

Quantum Sun and its extensions as models for MBL (and beyond)

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Thanks to:



Adith Sai A



Pedro RN Falcao



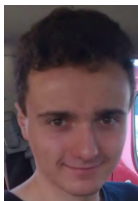
Konrad Pawlik



Maksym Prodius



N. Laflorencie



P. Sierant



Lev Vidmar



Maciej Lewenstein

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Piotr Sierant | MMA Fighter Pa...

P. Sierant



Lev Vidmar



Maciej Lewenstein

Many-body localization

many-body interacting systems – Eigenstate Thermalization Hypothesis
(local properties “thermalize” even for closed systems)

- MBL - robust counterexample
- memory about the initial state in time evolution
- eigenstates have area-law entanglement: $\text{System} = A + B$;
 $S_{ent}(t) = -\text{Tr} \rho_A \ln \rho_A$ $\rho_A = \text{Tr}_B \rho$
- slow logarithmic entanglement growth for initial separable state
- integrability - existence of LIOMs (lbits), Poisson levels spacing,
 $\delta_n = E_{n+1} - E_n$,
- adjacent gap ratio $r_n = \min[\delta_{n+1}, \delta_n] / \max[\delta_{n+1}, \delta_n]$, for Poisson
 $\bar{r} = 2 \ln 2 - 1 \approx 0.385$



Personal historical perspective - 1

- RMT E.P. Wigner, 1955 (but J. Wishart 1928 !)
- P.W. Anderson 1958
- I.V. Gornyi, A.D. Mirlin, D.G. Polyakov (2005); D.M. Basko, I.L. Aleiner, B. L. Altshuler (2006)
- ... many mainly numerical studies...
- J.Z. Imbrie 2016 "We prove that many-body localization follows from a physically reasonable assumption .. "
- F. Weiner, F. Evers, and S. Bera, Slow dynamics and strong finite-size effects in many-body localization with random and quasi-periodic potential (2019) 1904.06928
- J. Suntajs, J. Bonca, T. Prosen, & L. Vidmar, Quantum chaos challenges many-body localization (2020) 1905.06345



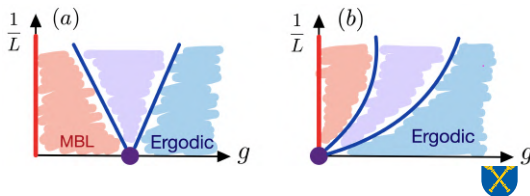
Personal historical perspective - 2

- D. Sels & A. Polkovnikov, A. Morningstar et al., P. Sierant & J. Z.
- W. De Roeck, L. Giacomin, F. Huveneers, and O. Prośniak, Absence of Normal Heat Conduction in Strongly Disordered Interacting Quantum Chains 2408.04338

"We prove that in strongly disordered, interacting, quantum chains, the conductance of a chain of length L vanishes faster than $1/L$. This means that transport is anomalous in such chains. This phenomenon was first claimed in BAA06, GMP05 and a pioneering treatment appeared in Imbrie16."

But in p.2 "The current paper does not rule out nor confirm the existence of MBL."

so where do we stand ?
 $g = 1/(\text{disorder strength})$



summary: P.Sierant, M. Lewenstein, A. Scardicchio, L. Vidmar, J.Z. , *Many-Body Localization in the Age of Classical Computing*, Rep. Prog. Phys. **88**, 026502 (2025).

Theory: spectral - e.g. Gap ratio

An universal measure: “gap ratio”:

$$r_n = \min\{\delta_{n+1}, \delta_n\} / \max\{\delta_{n+1}, \delta_n\},$$

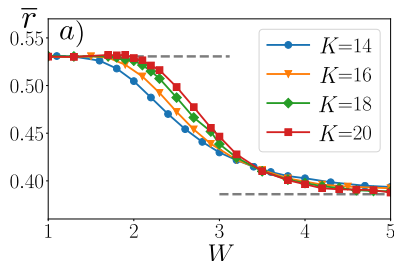
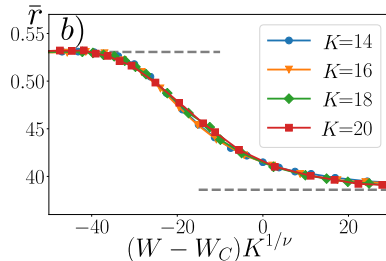
where $\delta_n = E_n - E_{n-1}$

Dimensionless!

