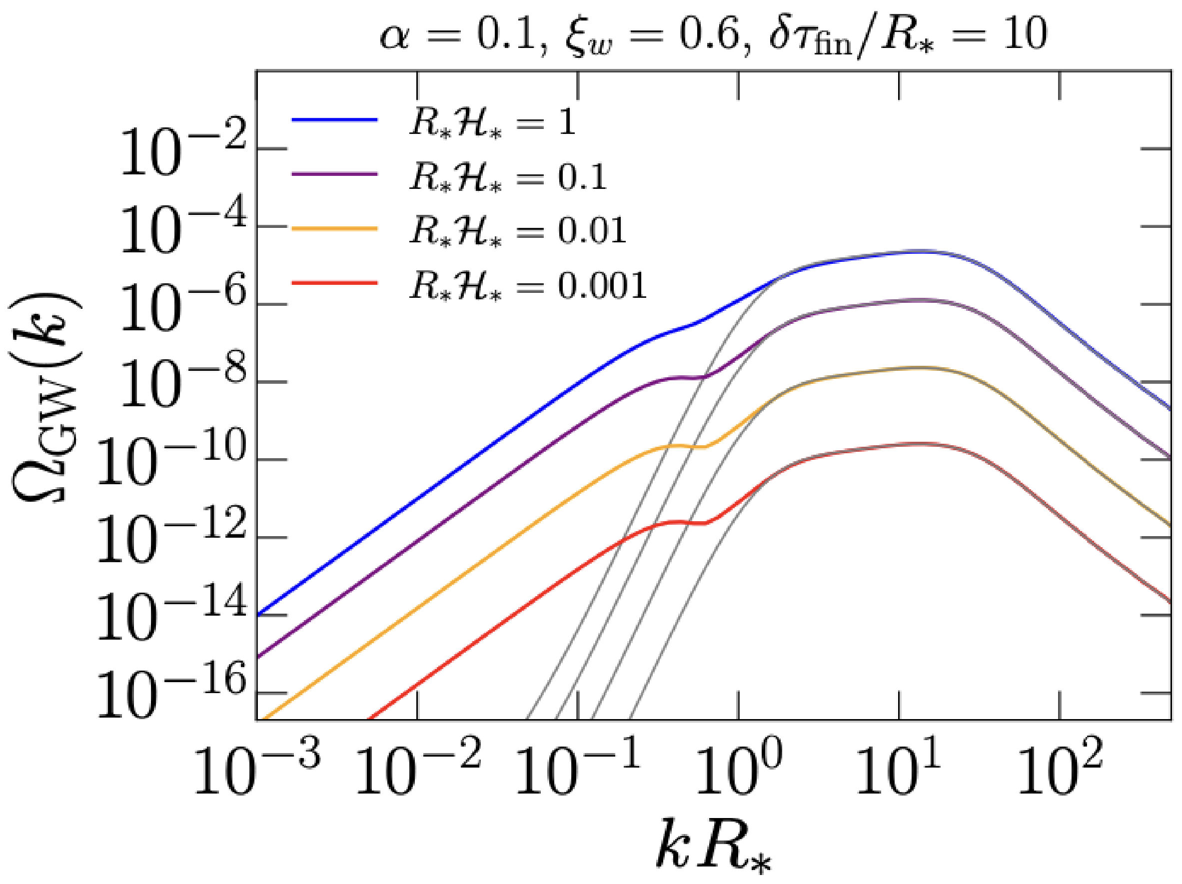
Hindmarsh & Hijazi [1909.10040]

Consequences for the gravitational wave spectrum

$$\Omega_{GW}(\tau_0, k) = 3\,\mathcal{T}_{GW} \iint_{\tau_*}^{\min[\tau_0,\tau_{fin}]} \frac{d\tau_1}{\tau_1}\frac{d\tau_2}{\tau_2} \cos k(\tau_0-\tau_1)\cos k(\tau_0-\tau_2)\, E_\Pi(k,\tau_1,\tau_2)$$

UETC for sound-waves with full $F_L(t,k)$

Hindmarsh & Hijazi [1909.10040]

