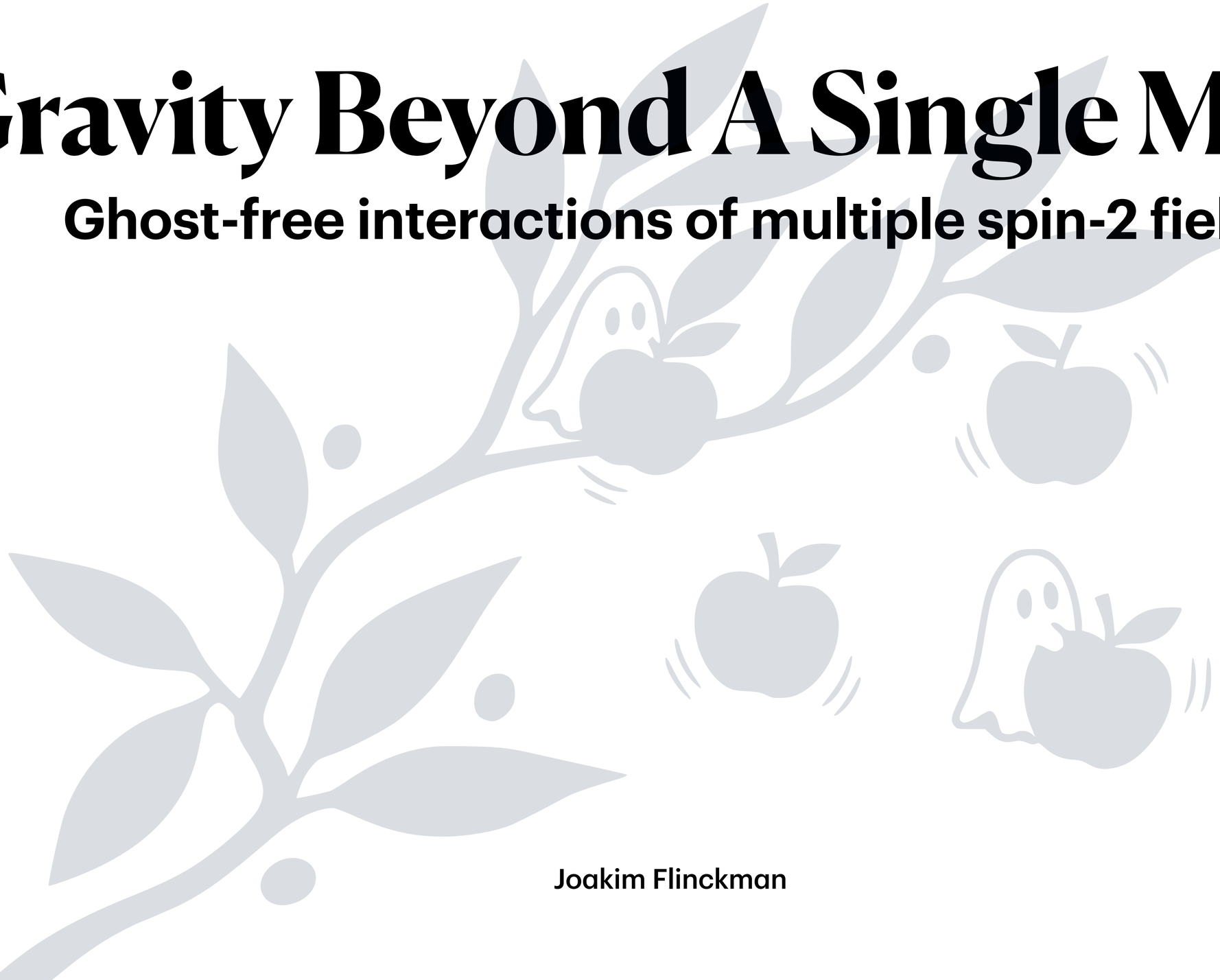
Gravity Beyond A Single Metric

Ghost-free interactions of multiple spin-2 fields



Joakim Flinckman

Extending General Relativity

- General Relativity – Extremely **successful** (empirical and theoretical)
- Open problems: DM, DE, Hubble Tension, Inflation, QG $\Longrightarrow$ Modify gravity?
- Viable modifications are **highly constrained** (empirical and theoretical)
- General Relativity – **Single** massless spin-2 field (classical graviton)
- Standard Model – Norm is **multiple spin-s** fields (e.g. YM) in multiplets

Could gravity be part of a **larger spin-2 structure**
similar to how electromagnetism is part of electroweak theory?