Standard model of MBL – disordered
XXX spin chain

$$H = J \sum_{i=1}^K \vec{S}_i \cdot \vec{S}_{i+1} + \sum_{i=1}^K h_i S_i^z,$$

random $h_i \in [-W, W]$



GOE like $\rightarrow$ Poisson like crossover: $W_c \approx 3.7$, $\nu \approx 0.9$ (breaking Harris bound)

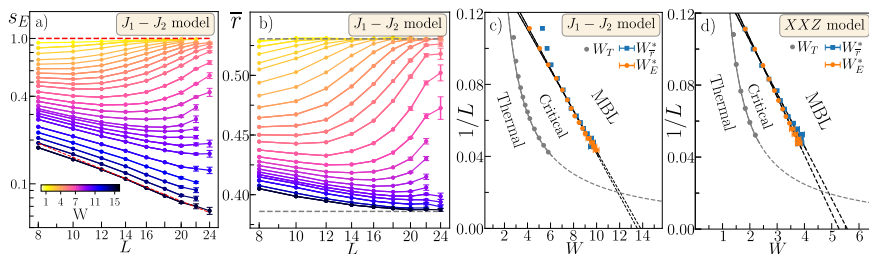
Typical BS result

Oganesyan and Huse 2007; D.J. Luitz, et al. 2015, J. Piotr Sierant, JZ 2018-20



Quantum chaos challenges MBL- drifts

Typical drifts: $W^* \propto a/L + W_c$ $W_T \propto cL$



Extrapolation to $L \rightarrow \infty$ based on fits - no free parameter. $L = 50$ needed

Piotr Sierant, Maciej Lewenstein, J.Z. Phys. Rev. Lett. **125** 156601 (2020)



Physical picture: Avalanche scenario

How a chaotic grain affects a localized system due to interactions? Theory

$$\hat{\mathcal{H}}_{\ell\text{-bit}} = \sum_i h_i \hat{\tau}_i^z + \sum_{i < j} J_{ij} \hat{\tau}_i^z \hat{\tau}_j^z + \sum_{i < j < k} J_{ijk} \hat{\tau}_i^z \hat{\tau}_j^z \hat{\tau}_k^z + \dots,$$

WOJCIECH DE ROECK AND FRANÇOIS HUENEERS

PHYSICAL REVIEW B **95**, 155129 (2017)

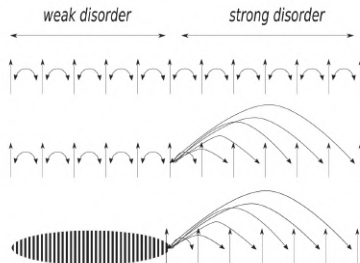


FIG. 2. (Top) A chain of weakly disordered spins—the ergodic grain—is coupled to a chain of strongly disordered spins—the MBL system. (Middle) The system is modelled by weakly disordered spins coupled to LIOMs (no coupling between the LIOMs any more). Note that all LIOMs are coupled only to the rightmost spin. (Bottom) The weakly disordered spins are sometimes modelled by a random matrix.

W. de Roeck and F. Huveneers, Phys. Rev. B **95**, 155129 (2017)

$$H = H_B + \sum_i^L h_i \hat{\tau}_i^z + \sum_i^L J_i \sigma_0^x \otimes \tau_i^x$$

$$h_i \in [1 - W, 1 + W]; J_i = g e^{-|i-1|/\xi}$$



Physical picture: Avalanche scenario 2

How a chaotic grain affects a localized system due to interactions? Model study



FIG. 1. LIOMs (blue) coupled to an ergodic grain (red) modeled by a Gaussian Orthogonal Ensemble (GOE) matrix.

