



# Quantum Connections in Sweden-16 Summer School

**Less is more:**

The power of vacuum quantum fluctuations

Qing-Dong Jiang



李政道研究所  
TSUNG-DAO LEE INSTITUTE

Quantum Connections in Sweden-16  
Summer School

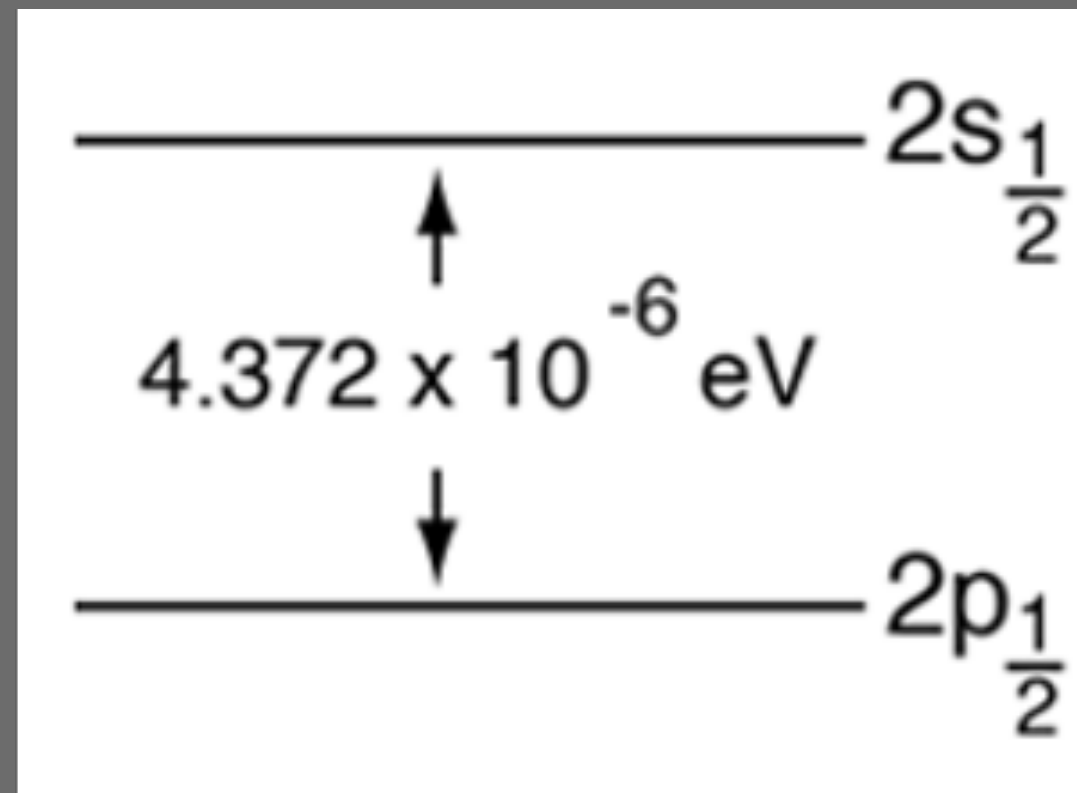
**Lecture 1: Renormalization and Casimir Physics**

**Lecture 2: Casimir Torque, Friction and Spectra**

**Lecture 3: Quantum atmosphere**

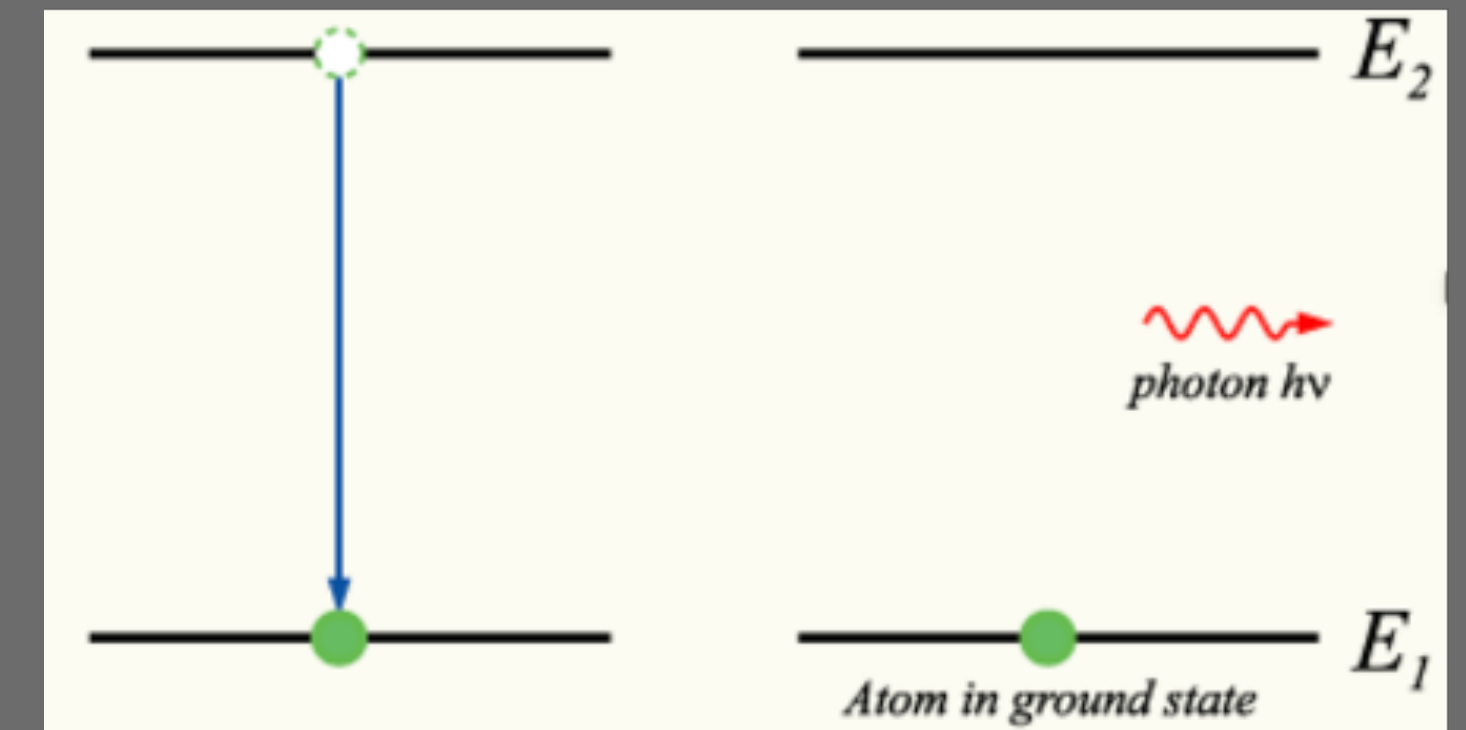
**Lecture 4: Cavity Quantum Materials**

# Visible Vacuum Quantum Fluctuations

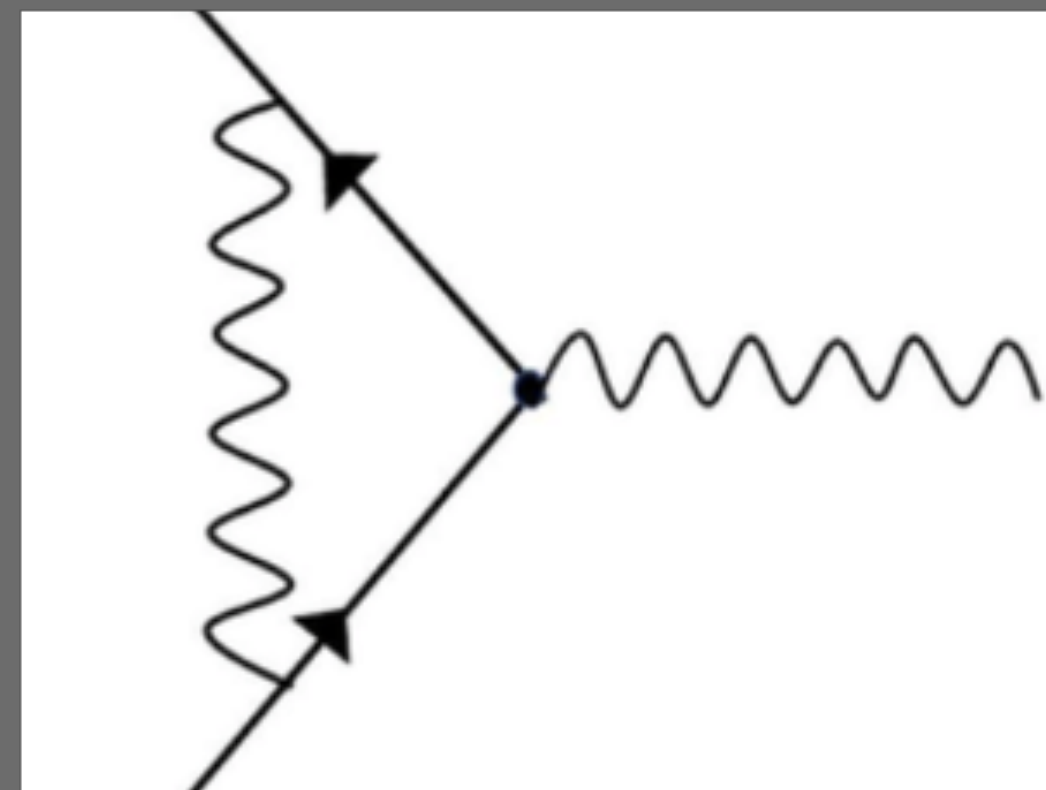
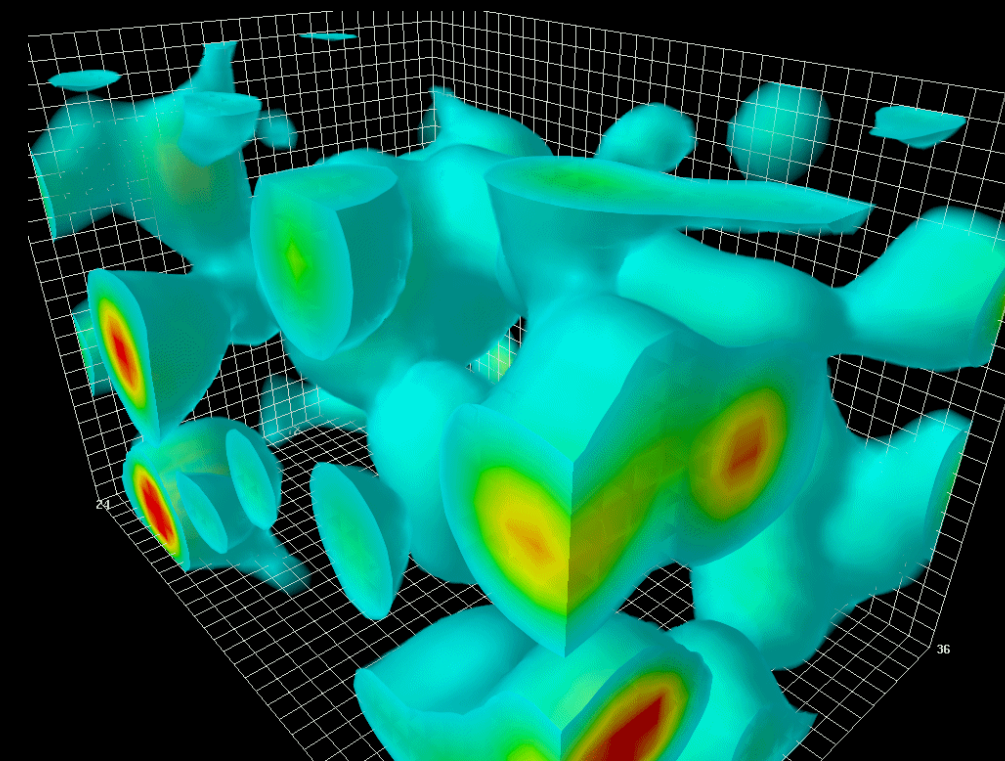


(1955)

Lamb shift

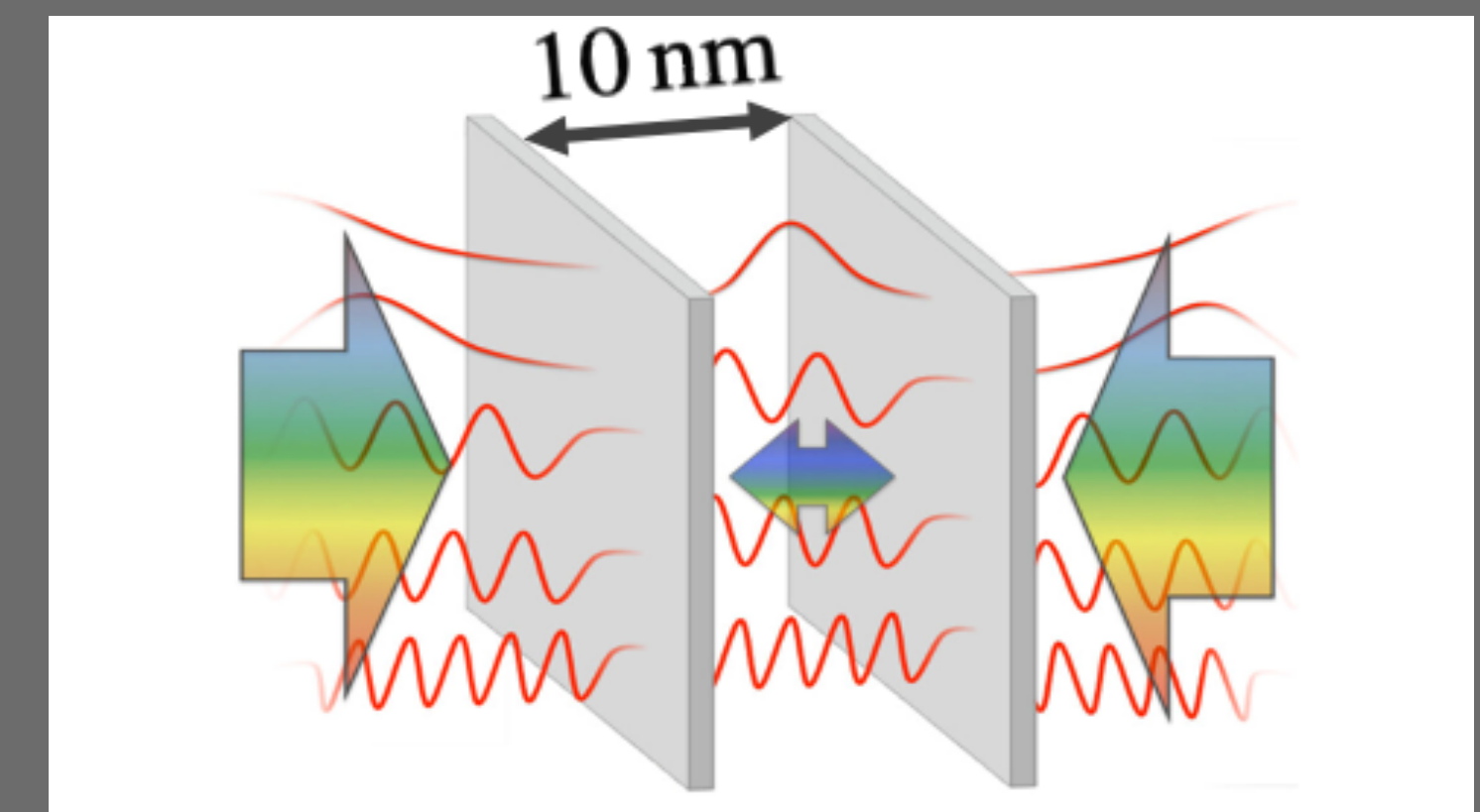


Spontaneous emission



(1955, 1965)

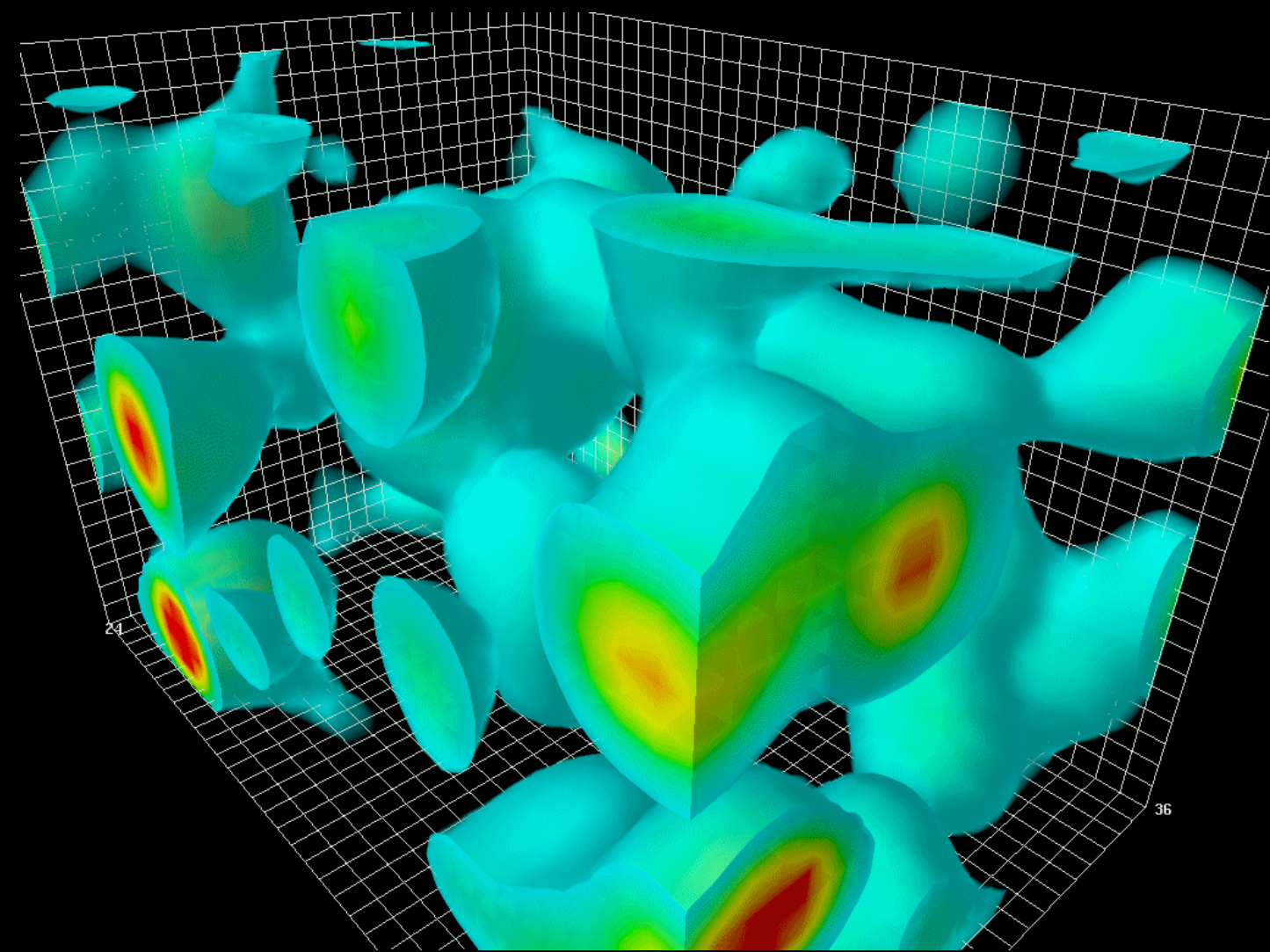
Anomalous magnetic moment



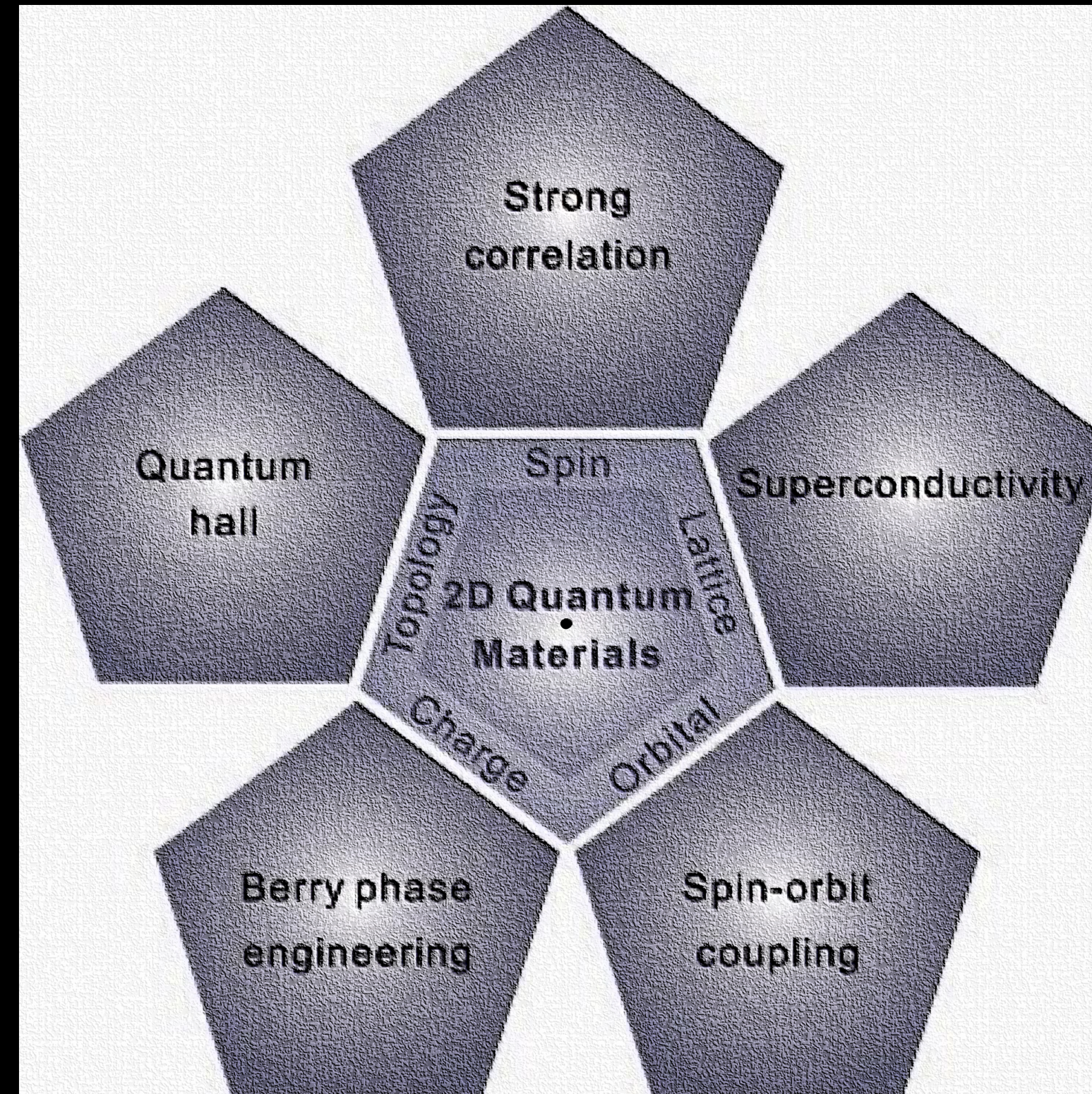
Casimir effect

# How quantum fluctuations can Shape Matter\*

\* npj Nanophotonics 2, 46 (2025)



