Quantum Connections in Sweden-16
Summer School

# Less is more:

## The power of vacuum quantum fluctuations

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Lecture 1: Renormalization and Casimir Physics

**Lecture 2: Casimir Torque, Friction and Spectra**

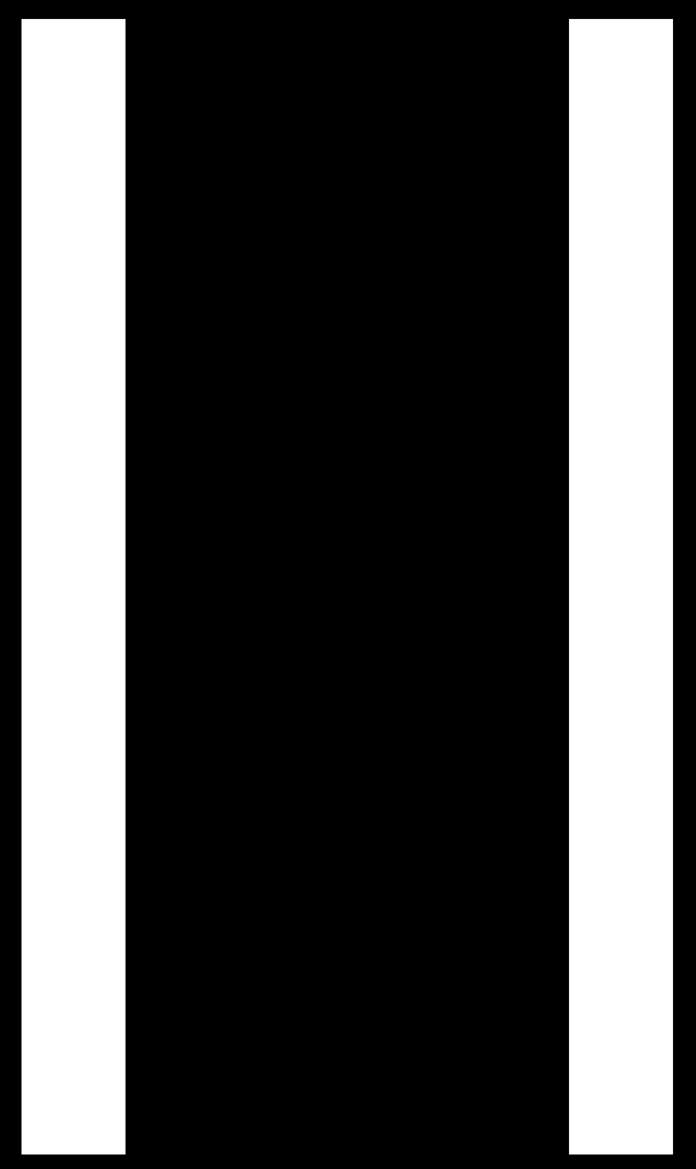
Lecture 3: Quantum atmosphere

**Lecture 4: Cavity Quantum Materials**

# (2) Casimir Torque

# Understanding Casimir effect from symmetry breaking

Casimir force:
translational symm. Breaking



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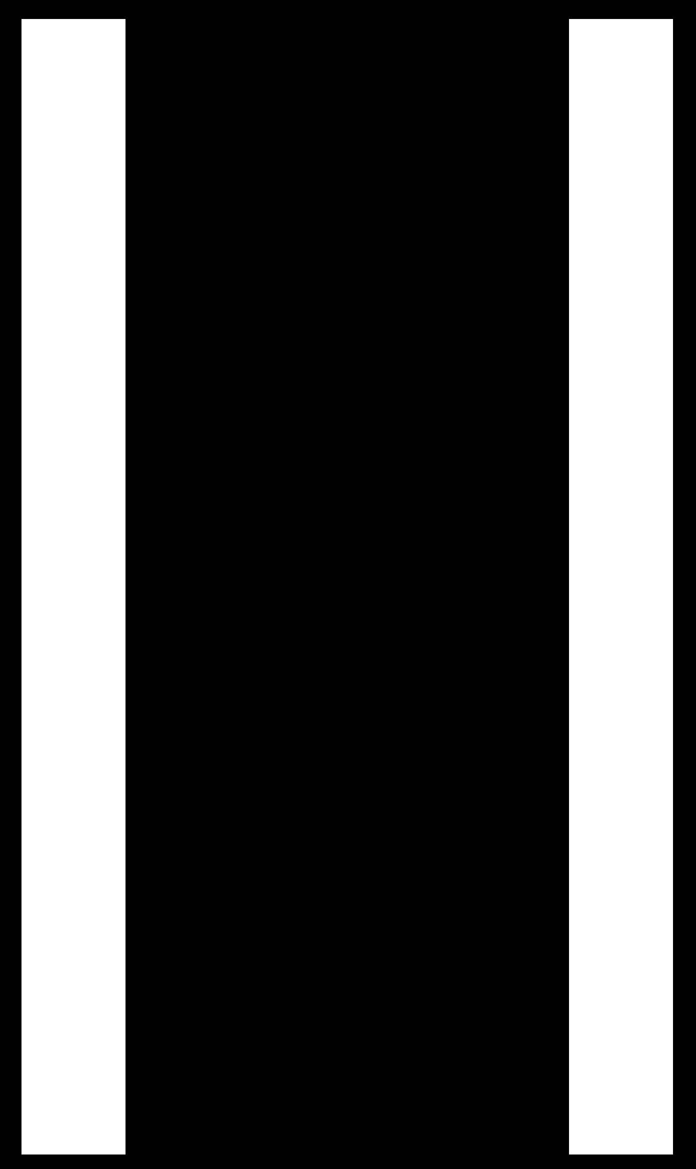

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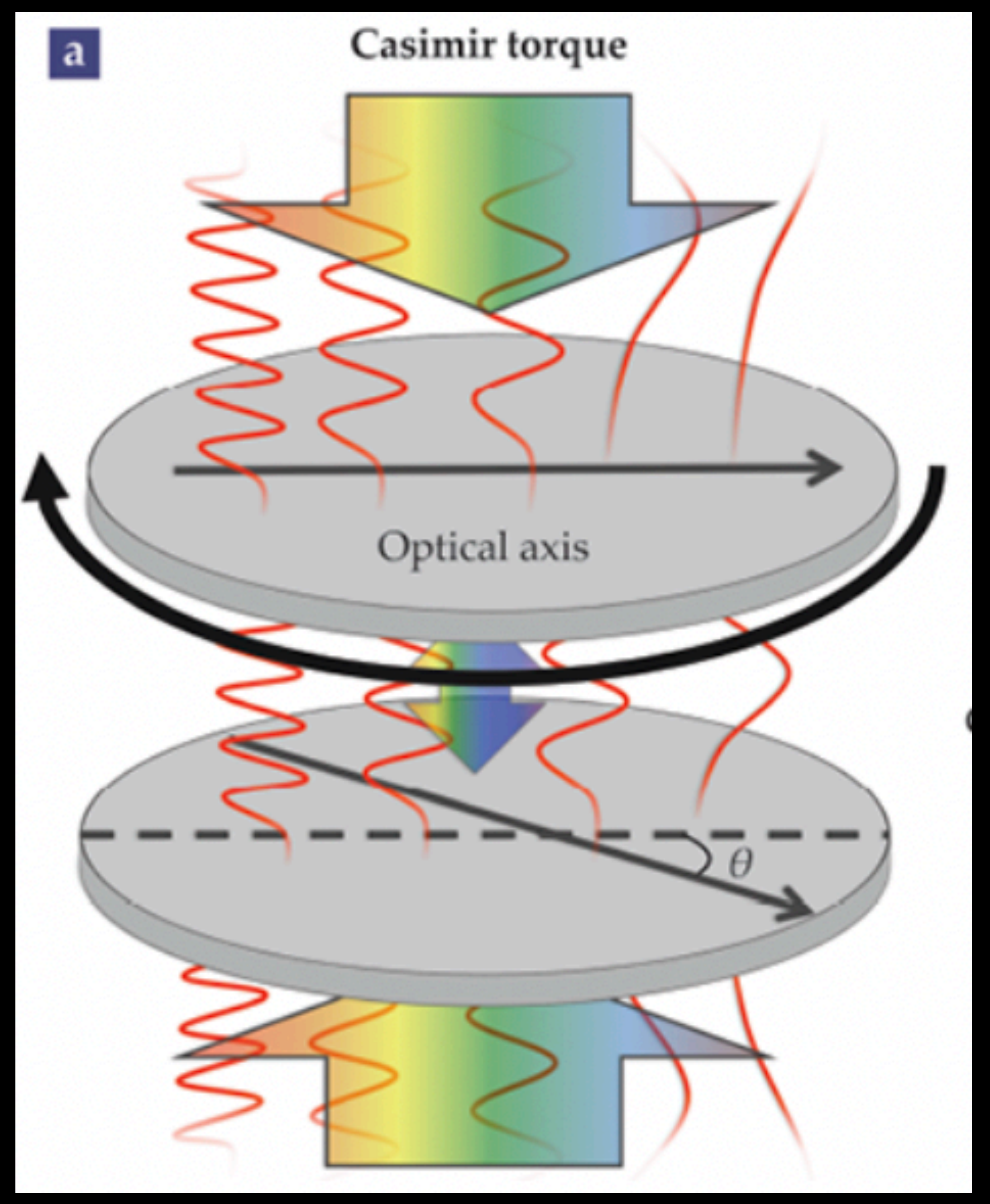


*Nature* **564**, 386 (2018)

# Understanding Casimir effect from symmetry breaking

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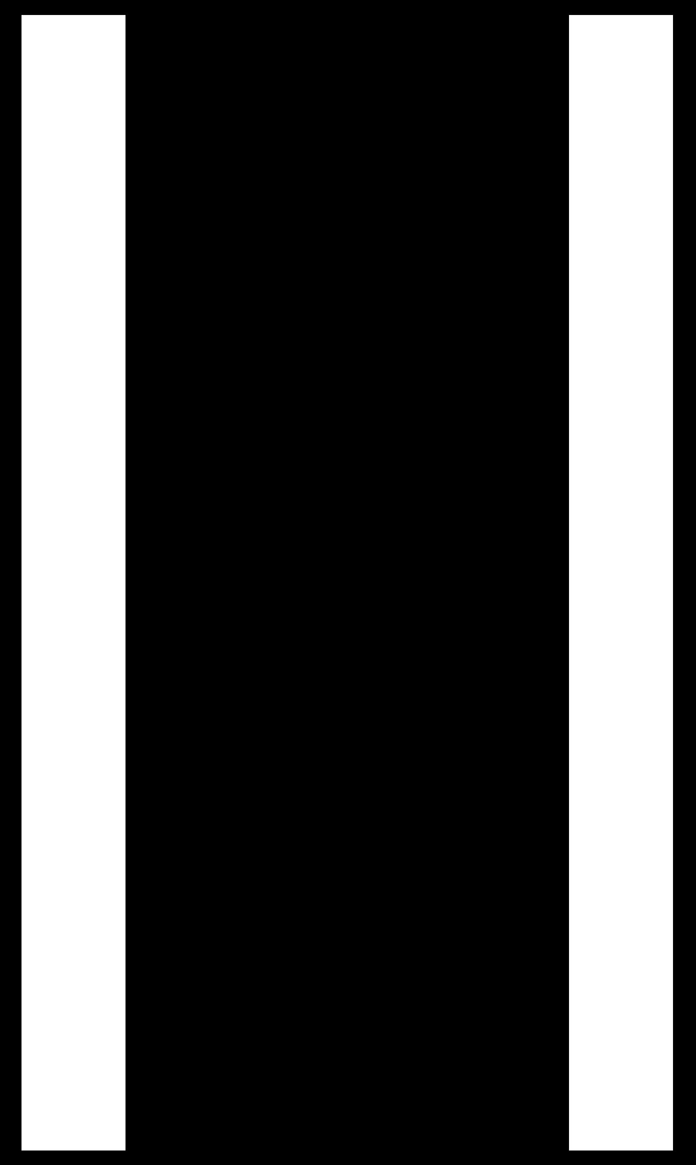

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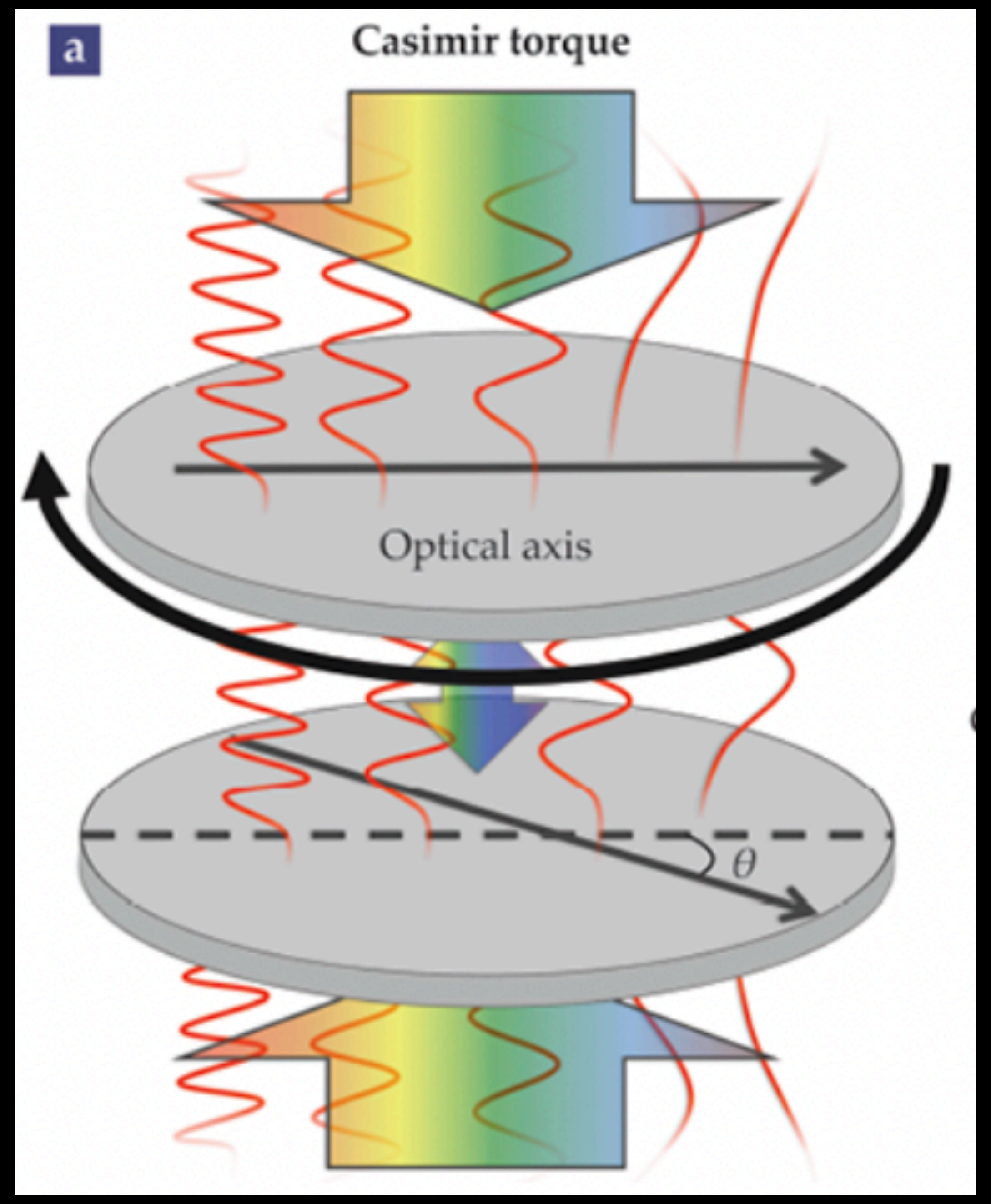



*Nature* **564**, 386 (2018)

Parsegian and Weiss, Barash, Cappaso, Munday…

# How to calculate Casimir torque?

Casimir Energy

$$E_c(\theta, d) = \frac{k_B T A}{4\pi^2} \sum_{n=0}^{\infty}{}' \int_0^{\infty} k_\parallel dk_\parallel \int_0^{2\pi} d\phi \; \ln\det\left(\mathbf{1} - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_n d}\right)$$