R - random 3×3 GOE matrix

$$H = R + \sum_{i=0}^{L_{\text{loc}}-1} \frac{h_i}{2} \sigma_i^z + \sum_{i=0}^{L_{\text{loc}}-1} \frac{g_0 \alpha^i}{4} \sigma_i^x \sigma_{-1}^x.$$

PRL **119**, 150602 (2017)

PHYSICAL REVIEW LETTERS

13 OCTOBER 2017

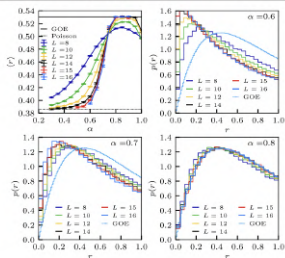


FIG. 2. Probability distributions and average of the adjacent gap ratio r for different coupling strengths and system sizes compared to the distribution obtained for large GOE matrices.

$$\alpha_c = 1/\sqrt{2}$$

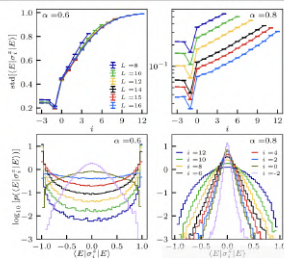


FIG. 3. Top: Standard deviation of the eigenvector expectation of the magnetization at site i ($i < 0$ is in the bath, and $i \geq 0$ are LIOMs) for various total length L . Bottom: Full distribution for $L = 16$ and different positions in the chain.

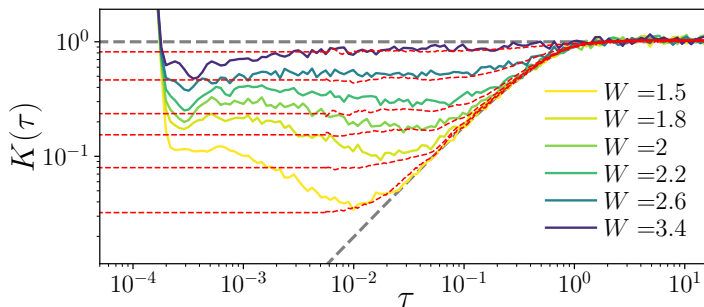
D. Luitz, W. de Roeck, and F. Huveneers, Phys. Rev. Lett. **111**, 160502 (2017)

Digression – Spectral form factor analysis

Different approach...

$$K(\tau) = \frac{1}{Z} \left\langle \left| \sum_j g(\epsilon_j) e^{-iE_j\tau} \right|^2 \right\rangle$$

(Heisenberg chain data)



Thouless time $\ll$ Heisenberg time – ergodic
Thouless time $>$ Heisenberg time – localized

analysis introduced by J. Šuntajs, J. Bonča, T. Prosen, and L. Vidmar Phys. Rev. E **102**, 062144 (2020)



0-dim Quantum Sun model

$$\hat{H} = R \otimes 1 + \sum_{j=1}^L \alpha^{u_j} \hat{S}_{n_j}^x \hat{S}_j^x + \sum_{j=1}^L h_j \hat{S}_j^z$$

R - $N \times N$ GOE matrix, R_c - $N \times N$ random $U(1)$ matrix
random $u_j \in [j - 0.2, j + 0.2]$, $h_j \in [0.5, 1.5]$

PHYSICAL REVIEW LETTERS **129**, 060602 (2022)

