Quantum Connections in Sweden 2026

- Lecture 1: Quantum Mechanics of Resonant Harmonic Detectors for Quantized Radiation Fields
- Lecture 2: Probing the Quantum Structure in Gravitational Radiation
- Lecture 3: Statistical Null-Tests of the Coherent State Hypothesis
- Lecture 4: Quantum Mechanics of the Ringdown

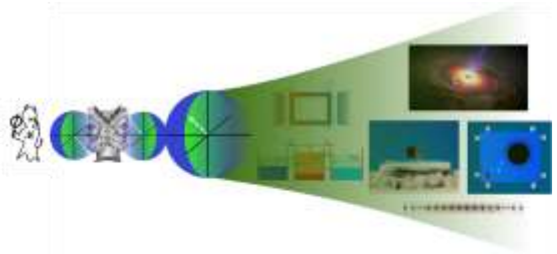
Speaker: Sreenath K. Manikandan,

Reader (F), Tata Institute of Fundamental Research Hyderabad

Email: skm@tifrh.res.in

Scientific Frontiers in Quantum Technologies

Tata Institute of Fundamental Research Hyderabad (TIFRH)



- **The Nature of Time and Clocks in Quantum Mechanics**
- Prithvi N. Bhatt (PhD student at TIFRH)
- Nishanth M (Summer student, IIT Bhubaneswar)
- Sayan Chakraborty (PhD student at TIFRH with Prof. Kabir Ramola)

- **Quantum Mechanics of Resonant Detectors For Quantized Radiation Fields**
- Dr. Athulya K. P. (Postdoctoral researcher at TIFRH)
- Dr. Vivek Pandey (Postdoctoral researcher at TIFRH)
- Shalin Jose (Incoming PARIMANA Postdoctoral fellow, now PhD student with Prof. Anil Shaji at IISER TVM)
- Sreemera (Summer student, University of Hyderabad)
- Jentisenla (Summer student, University of Hyderabad)
- Akshay Raj (Summer student, University of Hyderabad)
- Koundinya (High-school student)

- **Quantum Mechanics of Cooling**
- Aryan Soni (Project student at TIFRH)
- Pijush Varma (JRF at TIFRH with Prof. Karthik Raman)

For my lecture notes in quantum optics/TIFRH, please visit: https://sites.google.com/view/sciquantum/

References:

Lectures 1: Quantum Mechanics of Resonant Harmonic Detectors for Quantized Radiation Fields

- Sreenath K. Manikandan, and Frank Wilczek. "Testing the coherent-state description of radiation fields." Phys. Rev. A 111, 033705 (2025).

Lecture 2: Probing Quantum Structure in Gravitational Radiation

- Sreenath K. Manikandan, and Frank Wilczek. "Testing the coherent-state description of radiation fields." Phys. Rev. A 111, 033705 (2025).
- Germain Tobar*, Sreenath. K. Manikandan*, Thomas Beitel, & Igor Pikovski. “Detecting single gravitons with quantum sensing” Nature Communications 15, 7229 (2024) [equal first author]
- Victoria Shenderov, Mark Kanex, Thomas Beitel, Germain Tobar, Sreenath K. Manikandan, Igor Pikovski, Stimulated absorption of single gravitons: First light on quantum gravity, Annals of Physics, Volume 489,170448 (2026).
- Parikh, Maulik, Frank Wilczek, and George Zahariade. "The noise of gravitons." International Journal of Modern Physics D 29.14 (2020): 2042001.
- Parikh, Maulik, Frank Wilczek, and George Zahariade. "Quantum mechanics of gravitational waves." Physical Review Letters 127.8 (2021): 081602.
- Parikh, Maulik, Frank Wilczek, and George Zahariade. "Signatures of the quantization of gravity at gravitational wave detectors." Physical Review D 104.4 (2021): 046021.

Lecture 3: Statistical Null-Tests of the Coherent State Hypothesis

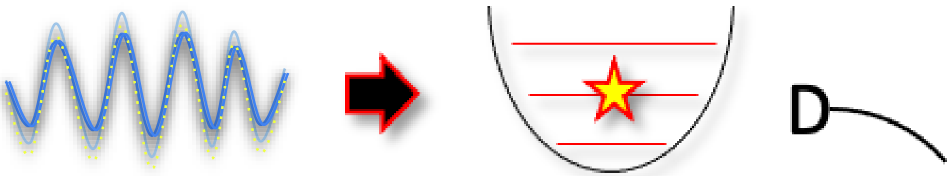
- Sreenath K. Manikandan, and Frank Wilczek. "Complementary Probes of Gravitational Radiation States" Phys. Rev. A 112, 043716 (2025).
- Sreenath K. Manikandan and Frank Wilczek. "Probing Quantum Structure in Gravitational Radiation." International Journal of Modern Physics D, May 22, 2025, 2543001. [First Prize at the Gravity Research Foundation’s essay competition 2025.]
- "Detector Correlations and Null Tests of the Coherent State Hypothesis." Sreenath K. Manikandan, and Frank Wilczek. International Journal of Modern Physics A: 2642001. (2026).
- Yuliya Bilinskaya and Sreenath K. Manikandan. "Quantum Sensing with Joint Emitter-Fluorescence Measurements." arXiv preprint arXiv:2604.11377 (2026).
- K. P. Athulya and Sreenath K. Manikandan. "Correlated Quantum Sensing at the Seemingly Classical Limit." arXiv preprint arXiv:2606.01673 (2026).

Lecture 4: Quantum Mechanics of the Ringdown

- Sreenath K. Manikandan, and Frank Wilczek. "Squeezed Quasinormal Modes from Nonlinear Gravitational Effects." arXiv preprint arXiv:2508.03380 (2025) [Accepted, International Journal of Modern Physics D]
- Das, A., Parikh, M., Wilczek, F., & Wutte, R. (2025). “Squeezed States in Gravity”. arXiv preprint arXiv:2512.20601.

Also see:

Kanno, Sugumi, Jiro Soda, and Akira Taniguchi. "Coherent State Description of Gravitational Waves from Binary Black Holes." Physical review letters 136.6 (2026): 061404.



Lecture 1: Quantum Mechanics of Resonant Harmonic Detectors for Quantized Radiation Fields

Speaker: Sreenath K. Manikandan,
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Based on:

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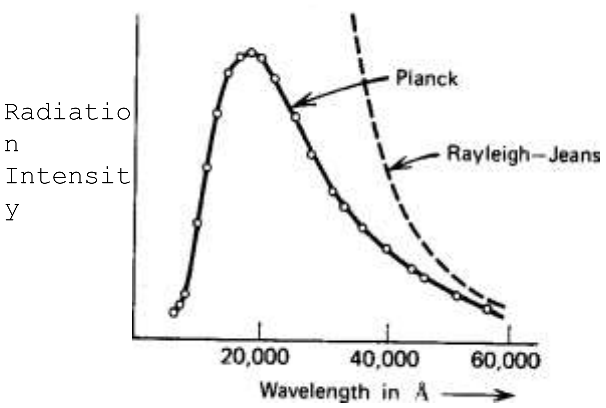
Photons as quantized packets of energy exchanged with matter

$$\frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/k_BT}-1}$$

❖ 1900: Planck's explanation for the black body radiation spectrum:



https://quantummechanics.ucsd.edu/ph130a/130_notes/node48.html

$$\frac{8\pi\nu^2}{c^3}k_BT \Leftarrow \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/k_BT}-1} \Rightarrow a\nu^3 e^{-\frac{b\nu}{k_BT}}$$

$$E = 0, h\nu, 2h\nu, 3h\nu\ \ldots$$

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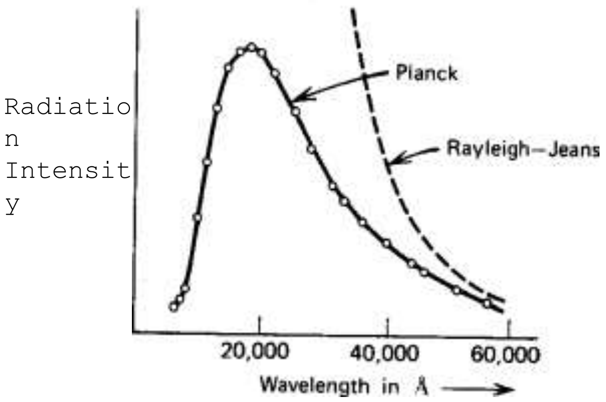

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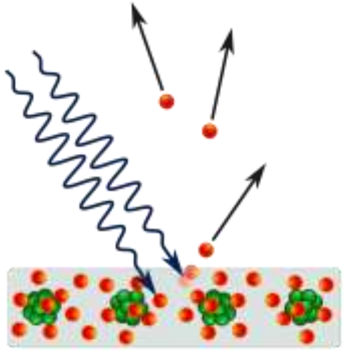
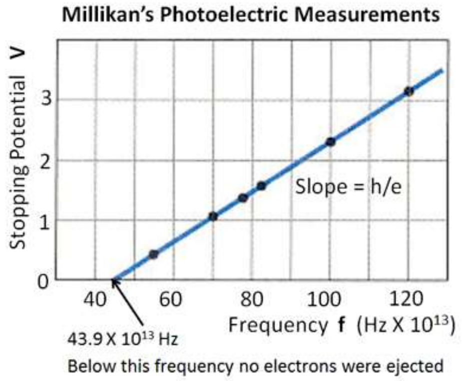
- 1905: Einstein's explanation for the photoelectric effect, and the light-quantum hypothesis:

Credits: Wikipedia





- 1916: Millikan experimentally tests the photoelectric effect.

Towards modern quantum optics

- Semiclassical limit of QED also works to capture some of the essential features of the photoelectric effect [Lamb Jr, W. E., & Scully, M. O. (1968).], also see [1]. The semiclassical limit of QED however is a convenient approximation, it violates energy conservation for example.

[1] Scully, M. O., & Sargent, M. (1972). The concept of the photon. Physics Today, 25(3), 38-47.

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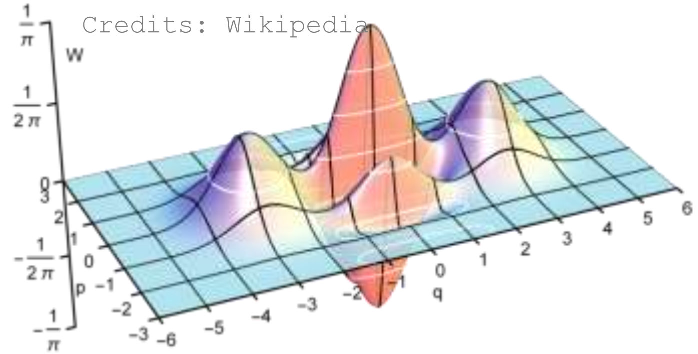
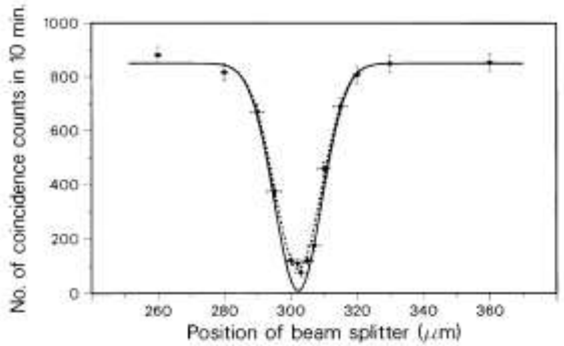
- Bell inequality violation experiments: the only loophole free test of non-classicality
- Sub-Poissonian statistics, Hong-Ou-Mandel effect, Wigner negativities, squeezing, non-classical correlations...



Wigner function

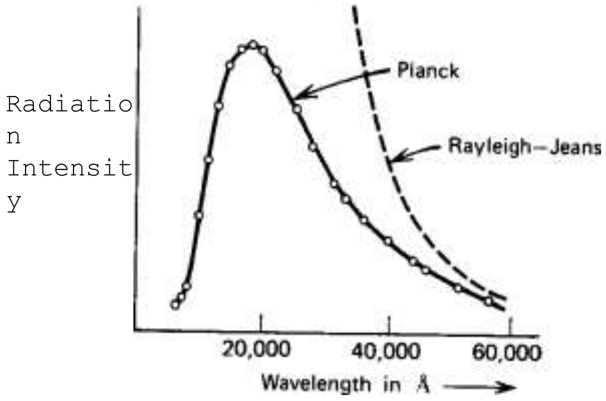


Hong-Ou-Mandel dip

C. K. Hong; Z. Y. Ou & L. Mandel (1987). Phys. Rev. Lett. 59 (18): 2044–2046.

[1] Scully, M. O., & Sargent, M. (1972). The concept of the photon. Physics Today, 25(3), 38–47.

The energy density of light comprise of discrete packets…

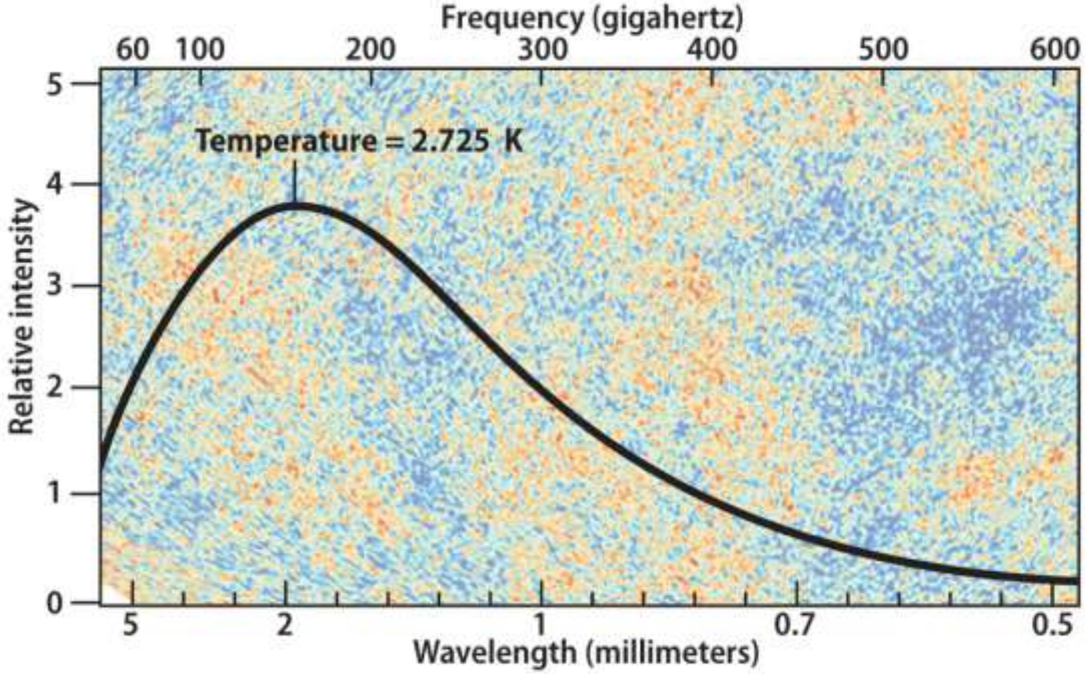


https://quantummechanics.ucsd.edu/ph130a/130_notes/node48.html

$$\frac{8\pi\nu^2}{c^3}k_BT \Leftarrow \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/k_BT}-1} \Rightarrow a\nu^3 e^{-\frac{b\nu}{k_BT}}$$

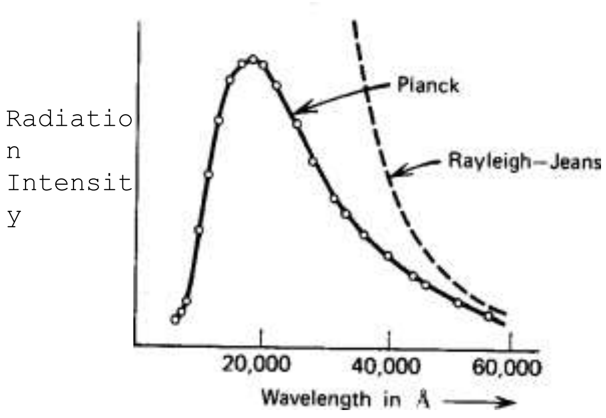
$$\Delta E = 0, h\nu, 2h\nu, 3h\nu \;\ldots$$