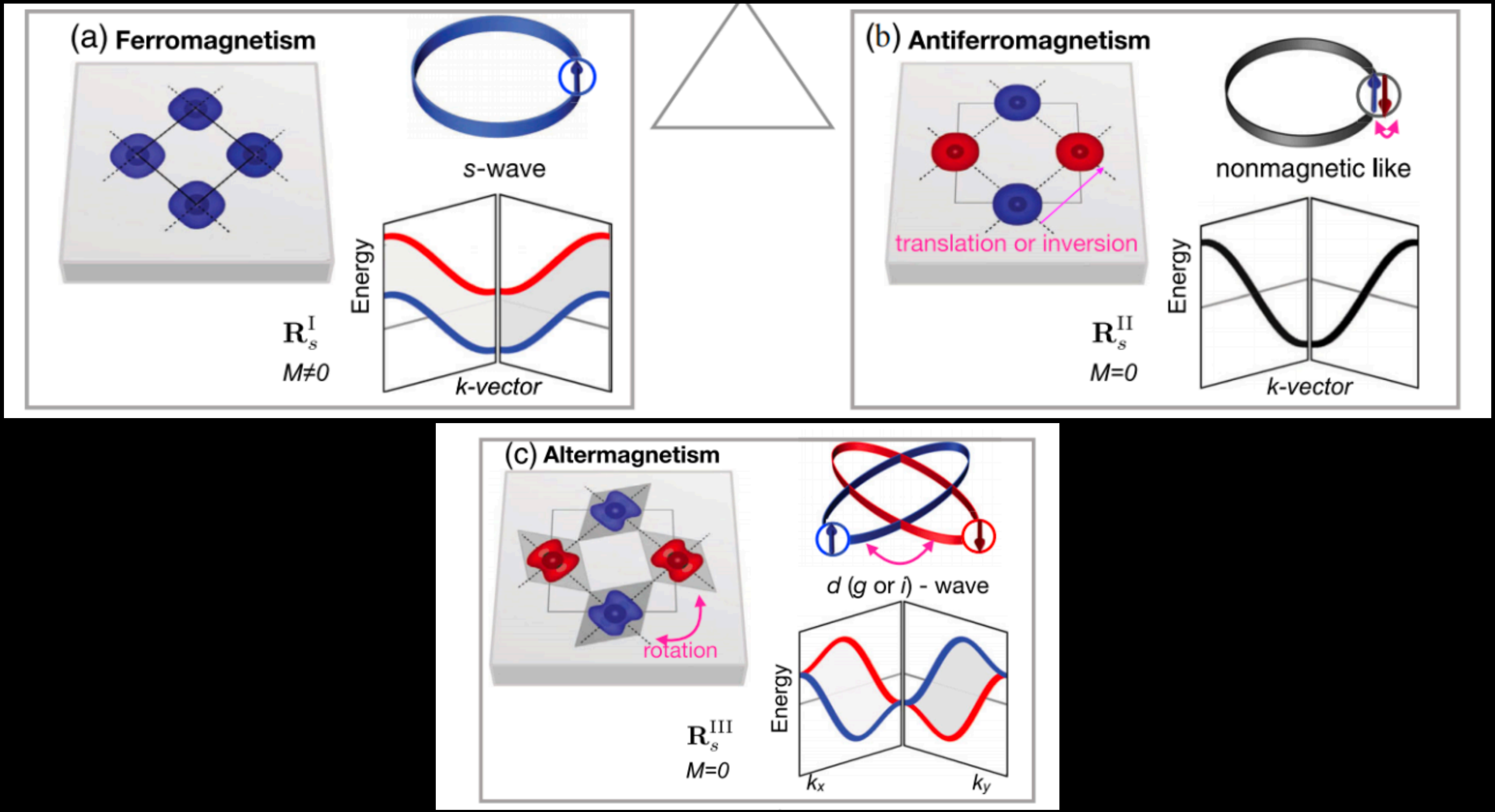
$\theta$-dependent

$$\mathbf{R}_i(i\xi_n, k_\parallel, \phi_i) = \begin{bmatrix} r_{i,ss}(i\xi_n, k_\parallel, \phi_i) & r_{i,sp}(i\xi_n, k_\parallel, \phi_i) \\ r_{i,ps}(i\xi_n, k_\parallel, \phi_i) & r_{i,pp}(i\xi_n, k_\parallel, \phi_i) \end{bmatrix}$$

Casimir Torque

$$\tau_c(\theta, d) = \frac{k_B T A}{4\pi^2} \sum_{n=0}^{\infty}{}' \int_0^{\infty} k_\parallel dk_\parallel \int_0^{2\pi} d\phi \; \mathrm{Tr}\left[\left(\mathbf{1} - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_n d}\right)^{-1} \mathbf{R}_1 \frac{\partial \mathbf{R}_2}{\partial \theta}\right] e^{-2K_n d}$$

# New twist of Casimir toque



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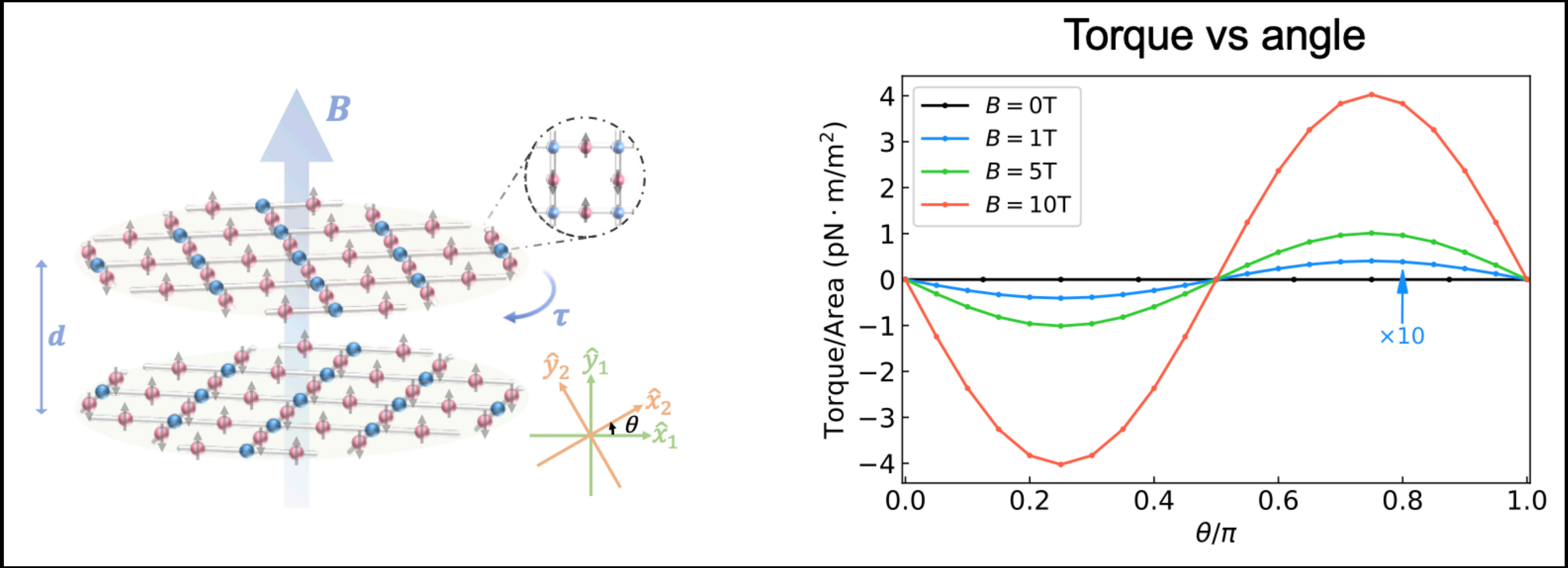
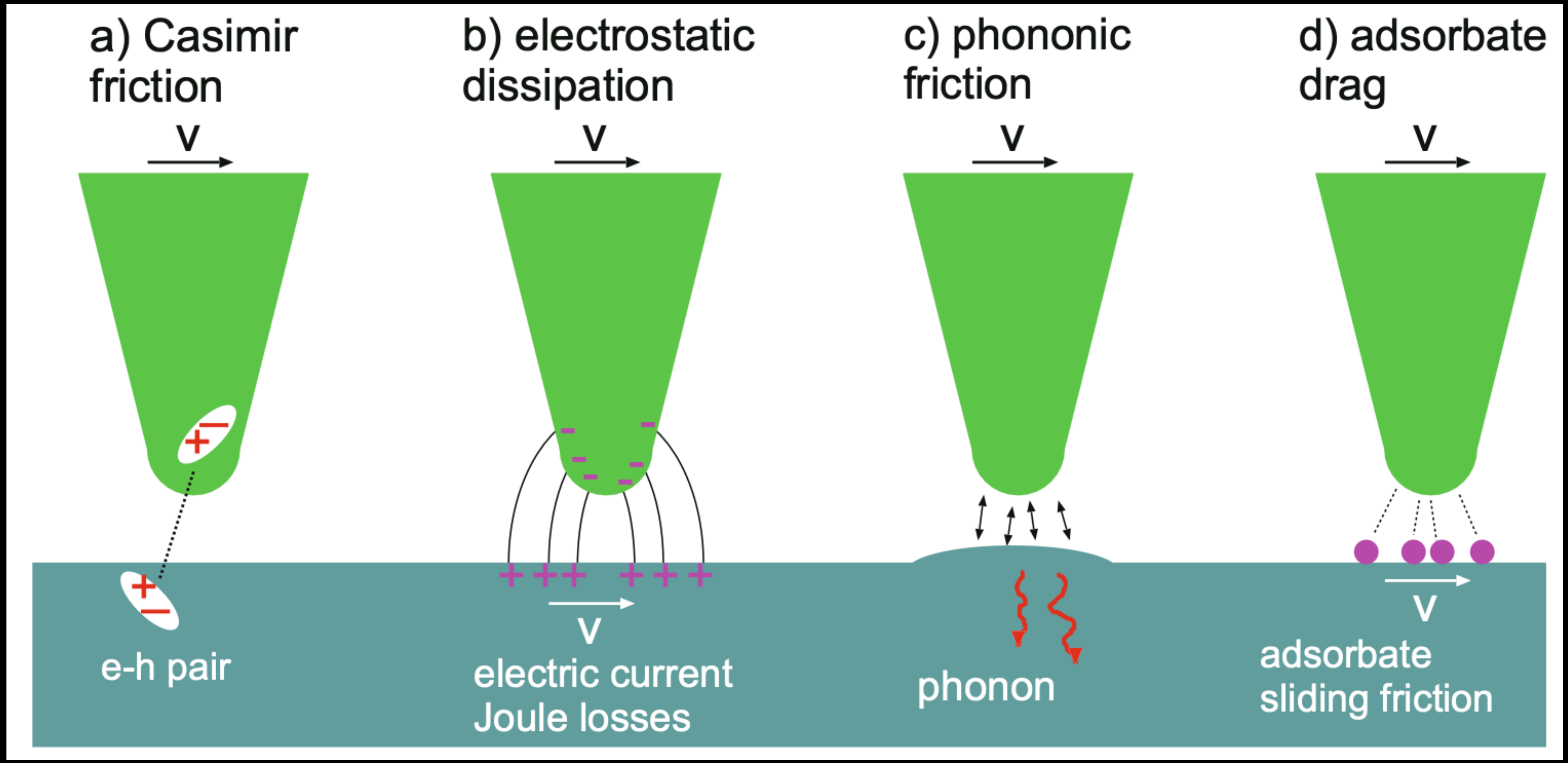