+



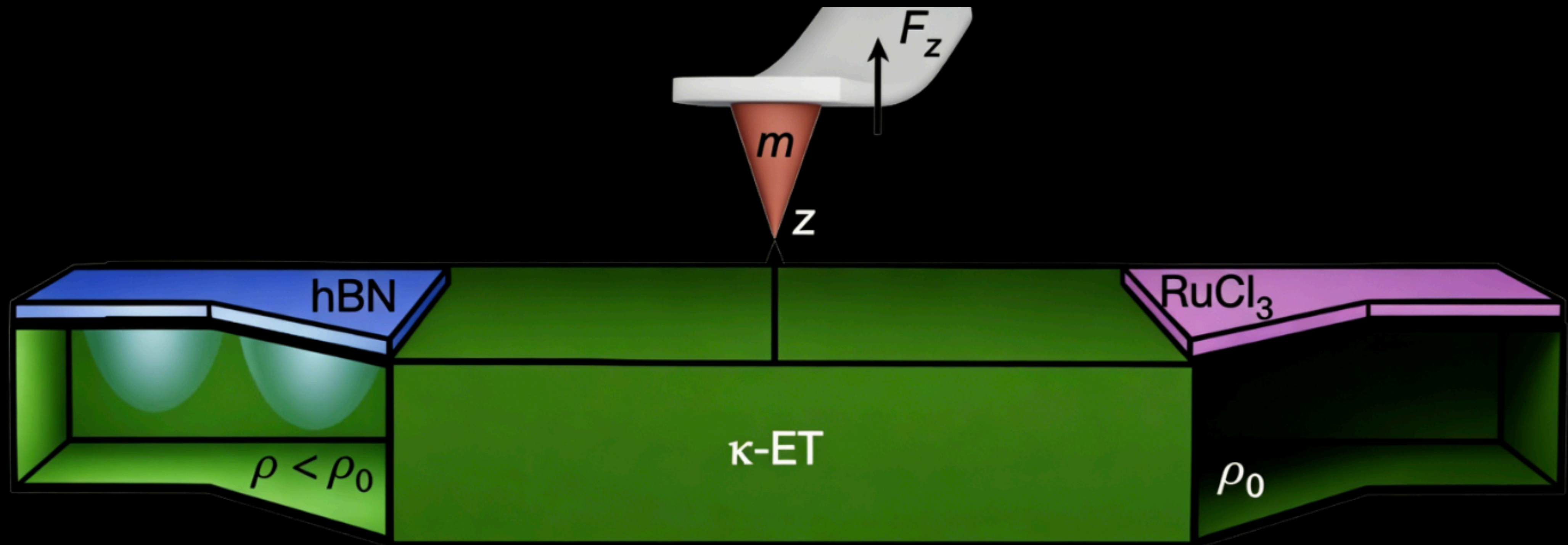
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Quantum Materials? **Symmetry & Topology & Interactions**

# How quantum fluctuations can Shape Matter\*

\* npj Nanophotonics 2, 46 (2025)



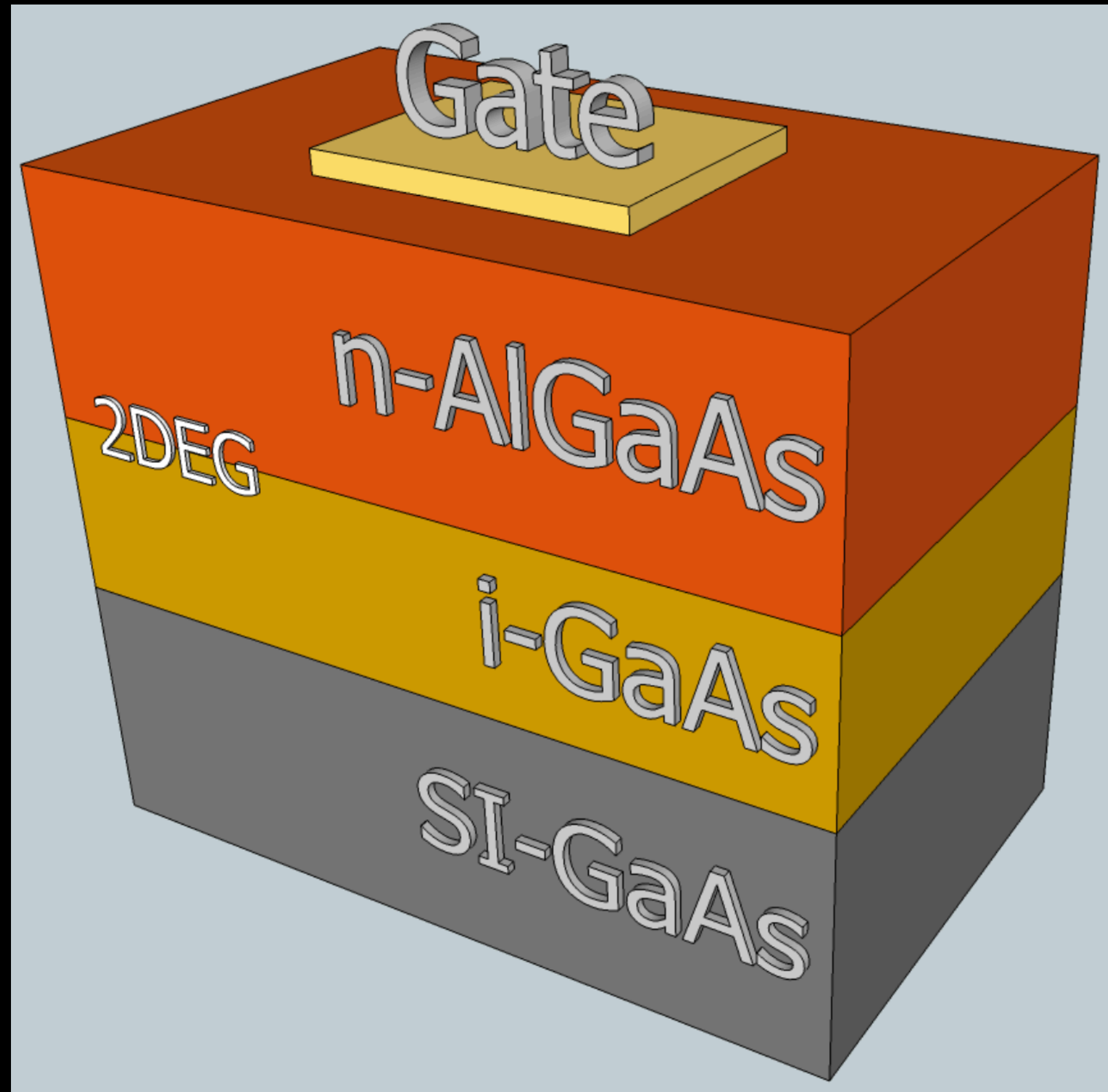
Basov group: Nature 650, 864 (2026)

# Outline

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- I. Vacuum modified topology
- II. Vacuum renormalized band structure
- III. Vacuum induced chiral spin liquids
- IV. Summary and outlooks

# Two dimensional electron gas



Very Clean

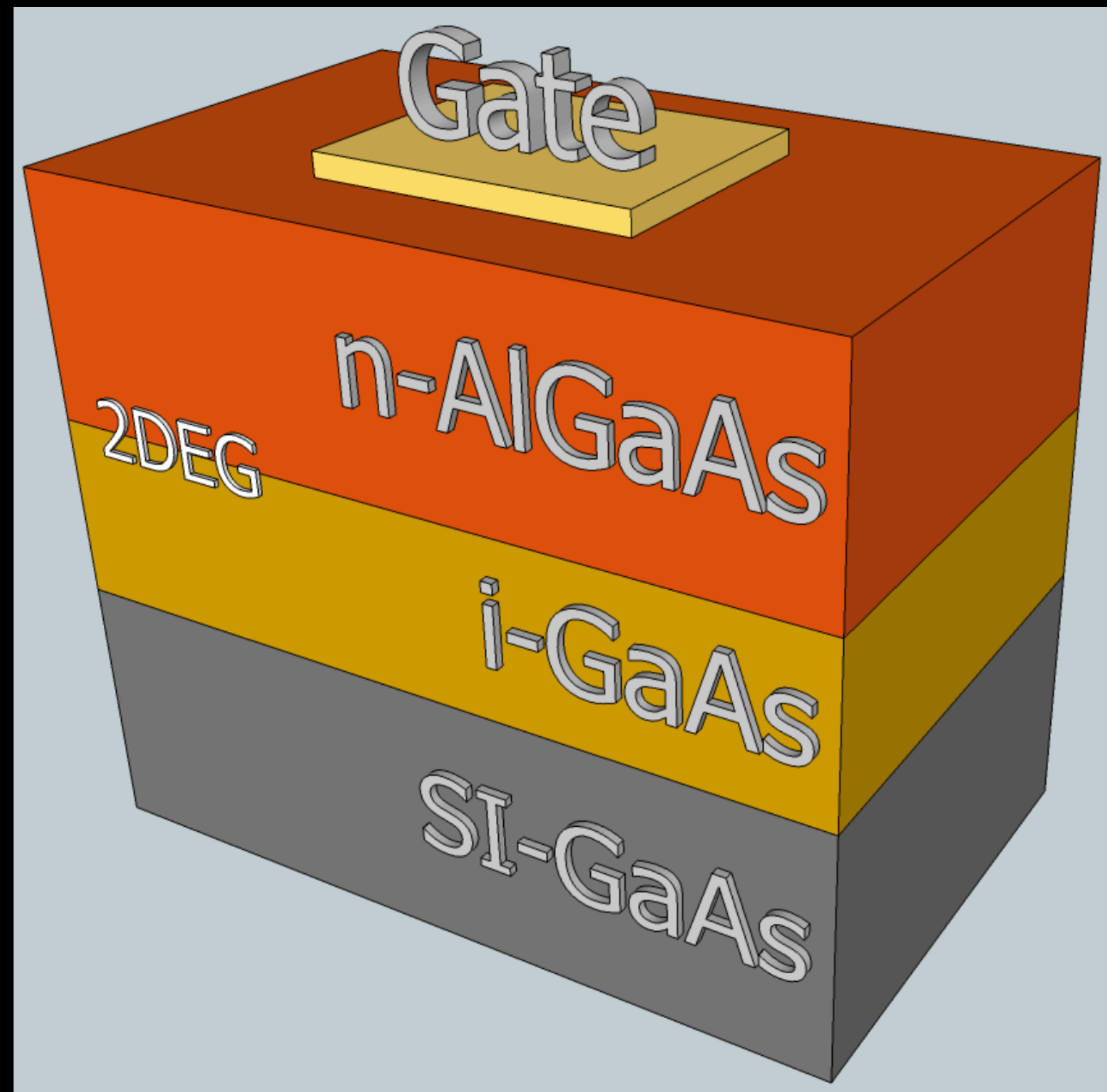
2 dimensional electron gas (2DEG)

Mean free path:

$$\ell = v_F \tau = \sqrt{2\pi n} \frac{\hbar \mu}{e} \approx 5.2 \mu\text{m} \times \mu [10^6 \text{ cm}^2/\text{Vs}] \sqrt{n [10^{11} \text{ cm}^{-2}]}$$

$$5.2 \mu\text{m} \sim 10^4 a_{\text{lattice}}$$

# Two dimensional electron gas



Very Clean

2 dimensional electron gas (2DEG)

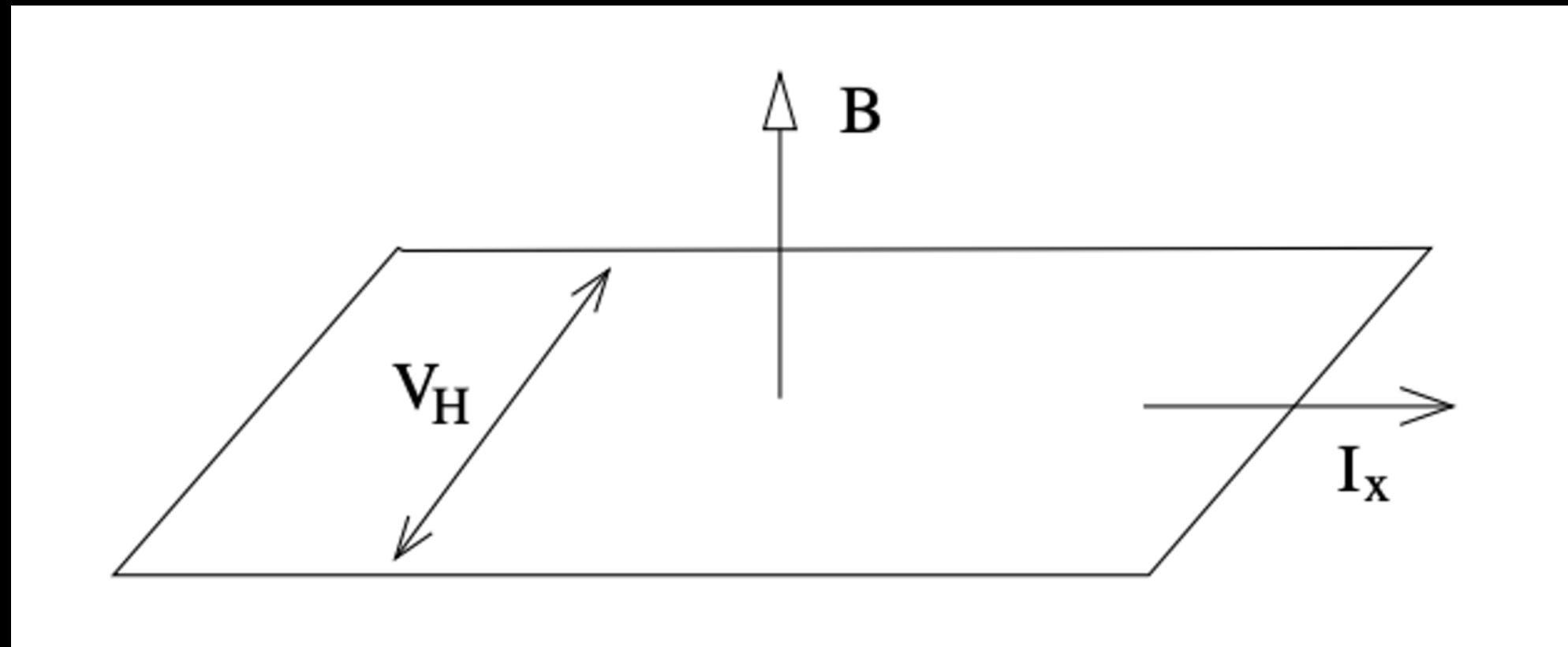
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$$5.2 \mu\text{m} \sim 10^4 a_{\text{lattice}}$$