A hot, metallic box with a small pinhole



Cosmic microwave background

…as a sum over the energies of quantum harmonic oscillators



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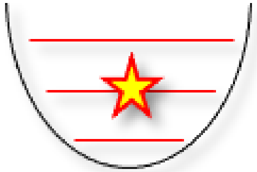
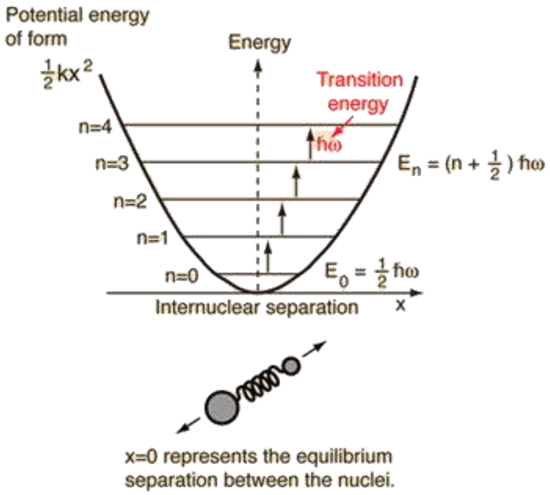


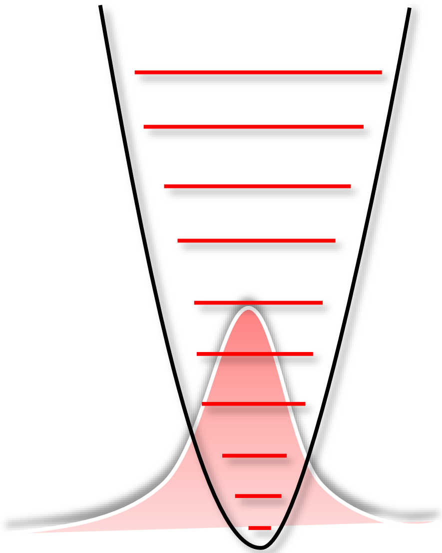
- $H_k = \hbar\omega_k\left(a^\dagger a + \frac{1}{2}\right)$
- $H_k|n\rangle = \hbar\omega_k\left(n + \frac{1}{2}\right)|n\rangle$
- Annihilation: $a|n\rangle = \sqrt{n}|n-1\rangle$
- Creation: $a^\dagger|n\rangle = \sqrt{n}|n+1\rangle$



http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/hosc2.html

Coherent states are right eigenstates of the annihilation operator

- Coherent states are the eigenstates of the annihilation operator: $a|\alpha\rangle = \alpha|\alpha\rangle$
- We want to get $a\sum_{n=0}^{\infty} c_n|n\rangle = \alpha\sum_{n=0}^{\infty} c_n|n\rangle$.
- Since $a|n\rangle = \sqrt{n}|n-1\rangle$, we get $a\sum_{n=0}^{\infty} c_n|n\rangle = \sum_{n=1}^{\infty} c_n\sqrt{n}|n-1\rangle = \alpha\sum_{n=0}^{\infty} c_n|n\rangle$
- $c_1 = \alpha c_0,\; c_2\sqrt{2} = \alpha c_1 = \alpha^2 c_0,\; c_3\sqrt{3} = \alpha c_2 = \frac{\alpha c_1}{\sqrt{2}} = \frac{\alpha^2 c_0}{\sqrt{2}}, c_k = \frac{\alpha^k c_0}{\sqrt{k!}}$.
- Normalized state: $\sum_n |c_n|^2 = 1$. This yields, $|\mathrm{c}_0|^2 \sum_n \frac{|\alpha|^{2n}}{n!} = |\mathrm{c}_0|^2 e^{|\alpha|^2} = 1$.
- $|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle$ that obeys $a|\alpha\rangle = \alpha|\alpha\rangle$.



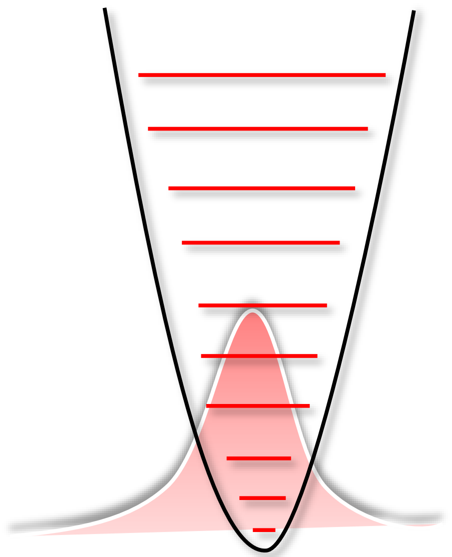
Coherent state occupation probabilities are Poissonian

❖ Eigenstates of the annihilation operator: $a|\alpha\rangle = \alpha|\alpha\rangle$

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$p(n) = \frac{e^{-|\alpha|^2} |\alpha|^{2n}}{n!}.$$

Poissonian probabilities



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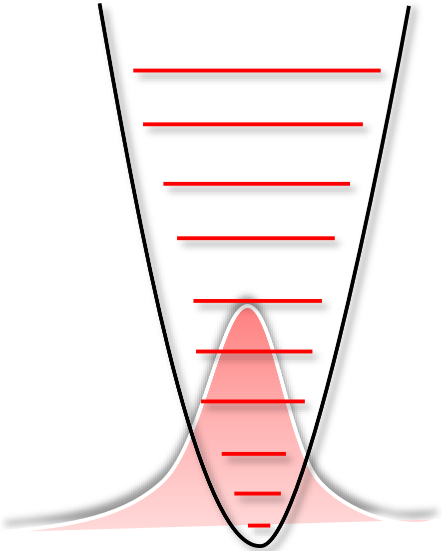

$$\langle n\rangle = \sum_n n\, p(n) = e^{-|\alpha|^2} \sum_n \frac{|\alpha|^{2n}}{(n-1)!} = e^{-|\alpha|^2} |\alpha|^2 \sum_n \frac{|\alpha|^{2(n-1)}}{(n-1)!} = e^{-|\alpha|^2} e^{|\alpha|^2} = |\alpha|^2$$

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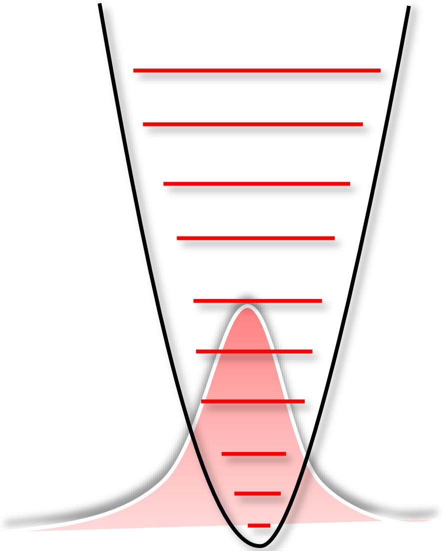

$$\langle n^2 \rangle = \langle n(n-1) + n \rangle = |\alpha|^2 + \sum_n n(n-1)p(n) = |\alpha|^2 + |\alpha|^4$$

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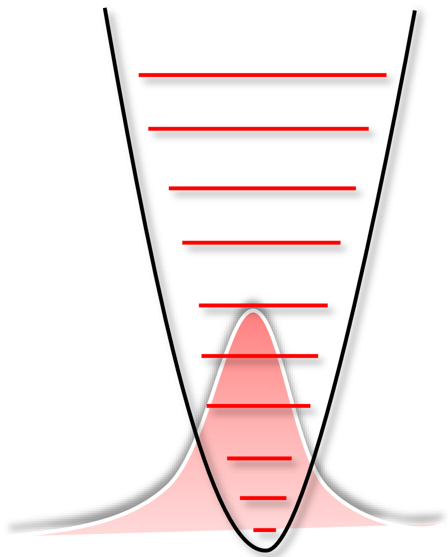

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$$\langle n^2 \rangle - \langle n \rangle^2 = |\alpha|^2 + |\alpha|^4 - (|\alpha|^2)^2 = |\alpha|^2 = \langle n \rangle$$

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The semi-classical limit/optical equivalence

- Plane waves in linearized quantum field theories (quantum optics, quantum acoustics, or linearized gravitational waves) can be decomposed generically as,

$$\hat{F} = \frac{1}{\sqrt{V}} \sum_{k,s} f_{k,s} (\hat{a}_{k,s} e^{ik.r - i\omega_k t} + \hat{a}^{\dagger}{}_{k,s} e^{-ik.r + i\omega_k t}),$$

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- For coherent states of the optical field we can substitute the field amplitude operators with complex field amplitudes,

$$\langle \{\alpha\} | \hat{F} | \{\alpha\} \rangle \rightarrow \frac{1}{\sqrt{V}} \sum_{k,s} f_{k,s} (\alpha_{k,s} e^{ik.r - i\omega_k t} + \alpha^*_{k,s} e^{-ik.r + i\omega_k t}) \,.$$

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$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

❖ This can be done because coherent states have zero variance (noise) for the field amplitude operators,

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \qquad \hat{a}^2|\alpha\rangle = \alpha^2|\alpha\rangle \qquad \langle\alpha|\hat{a}^2|\alpha\rangle - \langle\alpha|a|\alpha\rangle^2 = 0$$

$$\langle\alpha|\hat{a}^{\dagger} = \alpha^{*}\langle\alpha| \qquad \langle\alpha|(\hat{a}^{\dagger})^2 = \langle\alpha|(\alpha^{*})^2 \qquad \langle\alpha|(\hat{a}^{\dagger})^2|\alpha\rangle - \langle\alpha|\hat{a}^{\dagger}|\alpha\rangle^2 = 0.$$

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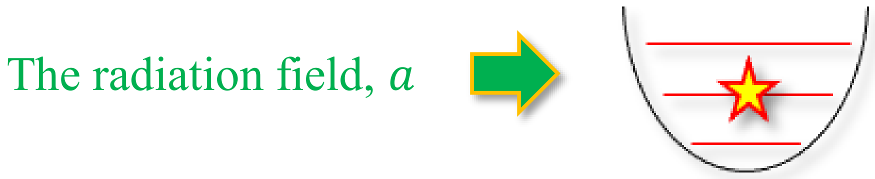
Quantum mechanics of a resonant harmonic detector for radiation fields

❖ The interaction (picture) Hamiltonian in the rotating wave approximation between the detector and the radiation field can be approximated as,

$$V_I = \hbar\sqrt{\gamma_0}\left[d(t)a^\dagger + d^\dagger(t)a\right]. \qquad \text{Assume} \quad \left[d(t), d^\dagger(t')\right] = \delta(t - t').$$



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○ Here γ_0 is the spontaneous emission rate of the detector for the radiation field, an intrinsic property of the detector.

❖ The time evolution operator can be approximated as $U_I = e^{-i/\hbar \int V_I(t')dt'} \approx e^{-\frac{i}{\hbar}H_I\Delta t}$ where,

$$H_I\Delta t \approx \hbar\sqrt{\gamma_0\Delta t}\left[ba^\dagger + b^\dagger a\right], \qquad b = \frac{1}{\sqrt{\Delta t}}\int_t^{t+\Delta t} dt' d(t') \quad \text{such that} \quad \left[b, b^\dagger\right] = 1.$$

The radiation field, a

○ Sreenath K. Manikandan, and Frank Wilczek. "Testing the coherent-state description of radiation fields." Phys. Rev. A 111, 033705 (2025).

Coherent states of the field creates coherent states in the detector

- Coherent state of the field, $a|\alpha\rangle = \alpha|\alpha\rangle$, coupled to the vacuum of the detector $\mathrm{b}|0\rangle = 0$, evolves as,

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Let us show this!

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$$\rho = \hat{1}\rho\hat{1} = \int d^2\alpha d^2\beta\langle\beta|\rho|\alpha\rangle|\beta\rangle\langle\alpha|$$
$$= \int d^2\alpha d^2\beta\langle\beta|\rho|\alpha\rangle D(\beta-\alpha)|\alpha\rangle\langle\alpha|$$

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Little detour: the P representation (or ϕ representation)

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Let us compute the P distribution function for a generic state!

C. L. Mehta, Phys. Rev. Lett., 18 (1967), 752.

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$$\langle -u|\rho|u\rangle = \int d^2\alpha \, P(\alpha) e^{-|\alpha|^2-|u|^2+\alpha^* u-u^*\alpha}$$

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Note that $\alpha^* u - u^* \alpha = 2i[Im(\alpha)Re(u) - Im(u)Re(\alpha)]$, suggesting that the above is a complex Fourier transform from $\alpha \to u$. We can therefore evaluate $P(\alpha)$ as the inverse Fourier transform of the LHS,

$$P(\alpha) = \frac{e^{|\alpha|^2}}{\pi^2}\int d^2u e^{|u|^2}\langle -u|\rho|u\rangle e^{\alpha^* u - u^* \alpha}$$

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❖ Let us evaluate the P representation for a fock state, $\rho = |n\rangle\langle n|$:

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$$= \frac{e^{|\alpha|^2}}{\pi^2 n!}\frac{\partial^n}{(\partial\alpha^*)^n}\frac{\partial^n}{(\partial\alpha)^n}\delta^2(\alpha)$$

- The P representation for a fock state, $\rho = |n\rangle\langle n|$ involves n^{th} derivatives of the Dirac delta function!

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- This yields the probability P_n of seeing n quantum jumps (clicks!) in the detector,

$$P_n = \frac{\sin^{2n}\left(\sqrt{\gamma_0\Delta t}\right)}{n!}\int d^2\alpha P(\alpha)|\alpha|^{2n}e^{-|\alpha|^2\sin^2\left(\sqrt{\gamma_0\Delta t}\right)}.$$

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❖ The linearized regime $\sin\left(\sqrt{\gamma_0\Delta t}\right) \approx \sqrt{\gamma_0 \Delta t}$ reproduces the standard result from Mandel and Wolf:

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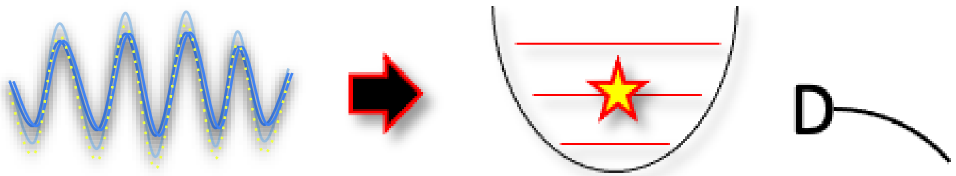

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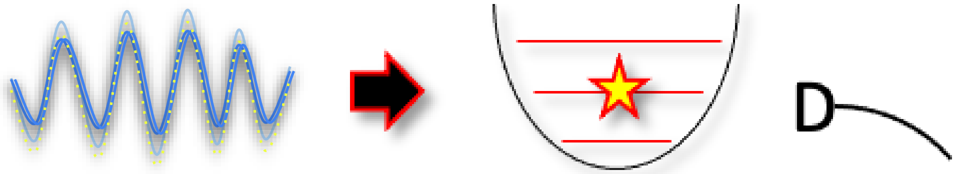
- Textbooks (e.g. Mandel and Wolf) arrives at the result considering several photodetectors interacting with the radiation field in sequence, to the linear order. [See M. Srinivas and E. Davies, Photon Counting Probabilities in Quantum Optics, Optica Acta: International Journal of Optics 28, 981 (1981) for a discussion on limitations of this result, and suggestions for improvements].



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Let us compute the average number of clicks!

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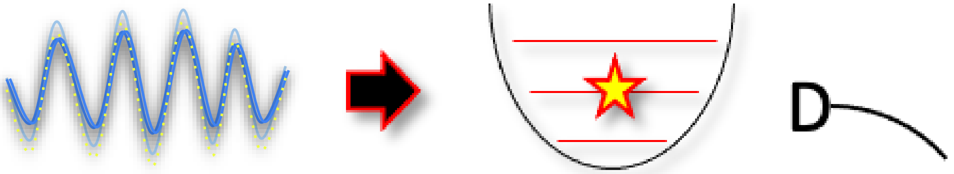
- Average number clicks: $\sum_n nP_n = \bar{n} = \sin^2\left(\sqrt{\gamma_0 \Delta t}\right) \int d^2\alpha P(\alpha) |\alpha|^2$



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Little detour: the optical equivalence theorem

- We can use the Sudarshan-Glauber P representation to compute expectation values of normally ordered operators as P function integrals:

$$\rho = \int d^2\alpha\, P(\alpha)|\alpha\rangle\langle\alpha|.$$

- $\left\langle \left(a^\dagger\right)^m a^n \right\rangle = tr\left(\rho \left(a^\dagger\right)^m a^n\right) = tr\left(\int d^2\alpha\, P(\alpha)|\alpha\rangle\langle\alpha| \left(a^\dagger\right)^m a^n\right)$

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$$= \int d^2\alpha\, P(\alpha) \sum_n \langle n|\alpha\rangle\langle\alpha|\left(a^\dagger\right)^m a^k|n\rangle = \int d^2\alpha\, P(\alpha) \sum_n \langle\alpha|\left(a^\dagger\right)^m a^k|n\rangle\langle n|\alpha\rangle$$

$$= \int d^2\alpha\, P(\alpha)\langle\alpha|\left(a^\dagger\right)^m a^k|\alpha\rangle = \int d^2\alpha\, P(\alpha)(\alpha^*)^m\alpha^k$$

Gerry C, Knight P. Introductory Quantum Optics. Cambridge University Press; 2004.