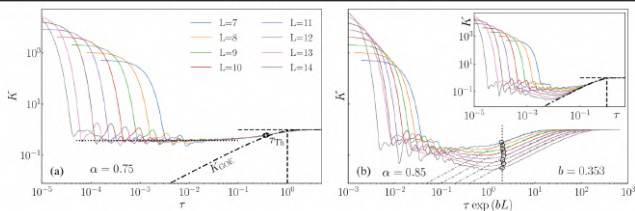


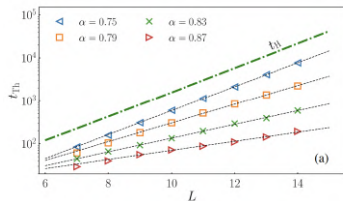
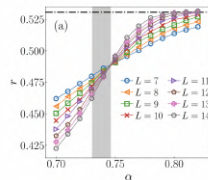
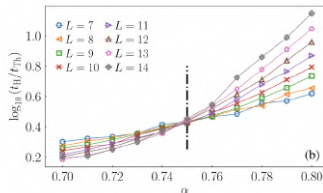
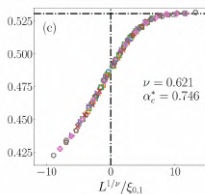
FIG. 2. Spectral form factor $K(\tau)$ at different system sizes L . Dashed-dotted lines denote the GOE results $K_{\text{GOE}}(\tau) = 2\tau - \tau \ln(1 + 2\tau)$, open circles denote the extracted values of τ_{Th} , and the vertical dashed line is the scaled Heisenberg time $\tau_H = 1$. (a) $K(\tau)$ at $\alpha = \alpha_c = 0.75$. The horizontal dotted line is the fit $K(\tau) \approx 0.38$. (b) $K(\tau)$ at $\alpha = 0.85$. The inset shows the raw data while in the main panel we rescale $\tau \rightarrow \tau \exp(bL)$, where $b = 0.353$, such that the rescaled values of τ_{Th} coincide. See [57] for a detailed description of the numerical extraction of τ_{Th} .

J. Suntajs, L. Vidmar, PRL **129**, 060602 (2022)



0-dim Quantum Sun model

$$\hat{H} = R \otimes 1 + \sum_{j=1}^L \alpha^{u_j} \hat{S}_{n_j}^x \hat{S}_j^x + \sum_{j=1}^L h_j \hat{S}_j^z$$



J. Suntajs, L. Vidmar, PRL **129**, 060602 (2022)



0-dim U(1) Quantum sun model

$$\hat{H}_c = R_c \otimes 1 + \sum_{j=1}^L \alpha^{u_j} (\hat{S}_{n_j}^x \hat{S}_j^x + \hat{S}_{n_j}^y \hat{S}_j^y) + \sum_{j=1}^L h_j \hat{S}_j^z$$

R - $N \times N$ GOE matrix, R_c - $N \times N$ random U(1) matrix
random $u_j \in [j - 0.2, j + 0.2]$, $h_j \in [0.5, 1.5]$

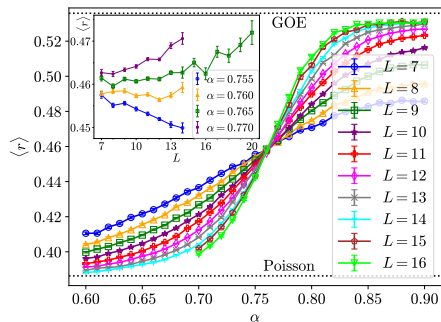
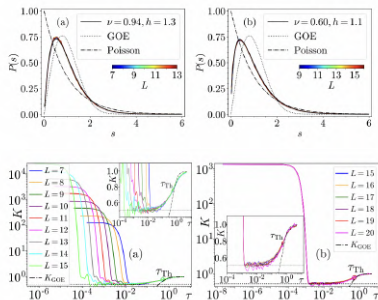


FIG. S.2. SFF in the U(1)-preserving QSM (5) for $\alpha = 0.765$.

