



what is the history of vector potentials? how did faraday and maxwell think about them?

I'll frame this historically rather than technically: Faraday's "lines of force," Maxwell's mechanical/vector machinery, then the later demotion and resurrection of potentials in quantum theory.

Thought for 12s >

The short story is: **Faraday almost had the vector potential physically, Maxwell made it mathematical and dynamical, later physics demoted it, and quantum mechanics promoted it again.**

Faraday did **not** write down a vector potential  $\mathbf{A}$ . His language was not analytic enough for that. But he introduced the key physical intuition: electromagnetic action is mediated by a **state of space**, represented by **lines of force**, not by instantaneous action at a distance. In his work on induction, he also spoke of an elusive "**electrotonic state**" of matter/space near magnets and currents. Historically, this is the ancestor of the vector potential. Bork's classic historical study summarizes the line of descent: Faraday's electrotonic state "eventually becomes the vector potential" in Maxwell's *Treatise*.  JSTOR

Maxwell took Faraday's qualitative picture very seriously. His first electromagnetic paper, *On Faraday's Lines of Force* in 1856, was explicitly an attempt to give Faraday's lines a mathematical form; contemporary histories note that Maxwell used fluid analogies there to model the lines of force mathematically.  Maths History +1 In the mature theory, Maxwell introduced a quantity essentially equal to the modern magnetic vector potential. In his notation it was  $\mathbf{A}$ , and he called it **electromagnetic momentum**. That name is revealing: Maxwell did not think of  $\mathbf{A}$  as a mere computational convenience, but as something like the momentum stored in, or associated with, the electromagnetic state of the medium.


In modern notation we write

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}.$$

Maxwell's way of thinking was especially close to the second equation. The time variation of  $\mathbf{A}$  was tied to electromotive force. That is why the vector potential was not secondary for him: it was a natural mathematical representative of Faraday's electrotonic state.

There is an important nuance. Maxwell knew, in effect, that  $\mathbf{A}$  is not unique. One can change

$$\mathbf{A} \rightarrow \mathbf{A} + \nabla \Lambda$$

without changing  $\mathbf{B}$ . The modern word for this is **gauge freedom**. C. N. Yang emphasized that already in Faraday's electrotonic state and Maxwell's vector potential, gauge freedom was "an unavoidable presence."  But Maxwell did not turn that ambiguity into a symmetry principle. That came much later.


After Maxwell, especially through Heaviside, Hertz, Lorentz, and the vector-calculus reformulation, the field strengths  $\mathbf{E}$ ,  $\mathbf{B}$  became the stars. The potentials were increasingly treated as auxiliary variables: useful for solving equations, but not directly physical. That attitude is very natural classically, because the Lorentz force on a charged particle is written in terms of  $\mathbf{E}$  and  $\mathbf{B}$ , not directly in terms of  $\mathbf{A}$ .

Then quantum mechanics changed the story. In Schrödinger/Dirac theory the charged particle couples to

$$\mathbf{p} - e\mathbf{A},$$

and the phase accumulated along a path contains

$$\exp \left( \frac{ie}{\hbar} \int \mathbf{A} \cdot d\mathbf{x} \right).$$

The Aharonov–Bohm effect made this unavoidable: charged particles can show interference effects in regions where  $\mathbf{E} = \mathbf{B} = 0$ , but where the enclosed magnetic flux, represented locally by  $\mathbf{A}$ , changes the phase. The original Aharonov–Bohm claim was precisely that potentials can have observable consequences in quantum theory even where fields vanish.  ResearchGate +1

So Faraday and Maxwell were, in a sense, closer to the modern gauge-theoretic viewpoint than the later force-field pedagogy suggests. Faraday imagined a real state of space. Maxwell mathematized that state using potentials and field energy. Nineteenth- and early twentieth-century practice then shifted attention to  $\mathbf{E}$ ,  $\mathbf{B}$ . Quantum theory and gauge theory restored the potentials to center stage, but with a new interpretation: not as uniquely defined mechanical ether variables, but as **connections**, whose gauge-invariant holonomies and phases carry physical content.

A compact summary would be:

Faraday: lines of force / electrotonic state;
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Maxwell: vector potential as electromagnetic momentum;
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Heaviside–Lorentz tradition: fields $\mathbf{E}$ , $\mathbf{B}$ as primary;
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That arc is deeply relevant to superconducting circuits: the “node flux” variables are not arbitrary bookkeeping devices, but descendants of this same potential-based formulation.

did Heaviside and Lorentz downplay Hamiltonians and Lagrangians?

Thought for a second >

Yes — **Heaviside did**, quite deliberately; **Lorentz less so**, but his mature electromagnetic theory was also not primarily Hamiltonian/Lagrangian in style.

**Heaviside** was central in recasting Maxwell’s theory into the compact field equations involving **E**, **B**, **D**, **H**. He stripped away much of Maxwell’s mechanical scaffolding: gears, vortices, elastic media, and also the potential-centered formulation. His attitude was operational and vectorial: use the directly physical field intensities and flux densities, write local differential equations, compute energy flow with the Poynting vector, and avoid what he regarded as unnecessary metaphysics. In that sense, Heaviside helped create the modern “Maxwell equations first” pedagogy — powerful, clean, but less naturally connected to variational mechanics.

So for Heaviside, yes: one can fairly say he **downplayed potentials and action principles** in favor of field equations.

**Lorentz** is subtler. Lorentz was a highly systematic theorist, and his electron theory certainly has dynamical content: charged particles acted on by fields, fields sourced by charges, forces given by what we now call the Lorentz force. But his preferred formulation was still not, in modern language, “start from an action”

$$S = \int L dt$$

and derive everything by variation. He worked more directly with field equations plus equations of motion. The Lorentz force law,

$$m\dot{\mathbf{v}} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}),$$

encourages exactly that style: fields first, force law second.

The important historical contrast is with **Larmor, Schwarzschild, Wiechert, Poincaré, Minkowski, Hilbert, Weyl**, and later the quantum/gauge-theory tradition, where potentials, variational principles, and spacetime geometry become increasingly central.

So a good schematic is:

Maxwell: potentials + mechanical models + field energy

Heaviside: local vector field equations, potentials demoted

Lorentz: fields + electrons + force law, not primarily variational

20th century: action principles and gauge potentials return

There is a nice irony here. The Heaviside-Lorentz formulation made electromagnetism look beautifully local and physical in terms of **E**, **B**. But the later Hamiltonian/Lagrangian and quantum formulations made the potentials indispensable again, because the canonical momentum is

$$\mathbf{p}_{\text{can}} = m\mathbf{v} + q\mathbf{A},$$

and the phase is controlled by

$$\exp\left(\frac{iq}{\hbar} \int \mathbf{A} \cdot d\mathbf{x}\right).$$

**So: Heaviside and Lorentz helped shift electromagnetism away from Maxwell's potential/mechanical/action-adjacent picture toward a force-field picture. But modern theory partially reverses that shift.**