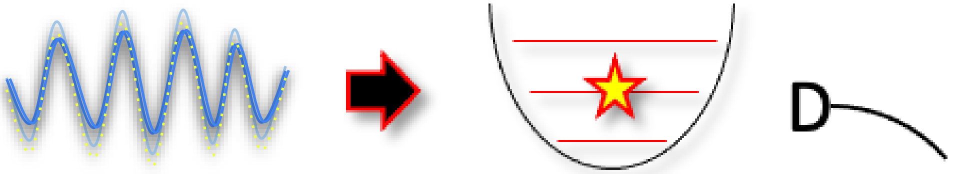
Good detectors are good emitters

- The probability P_n of seeing n quantum jumps (clicks!) in the detector,

$$P_n = \frac{\sin^{2n}\left(\sqrt{\gamma_0 \Delta t}\right)}{n!} \int d^2\alpha P(\alpha) |\alpha|^{2n} e^{-|\alpha|^2 \sin^2\left(\sqrt{\gamma_0 \Delta t}\right)}.$$

$$\hat{N} = a^\dagger a$$

- Average number clicks: $\sum_n n P_n = \bar{n} = \sin^2\left(\sqrt{\gamma_0 \Delta t}\right) \int d^2\alpha P(\alpha) |\alpha|^2 = \sin^2\left(\sqrt{\gamma_0 \Delta t}\right) \langle \hat{N} \rangle \approx \gamma_0 \Delta t \langle \hat{N} \rangle.$



o Sreenath K. Manikandan, and Frank Wilczek. "Testing the coherent-state description of radiation fields." Phys. Rev. A 111, 033705 (2025).

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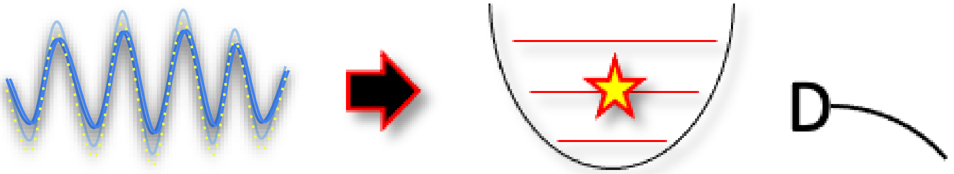
$$\bar{n}_{clicks} \approx \gamma_0 \Delta t \langle \hat{N} \rangle_{Field}$$

$\bar{n}_{clicks}$ = radiation quantum to click conversion efficiency × average number of radiation quantum.



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Let us compute the variance of clicks

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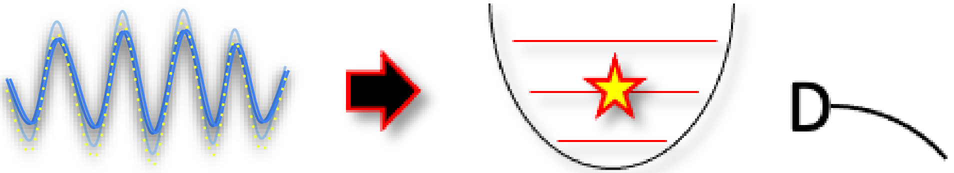

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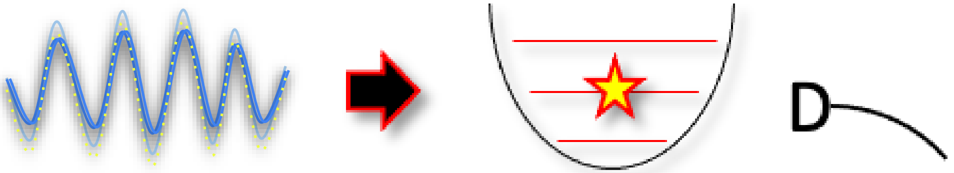

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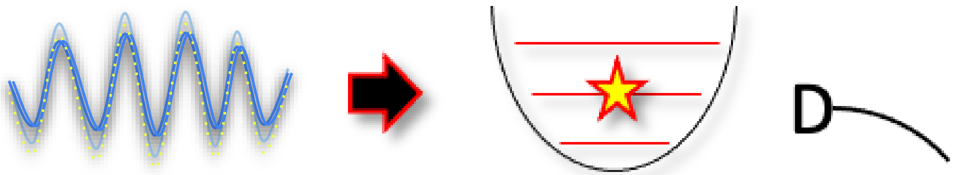
❖ Variance of counts is, $(\Delta n^2) = \overline{\mathrm{n}^2} - \bar{n}^2$

We obtain: $(\Delta n^2) = \bar{n} + (\gamma_0 \Delta t)^2 \left\{ \int d^2\alpha P(\alpha)|\alpha|^4 - \left[\int d^2\alpha P(\alpha)|\alpha|^2\right]^2 \right\}$



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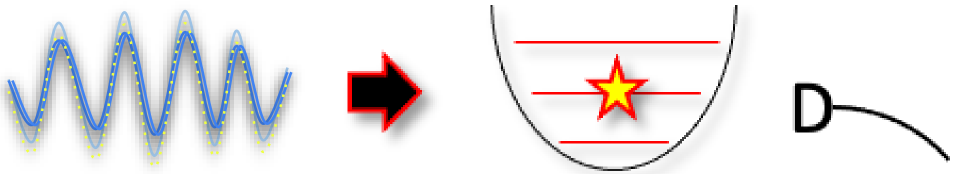

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$$= \bar{n} + (\gamma_0 \Delta t)^2 \left[\langle \hat{N}^2 \rangle - \langle \hat{N} \rangle - \langle \hat{N} \rangle^2 \right] = \bar{n} + (\gamma_0 \Delta t)^2 \left[\langle \Delta \hat{N}^2 \rangle - \langle \hat{N} \rangle \right]$$

[Variance minus the mean]

- Sreenath K. Manikandan, and Frank Wilczek. "Testing the coherent-state description of radiation fields." Phys. Rev. A 111, 033705 (2025).

The variance of clicks can also be computed

- The probability P_n of seeing n quantum jumps (clicks!) in the detector,

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$$\bar{n}_{clicks} \approx \gamma_0 \Delta t \langle \hat{N} \rangle_{Field}.$$

Varianceof clicks: $\sum_n n^2 P_n - \bar{n}^2 = \bar{n} + (\gamma_0 \Delta t)^2 Q \langle \hat{N} \rangle_{Field}$, $Q = \frac{\langle \Delta \hat{N}^2 \rangle - \langle \hat{N} \rangle}{\langle \hat{N} \rangle}$ for the field state.

- $-1 \leq \mathrm{Q} < 0$: Sub − Poissonian, $\mathrm{Q} = 0$: Poissonian, $\mathrm{Q} > 0$ super − Poissonian

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Global counting statistics: examples

$$|\beta\rangle = e^{-\frac{|\beta|^2}{2}} \sum_n \frac{\beta^n}{\sqrt{n!}} |n\rangle$$

❖ For coherent states, $|\beta\rangle$, the mean clicks equals the variance of clicks, $\langle \hat{N} \rangle = |\beta|^2 = \langle \Delta \hat{N}^2 \rangle$.

$$P(\alpha) = \delta(\alpha - \beta), \qquad \bar{n} = \gamma_0 \Delta t |\beta|^2, \qquad \langle \Delta n^2 \rangle = \gamma_0 \Delta t |\beta|^2, \qquad Q = 0$$

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- For a thermal state,

$$P(\alpha) = \frac{1}{\pi n_{th}} e^{-\frac{|\alpha|^2}{n_{th}}}, \qquad \bar{n} = \gamma_0 \Delta t\, n_{th},. \qquad (\Delta n^2) = \gamma_0\, \Delta t\, n_{th} + (\gamma_0 \Delta t)^2\, n_{th}^2, \qquad Q = n_{th}$$

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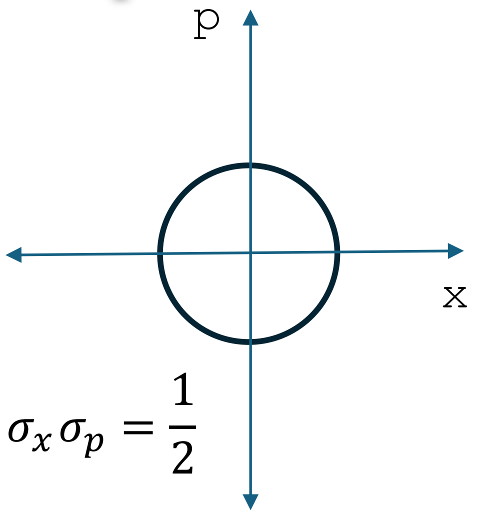
Coherent states saturate the uncertainty principle

❖ Let us define $X = \frac{a+a^\dagger}{\sqrt{2}}$, and $P = \frac{a-a^\dagger}{i\sqrt{2}}$. For coherent states, their variances are given by

$$\sigma_x^2 = \langle\alpha|(\Delta X)^2|\alpha\rangle = \langle\alpha|X^2|\alpha\rangle - \langle\alpha|X|\alpha\rangle^2 = \frac{1}{2}$$

$$\sigma_p^2 = \langle\alpha|(\Delta P)^2|\alpha\rangle = \langle\alpha|P^2|\alpha\rangle - \langle\alpha|P|\alpha\rangle^2 = \frac{1}{2}.$$



$$\sigma_x \sigma_p = \frac{1}{2}$$

Squeezed states are non-classical

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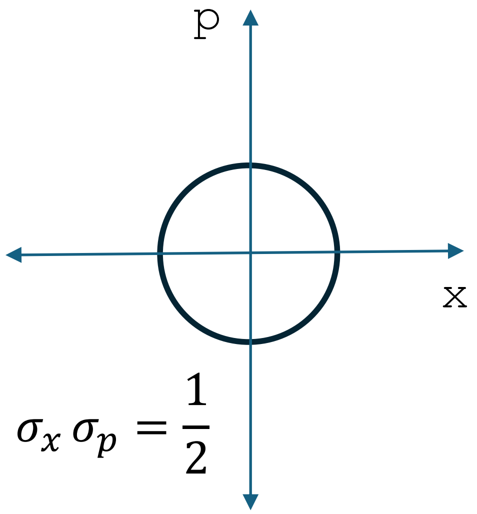
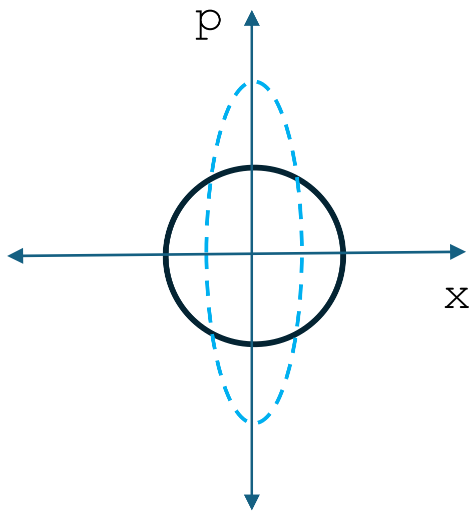


- For a generic state $\rho = \int d^2\alpha\, P(\alpha)|\alpha\rangle\langle\alpha|$,

$$\langle(\Delta X)^2\rangle = \frac{1}{2} + 2\int d^2\alpha P(\alpha)[Re(\alpha) - \langle Re(\alpha)\rangle]^2$$

$$\langle(\Delta P)^2\rangle = \frac{1}{2} + 2\int d^2\alpha P(\alpha)[Im(\alpha) - \langle Im(\alpha)\rangle]^2$$



- Squeezed states have below 1/2 noise in one of the quadratures, and this requires the P function to have some negativity

Global counting statistics: examples

- For coherent states, $|\beta\rangle$, the mean equals the variance, $\langle\hat{N}\rangle = |\beta|^2 = \langle\Delta\hat{N}^2\rangle$.

$$P(\alpha) = \delta(\alpha - \beta), \qquad \bar{n} = \gamma_0 \Delta t |\beta|^2, \qquad (\Delta n^2) = \gamma_0 \Delta t |\beta|^2, \qquad Q = 0$$

- For a thermal state,

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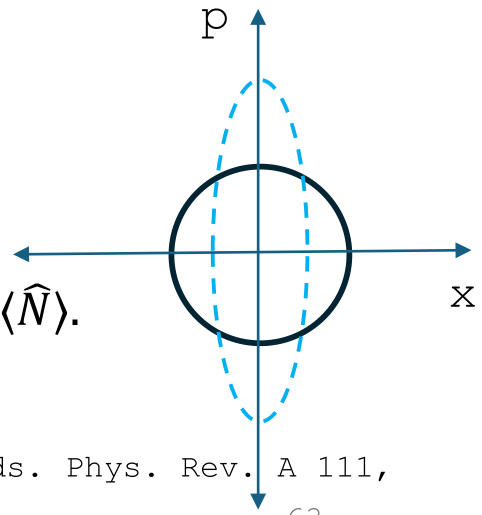
- For a highly squeezed vacuum state,



$$|\psi_{sq}\rangle = \frac{1}{\sqrt{\cosh r}} \sum_m \frac{(-\tanh r)^m \sqrt{2m!}}{2^m m!} |2m\rangle, \qquad \bar{n} = \gamma_0 \Delta t \, (\sinh r)^2,$$

$$(\Delta n^2) = \bar{n} + (\gamma_0 \Delta t)^2 \cosh 2r \, (\sinh r)^2, \qquad Q = \cosh 2r = 1 + 2\,(\sinh r)^2 = 1 + 2\langle\hat{N}\rangle \approx 2\langle\hat{N}\rangle.$$

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- For sub-Poissonian states $-1 \leq Q < 0$, and $Q = -1$ for Fock (number) states $|n\rangle$.

$$\bar{n} = \gamma_0\Delta t\,\langle\hat{N}\rangle., \qquad (\Delta n^2) = \gamma_0\,\Delta t\,\langle\hat{N}\rangle + (\gamma_0\Delta t)^2 Q\langle\hat{N}\rangle < \bar{n}$$

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Global counting statistics: Summary

❖ The probability P_n of seeing n quantum jumps (clicks!) in the detector,

$$P_n = \frac{\sin^{2n}\left(\sqrt{\gamma_0 \Delta t}\right)}{n!} \int d^2\alpha P(\alpha)|\alpha|^{2n} e^{-|\alpha|^2 \sin^2\left(\sqrt{\gamma_0 \Delta t}\right)}.$$

$$\bar{n}_{clicks} \approx \gamma_0 \Delta t \langle \hat{N} \rangle_{Field}.$$

Variance: $$\sum_n n^2 P_n - \bar{n}^2 = \bar{n} + (\gamma_0 \Delta t)^2 Q \langle \hat{N} \rangle_{Field},$$

$$Q = \frac{\langle \Delta \hat{N}^2 \rangle - \langle \hat{N} \rangle}{\langle \hat{N} \rangle}.$$

❖ $-1 \leq \mathrm{Q} < 0$: Sub − Poissonian, $\mathrm{Q} = 0$: Poissonian, $\mathrm{Q} > 0$ super − Poissonian

- Coherent states $Q = 0$: Poissonian clicks,
- Thermal states $Q = n_{th}$: super-Poissonian clicks,
- Squeezed vacuum states $Q \approx 2\langle \hat{N} \rangle$: more super-Poissonian than thermal states,
- Number states, $Q = -1$: sub-Poissonian statistics.

- Sreenath K. Manikandan, and Frank Wilczek. "Testing the coherent-state description of radiation fields." Phys. Rev. A 111, 033705 (2025).

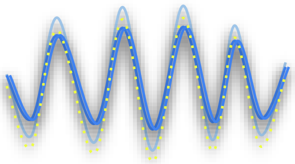
"I try to avoid hard work. When things look complicated, that is often a sign that there is a better way to do it."