2 Nobel prizes: Quantum Hall effect  
Fractional Quantum Hall effect

# Quantum Hall Effect



$$m \frac{d\mathbf{v}}{dt} = -e\mathbf{v} \times \mathbf{B}$$

$$R_H = h/e^2$$

## Magnetic Scales

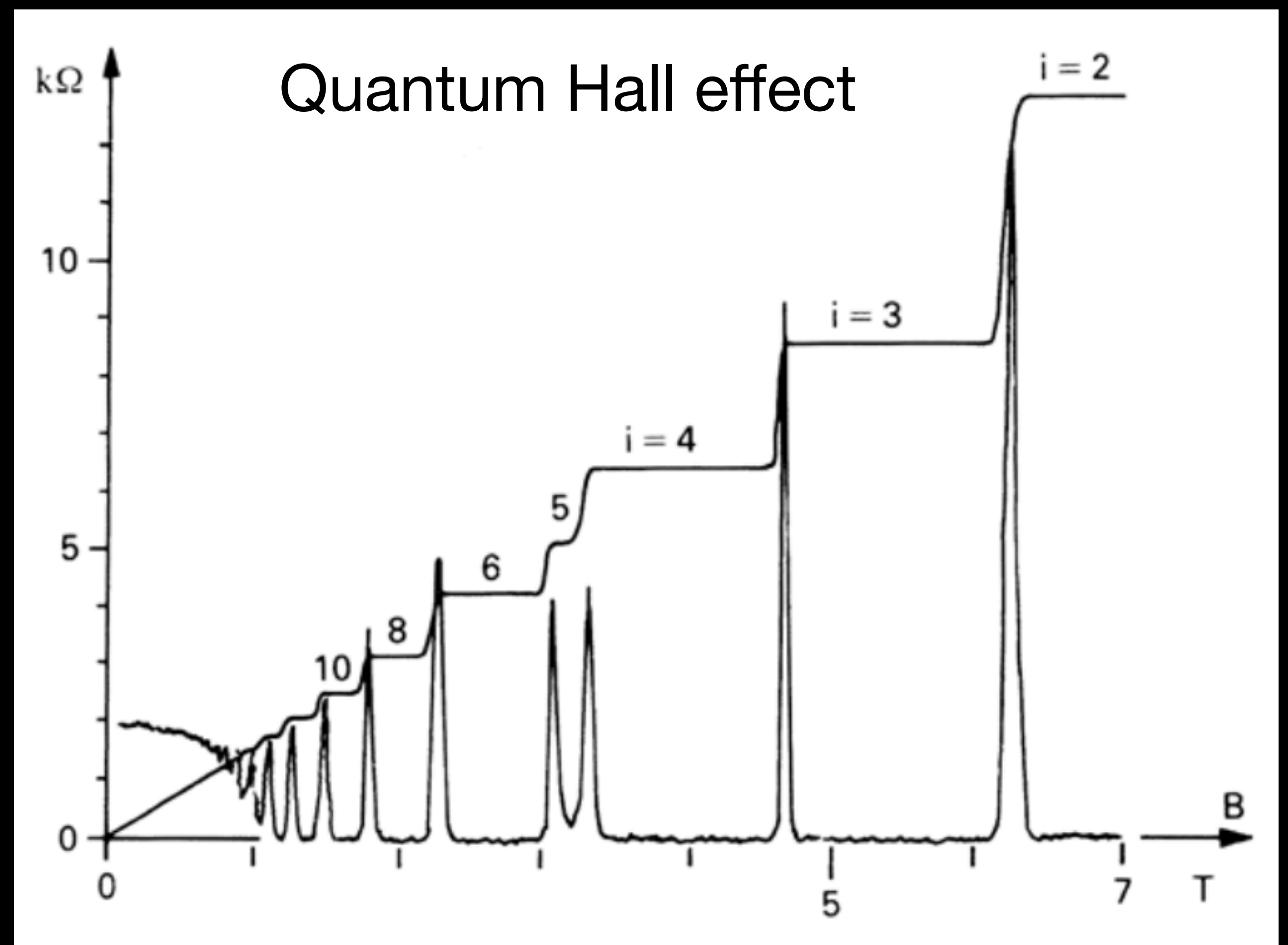
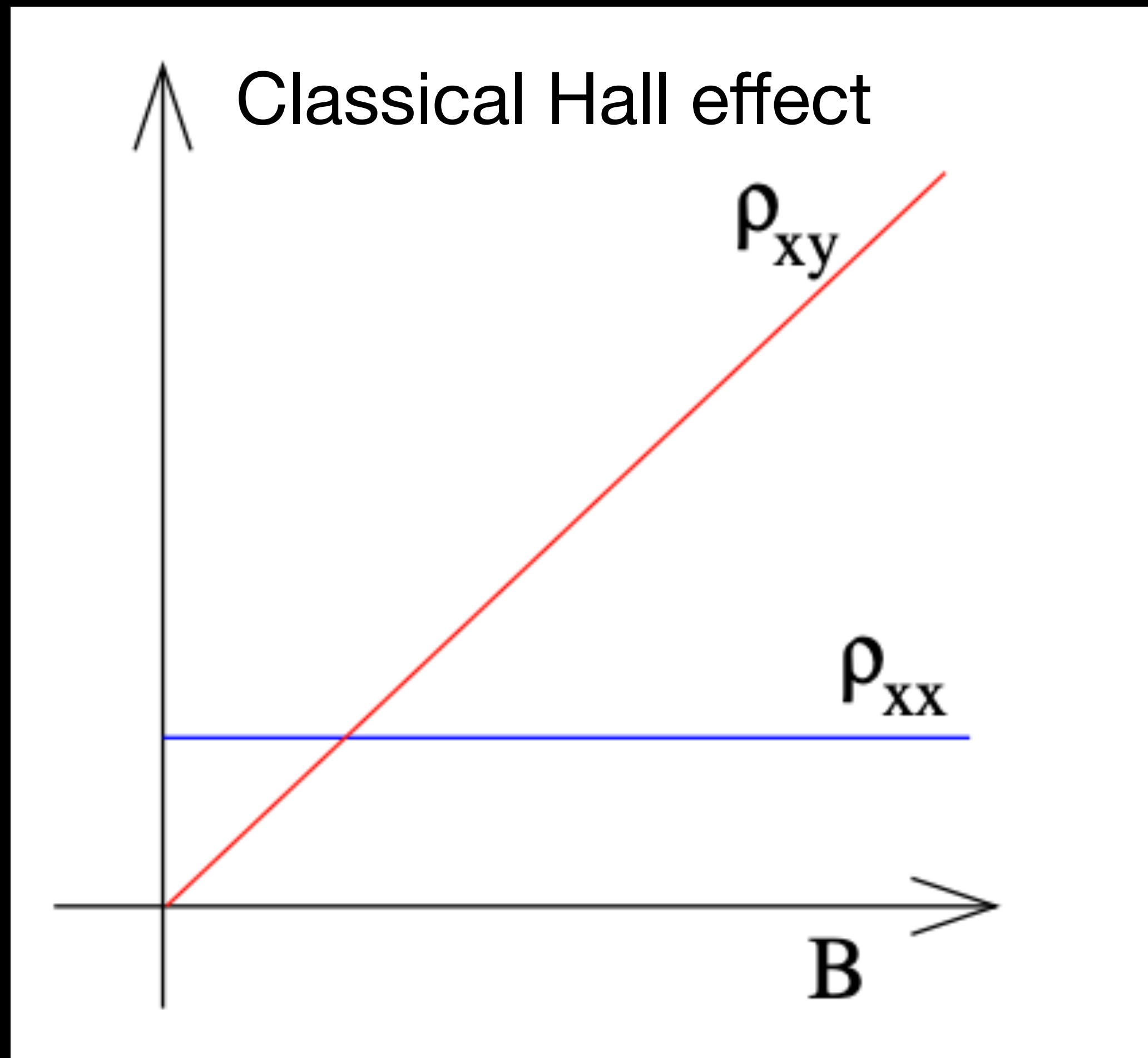
Cyclotron Frequency:  $\omega_B = \frac{eB}{m}$

Magnetic Length:  $l_B = \sqrt{\frac{\hbar}{eB}}$

Quantum of Flux:  $\Phi_0 = \frac{2\pi\hbar}{e}$

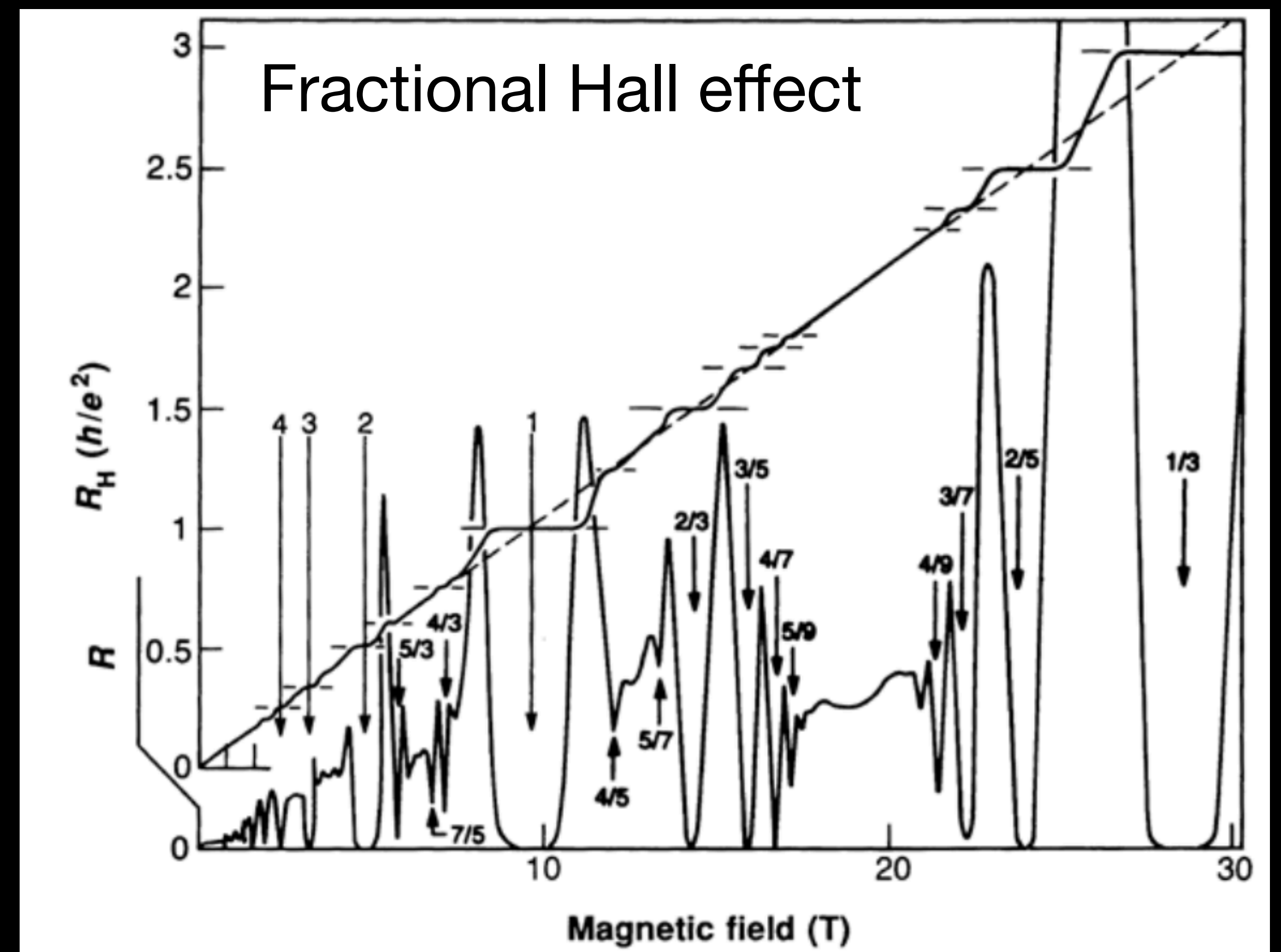
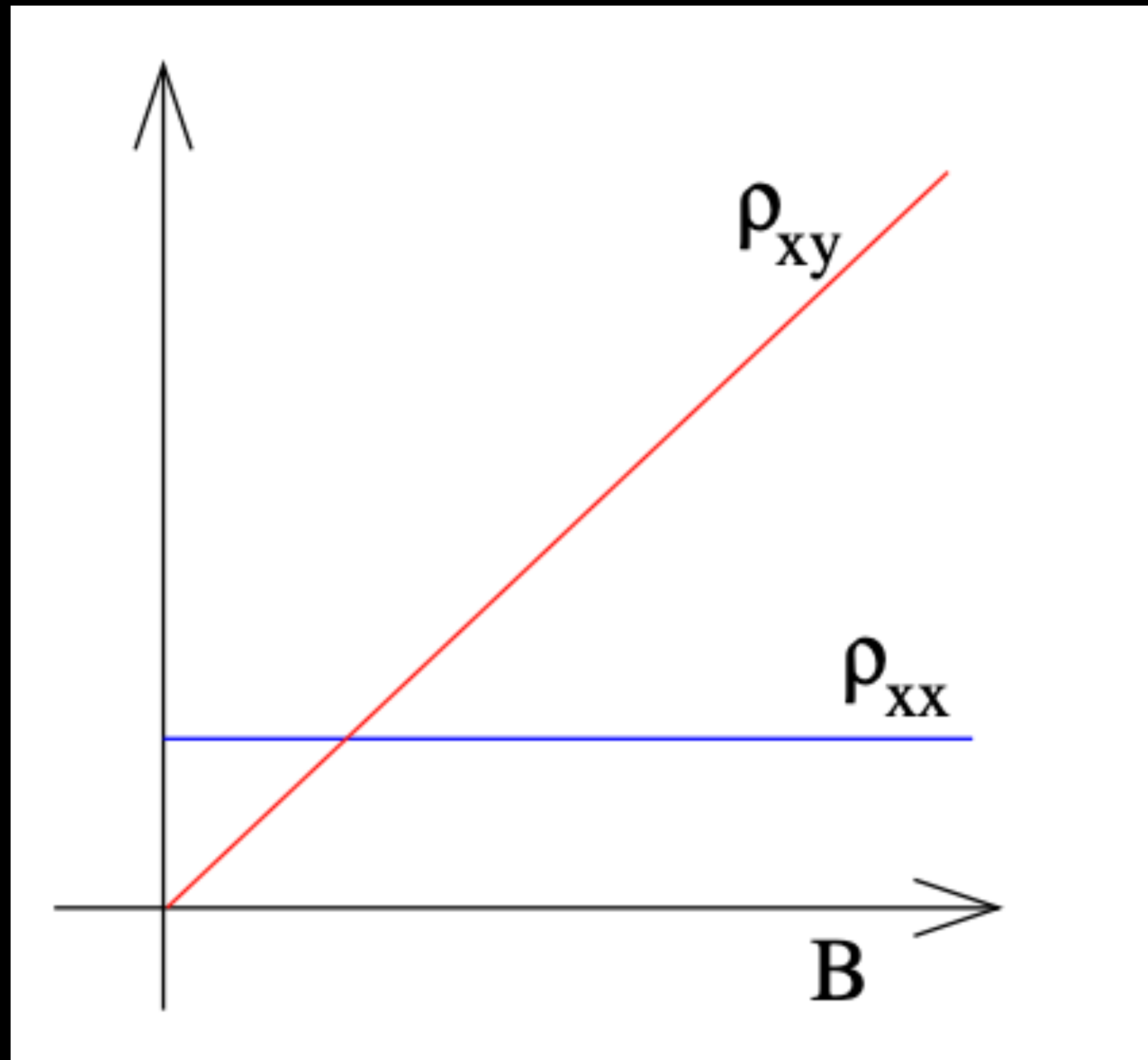
Hall Resistivity:  $\rho_{xy} = \frac{2\pi\hbar}{e^2} \frac{1}{\nu}$

# What is Quantum Hall Effect



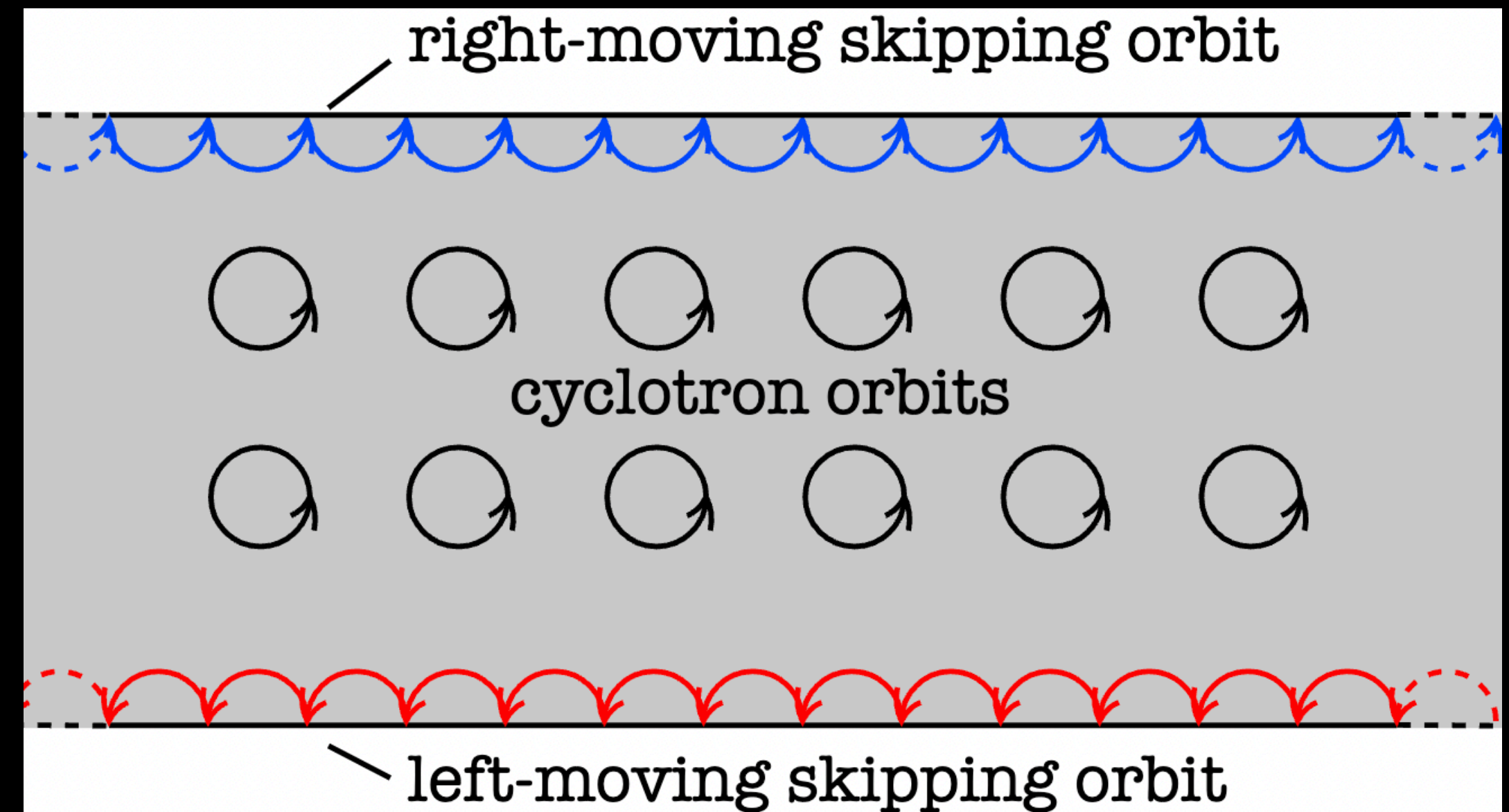
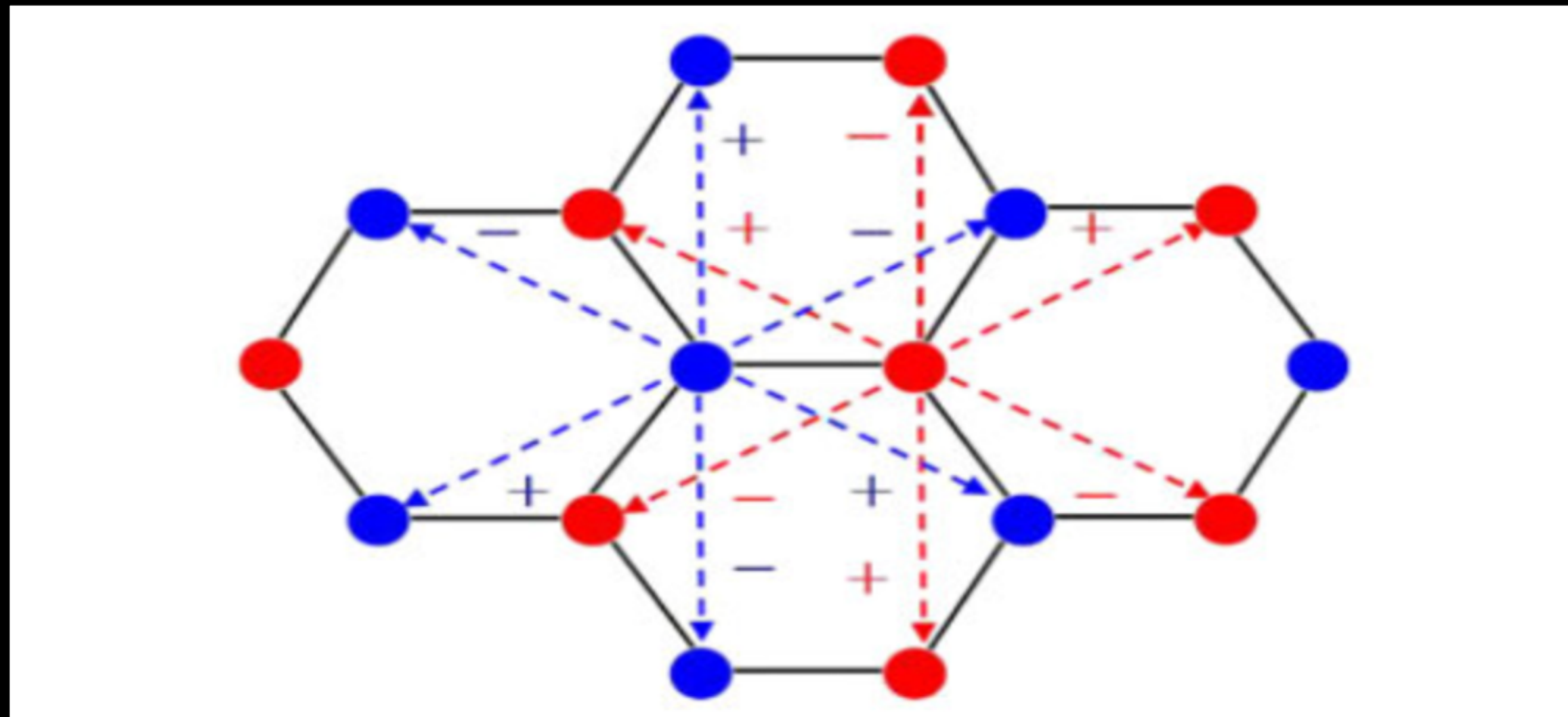
Nobel prize in physics 1985