Zixuan Dai, QDJ, arXiv:2601.14381

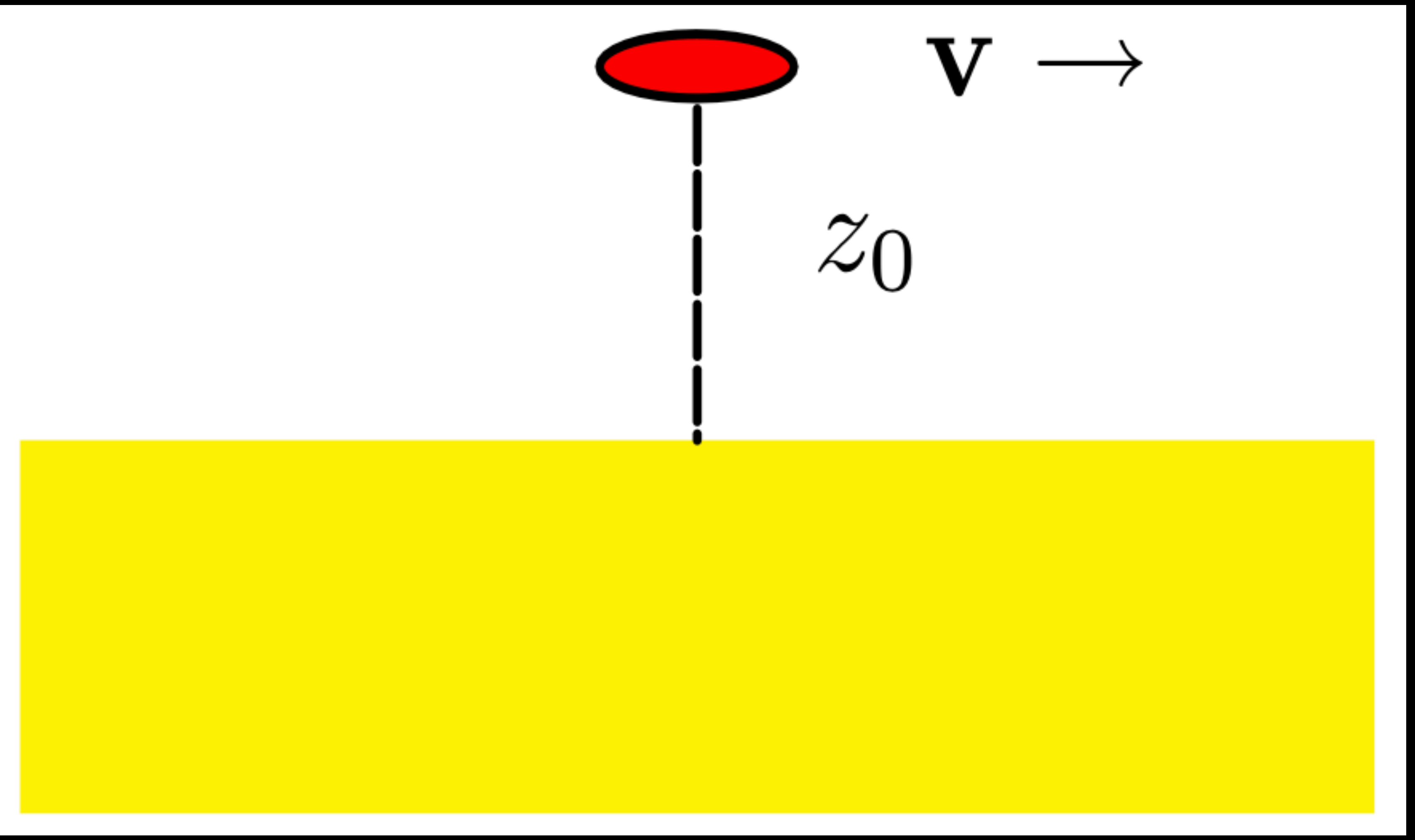
# (2) Casimir Friction

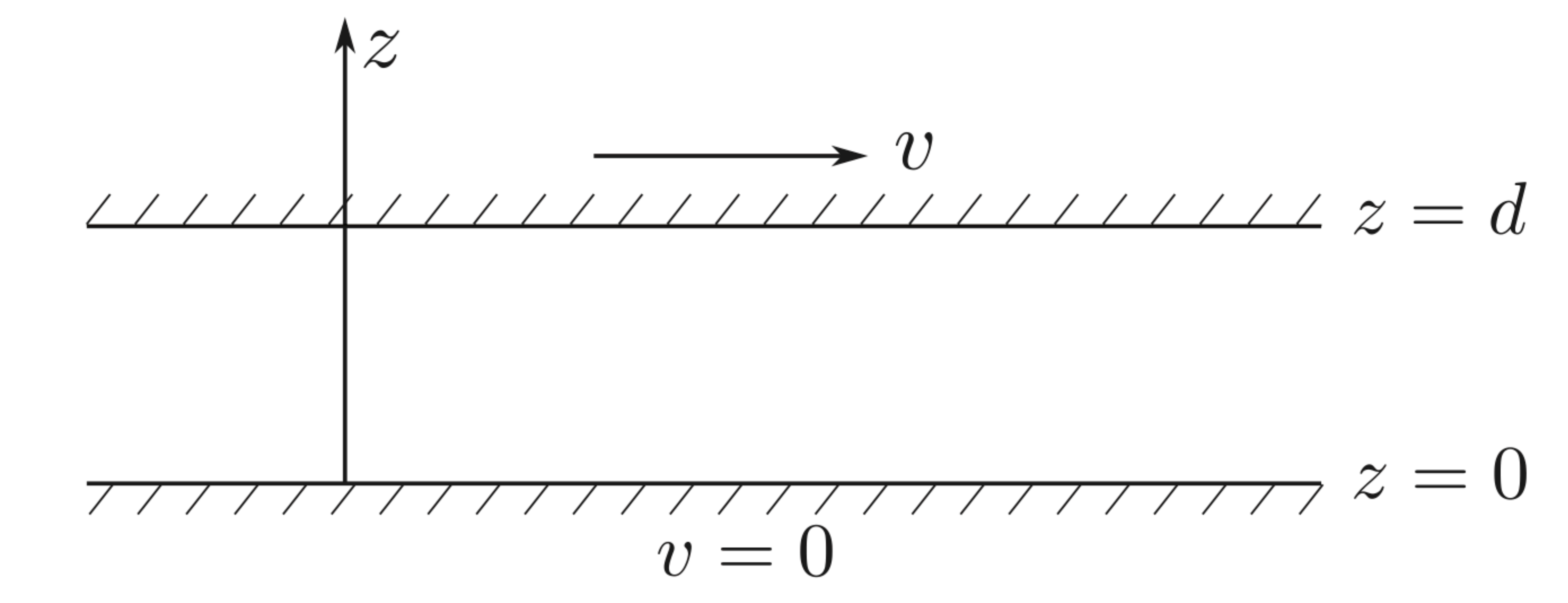
# Non-contact nanoscale friction




A. I. Volokitin, B. N. J. Persson, DOI 10.1007/978-3-662-53474-8

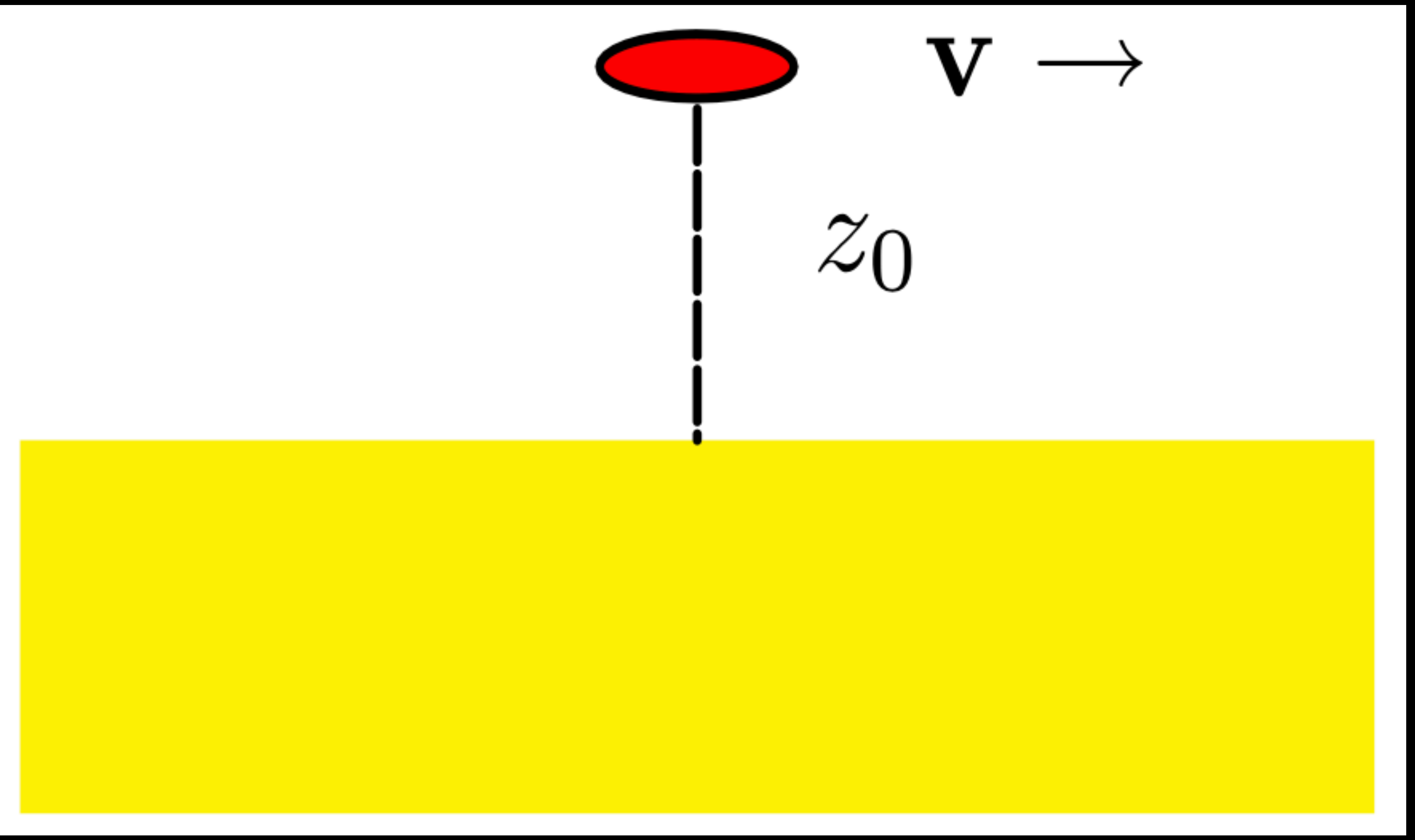
# Casimir friction is not a fiction: two simple models

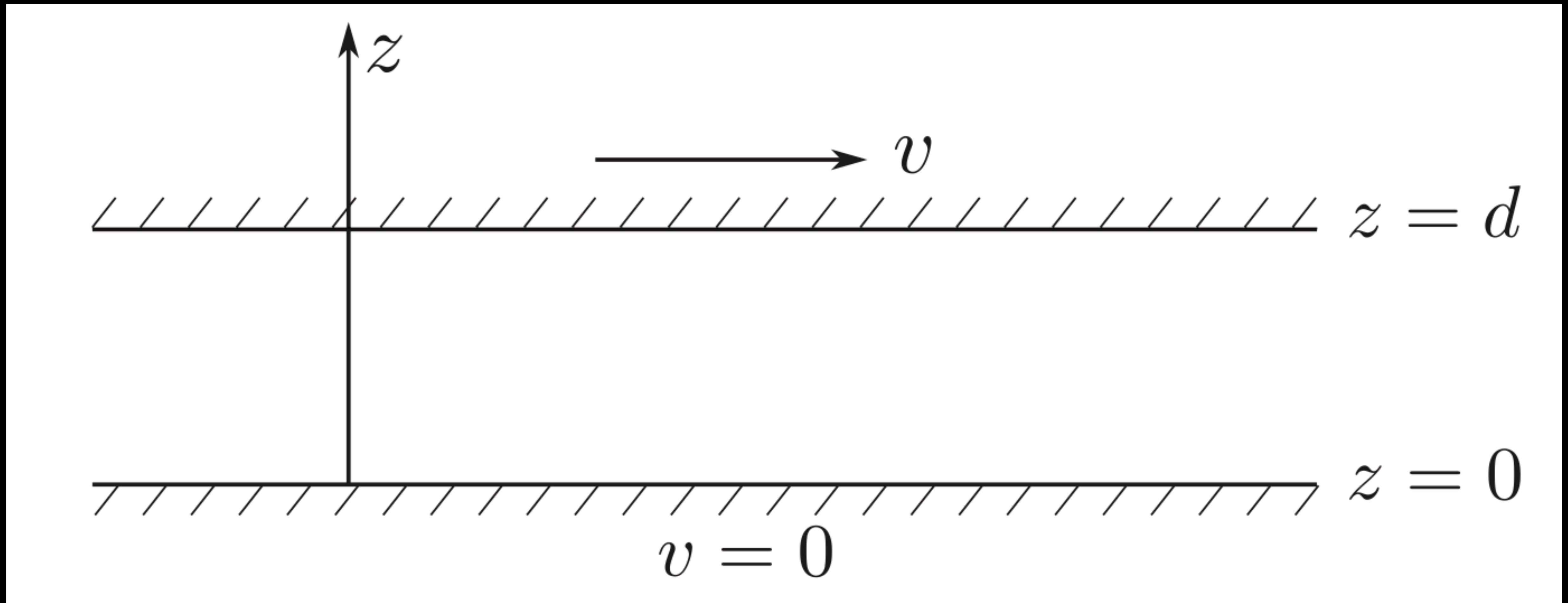



**Casimir friction**

# Casimir friction is not a fiction: two simple models




## Casimir friction

*Pendry,*

*Teodorovich,*

*Levitov,*

*Volokitin,*

*Persson,*

*Barton,*

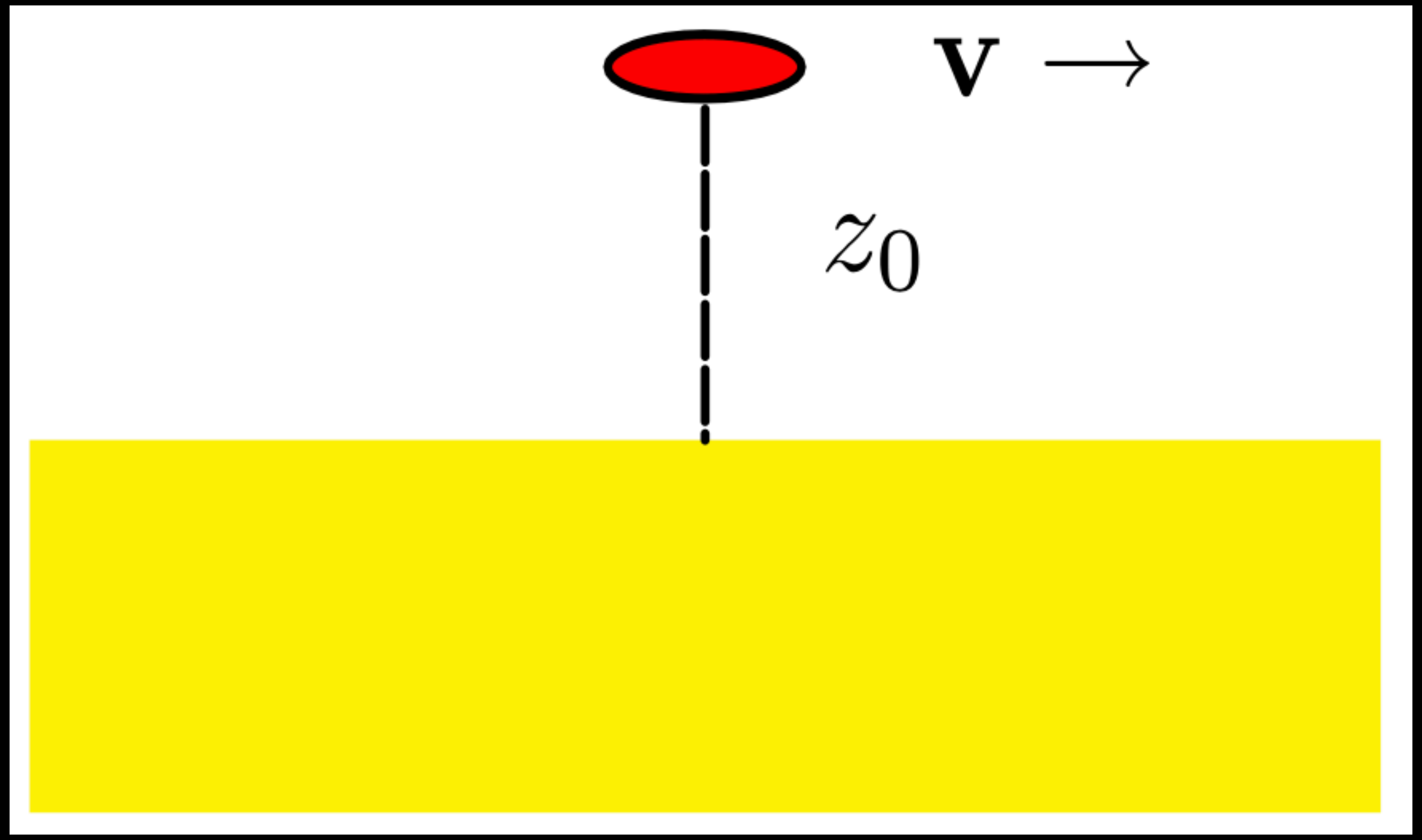
*Milton,*

*Høye and*

*Brevik…*

# Where is friction come from? (hotly debated)

Simplest model exhibits friction by exchange of photons



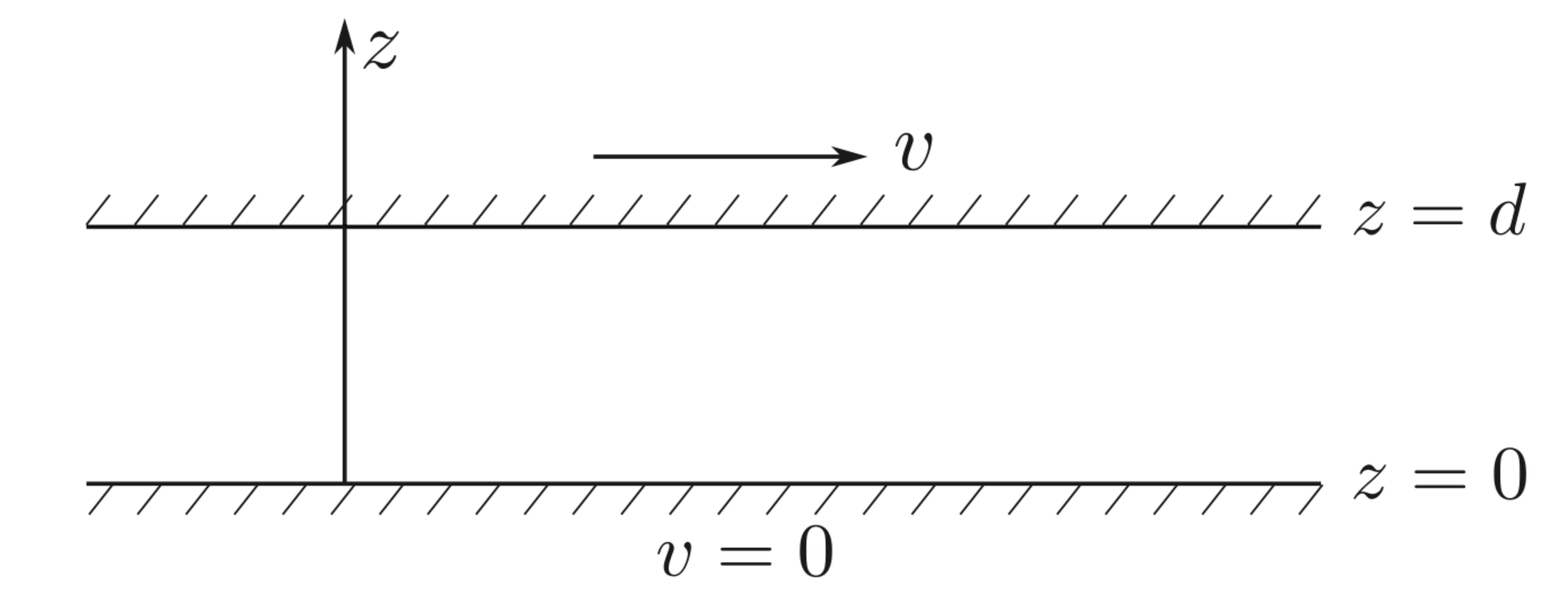

$$F = \frac{135\hbar\alpha^2 v^3}{4\pi^3\sigma^2 (2z_0)^{10}}$$


A. I. Volokitin, B. N. J. Persson, Rev. Mod. Phys. **79**, 1291 (2007)

J. Pendry, J. Phys.: Condens. Matter 9 10301 (1997)

Milton, Hoye, and Brevik, Symmetry 2016

# How to calculate Casimir friction between two plates?




$$\frac{F}{A} = \frac{5\hbar\varepsilon_0^2 v^3}{2^8\pi^2\sigma^2 d^6}, \quad v \ll \frac{d\sigma}{\varepsilon_0}$$

$$\frac{F}{A} = \frac{\hbar\sigma^2}{32 d^2\pi^2 v\varepsilon_0^2}\ln\frac{v\varepsilon_0}{2d\sigma}, \quad v \gg \frac{d\sigma}{\varepsilon_0}$$

**Finite T enhancement** $k_B T >> \hbar v/d$

$$\frac{F_T}{F_0} = \frac{16\pi^2}{15}\left(\frac{d k_B T}{\hbar v}\right)^2$$

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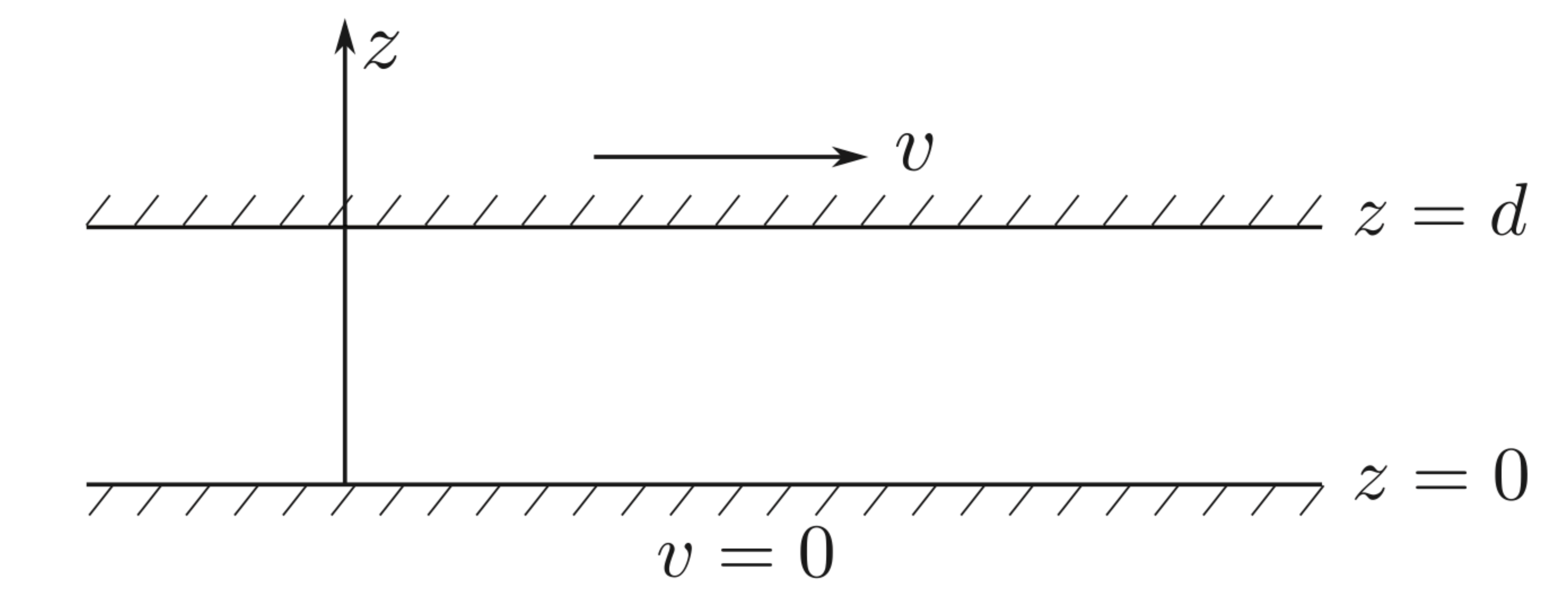