FRANK WILCZEK
Nobel Prize in Physics 2004

A simple ratio test for acoherence

- Coherent states have Poissonian statistics, evident from their representation in the number basis, which yields Poissonian probabilities that obey rigid relations between $p(0), p(1), p(2)$ ...

$$|\beta\rangle = e^{-\frac{|\beta|^2}{2}} \sum_n \frac{\beta^n}{\sqrt{n!}} |n\rangle , \qquad p(n) = \frac{e^{-|\beta|^2} |\beta|^{2n}}{n!} .$$



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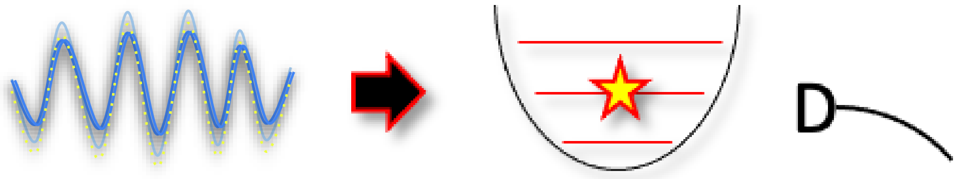
- For Poissonian statistics, $2p(0)p(2) = p(1)^2$. This is also satisfied by the probability of observing the quantum jumps in a resonant detector if the field is in a coherent state, $P(\alpha) = \delta(\alpha - \beta)$,

$$P_n = \frac{\sin^{2n}\left(\sqrt{\gamma_0 \Delta t}\right)}{n!} \int d^2\alpha P(\alpha) |\alpha|^{2n} e^{-|\alpha|^2 \sin^2\left(\sqrt{\gamma_0 \Delta t}\right)} .$$



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- For coherent states,

$$R = \frac{2P_0 P_2}{P_1^2} = 1.$$

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A simple ratio test for acoherence

❖ The probability of observing the quantum jumps in a resonant detector,

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❖ For thermal states,

$$R = \frac{2P_0 P_2}{P_1^2} = 2.$$

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- For highly squeezed vacuum states,

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$$R = \frac{2P_0P_2}{P_1^2} = 3.$$

❖ For number states, $|n\rangle$

$$R = \frac{2P_0P_2}{P_1^2} = 1 - \frac{1}{n}.$$

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What is this ratio?

$$R = \frac{2P_0P_2}{P_1^2} = ?$$

o Sreenath K. Manikandan, and Frank Wilczek. "Testing the coherent-state description of radiation fields." Phys. Rev. A 111, 033705 (2025).

Perturbative approach to the probability of clicks $\left(\kappa = \sqrt{\gamma_0 \Delta t}\right)$

$$
\begin{aligned}
P_0 &= 1 - \kappa^2 \langle s|a^\dagger a|s\rangle + \kappa^4 \langle s|\frac{1}{6}a^{\dagger 2}a^2 + \frac{1}{3}(a^\dagger a)^2|s\rangle \\
&- \kappa^6 \langle s|\frac{17}{360}a^\dagger a a^{\dagger 2}a^2 + \frac{17}{360}a^{\dagger 2}a^2 a^\dagger a + \frac{2}{45}(a^\dagger a)^3 + \frac{1}{60}a^{\dagger 3}a^3 + \frac{1}{90}a^{\dagger 2}a a^\dagger a^2|s\rangle
\end{aligned}
$$

- Sreenath K. Manikandan, and Frank Wilczek. "Testing the coherent-state description of radiation fields." Phys. Rev. A 111, 033705 (2025).

Perturbative approach to the probability of clicks $\left(\kappa = \sqrt{\gamma_0 \Delta t}\right)$

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P_0 &= 1 - \kappa^2 \langle s|a^\dagger a|s\rangle + \kappa^4 \langle s|\frac{1}{6}a^{\dagger 2}a^2 + \frac{1}{3}(a^\dagger a)^2|s\rangle \\
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\end{aligned}
$$

$$
\begin{aligned}
P_1 &= \kappa^2 \langle s|a^\dagger a|s\rangle - \kappa^4 \langle s|\frac{1}{3}(a^\dagger a)^2 + \frac{2}{3}a^{\dagger 2}a^2|s\rangle \\
&+ \kappa^6 \langle s|\frac{8}{45}a^{\dagger 2}a a^\dagger a^2 + \frac{4}{45}a^{\dagger 2}a^2 a^\dagger a + \frac{4}{45}a^\dagger a a^{\dagger 2}a^2 + \frac{2}{45}(a^\dagger a)^3 + \frac{1}{10}a^{\dagger 3}a^3|s\rangle
\end{aligned}
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\end{aligned}
$$

$$
\begin{aligned}
P_1 &= \kappa^2 \langle s|a^\dagger a|s\rangle - \kappa^4 \langle s|\frac{1}{3}(a^\dagger a)^2 + \frac{2}{3}a^{\dagger 2}a^2|s\rangle \\
&+ \kappa^6 \langle s|\frac{8}{45}a^{\dagger 2}a a^\dagger a^2 + \frac{4}{45}a^{\dagger 2}a^2 a^\dagger a + \frac{4}{45}a^\dagger a a^{\dagger 2}a^2 + \frac{2}{45}(a^\dagger a)^3 + \frac{1}{10}a^{\dagger 3}a^3|s\rangle
\end{aligned}
$$

$$
P_2 = \frac{\kappa^4}{2}\langle s|a^{\dagger 2}a^2|s\rangle - \kappa^6 \langle s|\frac{1}{4}a^{\dagger 3}a^3 + \frac{1}{6}a^{\dagger 2}a a^\dagger a^2 + \frac{1}{24}a^{\dagger 2}a^2 a^\dagger a + \frac{1}{24}a^\dagger a a^{\dagger 2}a^2|s\rangle
$$

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Perturbative approach to the probability of clicks

$$P_0 \approx 1$$

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$$R = \frac{2 P_0 P_2}{P_1^2} \approx \frac{\langle (a^\dagger)^2 a^2 \rangle}{\langle a^\dagger a \rangle^2}$$

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The second order coherence function for the radi

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A simple ratio test for acoherence

- For a generic quantum state,

$$R = \frac{2P_0P_2}{P_1^2} \approx 1 + \frac{Q}{\langle\hat{N}\rangle} = g^2(0); \qquad Q = \frac{\langle\Delta\hat{N}^2\rangle - \langle\hat{N}\rangle}{\langle\hat{N}\rangle}.$$

- R=1 for coherent states, R=2 for thermal states, R=3 for highly squeezed vacuum states.
- For a generic Gaussian state (Using Wigner functions, and Weyl transform),

$$R \approx \frac{4n_{\rm th}^2 - 8n_{\rm th}x_0^2\cos(\phi)\sinh(2r) + 8(2n_{\rm th}+1)\left(x_0^2-1\right)\cosh(2r)}{2\left((2n_{\rm th}+1)\cosh(2r) + x_0^2 - 1\right)^2} + \frac{3(2n_{\rm th}+1)^2\cosh(4r) + 4n_{\rm th} - 8x_0^2\cos(\phi)\sinh(r)\cosh(r) + 2x_0^4 - 8x_0^2 + 5}{2\left((2n_{\rm th}+1)\cosh(2r) + x_0^2 - 1\right)^2}.$$

x_0: the displacement, n_{th}: the thermal occupation, r: the degree of squeezing,
ϕ: the angle between the displacement direction and squeezing direction.

o Sreenath K. Manikandan, and Frank Wilczek. Testing the coherent-state description of radiation fields. Phys. Rev. A 111, 033705 (2025).

Quantum Connections in Sweden 2026

- ❖ Lecture 1: Quantum Mechanics of Resonant Harmonic Detectors for Quantized Radiation Fields
- ❖ Lecture 2: Probing the Quantum Structure in Gravitational Radiation
- ❖ Lecture 3: Statistical Null-Tests of the Coherent State Hypothesis
- ❖ Lecture 4: Quantum Mechanics of the Ringdown

Speaker: Sreenath K. Manikandan,

Reader (F), Tata Institute of Fundamental Research Hyderabad

Email: skm@tifrh.res.in

Lecture 2: Probing the Quantum Structure in Gravitational Radiation

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Based on:

- Sreenath K. Manikandan, and Frank Wilczek. "Testing the coherent-state description of radiation fields." Phys. Rev. A 111, 033705 (2025).
- Sreenath K. Manikandan, and Frank Wilczek. "Complementary Probes of Gravitational Radiation States" Phys. Rev. A 112, 043716 (2025).
- Sreenath K. Manikandan and Frank Wilczek. "Probing Quantum Structure in Gravitational Radiation." International Journal of Modern Physics D, May 22, 2025, 2543001. https://doi.org/10.1142/S0218271825430011. [First Prize at the Gravity Research Foundation's essay competition 2025.]
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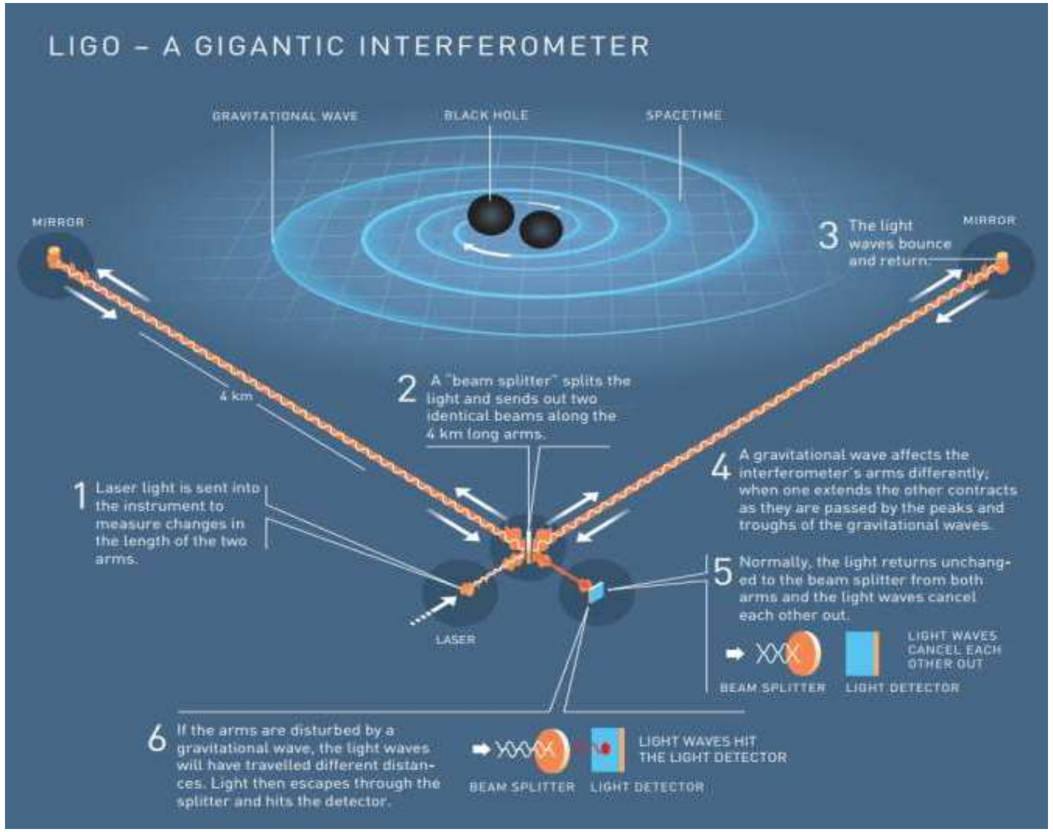
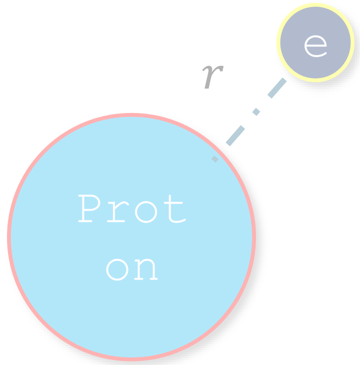
Gravitational waves change the relative distances between objects. We use this to detect them at LIGO.





$$h(t) \sim \frac{\delta L}{L} \sim 10^{-22}$$

Gravitational waves

- Solution to vacuum Einstein's equation. Einstein's equations can be derived from the action, $S = S_E + S_M$.

- $S_E = \frac{c^3}{16\pi G}\int d^4x\sqrt{-g}R$.

- The energy momentum tensor $T_{\mu\nu}$ is defined as $\delta S_M = \frac{1}{2c}\int d^4x\sqrt{-g}T_{\mu\nu}\delta g_{\mu\nu}$.

See Maggiore, M. (2007). Gravitational waves: Volume 1: Theory and experiments. OUP Oxford.

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- Variation of the total action w.r.t the metric gives the Einstein's equation,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}.$$

- Away from source, far-field, $T_{\mu\nu} = 0$.

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- Away from source, far-field, $T_{\mu\nu} = 0$.

- Huge local gauge invariance: $x^\mu \to x'^\mu(x)$: $g_{\mu\nu} \to g'_{\mu\nu}(x') = \frac{\partial x^\sigma}{\partial x'_\mu}\frac{\partial x^\rho}{\partial x'_\nu}g_{\mu\nu}(x)$

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Gravitational waves

- $g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$, there is a choice of coordinate such $|h_{\mu\nu}| \ll 1$ over a sufficiently large region of space.

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Gravitational waves

- $g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$, there is a choice of coordinate such $|h_{\mu\nu}| \ll 1$ over a sufficiently large region of space.

- Slowly varying diffeomorphisms, $x^{\mu} \to x'^{\mu} = x^{\mu} + \zeta^{\mu}(x)$: $h_{\mu\nu} \to h_{\mu\nu} - (\partial_{\mu}\zeta_{\nu} + \partial_{\nu}\zeta_{\mu})$ where $|\partial_{\mu}\zeta_{\nu}| \sim |h_{\mu\nu}|$ are a symmetry of the linearized theory.

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- We can choose ζ_ν such that $\partial^\mu \bar{h}_{\mu\nu} = 0$ (Lorentz gauge), so from 10 components of $h_{\mu\nu}$ we go to remaining 6 components.

See Maggiore, M. (2007). Gravitational waves: Volume 1: Theory and experime

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- Outside the source, the vacuum Einstein equations yield $\Box \bar{h}_{\mu\nu} = 0$, here $\Box = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2$.

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- The condition $\partial^{\mu}\bar{h}_{\mu\nu} = 0$ does not completely fix the gauge; we can do a further shift ζ^{μ} such that $\Box\zeta^{\mu} = 0$, and still obey $\partial^{\mu}\bar{h}_{\mu\nu} = 0$. This gives four more constraints, such that we are left with $6 - 4 = 2$ degrees of freedom.

- We can choose ζ^{0} such that $h^{\mu}{}_{\mu} = 0$: $\bar{h}_{\mu\nu} = h_{\mu\nu}$, and ζ^{i} such that $h^{0i} = 0$. That is, traceless and, combined with the zero divergence, transverse!

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Gravitational waves

- $g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$

- GWs in TT-gauge:

$$h^{0\mu} = 0, h^i_i = 0, \partial^j h_{ij} = 0.$$

- $\Box h_{\mu\nu} = 0$

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$$h_{ij} = \frac{1}{\sqrt{V}} \sum_{\boldsymbol{k},\lambda} e_{ij}^{\boldsymbol{k},\lambda}\, h_{\boldsymbol{k},\lambda}\, e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)} + cc$$

Normalization

Polarization

Polarization tensor

Fourier amplitudes

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Gravitational waves

- ❖ $g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$

$$h_{ij}^{\mathrm{TT}}(t,z) = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij} \cos[\omega(t - z/c)]\,,$$

or, more simply,

$$h_{ab}^{\mathrm{TT}}(t,z) = \begin{pmatrix} h_+ & h_\times \\ h_\times & -h_+ \end{pmatrix}_{ab} \cos[\omega(t - z/c)]\,,$$

where $a, b = 1, 2$ are indices in the transverse (x, y) plane;

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Quantum mechanics of gravitational waves in the linearized theory

- **Linearized gravity, low energy regime:** Bronstein 1935, Feynman 1963, Dyson 1969, Weinberg1972, Lightman 1973, Boughn & Rothman 2006.

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Bronstein 1935, Feynman 1963, Dyson 1969, Weinberg1972, Lightman 1973, Boughn & Rothman 2006.