# What is Quantum Hall Effect



Nobel prize in physics 1998

# Quantum Anomalous Hall Effect

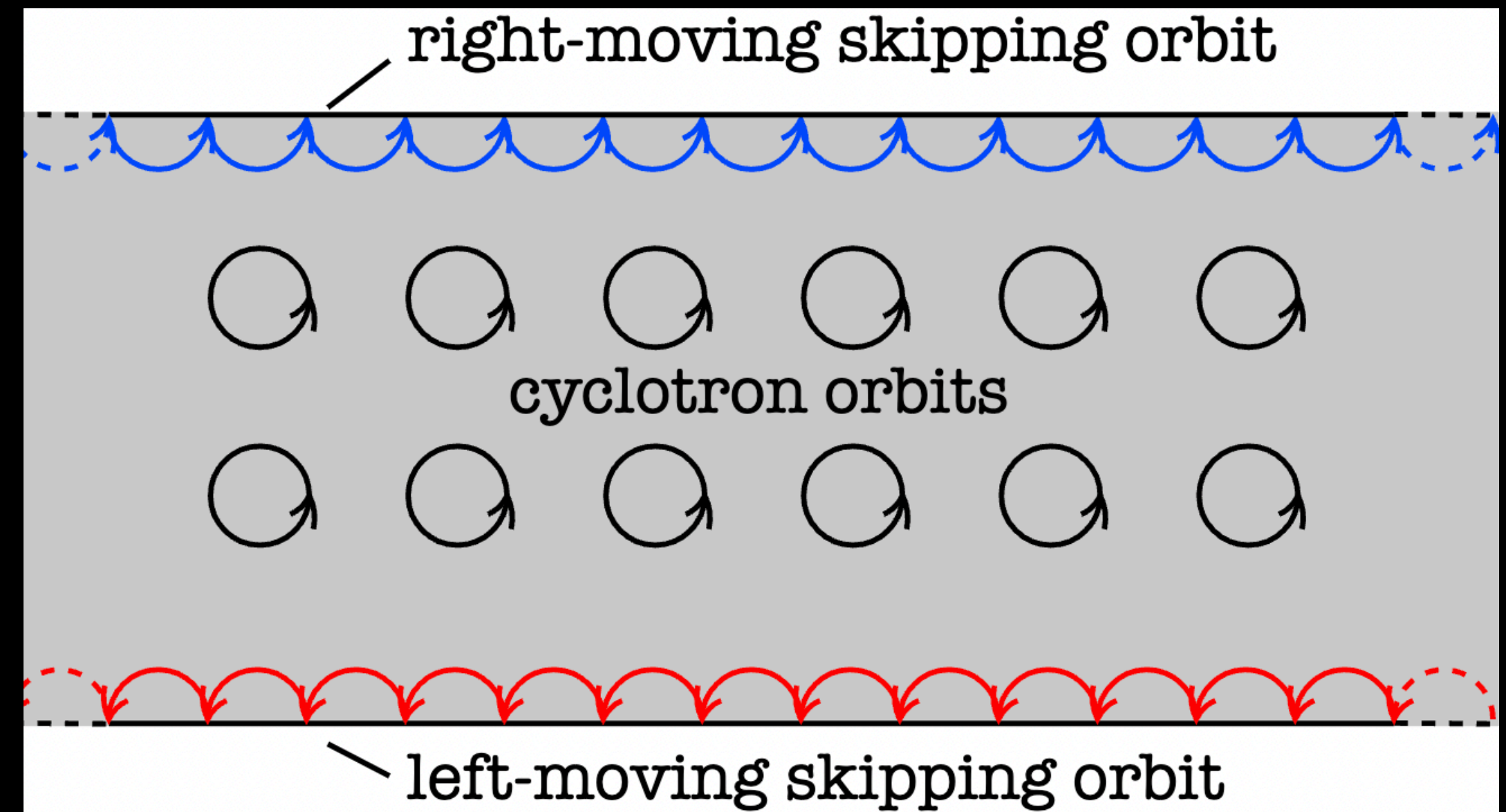
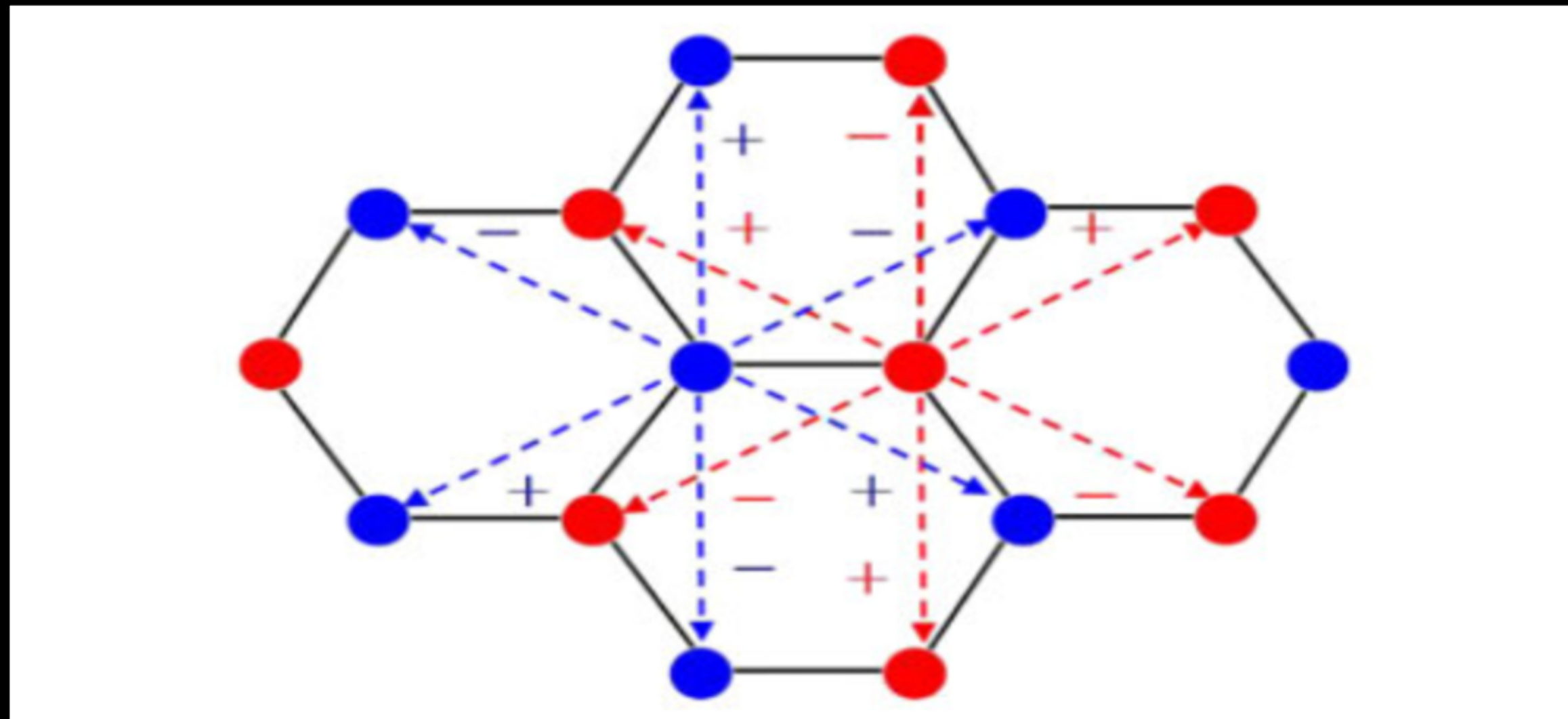


Average magnetic flux is ZERO

Time reversal breaking

Nobel prize in physics 2016

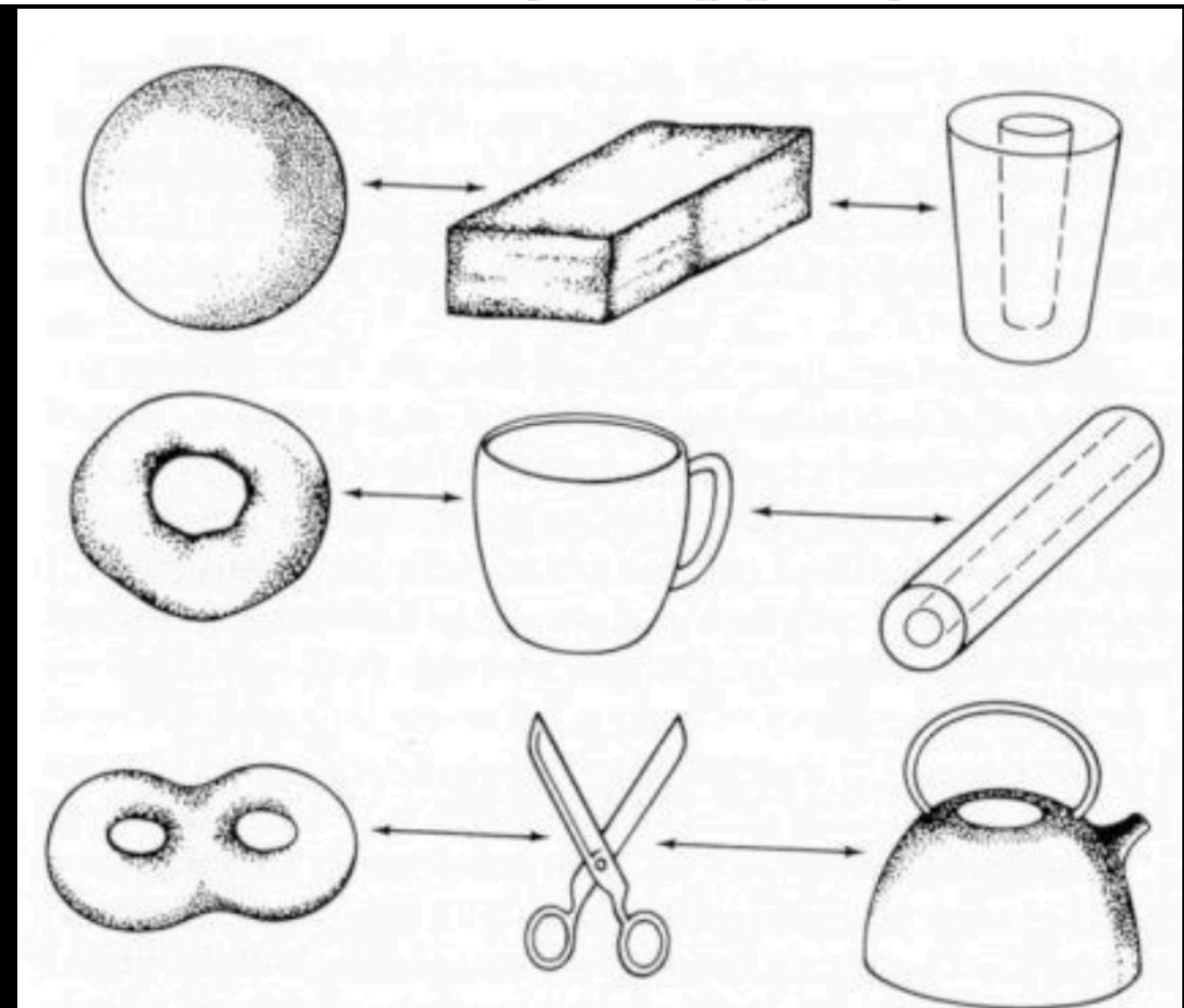
# Quantum Anomalous Hall Effect



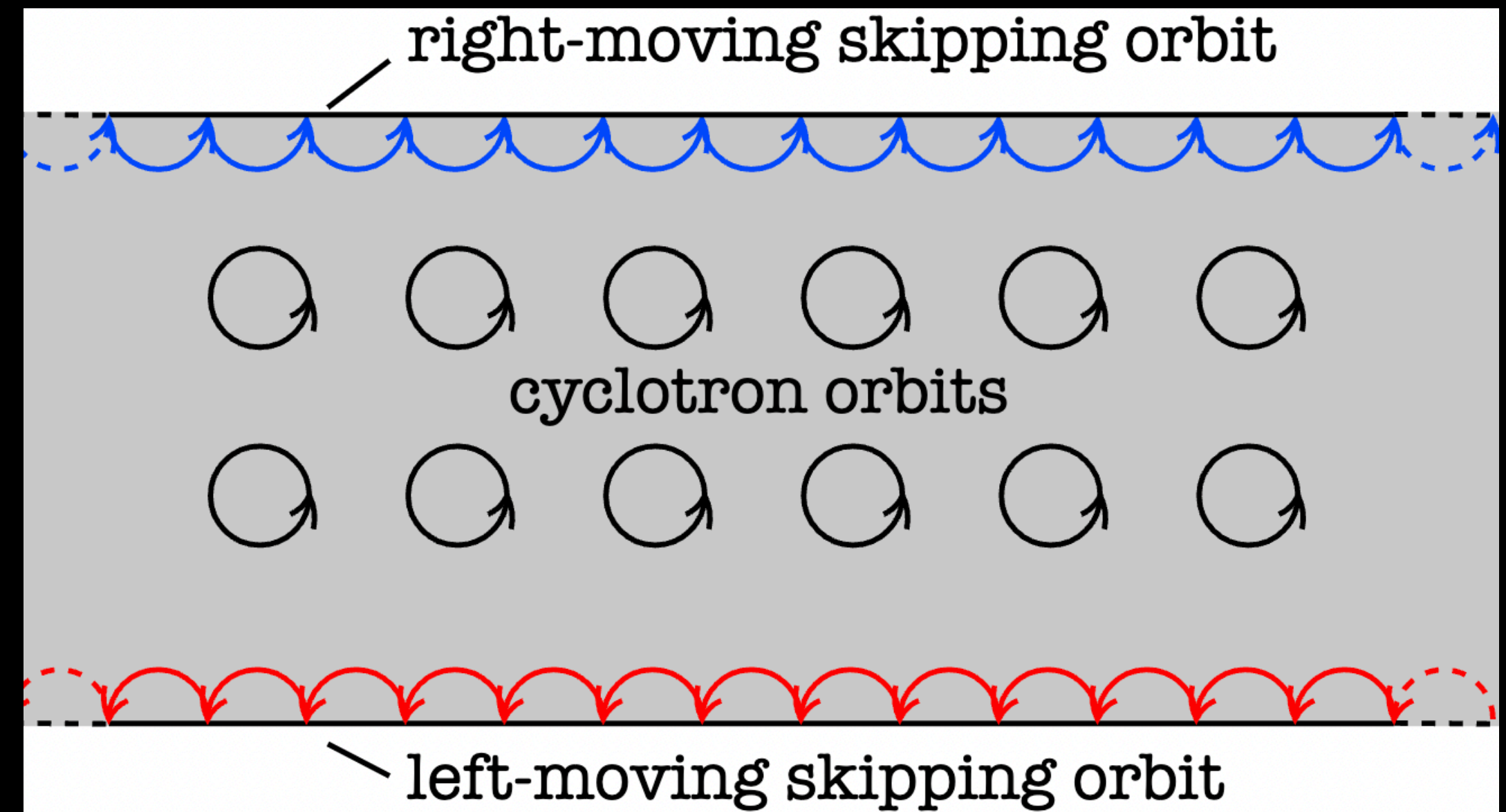
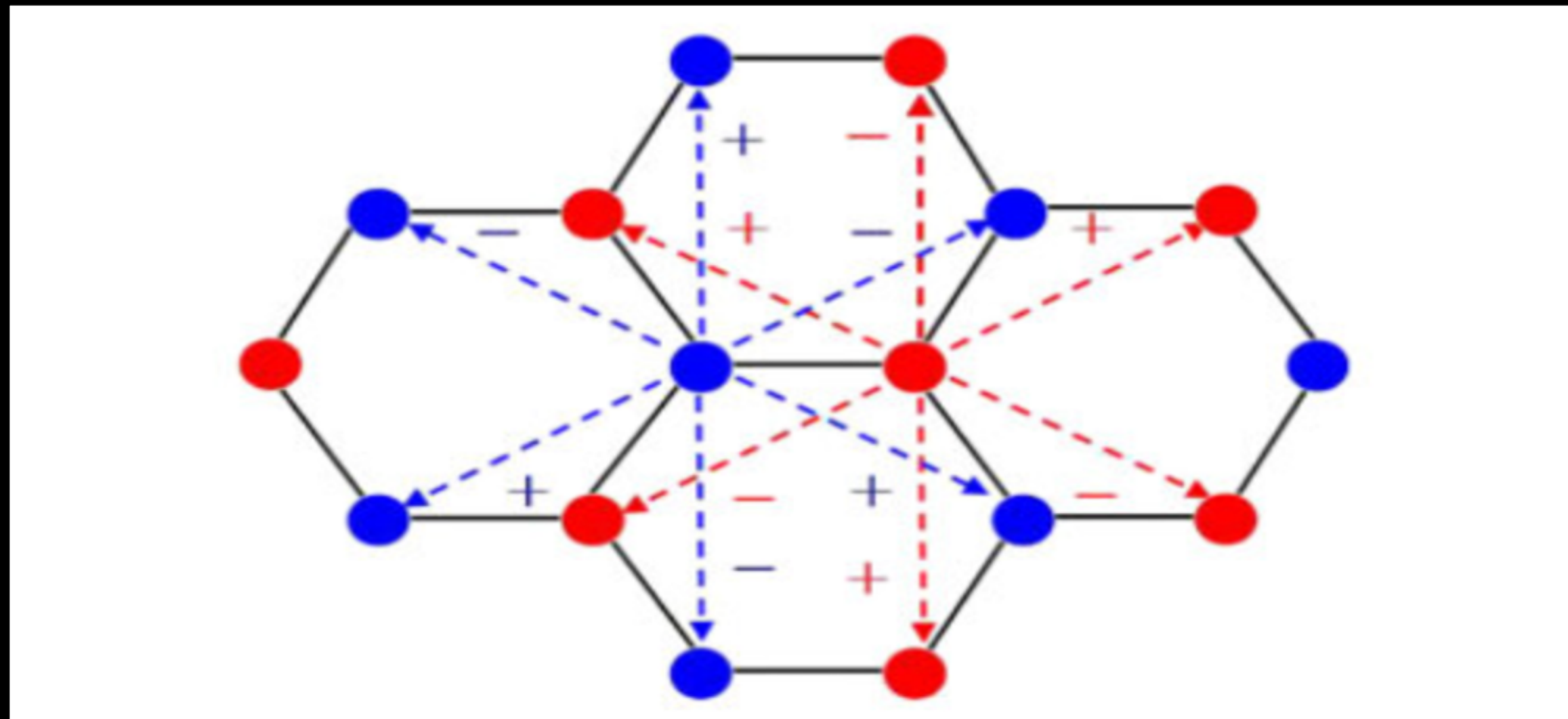
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# Quantum Anomalous Hall Effect

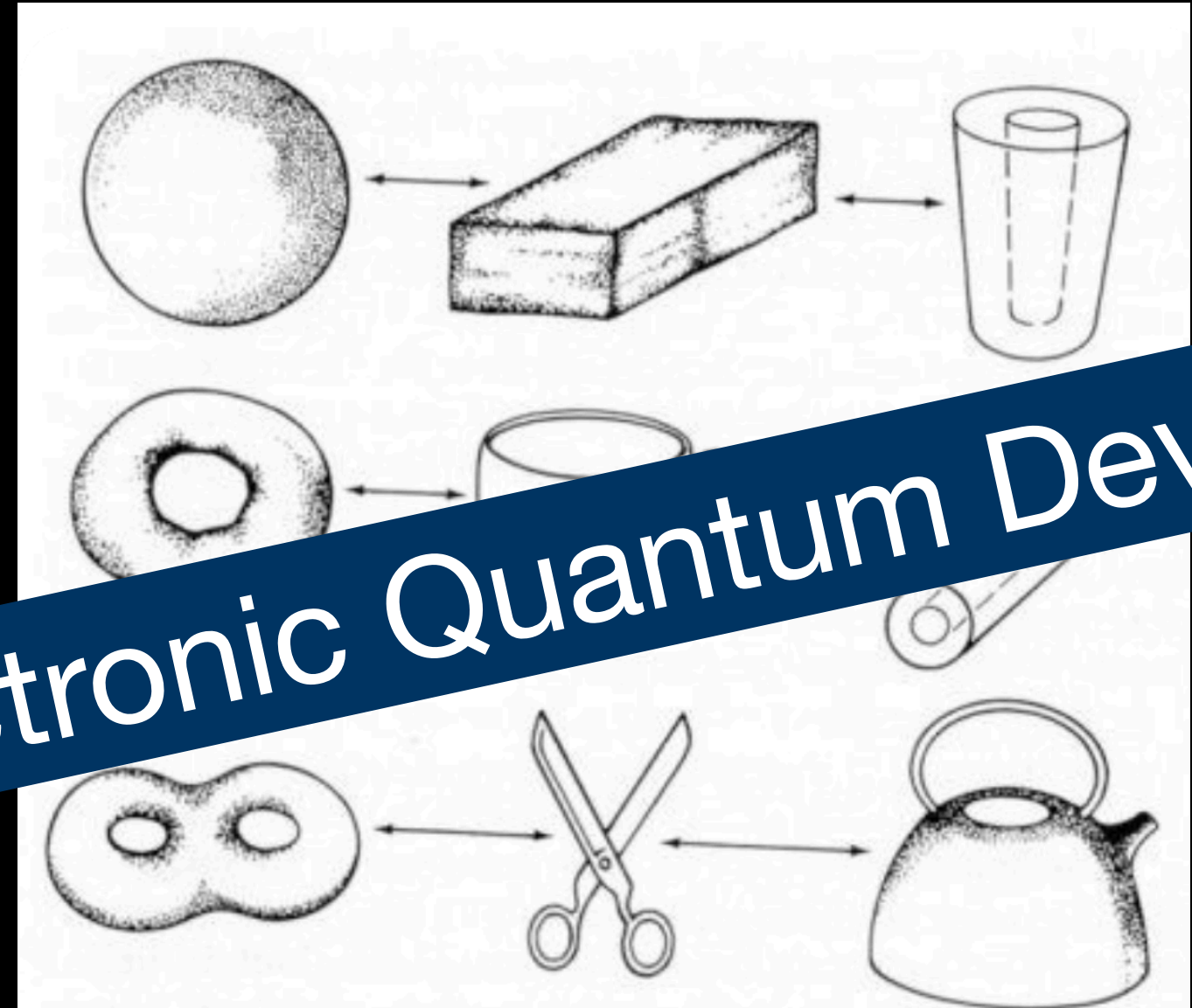


Average magnetic flux is ZERO

Time reversal breaking

Nobel prize in physics 2016

Electronic Quantum Devices



Are these quantum states **robust**  
to **vacuum** quantum fluctuations?