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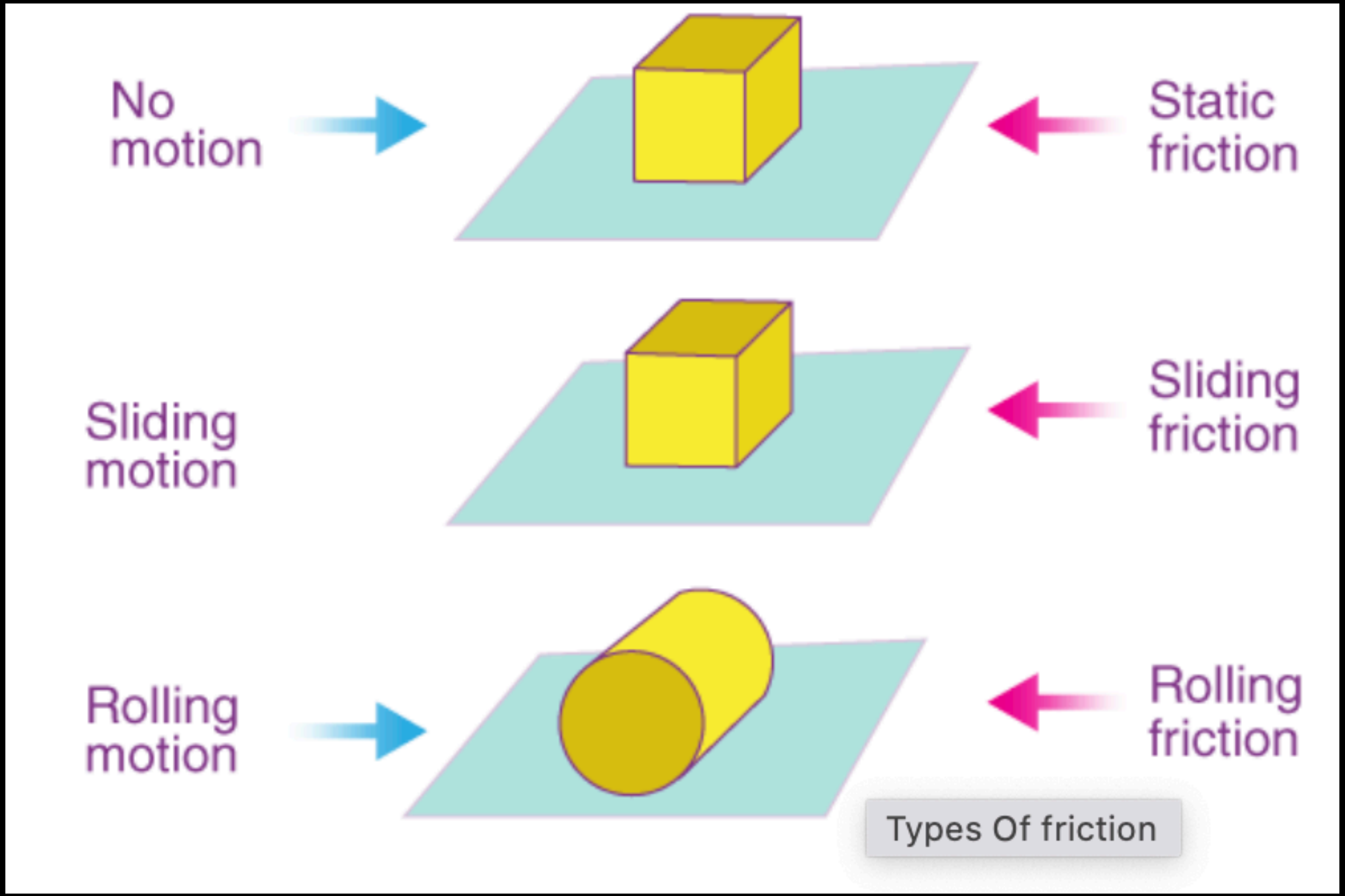
**Finite T enhancement** $k_B T >> \hbar v/d$

$$\frac{F_T}{F_0} = \frac{16\pi^2}{15}\left(\frac{dk_B T}{\hbar v}\right)^2$$

$$F_{x0} = \frac{2\hbar}{\pi}\frac{L^2}{(2\pi)^2}\int_0^{+\infty} k_x\, dk_x \int_{-\infty}^{+\infty} e^{-2kd}\, dk_y \times \left\{\int_0^{+\infty} \Im\left[R_{1,pp}(\omega + k_x v) - R_{1,pp}(\omega - k_x v)\right] \Im R_{2,pp}(\omega)\, d\omega\right\}$$

*J.B.Pendry 1997*

# Types of friction



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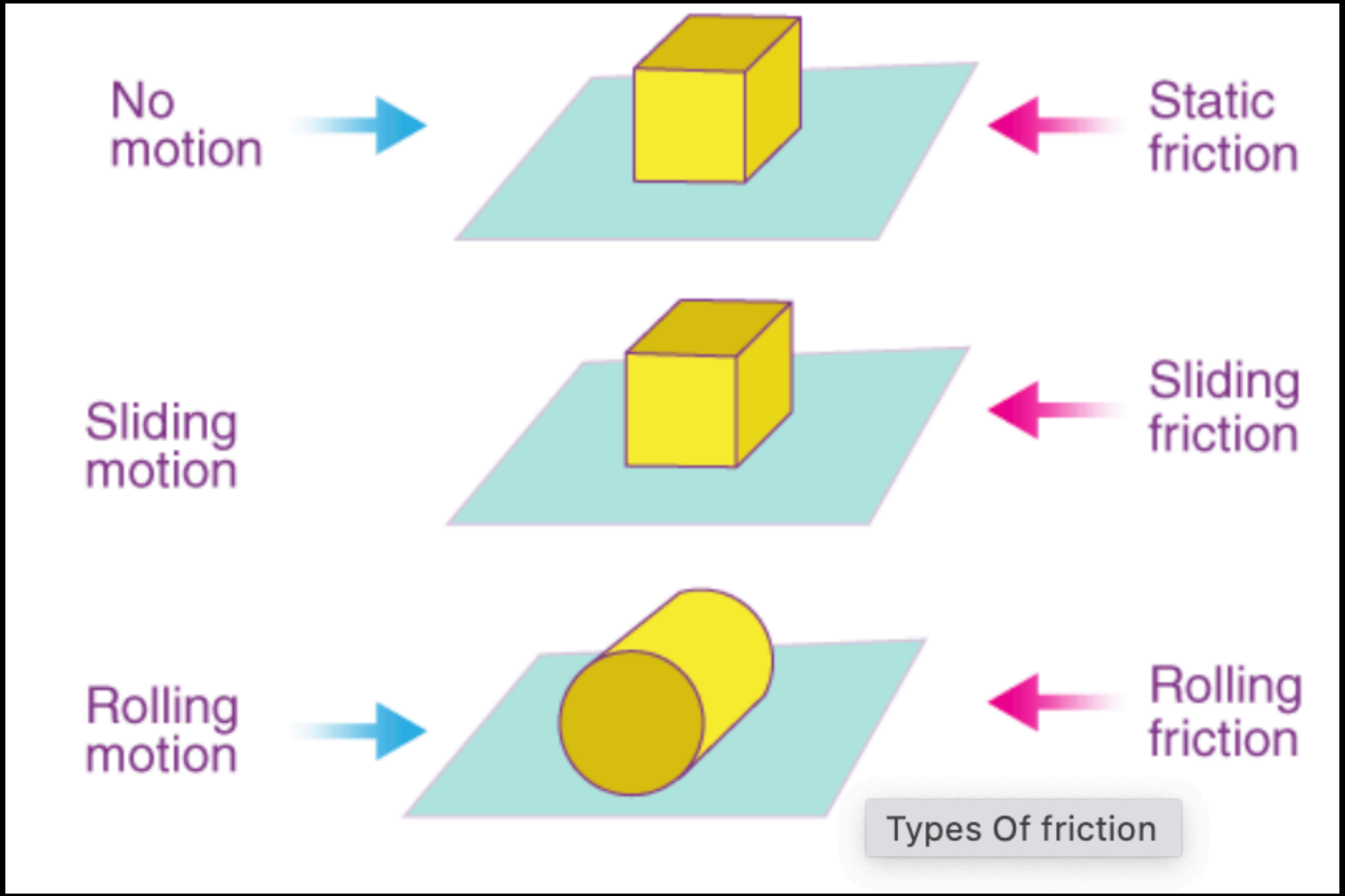
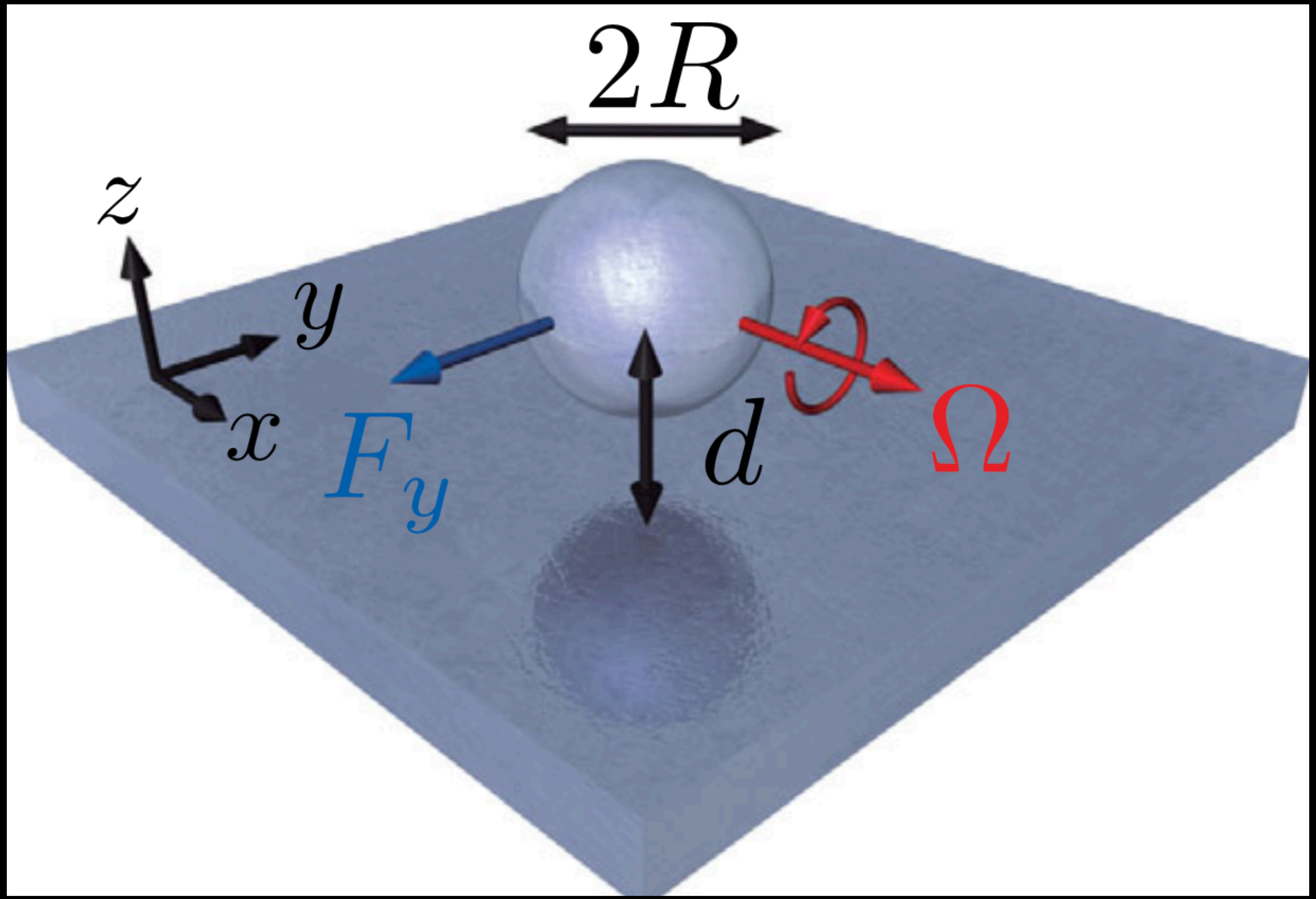
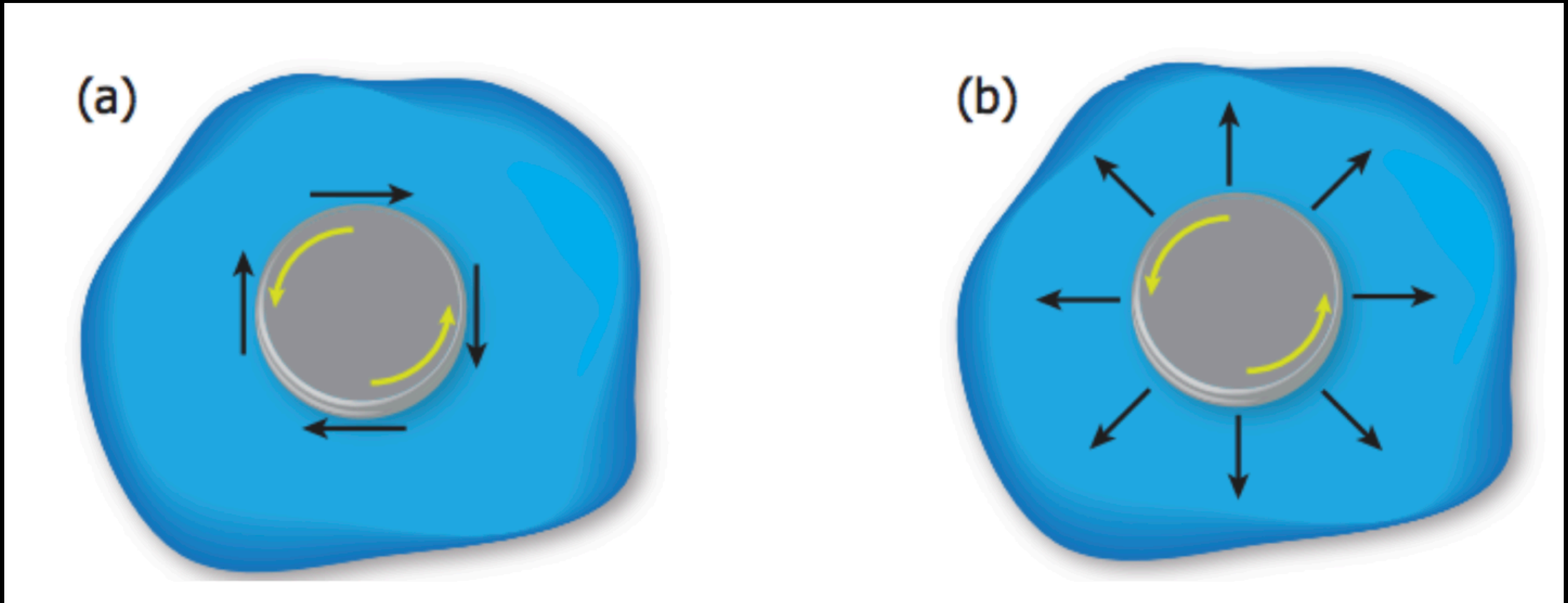