Quantization of gravitational waves in the linearized theory: the power of dimensional analysis

❖ Consider the temporal part of the wave equation $\Box h_{\mu\nu} = 0$ which obeys:

$$\frac{\partial^2 h(t)}{\partial t^2} + \omega^2 h^2(t) = 0$$

Bronstein 1935, Feynman 1963, Dyson 1969, Weinberg1972, Lightman 1973, Boughn & Rothman 2006.

Quantization of gravitational waves in the linearized theory: the power of dimensional analysis

$$\frac{\partial^2 h(t)}{\partial t^2} + \omega^2 h^2(t) = 0$$

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❖ The gravitational wave strain amplitude, h, is dimensionless.

$$h(t) \sim \frac{\delta L}{L} \sim 10^{-22}$$

Bronstein 1935, Feynman 1963, Dyson 1969, Weinberg1972, Lightman 1973, Boughn & Rothman 2006.

Quantization of gravitational waves in the linearized theory: the power of dimensional analysis

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- ❖ The Lagrangean, $L = \frac{\dot{h}^2}{2} - \omega^2 h^2$ gives the above equation of motion,

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❖ The Lagrangean, $L = \frac{\dot{h}^2}{2} - \omega^2 h^2$ gives the above equation of motion,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{h}}\right) - \frac{\partial L}{\partial h} = 0 \rightarrow \frac{\partial^2 h(t)}{\partial t^2} + \omega^2 h^2(t) = 0$$

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❖ The dimensions of L is, $[L] = T^{-2}$. Let us multiply L with μ: $[\mu] = ML^2$ such that $L' = \mu L$ has dimensions of energy. It is evident that μ is some mass-quadrupole moment.

Bronstein 1935, Feynman 1963, Dyson 1969, Weinberg1972, Lightman 1973, Boughn & Rothman 2006.

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❖ That makes sense because the sources of gravitational radiation are oscillating mass-quadrupoles!

Bronstein 1935, Feynman 1963, Dyson 1969, Weinberg1972, Lightman 1973, Boughn & Rothman 2006.

Quantization of gravitational waves in the linearized theory: the power of dimensional analysis

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- The dimensions of L is, $[L] = [T^{-2}]$. Let us multiply L with μ: $[\mu] = [ML^2]$ such that $L' = \mu L$ has dimensions of energy, $[\mathrm{E}] = [ML^2 T^{-2}]$. It is evident that μ is some quadrupole moment.

- We can construct one such μ at frequency ω and involving fundamental constants, G, c:

$$[\mu] \propto \left[\frac{c^5}{G\omega^3}\right] = [ML^2]$$

Bronstein 1935, Feynman 1963, Dyson 1969, Weinberg1972, Lightman 1973, Boughn & Rothman 2006.

Quantization of gravitational waves in the linearized theory: the power of dimensional analysis

$$[\mu] \propto \left[\frac{c^5}{G\omega^3}\right] = [ML^2]$$

- The Lagrangean, $L' = \mu L \propto \frac{c^5}{G\omega^3} L = \frac{c^5}{G\omega^3}\left(\frac{\dot{h}^2}{2} - \omega^2 h^2\right)$, also gives the same equation of motion, with the corresponding Hamiltonian for the effective single mode,

$$H = \frac{p_h^2}{2\mu} + \frac{1}{2}\mu\omega^2 h^2 \equiv \frac{1}{2}\mu\dot{h}^2 + \mu\omega^2 h^2.$$

Bronstein 1935, Feynman 1963, Dyson 1969, Weinberg1972, Lightman 1973, Boughn & Rothman 2006.

Quantization of gravitational waves in the linearized theory: the power of dimensional analysis

- The Lagrangean, $L' = \mu L \propto \frac{c^5}{G\omega^3} L = \frac{c^5}{G\omega^3}\left(\frac{\dot{h}^2}{2} - \omega^2 h^2\right)$, also gives the same equation of motion, with the corresponding Hamiltonian for the effective single mode,

$$H = \frac{p_h^2}{2\mu} + \frac{1}{2}\mu\omega^2 h^2 \equiv \frac{1}{2}\mu\dot{h}^2 + \mu\omega^2 h^2.$$

- We can estimate μ such that the kinetic part of the Hamiltonian agrees with the energy density of the gravitational wave in mode volume $(c/\omega)^3$ averaged over cycle,

$$\frac{c^2}{32\pi G} h^2\omega^2(c/\omega)^3 = \frac{1}{2}\mu\dot{h}^2 \ \rightarrow \mu = \frac{c^5}{16\pi G\omega^3}.$$

Bronstein 1935, Feynman 1963, Dyson 1969, Weinberg1972, Lightman 1973, Boughn & Rothman 2006.

Quantization of gravitational waves in the linearized theory: the power of dimensional analysis

$$[\mu] \propto \left[\frac{c^5}{G\omega^3}\right] = [ML^2]$$

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- We can now follow the canonical quantization procedure,

$$\hat{h}_\lambda(t) = \sqrt{\frac{\hbar}{2\mu\omega}}\left[\hat{a}_\lambda e^{i\omega t} + \hat{a}_\lambda^\dagger e^{-i\omega t}\right] = \frac{1}{\sqrt{2}}\sqrt{\frac{32\pi G\hbar\omega^2}{c^5}}\left[\hat{a}_\lambda e^{i\omega t} + \hat{a}_\lambda^\dagger e^{-i\omega t}\right]$$

Bronstein 1935, Feynman 1963, Dyson 1969, Weinberg1972, Lightman 1973, Boughn & Rothman 2006.

Estimating the number of gravitons in the semiclassical limit

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- The classical intensity of the field is,

$$h_\lambda(t)^2 \approx \left\langle \alpha \left| \left|\hat{h}(t)\right|^2 \right| \alpha \right\rangle = \frac{16\pi G\hbar\omega^2}{c^5}\langle\alpha|[2\hat{a}^\dagger\hat{a} + 1]|\alpha\rangle \approx \frac{32\pi G\hbar\omega^2}{c^5}|\alpha|_\lambda^2$$

Bronstein 1935, Feynman 1963, Dyson 1969, Weinberg1972, Lightman 1973, Boughn & Rothman 2006.

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- ❖ This correspond to the following number of gravitons,

$$N_\lambda \approx |\alpha|_\lambda^2 \approx \frac{h^2 c^5}{32\pi G\hbar\omega^2}$$

Bronstein 1935, Feynman 1963, Dyson 1969, Weinberg1972, Lightman 1973, Boughn & Rothman 2006.

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❖ For $h \approx 10^{-22}, \omega \sim 100Hz,\ N \approx 10^{36}$.

Bronstein 1935, Feynman 1963, Dyson 1969, Weinberg1972, Lightman 1973, Boughn & Rothman 2006.

 

Observing matter and light exchanging quantum of energy (i. e., photons) was an important first step in formulating the quantum theory of light.

Can we detect a graviton, as exchange of energy in discrete units between matter and gravitational waves?

 

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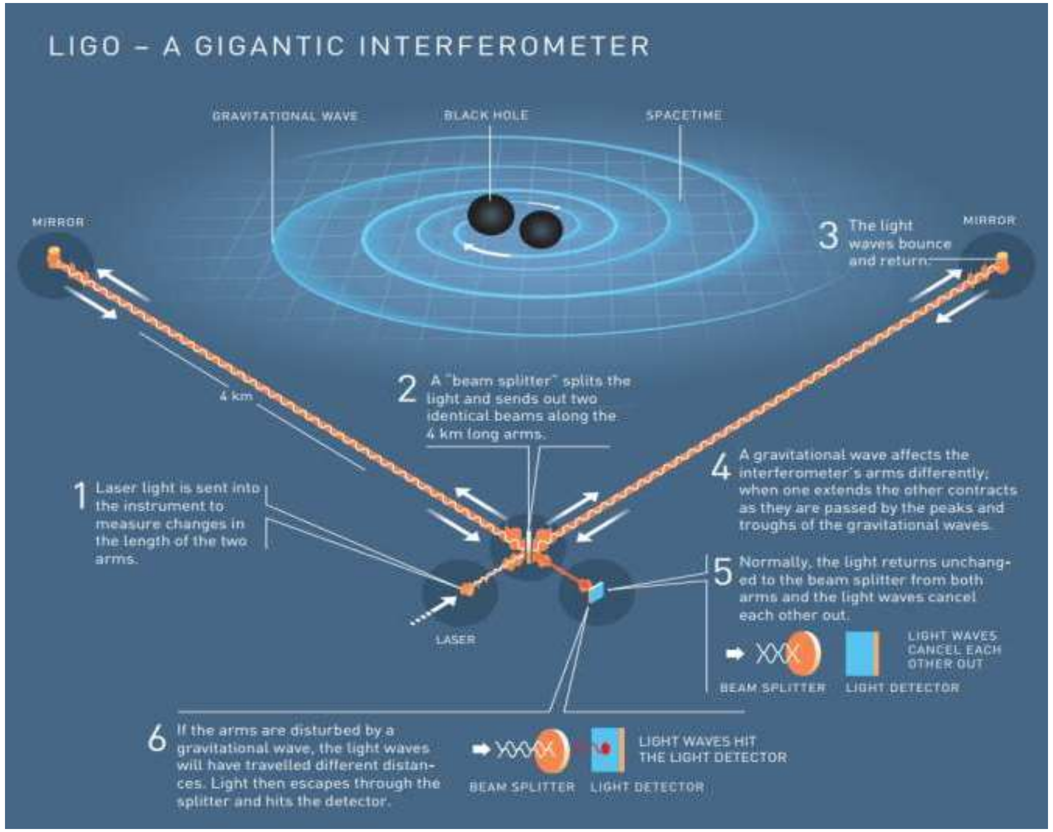
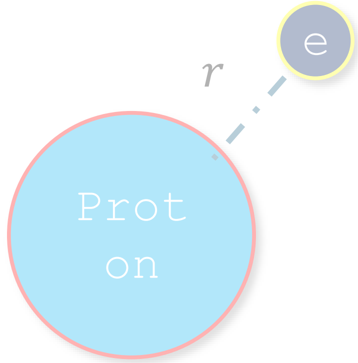
Can we detect a graviton, as exchange of energy in discrete units between matter and gravitational waves?

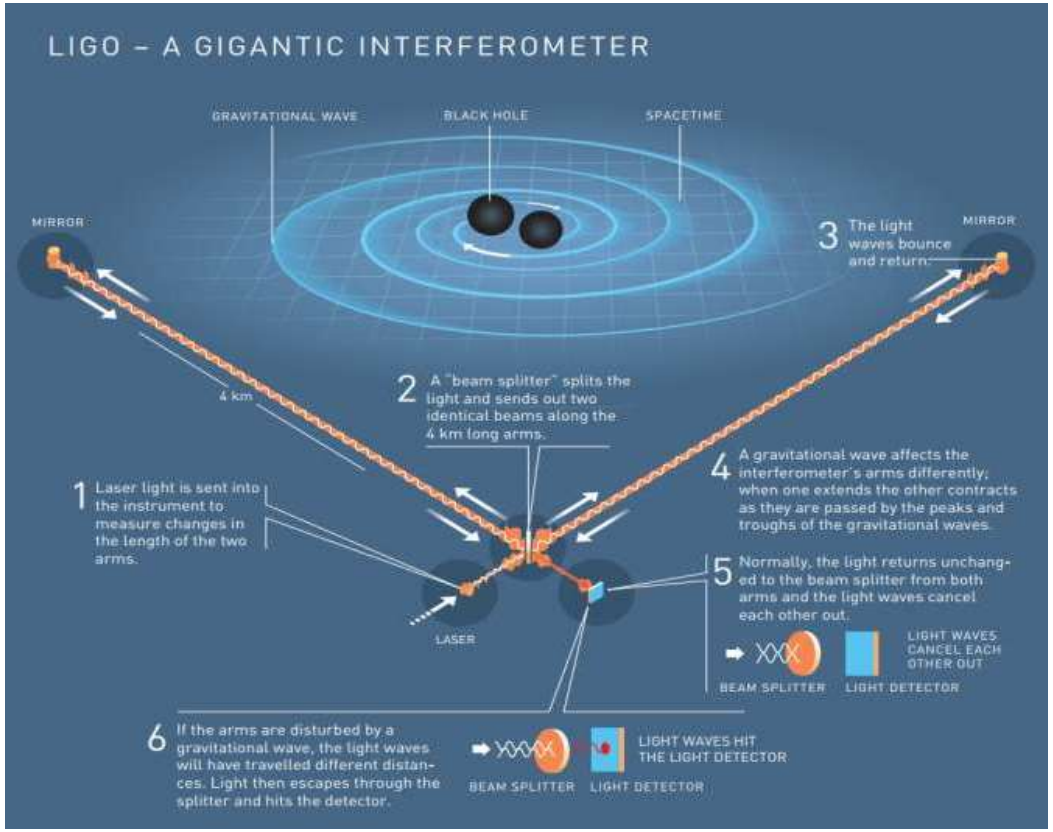
Gravitational waves change the relative distances between objects. We use this to detect them at LIGO.

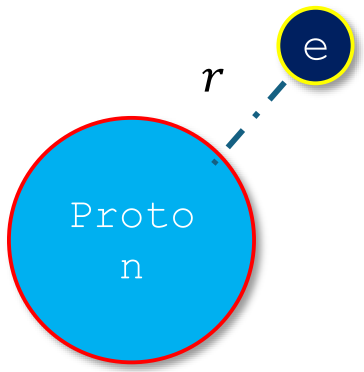




$$h(t) \sim \frac{\delta L}{L} \sim 10^{-22}$$

Gravitational waves will also change the relative distance between an electron and the proton in a Hydrogen atom





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Interaction of a gravitational wave with matter

- Interaction with matter: $\delta S_M = \frac{1}{2}\int d^4x\, \sqrt{-g}\,\delta g_{\mu\nu} T^{\mu\nu} \;\rightarrow L_{int} \approx \frac{1}{2} h_{\mu\nu} T^{\mu\nu}$
- For the reduced mass electron from the Hydrogen atom in (Fermi coordinates/proper detector frame):

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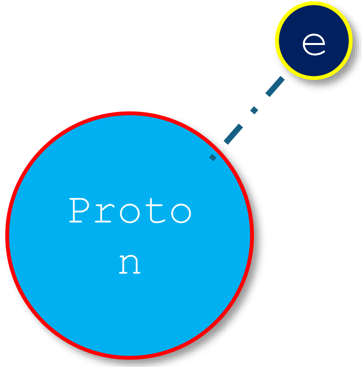

Boughn, S., & Rothman, T. (2006). Aspects of graviton detection: graviton emission and absorption by atomic hydrogen. Classical and Quantum Gravity, 23(20), 5839. Also see Maggiore, M. (2007). Gravitational waves:

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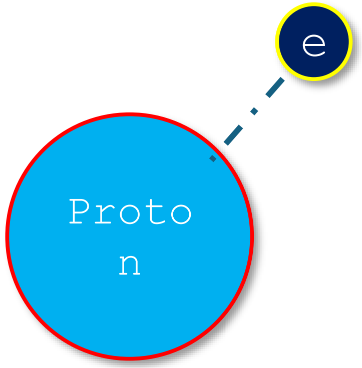


Quadrupole

$$H_{int} = -\frac{1}{2}h_{\mu\nu}T^{\mu\nu} \approx -\frac{1}{2}m_e h_{00} = \frac{1}{4}m_e\omega^2 h_{\boldsymbol{k},\lambda}\, e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}x^j x^k e_{jk} \;+\; \text{H. c.}$$

Boughn, S., & Rothman, T. (2006). Aspects of graviton detection: graviton emission and absorption by atomic hydrogen. Classical and Quantum Gravity, 23(20), 5839. Also see Maggiore, M. (2007). Gravitational waves:

Detecting single gravitons was thought to be impossible

- Weinberg computed the rate for spontaneous atomic processes involving a graviton, and obtained $\Gamma \approx 10^{-44} s^{-1}$ (actually $10^{-40} s^{-1}$), Boughn & Rothman 2006.