# Quantum Hall effect in a cavity

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## Breakdown of topological protection by cavity vacuum fields in the integer quantum Hall effect

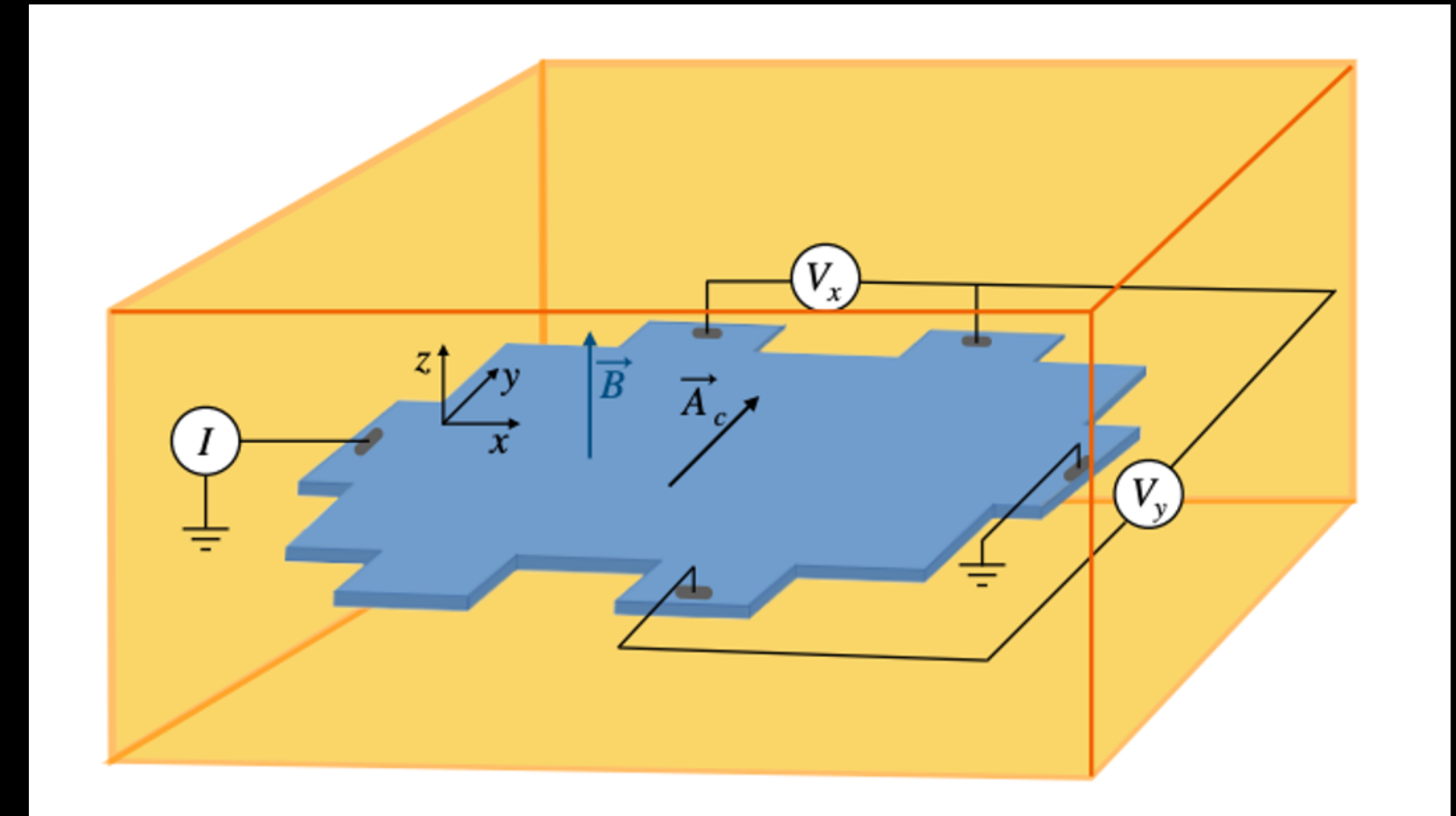
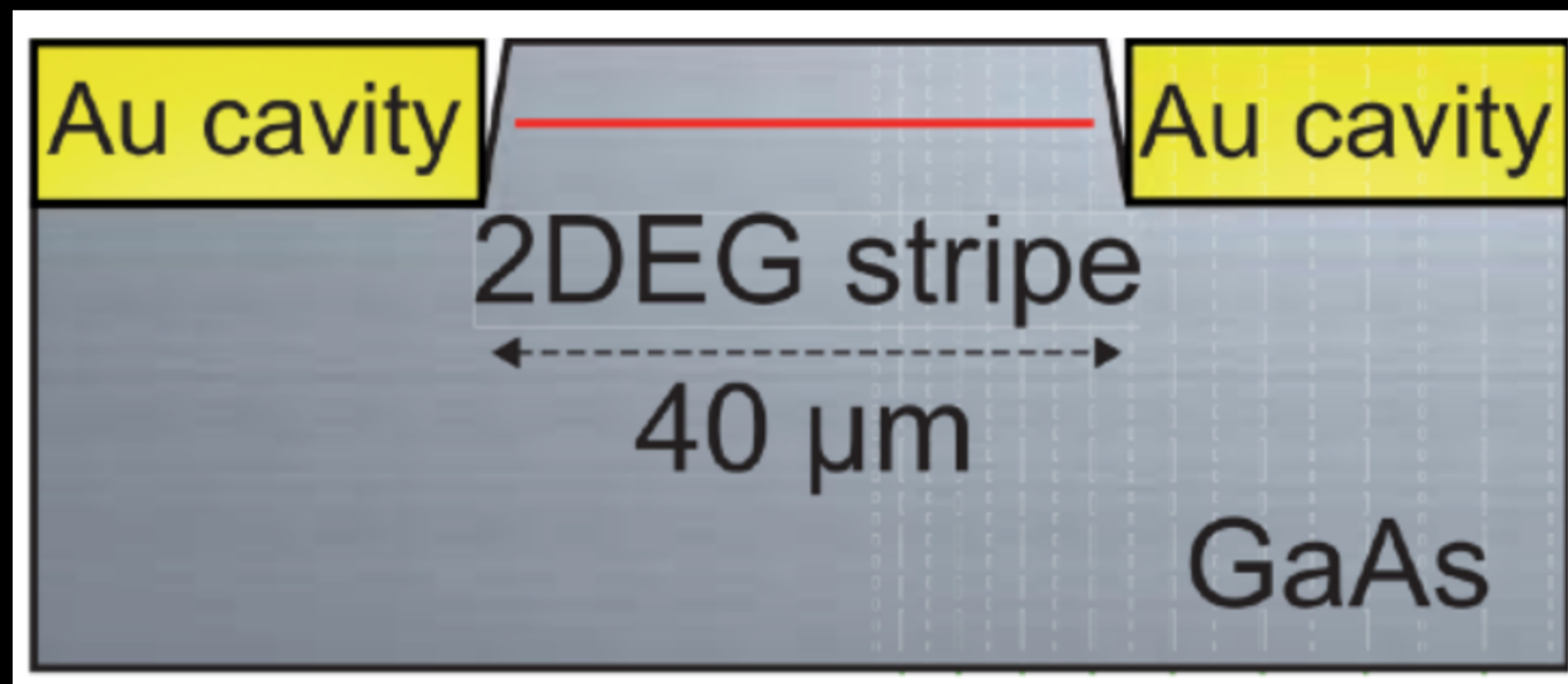
Article | [Open access](#) | Published: 14 May 2025

# Tunable vacuum-field control of fractional and integer quantum Hall phases

[Josefine Enkner](#) , [Lorenzo Graziotto](#), [Dalin Boriçi](#), [Felice Appugliese](#), [Christian Reichl](#), [Giacomo Scalari](#),  
[Nicolas Regnault](#), [Werner Wegscheider](#), [Cristiano Ciuti](#) & [Jérôme Faist](#) 

[Nature](#) **641**, 884–889 (2025) | [Cite this article](#)

# Quantum Hall effect in a cavity



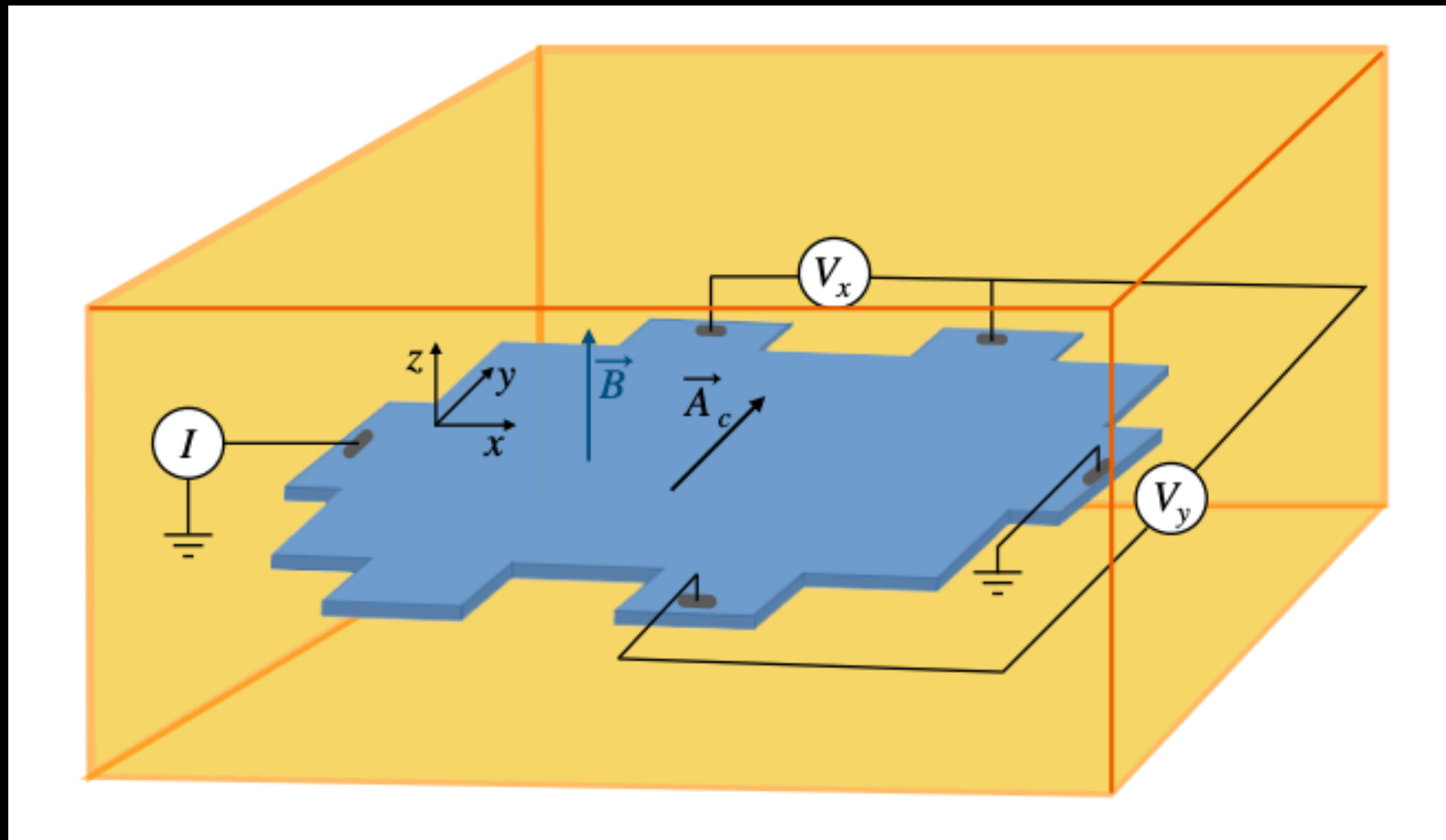
# Effective field theory for QHE

$$S_{\text{eff}}[A] = \frac{\nu e^2}{4\pi\hbar} \int dt d^2x \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

$$j^\mu = \frac{\delta S_{\text{eff}}}{\delta A_\mu} = \frac{\nu e^2}{2\pi\hbar} \epsilon^{\mu\nu\lambda} \partial_\nu A_\lambda$$

$$S[b, A] = \int d^3x \left[ \frac{1}{4\pi\nu} \epsilon^{\mu\nu\lambda} b_\mu \partial_\nu b_\lambda + \frac{e}{2\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu b_\lambda \right]$$

# Cavity Quantum Hall Hydrodynamics

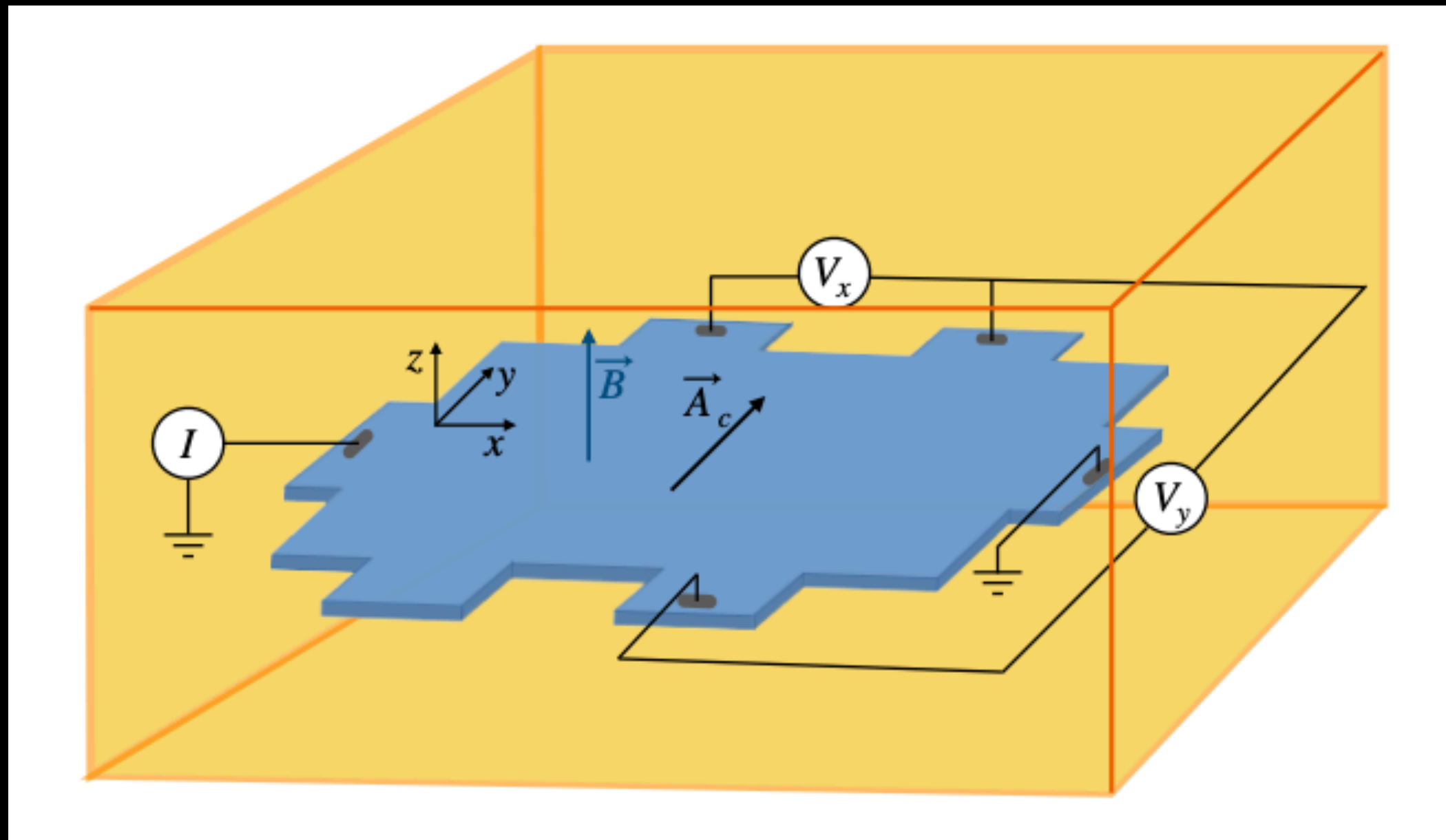


$$\mathcal{L}_H = -\frac{m}{4\pi} \epsilon^{\mu\nu\rho} b_\mu \partial_\nu b_\rho - \frac{e}{2\pi} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu b_\rho$$

$$+ \frac{m}{4\pi\omega_B} \vec{E}_b^2 - \frac{u}{2} B_b^2,$$

# Cavity Quantum Hall Hydrodynamics

## Result 1: Longitudinal conductance



$$j^x(\omega) = \frac{e^2}{2\pi} \frac{1}{m} E_y(\omega) + \frac{e^2}{2\pi} \frac{1}{m^2} \frac{2\alpha}{L_{\text{eff}}} \frac{i\omega}{\omega^2 - \omega_c^2} E_x(\omega),$$
$$j^y(\omega) = -\frac{e^2}{2\pi} \frac{1}{m} E_x(\omega),$$