**Rolling Casimir friction**

*Phys. Rev. Lett. 118, 133605 (2017)*

# More types of friction (motion-induced forces)

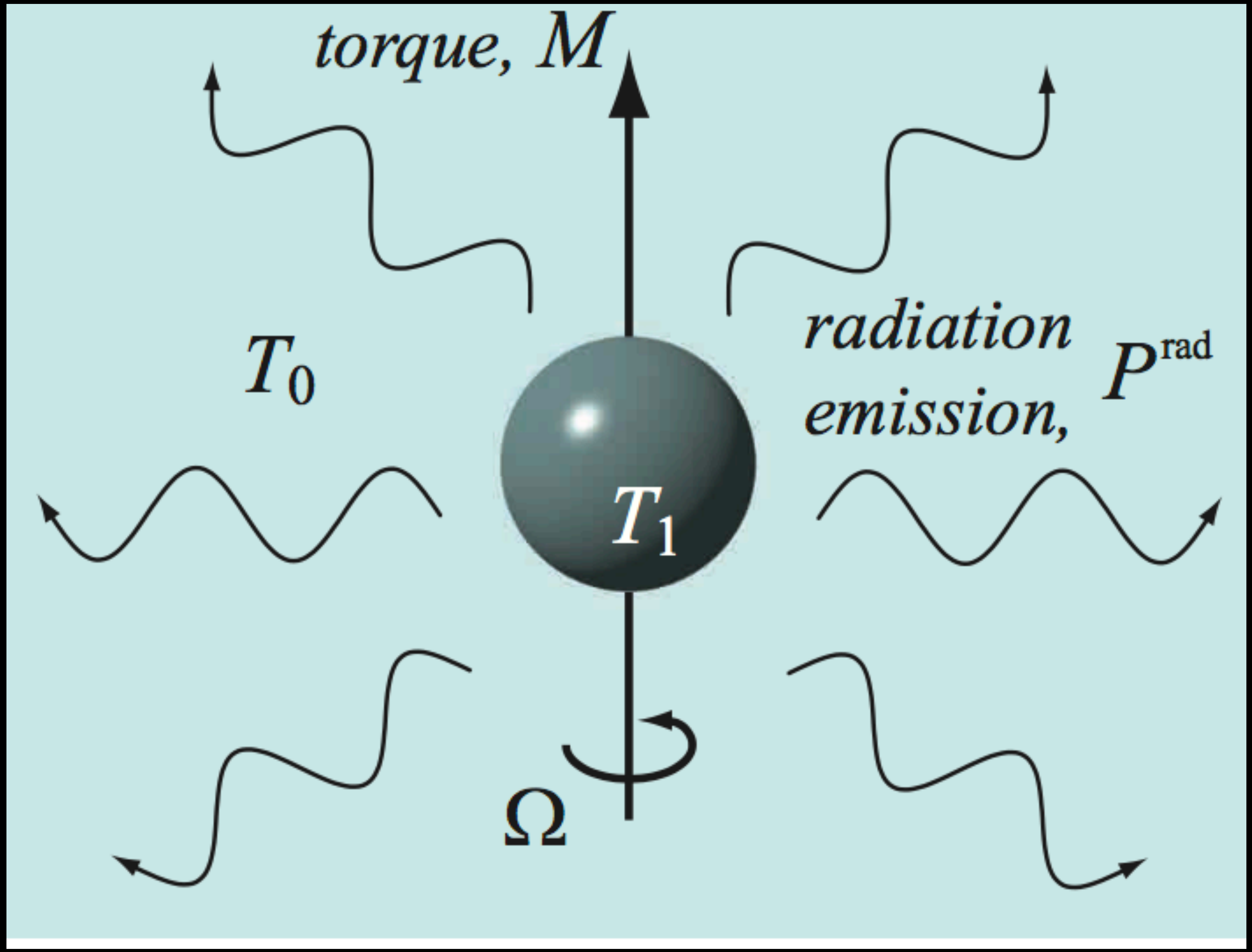



**Dissipative**

**Dissipationless**

*J. E. Avron, Journal of statistical physics 92, 543 (1998)*

# Can dissipionless Casimir friction be realized?




A. Manjavacas  2010

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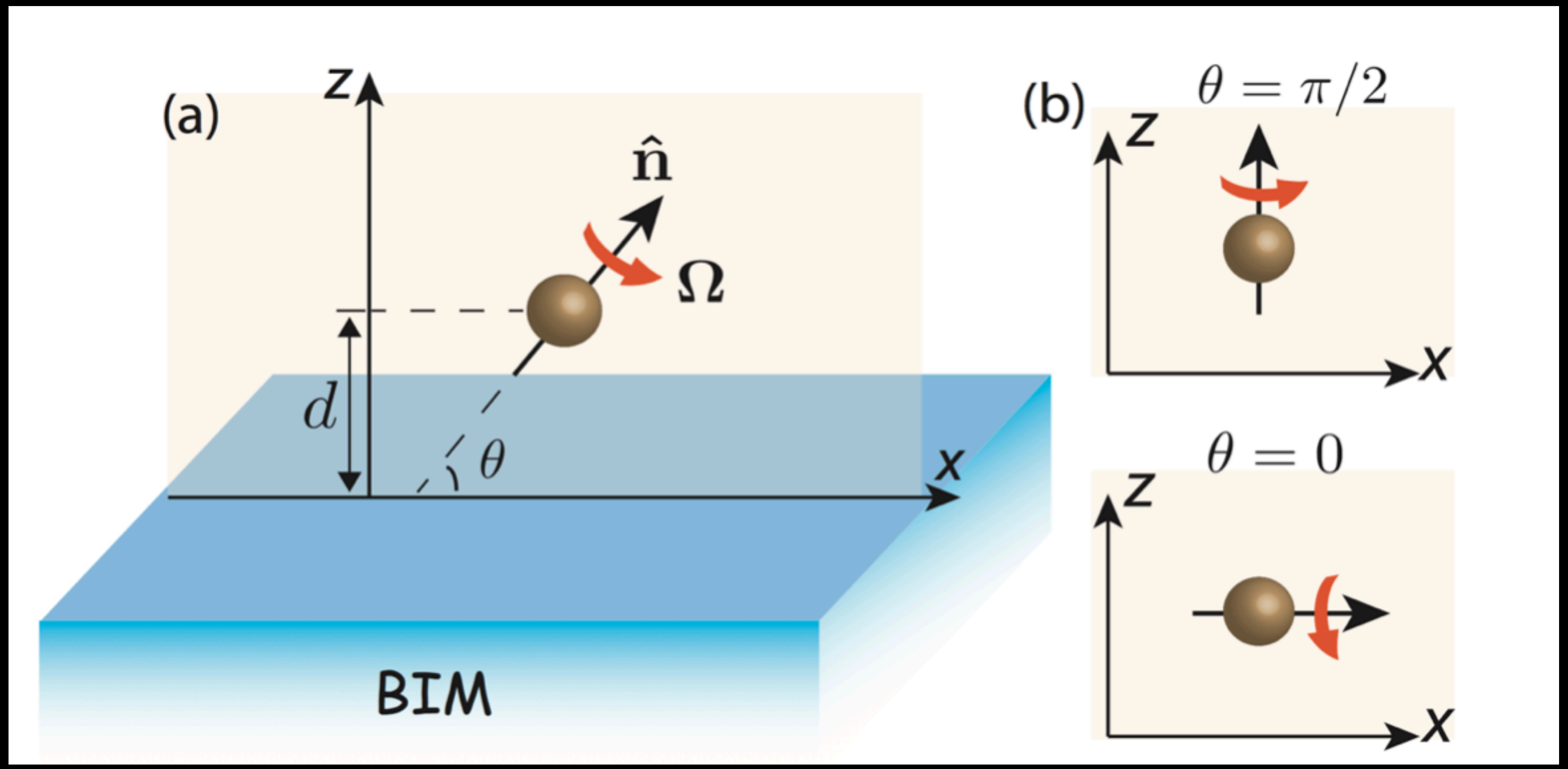


$$\mathbf{D} = \epsilon\mathbf{E} + (\chi - i\kappa)\sqrt{\epsilon_0\mu_0}\mathbf{H}$$
$$\mathbf{B} = \mu\mathbf{H} + (\chi + i\kappa)\sqrt{\epsilon_0\mu_0}\mathbf{E}$$

*Qing-Dong Jiang and Frank Wilczek,*
*Phys. Rev. B 99, 165402 (2019)*

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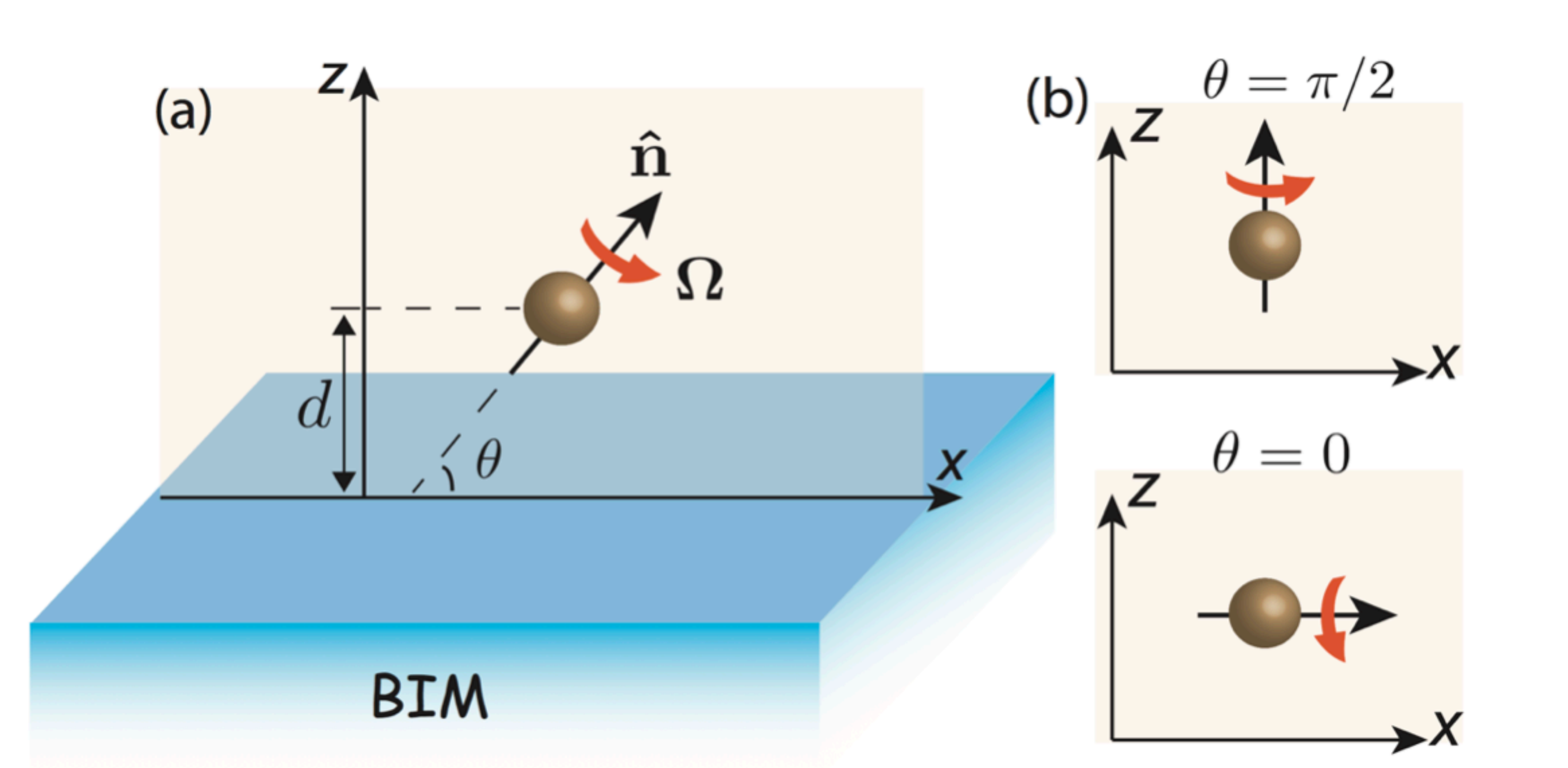


$$F_{\hat{n}} = \langle p_i^{fl}(t)\partial_{\hat{n}} E_i^{ind}(\mathbf{r}_0, t)\rangle + \langle p_i^{ind}(t)\partial_{\hat{n}} E_i^{fl}(\mathbf{r}_0, t)\rangle$$

$$E_i^{ind}(\mathbf{r}, \omega) = G_{ij}(\mathbf{r}, \mathbf{r}_0, \omega) p_j^{fl}(\omega)$$
$$p_i^{ind}(\omega) = \alpha_{ij}(\omega) E_j^{fl}(\mathbf{r}_0, \omega),$$

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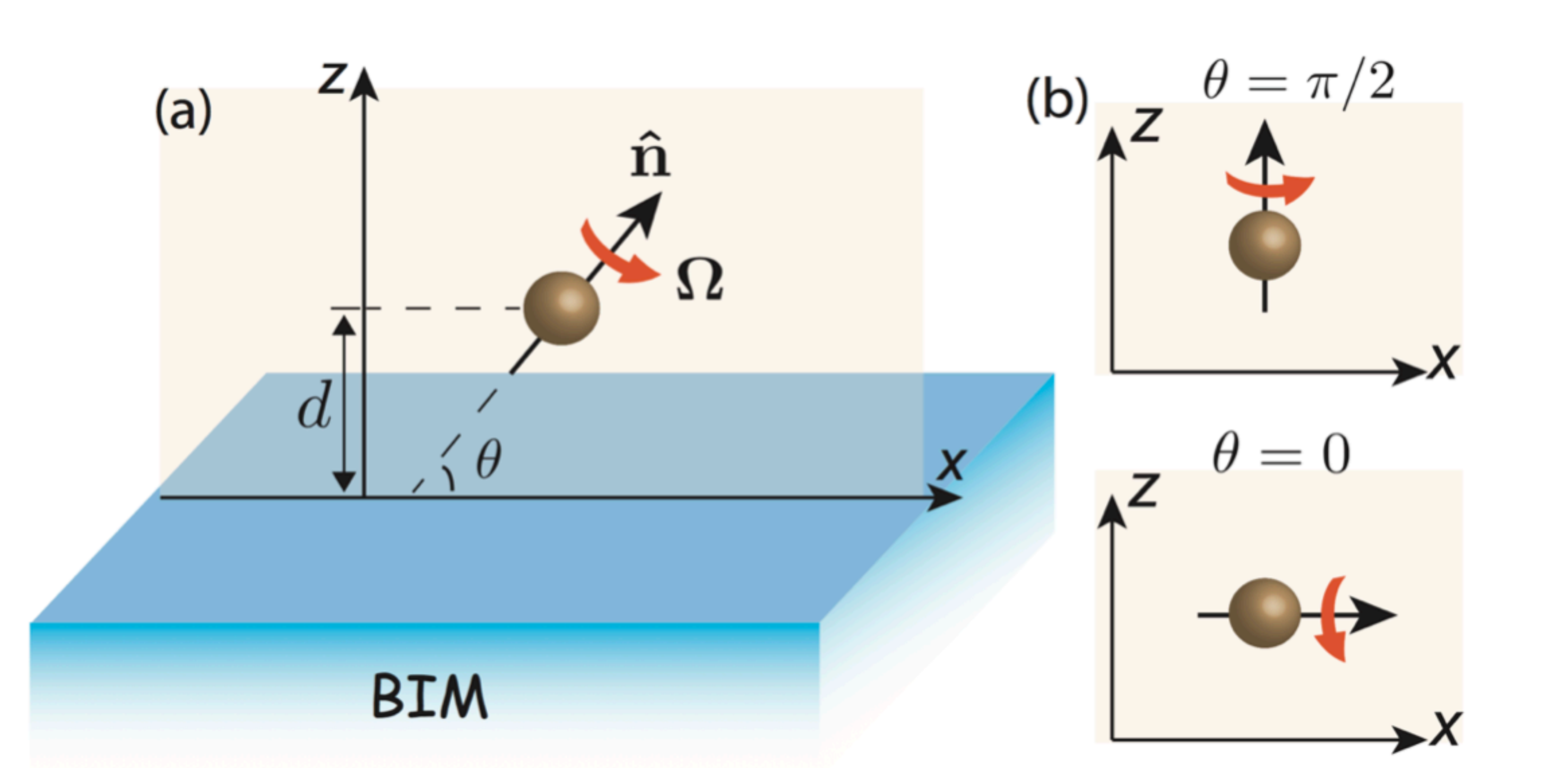


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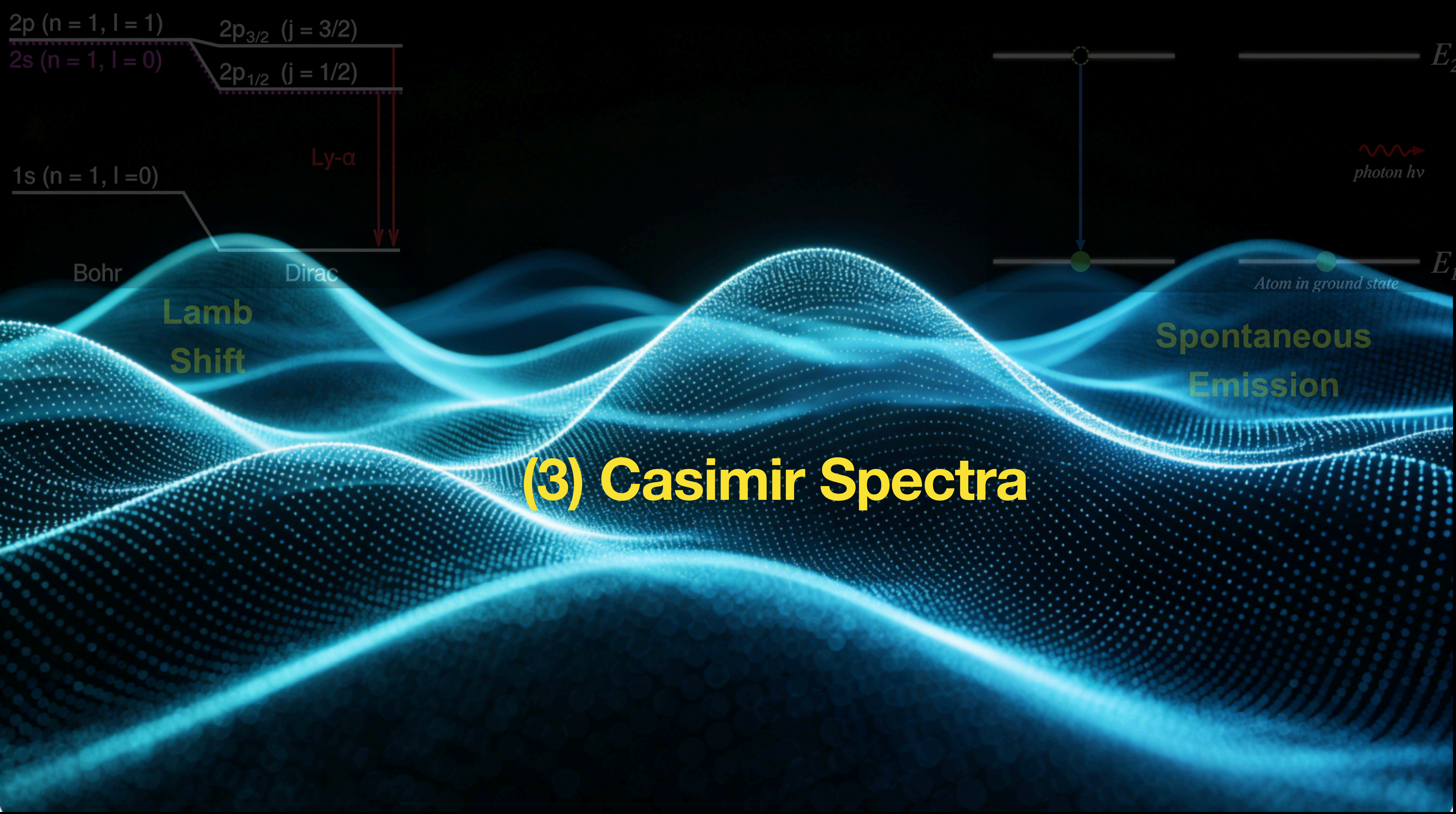
$$F_{x/z}(\Omega) = \frac{\hbar}{\pi}\int_0^\infty d\omega\,\mathrm{Im}\left\{\Sigma_{x/z}\right\} \times \left[\mathrm{Im}\,\alpha(\omega_+)N(\omega_+) - \mathrm{Im}\,\alpha(\omega_-)N(\omega_-)\right]$$