Weinberg, "Gravitation and Cosmology" 1972:

states. In particular, in the quadrupole approximation the total rate for an atom to make a transition $a \to b$ by emitting gravitational radiation is

$$\Gamma(a \to b) = \frac{2G\omega^5}{5\hbar}\left[D_{ij}^*(a \to b)D_{ij}(a \to b) - \tfrac{1}{3}|D_{ij}(a \to b)|^2\right] \quad (10.8.6)$$

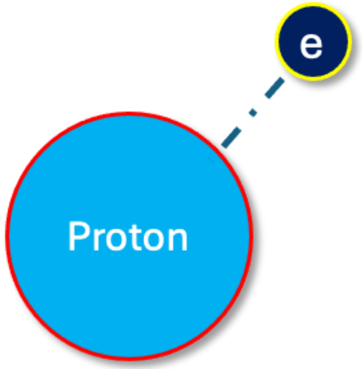
where

$$D_{ij}(a \to b) \equiv m_e \int \psi_b^*(\mathbf{x})x_i x_j \psi_a(\mathbf{x})\, d^3\mathbf{x} \quad (10.8.7)$$

with ψ_a, ψ_b the initial and final state wave functions. For instance, the rate for decay of the $3d(m = 2)$ state of the hydrogen atom into the $1s$ state with emission of one graviton is

$$\Gamma(3d \to 1s) = \frac{2^{23} G m_e{}^3 c}{3^7 5^{15} (137)^6 \hbar^2} = 2.5 \times 10^{-44}\ \text{sec}^{-1}$$

Needless to say, there is no chance of observing such a transition.



Actually $\Gamma = 5.7 \times 10^{-40}\ s^{-1}$

Boughn & Rothman 2006

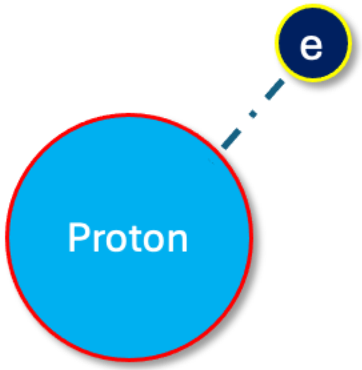
Detecting single gravitons was thought to be impossible

- **Linearized gravity, low energy regime:** Bronstein 1935, Feynman 1963, Dyson 1969, Weinberg1972, Lightman 1973, Boughn & Rothman 2006.

Quantize:

$$\hat{h}^{ij} = \sum_{\boldsymbol{k},\lambda} e^{ij}_{\boldsymbol{k},\lambda}\, h_{q\boldsymbol{k},\lambda} \hat{a}\, e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)} + cc$$

$$h_{q\boldsymbol{k},\lambda} = \sqrt{\frac{16\pi G\hbar}{c^2 \nu_{\boldsymbol{k}} V}}$$



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Graviton transition rate:

$$\Gamma_{atom}\,(3d2 \to 1s) = \frac{2\pi}{\hbar}\left|\langle 1s|\,\langle 1|\hat{H}_{int}|0\rangle\,|3d2\rangle\right|^2 \rho \qquad \rho = \frac{V\nu^2}{2\pi^2\hbar c^3}$$

Density of graviton states

$$\approx 10^{-40} s^{-1}$$

Boughn, S., & Rothman, T. (2006). Aspects of graviton detection: graviton emission and absorption by atomic hydrogen. Classical and Quantum Gravity, 23(20), 5839.; Weinberg, “Gravitation and Cosmology” 1972

Detecting single gravitons was thought to be impossible

- Dyson also looked at sensitivity required for LIGO to detect the strain amplitude corresponding to a single graviton:

- LIGO detects gravitational waves with $\sim 10^{36}$ gravitons: Need 36 orders of magnitude better sensitivity, in the Planck scale.
  - Energy density of GW: $\frac{c^2}{32\pi G} h_0^2 \omega^2$
  - Energy density of gravitons: $\frac{\hbar\omega}{L^3}$, where $L = \frac{c}{\omega}$.

- Graviton absorption cross-section: $\sigma \sim l_p^2$

- Dyson, F. (2013). Is a graviton detectable?. International Journal of Modern Physics A, 28(25), 1330041.
- Weinberg, "Gravitation and Cosmology" 1972
- Boughn, S., & Rothman, T. (2006). Aspects of graviton detection: graviton emission and absorption by atomic hydrogen. Classical and Quantum Gravity, 23(20), 5839.

Single gravitons can be detected



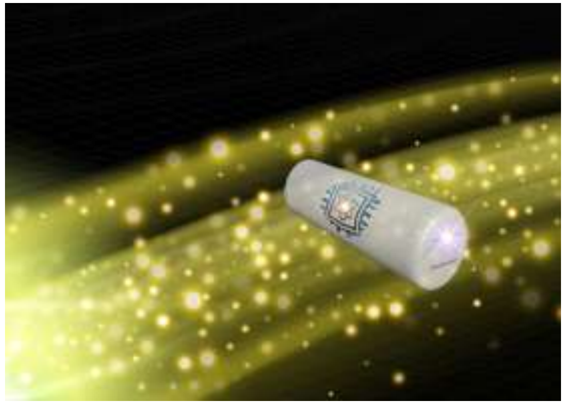
❖ Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, & Igor Pikovski, (2023). Nature Communications 15, 7229 (2024).

- ❑ We do not need a single graviton to arrive (or sensitivity at the single graviton level) to infer the exchange of single quantum of energy with the matter side, just as was the case with the photo-electric effect for photons.

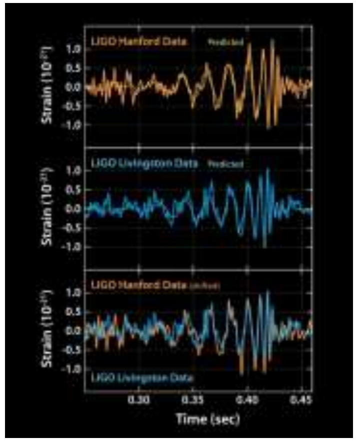
- ❑ To infer energy exchanges, we should measure quantized energy transitions on the matter side, so LIGO may not be the best detector for this purpose.

- ❑ We
  (1) focus on massive quantum systems, and
  (2) stimulated processes in the LIGO-band (kHz frequencies)
  (3) combined with quantum sensing of energy that allow us to infer single graviton exchange events

[*Equal first author]



We proposed to use
Weber bars with a quantum twist



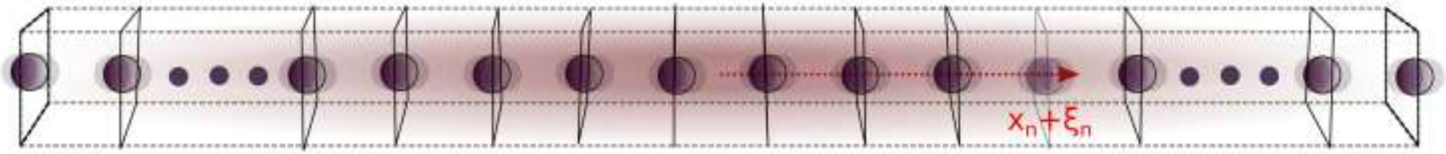
Acoustic bar resonators   LIGO events   Quantum sensing

- ❑ We (1) focus on massive quantum systems, and (2) stimulated processes in the LIGO-band, (3) combined with quantum sensing of energy levels that allow us to infer single graviton exchange events

- ❑ We use Weber bars with a quantum twist:

❑ Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing."

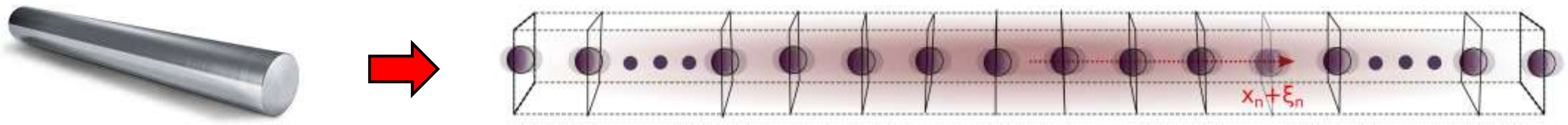
Acoustic modes of a Weber bar



- $N+1$ atoms with mass m, distance a apart, $M=m(N+1)$
- Vibrate with Debye frequency ω_D around their mean positions $x_j=aj/2$, j odd
- Local displacements $x=x_j+\xi_j$
- $\xi_j=\sum_{l=0,2\ldots}^{N-1}\chi_l\,(t)\cos\frac{jl\pi}{2N+2}+\sum_{l=1,3\ldots}^{N}\chi_l\,(t)\sin\frac{jl\pi}{2N+2}$, new collective modes $\ddot{\chi}_l=-\omega_l^2\chi_l$
- Total $E=\frac{1}{2}m\sum_{j=-N}^{N}\dot{\xi}_j^2+\frac{1}{2}m\omega_D^2\sum_{j=-N}^{N-2}\left(\xi_{j+2}-\xi_j\right)^2=\frac{M}{4}\sum_{l=0}^{N}\left(\dot{\chi}_j^2+\omega_l^2\chi_l^2\right)$
- Collective oscillators with mass $M/2$

- Grishchuk, L. P. (1992). Quantum mechanics of a solid-state bar gravitational antenna. Physical Review D, 45(8) 2601.
- Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with qua

A macroscopic interaction



$$H_{int} = -\int \frac{1}{2} h_{\mu\nu} T^{\mu\nu} \approx -m \sum_n \frac{1}{4} \ddot{h}_{xx}(t)(x_n+\xi_n)^2 \approx -\frac{ML\ddot{h}_{xx}(t)}{\pi^2} \sum_{l=1,3,5..} \frac{(-1)^{\frac{l-1}{2}}}{l^2} \chi_l - \frac{M\ddot{h}_{xx}(t)}{8} \sum_l \chi_l^2 .$$

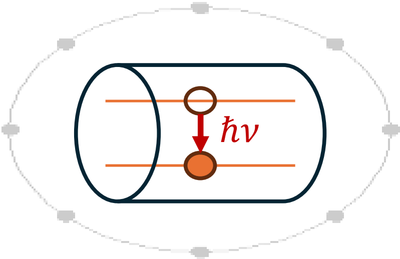
Leading contribution

Sub-leading correction: Weinberg-like effect on each atoms

❑ The leading coupling to GW increases with the mass of the bar Weber bar as well as length of the Weber bar.

❑ Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with sensing."
Nature Communications 15, 7229 (2024)

Rates for spontaneous and stimulated processes



Field in vacuum state.

$|0\rangle \to |1\rangle$

$$\Gamma_{spon}\,(1 \to 0) = \frac{2\pi}{\hbar}\left|\langle 1|\,\langle 0|\widehat{H}_{int}|1\rangle\,|0\rangle\right|^2 \rho = \frac{8GML^2\omega_l^4}{l^4\pi^4c^5} = \frac{8\pi G\rho_m R^2 v_s^4}{Lc^5}$$

ρ_m: mass density

$v_s = \frac{L\omega_l}{l\pi}$: sound speed,

$$\Gamma_{stim}\,(1 \to 0) = \frac{2\pi}{\hbar}\left|\langle 1|\,\langle \alpha|\widehat{H}_{int}|\alpha\rangle\,|0\rangle\right|^2 \rho = \frac{|\alpha|^2 8GML^2\omega_l^4}{l^4\pi^4c^5} \qquad |\alpha|^2 \approx N = \frac{h_0^2c^5}{32\pi G\hbar\omega_l^2}$$

$$\Gamma_{stim} = \frac{ML^2\omega_l^2}{4l^4\pi^5\hbar}h_0^2 = \frac{Mv_s^2}{4l^4\pi^3\hbar}h_0^2$$

Spontaneous emission rate for a Niobium cylinder:

$\rho_m = 8570\,\frac{kg}{m^3}$ $\qquad 2R = L = 1m$

$$\Gamma_{spon} = 10^{-33}s^{-1}$$

Much better than Weinberg (atom), but still small!

Stimulated absorption rate for an aluminium cylinder:

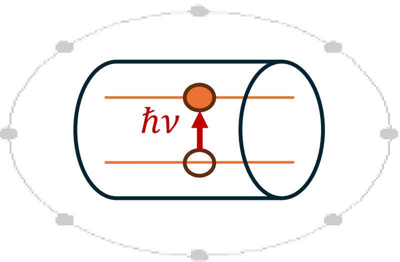
$h_0 = 5 \times 10^{-22}$ (GW150914) $\qquad v_s = 5.4\,\frac{km}{s}$

$M = 1800\,kg \qquad \Gamma_{stim} = 1\,Hz$

One graviton emitted/absorbed per seco

- Equal first author in: Tobar, Germain*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing."

Rates for spontaneous and stimulated processes



Field in a coherent state $|0\rangle \to |\alpha\rangle$.

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$v_s = \frac{L\omega_l}{l\pi}$: sound speed, ρ_m: mass density

$$\Gamma_{stim}\,(1 \to 0) = \frac{2\pi}{\hbar}\left|\langle 1|\,\langle \alpha|\widehat{H}_{int}|\alpha\rangle\,|0\rangle\right|^2 \rho = \frac{|\alpha|^2 8GML^2\omega_l^4}{l^4\pi^4c^5} \qquad |\alpha|^2 \approx N = \frac{h_0^2c^5}{32\pi G\hbar\omega_l^2}$$

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One graviton emitted/absorbed per second.

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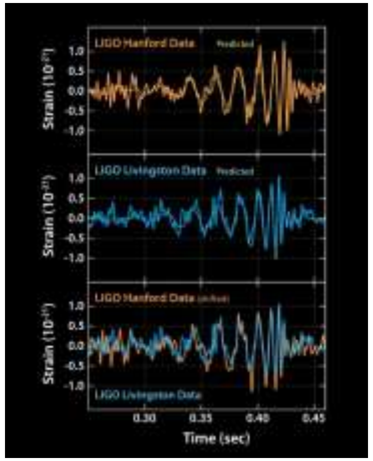
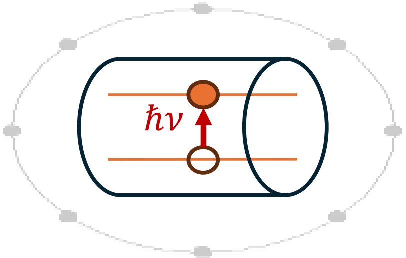
We solve the full, time-dependent problem

Interaction picture:





$$\hat{H} = \hbar\omega \hat{b}^\dagger \hat{b} + \frac{L}{\pi^2}\sqrt{\frac{M\hbar}{\omega}}\,\ddot{h}(t)\left(\hat{b} + \hat{b}^\dagger\right)$$

$$\hat{U}_{int} = \hat{\mathcal{T}} e^{-i\int_0^t ds\left(g(s)\hat{b}(s)+g^*(s)\hat{b}^\dagger(s)\right)}$$

Magnus expansion or Lie algebra or other methods:

$$\hat{U} = e^{-i\varphi} e^{-\omega t \hat{b}^\dagger \hat{b}} \hat{D}(\beta) \qquad \beta = -i\int_0^t ds\, g^*(s) e^{i\omega s}$$

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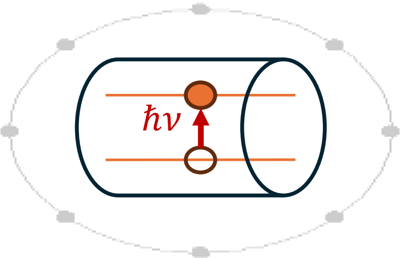
The full, time-dependent problem is solved and gives analytical predictions



$$|0\rangle \rightarrow \left|\beta(t)e^{-i\omega t}\right\rangle \qquad |\beta| = \frac{L}{\pi^2}\sqrt{\frac{M}{\omega\hbar}}\chi(h,\omega,t) \qquad \chi(h,\omega,t) = \left|\int_0^t ds\, \ddot{h}(s)e^{i\omega s}\right|$$

Single transition:

$$P_{0\rightarrow 1} = \left|\left\langle 1 \middle| \beta e^{-i\omega t}\right\rangle\right|^2 = e^{-|\beta|^2}|\beta|^2$$

$$P_{max} = \frac{1}{e} \rightarrow \sim 36\% \quad |\beta|_{max} = 1$$

Use LIGO data, or stationary phase method, or analytic ap

$$\chi \approx h_0\sqrt{\frac{5}{24}}\left(\frac{2c^3}{GM_c}\right)^{5/6}\omega^{1/6}$$

- ❑ GW chirp
- ❑ Binary source chirp mass, M_c.
- ❑ Slow transition through reson

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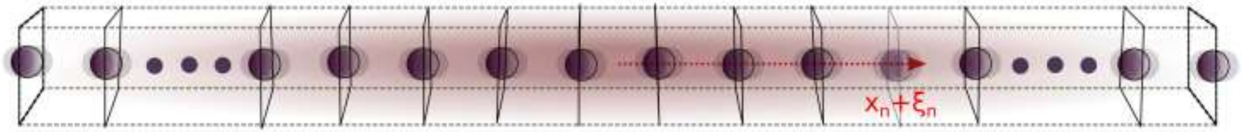
The optimum mass required that maximizes the probability of single graviton exchange



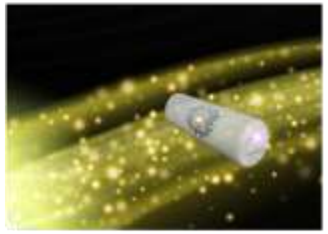
$$|0\rangle \rightarrow \left|\beta(t)e^{-i\omega t}\right\rangle \qquad P_{0\rightarrow 1} = \left|\left\langle 1\middle|\beta e^{-i\omega t}\right\rangle\right|^2 = e^{-|\beta|^2}|\beta|^2 \qquad P_{max} = \frac{1}{e} \approx 0.36$$

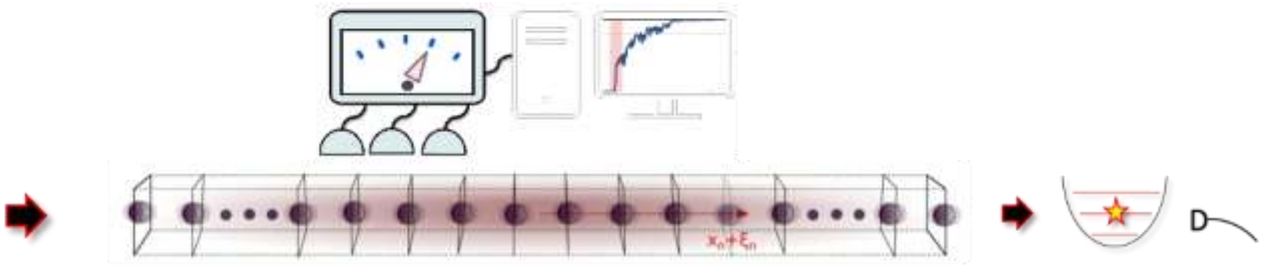
Ideal mass, optimized for single graviton exchange:

$$|\beta|_{max} = 1 \qquad M = \frac{\pi^2 \hbar \omega^3}{v_s^2 \chi(h, \omega, t)}$$

- Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing."