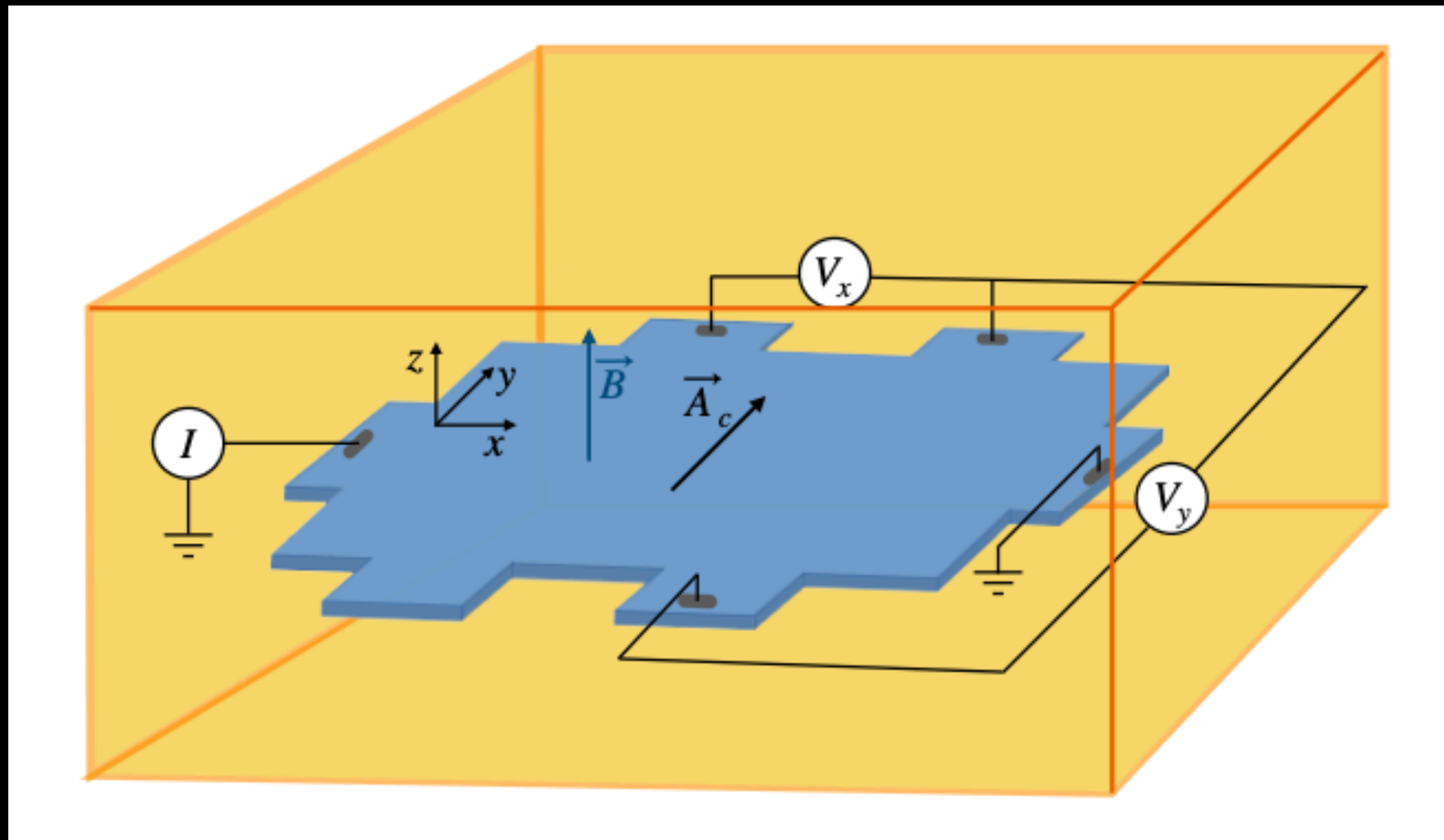
# Cavity Quantum Hall Hydrodynamics

## Result 1: Longitudinal conductance

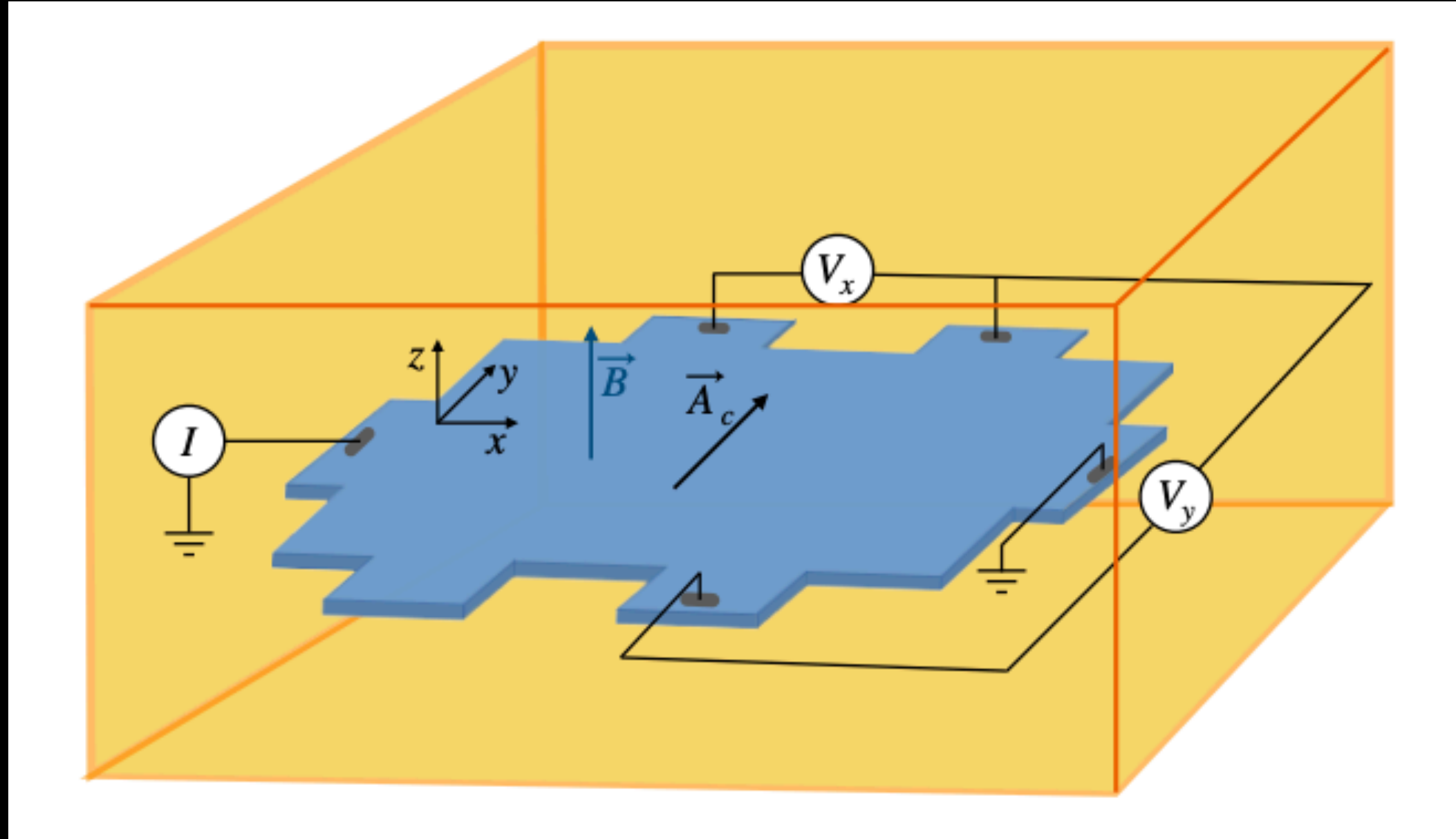
$$j^x(\omega) = \frac{e^2}{2\pi} \frac{1}{m} E_y(\omega) + \frac{e^2}{2\pi} \frac{1}{m^2} \frac{2\alpha}{L_{\text{eff}}} \frac{i\omega}{\omega^2 - \omega_c^2} E_x(\omega),$$
$$j^y(\omega) = -\frac{e^2}{2\pi} \frac{1}{m} E_x(\omega),$$

## Result 2: Kohn mode frequency

$$\omega = \omega_B + \frac{1}{m} \frac{\alpha}{L_{\text{eff}}}.$$



# Cavity Quantum Hall Hydrodynamics



G. Cardoso, L. Yang, T. H. Hansson, QDJ,  
Phys. Rev. B 113, 045108; Phys. Rev. B  
113, 045109 (2026)

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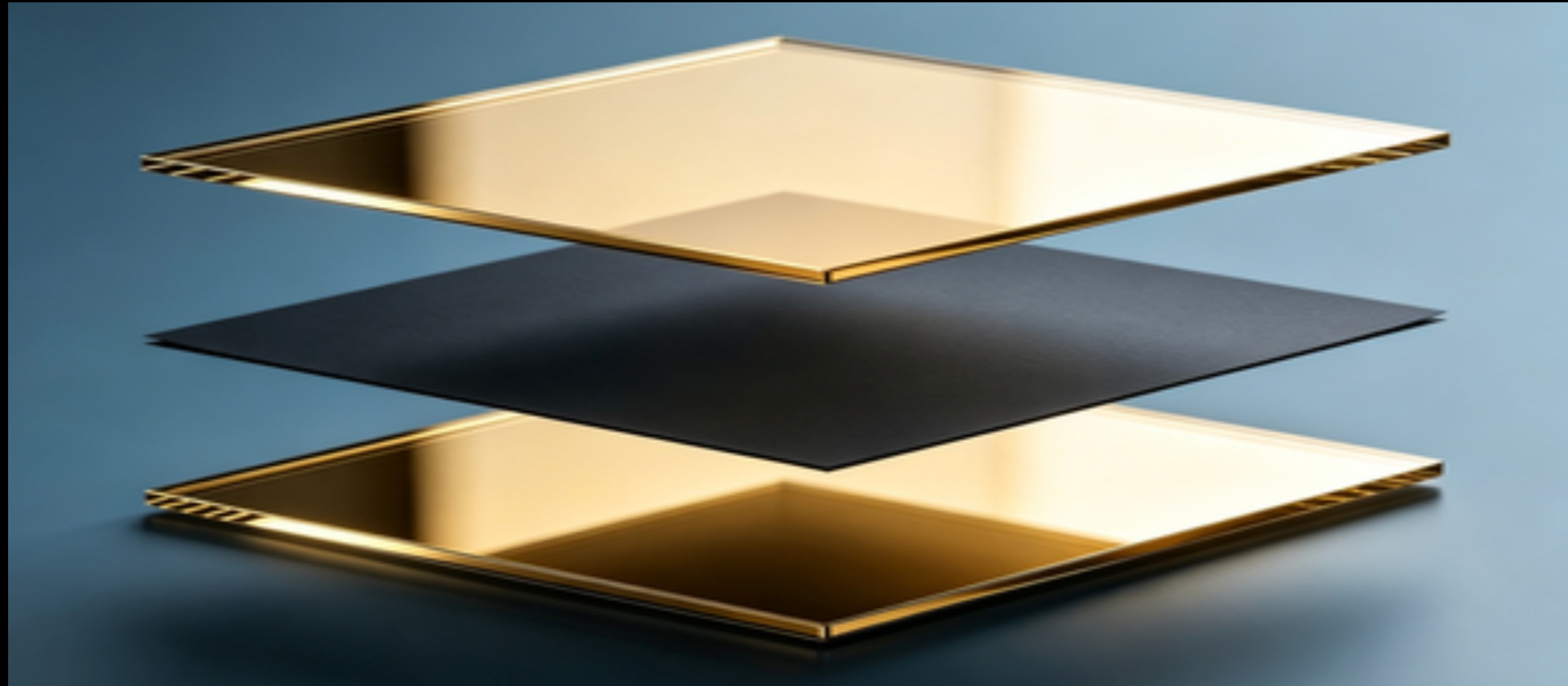
## Result 2: Kohn mode frequency

$$\omega = \omega_B + \frac{1}{m} \frac{\alpha}{L_{\text{eff}}}.$$

## Result 3: Photon electron mixing

Quantum Hall breaks down when  $\omega_c \sim \omega_B$

# Vacuum modification of QHE



$$\rho_L = \frac{1}{2}Z_0 \quad \rho_H = R_K/\nu \quad \longrightarrow \quad \sigma = \rho^{-1} = \frac{1}{\rho_{xx}^2 + \rho_{xy}^2} \begin{pmatrix} \rho_{xx} & -\rho_{xy} \\ \rho_{xy} & \rho_{xx} \end{pmatrix}$$

$$\sigma_H = 1/\rho_H \times [1 - (\rho_L/\rho_H)^2]$$

# Vacuum modification of QHE

$$S_b = \int d^3x \left[ -\frac{\nu}{4\pi} \epsilon^{\mu\nu\sigma} b_\mu \partial_\nu b_\sigma + \frac{e}{2\pi} \epsilon^{\mu\nu\sigma} (A_\mu + Q_\mu) \partial_\nu b_\sigma + j_\nu^\mu b_\mu \right]$$

$$\mathcal{S}_M = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \left\{ \vec{Q}(-p) \left[ (\omega/c)^2 \epsilon(\vec{p}, \omega) - q^2 - p_z^2 \right] \cdot \vec{Q}(p) + Q_0(-p) \epsilon(\vec{p}, \omega) p^2 Q_0(p) \right\}$$

$$\downarrow$$
$$\epsilon(\vec{p}, \omega) = \epsilon_0 - \frac{\tilde{\sigma}}{i\omega - Dp^2}$$

# Vacuum modification of QHE

$$S_b = \int d^3x \left[ -\frac{\nu}{4\pi} \epsilon^{\mu\nu\sigma} b_\mu \partial_\nu b_\sigma + \frac{e}{2\pi} \epsilon^{\mu\nu\sigma} (A_\mu + Q_\mu) \partial_\nu b_\sigma + j_\nu^\mu b_\mu \right]$$

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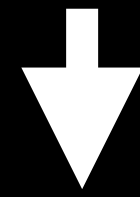
$$\downarrow$$

$$\epsilon(\vec{p}, \omega) = \epsilon_0 - \frac{\tilde{\sigma}}{i\omega - Dp^2}$$

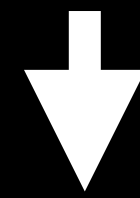
$$S_{tot} = \int \frac{d^3p}{(2\pi)^3} \left[ \frac{\alpha}{4\pi} \frac{i\omega^2}{\sqrt{\omega^2 + i\tilde{\sigma}\omega}} \delta^{ij} b_i b_j - \frac{\nu}{4\pi} i\omega \epsilon^{ij} b_i b_j - \frac{e}{2\pi} \epsilon^{ij} b_i E_j \right]$$

# Vacuum modification of QHE

$$S_{tot} = \int \frac{d^3p}{(2\pi)^3} \left[ \frac{\alpha}{4\pi} \frac{i\omega^2}{\sqrt{\omega^2 + i\tilde{\sigma}\omega}} \delta^{ij} b_i b_j - \frac{\nu}{4\pi} i\omega \epsilon^{ij} b_i b_j - \frac{e}{2\pi} \epsilon^{ij} b_i E_j \right]$$

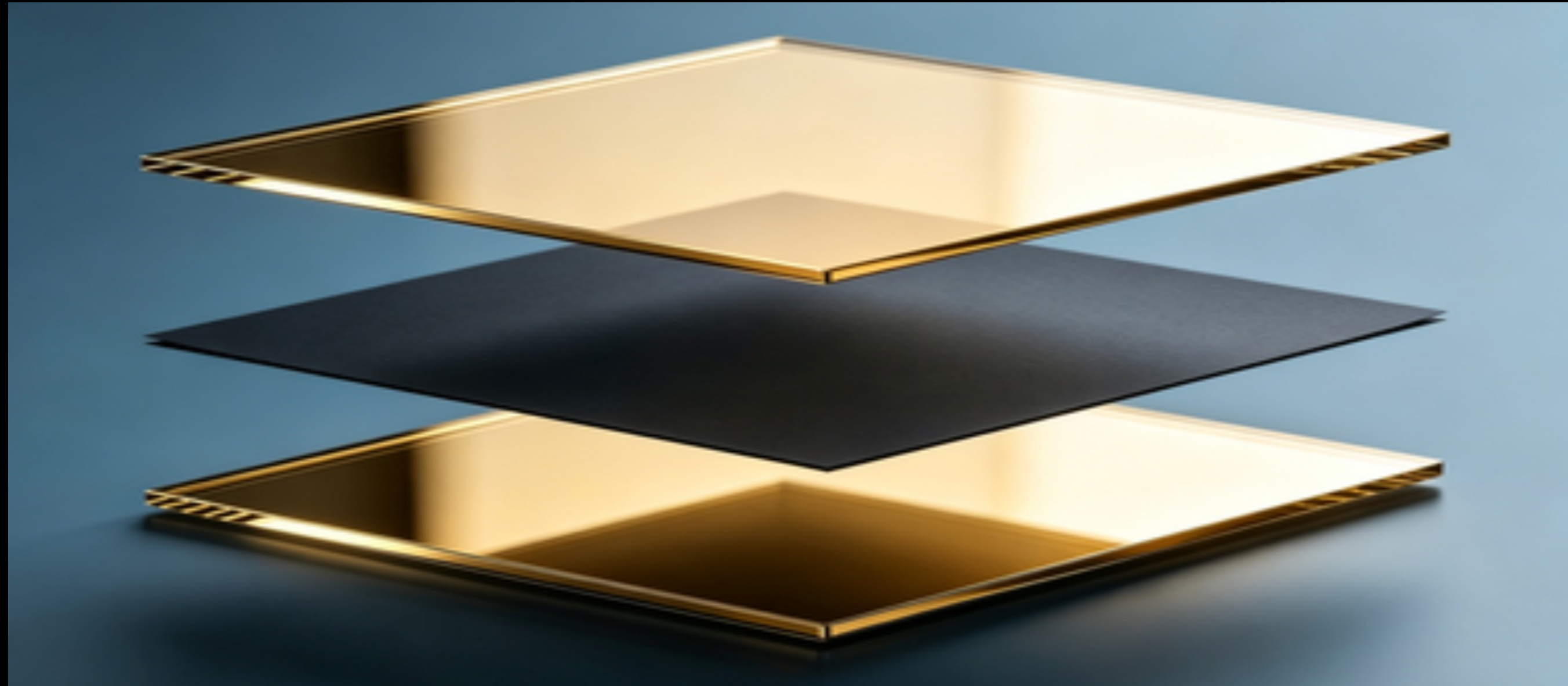


$$\frac{\alpha}{2\pi} \frac{i\omega^2}{\sqrt{\omega^2 + i\tilde{\sigma}\omega}} b^i - \frac{\nu}{2\pi c} i\omega \epsilon^{ij} b_j - \frac{e}{2\pi} \epsilon^{ij} E_j = 0$$



$$-\frac{\alpha}{e} \frac{\omega}{\sqrt{\omega^2 + i\tilde{\sigma}\omega}} \delta^{ij} J_j - \frac{\nu}{e} \epsilon^{ij} J_j + \frac{e}{2\pi} E^i = 0$$

# Vacuum modification of QHE

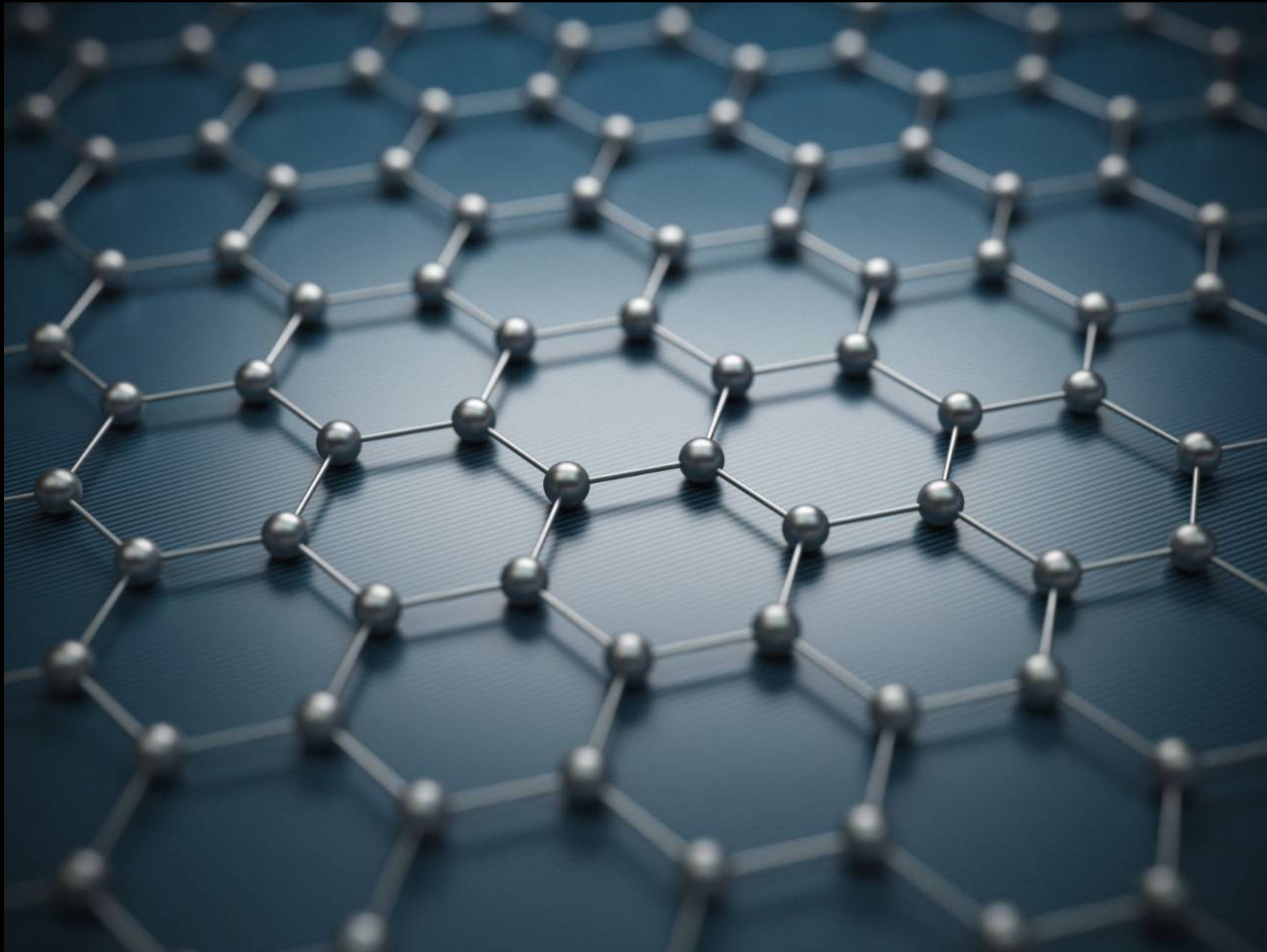


$$\rho_L = \frac{1}{2}Z_0 \quad \rho_H = R_K/\nu \quad \longrightarrow \quad \sigma = \rho^{-1} = \frac{1}{\rho_{xx}^2 + \rho_{xy}^2} \begin{pmatrix} \rho_{xx} & -\rho_{xy} \\ \rho_{xy} & \rho_{xx} \end{pmatrix}$$

$$\sigma_H = 1/\rho_H \times [1 - (\rho_L/\rho_H)^2]$$

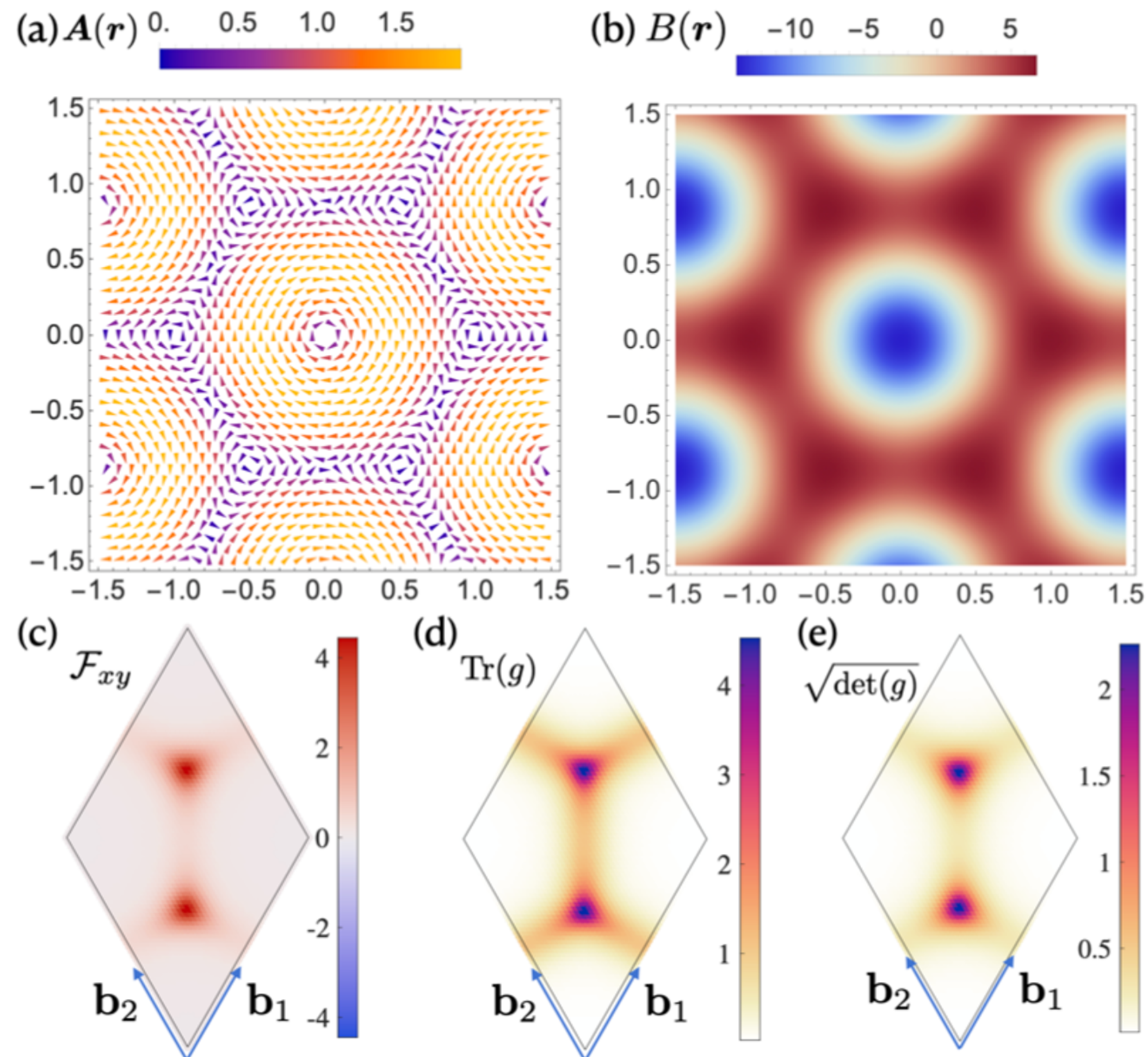
# Quantum Anomalous Hall in a chiral cavity