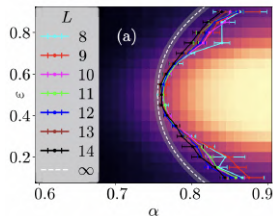
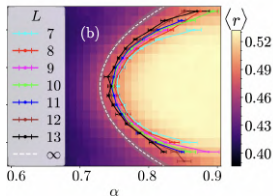
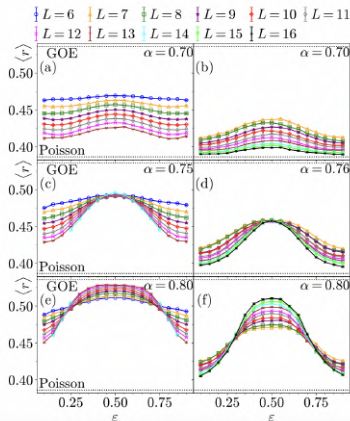
K.Pawlik, P.Sierant, L. Vidmar, J.Z. PRB109, L180201 (2024)

Mobility edge in Quantum sun model

hybridization $\rightarrow \alpha_c \propto \rho^{1/2L}$; $\rho(E=0) \propto 2^L$; $\alpha_c = 1/\sqrt{2}$

Typically $\rho(E) = \frac{2^L}{\sqrt{2\pi\sigma^2}} \exp(-E^2/2\sigma^2)$ with $\sigma = b\sqrt{L}$; $E_{\max} - E_{\min} = aL$

$$\alpha_c(\epsilon) = 1/\sqrt{2} \exp(a^2(\epsilon - 1/2)^2/4b^2)$$



K.Pawlik, P.Sierant, L. Vidmar, J.Z. PRB109, L180201 (2024)

Anderson Quantum Sun

Motivation: Introduce intra-chain hopping J into the localized chain to study how an ergodic bath destabilizes a **genuine 1D Anderson insulator**.

$$\mathcal{H}_{\text{AQS}} = \hat{H} + J \sum_{j=1}^{L-1} (\hat{S}_j^x \hat{S}_{j+1}^x + \hat{S}_j^y \hat{S}_{j+1}^y)$$

Two Competing Length Scales:

1 Anderson Localization Length:

$$\xi_{\text{AL}} = \frac{1}{\ln[1 + (W/W_0 J)^2]}$$

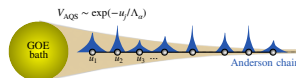
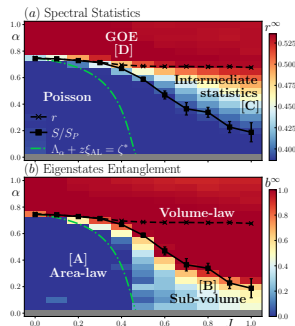
2 Sun Coupling Length: $\Lambda_\alpha = \frac{1}{|\ln \alpha|}$.

Generalized Avalanche Threshold

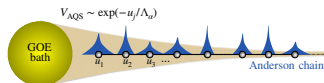
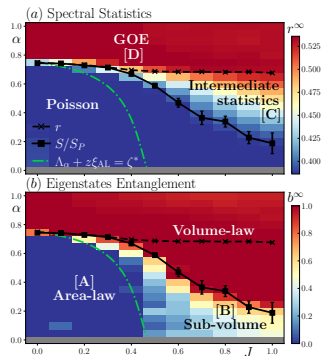
The crossover from [A] to [B] obeys:

$$\Lambda_\alpha + z \xi_{\text{AL}} = \zeta^*, \quad z \approx 2, \quad \zeta^* \approx 3.7$$

K.Pawlik, N. Laflorencie, J.Z. PRB113, L180203 (2026)



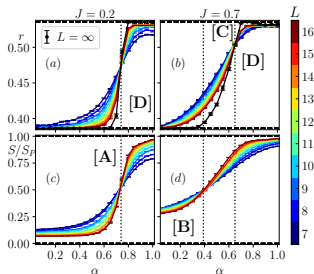
Anderson Quantum Sun 2



- **[A] Area-Law Regime:** Poisson statistics ($\langle r \rangle \approx 0.39$), area-law entanglement ($S \propto L^0$), short-range correlations.
- **[B] Sub-Volume Regime:** Poisson statistics coexists with **sub-volume entanglement** ($S \propto L^b$, $0 < b < 1$) and rare-event-dominated correlations.
- **[C] Intermediate Regime:** **Volume-law entanglement** ($b = 1$) coexists with **intermediate spectral statistics** ($0.39 < r < 0.53$).
- **[D] Fully Ergodic Phase:** GOE level repulsion ($\langle r \rangle \approx 0.53$), volume-law entanglement ($b = 1$), full ETH.