Continuous sensing of quantum jumps





Measurement model:

$$r(t) = \langle \hat{N}(t) \rangle + \sqrt{t_m}\,\zeta(t).$$

$$M_{\hat{N}}(r) = \left(\frac{2\pi t_m}{dt}\right)^{-\frac{1}{4}} \exp\left[-\frac{dt\left(r - \hat{N}\right)^2}{4t_m}\right]$$

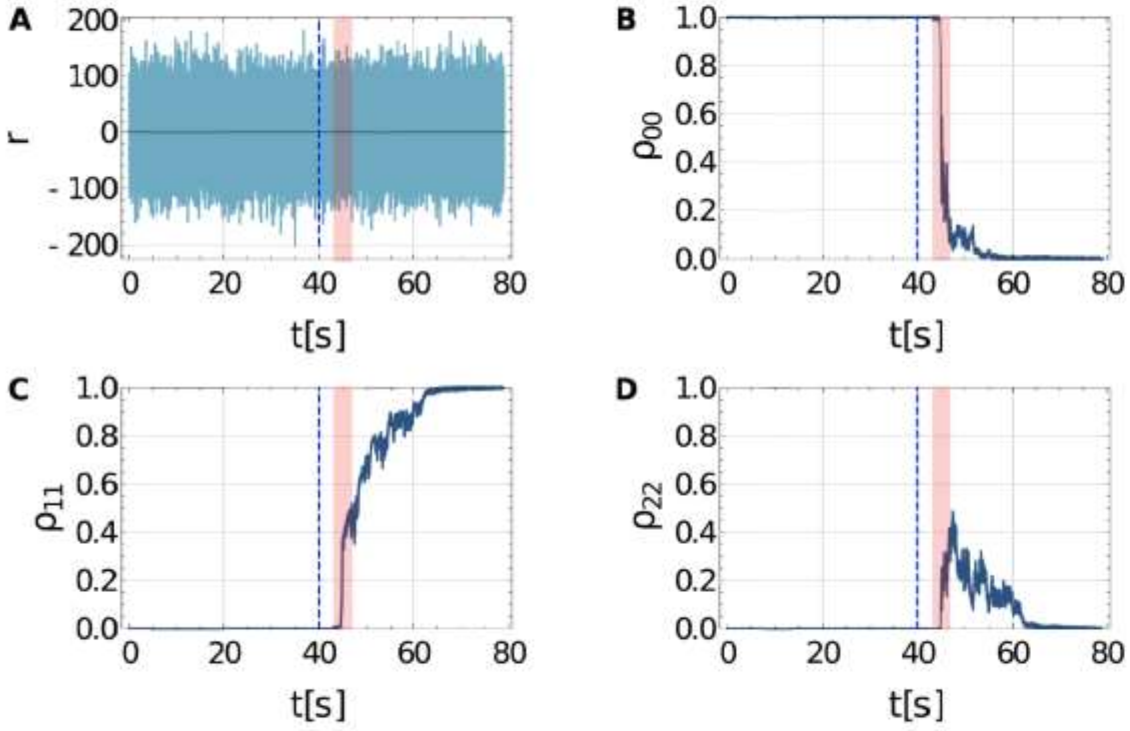
$$\rho(t+dt) = \frac{D[dt\beta'(t)dt]M_{\hat{N}}(r)\rho(t)M^{\dagger}_{\hat{N}}(r)D[-dt\beta'(t)]}{tr\{M_{\hat{N}}(r)\rho(t)M^{\dagger}_{\hat{N}}(r)\}}$$

Gradual collapse of the wavefunction

Image ref: Andrew N Jordan, Nature **502**, 177 (2013)

- Germain Tobar*, **Sreenath K. Manikandan***, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing." Nature Communications 15, 7229 (2024)
- Andrew N. Jordan, and Irfan A. Siddiqi. Quantum Measurement: Theory and Practice. Cambridge University Press, 2024.

Time-continuous and weak number-resolving quantum measurements of a Weber bar



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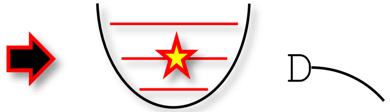
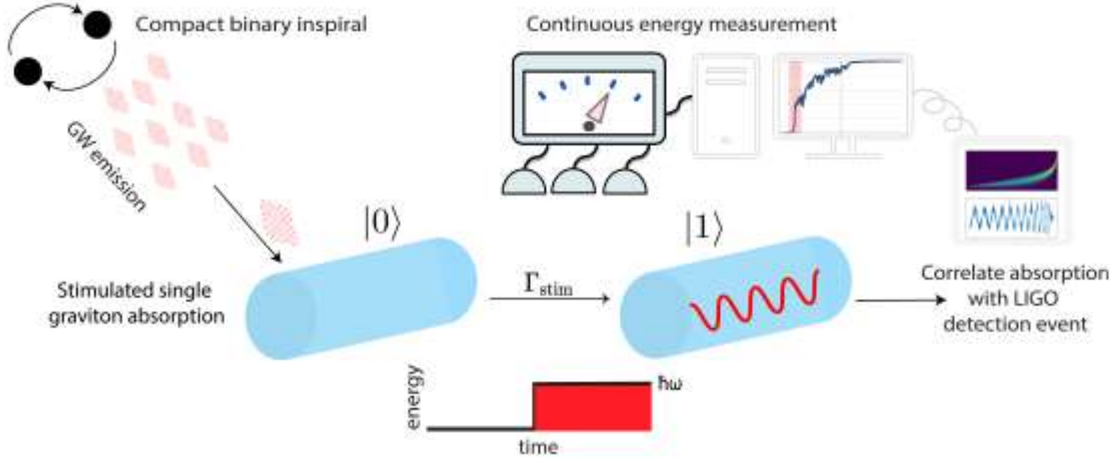


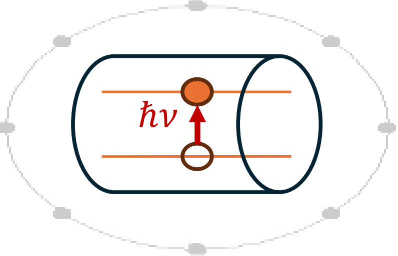
Image ref: Andrew N Jordan, Nature **502**, 177 (2013)

- ❑ Time-continuous energy measurement, outcomes r
- ❑ Fock state probabilities
- ❑ Blue dashed: re-cooling to ground state Pink: GW incidence

❑ Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing."
Nature Communications 15, 7229 (2024)

We can infer single graviton exchange events





- ❖ Absorption happens at resonance (RWA): $P_{0\to 1} \approx \dfrac{h_0^2 \omega^3 M L^2}{\hbar \pi^4 (\nu-\omega)^2} \sin^2 \dfrac{(\nu-\omega)t}{2}$.
- ❖ Classical LIGO can tell if there was a wave chirping through the resonant frequency.
- ❖ We see single excitations in the Weber bar: a gravito-phononic effect! Conservation of energy (which the conventional semi-classical limit of QED violates) suggests a quantum of energy (graviton) exchanged with the gravitational wave.
- ❖ Not proof of quantum theory of gravity. But exchange of energy quanta analogous to photo-electric indication for photons.

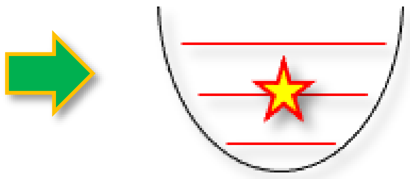
❑ Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing."

Quantum mechanics of a resonant harmonic detector for radiation fields

❖ The interaction (picture) Hamiltonian in the rotating wave approximation between the detector and the radiation field can be approximated as,

$$V_I = \hbar\sqrt{\gamma_0}\left[d(t)a^\dagger + d^\dagger(t)a\right]. \qquad \text{Assume} \quad \left[d(t), d^\dagger(t')\right] = \delta(t - t').$$

The gravitational radiation field, a

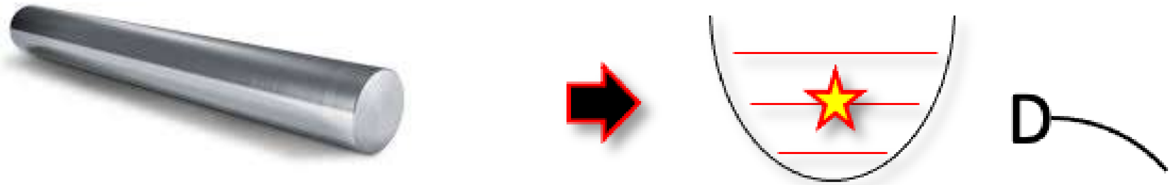


- Sreenath K. Manikandan, and Frank Wilczek. "Testing the coherent-state description of radiation fields." Phys. Rev. A 111, 033705 (2025).

Once the detector starts to click, the statistics of clicks tell us about the quantum state of gravitational radiation

❖ The global counting statistics (Here γ_0 is the spontaneous emission rate for gravitons by the detector):

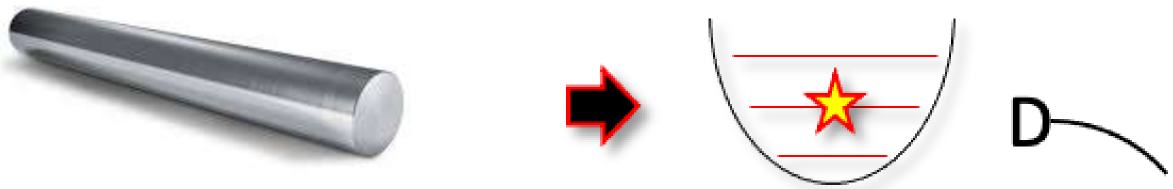
$$P_n = \frac{\sin^{2n}(\sqrt{\gamma_0 \Delta t})}{n!} \int d^2\alpha P(\alpha) |\alpha|^{2n} e^{-|\alpha|^2 \sin^2(\sqrt{\gamma_0 \Delta t})} \rightarrow \frac{(\gamma_0 \Delta t)^n}{n!} \int d^2\alpha P(\alpha) |\alpha|^{2n} e^{-|\alpha|^2 \gamma_0 \Delta t} .$$



o "Testing the coherent-state description of radiation fields." Sreenath K. Manikandan, and Frank Wilczek. Phys. Rev. A 111, 033705 (2025).

The small graviton to click conversion efficiency of the detector is compensated by huge influx of gravitons

❖ The spontaneous emission rate of a typical Weber bar detector for single gravitons, $\gamma_0 = \frac{8GML^2\omega^4}{\pi^4 c^5} \approx 10^{-33} s^{-1}$.



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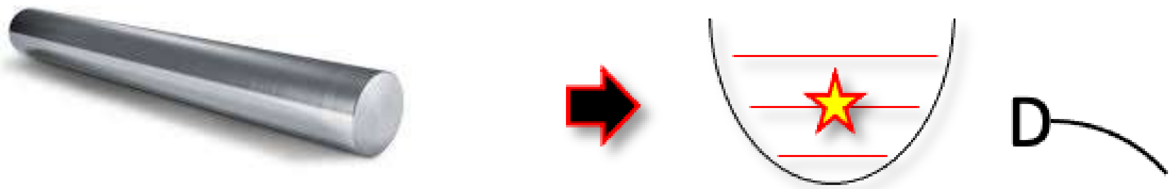
The small graviton to click conversion efficiency of the detector is compensated by huge influx of gravitons

❖ Let us use the number of gravitons in the LIGO band $\langle \hat{N} \rangle \approx 10^{36}$, and the duration $\Delta t \approx 1ms = 10^{-3}s$.



$$\gamma_0 \Delta t \sim 10^{-36}$$

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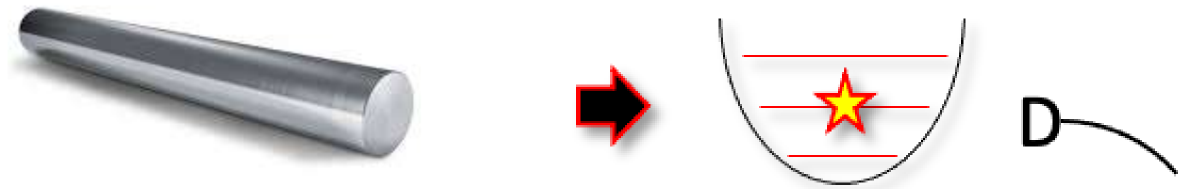
The small graviton to click conversion efficiency of the detector is compensated by huge influx of gravitons

Although the conversion efficiency, $\gamma_0 \Delta t$ is tiny, the number of gravitons is so huge, so that $\bar{n}_{clicks} \approx \gamma_0 \Delta t \langle \hat{N} \rangle_{Field} \approx 1$.



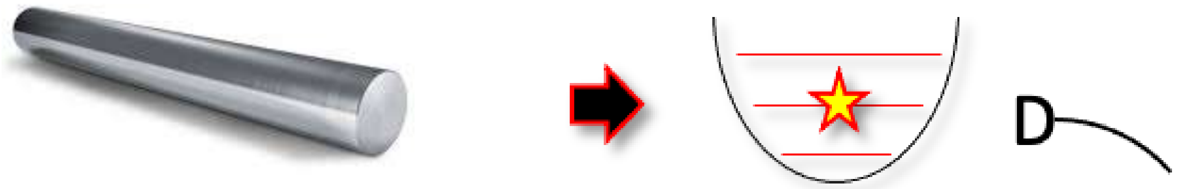
$$\gamma_0 \Delta t \sim 10^{-36}, \langle \hat{N} \rangle \sim 10^{36}$$

- "Testing the coherent-state description of radiation fields." Sreenath K. Manikandan, and Frank Wilczek. Phys. Rev. A 111, 033705 (2025)

Tests of acoherence for gravitational radiation

- For coherent states, $|\beta\rangle$, the mean equals the variance, $\langle\hat{N}\rangle = |\beta|^2 = \langle\Delta\hat{N}^2\rangle$.

$$\bar{n} = \gamma_0 \Delta t \langle\hat{N}\rangle \sim 1, \qquad \langle\Delta n^2\rangle = \gamma_0 \Delta t \langle\hat{N}\rangle \sim 1, \qquad Q = 0$$



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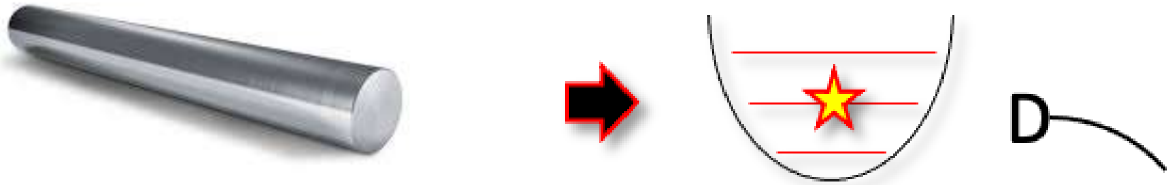
Tests of acoherence for gravitational radiation

- For a thermal state,

$$P(\alpha) = \frac{1}{\pi n_{th}} e^{-\frac{|\alpha|^2}{n_{th}}}, \qquad \bar{n} = \gamma_0 \Delta t \, n_{th} \sim 1, . \qquad (\Delta n^2) = \gamma_0 \, \Delta t \, n_{th} + (\gamma_0 \Delta t)^2 \, n_{th}^2 \sim 2, \qquad Q = n_{th}$$



- "Testing the coherent-state description of radiation fields." Sreenath K. Manikandan, and Frank Wilczek. Phys. Rev. A 111, 033705 (2025).