*Qing-Dong Jiang and Frank Wilczek, Phys. Rev. B 99, 165402 (2019)*

We find a dissipation-less Casimir friction

- we termed it axial Casimir force


2p (n = 1, l = 1)
2s (n = 1, l = 0)
2p3/2 (j = 3/2)
2p1/2 (j = 1/2)
Ly-α
1s (n = 1, l =0)
Bohr
Dirac
Lamb Shift
E2
E1
photon hv
Atom in ground state
Spontaneous Emission
(3) Casimir Spectra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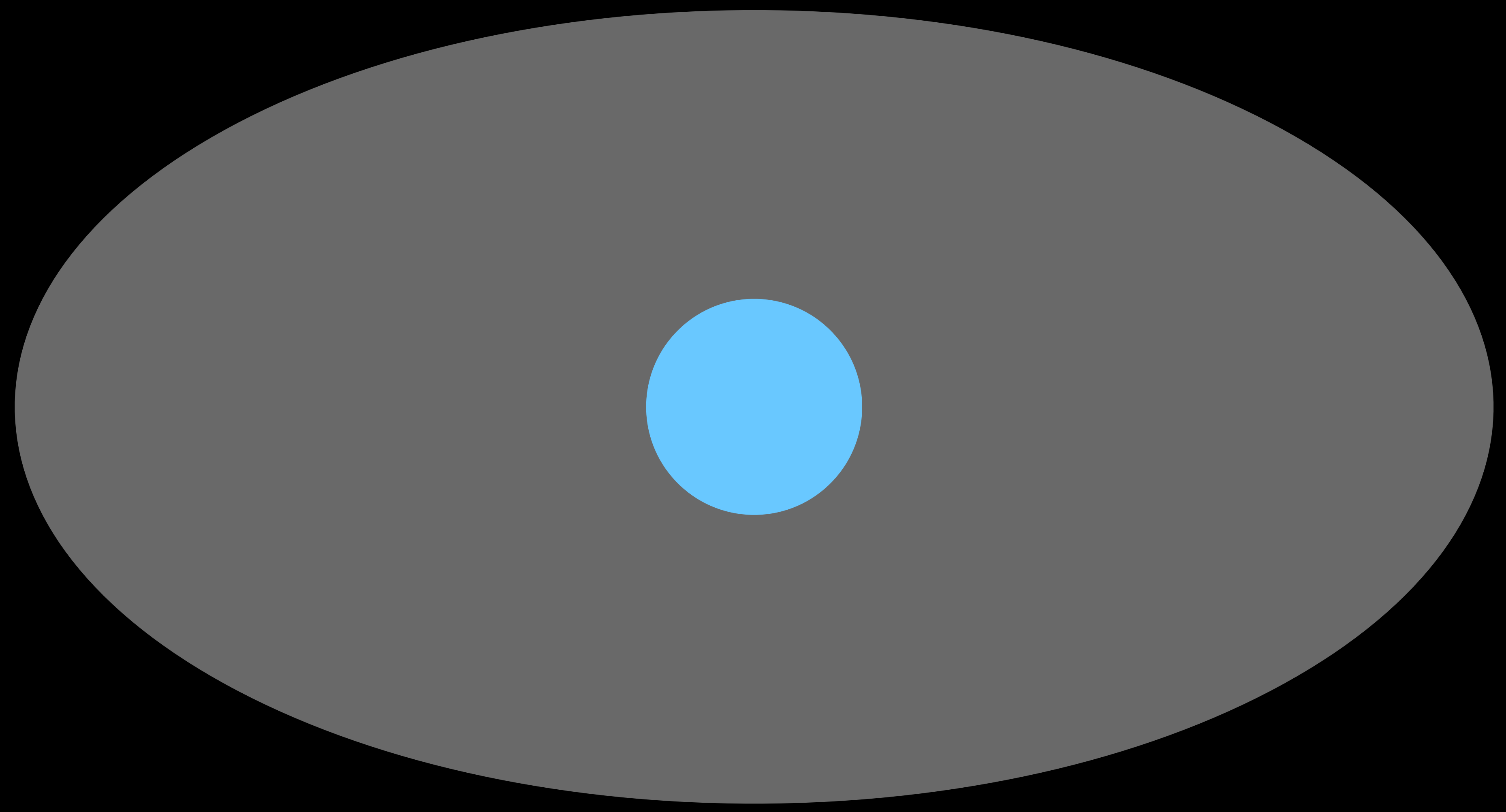
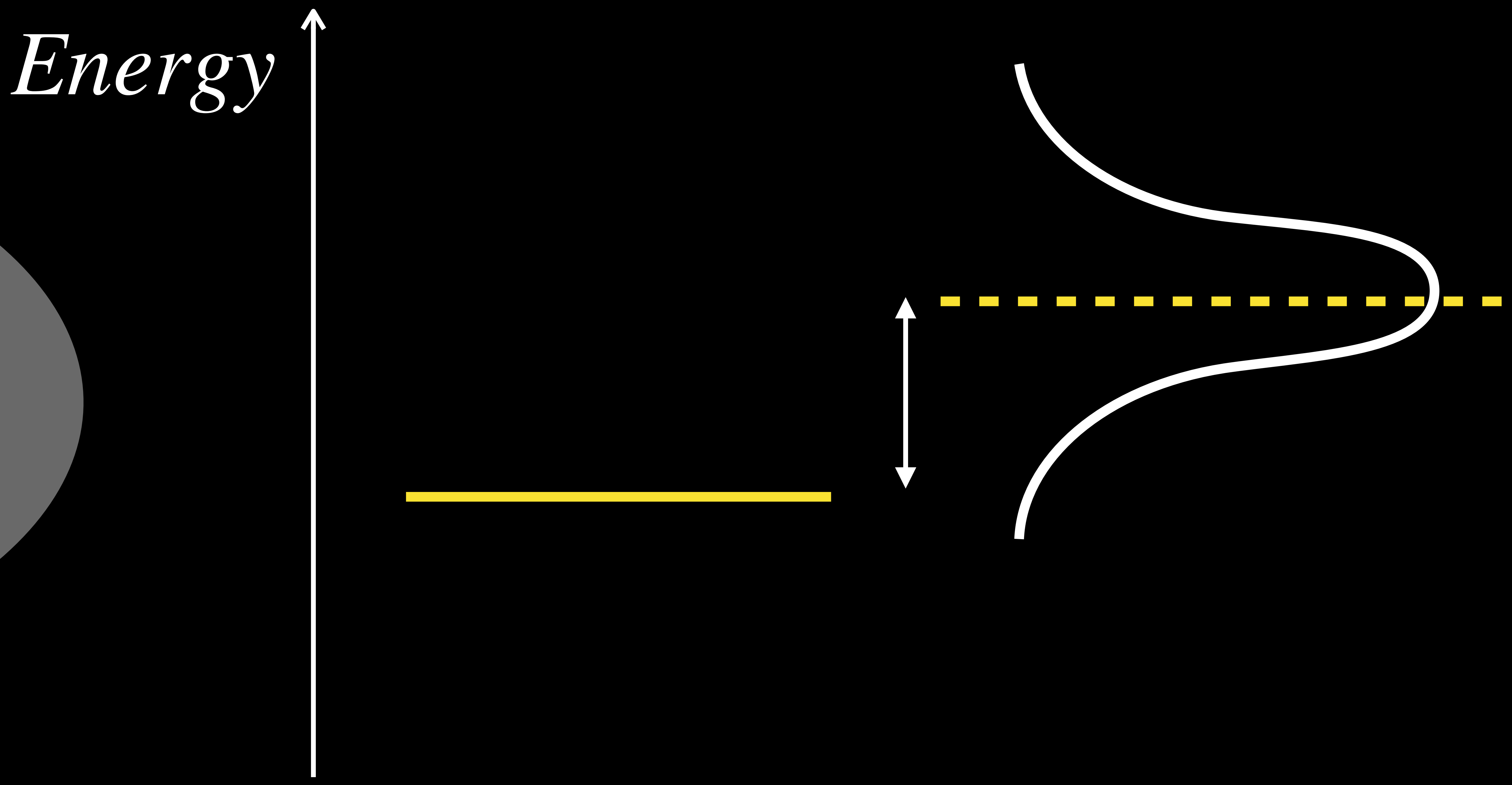
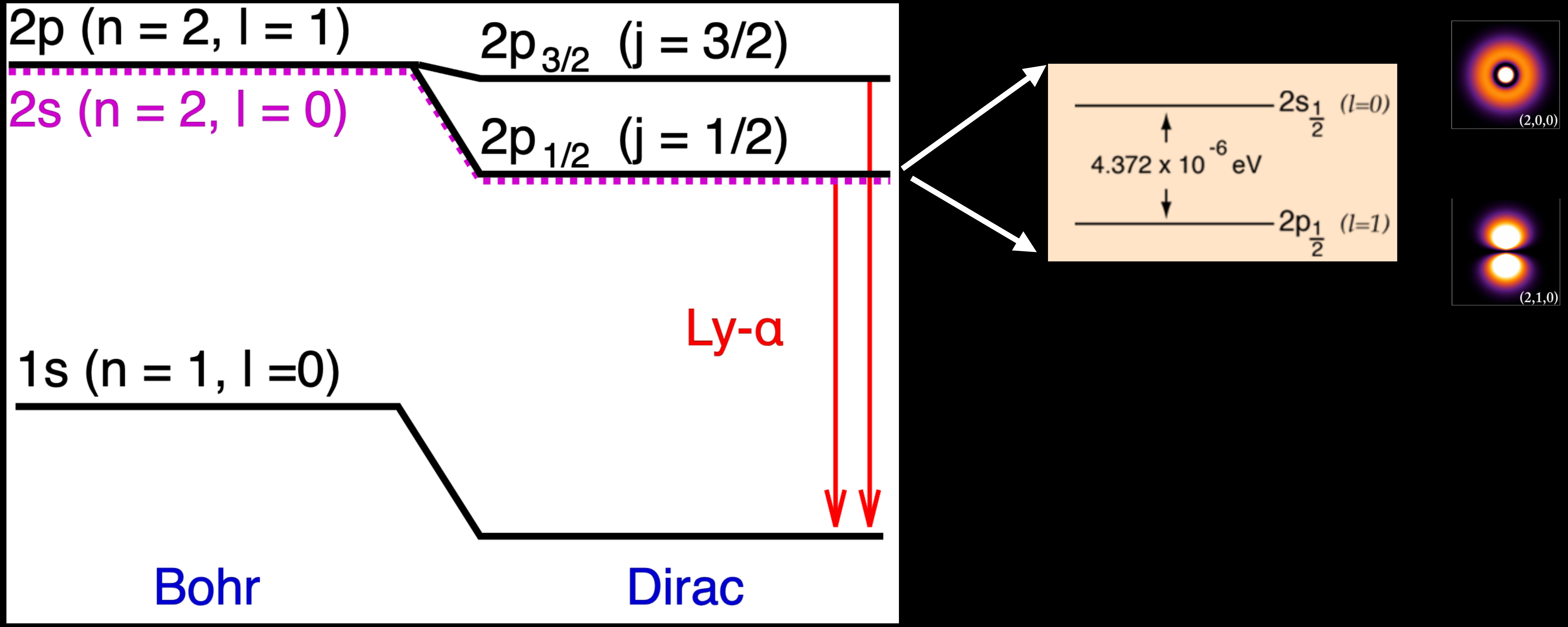
# Outline



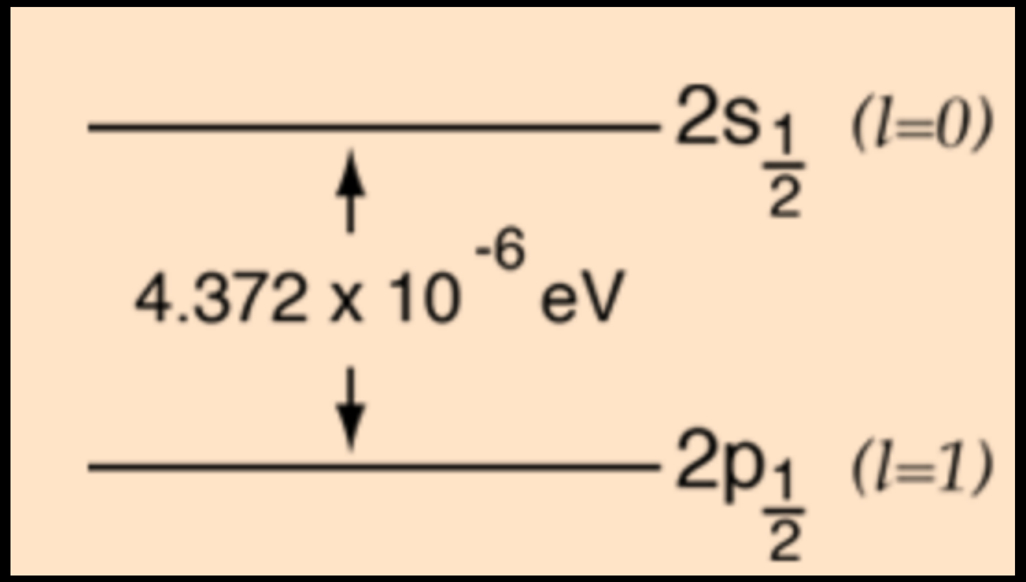


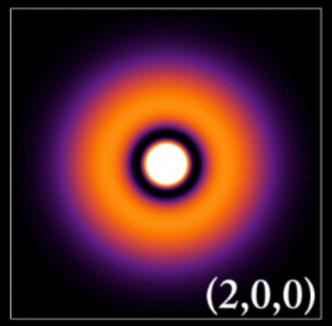

**Spectral shift, Spectral broadening**

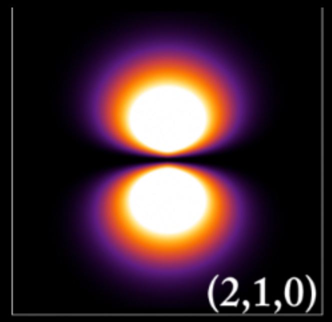
# Introduction to Lamb shift

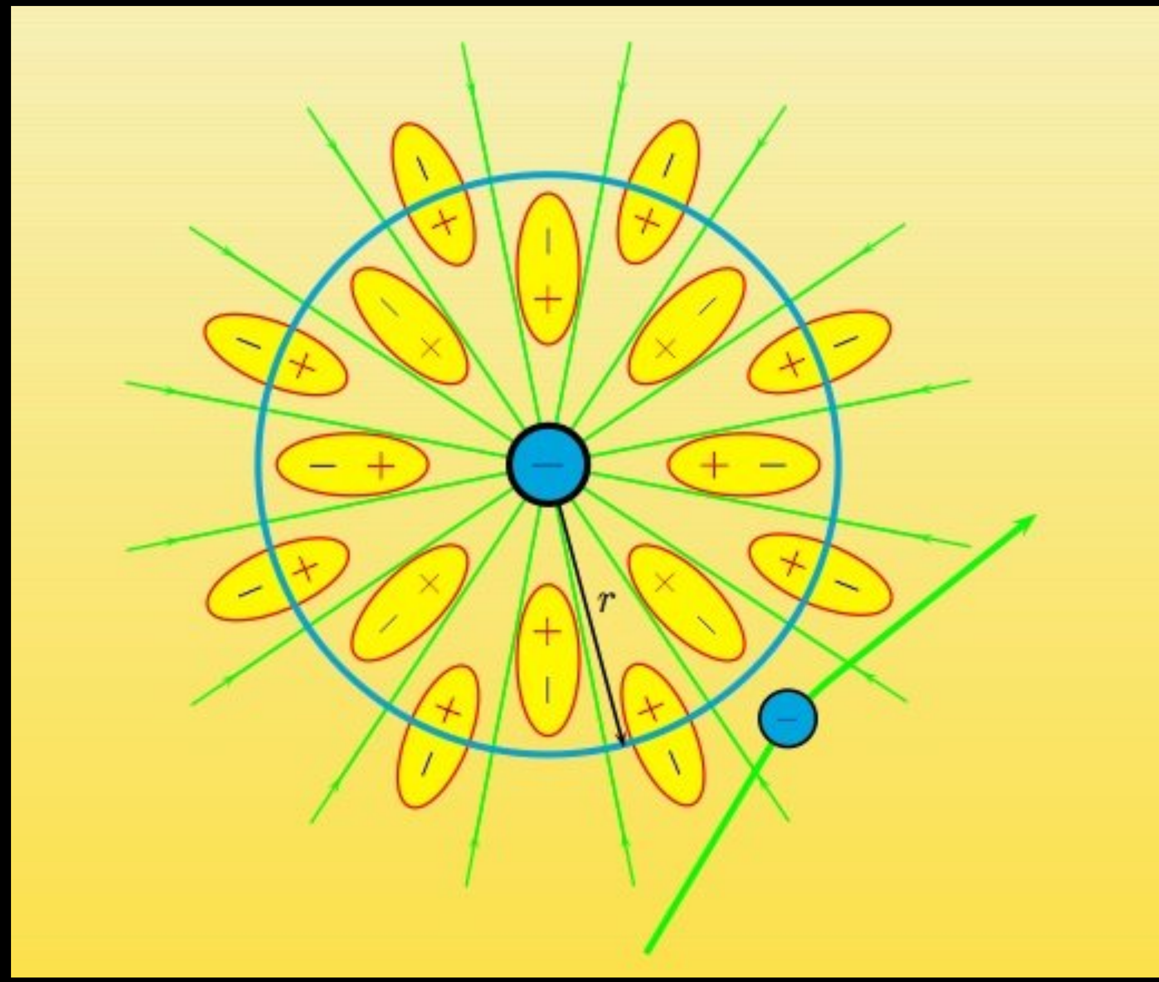


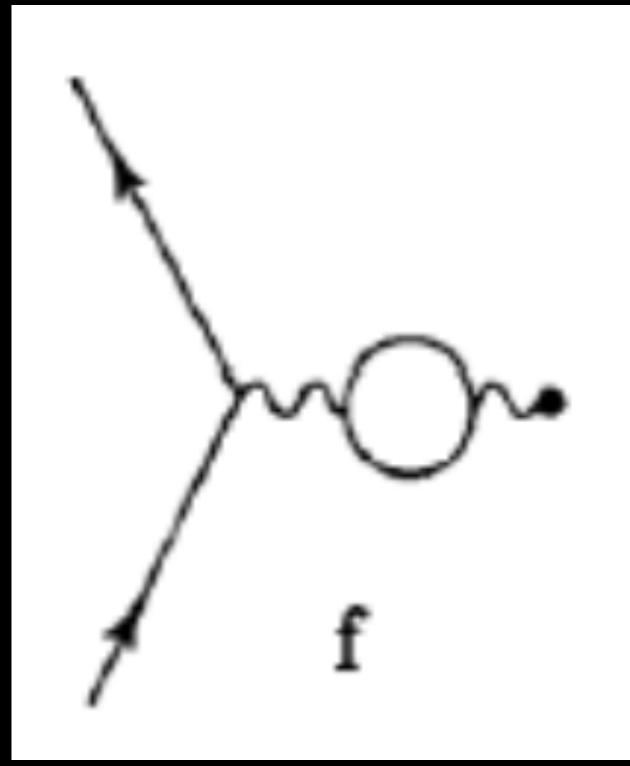
# Introduction to Lamb shift: Feynman diagram