## Emergent Haldane model



$$\hat{H}^C = \frac{(\hat{\mathbf{p}} + e\hat{\mathbf{A}})^2}{2m} + V(\mathbf{r}) + \hbar\omega_c \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right)$$
$$\hat{\mathbf{A}} = A_0 (\boldsymbol{\epsilon} \hat{a} + \boldsymbol{\epsilon}^* \hat{a}^\dagger)$$

# Graphene in a chiral cavity



$$\hat{H}^C = \frac{(\hat{\mathbf{p}} + e\hat{\mathbf{A}})^2}{2m} + V(\mathbf{r}) + \hbar\omega_c \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right)$$

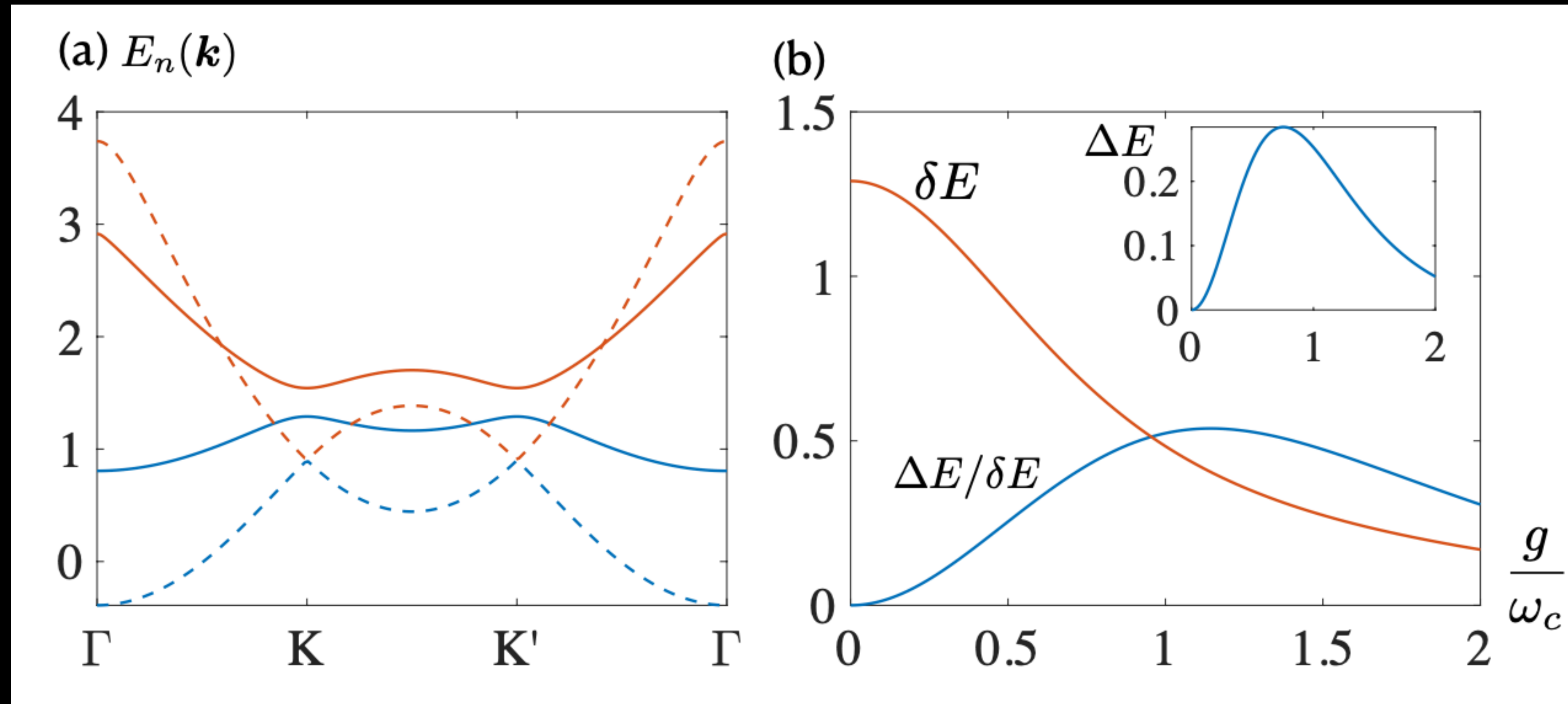
$$\hat{\mathbf{A}} = A_0 (\epsilon \hat{a} + \epsilon^* \hat{a}^\dagger)$$

$$\hat{H}^U = \langle 0_{\text{photon}} | \hat{U}^\dagger \hat{H}^C \hat{U} | 0_{\text{photon}} \rangle$$

$$= \frac{\hat{p}^2}{2m_{\text{eff}}} + \tilde{V} \left( \mathbf{r} + \frac{\xi^2}{2\hbar} \hat{\mathbf{p}} \times \mathbf{e}_z \right)$$

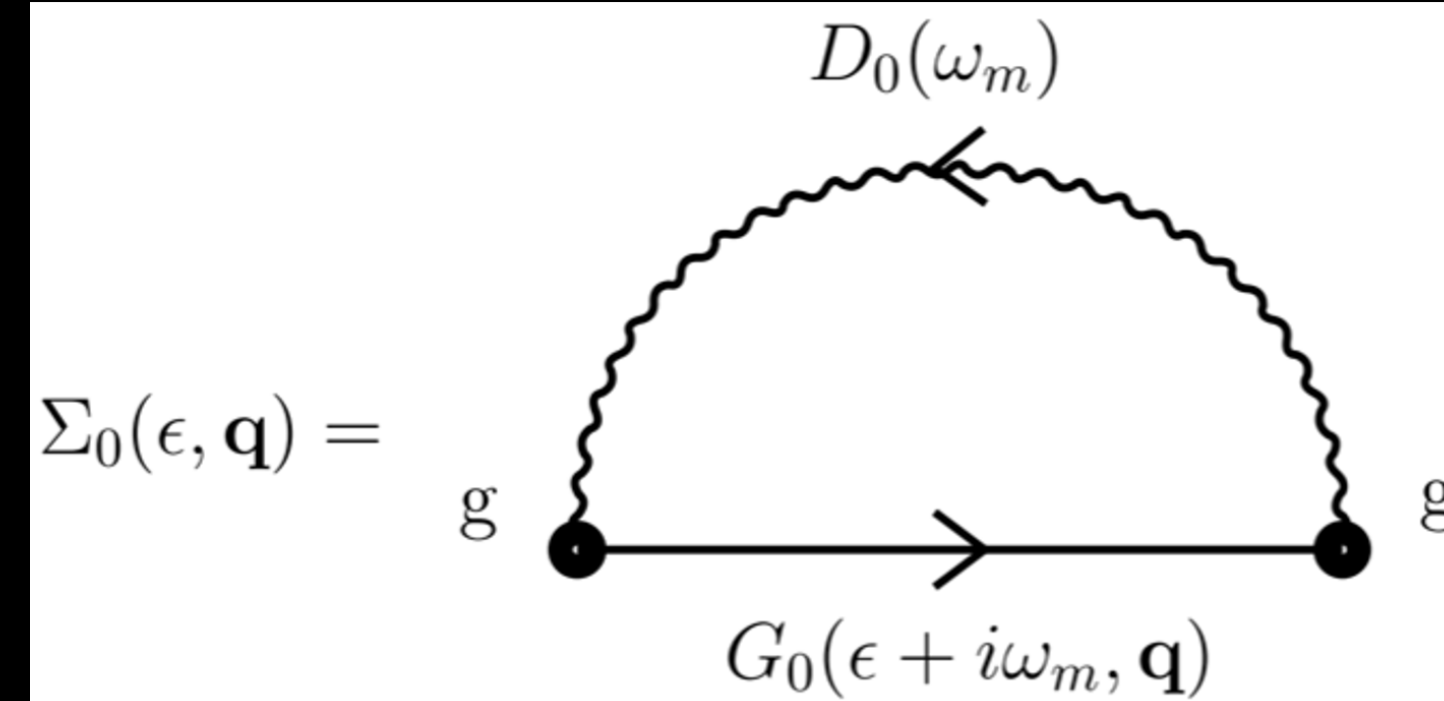
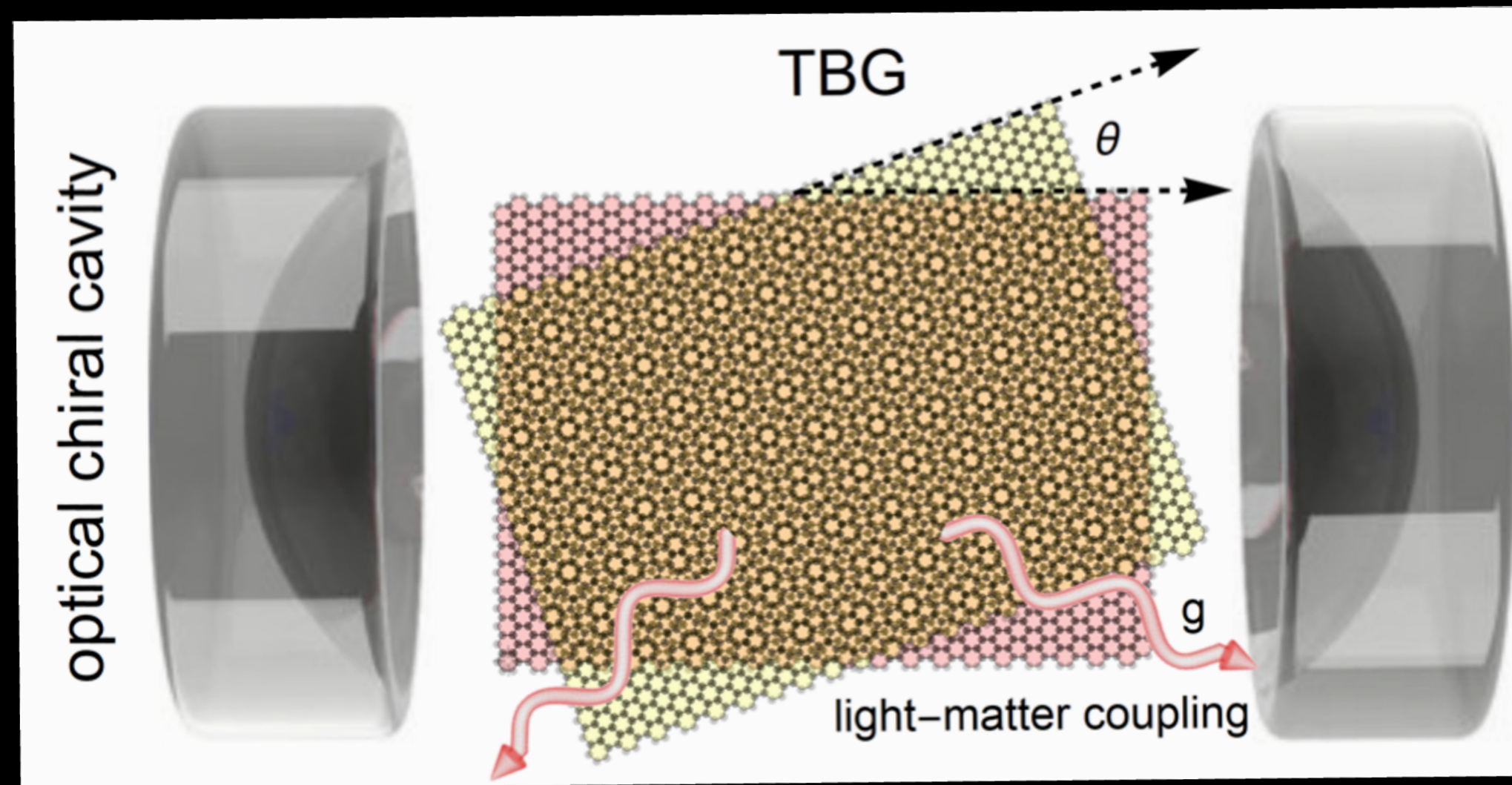
$$\hat{H}^U \approx \frac{[\hat{\mathbf{p}} + e\mathbf{A}(\mathbf{r})]^2}{2m_{\text{eff}}} + V_{\text{eff}}(\mathbf{r})$$

# Graphene in a chiral cavity



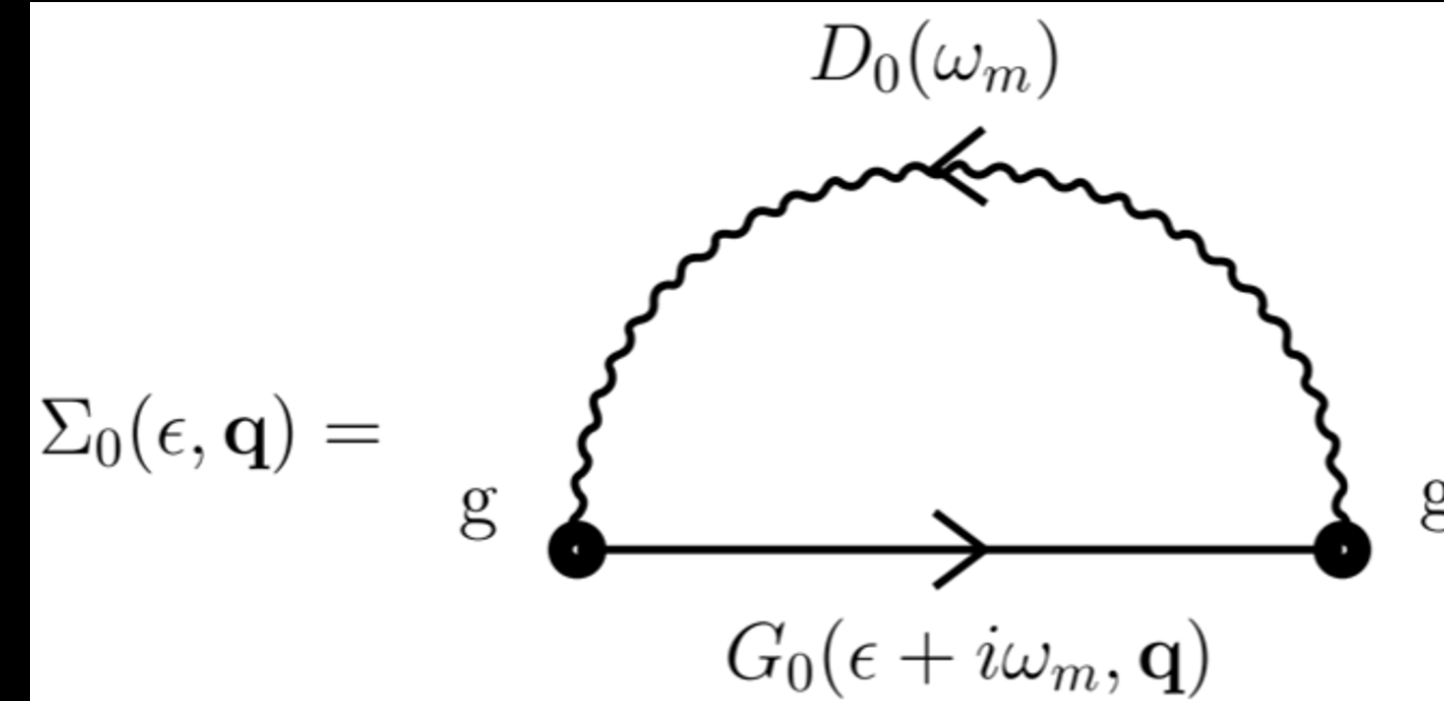
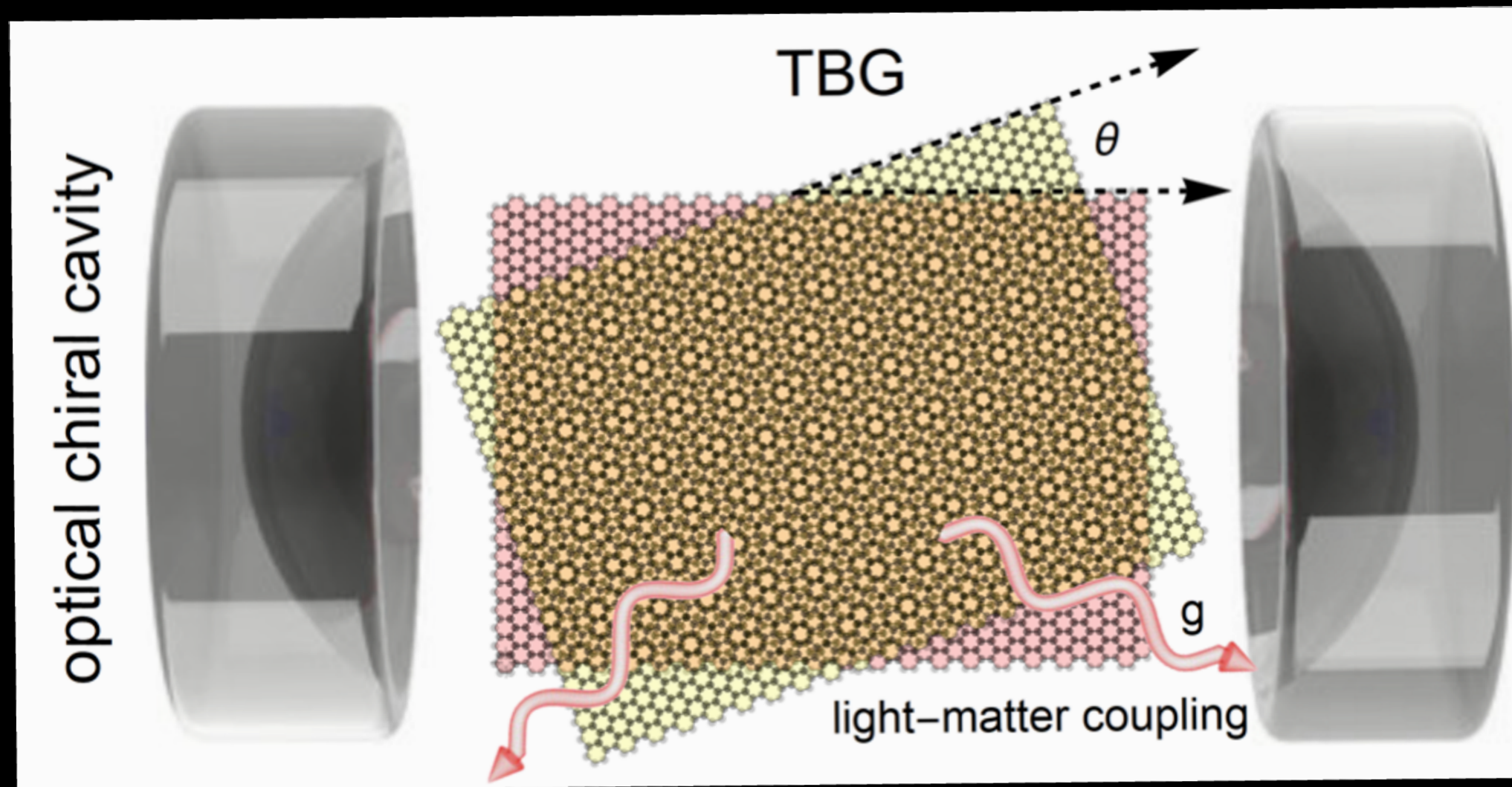
$$n_p(\mathbf{k}) = \langle \hat{a}^\dagger \hat{a} \rangle_C = \frac{\beta T_{\mathbf{k}}}{\hbar \omega_c},$$