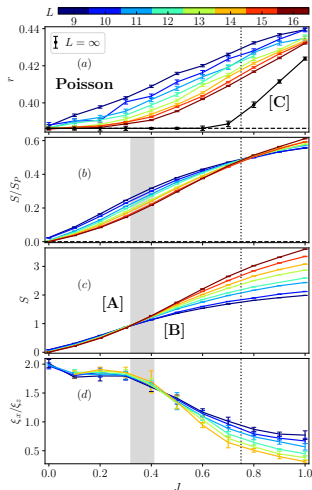
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Connected Correlation Functions:

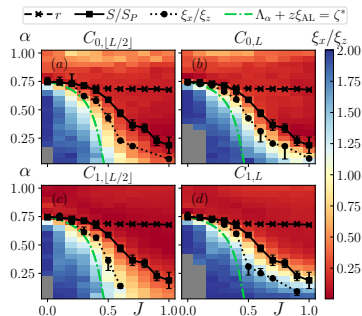
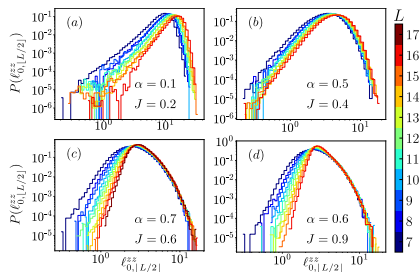
$$C_{ij}^{\gamma\gamma} = |\langle S_i^\gamma S_j^\gamma \rangle - \langle S_i^\gamma \rangle \langle S_j^\gamma \rangle|, \quad \gamma \in \{x, z\}$$

- In localized phase: $\xi_x/\xi_z \approx 2$ (JW).
- In Regime [B]: ξ_x/ξ_z drops rapidly below 2 with growing L , signaling ergodic instabilities.



K.Pawlik, N. Laflorencie, J.Z. PRB113, L180203 (2026)

$$C_{ij}^{\gamma\gamma} = |\langle S_i^\gamma S_j^\gamma \rangle - \langle S_i^\gamma \rangle \langle S_j^\gamma \rangle|, \quad \gamma \in \{x, z\}$$



$$\ell_{ij}^{zz} = -\ln |4C_{ij}^{zz}|$$



K.Pawlik, N. Laflorencie, J.Z. PRB113, L180203 (2016)

Summary of Progress

- ➊ **De Roeck & Huveneers (2017) / Luitz et al. (2017):** Formulated the quantum avalanche theory, showing that even tiny baths trigger total delocalization if $\alpha > 1/\sqrt{2}$.
- ➋ **Šuntajs & Vidmar (2022):** Established the 0D Quantum Sun toy model, revealing scale-invariant SFF and exponential Thouless time growth..
- ➌ **Pawlik, Sierant, Vidmar & J.Z. (2024):** Proved that Gaussian DoS enforces a robust, many-body mobility edge in both unconstrained and $U(1)$.
- ➍ **Pawlik, Laflorencie & JZ. (2026):** Introduced the Anderson Quantum Sun (AQS), uncovering:
 - Intermediate phase [C] with volume-law entanglement alongside non-GOE spectral statistics.
 - Subvolume phase [B] marked by Poisson statistics and rare-event systemwide resonances.
 - A generalized multi-scale avalanche threshold $\Lambda_\alpha + z\xi_{\text{AL}} = \zeta^*$.



Conclusions

- Thermodynamic limit of MBL remains an open question (in XXZ)
- Quasi-periodic seems to be easier localizable
- Floquet systems hint towards existence of MBL in the thermodynamic limit.
- Quantum sun artificial yet genuine MBL
- Bond-disordered systems interesting!

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