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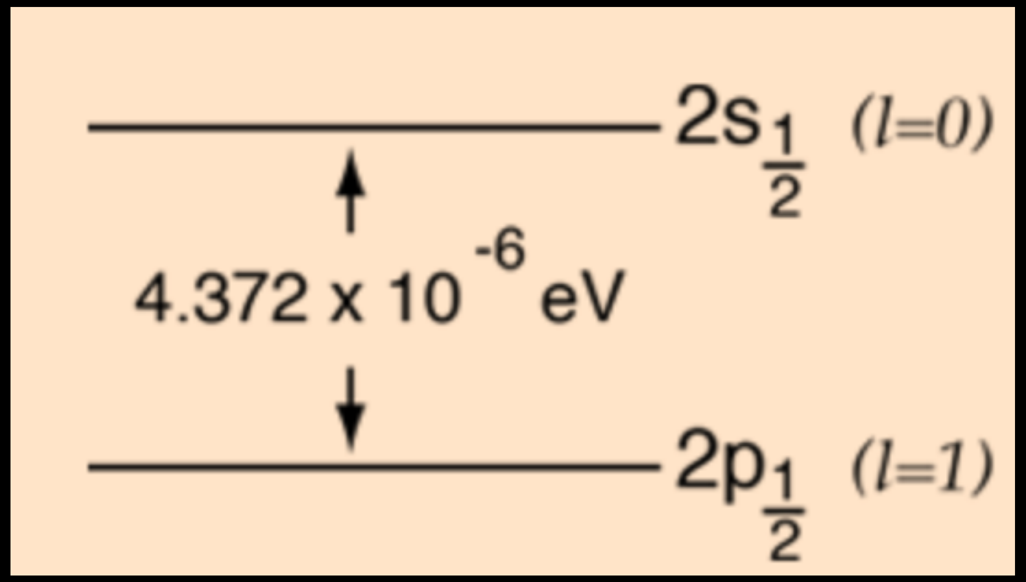
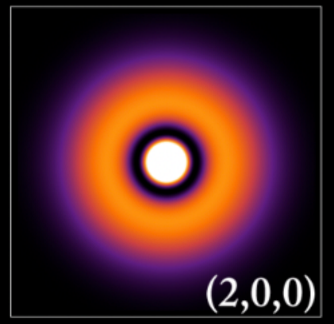
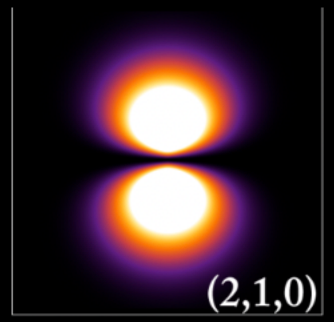
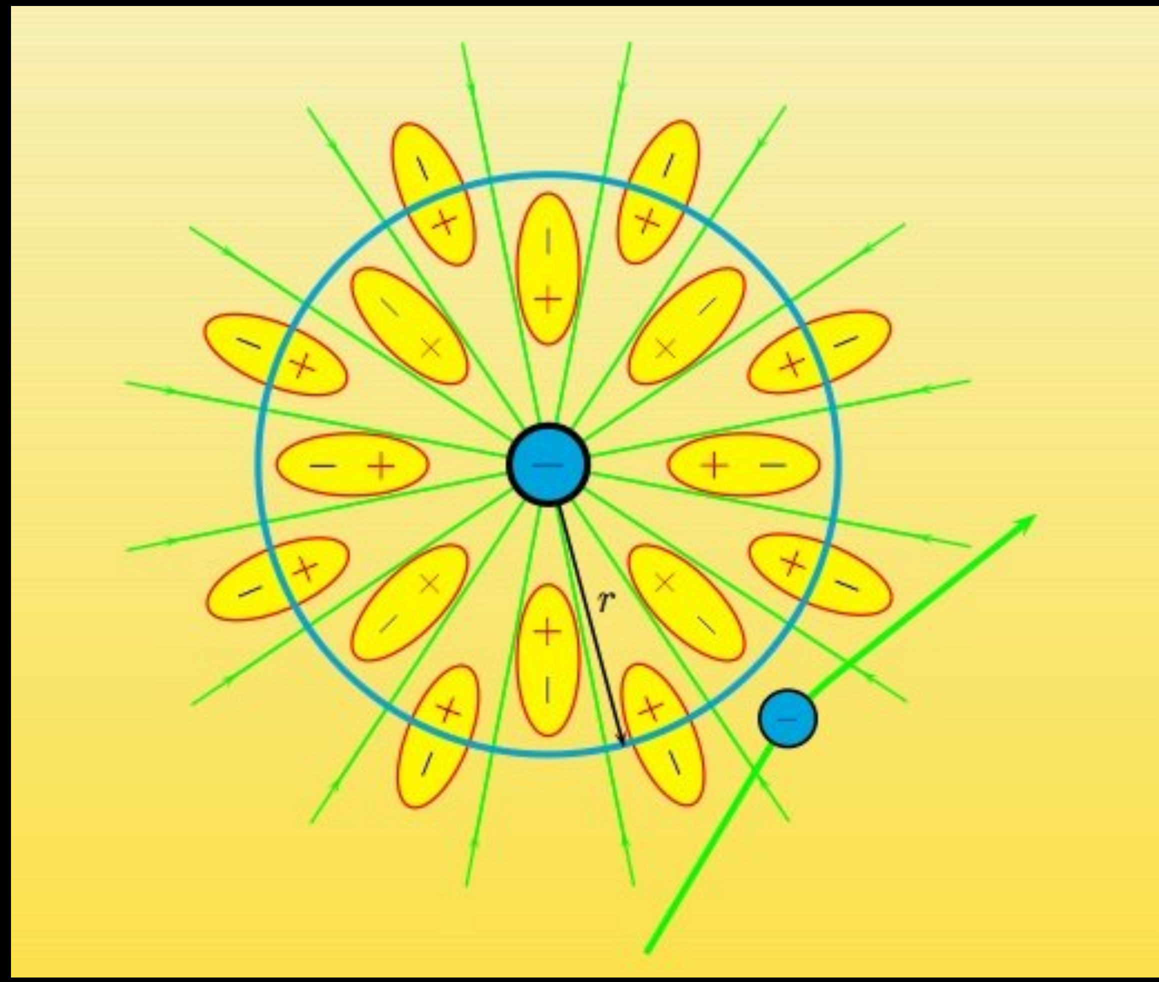
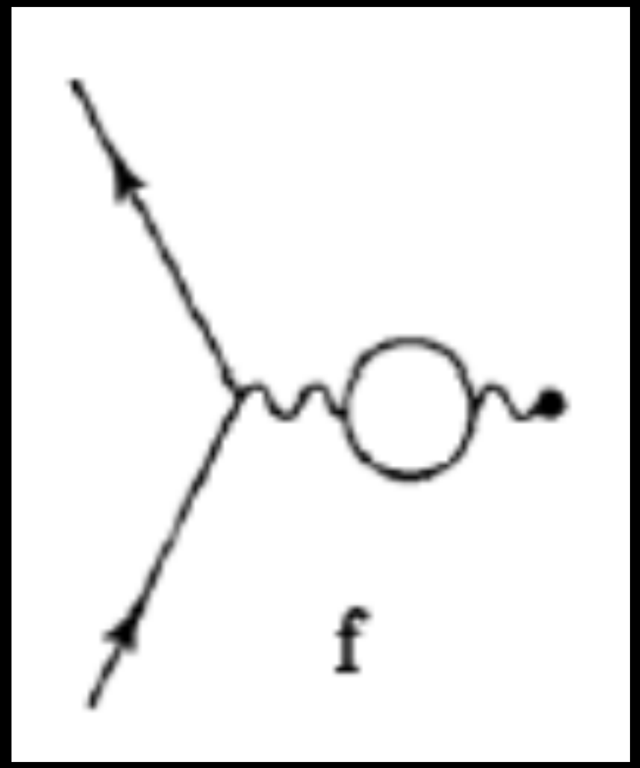


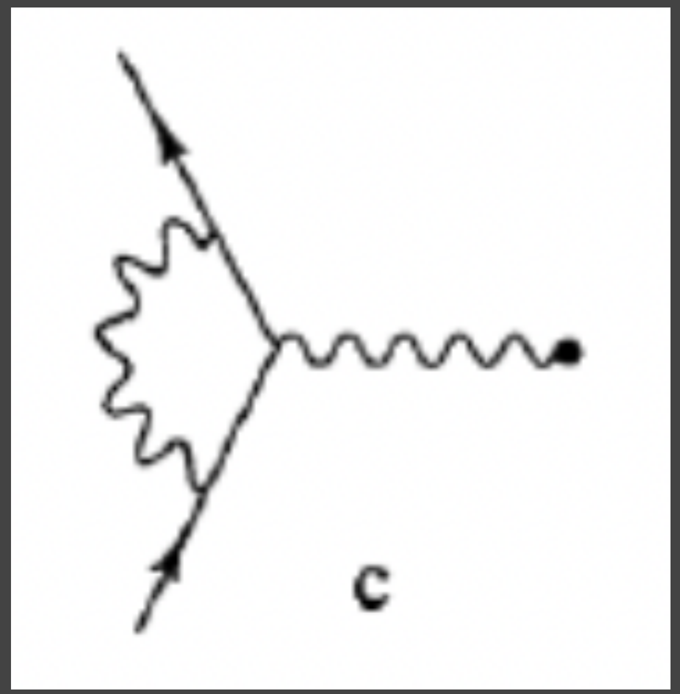






$$\Delta V = V(\vec{r} + \delta\vec{r}) - V(\vec{r}) = \delta\vec{r} \cdot \nabla V(\vec{r}) + \frac{1}{2}(\delta\vec{r} \cdot \nabla)^2 V(\vec{r}) + \cdots$$

$$\langle \delta\vec{r} \rangle_{\text{vac}} = 0,$$

$$\langle (\delta\vec{r} \cdot \nabla)^2 \rangle_{\text{vac}} = \frac{1}{3} \langle (\delta\vec{r})^2 \rangle_{\text{vac}} \nabla^2.$$



# Hans Bethe's 2nd order perturbation

$$\mathrm{Re}\left(\Delta_i^{(2)}\right) = Pr.\sum_{m\neq i}\frac{\left|V_{mi}\right|^2}{E_i - E_m}$$

$$\mathrm{Im}\left(\Delta_i^{(2)}\right) = = -\frac{\hbar}{2}\sum_{m+i} W_{i\to m}$$ (Fermi's golden rule)

$$H = p^2 + x^2 + \lambda x$$

$\langle G|\lambda x|G\rangle = 0$ First order vanishes Second

# Lamb shift: Feynman's derivation

A box of volume V containing N identical atoms per unit volume.

The frequencies of the allowed field modes in the box are:

$\omega \to \omega/n(\omega)$ Refractive index $n(\omega) \cong 1 + 2\pi N\alpha(\omega)$

Change in the zero-point energy:

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$$\Delta E = \sum_{\mathbf{k},\lambda} \left[ \frac{1}{2} \frac{\hbar \omega_k}{n(\omega_k)} - \frac{1}{2} \hbar \omega_k \right] \cong - \sum_{\mathbf{k},\lambda} \left[ n(\omega_k) - 1 \right] \frac{1}{2} \hbar \omega_k = -\pi \hbar N \sum_{\mathbf{k},\lambda} \omega_k \alpha(\omega_k)$$

$$\sum_{\mathbf{k},\lambda} \to (V/8\pi^3) \sum_{\lambda} \int d^3k$$

$$(N * V = 1)$$

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$$\Delta E = - \frac{\hbar}{\pi c^3} \int d\omega \omega^3 \alpha(\omega) \longleftarrow \sum_{\mathbf{k},\lambda} \to (V/8\pi^3) \sum_{\lambda} \int d^3k$$

$$(N * V = 1)$$

# Lamb shift: vacuum Stark shift

$$W = -(1/2)\alpha \mathbf{E}^2 .$$

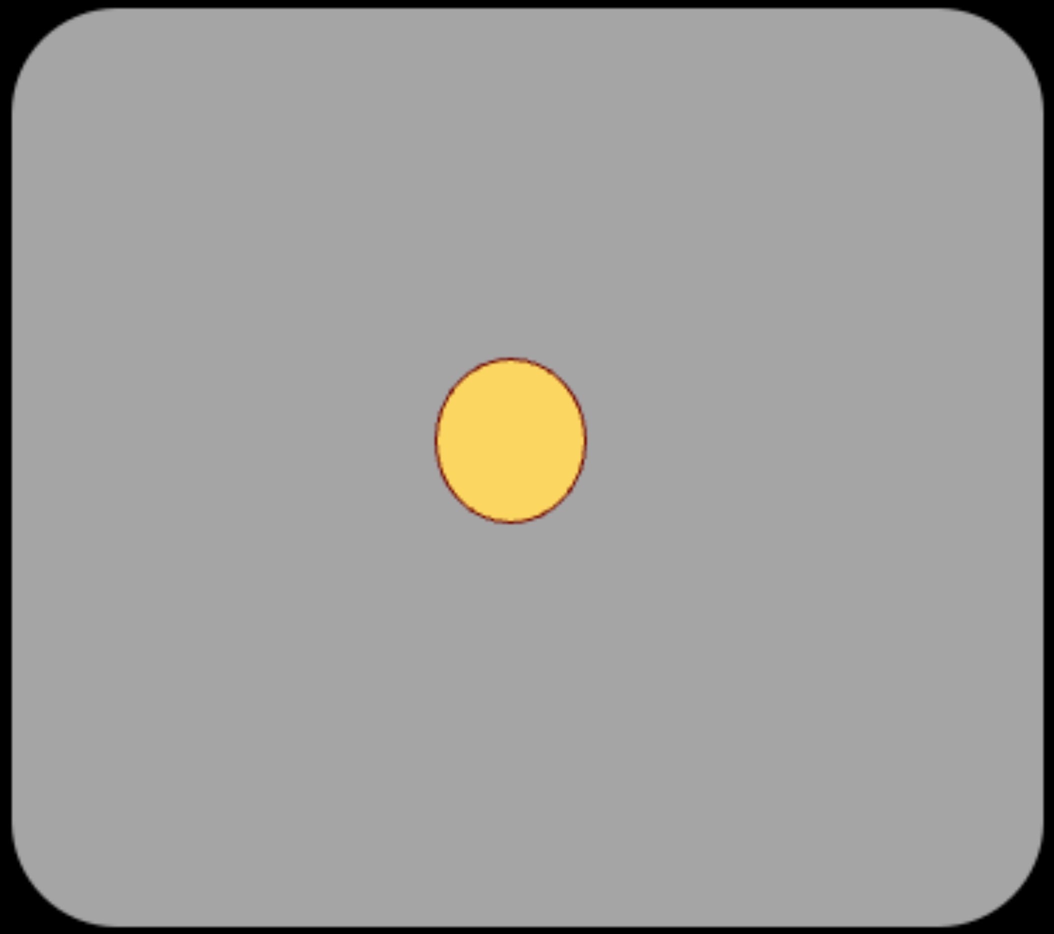
$$(1/4\pi)\mathbf{E}^2 = \rho_0(\omega)d\omega$$

$$\rho_0(\omega) = \frac{\omega^2}{\pi^2 c^3}\left(\frac{1}{2}\hbar\omega\right)$$

the Lamb shift can be regarded as a Stark shift caused by the vacuum electric field!