Consistency guided fundamental physics

- The **building blocks** of the Standard Model:
  - E.g. Dirac field & Yang–Mills (Glashow) **before** empirical indications
- General Relativity was formulated before empirical evidence
- These theories are **unique** given some theoretical **consistency conditions**
  - In our case: absence of **ghost fields**

Multi-Gravity

Multi-Gravity

- Theories of multiple spin-2 fields or **metrics,** $g^a_{\mu\nu} \qquad (a = 1, \ldots, N)$
- Quadratic spectrum: 1 **massless** & multiple **massive** spin-2 fields
- Generically propagate extra **ghostly scalar modes** 🚫
- Ghost-freedom is e**xtremely restrictive!**
  - $N = 2$: Bimetric theory is **only ghost-free theory** ✅
  - What about $N \geq 3$**!?**

Ghost-free Multi-Gravity

- A generic non-derivative spin-2 interaction + Ghost-freedom
  - **Necessary** conditions $\Longrightarrow$ **Unique** theory. [Flinckman & Hassan, 2604.07625, In preparation]
- Not **sufficient**: need non-linear analysis to show ghosts do not propagate
  - Proven **ghost-free** [Flinckman & Hassan, 2510.03014]
  - 1 massless & multiple healthy massive spin-2 fields. [Flinckman & Hassan, 2410.09439]

Ghost-freedom singles out a unique multi-gravity interaction!

Multi-Gravity Cosmology

- Massive modes in the **physical metric** $\Longrightarrow$ Modified gravity
- Additional **stable massive spin-2 fields** $\Longrightarrow$ Potential DM candidate
- Modified Friedmann equations, e.g. natural **dynamical DE** (bimetric [Högås, Mörtsell, 2507.03743])
- Bimetric Cosmology ($N = 2$), well-studied
- Beyond $N = 2$: Essentially **unexplored** [Flinckman, In preparation]
  - Provides an interesting framework with natural DM, DE and GR-style cosmologies

Questions?

Multi-Gravity Action

- Action:

$$\mathcal{S} = \sum_a \left[\mathcal{S}_{\rm EH}[g^a_{\mu\nu}] + \mathcal{S}_{\rm m}[g^a_{\mu\nu}, \psi^a]\right] - \int {\rm d}^4x V(g_1, g_2, \ldots, g_N)$$

- Hassan–Schmidt-May Interaction:

$$V = 2m^4 \det\left(\sum_I \beta^I e_I\right), \qquad g^I_{\mu\nu} = e^A_{I\,\mu}\eta_{AB}e^B_{I\,\nu}$$

Where is the ghost?

- No ghosts: $\mathcal{L} = -\dot{\phi}^2 + \ldots$ ✗ $\mathcal{L} = \dot{\phi}^2 + \ldots$ ✔
- Einstein–Hilbert action:

$$\mathcal{S} = m_{\rm pl}^2 \int \mathrm{d}^4x \sqrt{-g} R$$

- Ansatz: $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -N^2(t)\mathrm{d}t^2 + e^{\phi(t)}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$

$$\Longrightarrow \qquad \mathcal{S} = -\tfrac{3}{2} m_{\rm pl}^2 \int \mathrm{d}t\, e^{3\phi(t)/2} \frac{\dot{\phi}^2(t)}{N(t)}$$

$$H = -N \frac{e^{-3\phi/2}}{6m_{\rm pl}^2}\pi^2 - \lambda P$$

Cosmology

- Multi-Gravity FRLW:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} - \frac{\Lambda_1}{3} - \mathcal{V}(a_I) = \frac{\rho}{3m_1^2} \qquad\qquad \mathcal{P}_I(a_J) = 0$$

- Late universe or $\rho = 0 : \mathcal{V} \rightarrow$ positive CC (natural dS)

- Early universe or $\rho \rightarrow \infty : \mathcal{V} \rightarrow$ positive CC.

Mass Spectrum

- Around Einstein spacetimes, the mass spectrum can be identified
  - 1 massless & multiple non-tachyonic massive spin-2 fields

$$0 = \mu_1^2 \leq D_{11} \leq \mu_2^2 \leq D_{22} \leq ... \leq D_{\mathcal{N}-1\mathcal{N}-1} \leq \mu_{\mathcal{N}}^2 \leq D_{\mathcal{N}\mathcal{N}}$$

- Avoids the Higuchi bound (dS) $\quad m_{\rm FP}^2 > \frac{2}{3}\Lambda$
- Avoids the Breitenlohner-Freedman bound (AdS)