# Vacuum to engineer band structure of 2D materials

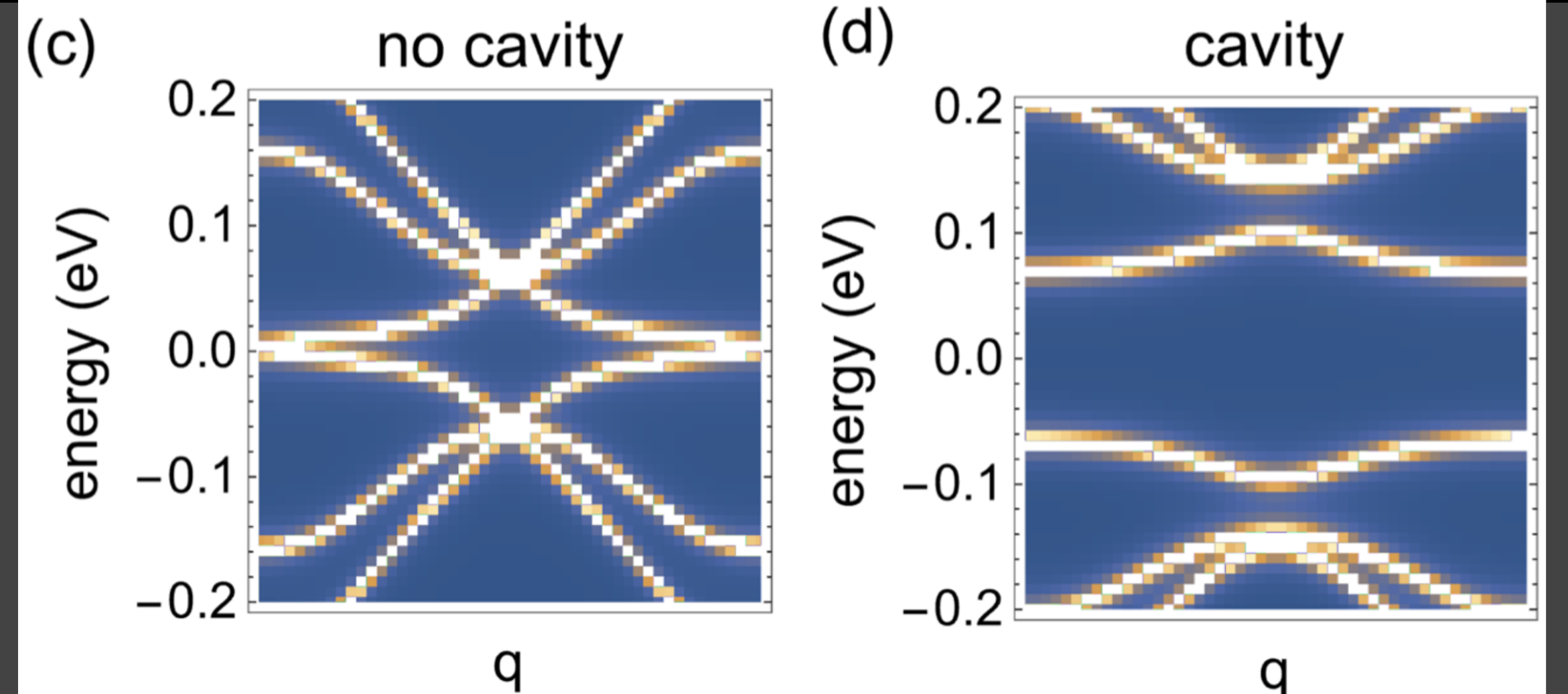
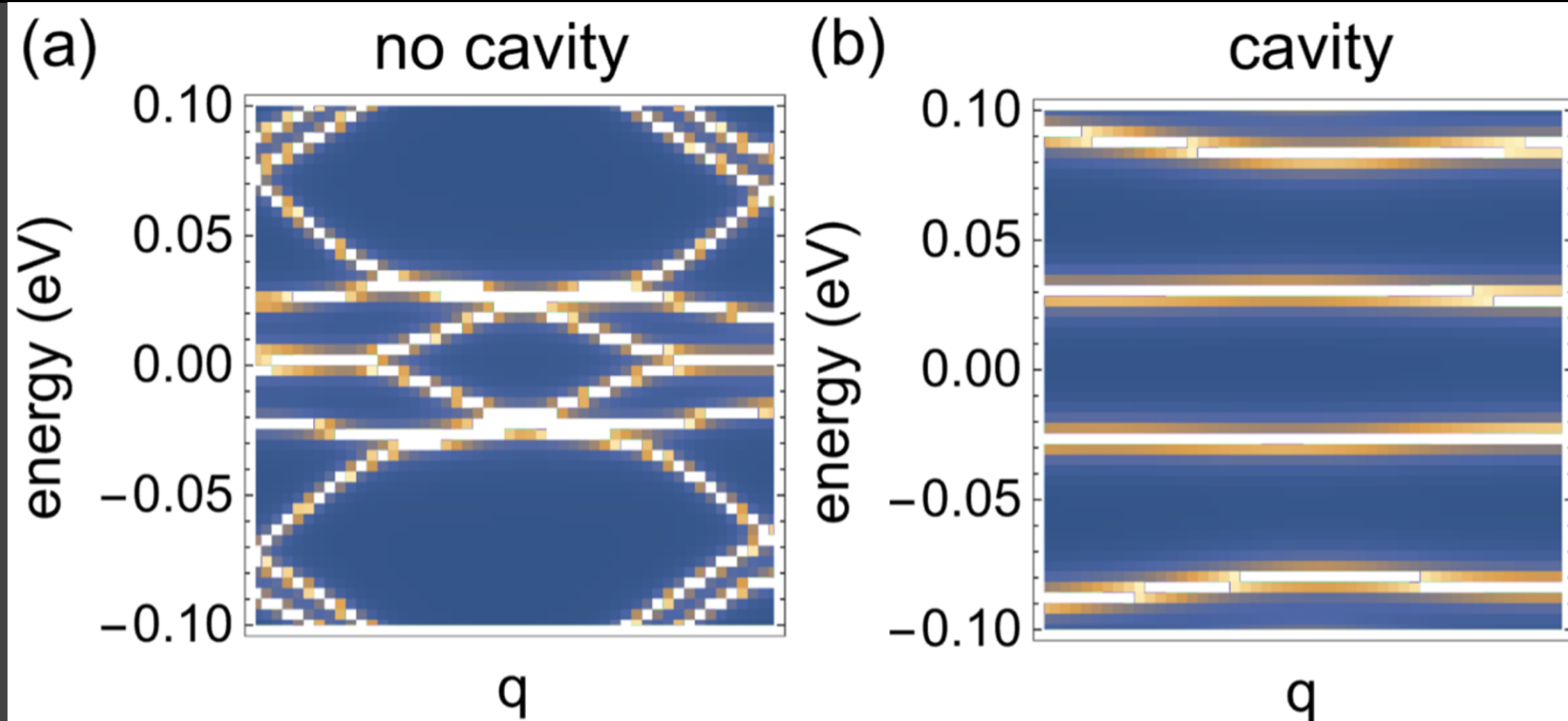


$$\Sigma_0(\epsilon, \mathbf{q}) = -\frac{g^2}{\beta} \sum_{m=1}^{\infty} G_0(\epsilon + i\omega_m, \mathbf{q}) D_0(\omega_m).$$

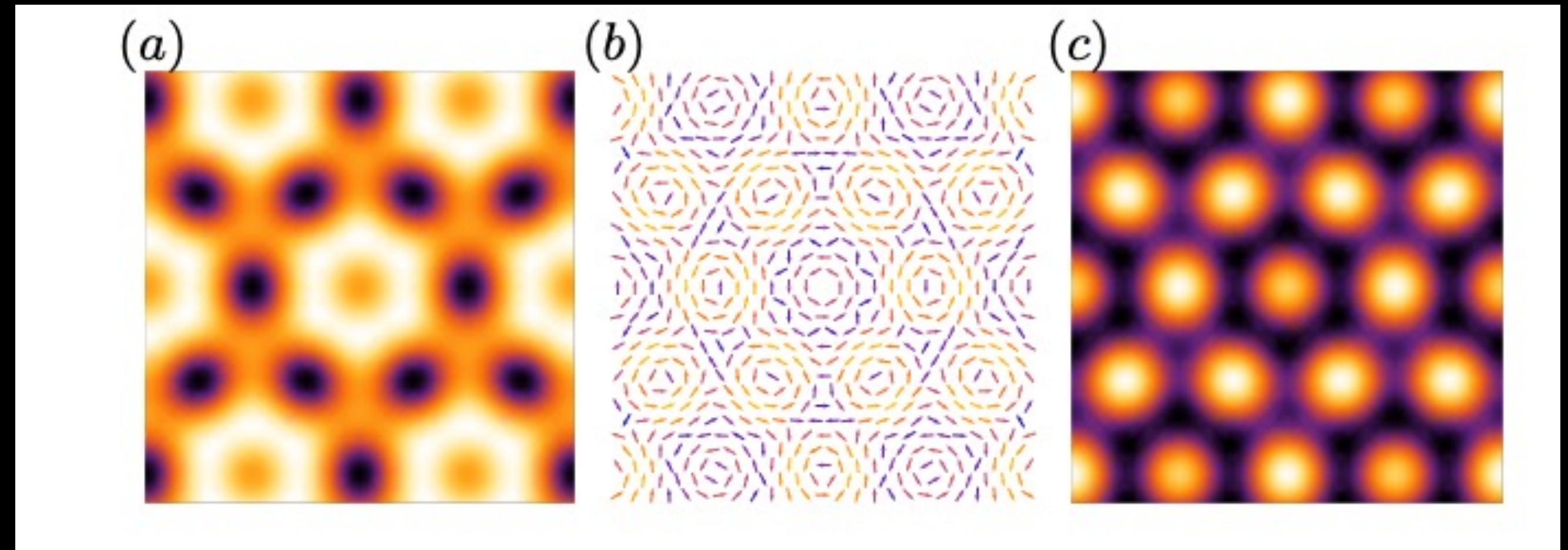
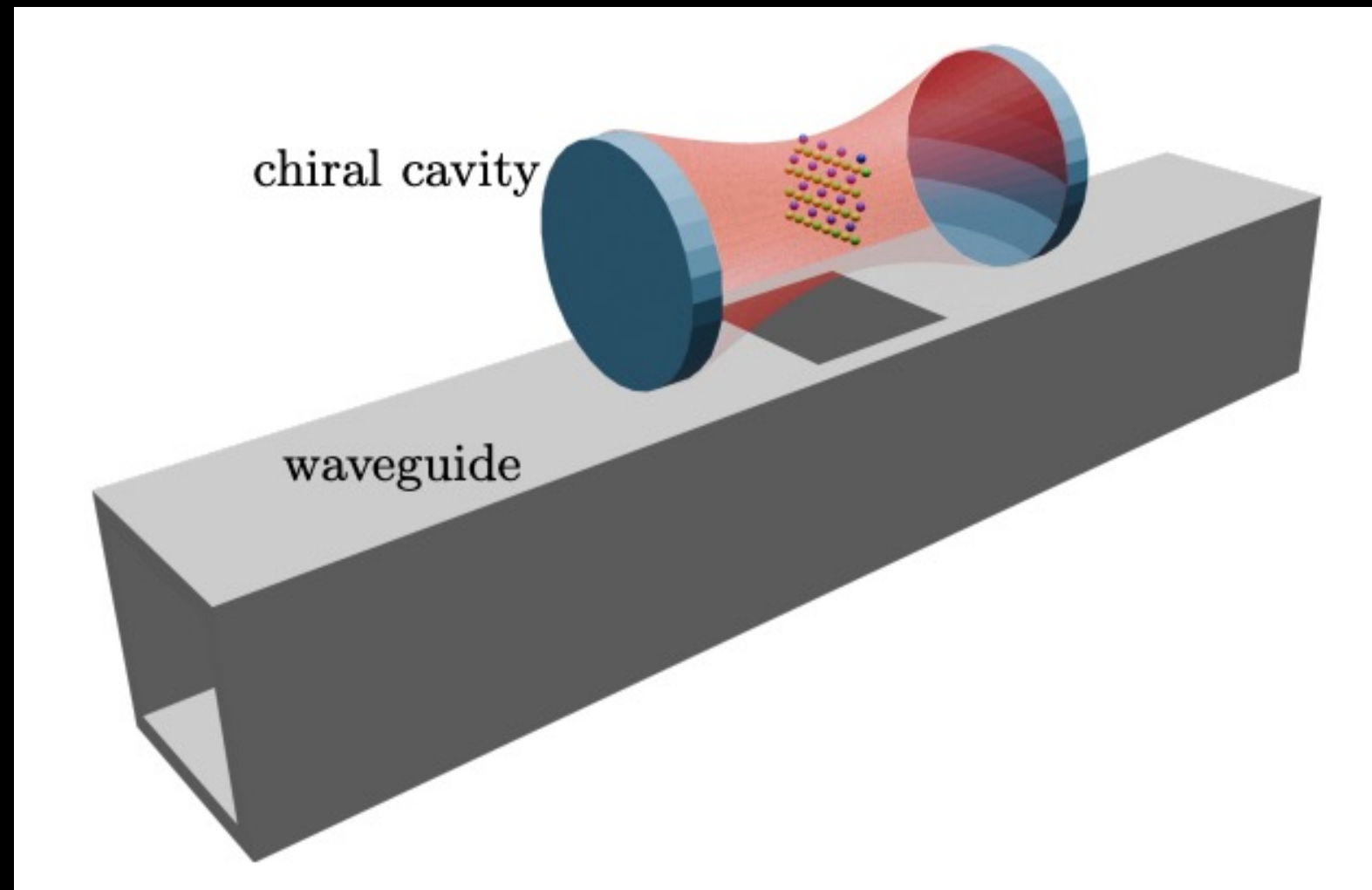
# Vacuum to engineer band structure of 2D materials



$$\Sigma_0(\epsilon, \mathbf{q}) = -\frac{g^2}{\beta} \sum_{m=1}^{\infty} G_0(\epsilon + i\omega_m, \mathbf{q}) D_0(\omega_m).$$



# Chiral virtual photons induce chiral spin liquid



$$H_{\text{AD}} = - \sum_{i,j,\sigma} t e^{i\varphi_{ij}} c_{i\sigma}^\dagger c_{j\sigma} + U' \sum_i n_{i\uparrow} n_{i\downarrow} + \sum_i B (n_{i\uparrow} - n_{i\downarrow})$$

$$H_{\text{eff}} = \sum_{\text{---}} \frac{2|t_{ij}|^2}{U'} (P_{ij} - 1) + \sum_{\triangle} \frac{6 \text{Im}(t_{ij} t_{jk} t_{ki})}{U'^2} i (P_{ijk} - h.c.).$$



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Quantum Connections in Sweden-16  
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**Lecture 1: Renormalization and Casimir Physics**

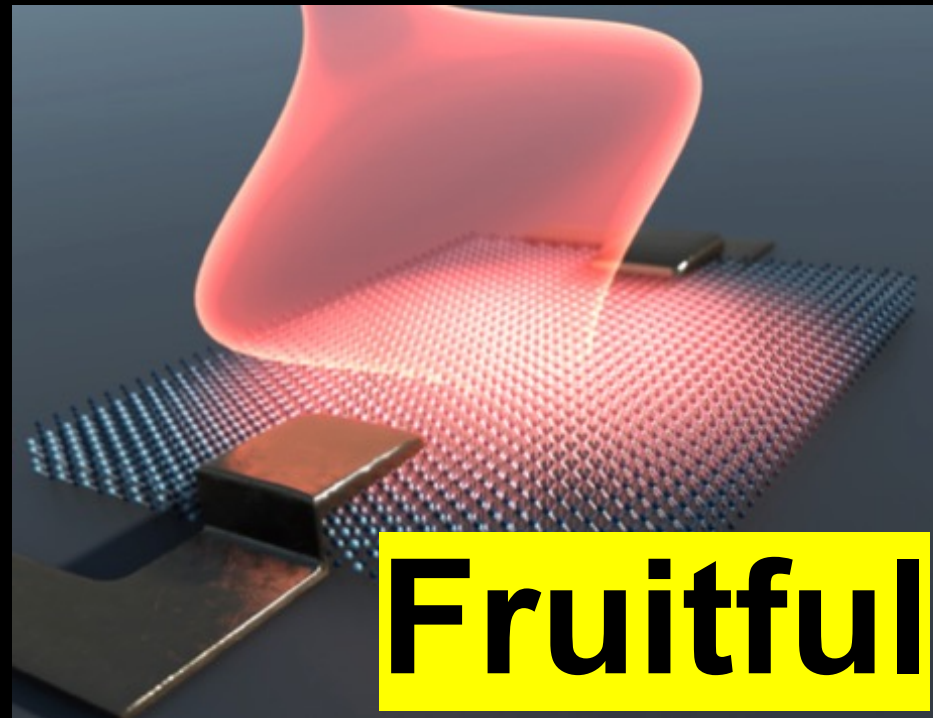
**Lecture 2: Casimir Torque, Friction and Spectra**

**Lecture 3: Quantum atmosphere**

**Lecture 4: Cavity Quantum Materials**

# The Rise of Vacuumronics

N  
Photon  
number



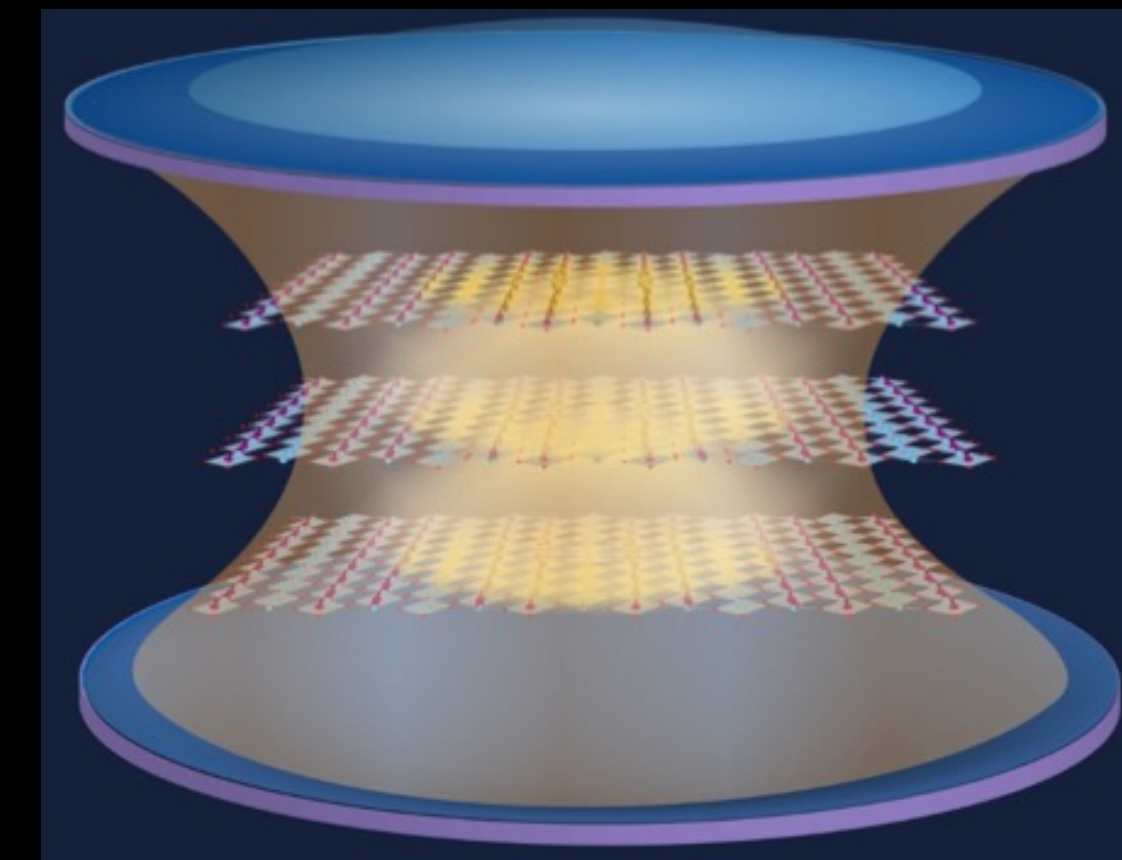
## Advantages

- No heating
- Equilibrium approach
- Strong coupling

### SOMETIMES

Less is More,  
when more is too much.

— Frank Lloyd Wright



Vacuum fluctuation strength

# Acknowledgement

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## Collaborators

Frank Wilczek (MIT), Hans Hansson (Stockholm University)

Cunyu Jiang, Matteo Baggioli, Zixuan Dai,

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# Summary