# Lamb shift: my derivation

$$H = \frac{1}{2m}(p - q\,A)^2 + V(\mathbf{r}) + \hbar\omega_c a^\dagger a$$



$$A = A_0\left(\boldsymbol{\varepsilon}^* a^\dagger + \boldsymbol{\varepsilon} a\right) \quad A_0 = \sqrt{\frac{\hbar}{2\epsilon_0 V \omega_c}}$$

$$\xi = \frac{g}{1+g^2}$$

$$H \rightarrow \quad H' = \frac{1}{m_{\text{eff}}} p^2 + V(\mathbf{r} + \xi\,\vec{\pi}) + \hbar\omega_{\text{eff}} a^\dagger a$$

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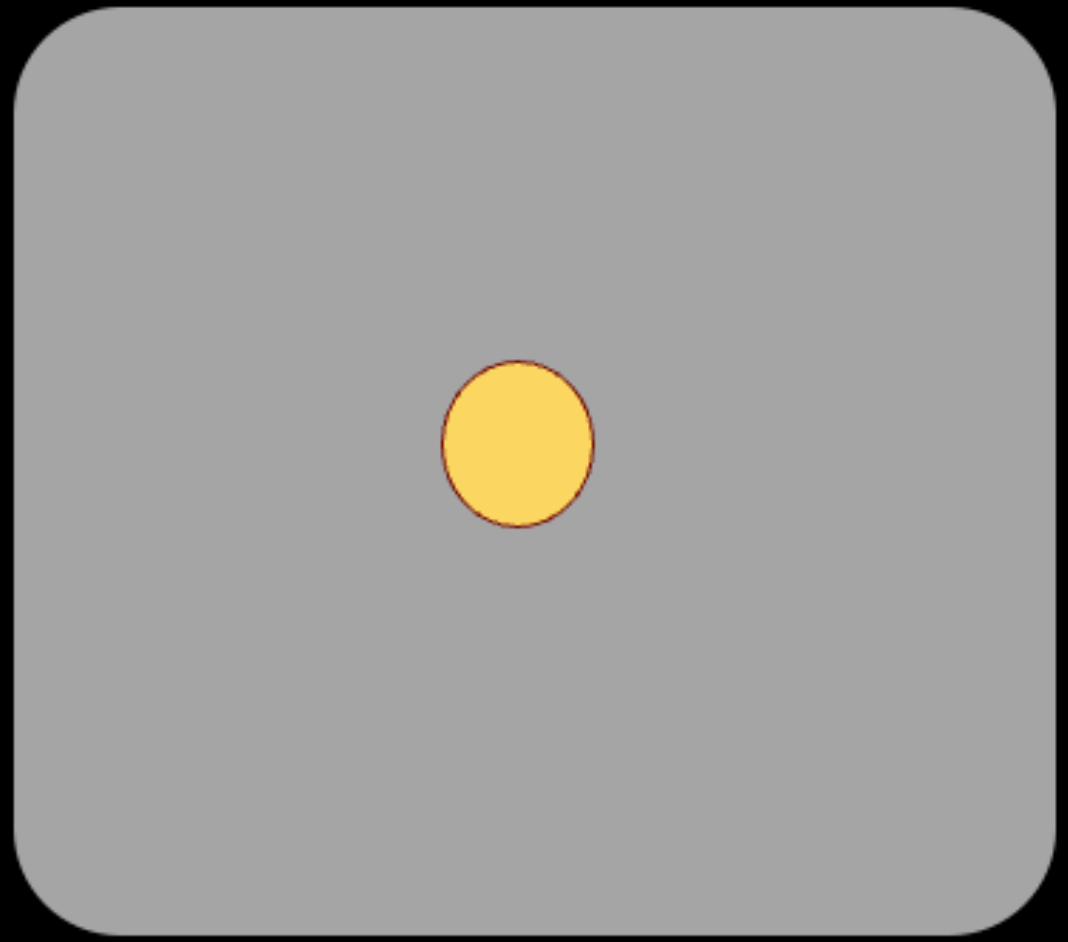

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$$\xi = \frac{g}{1+g^2}$$

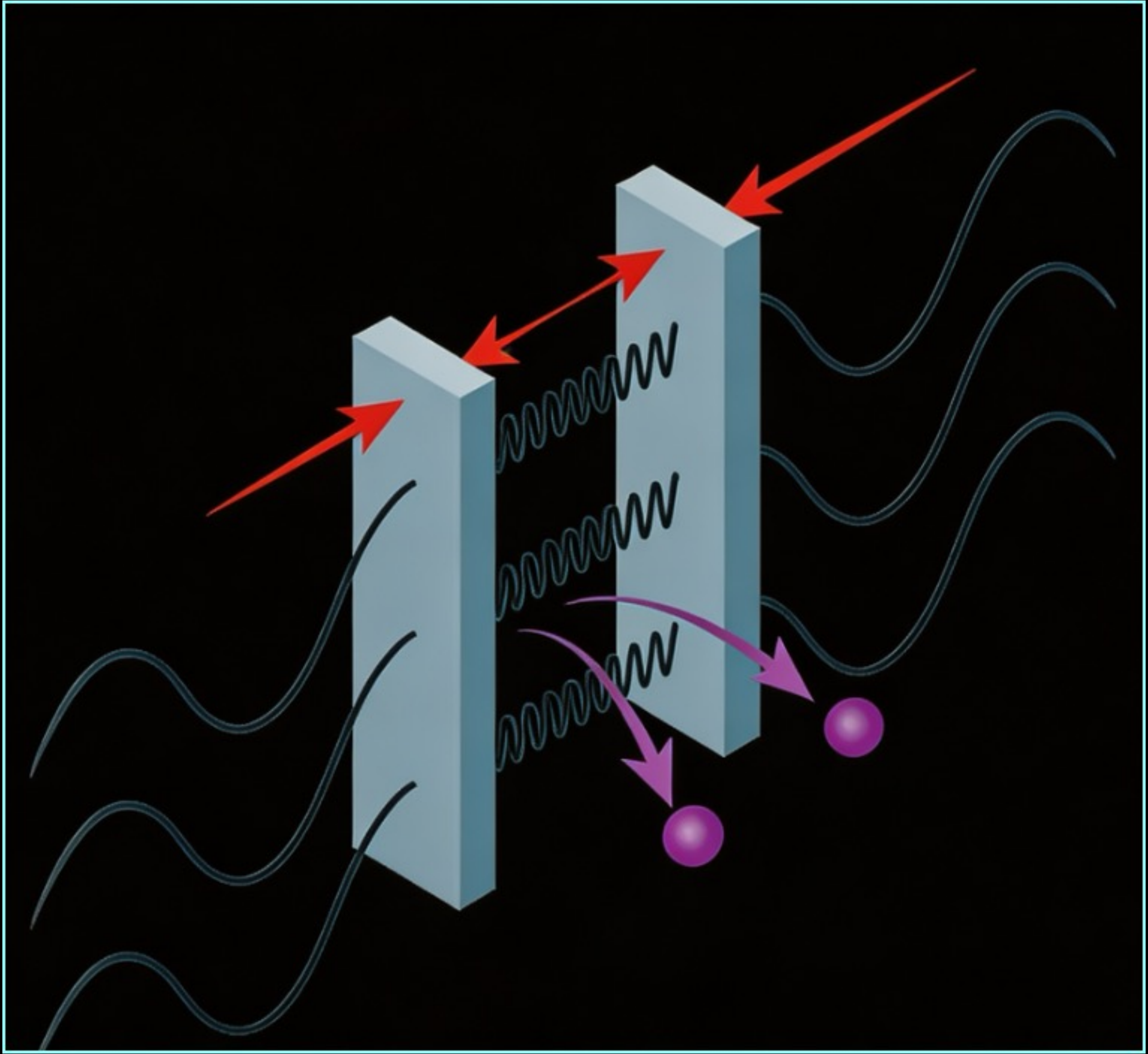
$$H \to \quad H' = \frac{1}{m_{\mathrm{eff}}} p^2 + V(\mathbf{r} + \ \xi\,\vec{\pi}) + \hbar\omega_{\mathrm{eff}} a^\dagger a$$

$$\Delta E_n^{\mathrm{CL}} = \frac{\xi^2}{4}\langle\psi_n|\nabla^2 V(r)|\psi_n\rangle$$

QDJ, Phys. Rev. B 111, 205405 (2025)

# (4) Dynamical Casimir Effect

# What's Dynamical Casimir Effect



### 01. Vacuum Fluctuations

The quantum vacuum is not empty, but a sea of virtual photon pairs that constantly pop into and out of existence spontaneously.

### 02. Non-Adiabatic Modulation

Rapidly altering a cavity's boundary (e.g., a vibrating mirror) at high speeds imparts external energy to these fleeting virtual pairs.

### 03. Energy Transfer to Reality

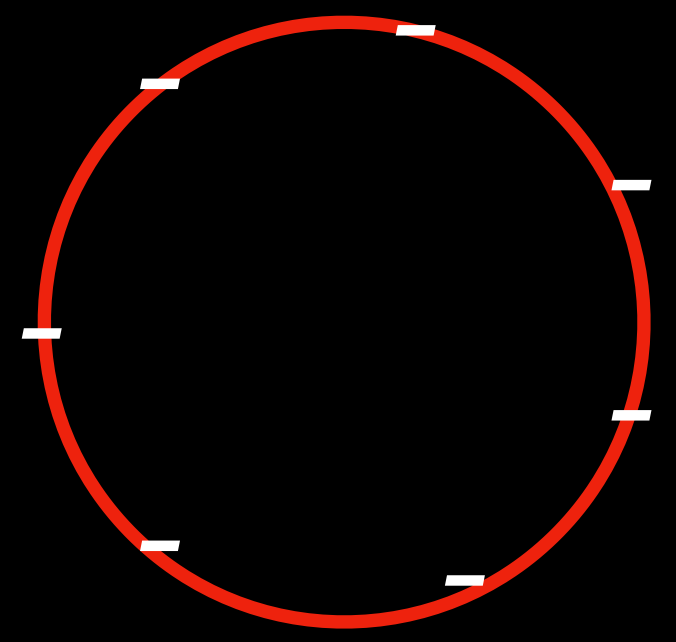
The injected energy "boosts" virtual pairs into **real, detectable photons** that escape the cavity as measurable radiation.

# Puzzles and Outlooks

# Puzzles and outlooks

A story

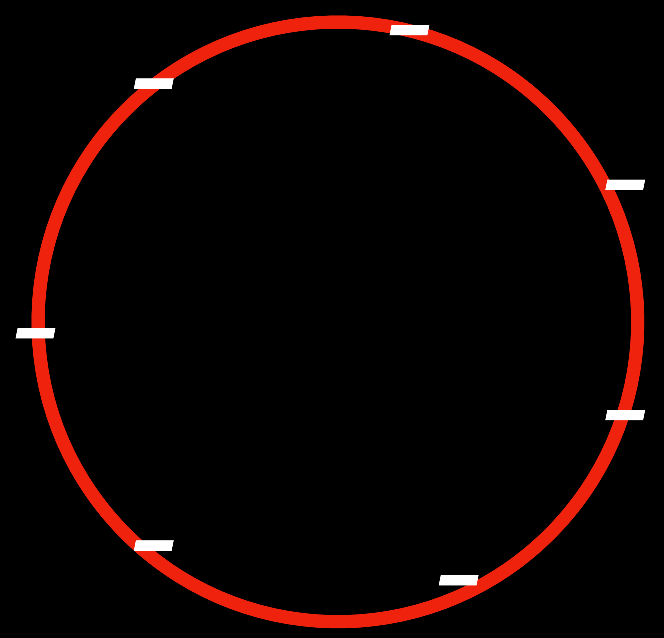
# Semiclassical model of electrons




$$E = \frac{e^2}{2a} - \frac{\eta}{a}\hbar c \qquad \eta = \frac{\alpha}{2} = \frac{1}{2}\frac{e^2}{\hbar c}$$

*Casimir, H. B. G. Physica XIX, 846 (1953)*

# Semiclassical model of electrons



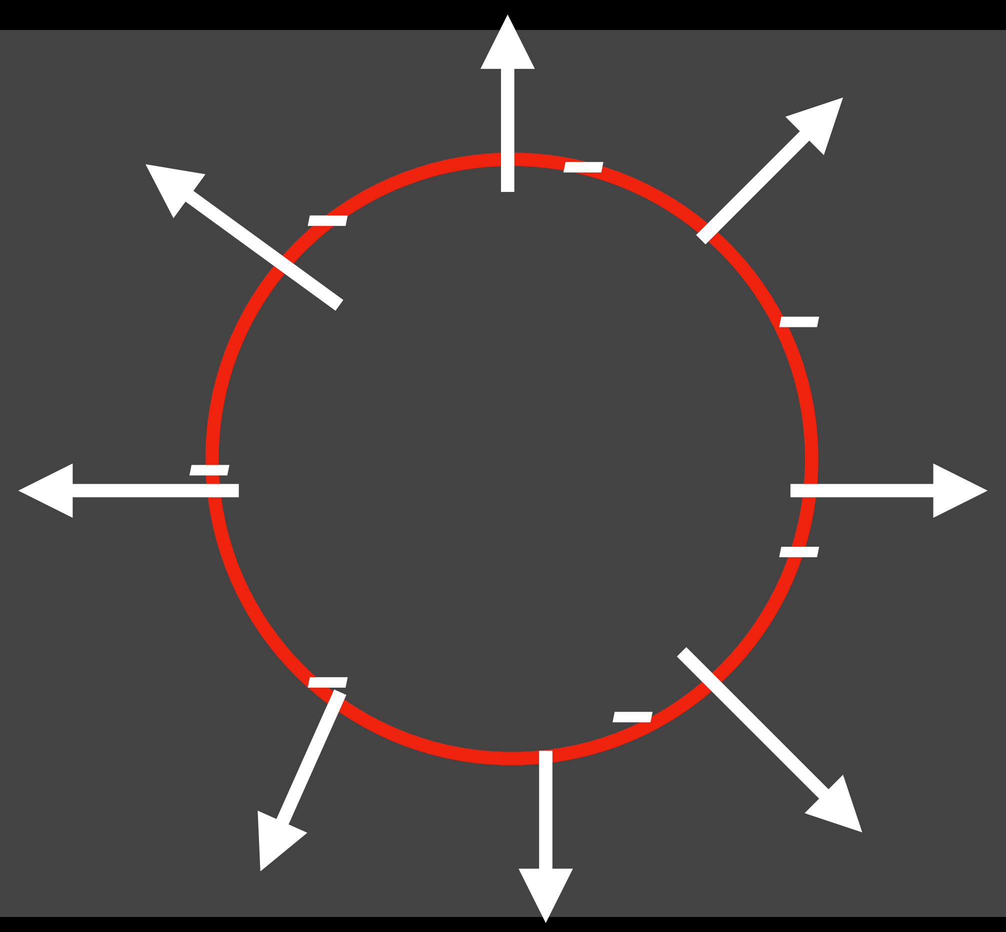
$$E = \frac{e^2}{2a} - \frac{\eta}{a}\hbar c \qquad \eta = \frac{\alpha}{2} = \frac{1}{2}\frac{e^2}{\hbar c}$$

*Casimir, H. B. G. Physica XIX, 846 (1953)*

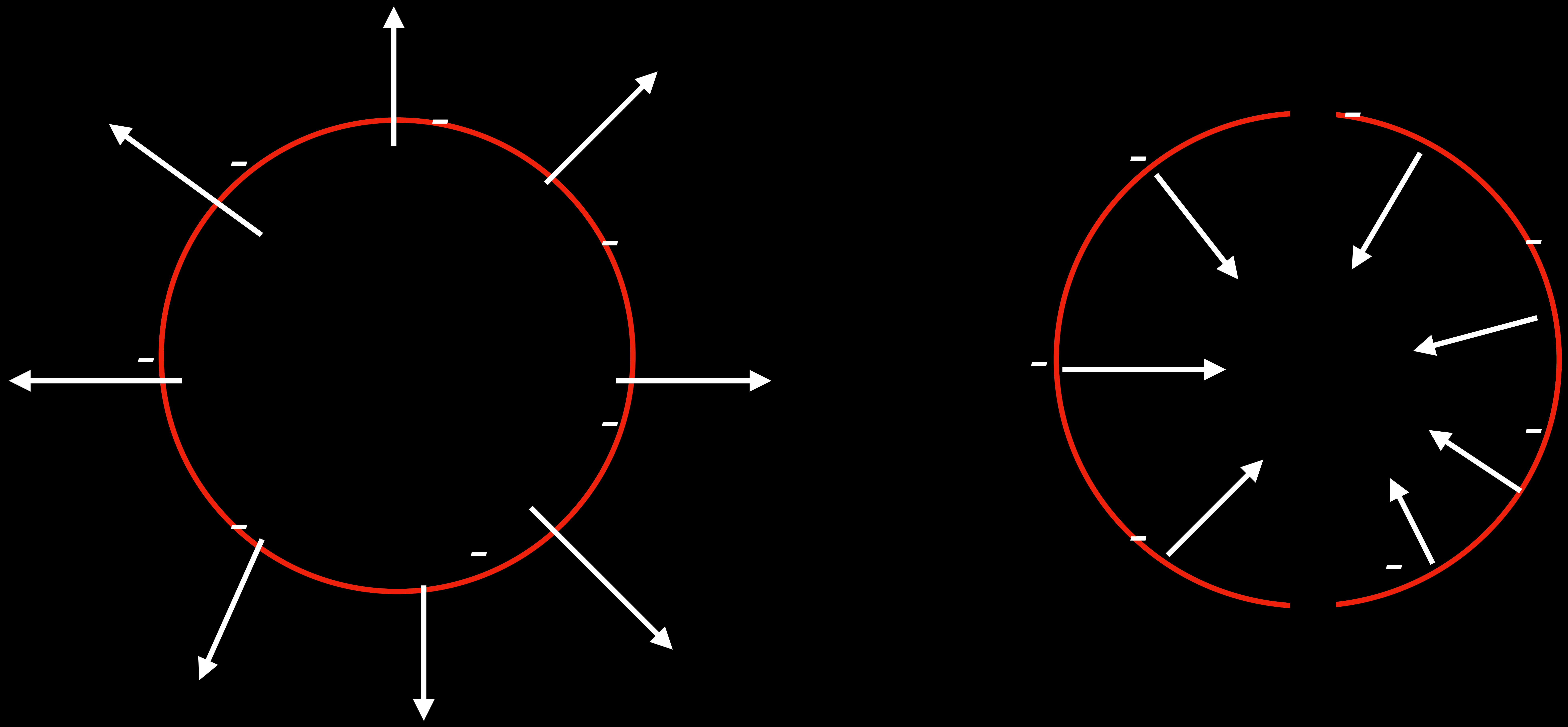
$$\eta = -0.04618$$

**Repulsion !!!**

*T. H. Boyer, Phys. Rev. 174, 1764 (1968)*



# Do you remember the no-go theorem?



## Open question

- the end of lecture 2

***Thank You!***

***Take home message?***

***May the energy be with you!***

# Appendices