The Larmor Tunneling Clock

Basic idea

The Larmor tunneling clock is a weak-measurement thought experiment, and in some forms an experimental protocol, for assigning a characteristic time to barrier penetration. A spin-1/2 particle tunnels through a one-dimensional barrier $V(x)$. A weak magnetic field is applied only inside the barrier region,

$$B_z(x) = B\,\Theta(x-a)\Theta(b-x),$$

so that the spin precesses only while the particle has support under the barrier. The spin is used as a clock: the amount of Larmor precession in the transmitted beam is interpreted as a tunneling-time observable.

The Hamiltonian is

$$H = \frac{p^2}{2m} + V(x) - \frac{\hbar\omega_L}{2}\sigma_z\,\chi_B(x), \qquad \chi_B(x) = \begin{cases} 1, & a < x < b, \\ 0, & \text{otherwise,} \end{cases}$$

where

$$\omega_L = \frac{g\mu B}{\hbar}$$

is the Larmor frequency. The field is taken to be weak, so that it minimally disturbs the tunneling probability.

Spin-dependent transmission amplitudes

Prepare the incoming spin along the x direction:

$$|{+x}\rangle = \frac{1}{\sqrt{2}}\left(|{+z}\rangle + |{-z}\rangle\right).$$

The two z-spin components see slightly different barriers, because their Zeeman energies have opposite signs. Equivalently, the transmitted amplitudes are

$$t_\pm(E) \simeq t\left(E \pm \frac{\hbar\omega_L}{2}\right),$$

up to a sign convention for which spin state is lowered in energy. Write the zero-field transmission amplitude as

$$t(E) = |t(E)|e^{i\varphi(E)}.$$

For small ω_L,

$$\ln t_\pm(E) = \ln t(E) \pm \frac{\hbar\omega_L}{2}\,\partial_E \ln t(E) + O(\omega_L^2).$$

The transmitted spinor is therefore

$$|\psi_T\rangle \propto t_+(E)|{+z}\rangle + t_-(E)|{-z}\rangle.$$

Two Larmor times

The weak field changes both the relative phase and the relative magnitude of the two spin components. These define two natural times:

$$\boxed{\tau_y = \hbar\,\partial_E \varphi(E)}$$

and

$$\boxed{\tau_z = \hbar\,\partial_E \ln|t(E)|.}$$

The first is read from ordinary Larmor precession in the transverse plane:

$$\Delta\varphi_{\text{spin}} = \arg t_+ - \arg t_- = \omega_L \tau_y.$$

Thus τ_y is often called the *Larmor precession time*.

The second comes from the unequal filtering of the two spin components:

$$\ln \frac{|t_+|}{|t_-|} = \omega_L \tau_z.$$

This produces a small spin polarization along z. For an initially x-polarized spin, to leading order in ω_L,

$$\langle \sigma_y \rangle_T \simeq \omega_L \tau_y, \qquad \langle \sigma_z \rangle_T \simeq \omega_L \tau_z,$$

up to sign conventions set by the definition of $|{+z}\rangle$ and the direction of B.

Relation to dwell time

The ordinary dwell time in a region $[a, b]$ is

$$\tau_{\text{dwell}} = \frac{\displaystyle\int_a^b dx\,|\psi(x)|^2}{j_{\text{in}}},$$

or, for transmitted particles, a corresponding postselected dwell time with the transmitted flux in the denominator. The Larmor construction provides an operational way to access such time information, but it also reveals that "the tunneling time" is not a single number independent of the measurement protocol. The transmitted spin contains both a phase response and an amplitude-filtering response.

This distinction is especially clear in the semiclassical tunneling limit.

WKB interpretation

For a classically forbidden region, define

$$\kappa(x) = \frac{\sqrt{2m[V(x) - E]}}{\hbar}.$$

In the WKB approximation,

$$|t(E)| \sim \exp\left[-\int_a^b dx\,\kappa(x)\right].$$

Therefore

$$\tau_z = \hbar\,\partial_E \ln|t(E)| \simeq \int_a^b dx\,\frac{m}{\hbar\kappa(x)}.$$

Equivalently, writing the imaginary-time velocity as

$$v_E(x) = \frac{\hbar\kappa(x)}{m},$$

one has

$$\boxed{\tau_z \simeq \int_a^b \frac{dx}{v_E(x)}.}$$

Thus the spin-filtering component of the Larmor clock naturally measures the Euclidean, or imaginary-time, duration of under-barrier motion. For a rectangular barrier of width $L = b - a$ and height $V_0 > E$,

$$\kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar}, \qquad \tau_z \simeq \frac{mL}{\hbar\kappa} = L\sqrt{\frac{m}{2(V_0 - E)}}.$$

By contrast, the phase time

$$\tau_y = \hbar\,\partial_E\varphi$$

is sensitive to the phase of the complex transmission amplitude. In opaque barriers it can show Hartman-type saturation, while τ_z continues to grow with barrier width. This is not a contradiction; the two quantities are different weak values of time-related observables.

Physical summary

The Larmor clock does not simply answer the question

"How long was the particle inside the barrier?"

with a unique classical time. Instead it gives a controlled operational decomposition:

$$\boxed{\text{phase rotation} \longleftrightarrow \tau_y = \hbar\,\partial_E \arg t,}$$

$$\boxed{\text{spin filtering} \longleftrightarrow \tau_z = \hbar\,\partial_E \ln|t|.}$$

For semiclassical tunneling, the filtering time τ_z is closely tied to the imaginary-time instanton picture. This makes the Larmor clock a useful probe of the internal structure of tunneling: the spin does not merely record whether tunneling occurred, but samples how the amplitude was built up under the barrier.

Connection to instanton language

In an instanton treatment, the tunneling amplitude has the schematic form

$$t(E) \sim A(E)e^{-S_E(E)/\hbar + i\varphi(E)}.$$

Then

$$\tau_z = \hbar\,\partial_E \ln|t| \simeq -\partial_E S_E(E) + \hbar\,\partial_E \ln A(E).$$

At leading semiclassical order,

$$\boxed{\tau_z \simeq -\partial_E S_E.}$$

This is precisely the usual Hamilton–Jacobi relation between an action and a time, continued to Euclidean motion. Thus the Larmor clock gives a concrete spin-readout version of a simple semiclassical statement: the energy derivative of the tunneling exponent measures how long the Euclidean trajectory lingers under the barrier.