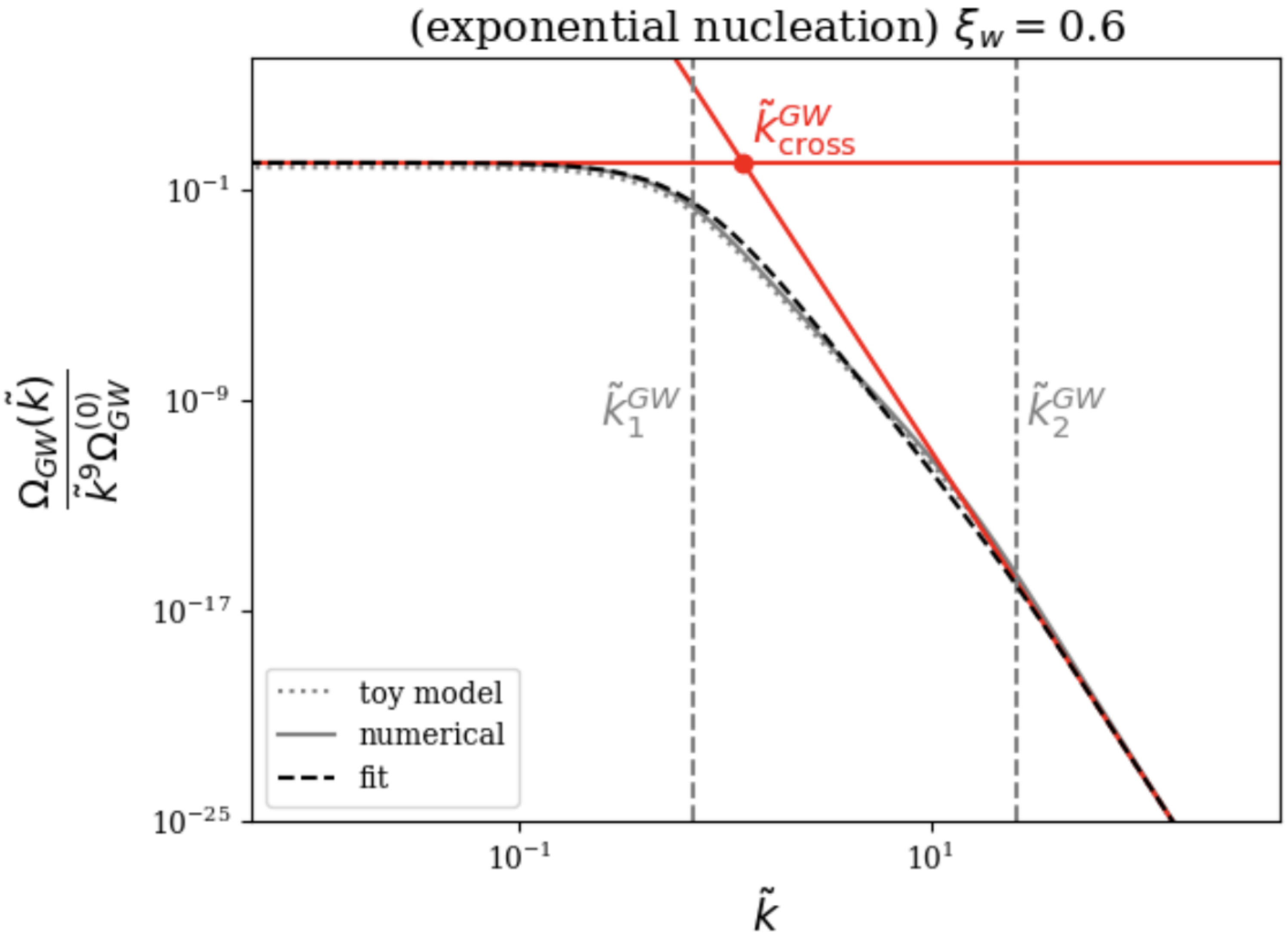


In the limit of long duration of the sound-waves a good approximation for the spectral peak is given by

$$\Omega_{\rm GW}^{\rm HH19}(K) = \frac{3\pi}{2}K^2\,\Upsilon(\tau_{\rm fin})\,\frac{\mathcal{H}_* R_*}{c_{\rm s}}\,\bar{w}^2\,\mathcal{T}_{\rm GW}\left(\frac{\Omega_{\rm K}}{\mathcal{K}}\right)^2 \times \int_{P_-}^{P_+} P\zeta_{\rm kin}(P)(1-z^2)^2 \times \frac{\zeta_{\rm kin}(K/c_{\rm s}-P)}{(K/c_{\rm s}-P)^3}\,{\rm d}P. \qquad \text{(B3)}$$

Roper Pol, Procacci, Caprini [2308.12943]

Consequences for the gravitational wave spectrum



[Preliminary results]

$$\tilde{k}_1^{GW} \approx 1.2 \times \tilde{k}_1$$

$$\tilde{k}_2^{GW} \approx 1.2 \times \tilde{k}_2$$

Peak in the GW spectrum related to (long sound-wave duration limit):

- sound-shell thickness (deflagrations & detonations)
- discontinuities in the self-similar profiles (hybrids)

Conclusions

- The GW spectrum from sound waves (in the sound shell model) can be understood from the properties of the self-similar profiles and of the bubble nucleation history

- For hybrids the GW peak scale is related to the distance between discontinuities instead of the sound-shell thickness (broader spectrum around the peak)

- An accurate characterization of the full GW spectrum in terms of phase transition parameters is necessary for parameter reconstruction at future experiments (e. g. LISA)

Conclusions

- The GW spectrum from sound waves (in the sound shell model) can be understood from the properties of the self-similar profiles and of the bubble nucleation history

- For hybrids the GW peak scale is related to the distance between discontinuities instead of the sound-shell thickness (broader spectrum around the peak)

- An accurate characterization of the full GW spectrum in terms of phase transition parameters is necessary for parameter reconstruction at future experiments (e. g. LISA)

Thanks for your attention!