Tests of acoherence for gravitational radiation

❖ For a highly squeezed vacuum state,

$$|\psi_{sq}\rangle = \frac{1}{\sqrt{\cosh r}} \sum_m \frac{(-\tanh r)^m \sqrt{2m!}}{2^m m!} |2m\rangle , \qquad Q = \cosh 2r = 1 + 2\,(\sinh r)^2 = 1 + 2\langle \hat{N} \rangle \approx 2\langle \hat{N} \rangle.$$

$$\bar{n} = \gamma_0 \Delta t\,(\sinh r)^2 \sim 1 , \qquad (\Delta n^2) = \bar{n} + (\gamma_0 \Delta t)^2 Q\,(\sinh r)^2 \sim 3.$$

o "Testing the coherent-state description of radiation fields." Sreenath K. Manikandan, and Frank Wilczek. Phys. Rev. A 111, 033705 (2025).

The ratio test can also be informative as we only hope to get a few clicks

❖ The probability of observing the quantum jumps in a resonant detector,

$$P_n = \frac{\sin^{2n}\left(\sqrt{\gamma_0 \Delta t}\right)}{n!} \int d^2\alpha P(\alpha) |\alpha|^{2n} \mathrm{e}^{-|\alpha|^2 \sin^2\left(\sqrt{\gamma_0 \Delta t}\right)}.$$

❖ For coherent states,

$$R = \frac{2P_0 P_2}{P_1^2} = 1.$$

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- For thermal states,

$$R = \frac{2P_0P_2}{P_1^2} = 2.$$

- For highly squeezed vacuum states, $$R = \frac{2P_0P_2}{P_1^2} = 3.$$

- For number states, $|n\rangle$

$$R = \frac{2P_0P_2}{P_1^2} = 1 - \frac{1}{n}.$$

o Sreenath K. Manikandan, and Frank Wilczek. "Testing the coherent-state description of radiation fields." Phys. Rev. A 111, 033705 (2025).

The ratio test can also be informative as we only hope to get a few clicks

- For a generic quantum state,

$$R = \frac{2P_0P_2}{P_1^2} \approx 1 + \frac{Q}{\langle n\rangle} = g^2(0); \qquad Q = \frac{\langle \Delta n^2\rangle - \langle n\rangle}{\langle n\rangle}.$$

- R=1 for coherent states, R=2 for thermal states, R=3 for highly squeezed vacuum states.
- For a generic Gaussian state (Using Wigner functions, and Weyl transform),

$$R \approx \frac{4n_{\rm th}^2 - 8n_{\rm th}x_0^2\cos(\phi)\sinh(2r) + 8(2n_{\rm th}+1)\left(x_0^2-1\right)\cosh(2r)}{2\left((2n_{\rm th}+1)\cosh(2r)+x_0^2-1\right)^2} + \frac{3(2n_{\rm th}+1)^2\cosh(4r) + 4n_{\rm th} - 8x_0^2\cos(\phi)\sinh(r)\cosh(r) + 2x_0^4 - 8x_0^2 + 5}{2\left((2n_{\rm th}+1)\cosh(2r)+x_0^2-1\right)^2}.$$

x_0: the displacement, n_{th}: the thermal occupation, r: the degree of squeezing, ϕ: the angle between the displacement direction and squeezing direction.

o Sreenath K. Manikandan, and Frank Wilczek. Testing the coherent-state description of radiation fields. Phys. Rev. A 111, 033705 (2025).

Requirements:

GW Source	GW170817 (NS-NS merger)	GW170817 (NS-NS merger)	GW170608 (BH-BH merger)	GW150914 (BH-BH merger)	J1301+0833 (black-widow pulsar)	J1748−2446ad (fast-spinning pulsar)	A0620-00 (BH Super-radiance)	Primordial (rare BH-BH merger)
$f = \frac{\omega}{2\pi}$	100 Hz	150 Hz	175 Hz	200 Hz	1085 Hz	1433 Hz	33 kHz	5.5 MHz
$h_0(f)$	2×10^{-22}	2×10^{-22}	2×10^{-22}	10^{-21}	$< 10^{-25}$	$< 10^{-25}$	3×10^{-21}	10^{-16}
M_c	$1.19\ M_\odot$	$1.19\ M_\odot$	$7.9\ M_\odot$	$28.6\ M_\odot$	Continuous	Continuous	Continuous	$5 \times 10^{-4}\ M_\odot$
Material	Beryllium	Aluminum	Niobium	CuAl6%	Niobium	Superfluid He-4	Sapphire	Quartz
v_0	13 km/s	5.4 km/s	5 km/s	4.1 km/s	5 km/s	238 m/s	10 km/s	6.3 km/s
T	1 mK	1 mK	1 mK	1 mK	0.1 μK	0.1 μK	0.6 K	0.6 mK
Q-factor	10^{10}	10^{10}	10^{10}	10^{10}	10^{10}	10^{13}	10^{10}	10^{10}
M	~ 15 kg	~ 250 kg	~ 9 t	~ 6 t	> 52 t	> 20 t	~ 100 kg	~ 10 g

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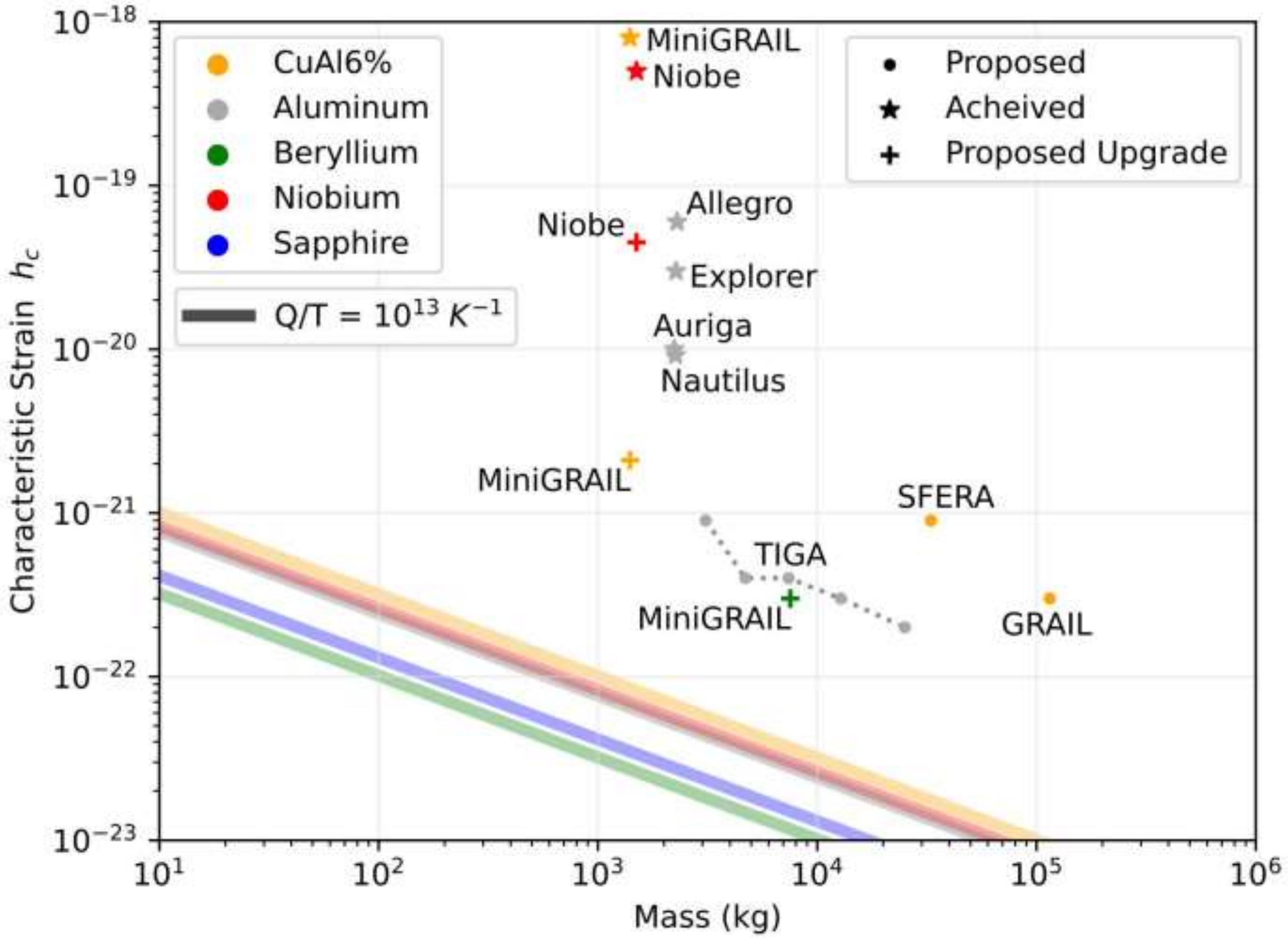
Requirements: $\omega \bar{n} Q^{-1} \leq \Gamma_{stim}$.



Credit: Caltech/MIT/LIGO Lab

Achieves occupation numbers close to $n \approx 10$ for a 10kg mirror.

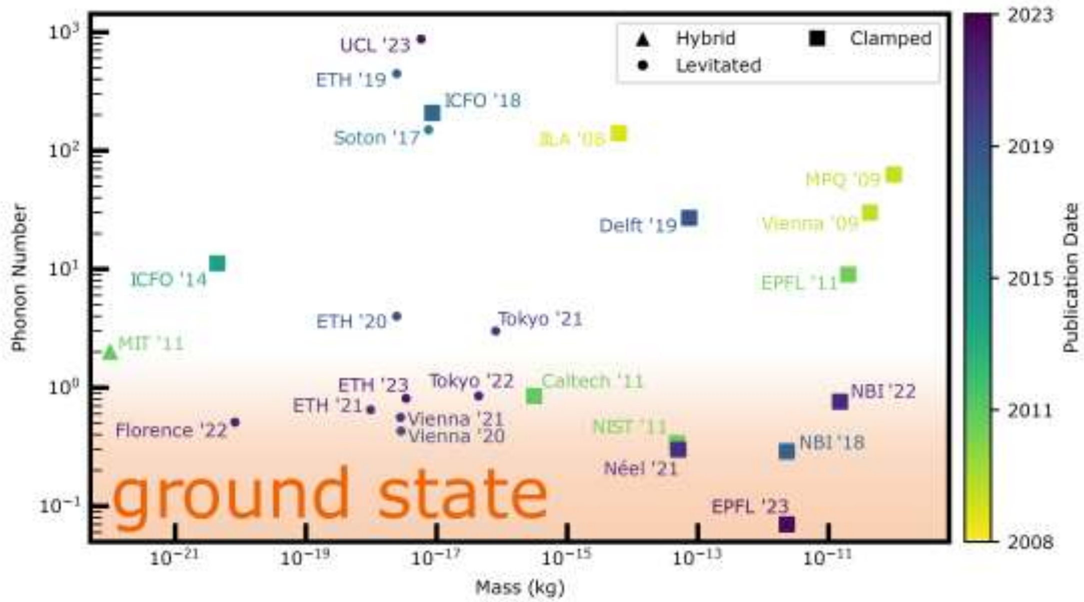
Credit: openclipart.org

"Cooling an elephant to the ground state"

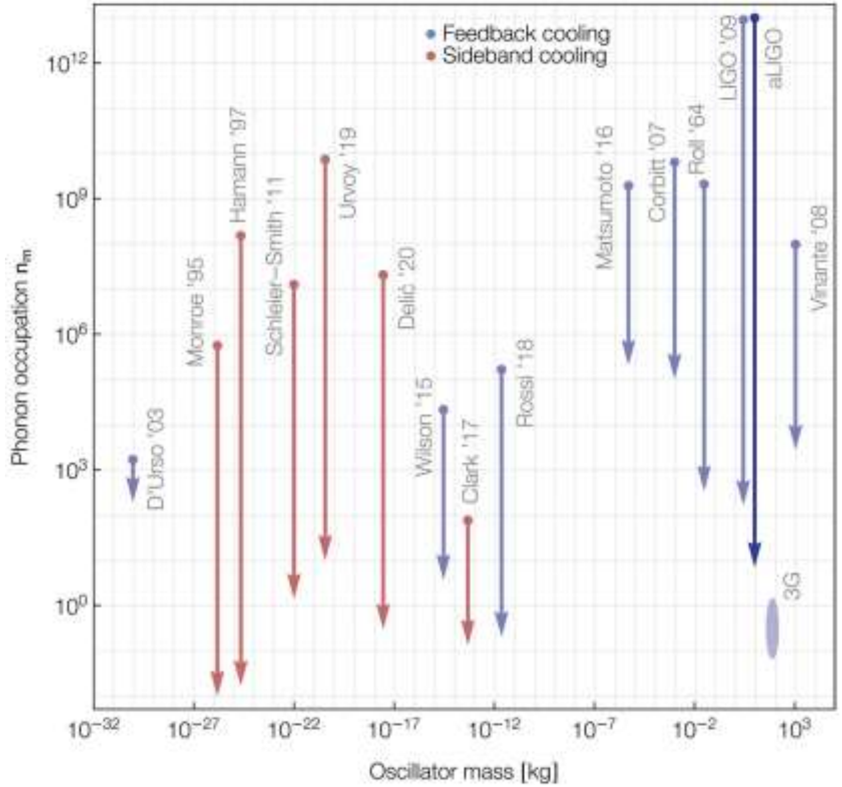
- Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing." Nature Communications 15, 7229 (2024)
- Sreenath. K. Manikandan, & Sofia Qvarfort (2023). Optimal quantum parametric feedback cooling. Physical Review A, 107(2), 023516.
- Whittle, Chris, Evan D. Hall, Sheila Dwyer, Nergis Mavalvala, Vivishek Sudhir, R. Abbott, A. Ananyeva et al. "Approaching the motional ground state...

Near-ground state cooling of larger and larger masses:

Brings opportunities for novel cooling principles and tests of fundamental physics!



Bose, Sougato, Ivette Fuentes, Andrew A. Geraci, Saba Mehsar Khan, Sofia Qvarfort, Markus Rademacher, Muddassar Rashid, Marko Toroš, Hendrik Ulbricht, and Clara C. Wanjura. "Massive quantum systems as interfaces of quantum mechanics and gravity." Reviews of Modern Physics 97, no. 1 (2025): 015003.



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Whittle, Chris, Evan D. Hall, Sheila Dwyer, Nergis Mavalvala, Vivishek Sudhir, R. Abbott, A. Ananyeva et al. "Approaching the motional ground state of a 10-kg object." Science 372, no. 6548 (2021): 1333-1336.

- Chu, Yiwen, Prashanta Kharel, Taekwan Yoon, Luigi Frunzio, Peter T. Rakich, and Robert J. Schoelkopf. "Creation and control of multi-phonon Fock states in a bulk acoustic-wave resonator." Nature 563, no. 7733 (2018): 666-670.
- Schrinski, Björn, Yu Yang, Uwe von Lüpke, Marius Bild, Yiwen Chu, Klaus Hornberger, Stefan Nimmrichter, and Matteo Fadel. "Macroscopic quantum test with bulk acoustic wave resonators." Physical Review Letters 130, no. 13 (2023): 133604.